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Strength Failure Conditions of the Various Structural Materials

- Is there Some Common Basis existing ? -

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Strength Failure Conditions
of the Various Structural Materials
- Is there Some Common Basis existing ? -

Contents of Presentation: (ca. 60 min)

Motivation for the Work

Existing Links in the Mechanical Behaviour show up: ***Different structural materials***

- ***can possess similar material behaviour or***
- ***can belong to the same class of material symmetry***

Consequence:

- ***The same strength failure function F can be used for different materials***
- ***More information is available for the pre-dimensioning and modelling***

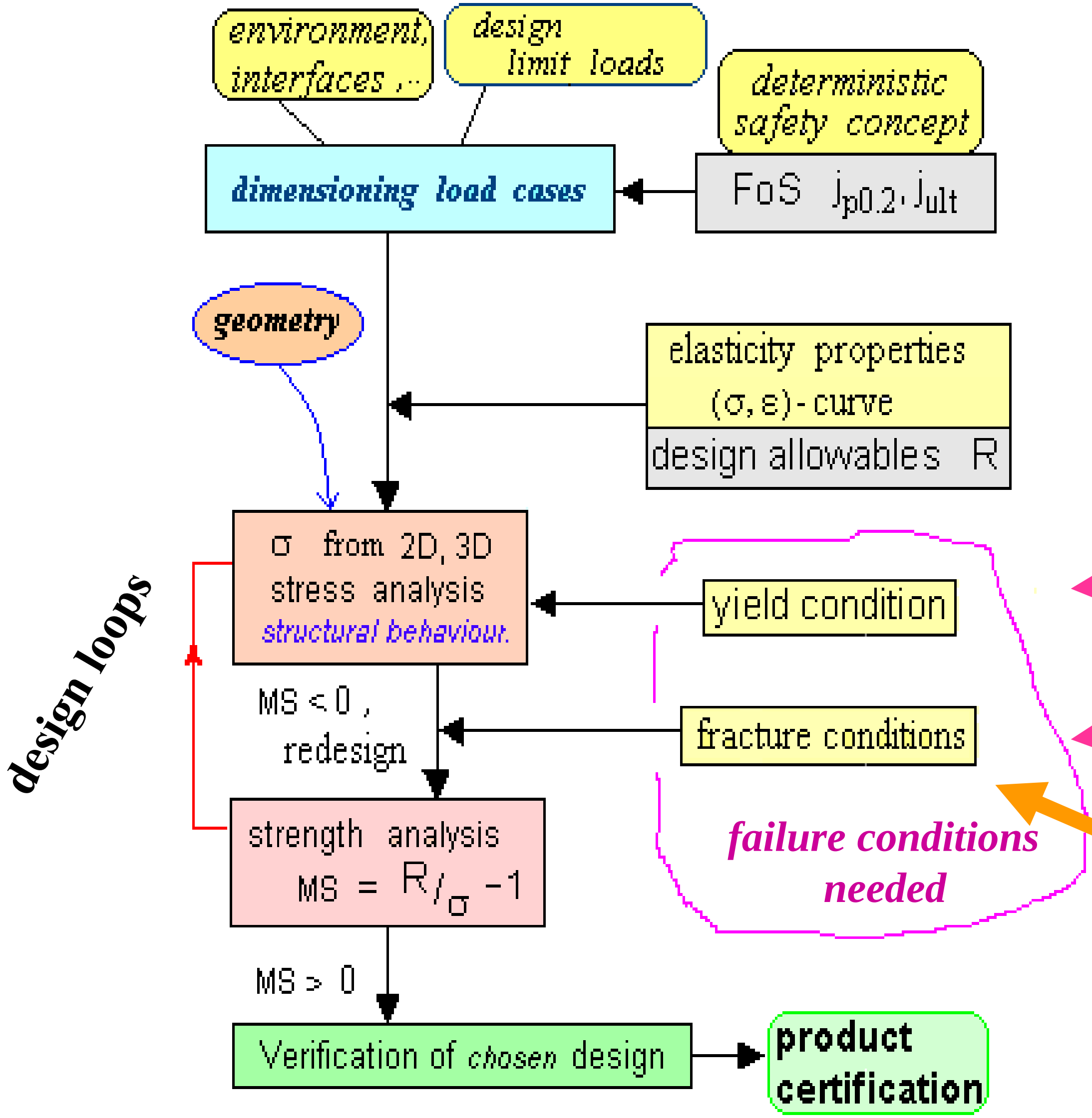
- in case of a newly applied material -

from experimental results of a <u>similarly behaving</u> material

MESSAGE: Let us use these benefits!

1 Introduction to Design Verification

1.1 Static Structural Analysis *Flow Chart* *(isotropic case for simplification)*



FoS := (design) Factor of Safety.

MS := Margin of Safety

in aerospace

Design Verification for:

Design Yield Load (DYL) flight load level

Design Ultimate Load (DUL) ≈ fracture load level

TOPIC here

How can we demonstrate static strength ??

1 Introduction to Design Verification

1.2 Strength Failure Conditions: Prerequisites for their formulation

by the application of strength failure conditions mandatory for the prediction of *Onset of Yielding* + *Onset of Fracture* of non-cracked materials.

Failure Conditions shall

- ***assess multi-axial stress states in the critical material point***

  by **utilizing the uniaxial strength values R** and an **equivalent stress σ_{eq}, representing a distinct actual multi-axial stress state.**

  for *** dense & porous,**
  *** ductile & brittle behaving materials,**

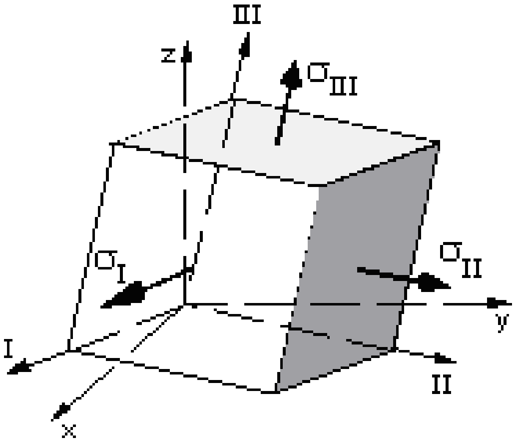
  for *** isotropic material**
  *** transversally-isotropic material (UD := uni-directional material)**
  *** rhombically-anisotropic material (fabrics) + ‘higher‘ textiles etc.**

- ***allow for inserting stresses from the utilized various coordinate systems into stress-formulated failure conditions, -and if possible- invariant-based.***

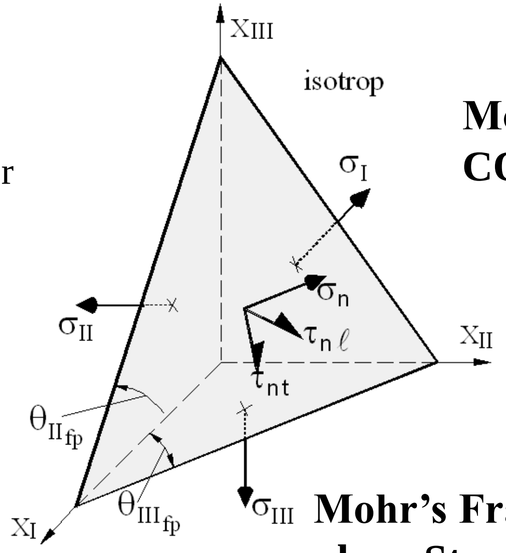
Which kinds of stresses may have to be inserted?

2 Stress States and Invariants

2.1 Isotropic Material (3D stress state), viewing Stresses & Invariants



The stress states in the various COS can be transferred into each other



Mohr's COS

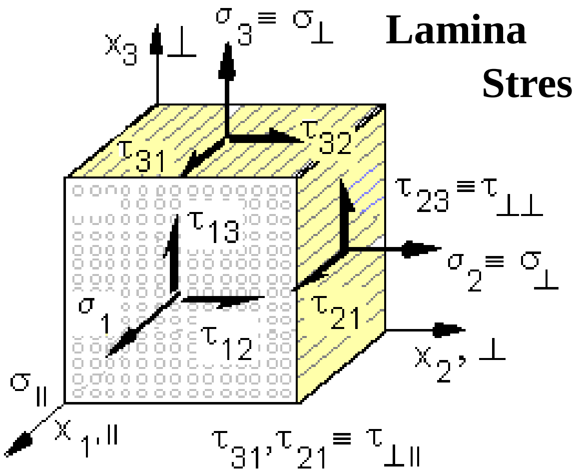
Principal Stresses	**Structural Component Stresses**	**Mohr's Fracture plane Stresses**
$\{\sigma\}_{principal} = (\sigma_I\,,\ \sigma_{II}\,,\ \sigma_{III}\,)^T$	$\{\sigma\}_{comp} = (\sigma_x\,,\ \sigma_y\,,\ \sigma_z\,,\ \tau_{yz}\,,\ \tau_{xz}\,,\ \tau_{xy}\,)^T$	$\{\sigma\}_{Mohr} = (\sigma_\ell\,,\ \sigma_n\,,\ \sigma_t\,,\ \tau_{nt}\,,\ \tau_{t\ell}\,,\ \tau_{\ell n}\,)^T$
$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III}\,)^T = 3\sigma_{oct} \equiv f(\sigma),$	**'isotropic invariants' !** $I_1 = (\sigma_x + \sigma_x + \sigma_z\,)^T$	$I_1 = (\sigma_\ell + \sigma_n + \sigma_t\,)^T$
$6J_2 = (\sigma_I - \sigma_{II}\,)^2 + (\sigma_{II} - \sigma_{III}\,)^2 + (\sigma_{III} - \sigma_I\,)^2$ $= 4(\tau_{III}{}^2 + \tau_{II}{}^2 + \tau_I{}^2\,) = 9\tau_{oct}{}^2 \equiv f(\tau)$	$6J_2 = (\sigma_y - \sigma_z\,)^2 + (\sigma_z - \sigma_x\,)^2 + (\sigma_x - \sigma_y\,)^2$ $+6(\tau_{yz}{}^2 + \tau_{zx}{}^2 + \tau_{xy}{}^2\,)\ \ (Mises, HMH\,)$	$6J_2 = (\sigma_n - \sigma_t\,)^2 + (\sigma_t - \sigma_\ell\,)^2 + (\sigma_\ell - \sigma_n\,)^2$ $+6(\tau_{nt}{}^2 + \tau_{t\ell}{}^2 + \tau_{n\ell}{}^2\,)$

$$27J_3 = \ (2\sigma_I - \sigma_{II} - \sigma_{III}\,)(2\sigma_{II} - \sigma_I - \sigma_{III}\,)(2\sigma_{III} - \sigma_I - \sigma_{II}\,),\quad I_\sigma = 4J_2 - I_1{}^2/3,\quad \sigma_{mean} = I_1/3$$

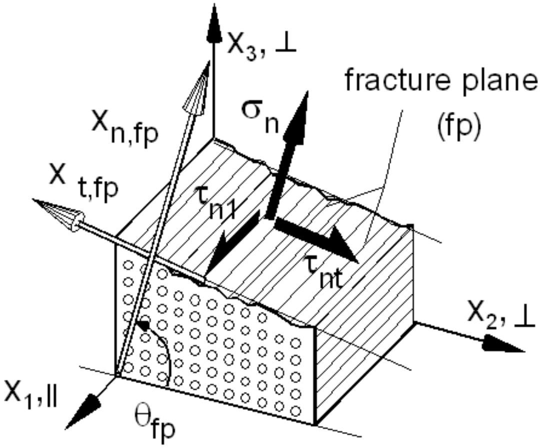
2 Stress States and Invariants

2.2 Transversely-Isotropic Material (◄ Uni-Direct. Fibre-Reinforced Plastics)

Transformation of lamina stresses into the quasi-isotropic plane stresses



Lamina Stresses



Mohr, Puck, Hashin: Fracture is determined by the (Mohr) stresses in the fracture plane

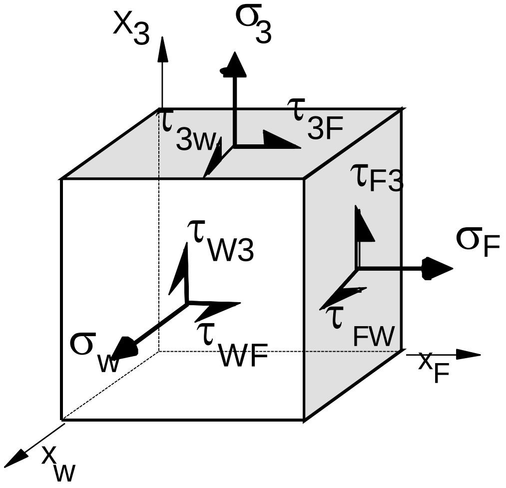
$\{\sigma\}^{quasi-isotropic\ plane}_{principal} =$	$\{\sigma\}_{lamina} =$	$\{\sigma\}_{Mohr} =$
$=(\sigma_1,\sigma_2^p,\sigma_3^p,0,\tau_{31}^p,\tau_{21}^p)^T$	$=(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{31},\tau_{21})^T$	$(\sigma_\ell,\ \sigma_n,\ \sigma_t,\ \tau_{nt},\ \tau_{t\ell},\ \tau_{\ell n})^T$
$I_1 = \sigma_1\ ,\quad I_2 = \sigma_2^p+\sigma_3^p$	$I_1 = \sigma_1,\quad I_2 = \sigma_2+\sigma_3$	$I_1 = \sigma_1,\quad I_2 = \sigma_n+\sigma_t$
$I_3 = \tau_{31}^{p\ 2}+\tau_{21}^{p\ 2}$	$I_3 = \tau_{31}^2+\tau_{21}^2$ **'UD invariants'!** *[Boehler]*	$I_3 = \tau_{t\ell}^2+\tau_{n\ell}^2$
$I_4 = (\sigma_2^p-\sigma_3^p)^2+0$	$I_4 = (\sigma_2-\sigma_3)^2+4\tau_{23}^2$	$I_4 = (\sigma_n-\sigma_t)^2+4\tau_{nt}^2$
$I_5 = (\sigma_2^p-\sigma_3^p)(\tau_{31}^{p\ 2}-\tau_{21}^{p\ 2})+\ 0$	$I_5 = (\sigma_2-\sigma_3)(\tau_{31}^2-\tau_{21}^2)-4\tau_{23}\tau_{31}\tau_{21}$	$I_5 = (\sigma_n-\sigma_t)(\tau_{t\ell}^2-\tau_{n\ell}^2)-4\tau_{nt}\tau_{t\ell}\tau_{n\ell}$

Invariant := Combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system. Good for an optimum formulation of *desired scalar Failure Conditions.*

2 Stress States and Invariants

2.3 Orthotropic Material (rhombically-anisotropic ◄ woven fabric)

Homogenized = smeared
***woven fabrics* material element**



Warp (W), Fill(F).

3D stress state:
Here, just a formulation in fabrics lamina stresses makes sense!

$$\{\sigma\}_{lamina} = (\sigma_W, \sigma_F, \sigma_3, \tau_{3F}, \tau_{3W}, \tau_{FW})^T$$

Fabrics invariants ! *[Boehler]*:

$I_1 = \sigma_W$, $I_2 = \sigma_F$, , $I_3 = \sigma_3$,
$I_4 = \tau_{3F}$, $I_5 = \tau_{3W}$, $I_6 = \tau_{FW}$

more, -however simple- invariants necessary

3 Observed Strength Failure Modes and Strengths

3.1a Isotropic Material *brittle* , *dense*

if brittle: failure = fracture

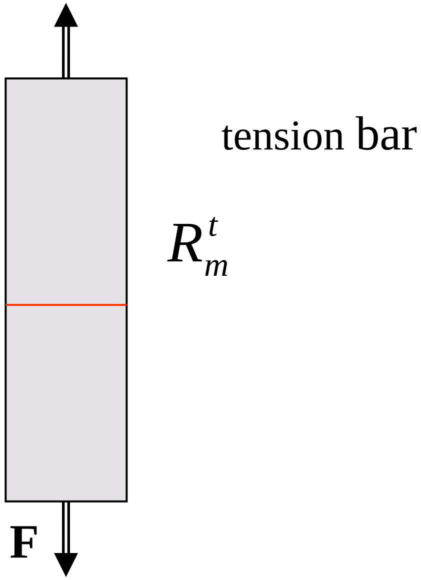
Cleavage fracture (NF) (Spaltbruch, Trennbruch) :

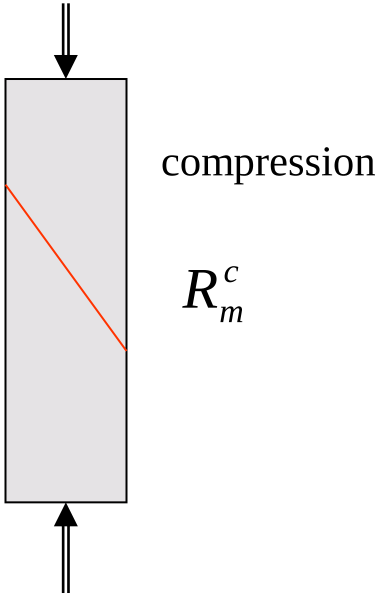
- poor deformation before fracture
- ‘smooth’ fracture surface

Shear fracture (SF) :

- shear deformation before fracture

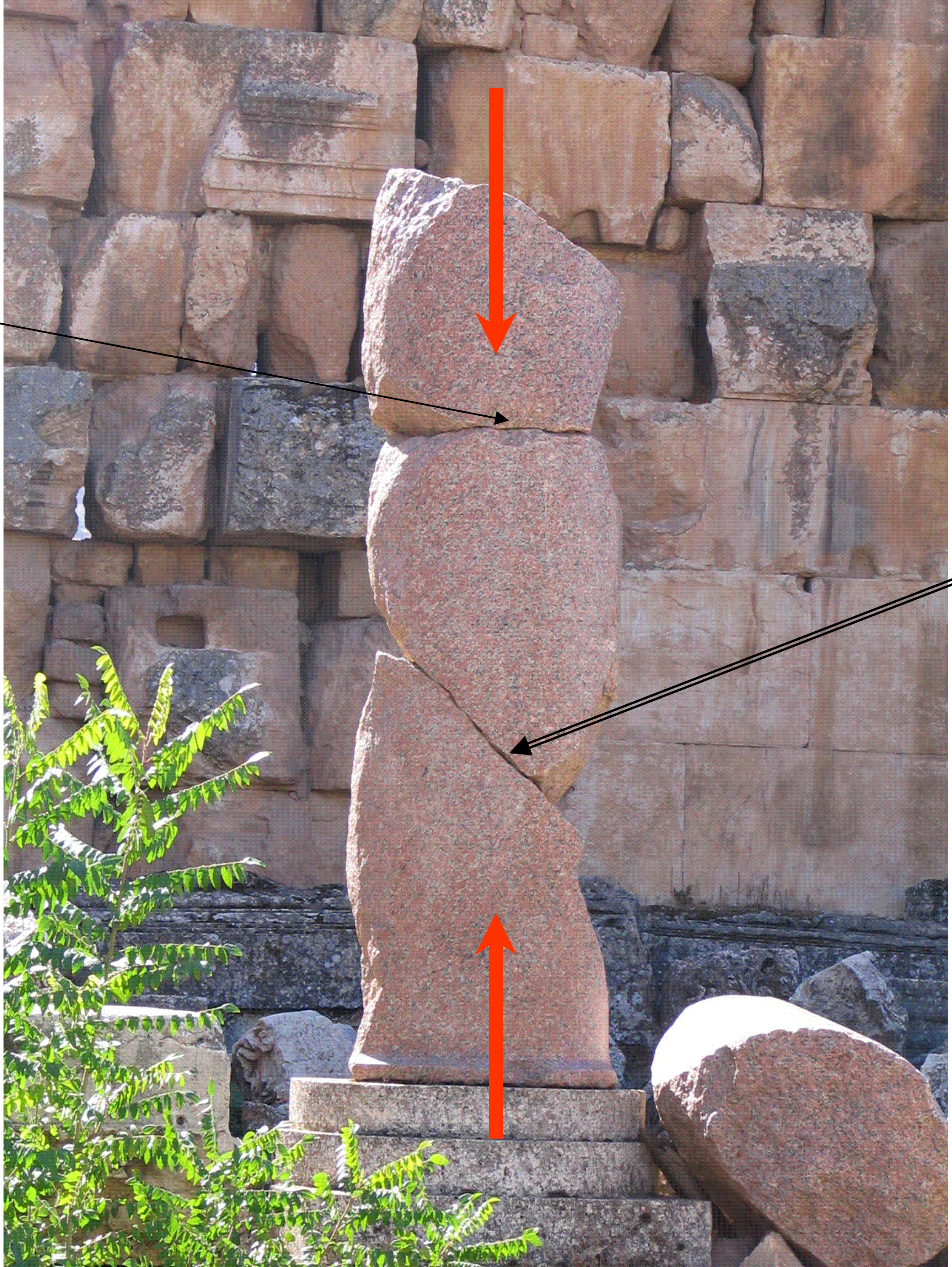
helpful for the later **choice of invariants**





▶ **2 strengths** to be measured

just a joint



Example SF : R_m^c

Shear Fracture plane under compression

(**Mohr-Coulomb,** acting *at* **a** ***rock material*** *column,*

at Baalbek, Libanon)

3 Observed Strength Failure Modes and Strengths

3.1b Isotropic Material *brittle, porous*

if brittle: failure = fracture

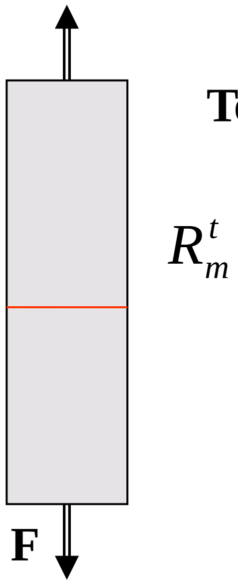
Normal Fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- rough fracture surface

Crushing Fracture (CrF): ⇐ **SF**

- volumetric deformation before fracture

helpful for the later **choice of invariants**

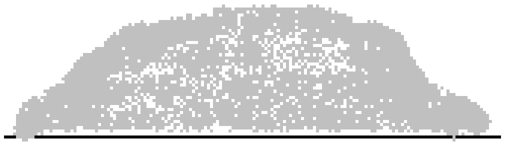


Tension

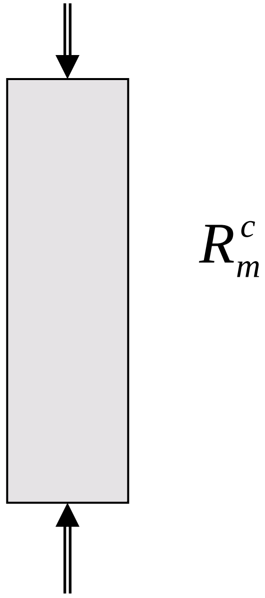
Compression

result of the compression test

= *hill of fragments* (crumbs)



= decomposition of texture



▶ **2 strengths** to be measured

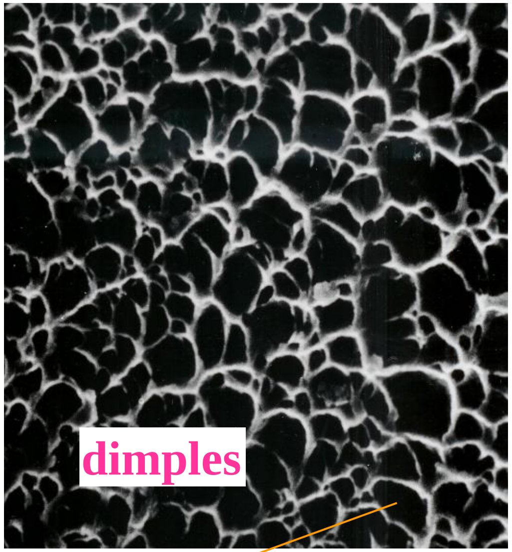
3 Observed Strength Failure Modes and Strengths

audience familiar ??

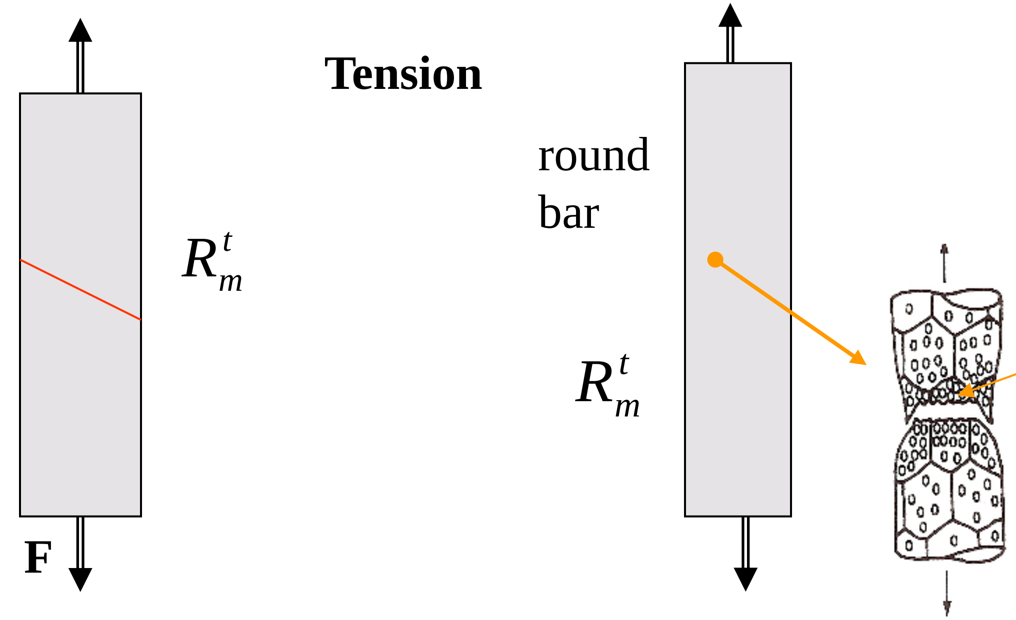
3.1c Isotropic Material *dense, ductile (most of the aerospace nmaterials)*

Shear fracture (SF) :

- *shear deformation* before fracture (maximum load)
- later in addition, *volume change* before rupture (‘Gurson domain’)
- dimples under tension.



dimples



first a **diffuse** and later **local** ***necking*** + **void growth**

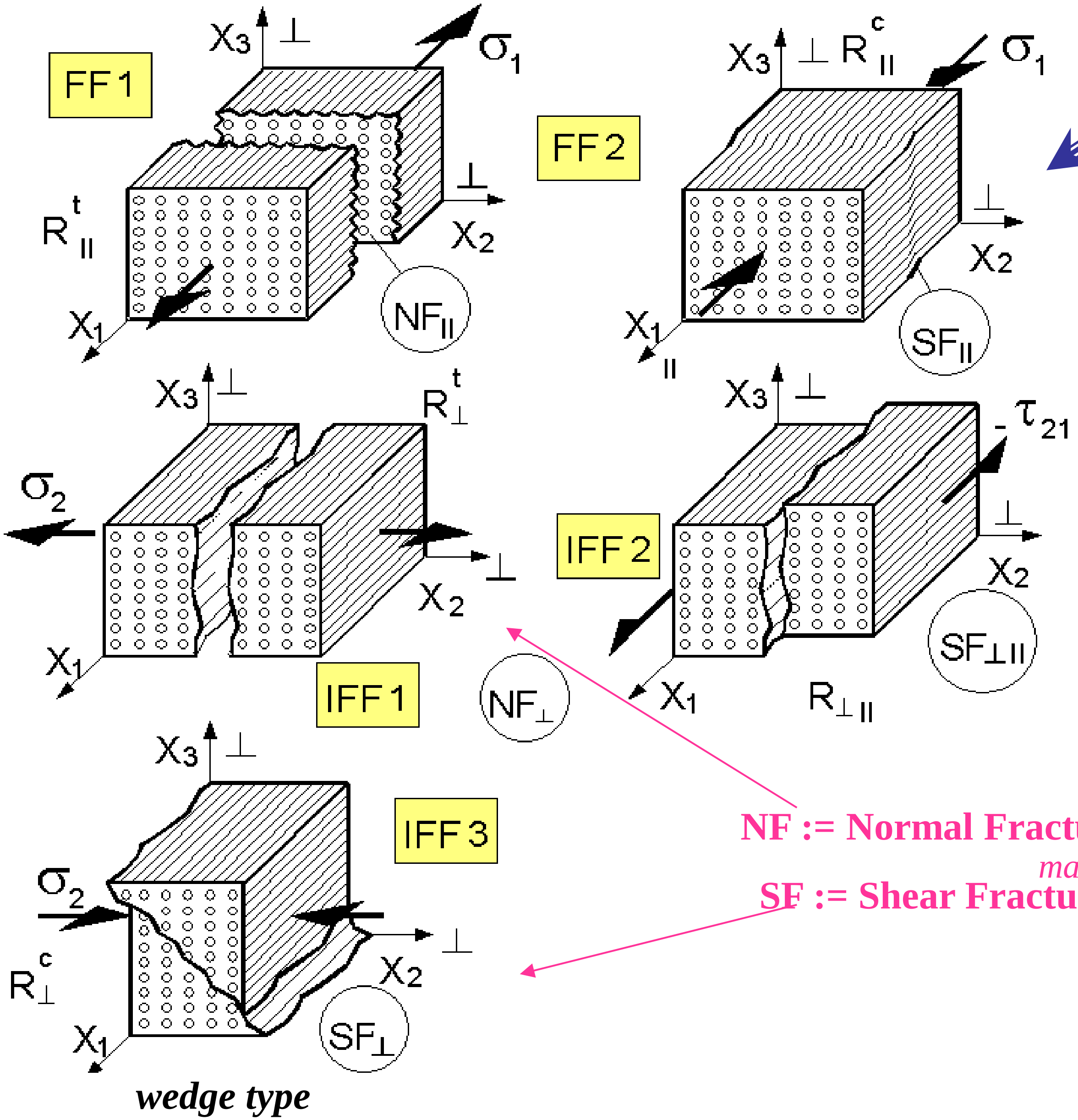
►

- **1 strength, R_m^t** to be measured **(= *load-controlled* value),**
- **R_m^c is neither existing nor necessary for design ,**

$R_{c0.2}$ is the design driving strength.

3 Observed Strength Failure Modes and Strengths

3.2a Transversely-Isotropic Material (UD) *brittle. Scheme*



Fractography of test specimens reveals:

- ▶ 5 Fracture modes exist

in a UD Laminae.

= 2 FF (Fibre Failure)

\+ 3 IFF (Inter Fibre Failure)

▶ 5 strengths

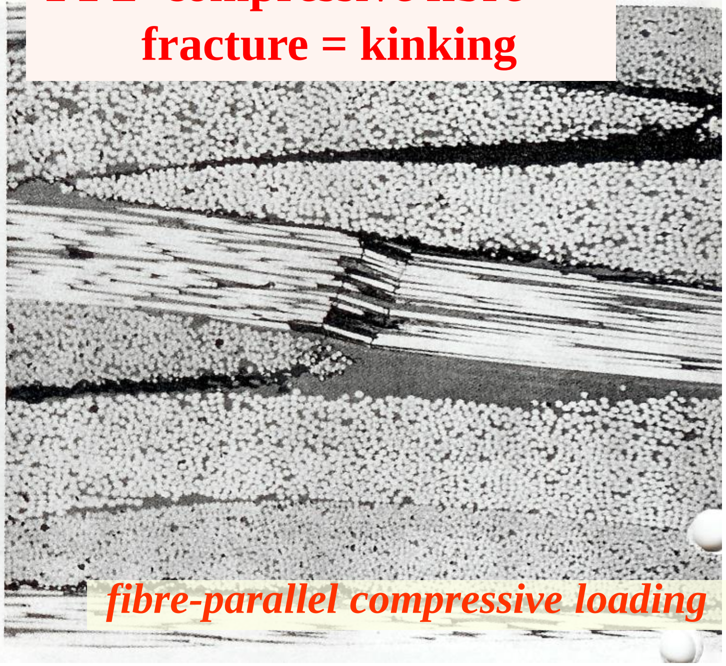
to be measured

NF := Normal Fracture

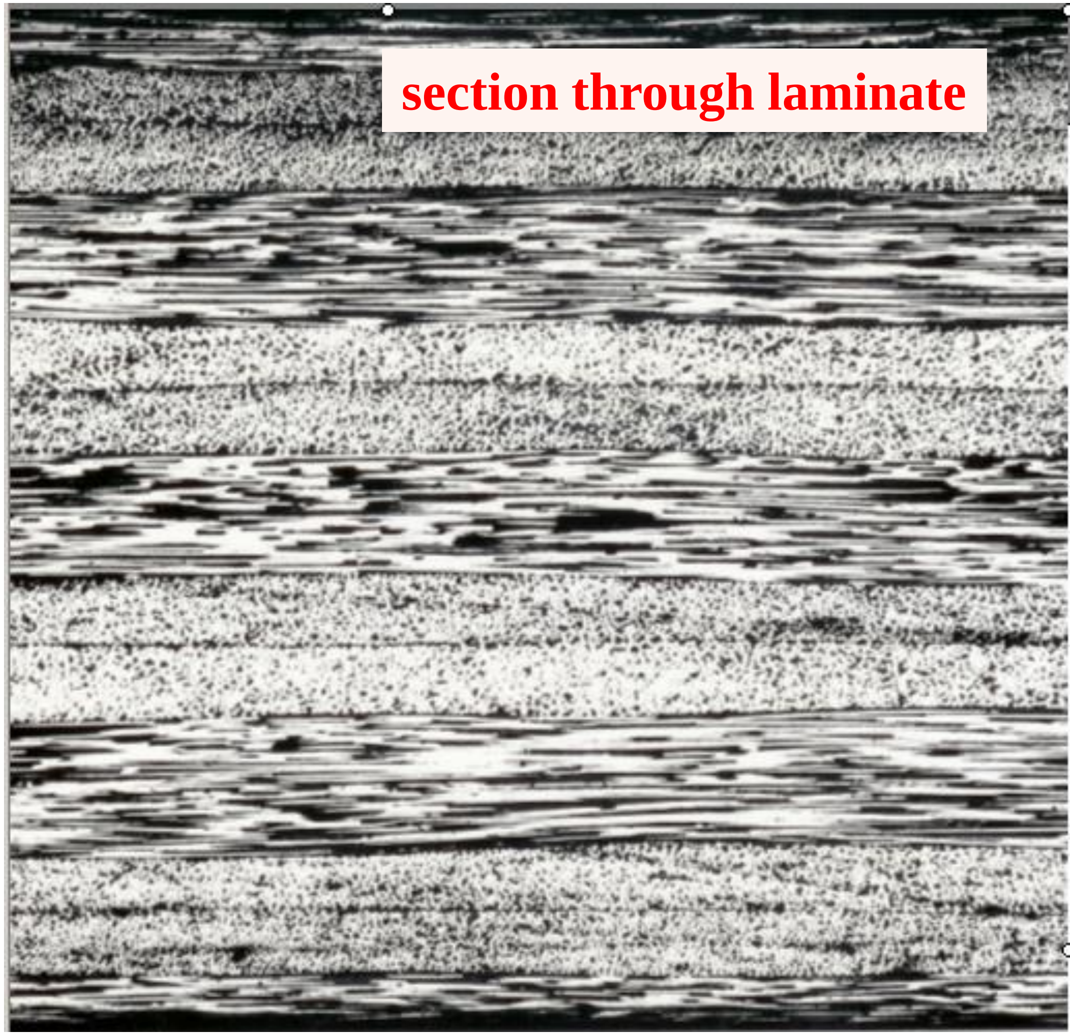
macroscopically:

SF := Shear Fracture

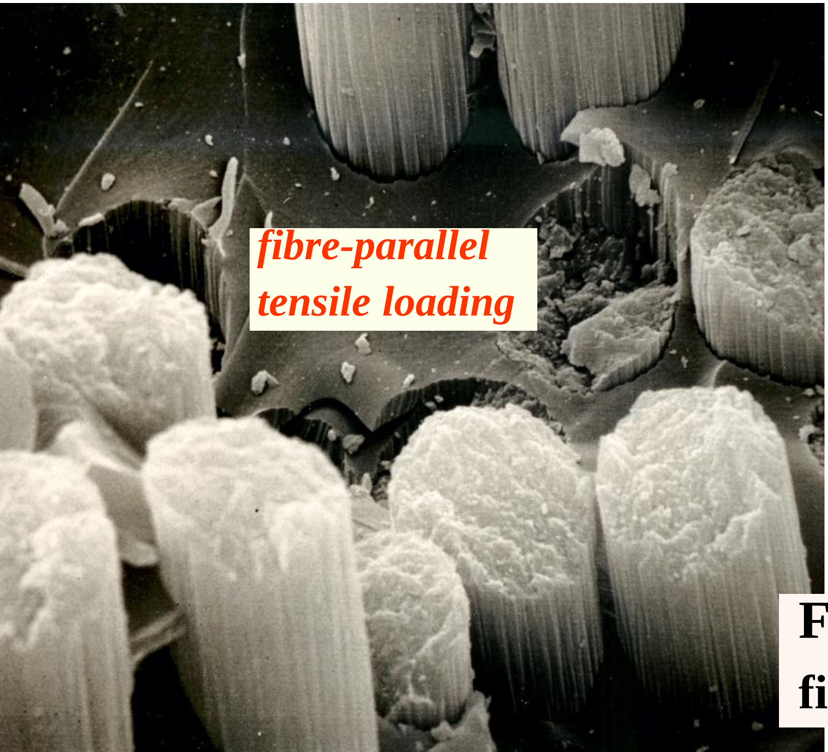
FF2 compressive fibre fracture = kinking



fibre-parallel compressive loading



section through laminate



fibre-parallel tensile loading

FF1 tensile fibre fracture

3 Observed Strength Failure Modes and Strengths

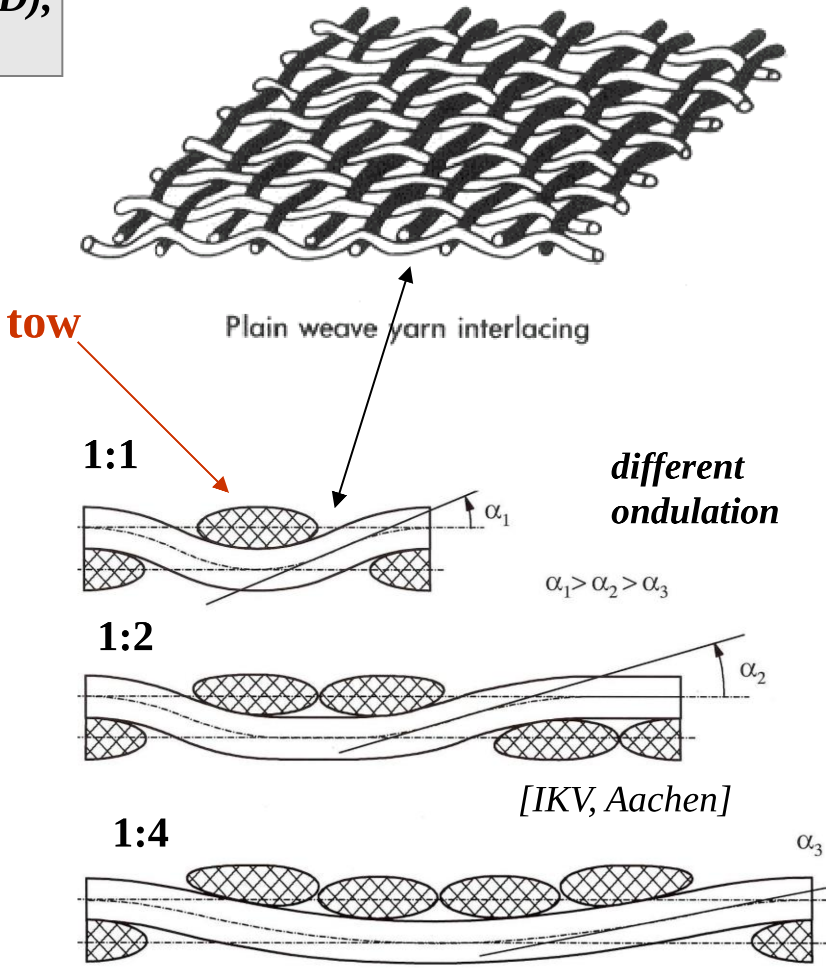
3.3 Woven fabrics

next level above UD

Fibre preforms : from ***roving, tape, weave, braid (2D, 3D), knit, stitch,*** or mixed as in a ***pre-form hybrid***

Fractography exhibits no clear failure modes.

In this material case always multiple cracking is caused under tension, compression, bending, shear !



different ondulation

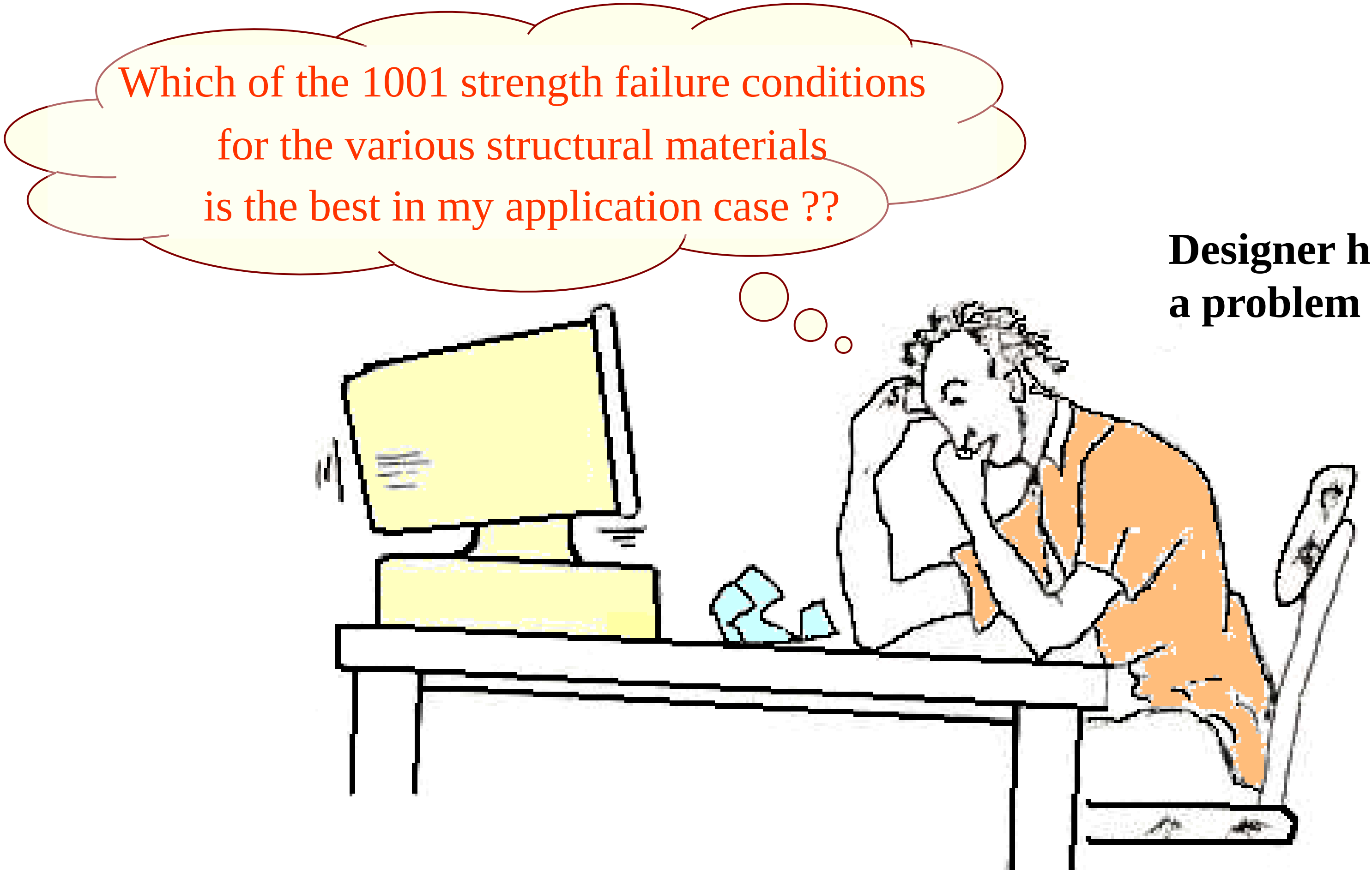
[IKV, Aachen]

approximately UD-describable

Lessons learned:

- Strengths have to be <u>defined</u> according to material symmetry
- Modelling depends on fabrics type !

▶ **9 (6** *if F=W* **) strengths** to be measured



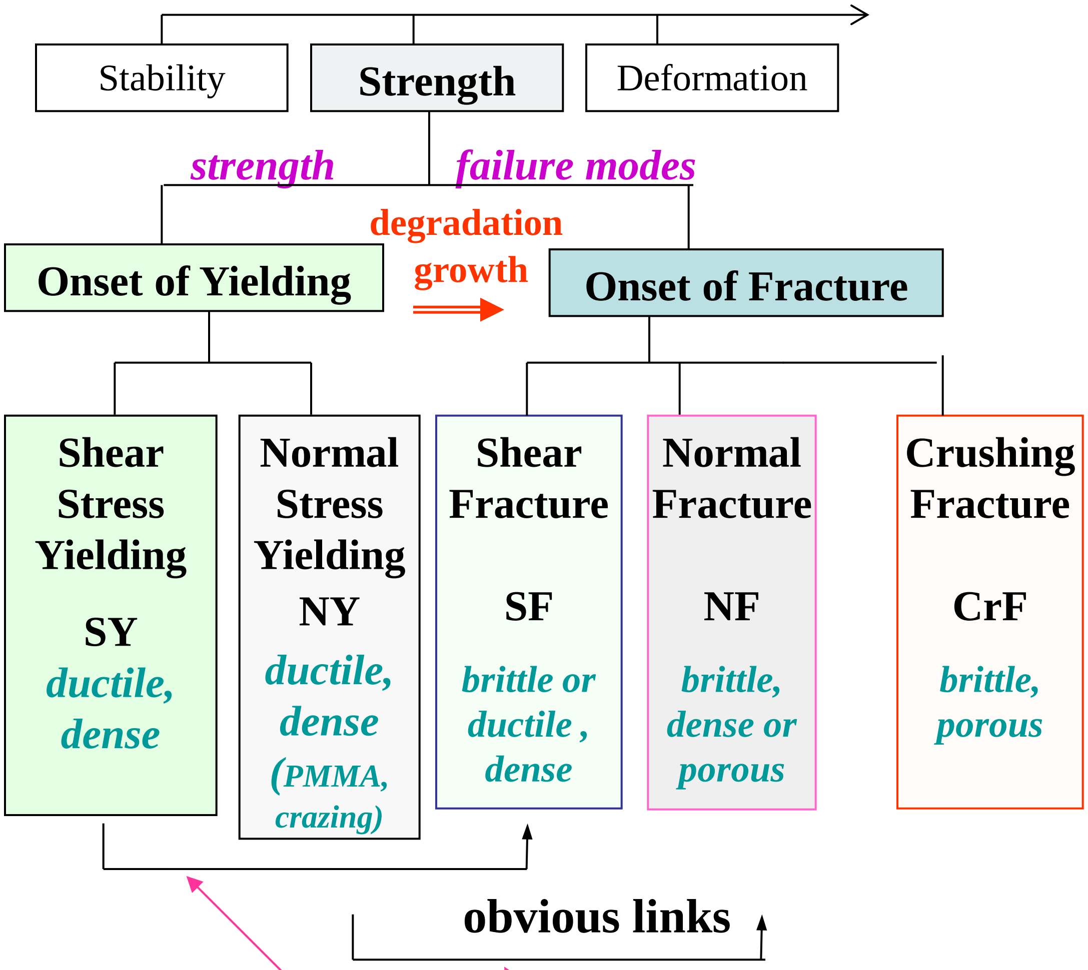
Designer has a problem !

***** *Is there a possibility to find a procedure to figure out failure conditions which are simple, however, describe physics of each failure mechanism sufficiently well ?*

***** *Can one help him by thinking about a systematization ?*

4 Attempt for a Systematization

4.1a Scheme of Strength Failures for *isotropic materials*



Stability

Strength

Deformation

strength *failure modes*

Onset of Yielding

degradation growth

Onset of Fracture

Shear Stress Yielding

SY

ductile, dense

Normal Stress Yielding

NY

ductile, dense (PMMA, crazing)

Shear Fracture

SF

brittle or ductile , dense

Normal Fracture

NF

brittle, dense or porous

Crushing Fracture

CrF

brittle, porous

obvious links

The growing yield body (SY or NY)

is confined by the fracture surface (SF or NF)**!**

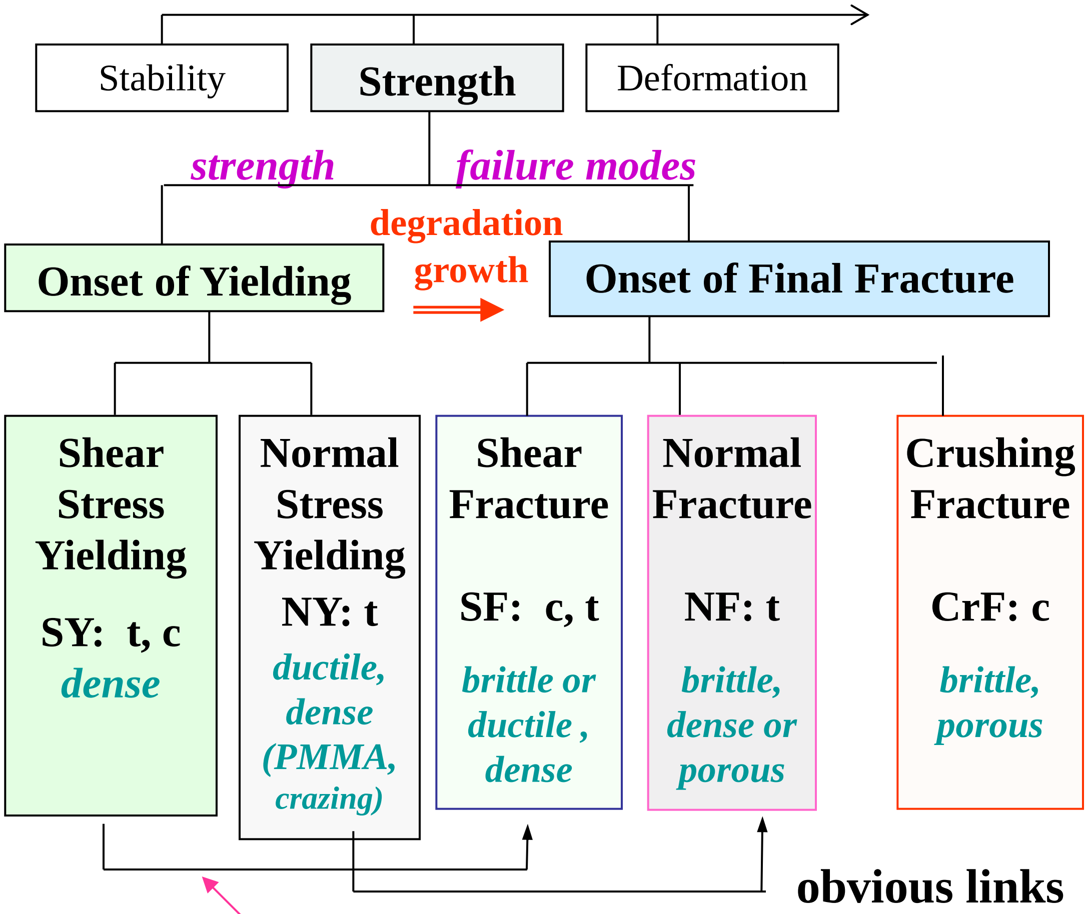
◄ = ***kinds of fracture***

Lesson learned from Mapping Test Data:

The same mathematical form of a failure condition holds - from onset of yielding to onset of fracture - if the physical mechanism remains !

4 Attempt for a Systematization

4.1a Scheme of Strength Failures for *isotropic materials*



Stability

Strength

Deformation

strength *failure modes*

degradation growth

Onset of Yielding

Onset of Final Fracture

Shear Stress Yielding

SY: t, c

dense

Normal Stress Yielding

NY: t

ductile, dense (PMMA, crazing)

Shear Fracture

SF: c, t

brittle or ductile , dense

Normal Fracture

NF: t

brittle, dense or porous

Crushing Fracture

CrF: c

brittle, porous

obvious links

The growing yield body (SY or NY)

is confined by the fracture surface (SF or NF) **!**

◄ = ***kinds of fracture***

CrF replaces SF

t:= tensile;

c:= compressive

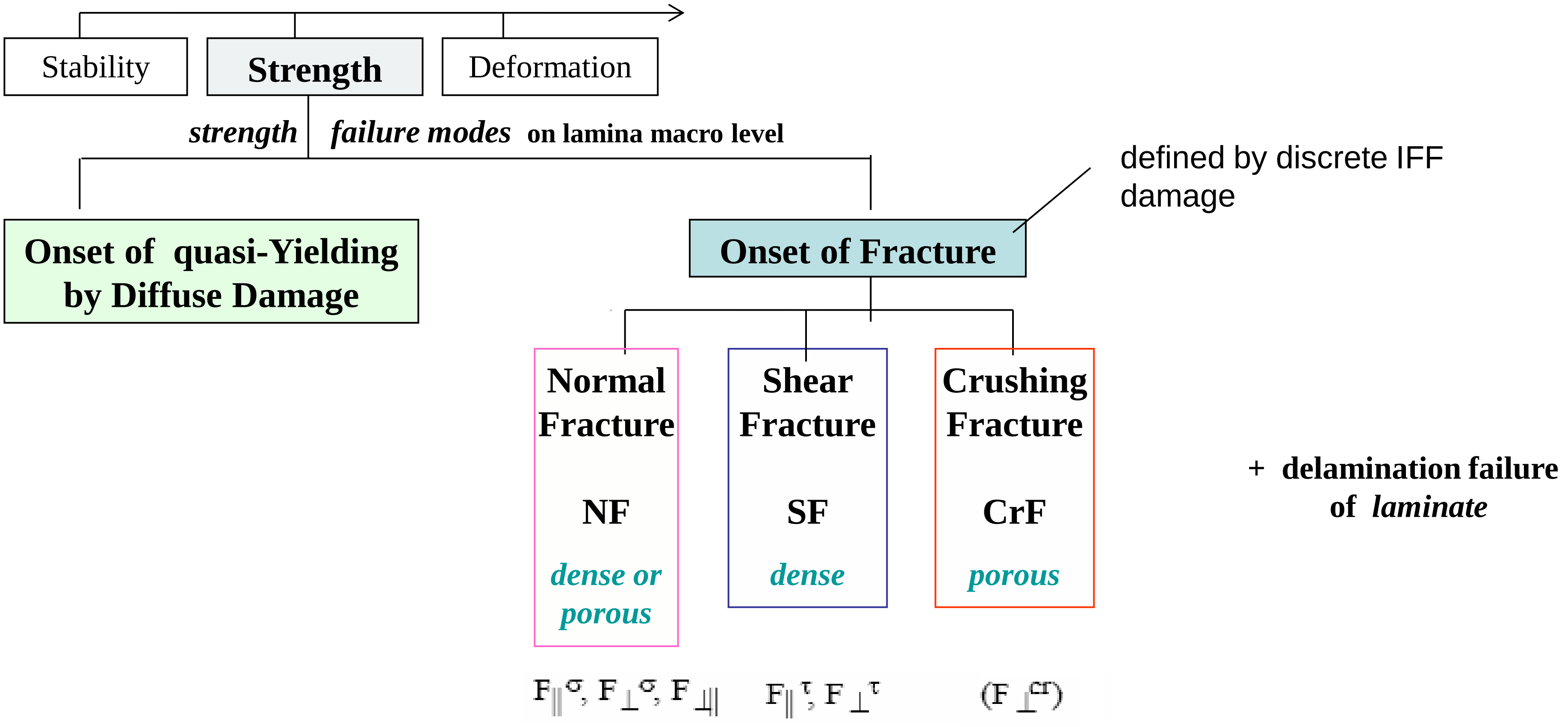
Lesson learned from Mapping Test Data:

Same mathematical form of a failure condition holds

- *from onset of yielding to onset of fracture - if the physical mechanism remains*
- *for a ductile steel in gigh tensile domain (pores initiated) and porous concrete in compression*

4 Attempt for a Systematization

4.1b Scheme of Strength Failures for *the brittle UD lamina (ply) material*



Stability

Strength

Deformation

strength ***failure modes*** **on lamina macro level**

Onset of quasi-Yielding by Diffuse Damage

Onset of Fracture

defined by discrete IFF damage

Normal Fracture

NF

dense or porous

Shear Fracture

SF

dense

Crushing Fracture

CrF

porous

+ **delamination failure of *laminate***

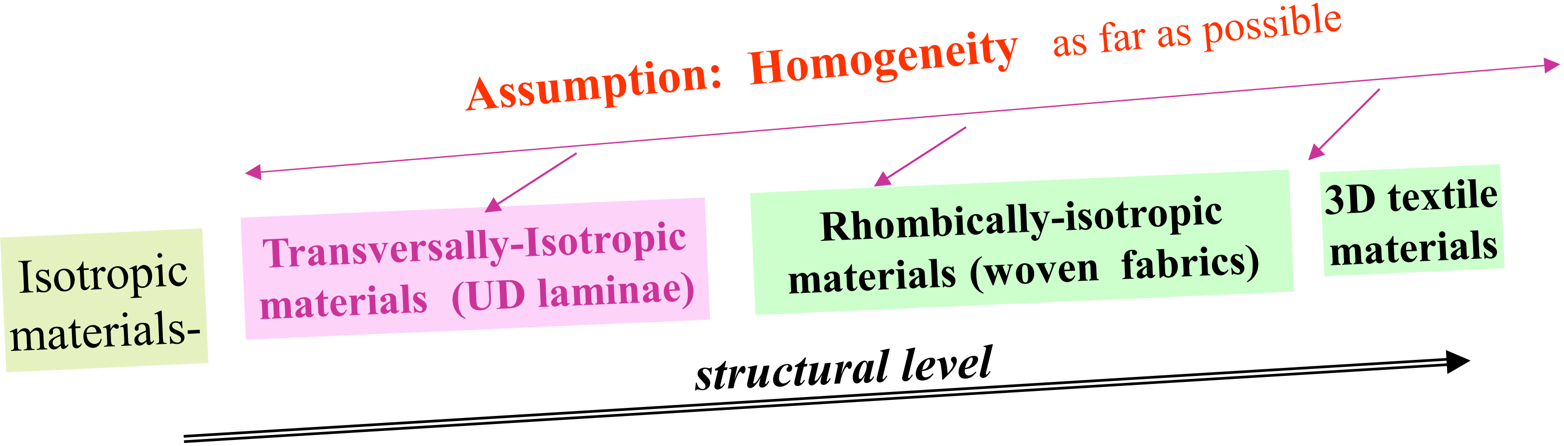
$F_{\parallel}^{\sigma}, F_{\perp}^{\sigma}, F_{\perp\parallel}$ $F_{\parallel}^{\tau}, F_{\perp}^{\tau}$ $(F_{\perp}^{cr})$

Lessons learned from inspection:

- *There are coincidences between brittle UD <u>laminae</u> and brittle isotropic materials*
- *Degradation begins with onset of diffuse damage (hardening) until IFF1, IFF3*
- *Fracture failure occurs with FF1. FF2, and IFF2*
- *Increased diffuse damaging occurs in the* laminate *beyond onset of the first IFF*

4 Attempt for a Systematization

4.2 Material Homogenizing (smearing) + Modelling, Material Symmetry



Assumption: Homogeneity as far as possible

Isotropic materials-

Transversally-Isotropic materials (UD laminae)

Rhombically-isotropic materials (woven fabrics)

3D textile materials

structural level

Material symmetry shows:

Number of strengths ≡ number of elasticity properties !

Application of material symmetry

- ***Requires that homogeneity is a valid assessment for the <u>task-determined</u> model ,*** but,
- ***Just the minimum number of properties has to be measured* (proposes benefits) !**

It' worthwhile to structure the establishment of strength failure conditions

4 Attempt for a Systematization

4.3 Proposed Classification of Homogenized (assumption) Materials

A Classification helps to structure the Modelling Procedure:

Failure Type *Consistency*	**brittle, semi-brittle** Design Ultimate Load	**(quasi-) ductile** Design Yield Load
dense	fibre re-inforced plastics , mat, woven fabrics, grey cast iron, matrix material, amorphous glass C90-1,.	Glare, ARALL, metal alloys braided textiles
porous	foam, fibre re-inforced ceramics	sponge

design driving

failure: fracture functional or usability limit

Conclusion:

Modelling, Struct. Analysis and Design Verification strongly depend on material behaviour + consistency

5 Short Derivation of the Failure Mode Concept (FMC)

5.1 General on Global Formulation & Mode-wise Formulation

- **A failure condition is the mathematical formulation , F = 1, of the failure surface:.**

1 global failure condition : $F\,(\{\sigma\}, \{R\}) = 1$ **(usual formulation) ;**

= 'fully interactive conditions'
which include several modes

***Several mode* failure conditions** : $F\,(\{\sigma\}, R^{mode}) = 1$ **(used in Cuntze's FMC).**

mode-associated strength

Lesson learned from application of global failure conditions:

A change, necessary in one failure mode domain, has an impact on other physically not related failure mode domains , however in general, not on the safe side.

F >=< 1 failure criterion.

5 Short Derivation of the Failure Mode Concept (FMC)

5.2 Fundamentals of the FMC (example: UD material)

Remember:

- **Each of these fracture failure modes was linked to one strength**
- **Symmetry of a material showed :** ***Number of strengths =*** $R_{||}^{t}, R_{||}^{c}, R_{\perp ||}, R_{\perp}^{t}, R_{\perp}^{c}$

number of elasticity properties ! $E_{\|}, E_{\perp}, G_{\|\perp}, \nu_{\perp\|}, \nu_{\perp\perp}$

example UD:

Due to the facts above the

FMC postulates in its ‘*Phenomenological Engineering Approach*’ :

► Number of failure modes = number of strengths, too !

e.g.: isotropic = 2 or transversely-isotropic (UD) = 5

5. Short Derivation of the *Failure Mode Concept (FMC)*

5.3 Driving idea behind the FMC

A possibility exists to *more generally* formulate failure conditions

- **failure mode-wise** *(shear yielding etc.)*
- **stress invariant-based** *(J_2 etc.)*

Mises, Hashin, Puck etc.

Mises, Tsai, Hashin, Christensen, etc.

5. Short Derivation of the *Failure Mode Concept (FMC)*

5.4 Detail Aspects

- **1 failure *condition* represents 1 Failure Mode** *(interaction of acting stresses)*.
- **Interaction of adjacent Failure Modes by a *series failure system* model**

$$(Eff)^m = (Eff^{mode1})^m + (Eff^{mode2})^m + \ldots + \ldots\ldots = 1 .$$

with Stress Effort *Eff* := portion of load-carrying capacity of the material $\equiv \sigma_{eq}^{mode} / R^{mode}$

and Interaction coefficient m **.**

5. Short Derivation of the *Failure Mode Concept (FMC)*

5.5 Interaction of the Strength Failure Modes (example: UD, the 3 IFF)

Stress efforts of the 3 pure IFF modes
= 3 straight lines :

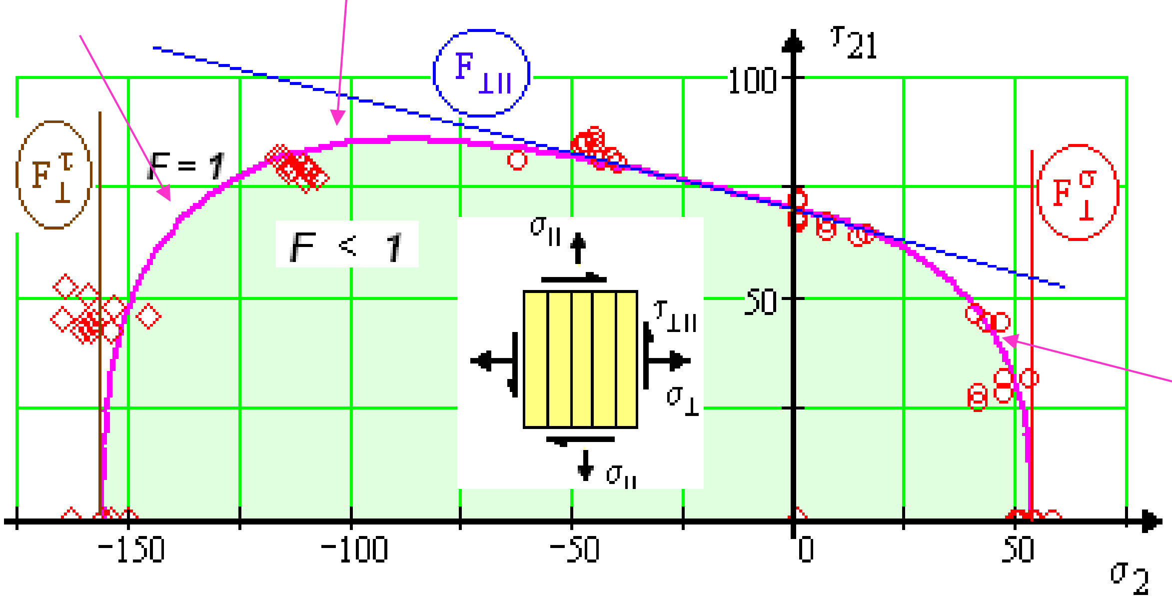
$$Eff_{\perp}^{\sigma} = \frac{\sigma_2}{\bar{R}_{\perp}^{t}}, \qquad Eff_{\perp||} = \frac{|\tau_{21}|}{\bar{R}_{\perp||} - \mu_{\perp||}\cdot\sigma_2}, \qquad Eff_{\perp}^{\tau} = \frac{-\sigma_2}{\bar{R}_{\perp}^{c}}.$$

All failure modes, 3 IFF + 2 FF, are interacted in one single (*global*) failure equation

magenta curve ;

$$Eff^{\,m} = (\cancel{Eff_{||}^{\tau}})^m + (\cancel{Eff_{||}^{\sigma}})^m + (Eff_{\perp}^{\sigma})^m + (Eff_{\perp||})^m + (Eff_{\perp}^{\tau})^m = 1.$$

by above series failure system model



** for UD laminae m =2.5 - 3*

** the same value m is applied*
for all interaction zones

IFF curves: (σ_2, τ_{21}) ***.****Hoop wound GFRP tube: E-glass/LY556/HT976*

5. Short Derivation of the *Failure Mode Concept (FMC)*

5.6 Reasons for Chosing Invariants when Generating Failure Conditions

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* Beltrami : “At ‘Onset of Yielding’ the material possesses a distinct ***strain energy*** composed of *dilatational energy* (I_1^2) and *distortional energy* ($J_2 \equiv Mises$) ”.

* So, from Beltrami, Mises (HMH), and Mohr / Coulomb (friction) can be concluded:

Each invariant term in the *failure function* ***F*** may be dedicated to one physical mechanism in the solid = cubic material element:

- volume change : I_1^2 … ***(dilatational energy)***
- shape change : J_2 (Mises) … ***(distortional energy)***

and - friction : I_1 … ***(friction energy)***

Stress Invariants: ***isotropic materials*** *and*

I_1^2 , I_2^2

I_3 , I_4

I_2

: UD materials !

Mohr-Coulomb

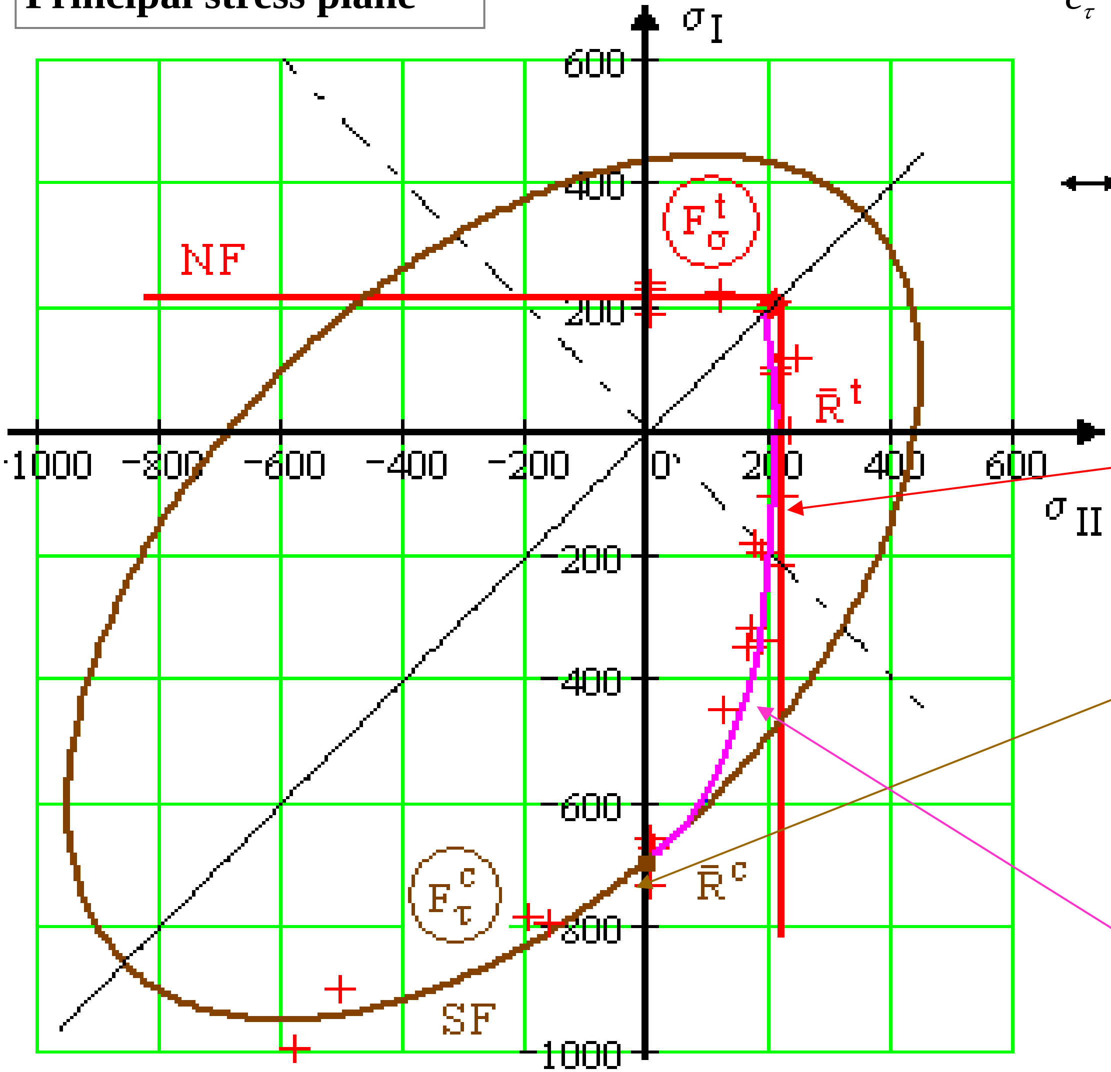
Remember:

These I_1 are different !

6 Visualisation of some Derived Failure Conditions

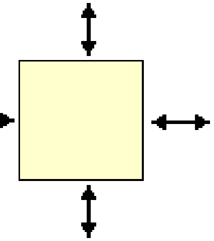
6.1 Grey Cast Iron (brittle, dense, microflaw-rich), *Principal stress plane*

Principal stress plane



$c_\tau^c = a_\tau^c - 1, \quad a_\tau^c = 1.58 \quad m = 3.1 \quad \bar{R}^t = 215 MPa;$

$\bar{R}^c = 690 MPa$



$$\{\sigma\}_{principal} = (\sigma_I, \ \sigma_{II}, \ 0)^T$$

$$F_\sigma^t = \frac{\sqrt{I_\sigma} + I_1}{2 \cdot \bar{R}^t} = 1$$ deformationless

$$F_\tau^c = a_\tau^c \frac{3J_2}{\bar{R}^{c2}} + c_\tau^c \frac{I_1}{3\bar{R}^c} = 1$$

shear friction

= 2 Mode Failure Conditions

Interaction zone

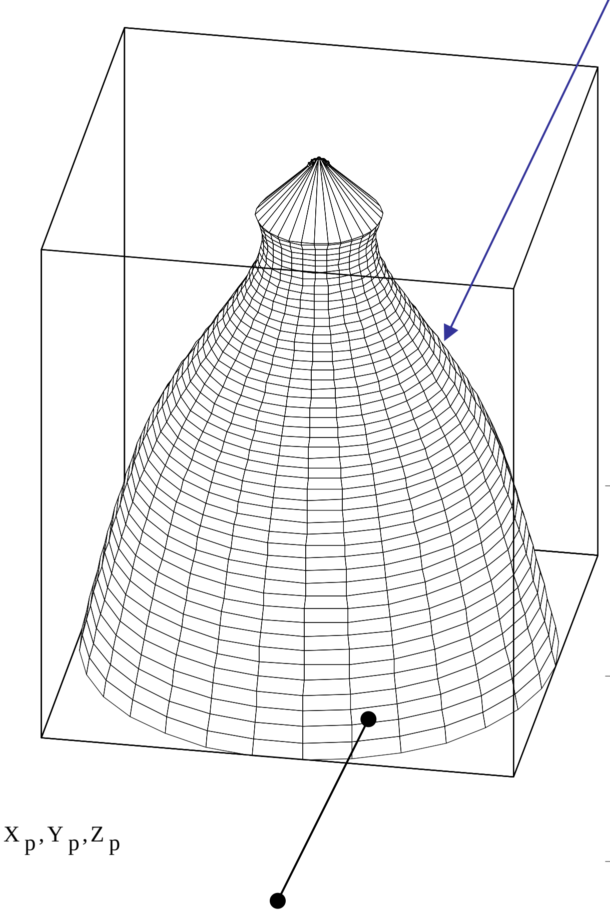
Lessons learned: Basically, *<u>Dense</u> concrete and Glass C 90 will have same failure condition*

6 Visualisation of some Derived Failure Conditions

6.1b Grey Cast Iron (brittle, dense, microflaw-rich), Spatial visualization

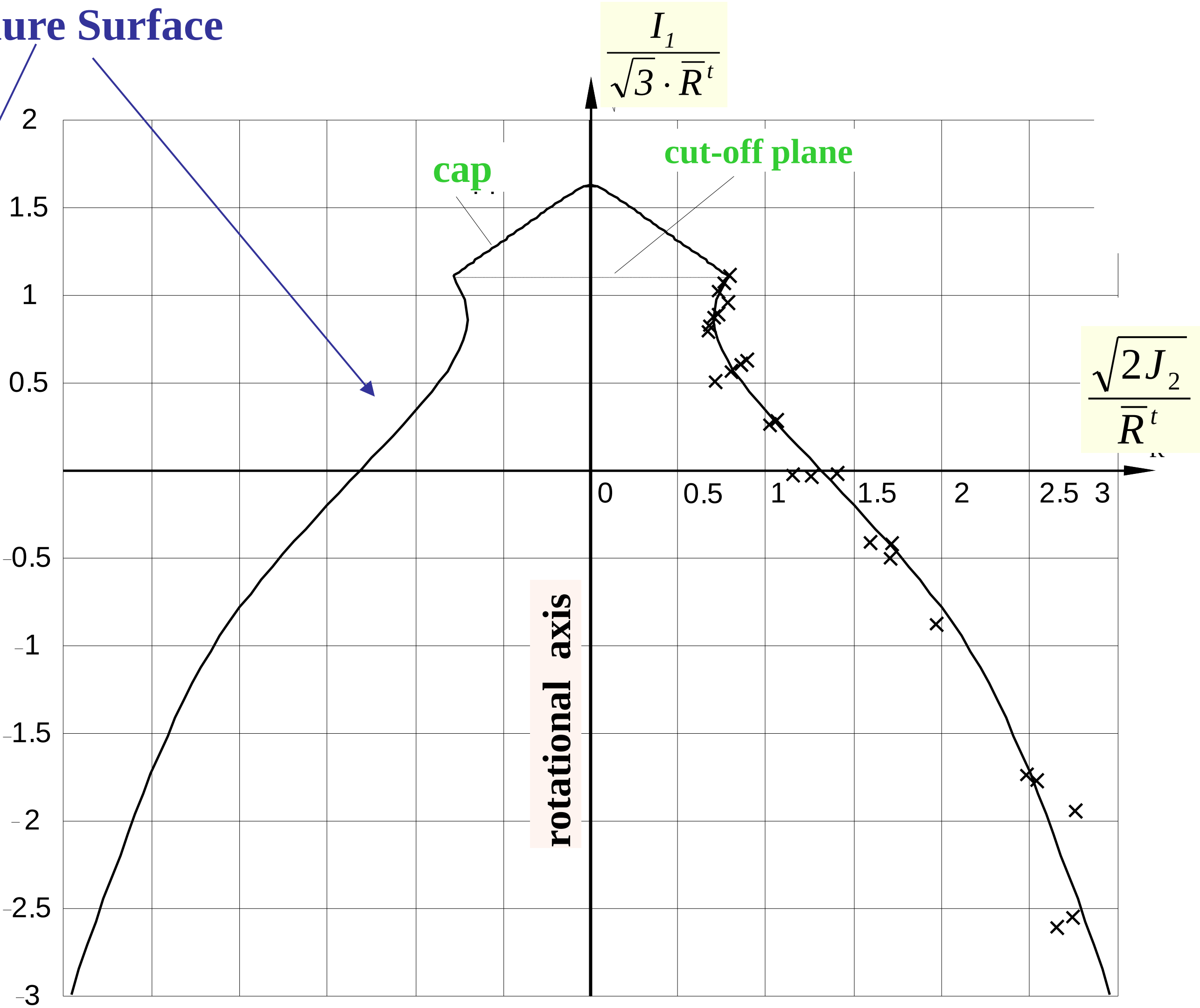
3D: in Lode coordinates

Failure Surface



neglecting difference between tensile and compressive meridian (see concrete)

potential closing surfaces



2D test data, filled in Lode diagram

6 Visualisation of some Derived Failure Conditions

6.2a Concrete (isotropic, slightly porous) *Kupfer's data*

Octahedral stresses (B-B view)

$$F^t_\sigma = \frac{\sqrt{I_\sigma} + I_1}{2\bar{R}^t_m} = Eff^t_\sigma = 1$$

deformation poor

hyperbola

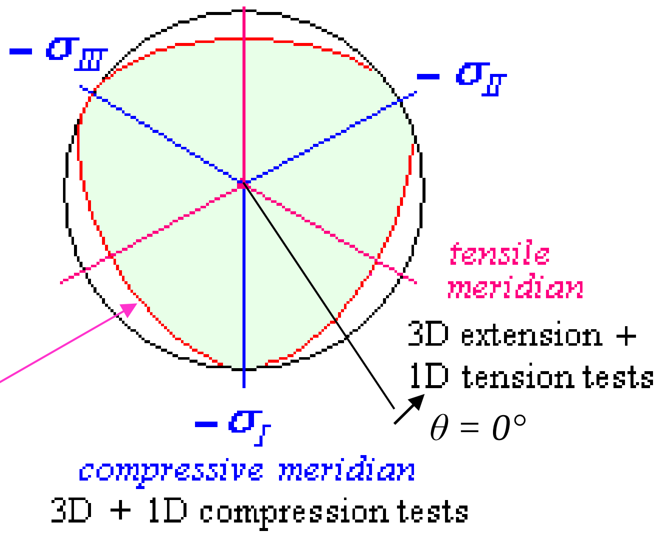
shape + volume change + friction: Mohr-Coulomb :

$$F^c_\tau = a^c_\tau \frac{3J_2\cdot\Theta}{\bar{R}^{c\,2}_m} + b^c_\tau \frac{I_1^{\,2}}{\bar{R}^{c\,2}_m} + c^c_\tau \frac{I_1}{\bar{R}^c_m} = 1$$

(closed failure surface)

paraboloide

Isotropic materials possess 120° symmetry :



Lessons learned from test data viewing**:**

- *Course of concrete test data shows a big bandwidth*
- *The reason for the bandwidth is not only the test scatter but the stress-state dependent 'double' failure probability causing non-coaxiality in the octahedral plane.*
  The difference between the so-called tensile (extension) meridian and the compression meridian is to be considered.

Basically, the differences in the octahedral (deviatoric) plane can be described by :

$$\Theta \Rightarrow \sqrt[3]{1 + d\cdot\sin(3\theta)}$$

$$\sin(3\theta) = 3\sqrt{3}J_3/(2J_2^{3/2}),$$

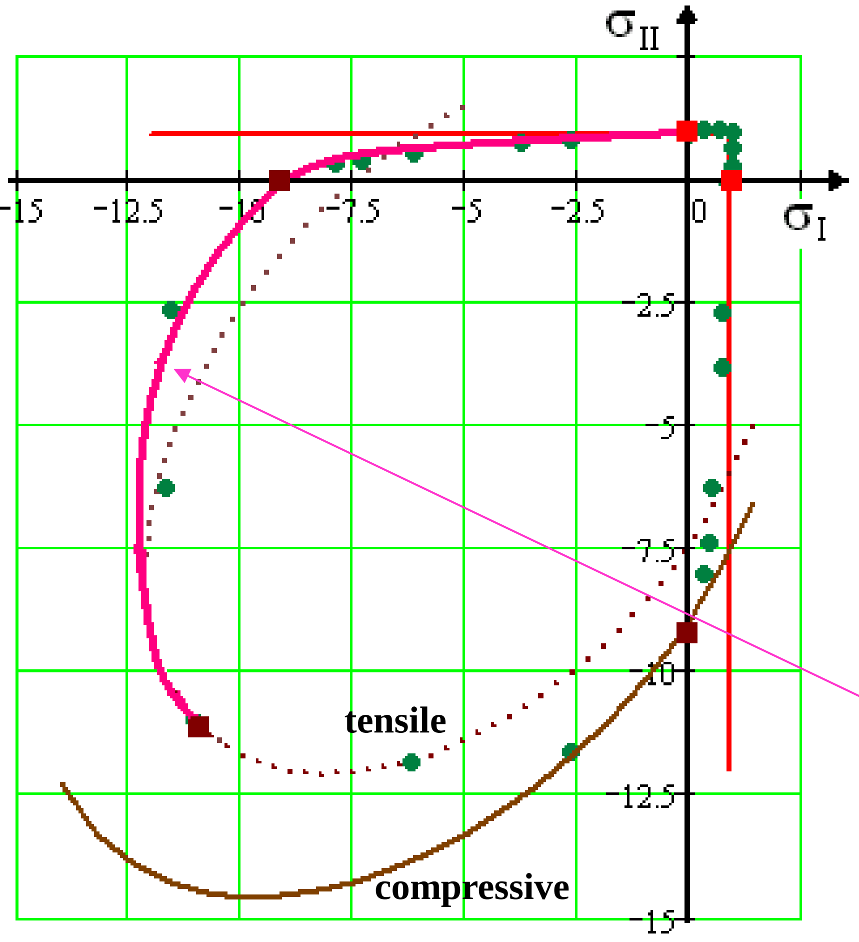
[de Boer, et al] $d \le 0.5$, *convex*

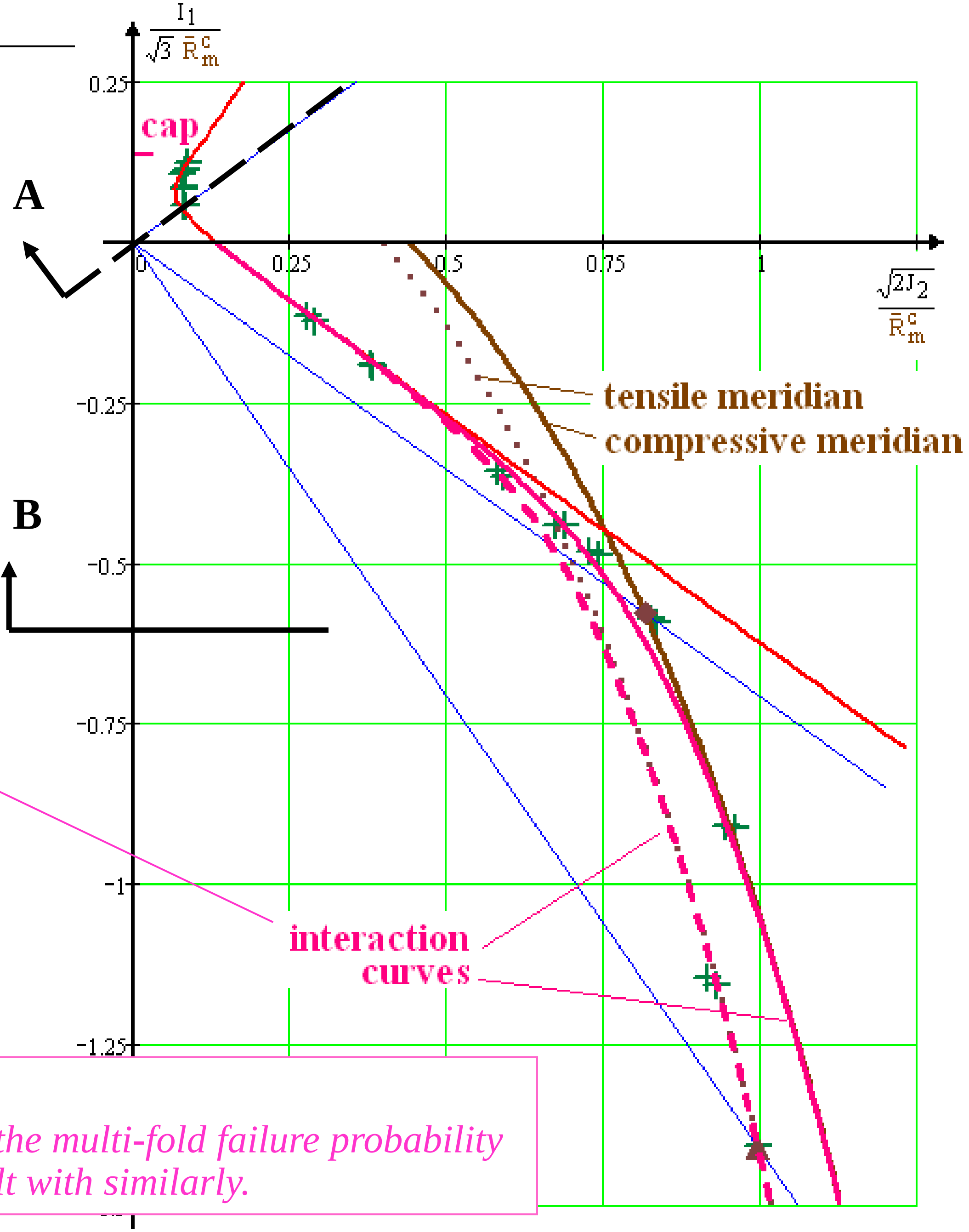
Remark Cuntze: J_3 practically describes the effect of the doubly acting failure mode, no relation to new special mechanism.

6 Visualisation of some Derived Failure Conditions

6.2b Concrete

Principal stresses (A-A view):





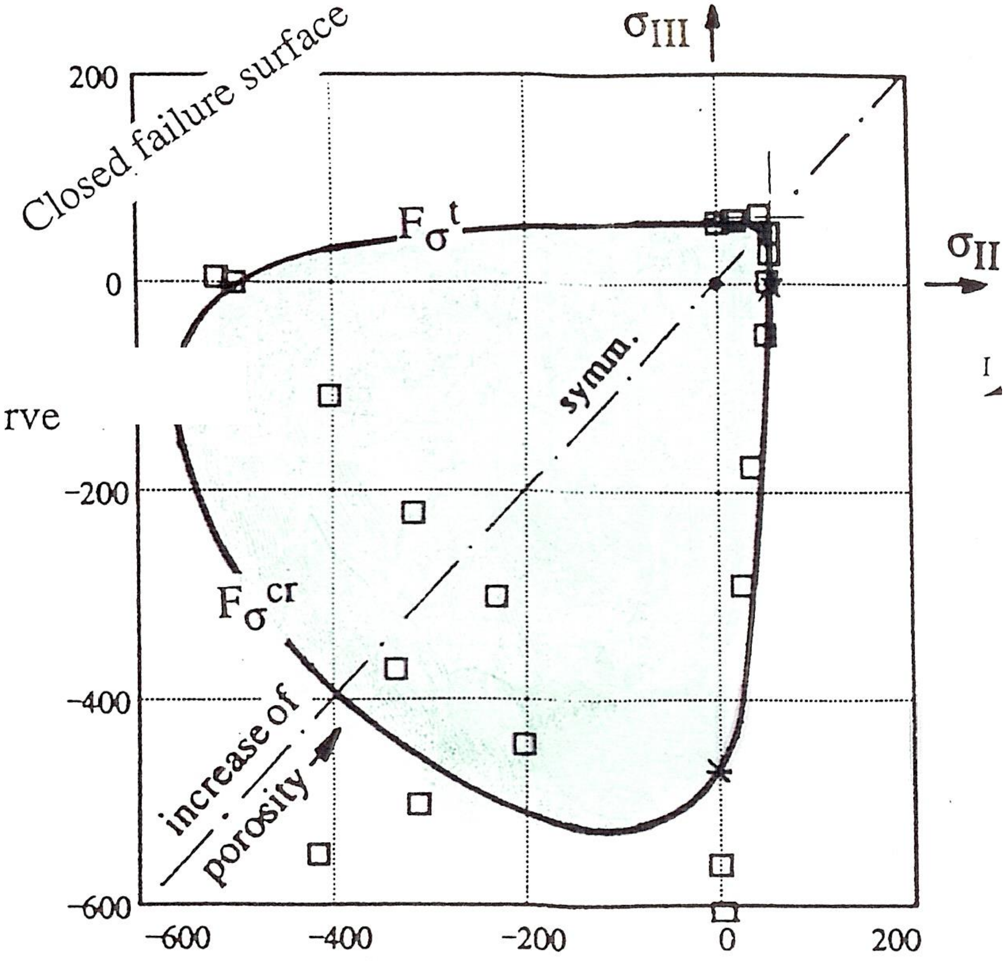
Lessons learned :

- *J_3 considers -as an engineering approach- the multi-fold failure probability*
- *Stone material or grey cast iron can be dealt with similarly.*

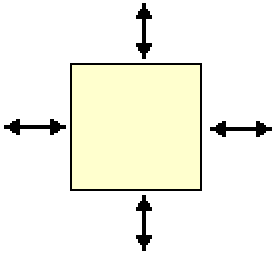
6 Visualisation of some Derived Failure Conditions

6.3 Monolithic Ceramics (brittle, porous isotropic material)

Principal stress plane



$$c^{cr} = a^{cr} - 1$$ **[Kowalchuk]**



$$F_\sigma^t = \frac{\sqrt{I_\sigma} + I_1}{2 \cdot \bar{R}^t} = 1$$

$$F^{cr} = a^{cr} \frac{3J_2}{\bar{R}_m^{c\,2}} + \; + c^{cr} \left(\frac{I_1}{\bar{R}_m^c} \right)^2 = Eff^{cr} = 1$$

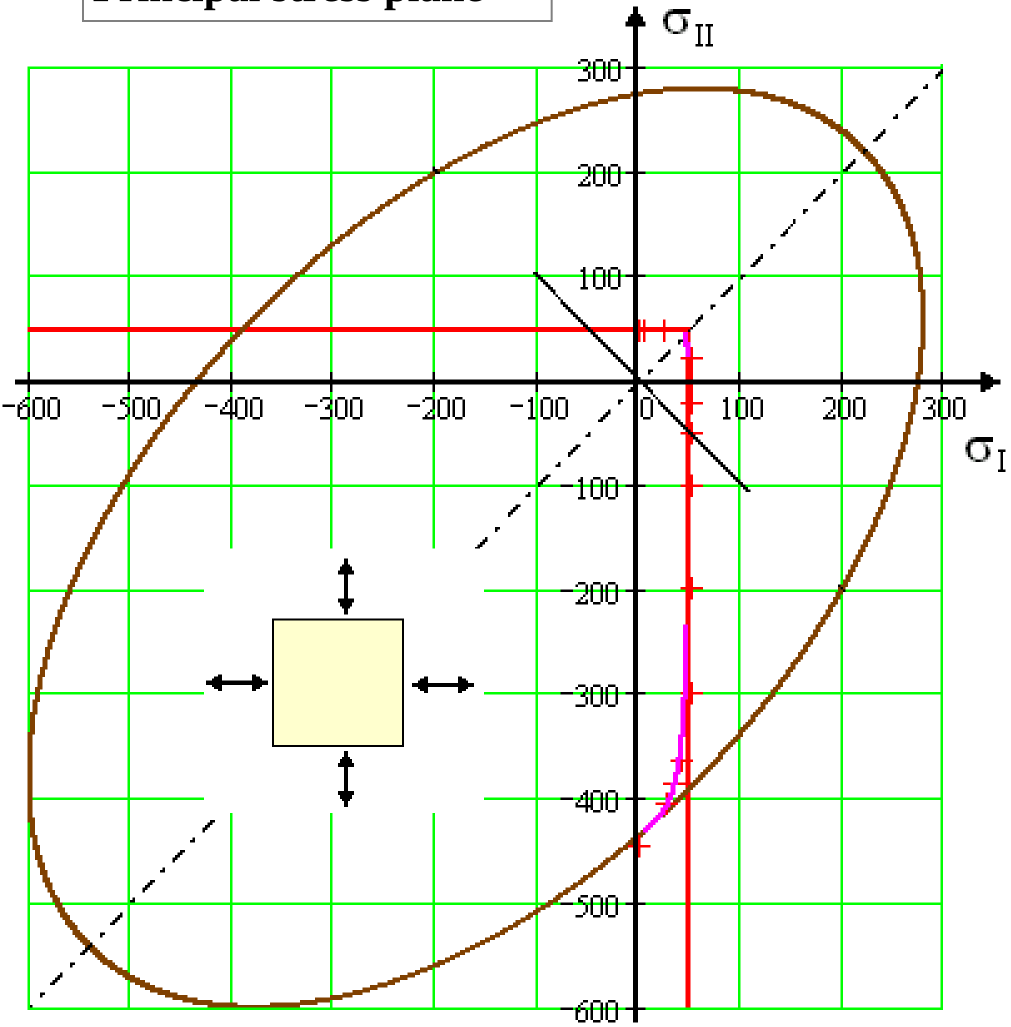
shear **volume**

Lessons learned: ***Same failure condition as very porous concrete***

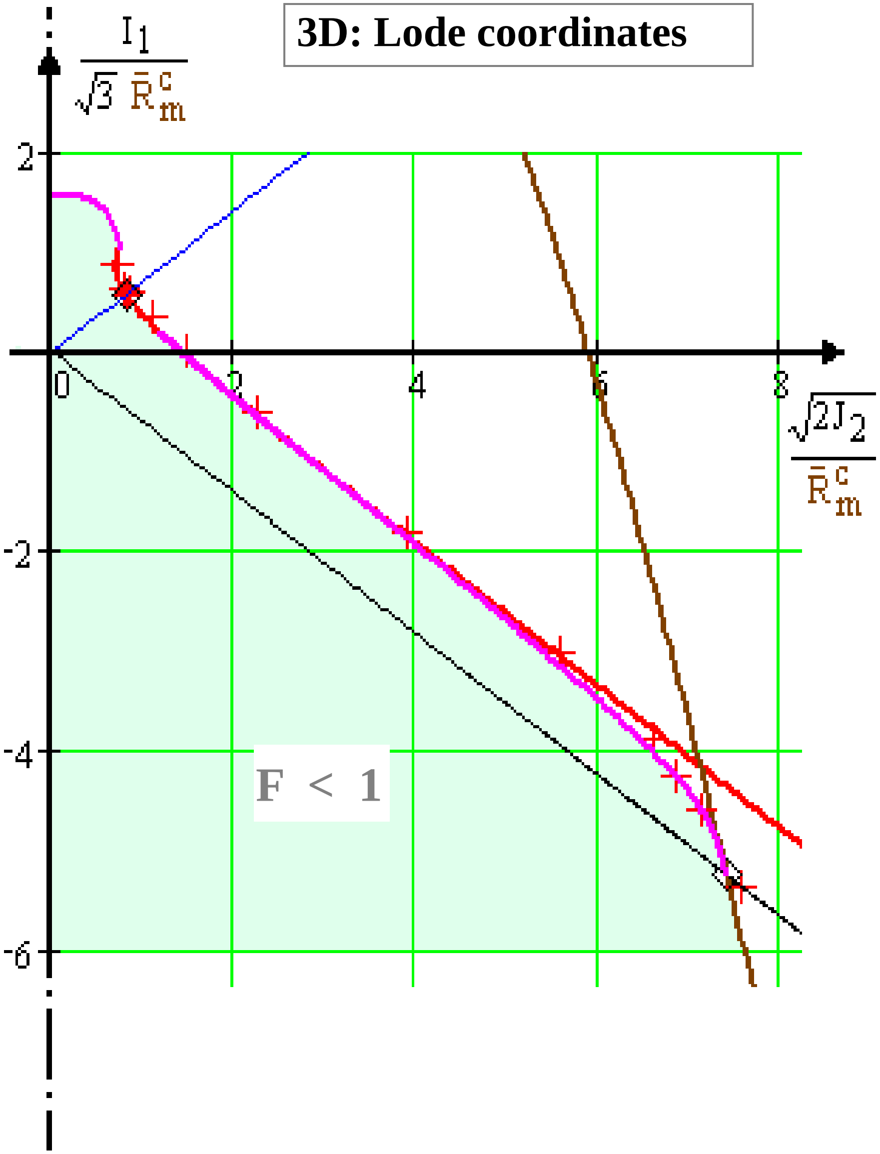
6 Visualisation of some Derived Failure Conditions

6.4 Glass C 90 (brittle, dense isotropic material)

Principal stress plane



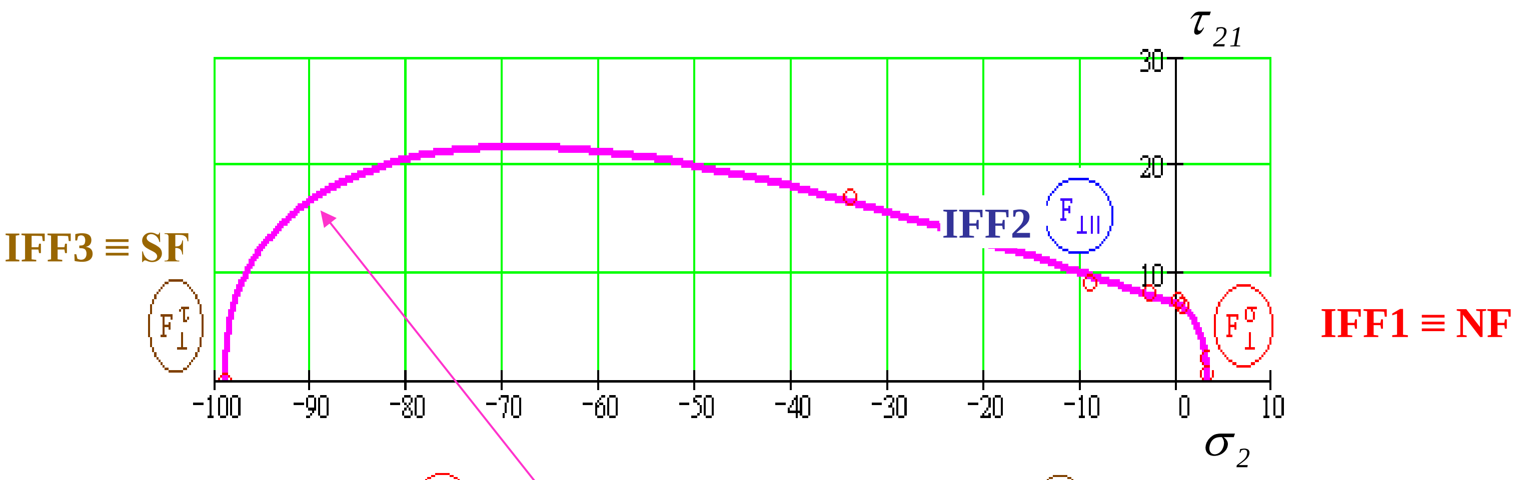
3D: Lode coordinates



6 Visualisation of some Derived Failure Conditions

6.5 UD Ceramic Fibre-Reinforced Ceramics (C/C) (brittle, porous, tape)

$\{\bar{R}\} = (\bar{R}_{||}^t, \bar{R}_{||}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp||}) = (-, -, 3, 99, 7)^T, \; m = 2.3, \; \mu_{\perp||} = 0.3$ *[Diss. B. Thielicke, 1997]*



$F_\perp^\sigma$ $F_{\perp||}$ $F_\perp^\tau$

interaction equation :

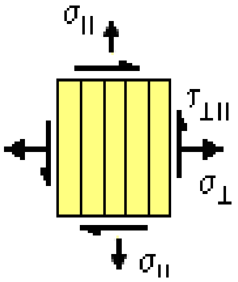
$$\left(\frac{\sigma_2}{\bar{R}_\perp^t}\right)^m + \left(\frac{|\tau_{21}|}{\bar{R}_{\perp||} - \mu_{\perp||} \cdot \sigma_2}\right)^m + \left(\frac{-\sigma_2}{\bar{R}_\perp^c}\right)^m = 1$$

deformationless

friction

Mohr-Coulomb

shear

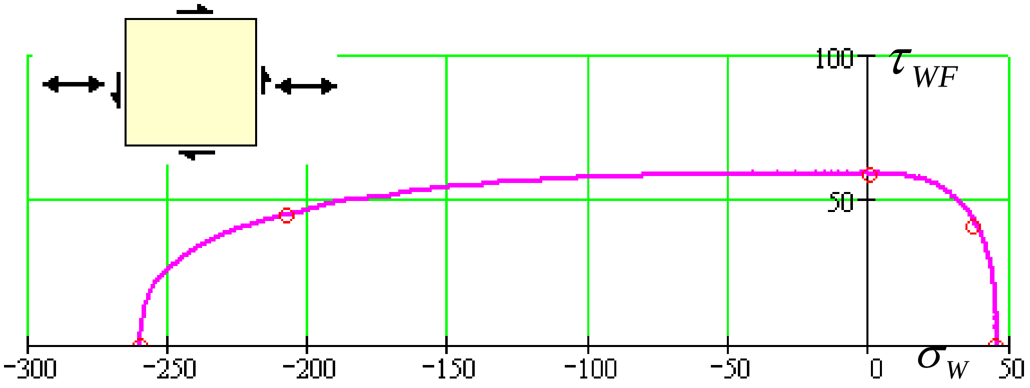


Invariants applied: I3, I2 I4 , I2

Lesson learned: ***Same failure condition as with UD-FRP***

6 Visualisation of some Derived Failure Conditions

6.8 Fabric Ceramic Fibre-Reinforced Ceramics (CFRC) (brittle, porous)



C/C-SiC, T= 1600°C

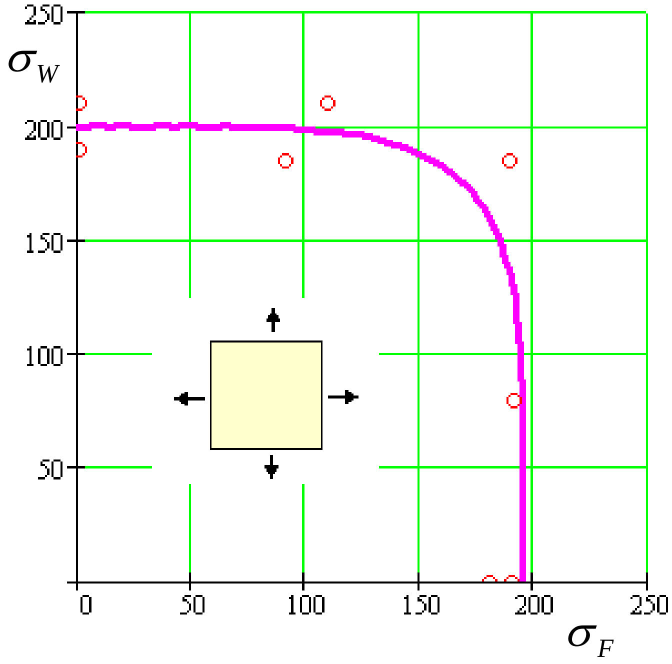
[Geiwitz/Theuer/Ahrendts 1997] ,

*tension/compression-torsion-**tube??***

$$\{\bar{R}\} = (\bar{R}^t_{||}, \bar{R}^c_{||}, \bar{R}^t_{\perp}, \bar{R}^c_{\perp}, \bar{R}_{\perp||}) = (-, -, 45, 260, 59)^T$$

$m = 2.8$

$$(\frac{\sigma_W}{\bar{R}^t_W})^m + (\frac{-\sigma_W}{\bar{R}^c_W})^m + (\frac{\tau_{WF}^{\ 2}}{\bar{R}_{WF}^{\ 2}})^m = 1$$



$$\{\bar{R}\} = (\bar{R}^t_W, \bar{R}^c_W, \bar{R}^t_F, \bar{R}^c_F, \bar{R}_{WF}, \bar{R}^t_3, \bar{R}^c_3, \bar{R}_{3F}, \bar{R}_{3W})^T$$

$\{\bar{R}\}$ = *vector of mean strength values*

C/SiC, ambient temperature *[MAN-Technologie, 1996],*

tension/tension tube

$$\{\bar{R}\} = (200, -, 195, -, -,,,,)^T, \ m = 5$$

$$(\frac{\sigma_W}{\bar{R}^t_W})^m + (\frac{\sigma_F}{\bar{R}^t_F})^m = 1$$

NOTE: For woven fabrics test information for a real validation is not yet available!

Main Conclusions from *Failure Mode Concept* Applications

- **FMC is an efficient concept, that improves prediction , simplifies design verification**
- **Simply applicable to brittle/ductile, dense /porous, isotropic /anisotropic material**
  - if clear failure modes can be identified and
  - if the homogenized material element experiences a *volume* or *shape change* or *friction*
- **Delivers a global formulation of '*<u>individually'</u>* *combined independent failure modes,* without the well-known drawbacks of global failure conditions** which *mathematically combine in-dependent failure modes* .

Many material behaviour Links/Relationships have been outlined :

Example: basically, a compressed brittle *porous* concrete can be described like
a tensioned ductile *porous* metal ('Gurson' domain)

Final Note on ‘Validation of Failure Conditions’: and on reducing *Gaps* between *Predictions* and *Test Results*

- **Check** by *Engineering Judgement* +
- **Analyse your Analysis !**

  Do the chosen models (structural, material, numerical) respect the quality, required by the posed task?
- **Test your Test !**

  Is the test specimen well designed?

  Is the performed experiment of a good quality?

  Is the evaluation of the test results carefully done?
- **“Think** (Utilize) **Material Behaviour Links” !**

Keep in mind !

- ***Experimental results can be far away from the reality like a bad theoretical model.***
- ***Theory creates a model of the reality, ‘only’,*** and ***1 Experiment is ‘just’ 1 realisation of the reality.***

Development and application of the FMC was never funded !

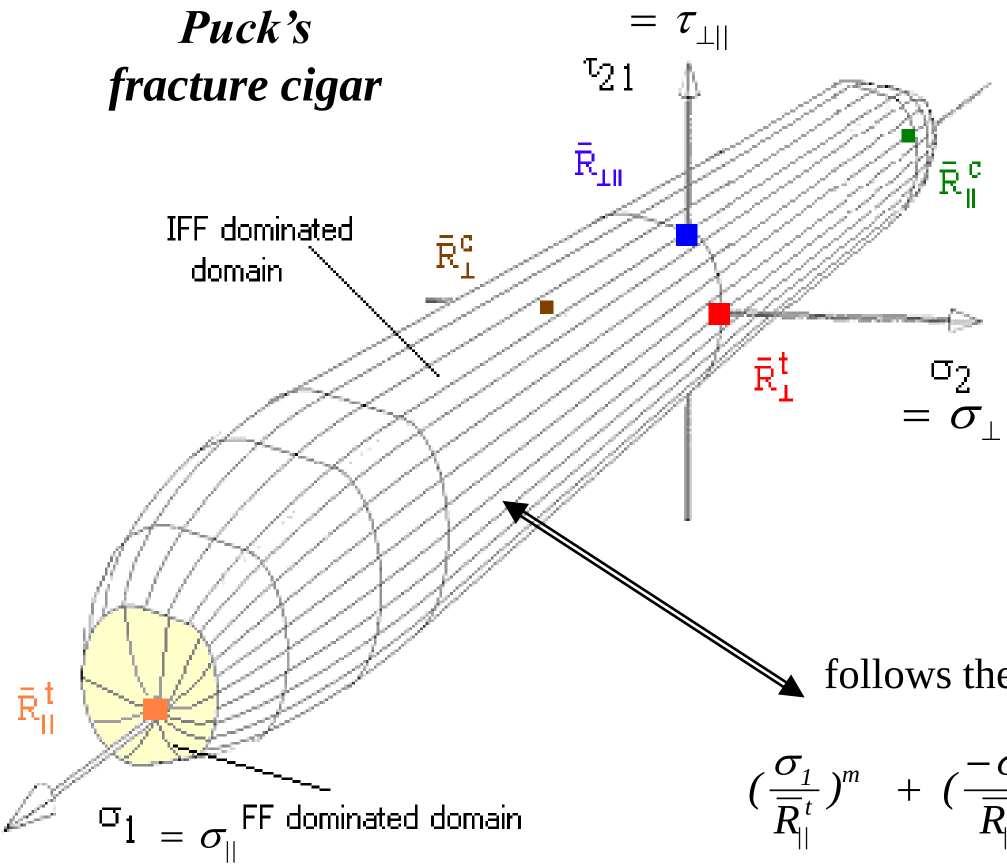
Requirements for the Development of Failure Conditions

Failure conditions are demanded to :

- **simply formulated + numerically robust**
- **physically-based, and**
- **practically just need the (few) information on the strengths available at pre-dimensioning.** Further probablly necessary parameters shall be assessable.
- **be a mathematically homogeneous function,**

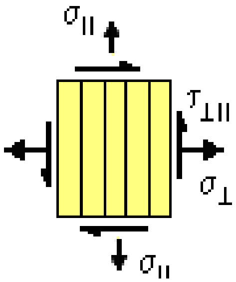
UD lamina FRP Failure Conditions (brittle, dense, flaw-rich), 2D Fracture)

Puck's fracture cigar



Carbon Fibre-Reinforced Plastics (CFRP)

$$\{\sigma\}_{lamina} = (\sigma_1, \sigma_2, 0, 0, 0, \tau_{21})^T$$



After inserting above stresses into the invariants

$$I_1 = \sigma_1$$

$$\cong \nu_f \cdot \sigma_{1f}{}^t = \nu_f \cdot \varepsilon_1{}^t \cdot E_{1f} = \varepsilon_1{}^t \cdot E_\parallel$$

$$I_2 = \sigma_2 + \sigma_3, \qquad I_3 = \tau_{31}^2 + \tau_{21}^2$$

$$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2$$

follows the ***2D failure condition* ≡ *failure surface :***

$$\left(\frac{\sigma_1}{\bar{R}_\parallel^t}\right)^m + \left(\frac{-\sigma_1}{\bar{R}_\parallel^c}\right)^m + \left(\frac{\sigma_2}{\bar{R}_\perp^t}\right)^m + \left(\frac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2}\right)^m + \left(\frac{-\sigma_2}{\bar{R}_\perp^c}\right)^m = 1$$

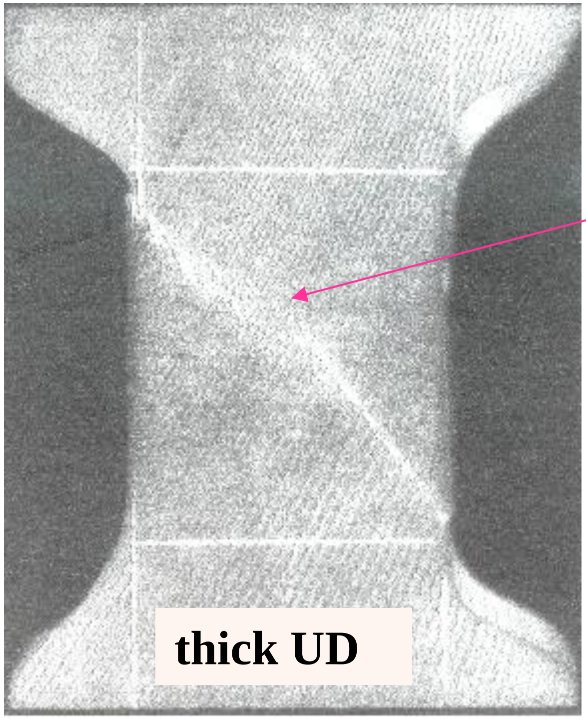
friction

$$\{\bar{R}\} = (\bar{R}_\parallel^t, \bar{R}_\parallel^c, \bar{R}_\perp^t, \bar{R}_\perp^c, \bar{R}_{\perp\parallel}) = (-, -, 45, 260, 59)^T, \; m \approx 2.8, \; \mu_{\perp\parallel} \approx 0.2$$

examples: see WWFE

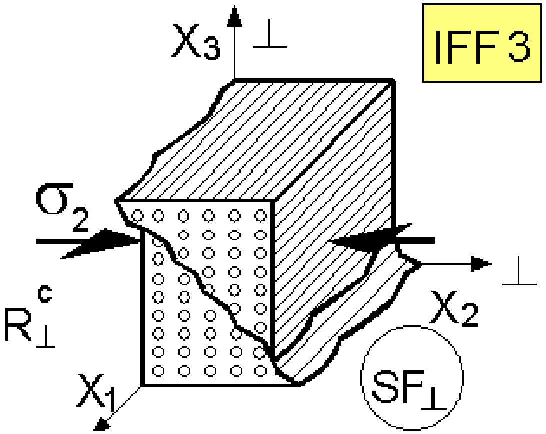
Lesson learned: *Same failure condition as UD-CMC*

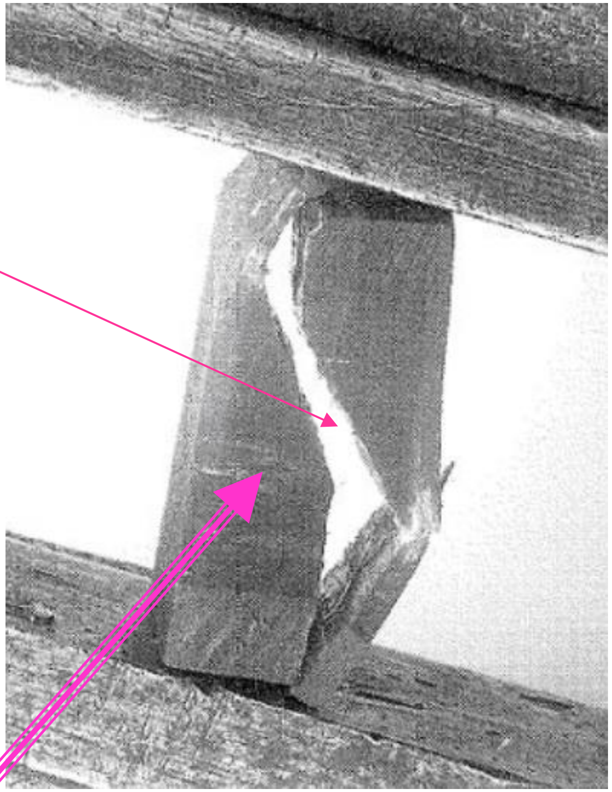
Transversely-Isotropic Material (UD). Observed Puck's *Wedge Failure Mode*



macroscopically:

SF := Shear Fracture





Laminate:

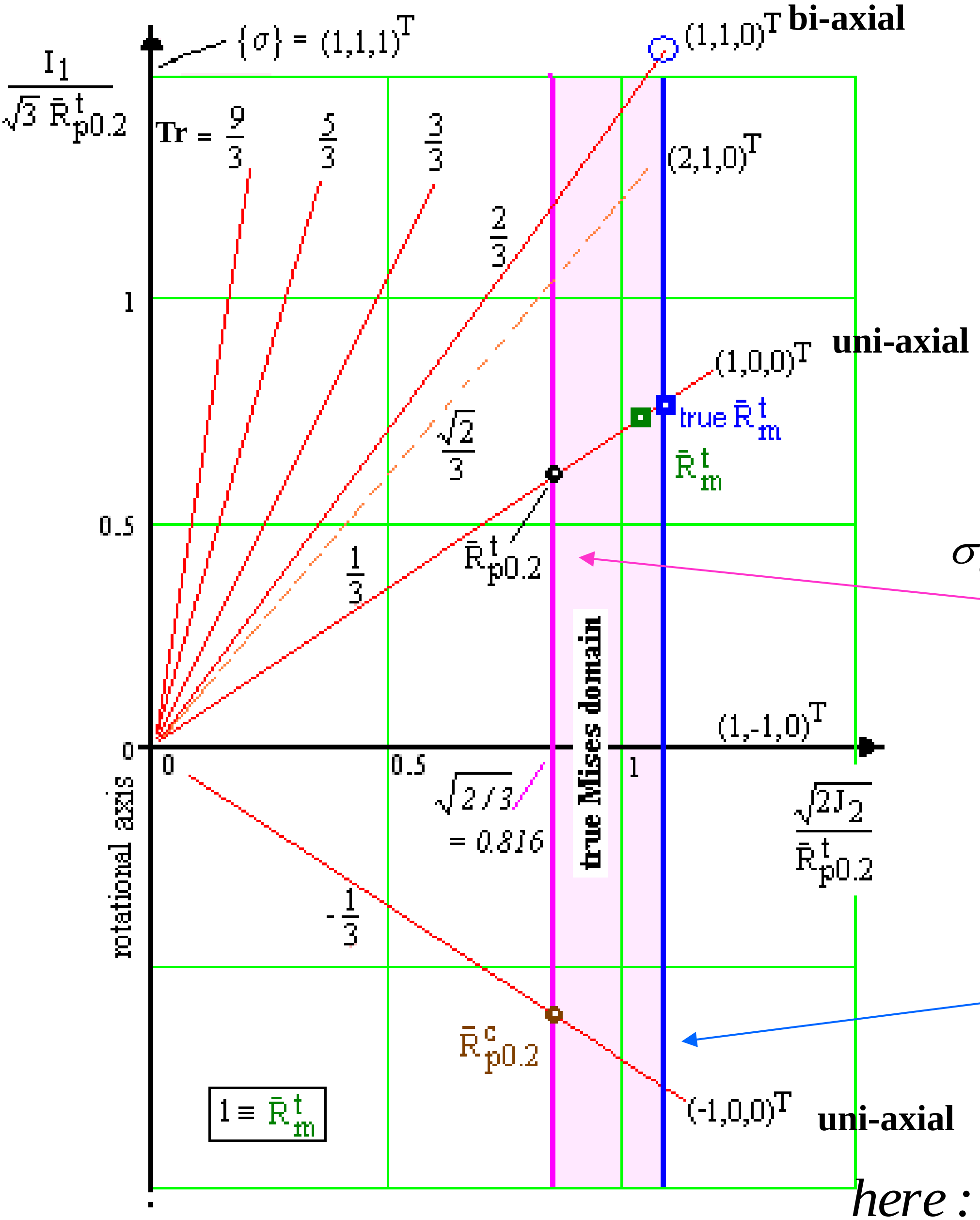
[±30/90/±30], tube

Fig. 2.15 in [Puck 1996]

Lessons learned from component tests:

Wedge failure IFF3 might be hazardous like FF.

Practical Stress State Regimes, Triaxiality, and Lode Coordinates



(example: isotropic material)

3D: Lode coordinates

Triaxiality: $Tr = \sigma_{mean} / \sigma_{eq}^{Mises}$

Practical regime of Triaxiality:

$Tr < 1$ *(sharp notch =1)*

$$\sigma_{eq}^{Mises} = \sqrt{3J_2} = \overline{R}_{p0.2}$$

Mises Cylinder:

Onset of full yielding,
subsequent yield surfaces

Lesson learned:

The fracture failure surface confines the yield surface!

here : $\overline{R}_m^t = R_m^c$