Cuntzes ‘Failure Mode Concept’ applicable to Static and Cyclic Strength Prediction of Isotropic, Transversely-isotropic and Orthotropic Materials

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retired from MAN-Technologie, now linked to Carbon Composites e.V. Augsburg

results of non-funded private research

zum Vortragenden:

1964: **Diplom** **Statiker**

1968: **Dr.-Ing.** **Strukturdynamik**

1978: **Dr.-Ing. habil.** **Mechanik des Leichtbaus** (Composites)

1968- 1970: Institut Luft- und Raumfahrt (DLR)

1970-2004: MAN-Technologie (Raumfahrt, Wind-, Sonnenenergie-, Kernenergie, ..)

1980-2002: Dozent an der Universität der Bundeswehr

jetzt: Ingenieur, Unruheständler + Simulant und Leiter der AGs ***Engineering, Faserverstärkung im Bauwesen*** **beim Carbon Composites e. V.**

VDI 2014, HSB, ESA-Standards und Handbücher, Gewinner WWFE-I

Gutachter für BMFT, BMBF, DFG

Worked in the areas:

Finite Element Analysis, Structural and Rotor dynamics,

Structural reliability and Safety concepts, Development policy,

Failure hypotheses (isotropic + composites),

Composite Fatigue, Fracture mechanics, and Damage mechanics.

Motivation for this Scientific Work

DRIVER: ***Author's industrial experience at MAN-Technologie with structural material applications, range 4 K - 2000 K, experienced in***

ARIANE 1-5 launchers, cryogenic tanks, heat exchanger in solar towers (GAST Almeria), wind energy rotors (GROWIAN), Antennas, ATV (JulesVerne), Crew Rescue Vehicle (CMC) for ISS, Gasultra-Centrifuges, ….

Existing Links in the Mechanical Behaviour show up: ***Different structural materials***

- ***can possess similar material behaviour or***
- ***can belong to the same class of material symmetry .***

similarity aspect

Welcomed Consequence:

**** The same strength failure function F can be used for different materials***

**** More information is available for pre-dimensioning + modelling***

- in case of a newly applied material -

from experimental results of a <u>similarly behaving</u> material.

MESSAGE: **Let's use these benefits!**

Einschränkung des Themas

1 Stress (local material point): verification by a strength

static prediction of onset of delamination

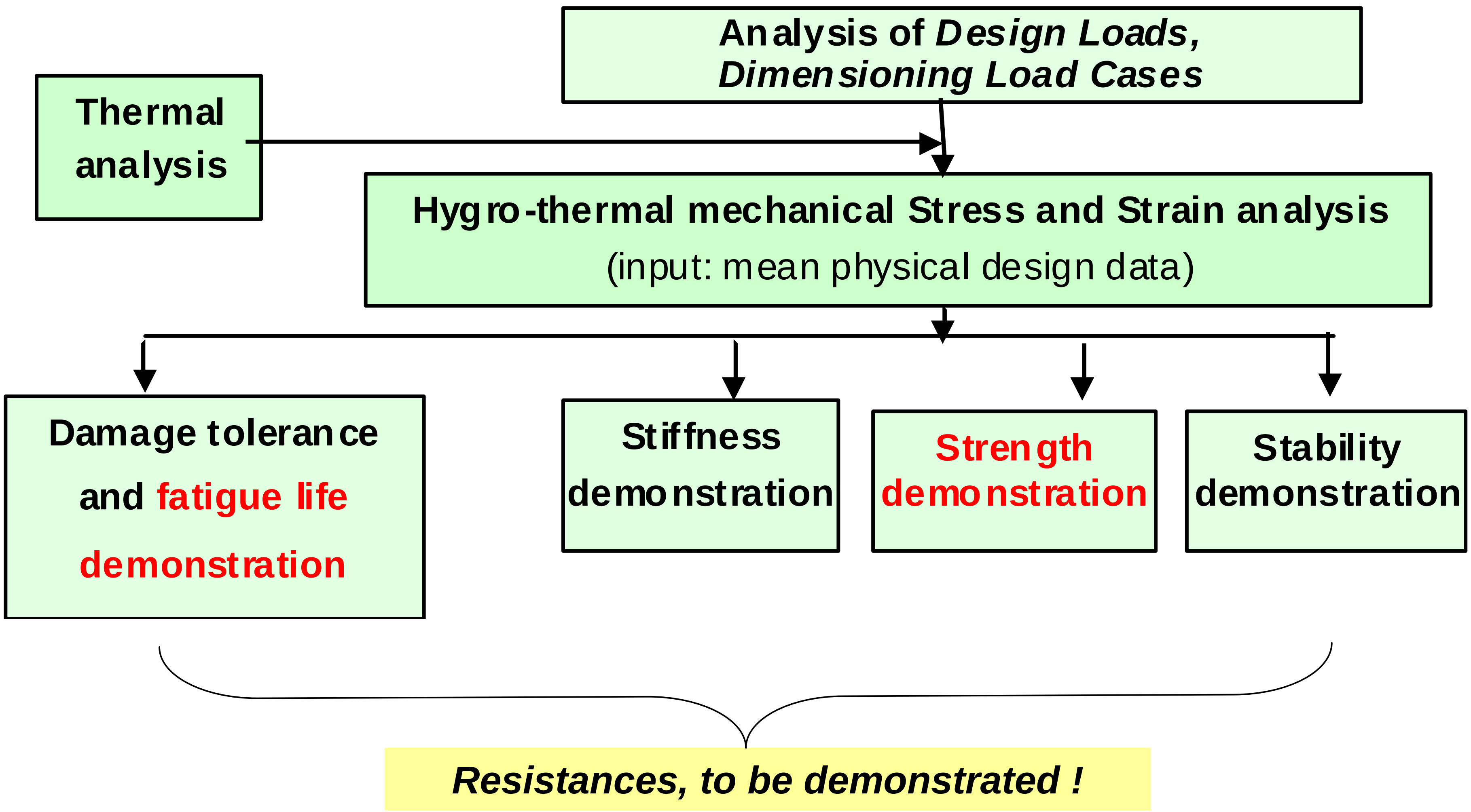
2 Stress concentration (stress peak at a joint): verification by a notch strength

(Neuber)‘, *Verfahren der kritischen Abstände‘ bei FKV*

3 Stress intensity (delamination = crack): verification by a fracture toughness

- prediction whether a delamination is instabile

- predictiing delamination growth (propagation)


Analysis of Design Loads,
Dimensioning Load Cases
Thermal analysis
Hygro-thermal mechanical Stress and Strain analysis
(input: mean physical design data)
Damage tolerance and fatigue life demonstration
Stiffness demonstration
Strength demonstration
Stability demonstration
Resistances, to be demonstrated !

Intention of the Talk

**** To draw attention to :***

- material behaviour (ductile, brittle, intermediate),

- material consistency (dense, porous),

- material element behaviour (volume change, shape change, friction).

**** To show***

- some Basic Ideas of Cuntze's FMC-derived UD failure conditions

- some Lessons Learnt when applying them to test data

- a Novel Idea to transfer Static findings to Cyclic Behaviour

**** Basically addressed will be uni-directional (UD) material***

For prediction of ***Onset of Yielding*** + ***Onset of Fracture*** for non-cracked materials.

What are Failure Conditions for? *They shall*

• ***assess multi-axial stress states in the critical material point,*** by

$$\frac{\sigma_{eq}}{R} = \frac{\sigma_{eq}^{mode}}{R^{mode}}$$

utilizing the uniaxial strength values R and - if possible - **equivalent stress σ_{eq}, representing a distinct multi-axial stress state.**

for *** dense & porous,**

*** ductile & brittle behaving materials,**

$R_{p0.2} \cong R_{c0.2}$ $R_m^{\ c} \geq 3R_m^{\ t}$

for *** isotropic material**

*** transversally-isotropic material (UD := uni-directional material)**

*** rhombically-anisotropic material (woven fabrics, non-crimped fabrics, braided + stitched + z-pin textiles, …)**

1.3 State of the Art in Static Strength Analysis of UD Laminas (plies)

Information collected as Participant of World-Wide-Failure-Exercise (WWFE),
since 1991 running

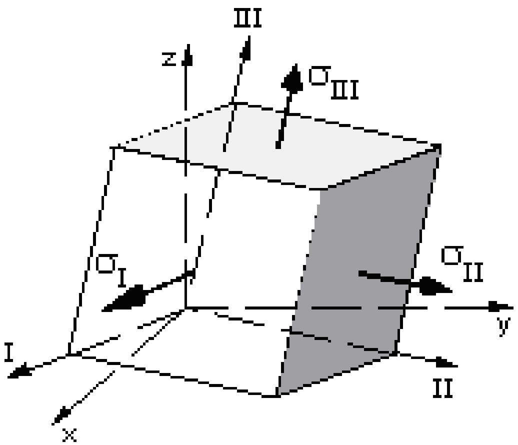
- **WWFE-I : 2D *Failure mode–based* strength failure conditions could be validated !**

- **WWFE-II : 3D Failure mode–based strength failure conditions cannot be fully validated** *due to a lack of*

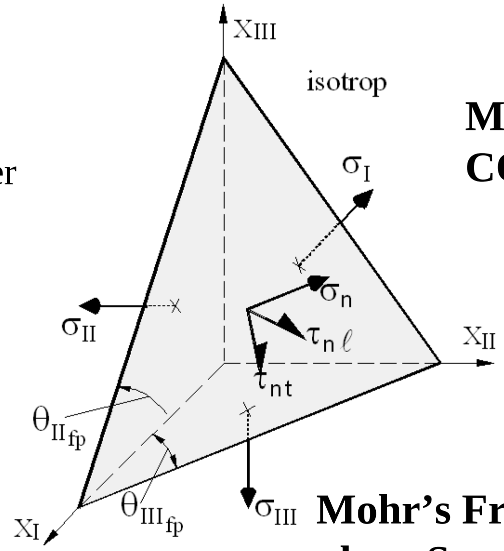
  sufficient reliable test data in several 3D stress domains

Even for isotropic materials not all conditions used are validated !

2.1 3D Stress states and Invariants - Isotropic Material



The stress states in the various COS can be transferred into each other



Mohr's COS

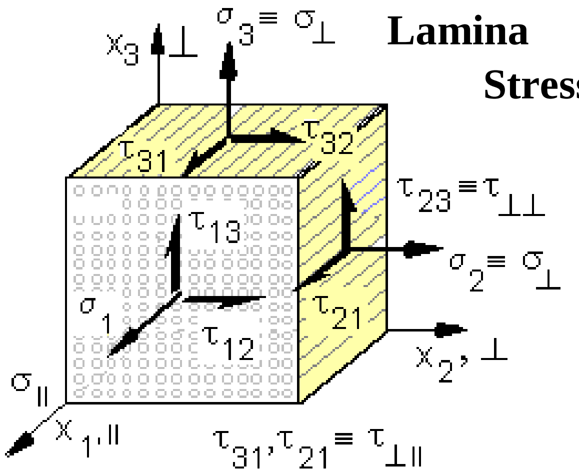
Mohr's Fracture plane Stresses

Principal Stresses	**Structural Component Stresses**	
$\{\sigma\}_{principal} = (\sigma_I\,,\ \sigma_{II}\,,\ \sigma_{III}\,)^T$	$\{\sigma\}_{comp} = (\sigma_x\,,\ \sigma_y\,,\ \sigma_z\,,\ \tau_{yz}\,,\ \tau_{xz}\,,\ \tau_{xy}\,)^T$	$\{\sigma\}_{Mohr} = (\sigma_\ell\,,\ \sigma_n\,,\ \sigma_t\,,\ \tau_{nt}\,,\ \tau_{t\ell}\,,\ \tau_{\ell n}\,)^T$
$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = 3\sigma_{oct} \equiv f(\sigma),$	**'isotropic' invariants !** $I_1 = (\sigma_x + \sigma_x + \sigma_z)^T$	$I_1 = (\sigma_\ell + \sigma_n + \sigma_t)^T$
$6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2$ $= 4(\tau_{III}{}^2 + \tau_{II}{}^2 + \tau_I{}^2) = 9\tau_{oct}{}^2 \equiv f(\tau)$	$6J_2 = (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2$ $+6(\tau_{yz}{}^2 + \tau_{zx}{}^2 + \tau_{xy}{}^2)$ (Mises,HMH)	$6J_2 = (\sigma_n - \sigma_t)^2 + (\sigma_t - \sigma_\ell)^2 + (\sigma_\ell - \sigma_n)^2$ $+6(\tau_{nt}{}^2 + \tau_{t\ell}{}^2 + \tau_{n\ell}{}^2)$

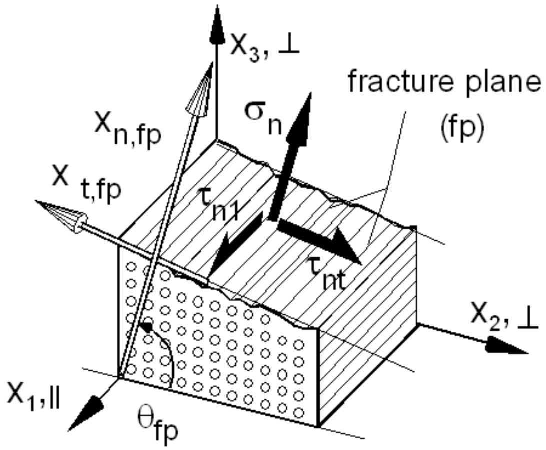
$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III})(2\sigma_{II} - \sigma_I - \sigma_{III})(2\sigma_{III} - \sigma_I - \sigma_{II}),\quad I_\sigma = 4J_2 - I_1^2/3,\quad \sigma_{mean} = I_1/3$$

2.2 3D Stress states and Invariants - Transversely-Isotropic UD-Material

Transformation of lamina stresses into the quasi-isotropic plane stresses



Lamina Stresses



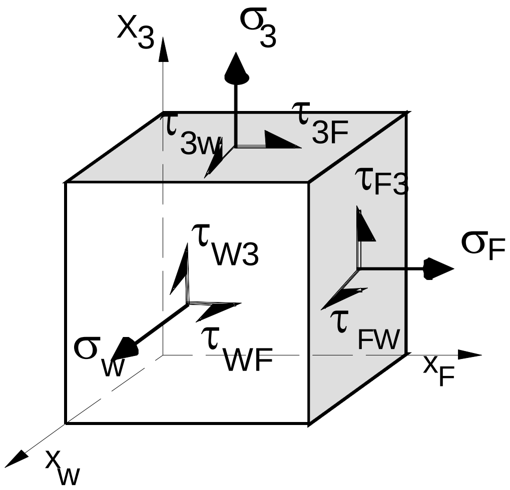
Mohr, Puck, Hashin: Fracture is determined by the (Mohr) stresses in the fracture plane !

$\{\sigma\}^{quasi-isotropic\ \ plane}_{principal} =$ $= (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$	$\{\sigma\}_{lamina} =$ $= (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$	$\{\sigma\}_{Mohr} =$ $(\sigma_\ell, \ \sigma_n, \ \sigma_t, \ \tau_{nt}, \ \tau_{t\ell}, \ \tau_{\ell n})^T$
$I_1 = \sigma_1, \quad I_2 = \sigma_2^p + \sigma_3^p$ $I_3 = \tau_{31}^{p\,2} + \tau_{21}^{p\,2}$	$I_1 = \sigma_1, \quad I_2 = \sigma_2 + \sigma_3$ $I_3 = \tau_{31}^2 + \tau_{21}^2$ **'UD invariants'!** *[Boehler]*	$I_1 = \sigma_1, \quad I_2 = \sigma_n + \sigma_t$ $I_3 = \tau_{t\ell}^2 + \tau_{n\ell}^2$
$I_4 = (\sigma_2^p - \sigma_3^p)^2 + 0$ $I_5 = (\sigma_2^p - \sigma_3^p)(\tau_{31}^{p\,2} - \tau_{21}^{p\,2}) + 0$	$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2$ $I_5 = (\sigma_2 - \sigma_3)(\tau_{31}^2 - \tau_{21}^2) - 4\tau_{23}\tau_{31}\tau_{21}$	$I_4 = (\sigma_n - \sigma_t)^2 + 4\tau_{nt}^2$ $I_5 = (\sigma_n - \sigma_t)(\tau_{t\ell}^2 - \tau_{n\ell}^2) - 4\tau_{nt}\tau_{t\ell}\tau_{n\ell}$

Invariant := Combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system. Good for an optimum formulation of *desired scalar Failure Conditions.*

rhombically-anisotropic ◄ woven fabric)

Homogenized = smeared
***woven fabrics* material element**



Warp (W), Fill (F)=Weft

3D stress state:
Here, just a formulation in fabrics lamina stresses makes sense!

$$\{\sigma\}_{lamina} = (\sigma_W, \sigma_F, \sigma_3, \tau_{3F}, \tau_{3W}, \tau_{FW})^T$$

Fabrics invariants ! *[Boehler]*:

$$I_1 = \sigma_W,\ I_2 = \sigma_F,\ I_3 = \sigma_3,$$
$$I_4 = \tau_{3F},\ I_5 = \tau_{3W},\ I_6 = \tau_{FW}$$

more, -however simple- invariants necessary

2.3a Observed Strength Failure Modes, Strengths - Isotropic Material, *brittle*, *dense*

Which failure types (brittle or ductile) are observed ?

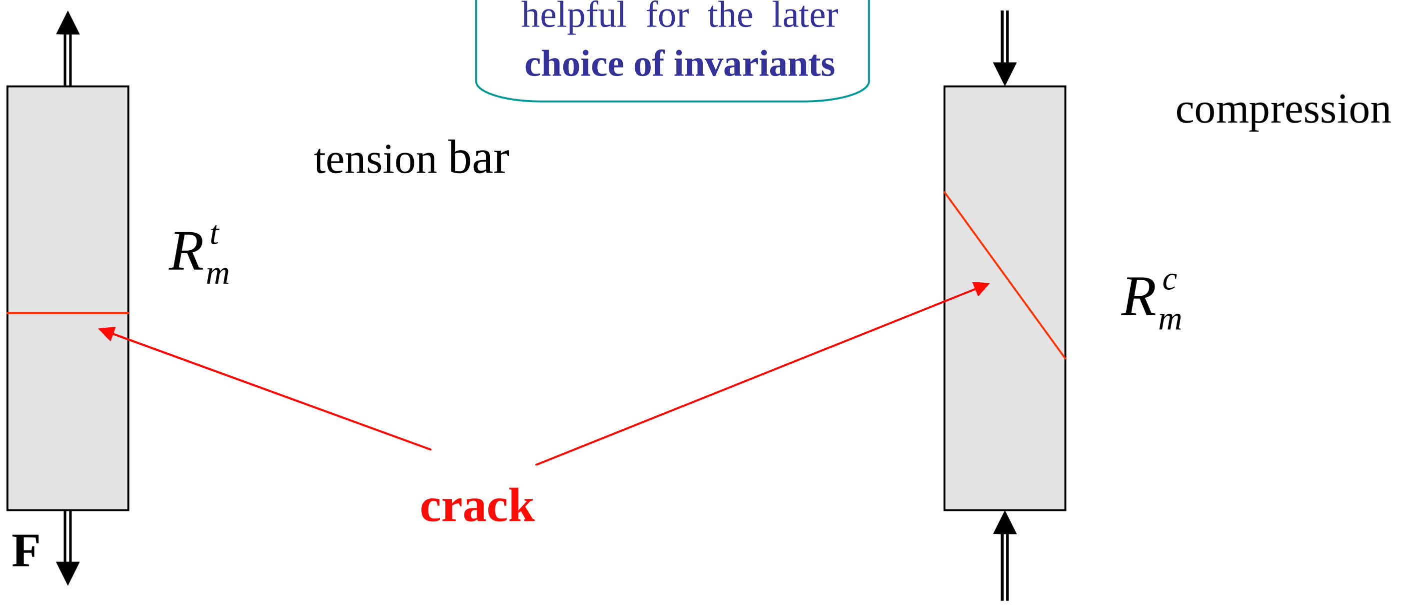
Cleavage fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- 'smooth' fracture surface

Shear fracture (SF) :

- shear deformation before fracture

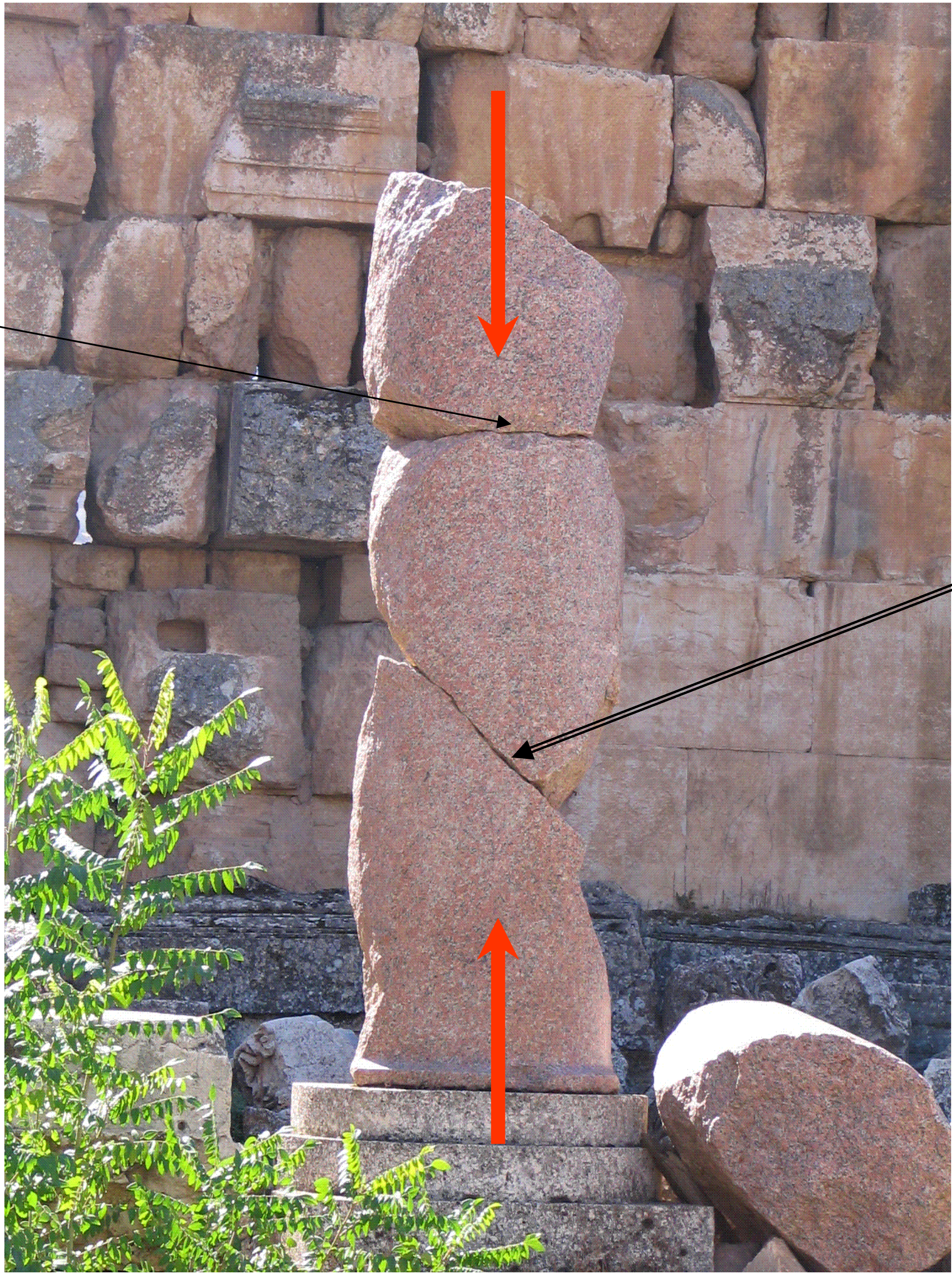
knowledge is helpful for the later **choice of invariants**



conclusion: ► **2 strengths** to be measured

if brittle: failure = fracture

just a joint



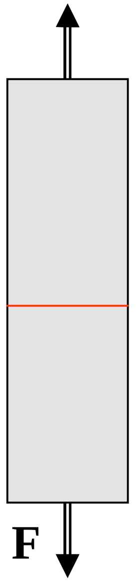
Example SF : R_m^c

Shear Fracture plane under compression

(**Mohr-Coulomb,** acting ***at a*** ***rock material*** *column,* *at Baalbek, Libanon)*

Normal Fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- rough fracture surface



Tension

R_m^t

Crushing Fracture (CrF): ← **SF**

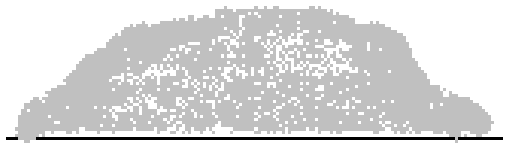
- volumetric deformation before fracture

helpful for the later **choice of invariants**

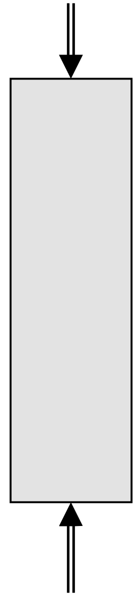
Compression

result of the compression test

= *hill of fragments* (crumbs)



= decomposition of texture



R_m^c

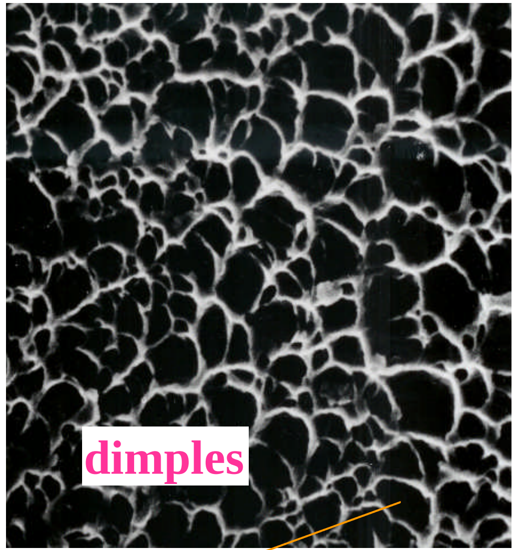
► **2 strengths** to be measured

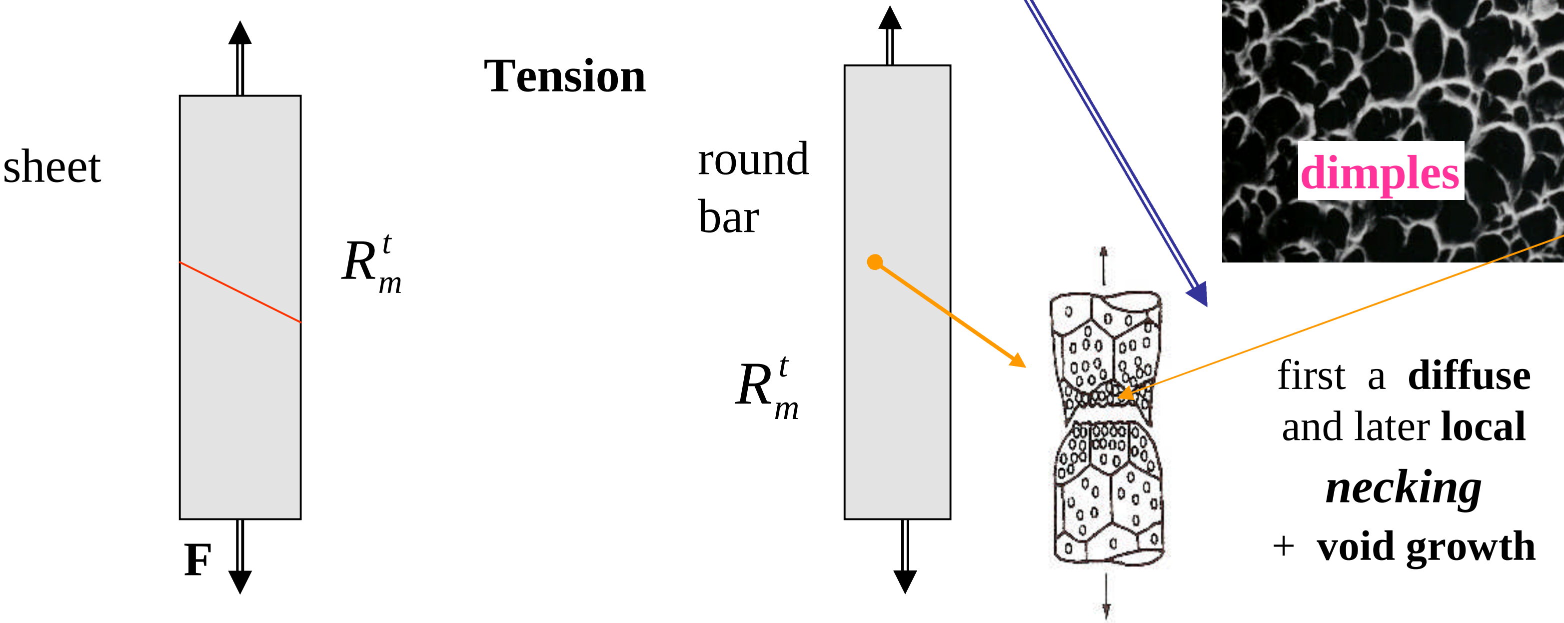
if brittle: failure = fracture failure

2.3c Observed Strength Failure Modes, Strengths - Isotropic Material, ductile, dense

Shear fracture (SF) :

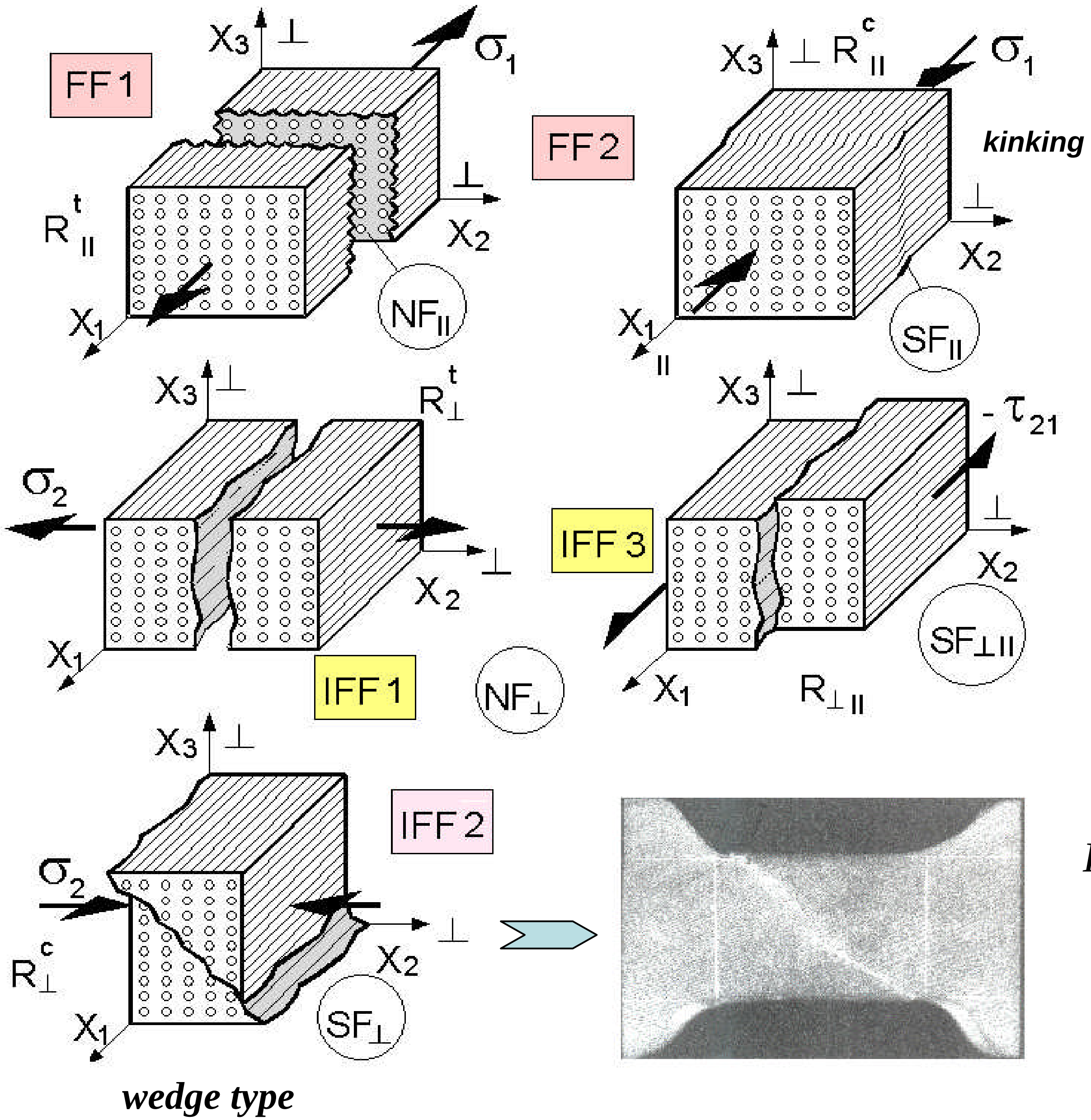
- *shear deformation* observed before fracture (maximum load)
- later in addition, *volume change* before rupture (‘Gurson domain’)
- dimples under tension.





►	**1 strength, R_m^t** to be measured **(=** ***load-controlled*** **value),**
•	**R_m^c is neither existing nor necessary for design ,** **$R_{c0.2}$ is the design driving strength.**

2.4 Observed Strength Failure Modes, Strengths - UD Material, brittle



t = tension
c = compression

- 5 Fracture modes exist

  = 2 FF (Fibre Failure)
  + 3 IFF (Inter Fibre Failure)

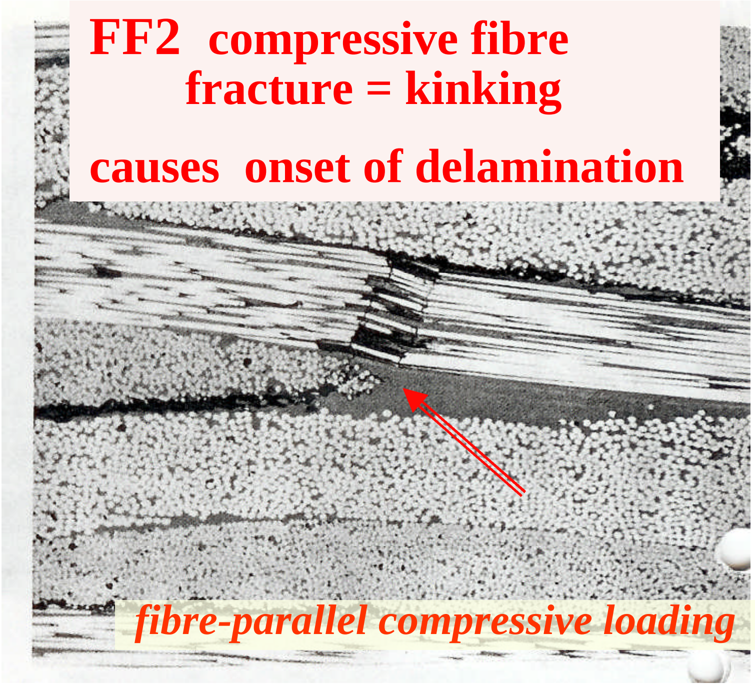
Fracture Types:

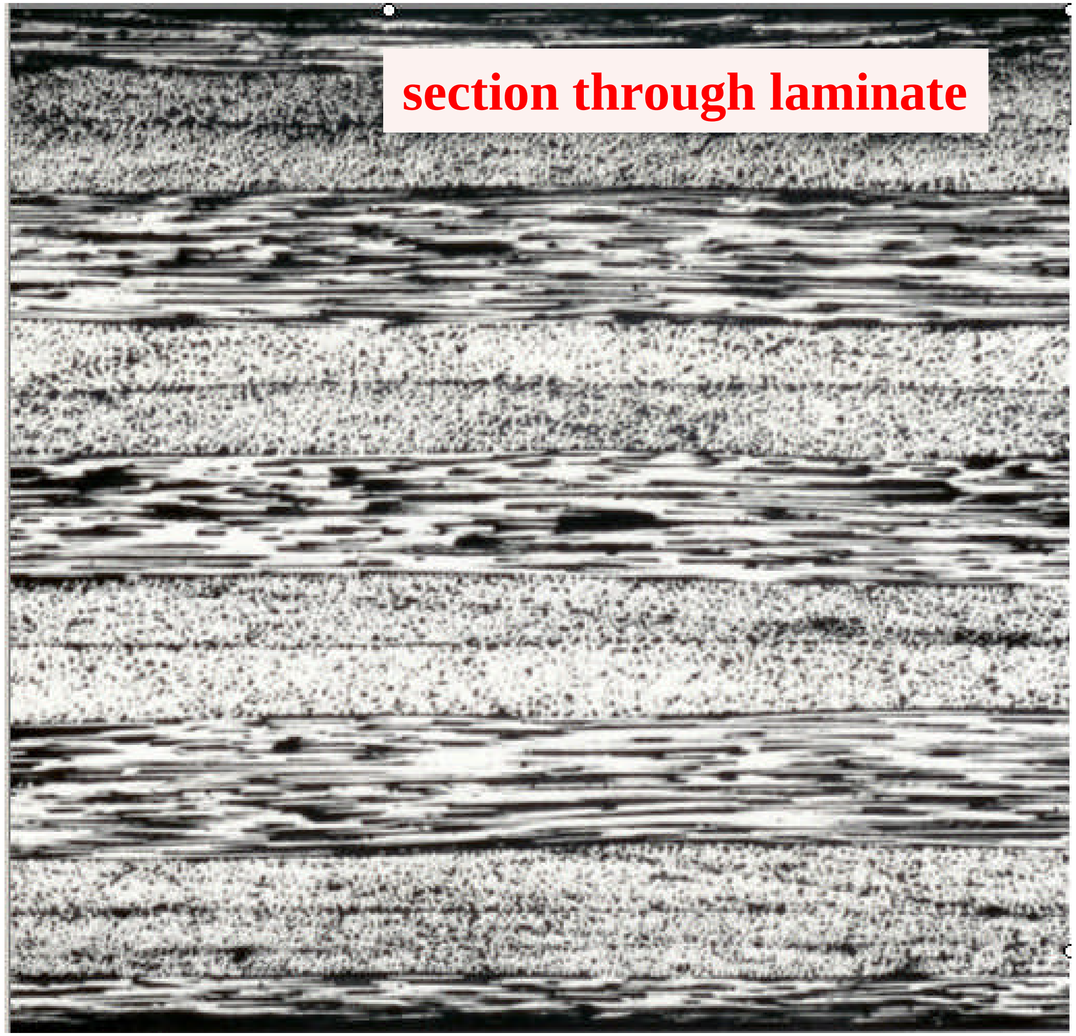
NF := Normal Fracture

SF := Shear Fracture

Strengths:

$$R_{\parallel}^{t}\,(=X^{t}),\;\; R_{\parallel}^{c}\,(=X^{c}),\;\; R_{\perp}^{t}\,(=Y^{t}),$$
$$R_{\perp}^{c}\,(=Y^{c}),\;\; R_{\perp\parallel}\,(=S)$$





FF1 tensile

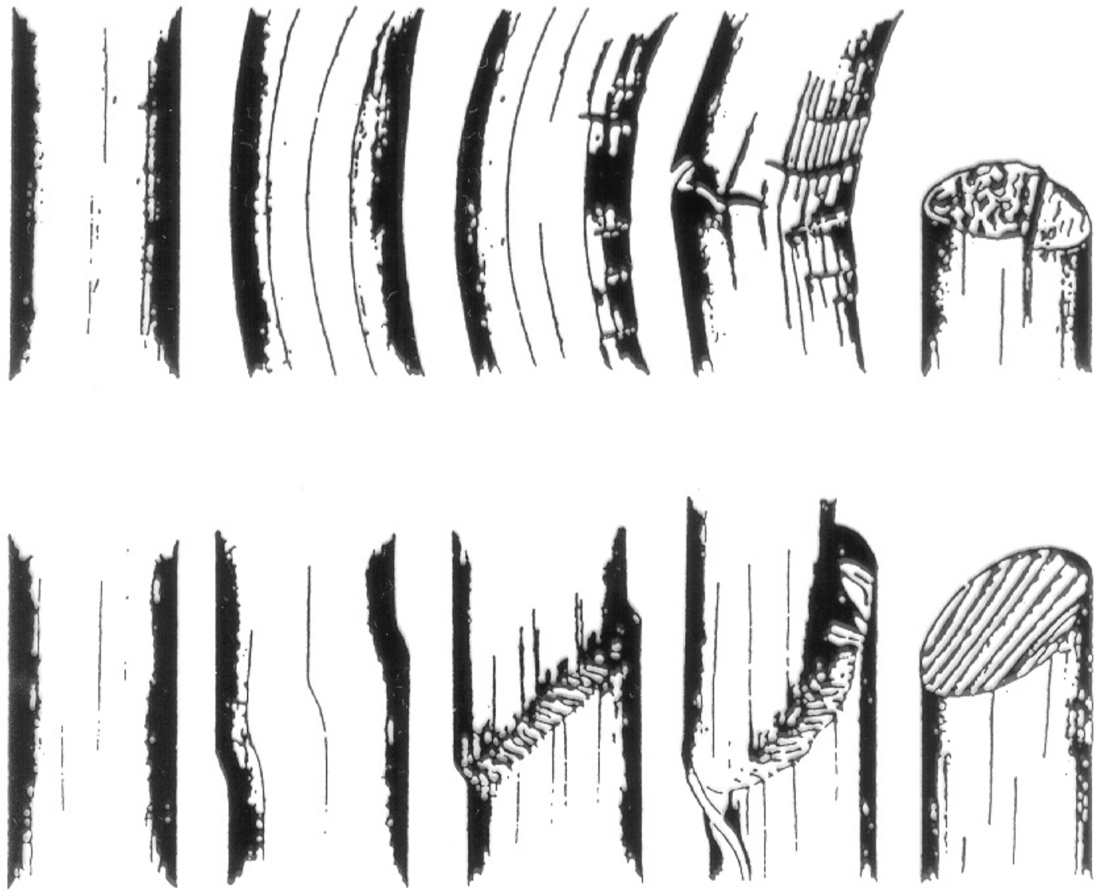
fibre fracture (pull-out)

Fractography pictures as proofs

Failure mechanisms of compressed carbon filaments

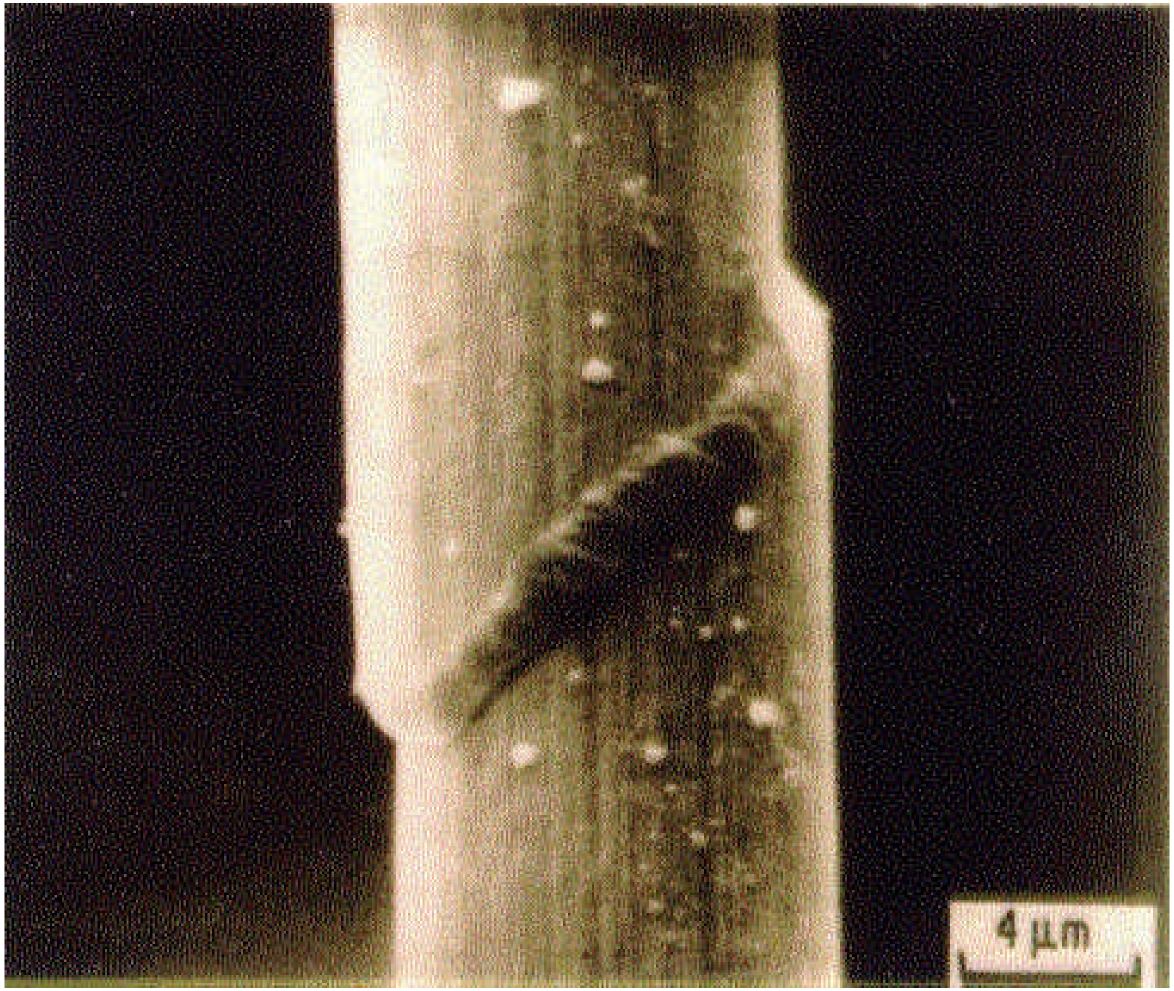
PAN

Mesophase
pitch



Courtesy: K. Schulte, TUHH

Compressed carbon pitch filament



Shear

Fracture

Shear band of a C-fibre (mesophase pitch) during compression.

Courtesy: K. Schulte, TUHH

Basics of the General Failure Mode concept (FMC)

2.1 Information available when generating UD Strength Failure Conditions

1 If a UD- material element can be homogenized to an ideal (frictionless) crystal,

then, material symmetry demands for this transversely-isotropic material

- *5 strengths*, 5 e*lastic ‘constants’ , etc.*
- *2 physical parameters (such as coefficients of thermal expansition, friction, .)*

2 Mohr-Coulomb requires for the real crystal another inherent parameter:

- the *physical parameter ‘material friction’* μ *value*

3 Fracture morphology witnesses:

- Each strength failure corresponds to a distinct farcture *failure mode* and to a *fracture type* as Normal Fracture (NF) or Shear Fracture (SF).

Therefore,
the FMC strictly employs single *independent* failure modes.

Formulations of Failure Conditions

Various Structural Materials

- Isn't it basically just *Beltrami* and *Mohr-Coulomb*? -- *Is there Some Common Basis existing ? -*

Hencky-
Mises-
Huber

Richard von Mises
1883-1953
Mathematician

Eugenio Beltrami
1835-1900
Mathematician

Otto Mohr
1835-1918
Civil Engineer

Charles de Coulomb
1736-1806
Physician

'Onset of Yielding'

'Onset of Cracking (fracture)'

2.7 Reasons for Chosing Invariants when generating Failure Conditions

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* Beltrami : "At 'Onset of Yielding' the material possesses a distinct ***strain energy*** composed of *dilatational energy* (I_1^2) and *distortional energy* (J_2≡*Mises*)".

* So, from Beltrami, Mises (HMH), and Mohr / Coulomb (friction) can be concluded:

Each invariant term in the *failure function* ***F*** may be dedicated to one physical mechanism in the solid = cubic material element:

	- volume change	: I_1^2	… ***(dilatational energy)***	I_1^2 , I_2^2
	- shape change	: J_2 (Mises)	… ***(distortional energy)***	I_3 , I_4
and	- friction	: I_1	… ***(friction energy)***	I_2

Stress Invariants: ***isotropic materials*** *and* ***: UD materials !***

Mohr-Coulomb

Remember:
These I_1 are different !

Short Derivation of the *Failure Mode Concept (FMC)*

3.1 Driving idea behind the FMC

A possibility exists to *more generally* formulate failure conditions

- **failure mode-wise** *(shear yielding etc.)* — **Mises, Hashin, Puck etc.**
- **stress invariant-based** *(J_2 etc.)* — **Mises, Tsai, Hashin, Christensen, etc.**

Cuntze‘s FMC considers both !

3.2 Introduction of the 'Material Stressing Effort' *Eff*

Material stressing effort = portion of load-carrying capacity of the material

Necessary for non-linear analyses

Each active failure mode contributes to the

global material stressing effort by its Eff^{mode}

Of course, accumulation of the mode efforts is to be performed,

to represent interaction, according to $Eff\,(Eff^{\text{modes}})$

Material stressing effort = Werkstoff-Anstrengung in German

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting failure danger portions Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots.} = \mathbf{1 = 100\%}, \textit{ if failure}$$

with mode-interaction exponent *2.5 < m < 3 from mapping experience*

and

modal material stressing effort

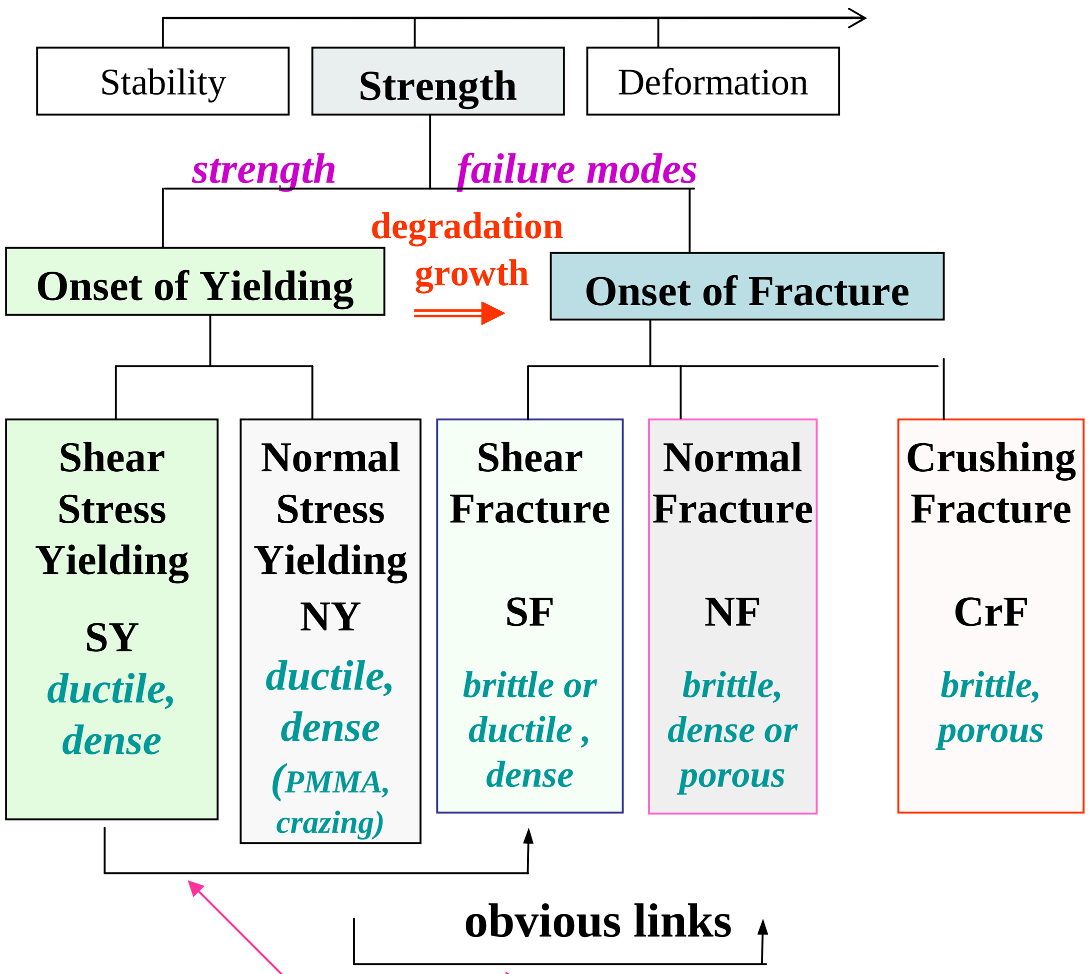
equivalent mode stress

mode associated average strength

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / \overline{R}^{\text{mode}}$$

Short Derivation of the *Failure Mode Concept (FMC)*

3.4 Scheme of Strength Failures for *isotropic materials*



The growing yield body (SY or NY)

is confined by the fracture surface (SF or NF)**!**

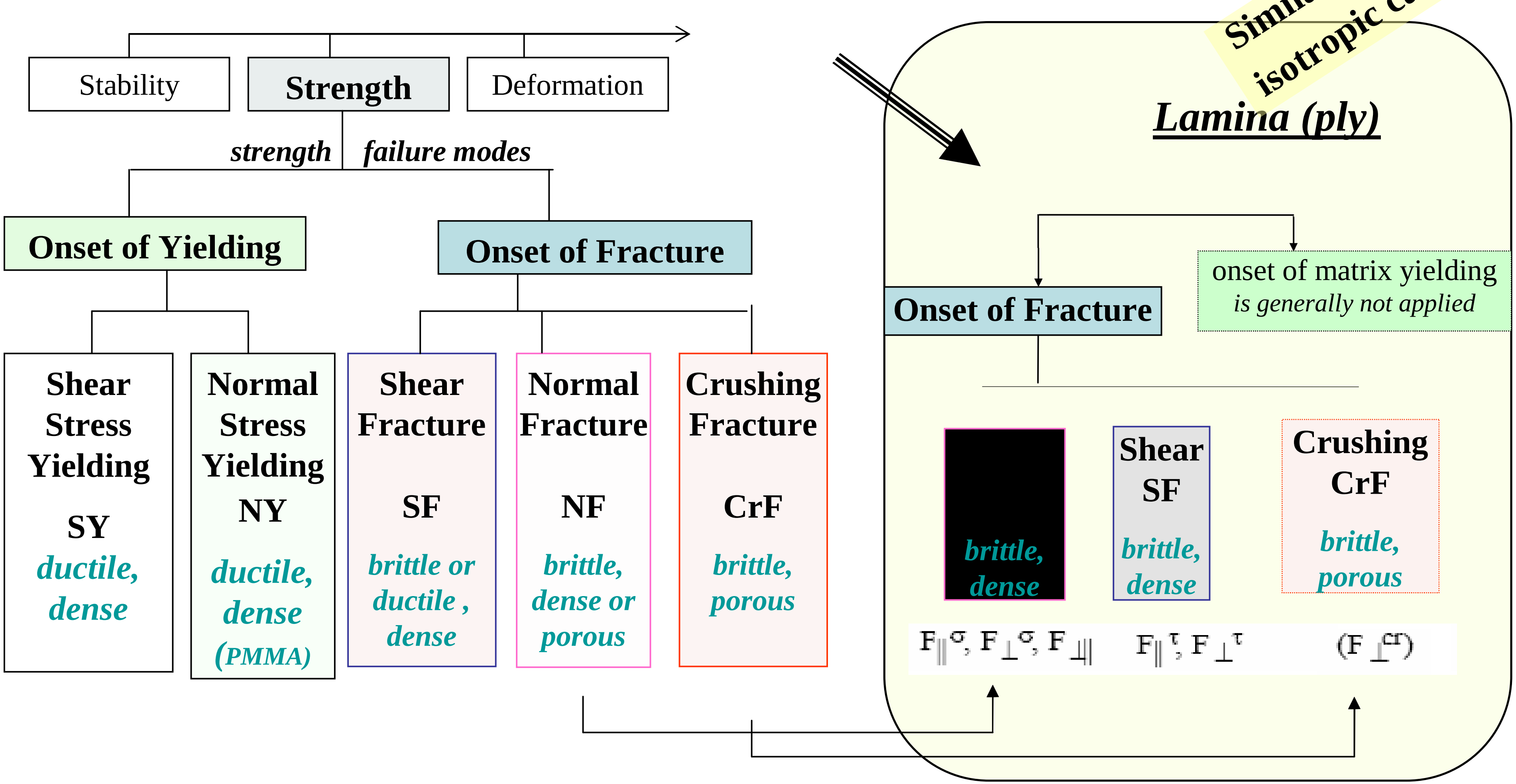
◄ = ***kinds of fracture***

Lesson learned from Mapping Test Data:

The same mathematical form of a failure condition holds - from 'onset of yielding' to 'onset of fracture' - if the physical mechanism remains !

Short Derivation of the *Failure Mode Concept (FMC)*

3.5 Scheme of Strength Failures for *the brittle UD laminae*



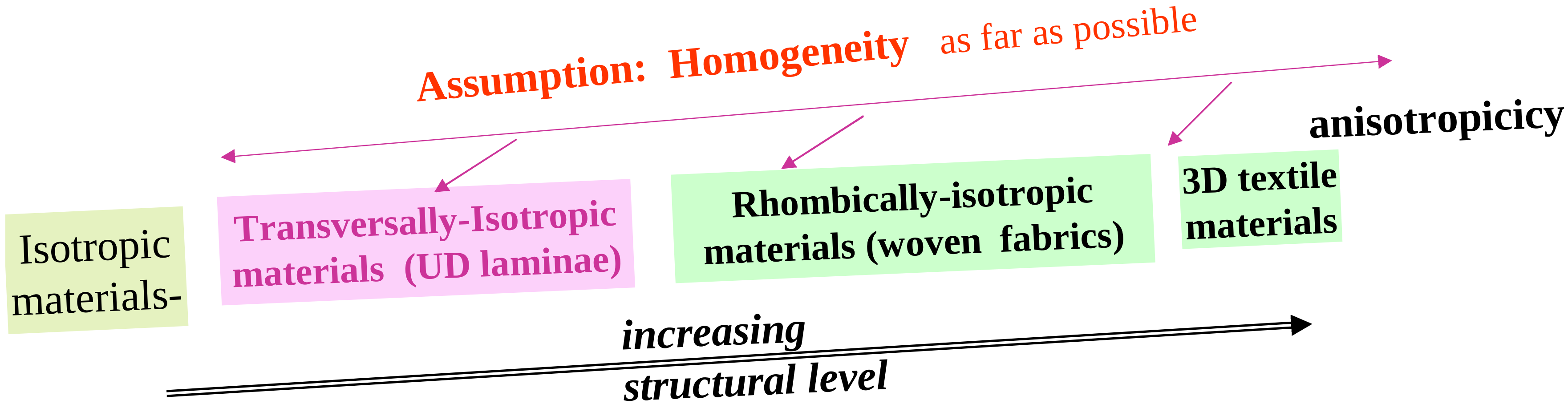
\+ **delamination failure of *laminate***

Lessons learned:

** There are coincidences between brittle UD laminae and brittle isotropic materials*

** Increased degradation occurs in the* laminate *beyond onset of the first IFF*

3.6 Material Homogenizing (smearing) + Modelling, Material Symmetry



Material symmetry shows:

Number of strengths ≡ number of elasticity properties !

Application of material symmetry:

- *Requires that homogeneity is a valid assessment for the <u>task-determined</u> model ,*

but, if applicable

- A *minimum number of properties has to be measured, only* (cost + time benefits) !

It's worthwhile to structure the establishment of strength failure conditions

3.7 Proposed Classification of Homogenized (assumption) Materials

A Classification helps to structure the Modelling Procedure:

Failure Type / Consistency	brittle, semi-brittle Design Ultimate Load	(quasi-) ductile Design Yield Load ← *design driving*
dense	fibre re-inforced plastics , mat, woven fabrics, grey cast iron, matrix material, amorphous glass C90-1,.	Glare, ARALL, metal alloys braided textiles
porous	foam, fibre re-inforced ceramics	sponge

failure: fracture — functional or usability limit

Conclusion:

Modelling, and Struct. Analysis + Design Verification strongly depend on material behaviour + consistency

4.1 Types of Strength Failure Conditions

1 Global ***strength*** **failure condition** :	**F ({σ}, {*R*}) = 1**	(usual formulation)
Set of Modal **strength failure conditions:**	**F ({σ}, R^{mode}) = 1**	(addressed in FMC)

Test data mapping : $R \Rightarrow \bar{R}$ average strength value (here addressed)

Design Verification : R strength design allowable,

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of stresses

$$\{R\} = (R_{\parallel}^t, R_{\parallel}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\parallel})^T$$

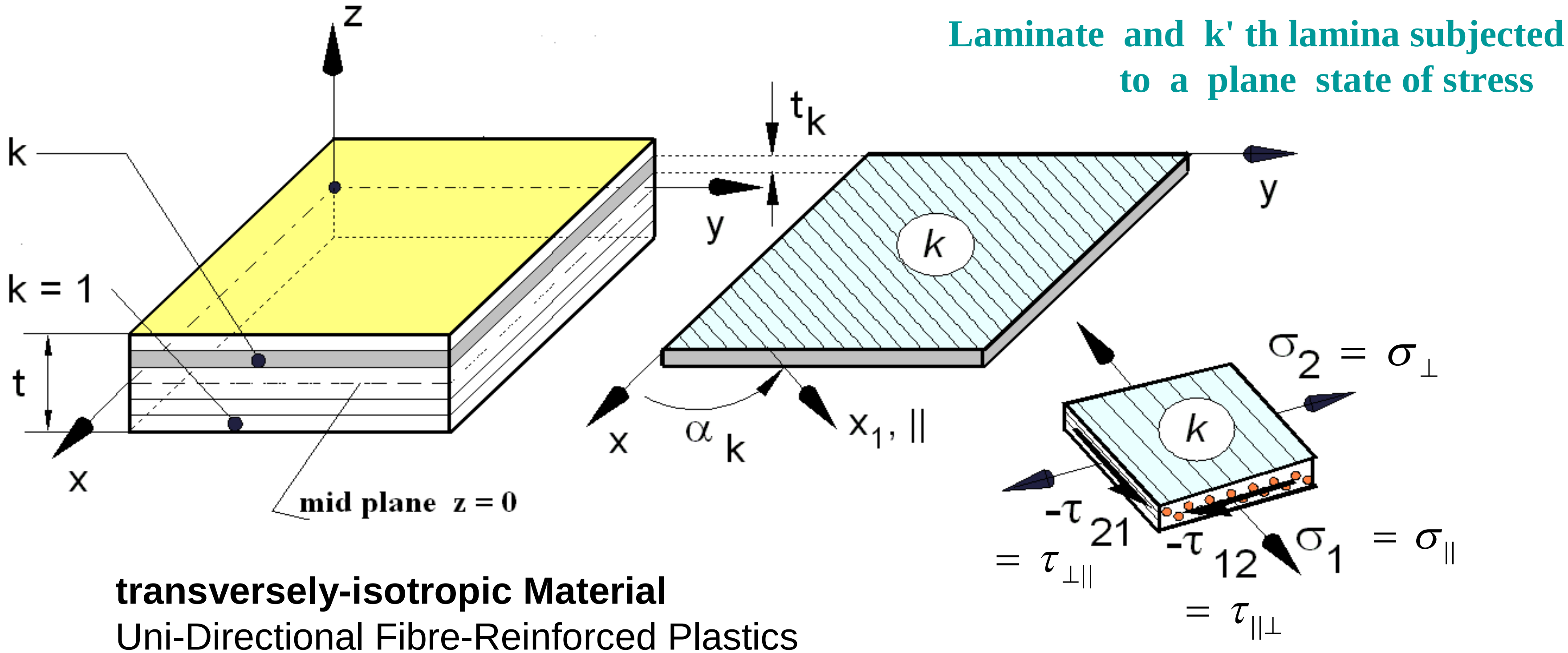
vector of strengths

Strength Failure Conditions are demanded to be :

- **simply formulated , numerically robust,**
- **physically-based, and therefore, need only few information during pre-dimensioning**
- **shall allow for a simple determination of the design driving (material) reserve factor**
- **ply-oriented in the case of UD composites.**

4.3 Stress State, Strengths, and Elasticity Properties of UD material

Lamina (ply) = homogenized (smeared) material = building block of the Laminate !

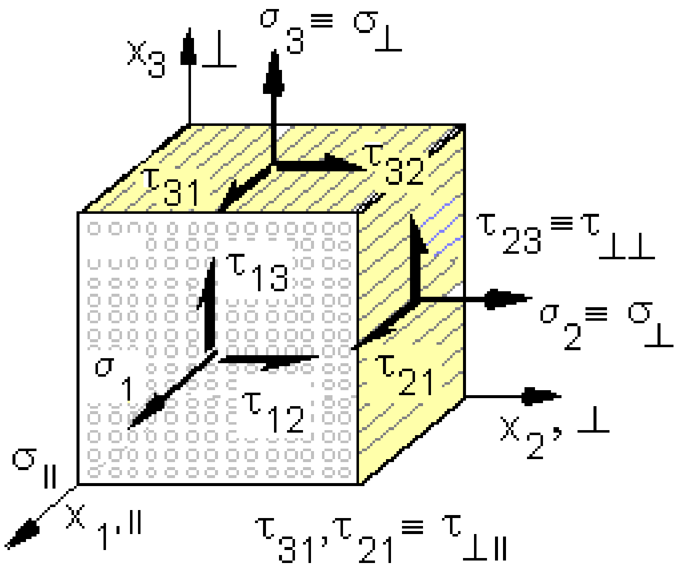


transversely-isotropic Material
Uni-Directional Fibre-Reinforced Plastics

5 strengths : $R_\|^t\,(=X^t),\;\; R_\|^c\,(=X^c),\;\; R_\perp^t\,(=Y^t),\;\; R_\perp^c\,(=Y^c),\;\; R_{\perp\|}\,(=S)$

5 elasticity properties : $E_\|, E_\perp, G_{\|\perp}, \nu_{\perp\|},\;$ (and $\nu_{\perp\perp}$, *if* 3D)

EN: Use of letter R required !



Lamina (ply) stress vector

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

'UD invariants'

$$I_1 = \sigma_1, \quad I_2 = \sigma_2 + \sigma_3,$$

$$I_3 = \tau_{31}^2 + \tau_{21}^2$$

$$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2$$

$$I_5 = (\sigma_2 - \sigma_3)(\tau_{31}^2 - \tau_{21}^2) - 4\tau_{23}\tau_{31}\tau_{21}$$

Next step: **Formulation of 5 invariant-described strength conditions** (not shown)

After:

* **replacement of the 5 'UD invariants' by the stresses, they are composed of, and**
* **some simplifications, and re-formulations to by-pass possible numerical problems**

above derived **FMC-based set of UD strength failure conditions** reads

FMC-based UD Strength Failure Conditions

4.5 Set of Modal 3D UD Strength Failure Conditions

[Cun04, Cun11]

FF1 $Eff^{\|\sigma} = \breve{\sigma}_1 / \bar{R}^t_\| = \sigma^{\|\sigma}_{eq} / \bar{R}^t_\|$, $\breve{\sigma}_1 \cong \varepsilon_1^t \cdot E_\|$ strains from FEA

FF2 $Eff^{\|\tau} = -\breve{\sigma}_1 / \bar{R}^c_\| = +\sigma^{\|\tau}_{eq} / \bar{R}^c_\|$, $\breve{\sigma}_1 \cong \varepsilon_1^c \cdot E_\|$

2 filament modes

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2}\,]/2\bar{R}^t_\perp = \sigma^{\perp\sigma}_{eq} / \bar{R}^t_\perp$

IFF2 $Eff^{\perp\tau} = [(b_{\perp\perp} - 1)\cdot(\sigma_2 + \sigma_3) + b_{\perp\perp}\sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2}\,] / \bar{R}^c_\perp = +\sigma^{\perp\tau}_{eq} / \bar{R}^c_\perp$

3 matrix modes

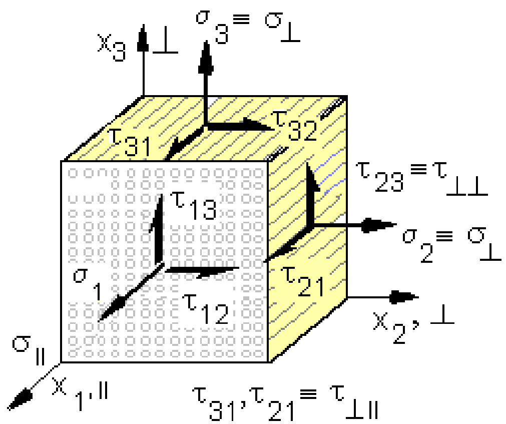
IFF3 $Eff^{\perp\|} = \{[b_{\perp\|} \cdot I_{23-5} + (\sqrt{b_{\perp\|}^2 \cdot I_{23-5}^2 + 4 \cdot \bar{R}_{\perp\|}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,]/(2 \cdot \bar{R}_{\perp\|}^3))\}^{0.5} = \sigma^{\perp\|}_{eq} / \bar{R}_{\perp\|}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

Modes-Interaction :

with mode-interaction coefficient $2.5 < m < 3.1$ **from mapping test data**

$$Eff^{m} = (Eff^{\|\tau})^m + (Eff^{\|\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$



Typical friction value data range:

$$0.05 < \mu_{\perp\|} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$$

$$b_{\perp\|} = \mu_{\perp\|}, \quad b_{\perp\perp} \cong 1/(1 - \mu_{\perp\perp})$$

4.6 Pre-design Input for 3D FMC-based Strength Failure Conditions

	Test Data Mapping	Design Verification
• **5 strengths :**	$\{\bar{R}\} = (\bar{R}_{\parallel}^{t}, \bar{R}_{\parallel}^{c}, \bar{R}_{\perp}^{t}, \bar{R}_{\perp}^{c}, \bar{R}_{\perp\parallel})^{T}$	$\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^{T}$
	average (typical) values	**strength design allowables**

• **2 friction values :** $\mu_{\perp\parallel}$ **for 2D ,** $\mu_{\perp\parallel}, \ \mu_{\perp\perp}$ **for 3D**

$\mu_{\perp\parallel} = 0.1$ $\qquad \mu_{\perp\perp} = 0.1$

• **1 mode-interaction coefficient :** *m = 2.6 .*

values, recommended for pre-design

Benefits of these <u>modal</u> strength failure conditions :

* No more input required than for the usually applied *<u>global</u> strength failure conditions* (such as Tsai-Wu) **!**

* Have not the draw-backs of the global conditions that do not use the physically necessary friction !

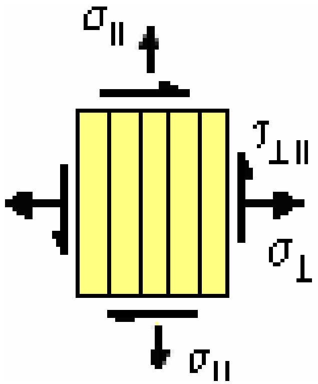
4.7 Application to a 2D Stress State

$$\{\sigma\} = (0, \sigma_2, 0, 0, 0, \tau_{21})^T$$

UD E-glass/LY556/T976/DY070

Hoop wound tube

loading situation



***FF 1*:** $$\frac{\varepsilon_1 \cdot E_{\|}}{\bar{R}^t_{\|}} = 1,$$

***FF 2*:** $$\frac{-\varepsilon_1 \cdot E_{\|}}{\bar{R}^c_{\|}} = 1,$$

***IFF 1*:** $$\frac{\sigma_2}{\bar{R}^t_{\perp}} = 1$$

***IFF 2*:** $$\frac{-\sigma_2}{\bar{R}^c_{\perp}} = 1$$

***IFF 3 (2D)*:** $$\frac{|\tau_{21}|}{\bar{R}_{\perp\|} - \mu_{\perp\|} \cdot \sigma_2} = 1$$

simplified 2D approach:

$$m = 2.5, \quad \mu_{\perp\|} = 0.3$$

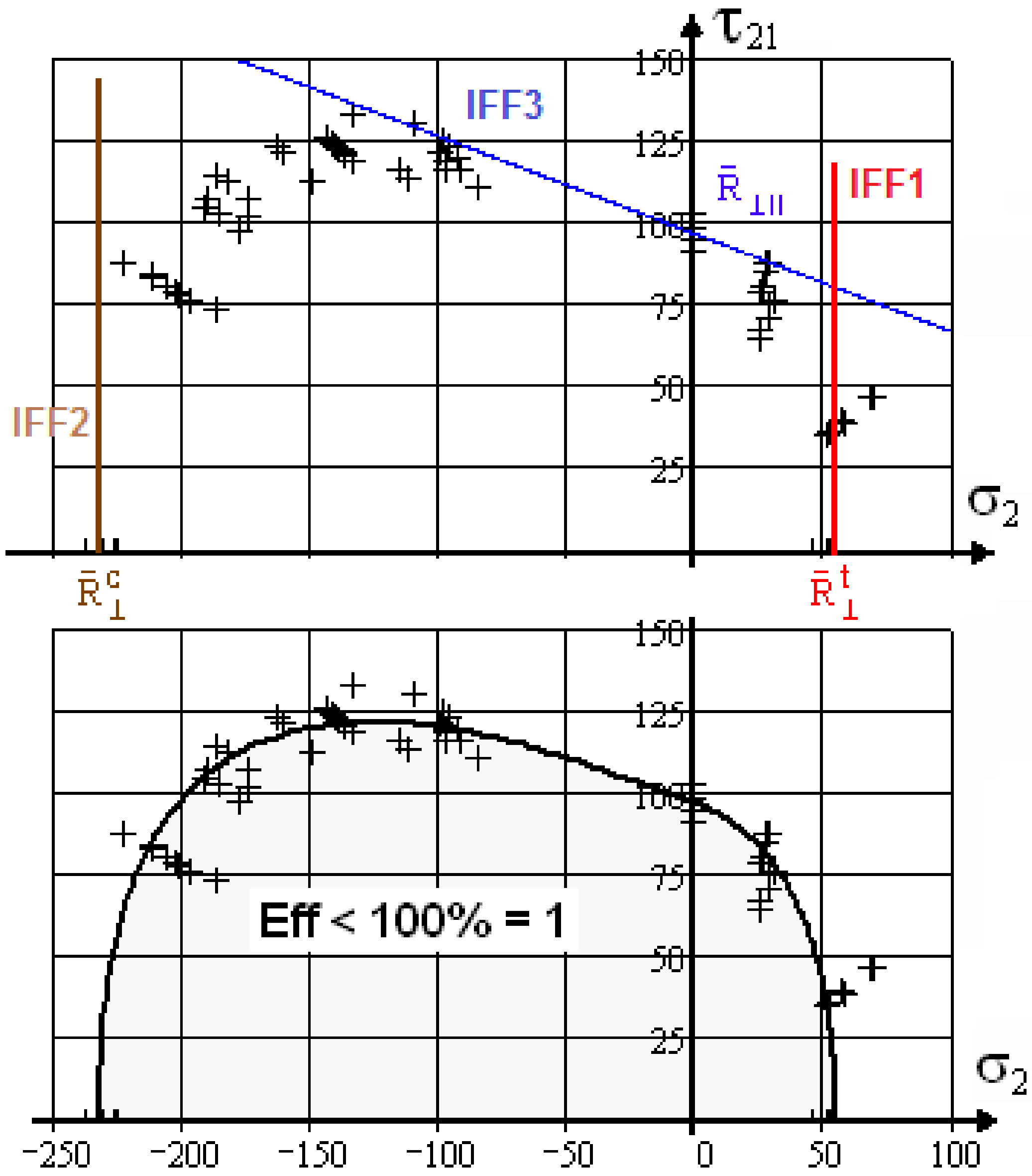
= 2 FF + 3 IFF = 5 UD (material) failure modes

FMC-based UD Strength Failure Conditions

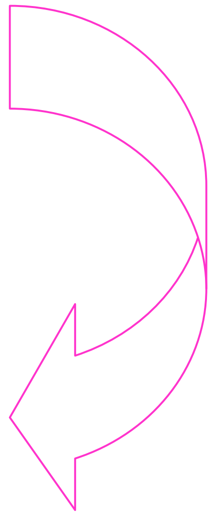
4.8 Visualization of Failure Modes Interaction

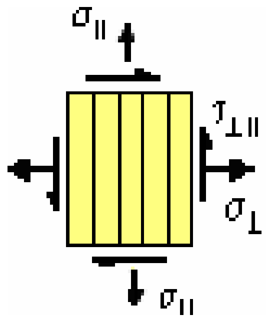
$$\breve{\sigma}_1 = 0$$

$$\tau_{21}(\sigma_2) \text{ or } \{\sigma\} = (0, \sigma_2, 0, 0, 0, \tau_{21})^T$$



Mapping of course of IFF test data in a pure mode domain by the *single Mode Failure Condition.* *3 IFF pure modes = straight lines !.*

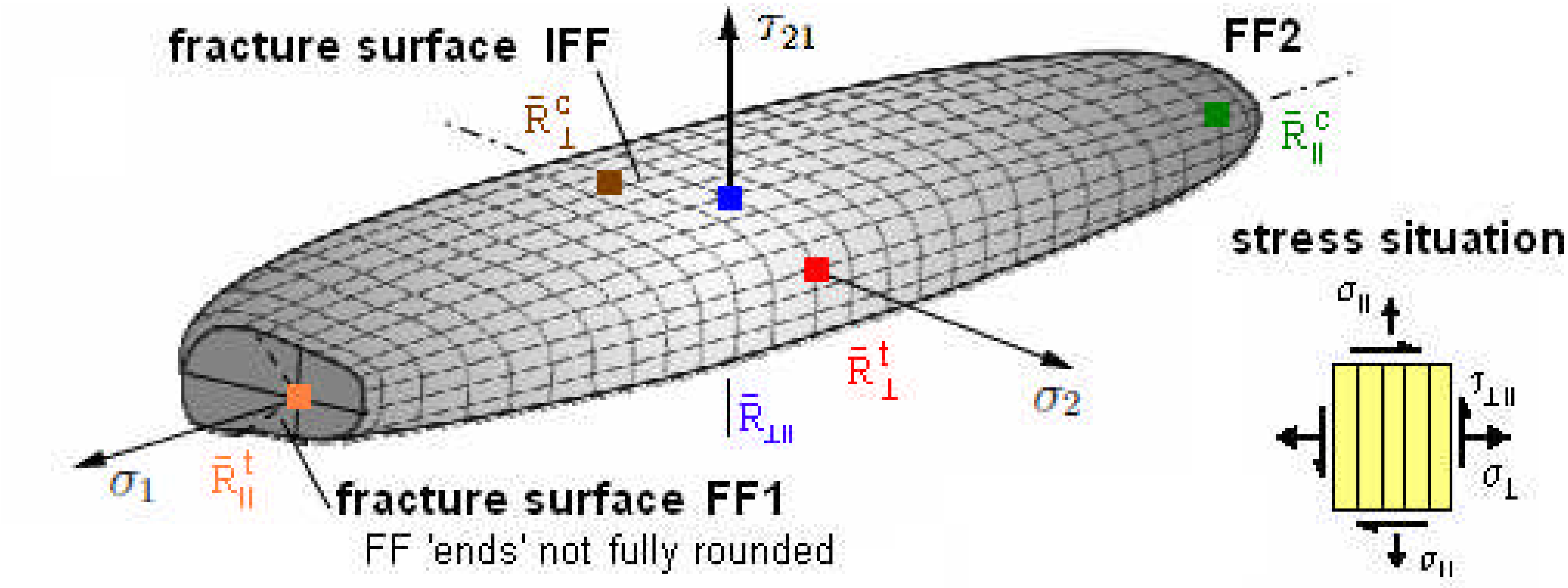




Mapping of course of test data by *Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

4.9 Visualization of Set of FMC-based 2D Strength Failure Conditions



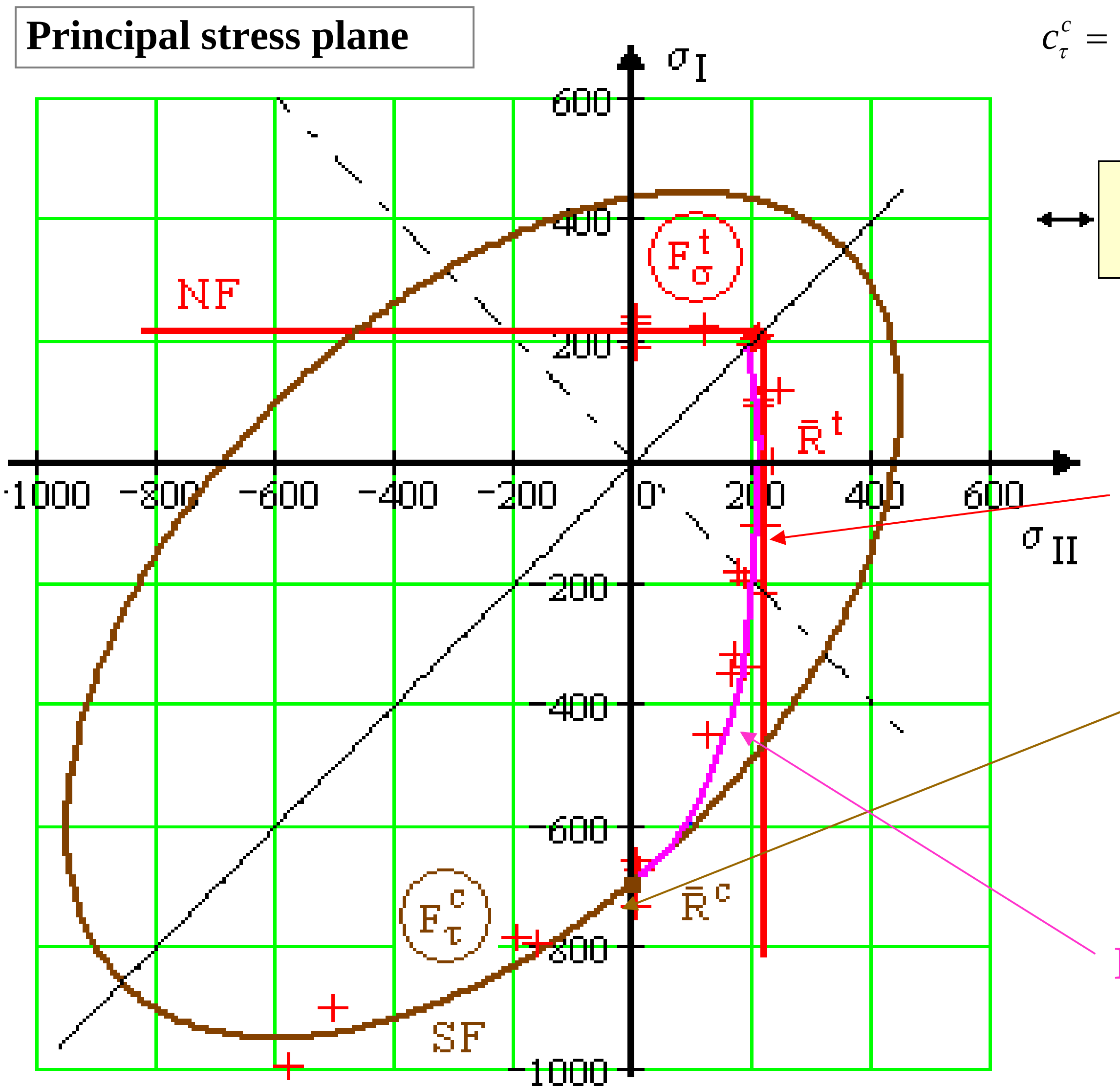
$$\{\sigma\} = (\sigma_1, \sigma_2, 0, 0, 0, \tau_{21})^T$$

Mode interaction fracture failure surface *of FRP UD lamina (courtesy W. Becker) . Mapping: Average strengths indicated*

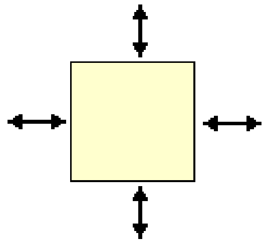
$$Eff^{\,m} = (Eff_{\parallel}^{\tau})^m + (Eff_{\parallel}^{\sigma})^m + (Eff_{\perp}^{\sigma})^m + (Eff_{\perp}^{\tau})^m + (Eff_{\perp\parallel})^m = 1$$

Application to Static Testv Data of Various Materials

5.1 Grey Cast Iron (brittle, dense, microflaw-rich), *Principal stress plane*



$c_\tau^c = a_\tau^c - 1, \quad a_\tau^c = 1.58 \qquad m = 3.1 \qquad \bar{R}^t = 215\,MPa\,; \quad \bar{R}^c = 690\,MPa$



$$\{\sigma\}_{principal} = (\sigma_I\,,\ \sigma_{II}\,,\ 0)^T$$

$$F_\sigma^t = \frac{\sqrt{I_\sigma} + I_1}{2 \cdot \bar{R}^t} = 1 \quad \text{deformationless}$$

$$F_\tau^c = a_\tau^c \frac{3J_2}{\bar{R}^{c2}} + c_\tau^c \frac{I_1}{3\bar{R}^c} = 1$$

shear change   friction

= 2 Mode Failure Conditions

Interaction zone

Lessons learned: Basically, *Dense concrete and Glass C 90 will have same failure condition*

Application to Static Testv Data of Various Materials

5.2a Concrete (isotropic, slightly porous) *Kupfer's data*

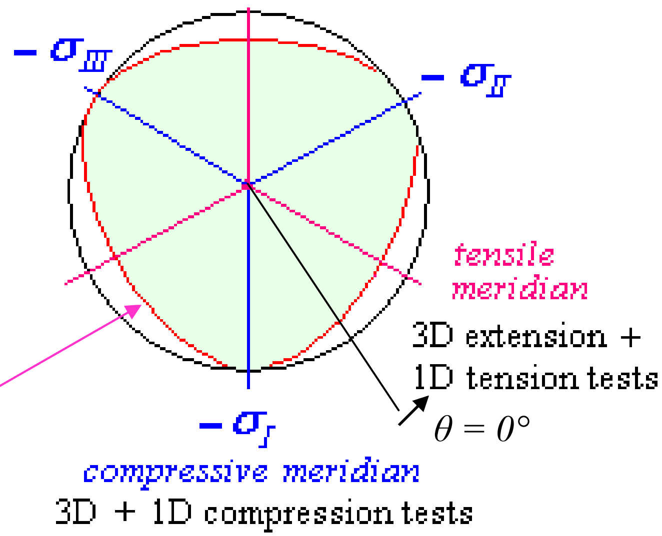
Octahedral stresses (B-B view)

$$F_\sigma^t = \frac{\sqrt{I_\sigma} + I_1}{2\overline{R}_m^t} = Eff_\sigma^t = 1$$

deformation poor

hyperbola

Isotropic materials possess 120° symmetry :



shape + volume change + friction: Mohr-Coulomb :

$$F_\tau^c = a_\tau^c \frac{3J_2 \cdot \Theta}{\overline{R}_m^{c\,2}} + b_\tau^c \frac{I_1^{\,2}}{\overline{R}_m^{c\,2}} + c_\tau^c \frac{I_1}{\overline{R}_m^c} = 1$$

(closed failure surface)

paraboloide

Lessons learned from test data viewing**:**

- *Course of concrete test data shows a big bandwidth*
- *The reason for the bandwidth is not only the test scatter but the stress-state dependent 'double' failure probability causing non-coaxiality in the octahedral plane. The difference between the so-called tensile (extension) meridian and the compression meridian is to be considered.*

Basically, the differences in the octahedral (deviatoric) plane can be described by :

$$\Theta \Rightarrow \sqrt[3]{1 + d \cdot \sin(3\theta)}$$

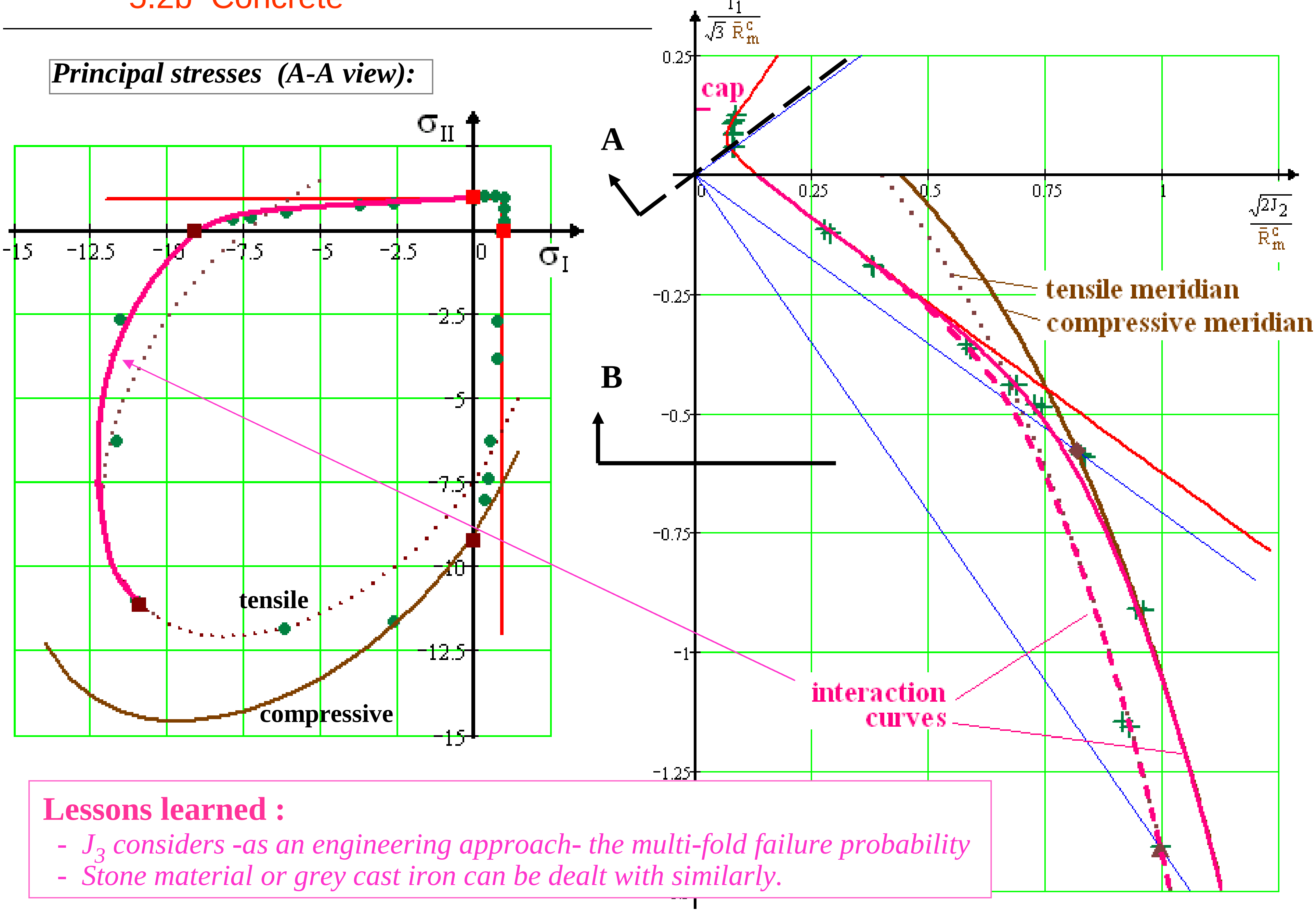
$$\sin(3\theta) = 3\sqrt{3} J_3 / (2 J_2^{\,3/2}),$$

[de Boer, et al] $d \le 0.5$, convex

Remark Cuntze: J_3 practically describes the effect of the doubly acting failure mode, no relation to new special mechanism.

Application to Static Testv Data of Various Materials

5.2b Concrete



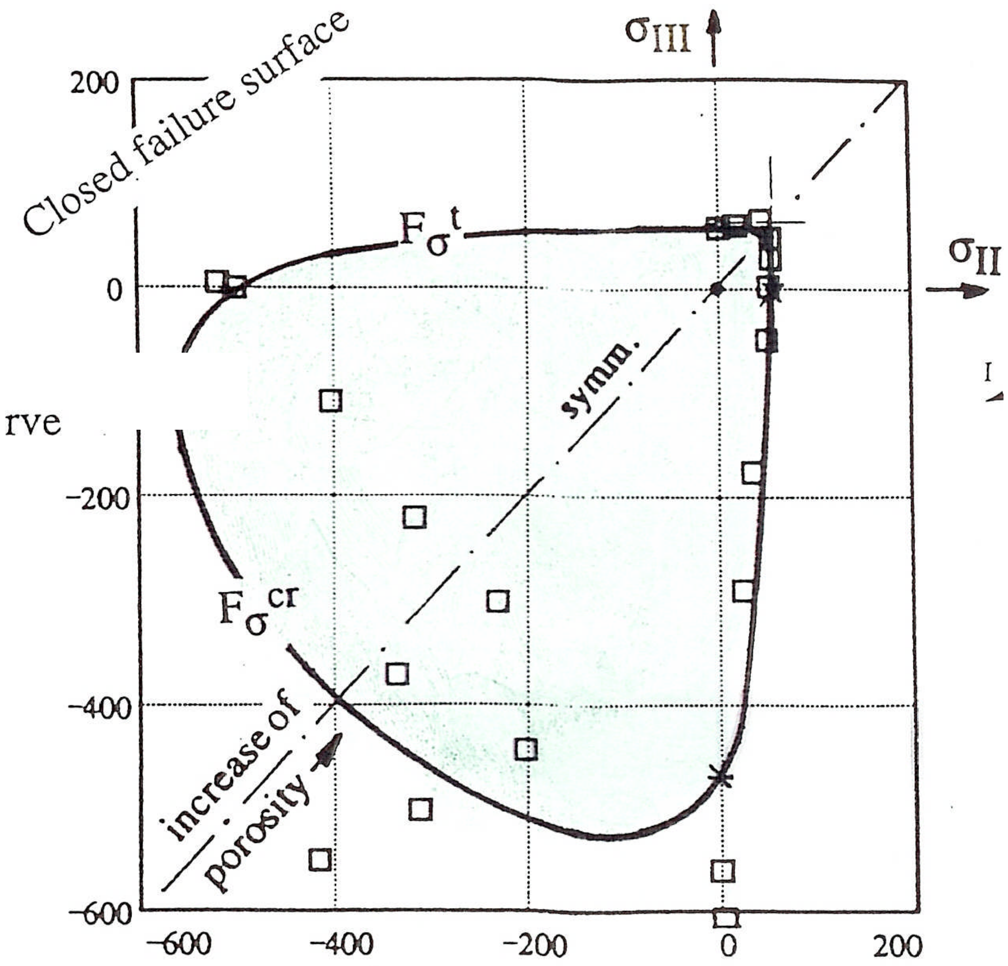
Lessons learned :

- *J_3 considers -as an engineering approach- the multi-fold failure probability*
- *Stone material or grey cast iron can be dealt with similarly.*

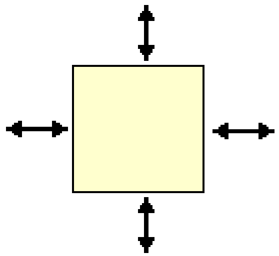
Application to Static Testv Data of Various Materials

5.3 Monolithic Ceramics (brittle, porous isotropic material)

Principal stress plane



$c^{cr} = a^{cr} - 1$ ***[Kowalchuk]***



$$F_\sigma^t = \frac{\sqrt{I_\sigma} + I_1}{2 \cdot \bar{R}^t} = 1$$

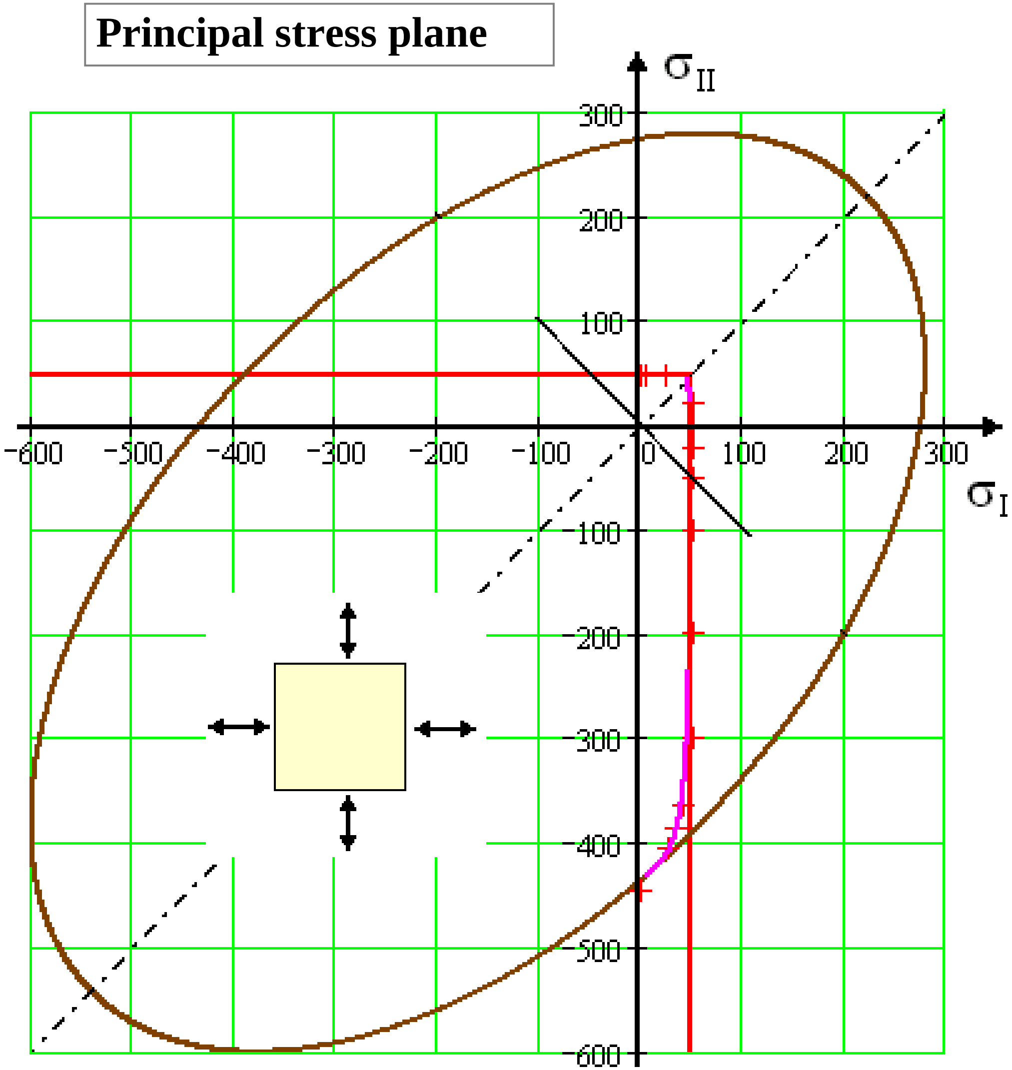
$$F^{cr} = a^{cr} \frac{3J_2}{\bar{R}_m^{c\,2}} + \; + c^{cr} \left(\frac{I_1}{\bar{R}_m^c} \right)^2 = Eff^{cr} = 1$$

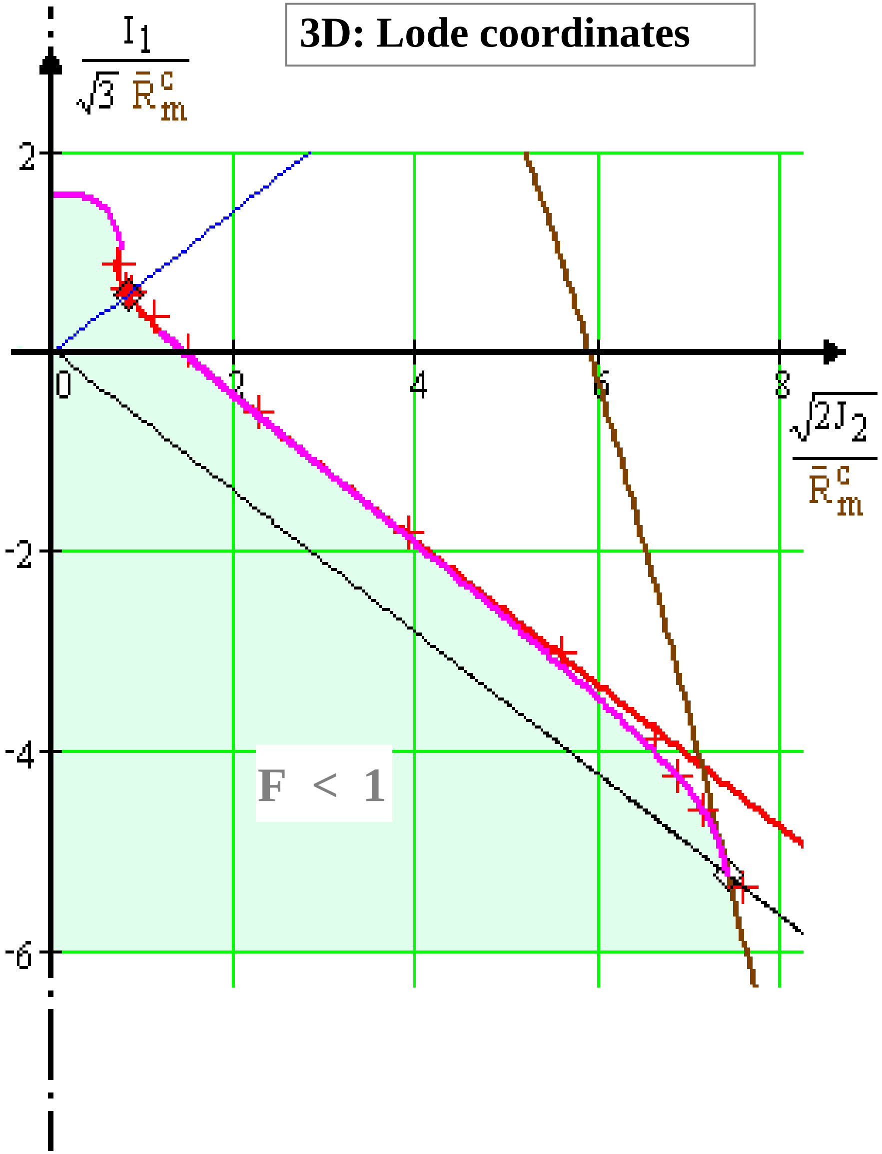
shear change **volume change**

Lessons learned: ***Same failure condition as very porous concrete***

Application to Static Testv Data of Various Materials

5.4 Glass C 90 (brittle, dense isotropic material)



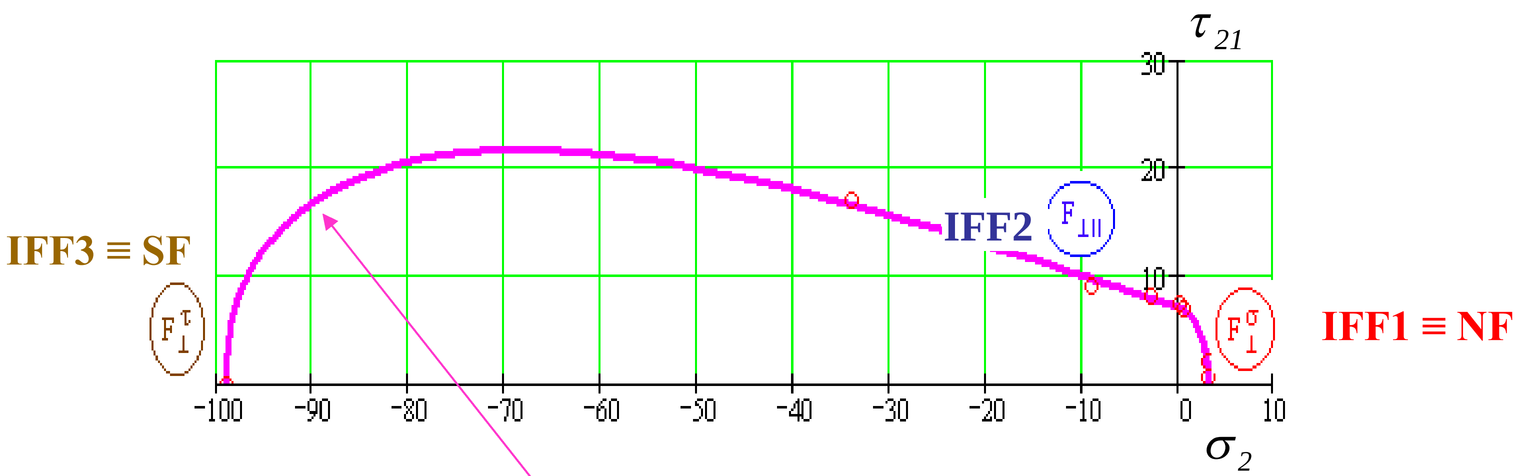


Application to Static Test Data of Various Materials

5.5 UD Ceramic Fibre-Reinforced Ceramics (C/C) (brittle, porous, tape)

$$\{\bar{R}\} = (\bar{R}_{\parallel}^{t},\ \bar{R}_{\parallel}^{c},\ \bar{R}_{\perp}^{t},\ \bar{R}_{\perp}^{c},\ \bar{R}_{\perp\parallel}) = (-,\ -,\ 3,\ 99,\ 7)^{T},\ m = 2.3,\ \mu_{\perp\parallel} = 0.3$$

[Diss. B. Thielicke, 1997]

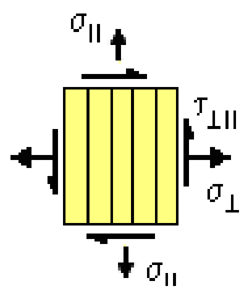


$F_{\perp}^{\sigma}$ $F_{\perp\parallel}$ $F_{\perp}^{\tau}$

Interaction equation :

$$\left(\frac{\sigma_2}{\bar{R}_{\perp}^{t}}\right)^m + \left(\frac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel}\cdot\sigma_2}\right)^m + \left(\frac{-\sigma_2}{\bar{R}_{\perp}^{c}}\right)^m = 1$$

deformationless — friction (Mohr-Coulomb) — shear

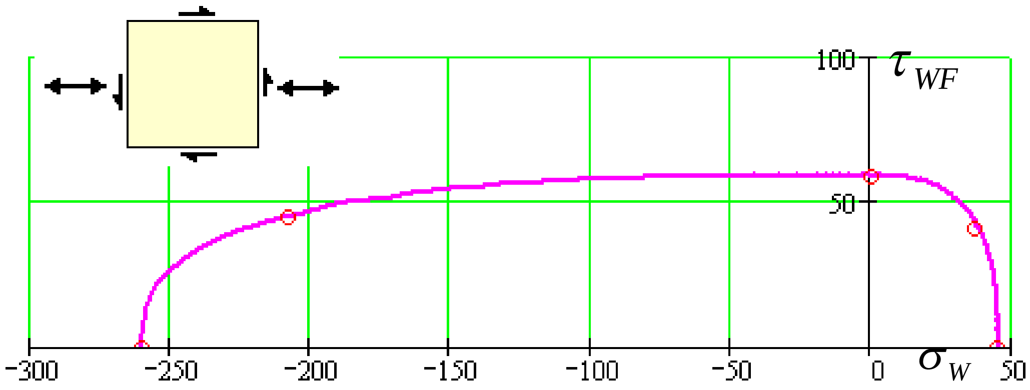


Invariants applied: I3, I2 — I4 , I2

Lesson learned: ***Same failure condition as with UD-FRP***

Application to Static Test Data of Various Materials

5.6 Fabric Ceramic Fibre-Reinforced Ceramics (CFRC) (brittle, porous)



C/C-SiC, T= 1600°C

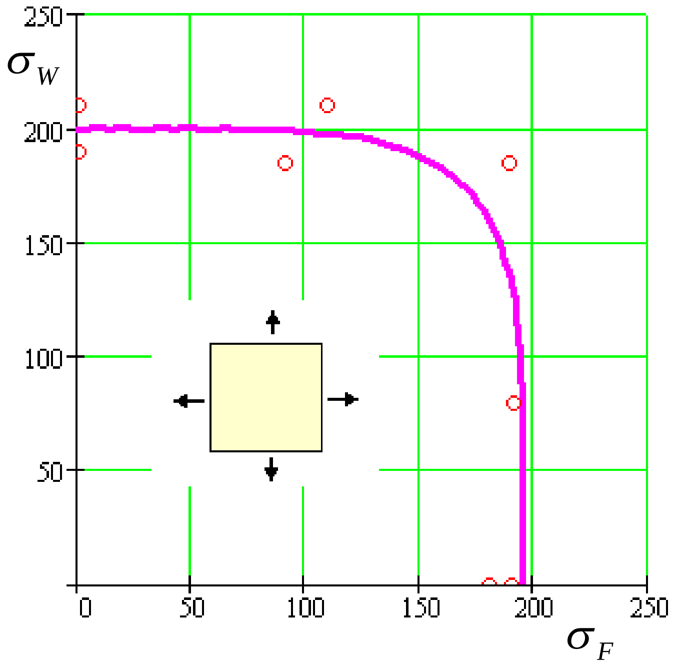
[Geiwitz/Theuer/Ahrendts 1997] ,

*tension/compression-torsion-**tube??***

$$\{\overline{R}\} = (\overline{R}_{||}^t, \overline{R}_{||}^c, \overline{R}_{\perp}^t, \overline{R}_{\perp}^c, \overline{R}_{\perp||}) = (-, -, 45, 260, 59)^T$$

$m = 2.8$

$$\left(\frac{\sigma_W}{\overline{R}_W^t}\right)^m + \left(\frac{-\sigma_W}{\overline{R}_W^c}\right)^m + \left(\frac{\tau_{WF}^2}{\overline{R}_{WF}^2}\right)^m = 1$$



$$\{\overline{R}\} = (\overline{R}_W^t, \overline{R}_W^c, \overline{R}_F^t, \overline{R}_F^c, \overline{R}_{WF}, \overline{R}_3^t, \overline{R}_3^c, \overline{R}_{3F}, \overline{R}_{3W})^T$$

$\{\overline{R}\}$ = *vector of mean strength values*

C/SiC, ambient temperature *[MAN-Technologie, 1996],*

tension/tension tube

$$\{\overline{R}\} = (200, -, 195, -, -,,,,)^T, \; m = 5$$

$$\left(\frac{\sigma_W}{\overline{R}_W^t}\right)^m + \left(\frac{\sigma_F}{\overline{R}_F^t}\right)^m = 1$$

NOTE: For woven fabrics enough test information for a real validation is not yet available!

The World-Wide-Failure-Exercises

Organizer‘s (QinetiQ , UK) Objective: ***‘Testing Failure Theories to the full !‘***

Structure of the World-Wide-Failure-Exercises :

Part A of a WWFE: ***Predictions*** on provided strength data, only

Part B of a WWFE: ***Comparison Theory-Test*** with Failure Stress test data‘
Here addressed, only.

WWFE-I: 2D Test Data, provided for 14 Test Cases

WWFE-II: 3D Test Data, provided for 12 Test Cases

Organizer‘s (QinetiQ , UK) Objective: ‘Testing Failure Theories to the full !‘

Parts of a Failure Theory :

1) Strength Failure Conditions *(can be validated by UD test data sets, only)*

2) Use of stress-strain curve in hardening and softening *(after IFF)* domain

3) Analysis program that tackles non-linear laminate behaviour.

Structure of the World-Wide-Failure-Exercises :

Part A of a WWFE: ***Predictions*** on provided strength data, only

Part B of a WWFE: ***Comparison Theory-Test*** with Failure Stress test data‘.

WWFE-I: 2D Test Data, provided for 14 Test Cases

TC1-TC3 ***UD lamina : (multi-axial) failure stress envelopes***

TC4-TC14 ***endless fibre-reinforced Laminates***
(quasi-isotropic, angle-ply, cross-ply):
failure stress envelopes *and* ***stress-strain curves .***

WWFE-II: 3D Test Data, provided for 12 Test Cases *involving hydrostatic pressures up to > 10000 bar = 1000 MPa*

TC1 ***epoxide matrix,***

TC2-TC7 ***UD lamina***

TC8-TC12 ***laminates*** **.**

6.3 Introduction to Problems with Provided Part B Test Data

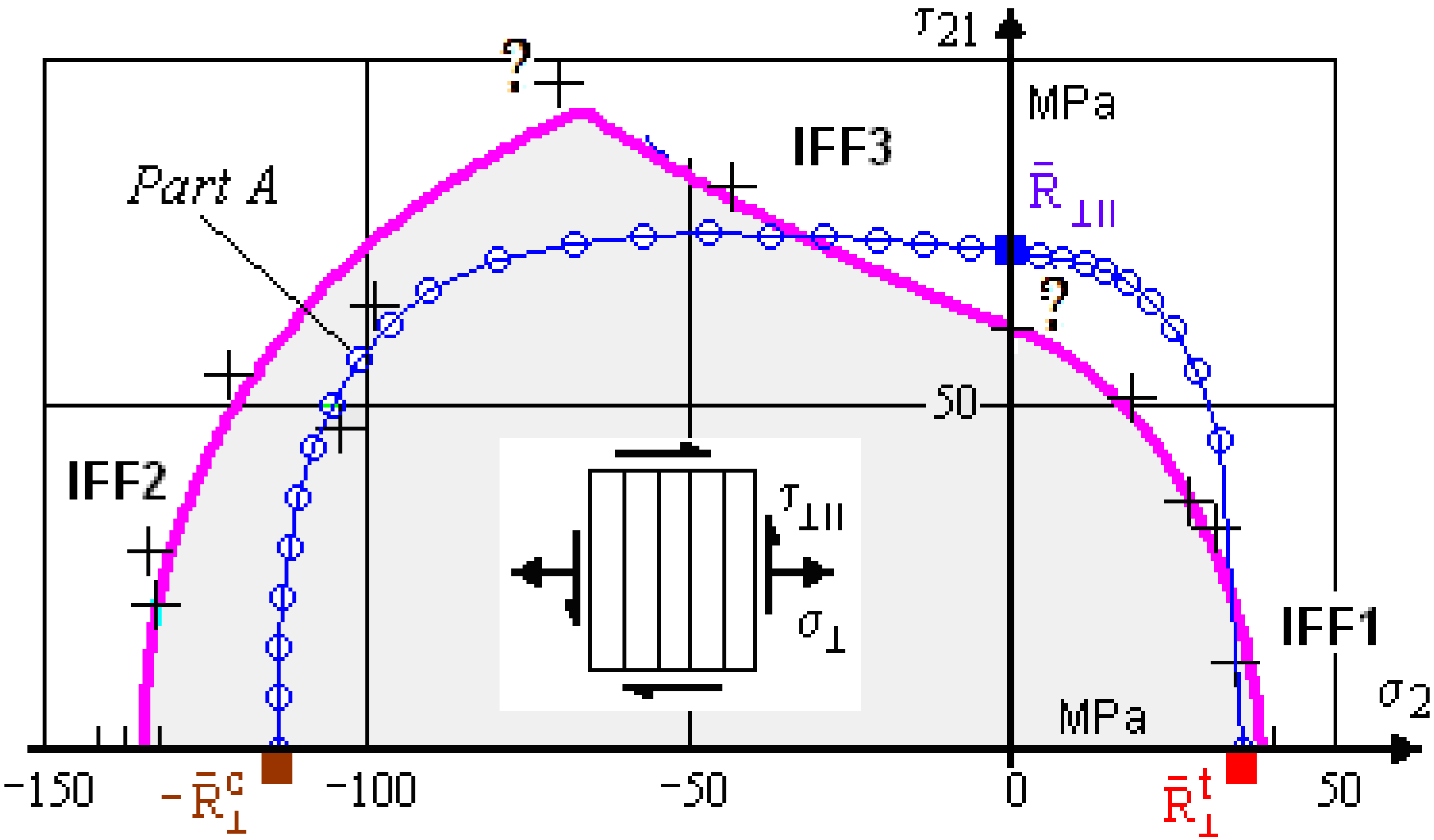
- Often, interpretation *(very effortful)* of provided test data was not possible
- Sometimes test records are not reliable or not obtainable
- Physically necessary friction values could not be provided

  (were estimated from the courses of test data)
- Parts of provided test data not applicable *(0°tube data)*
- Doubtful evaluation and presentation of the provided test
- Limits of the applicability of a strength failure condition
  - * structural failure occurs, not material failure anymore

    (instability of tube test specimen under compression)
  - * filament-upon-filament compression within an ultrahighly compressed stack
  - *

6.4 Test Case 1, WWFE-I, *IFF curve*

$\tau_{21}(\sigma_2)$



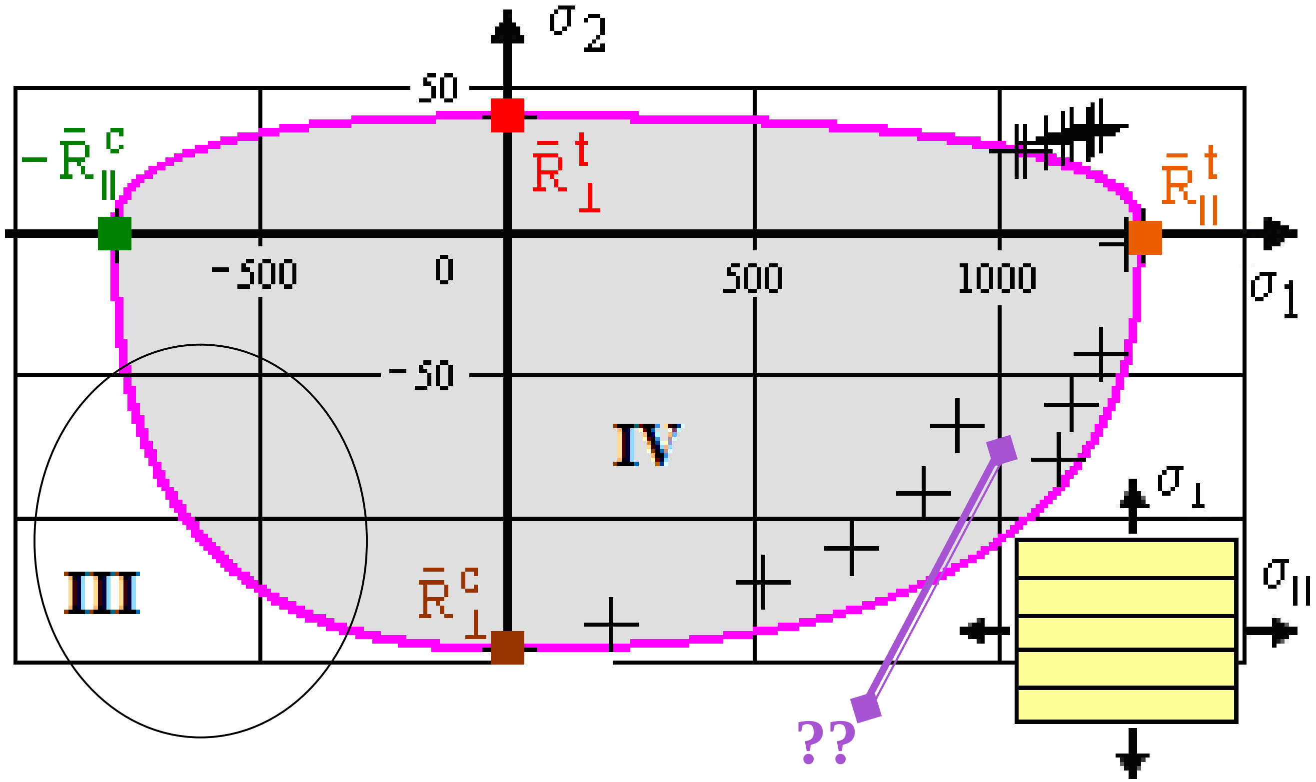
Part A, prediction: strength data provided, only. No friction value (slope) $\mu_{\perp\parallel}$

Part B, comparison: strength points altered, 2 doubtful (?) single failure stress points

6.5 Test Case 3, WWFE-I

$$\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$$

$$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$$



Hoop wound tube
UD-lamina.
E-glass/MY750epoxy +

$\sigma_1 = \sigma_{hoop}$

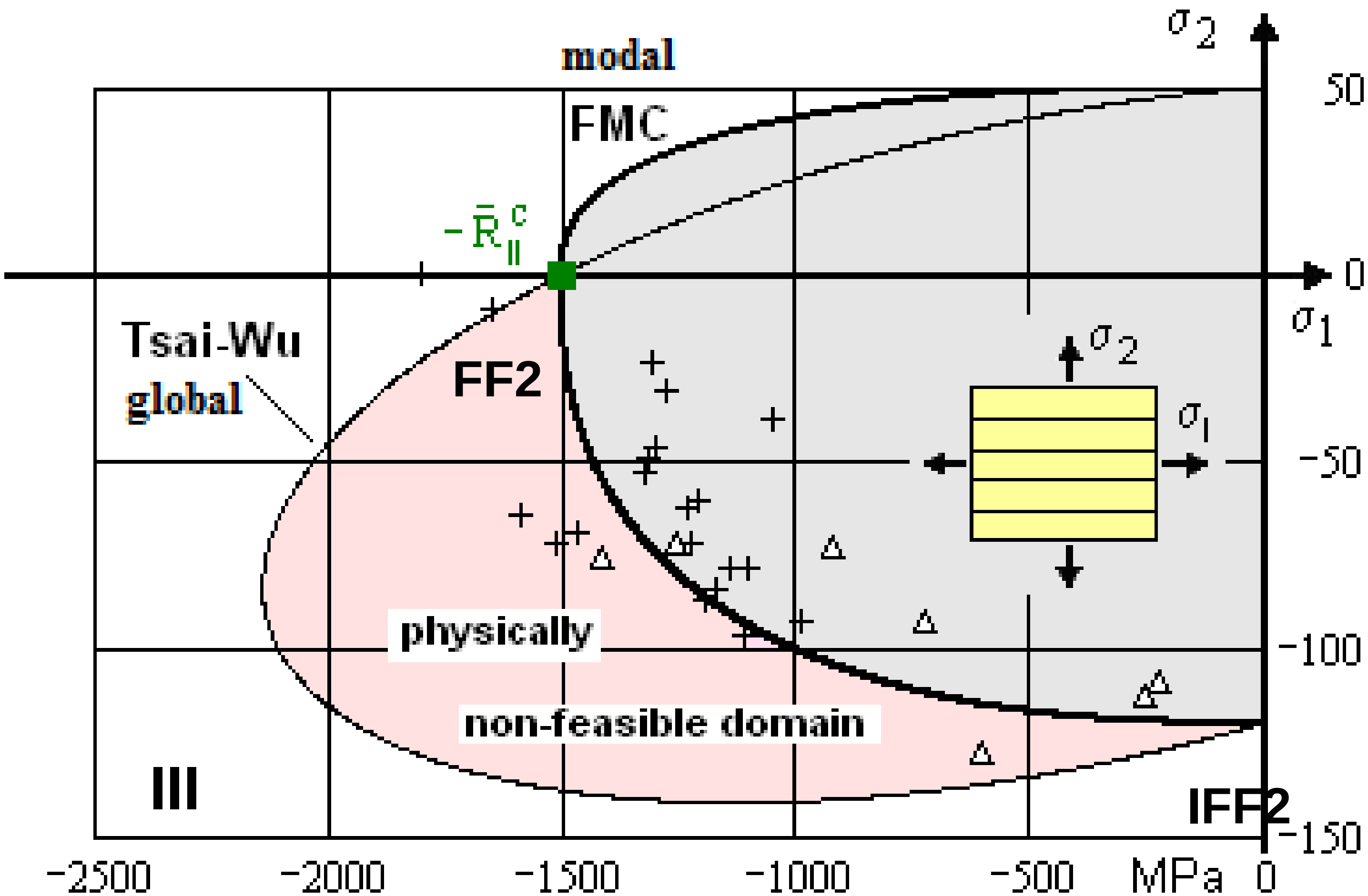
$\sigma_2 = \sigma_{axial}$

Part A: Data of strength points provided, only

Part B: Test data in quadrant IV show discrepancy

No data for quadrants II, III was be provided ! But, ..

6.6 Mapping in the 'Tsai-Wu non-feasible domain' (quadrant III)

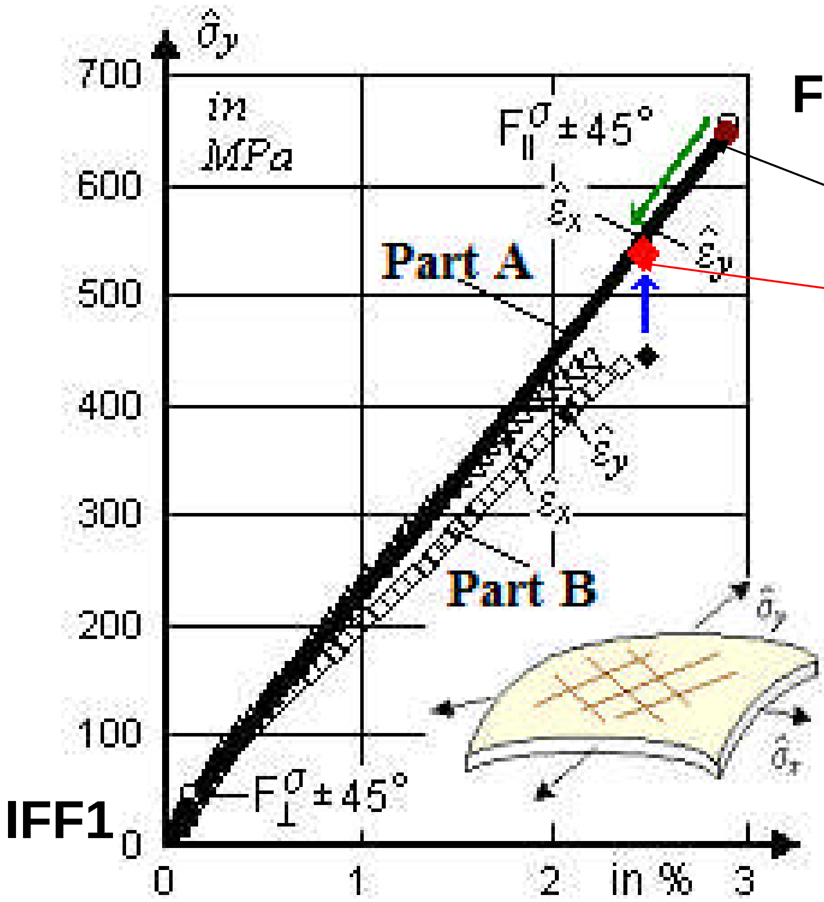


Data: courtesy IKV Aachen, Knops

Lesson Learnt: The FMC maps correctly as it is no *Global* formulation !

6.7 Test Case 13, WWFE-I, Laminate Stress-Strain Curve

$\hat{\sigma}_y : \hat{\sigma}_x = 1 : 1$



$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$

Loading of tube: internal pressure + axial tension.

Laminate: E-glass/MY750. [+45/-45/45/-45]-

Bulging (widening) reported in experiment.

- Final blind prediction point.
- Maximum test value *after* correction and shifting.

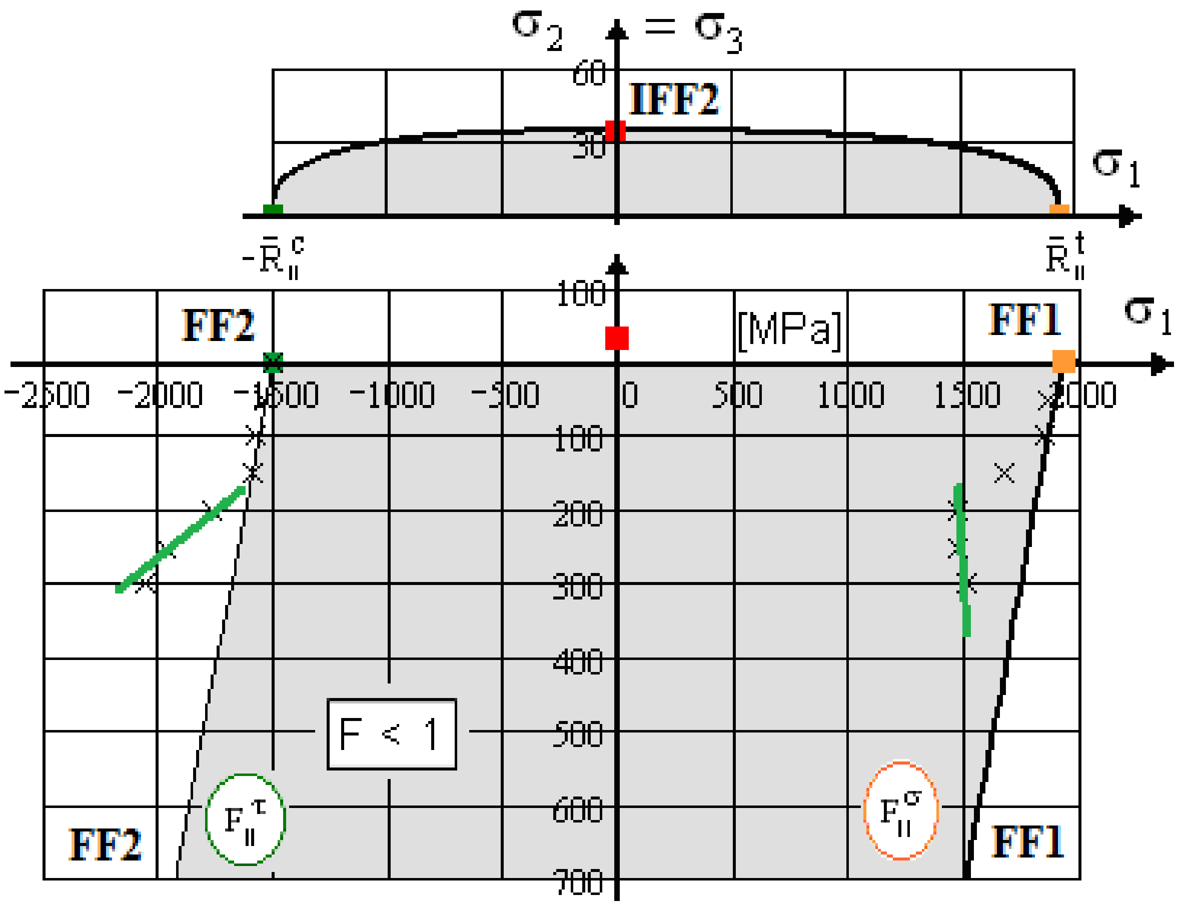
Mapping quality very good after re-evaluation !

Part A: Data of strength points and fracture strains was provided

Part B: Provided test data information made to *reduce the fracture strain* and to *increase the failure stress* after assessing the widening of the tube .

6.8 Test Case 6, WWFE-II, UD test specimen

$\sigma_1(\sigma_2 = \sigma_3)$



Was p_{hyd} correctly considered ?

No mapping possible! No explanation for differences of the slopes !

Not acceptable for model validation and design verification!

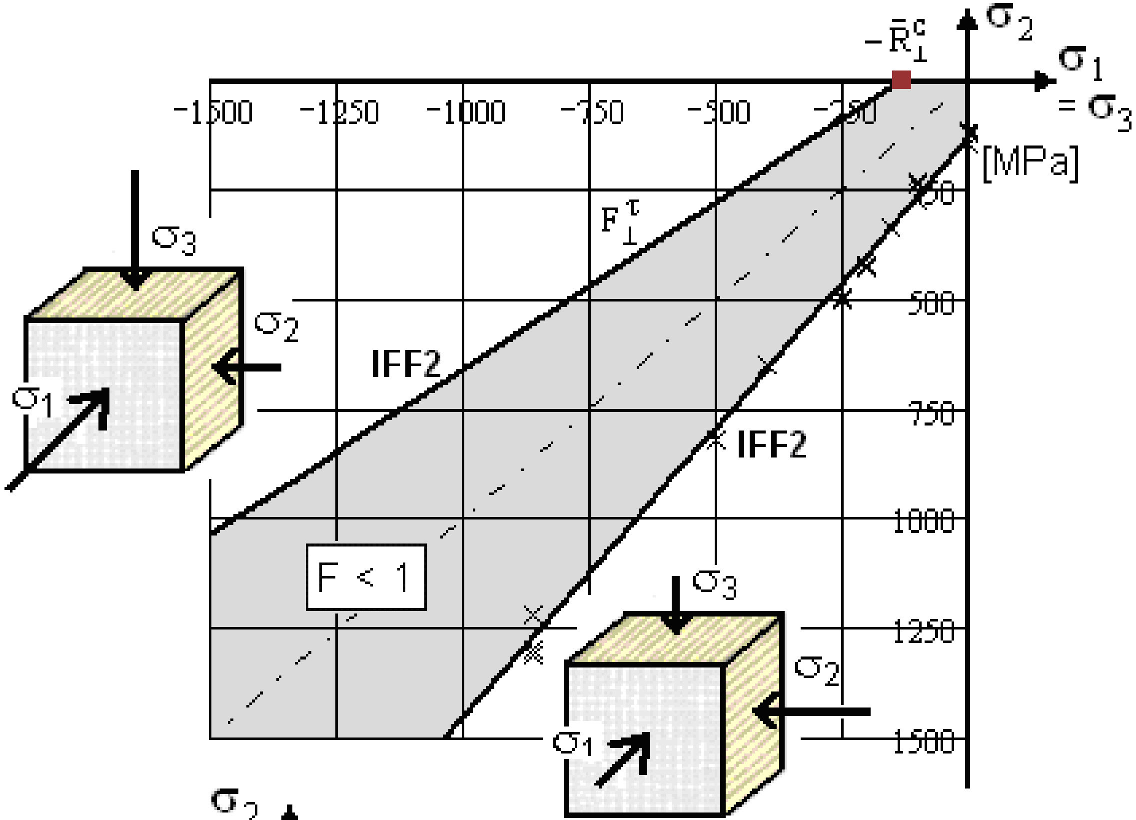
6.9 Test Case 5, WWFE-II, UD test specimen

$\sigma_2(\sigma_1 = \sigma_3)$

UD E-glass/MY750epoxy.

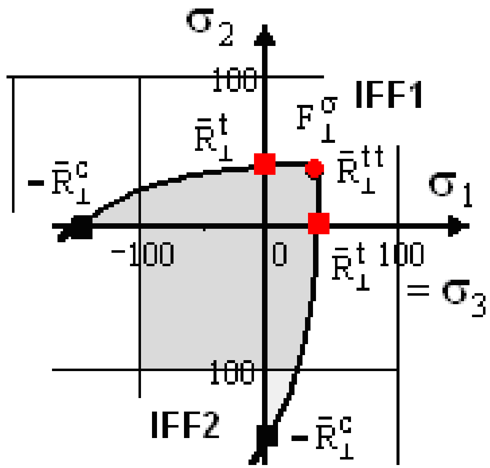
$\nu_{\perp\|} = 0.28 \quad b_{\perp\perp} = 1.16 \quad m = 2.8$

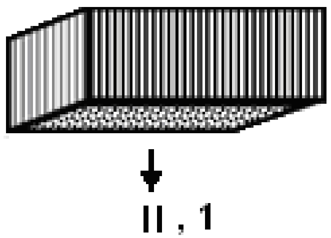
$\{\overline{R}\} = (1280,\ 800,\ 40,\ 132,\ 73)^T\ MPa$



Good Mapping,

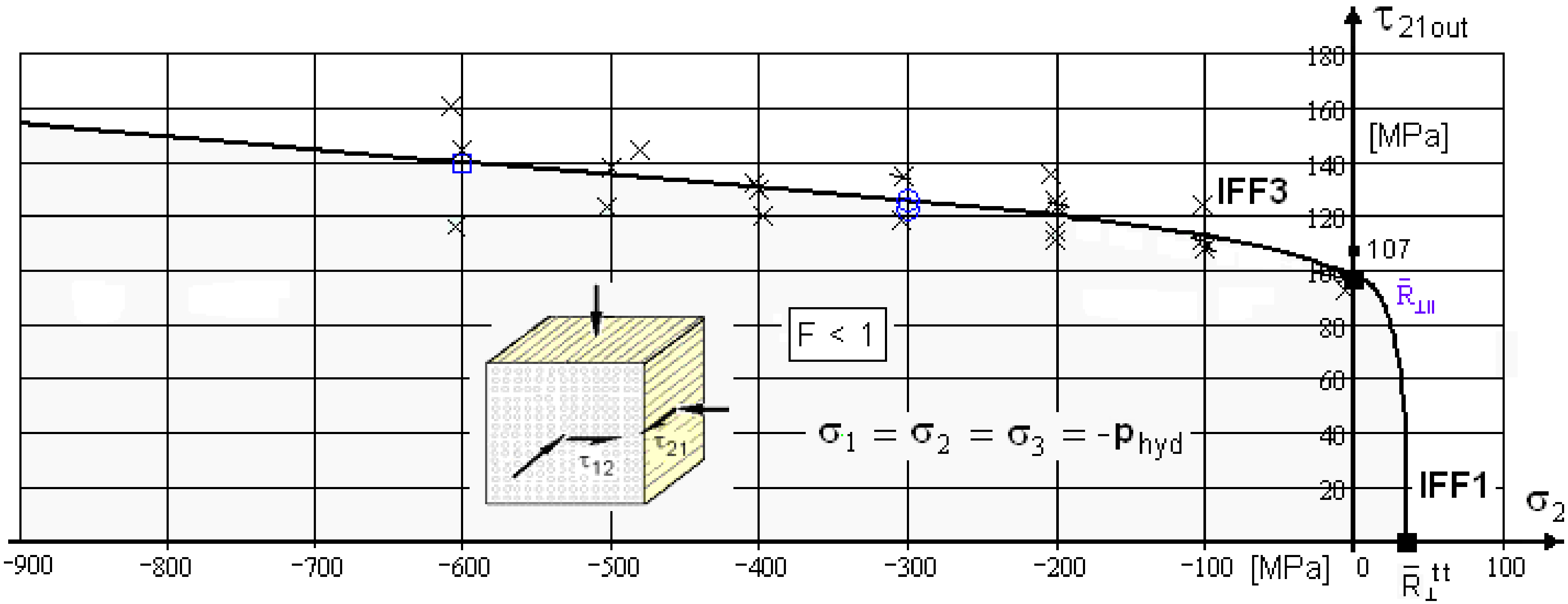
<u>after</u> re-evaluation of test data !





6.10 Test Case 3, WWFE-II, (non-)Failure Envelope

$$\tau_{21}(\sigma_2 = -p_{hyd})$$



Good Mapping,

after re-evaluation of provided data

and novel physical interpretation of test data !

redundant, isolated

Isolated and Embedded Laminas (test case 3)

Isolated behaviour:

Embedded behaviour:

IFF 1 :

weakest link problem

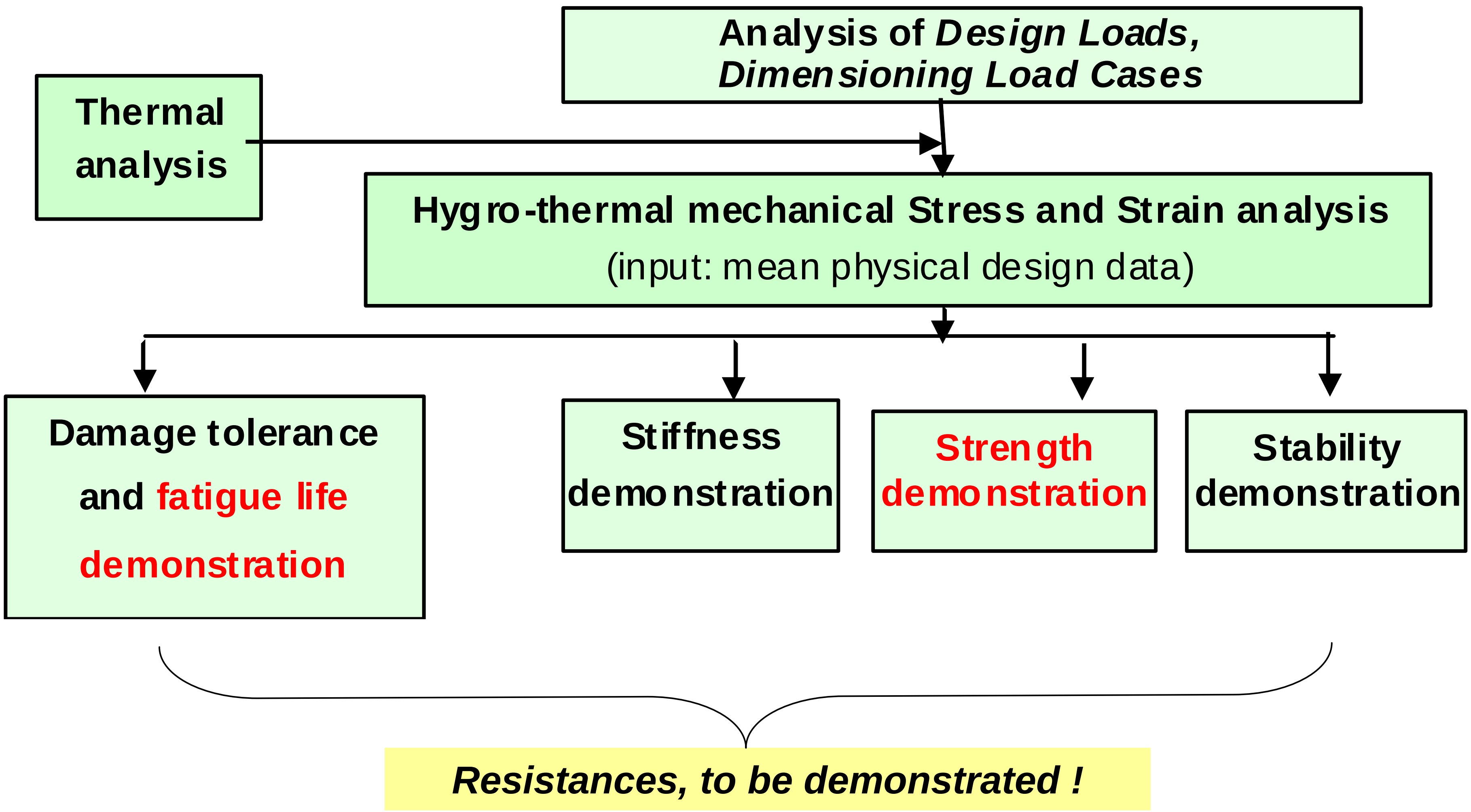
$$\{R\} = (R_{\|}^{t}, R_{\|}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\|})^{T}$$

Strengths are weakest-link data !

redundancy problem

'healing' versus 'notching'

of neighbour laminas


Analysis of *Design Loads, Dimensioning Load Cases*
Thermal analysis
Hygro-thermal mechanical Stress and Strain analysis
(input: mean physical design data)
Damage tolerance and fatigue life demonstration
Stiffness demonstration
Strength demonstration
Stability demonstration
Resistances, to be demonstrated !

7.0 Lesson Learnt from WWFE for Cyclic Loading Investigations

As the application of the <u>failure mode-wise</u> thinking

turned out to be very promising in the “static” WWFE-I

it was transferred to “cyclic loading”.

The novel idea is to use this <u>failure mode-wise</u> approach too, for :

- determining the diffuse micro-damaging portions, but also

- modelling the loading cycles (fully new way for materials).

Mind: The to be used Failure Surface of the static case shrinks with increasing damage in the cyclic case. Loading sequence

7.1 Introduction 1

- **Standard Lifetime Prediction Methods for metals:** use

Letter *R* also here standard !

  - ***S*-*N* curves,** dependent on the stress ratio $R = \sigma_{min} / \sigma_{max}$

    S := cyclic stress range = $\Delta\sigma$, N:= number of cycles to failure

  - '**Constant Life Goodman Diagram**' to account for the mean stress effect .

In case of <u>***ductile***</u> behaving <u>metals</u>

**** 'Slip band shear yielding' occurs under cyclic tensile,***

under compressive, and under shear stress !

**** This shear stress–caused yielding can be described by <u>one</u> yield failure condition !***

(Formulation is in normal stresses, but the shear stress is the damaging driver).

But, <u>*semi-brittle, brittle*</u> behaving <u>materials</u> experience

several failure modes or mechanisms

Consequence: <u>More than one</u> failure condition is to be employed !

<u>Asssumption</u>: *Static failure conditions can be used.*

„Ermüdung ist die schwarze Kunst, finanzielle Schwarze Löcher zu produzieren“.

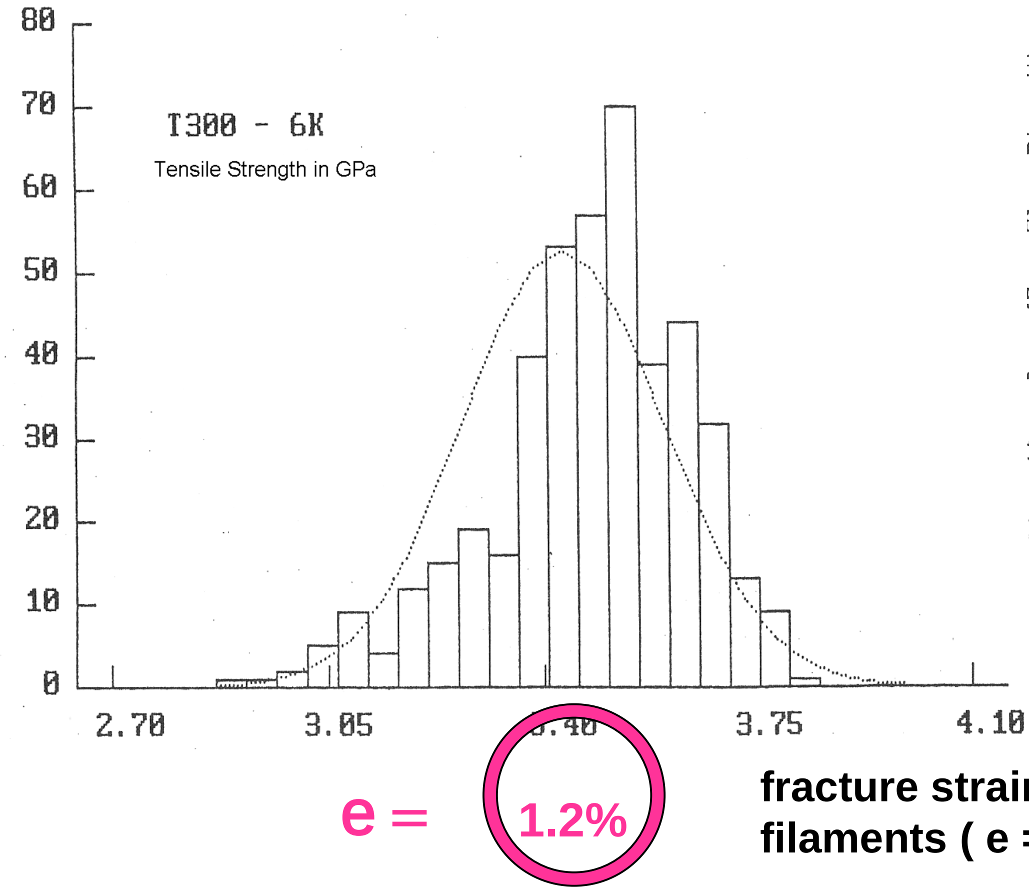
[J. Draper]

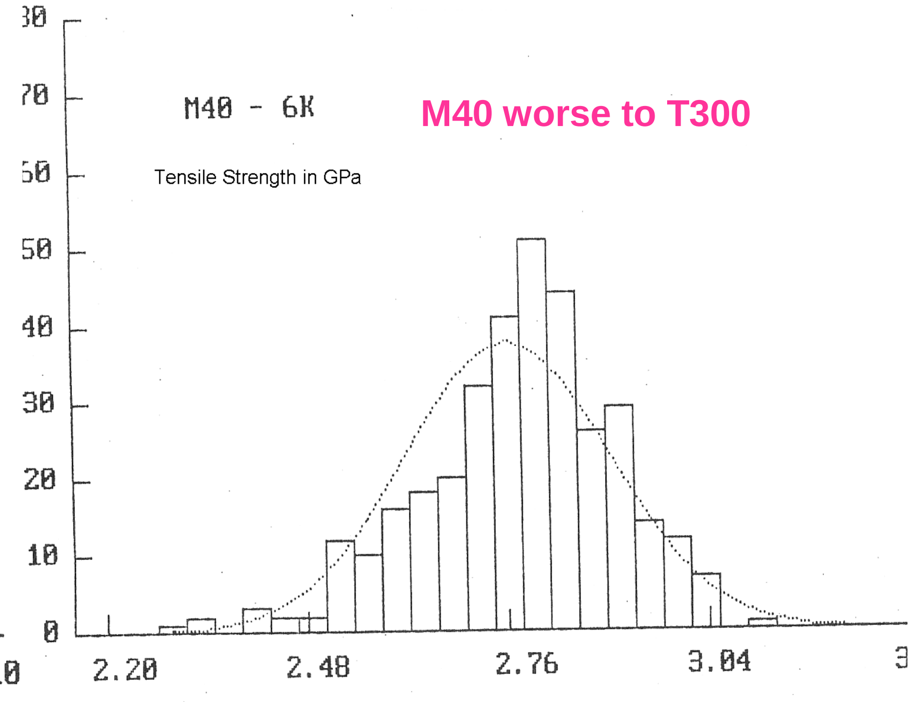
7.2 Driver of the Investigation

Increase of the usual *Design Limit Strain* from classical about 0.3% to > 0.5%

will increase damaging caused by

1) Matrix micro-cracking (IFF) + 2) First filament breaks





fracture strain distribution of filaments (e = R/E transformed)

Cyclic fatigue life consists of three phases:

1. **Growth of diffuse damage up to discrete damage**

   Main phase for determination of accumulating damage portions (Schädigungen)

2. **Stabile local discrete (macro-)damage growth** (delamination for predictions in DTA)

3. **Final instabile fracture** due to delamination criticality.

Traditional fatigue verification (not just for isotropic metals):

Stress amplitude procedure with *mean stress correction*

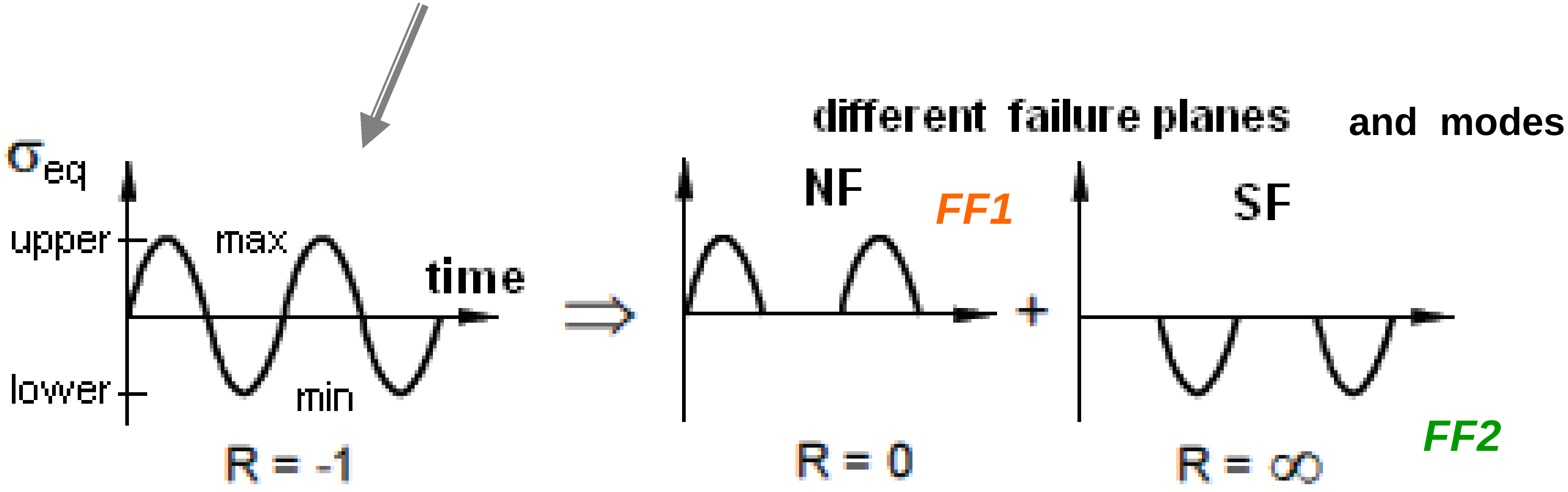
may to be replaced by a

Full stress state procedure with *failure mode reflection*.

7.4 Novel failure mode-wise modelling of Loading Cycles

For simply displaying the approach it is chosen :

- **the usually ‘fiber-dominated‘ laminate** and
- ***R = -1 loading***



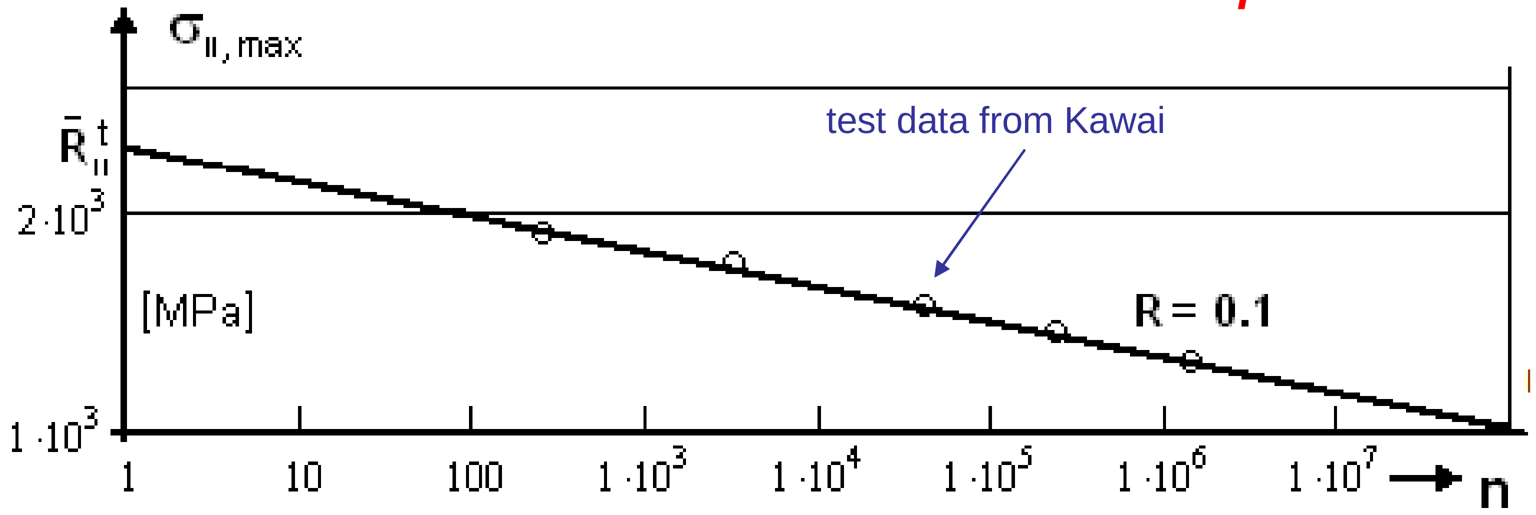
NF := Normal Fracture, SF := Shear Fracture

I : Failure mode-linked apportionment of cyclic loading

specific rain-fall procedure to be applied, $\sigma_m = 0$

7.5 Mapping of S-N data and Mode-representative *Master S-N curve*

Example: FF1 failure mode



***II* : S-N curve may be mapped by a straight line in a *log-log* graph**

$$\sigma_{\|,\ \max}^{Master}(n) \approx \overline{R}_{\|}^{t} \cdot n^{c_{Master}}$$

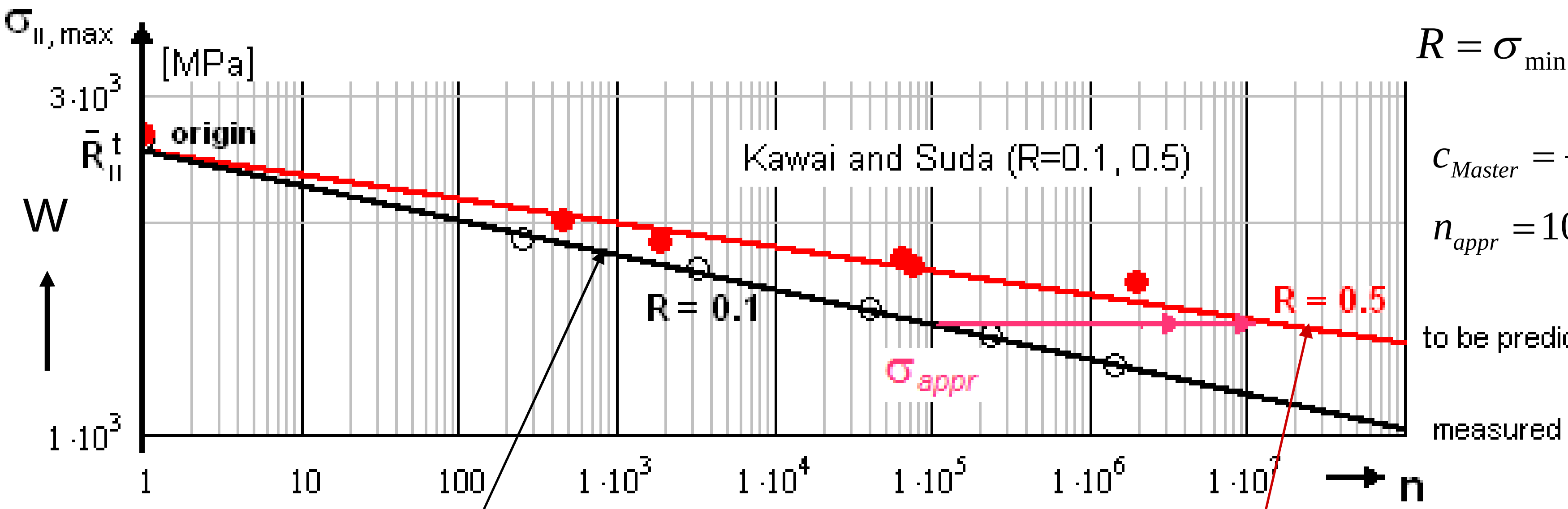
Measured curve used

as moe-representative ***Master S-N curve*** for ***FF1***

FF1 strength

Ely 96, Sho08, Cun09

7.6 Prediction of needed other *FF1* S-N curves from Master *FF1* Curve



$R = \sigma_{\min} / \sigma_{\max}$

$c_{Master} = -\ 0.049$

$n_{appr} = 10^5\ cycles$

Slope ?

Given : $\sigma_{\|,\ \max}^{Master}(n) \approx \overline{R}_{\|}^{t} \cdot n^{c_{Master}}$ **Searched :** $\sigma_{\|,\max}^{pred} = \overline{R}_{\|}^{t} \cdot n^{c_{pred}}$

Slope of R = 0.5 ?

strength

Assumption:

Neglecting heat loss, damaging is proportional to the supplied strain energy

III : A distinct strain energy level will be reached for R > 0.1 at higher cycles

7.7 Use of Strain Energy Equilibrium

[Cun09]

Logic behind: *Fatigue strain energy, required to generate a distinct damage state is equal to the strain energy, which is necessary under monotonic loading to obtain the same damage state.*

This energy can be formulated as: $\Delta W = \frac{1}{2} \cdot (\sigma_{\max} \cdot \varepsilon_{\max} - \sigma_{\min} \cdot \varepsilon_{\min})$

Hooke, <u>ignoring</u> non-linearity $\sigma = \varepsilon \cdot E \Longrightarrow \Delta W = \frac{1}{2 \cdot E} \cdot (\sigma_{\max}{}^2 - \sigma_{\min}{}^2) = \frac{1}{2 \cdot E} \cdot \sigma_{\max}{}^2 \cdot (1 - R^2)$

Advantageous is a <u>normalized</u> strain energy [Sho06] with a re-formulation by stress ratio R:

$$\Delta W \cdot 2 \cdot E = \sigma_{\|,\max}^{Master\,2} \cdot (1 - R_{Master}{}^2) = \sigma_{\|,\max}^{pred\,2} \cdot (1 - R_{pred}{}^2) \quad = \quad const$$

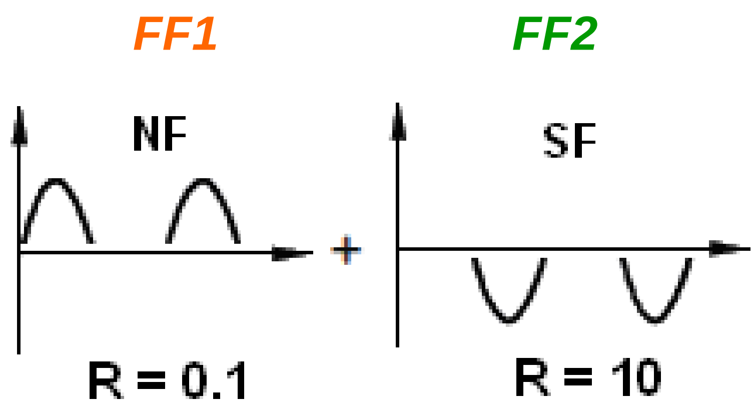
<u>Example: Fibre-dominated,, one mode, tension FF1</u>

$$\sigma_{\|,\max}^{Master\,2} \cdot (1 - R_{Master}{}^2) = \sigma_{\|,\max}^{pred\,2} \cdot (1 - R_{pred}{}^2)$$

$$(\bar{R}_{\|,\max}^{t} \cdot n^{c_{Master}})^2 \cdot (1 - R_{Master}{}^2) = (\bar{R}_{\|,\max}^{t} \cdot n^{c_{pred}})^2 \cdot (1 - R_{pred}{}^2)$$

$$c_{pred} = c_{Master} + \frac{0.5}{\ln(n_{appr})} \cdot \frac{1 - R_{Master}{}^2}{1 - R_{pred}{}^2} = -\ 0.034$$

7.8 Miner-Accumulation of Damaging Portions



Simple Example:

R = -1 stressing

$$D\ (FF1, FF2) = NF : (n_1 / N_1 + n_2 / N_2 + n_3 / N_3) + SF : (n_4 / N_4)$$

$$+ \quad D\ (IFF1, IFF2, IFF3) \ = \ D \ \leq \ D_{feasible}$$

from test experience

7.9 Choice of Test Specimens, Stress Combinations and Loading Types

Demands on test specimens: Consideration of
embedding of ply, ply-thickness effect, fibre volume fraction,
stacking sequence, loadings

1 : Flat coupon material *test specimens* (relatively cheap compared to tubes)

2 : Tension/compression-torsion tube *test specimens* $(\sigma_1, \sigma_2, \tau_{21})$

3 : Sub-laminate *test specimens* *(with internal proof ply and outer supporting plies)*

4 : Flat off-axis coupons (shortcomings 'free edge effect' + bi-axial stiffness loss not accurately considered)

5 : 3D stress state. See WWFE-II .

To be tested: Combinations of stresses (3D or 2D state of stresses)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T \quad \Rightarrow \quad \sigma_{\parallel}^t,\ \sigma_{\parallel}^c,\ \sigma_{\perp}^t,\ \sigma_{\perp}^c,\ \tau_{\perp\parallel}$$ **basic stresses**

Model VALIDATION: Loading types applied for the *operational lifetime estimation* are

- ***Constant-amplitude loading*** : delivers S-N curves (Wöhler curve)
- ***Block-loading*** (if appropriate) : for a more realistic fatigue life estimation
- ***Random spectrum loading*** : fatigue life (Gaßner) curve

7.10 To be monitored during Testing

1. Growth of diffuse damage hardening branch of the *material* lamina $\sigma-\varepsilon$ curve

until forming of discrete micro-cracks at Inter Fibre Failure (IFF)

2. Growth of discrete micro-cracks softening branch of the material lamina

until characteristic damage state (CDS) incl. growth of micro-delaminations

and delamination onset through 3D stress concentrations and σ_3^t

Effects of the negative neighbour-lamina notching are to be regarded and the positive embedding effect as well

3**. Growth of** delamination of the *structural element* laminate

Growth or no-growth of delamination (crack propagation).

Assessment Tools: fracture toughness to be determined

Damage Tolerance and Mixed-mode Fracture Mechanics.

•Initial failure depends on the cycles-dependent shrinking of the IFF body determined by the degrading residual strength.
•A laminate is a random but not deterministic *failure system* of its building blocks, the laminas.

Conclusions I: Application to UD lamina-composed Laminates

FMC-based Static Strength Failure Conditions :

1) 2D stress case: Test data mapping was successful , Validation achieved
2) 3D stress case: Looks promising as far as reliable 3D test data was available.
   To be done: Generation of missing 3D strength test data.

- Prediction is not possible if physically necessary friction values must be considered.
<u>Global</u> conditions do not consider them, therefore have shortcomings

- Validation of failure conditions requires a <u>uniform stress field in the critical domain</u>.
This was be not always given for the WWFE test cases.

Lesson Learnt:

Generating reliable 3D test data is a bigger challenge than generating a theory !

Specifically for WWFE-II is valid: **One will seldom obtain a prediction that is so dense to the test result** as Hippo & Croco show below.

In this context, the engineer shall be reminded:

**** Test results can be far away from the reality like a bad theoretical model;***
**** Theory creates a model of the reality,***
whereas an experiment is one realisation of the reality, 'only' !

Conclusions from the Beltrami-based *Failure Mode Concept* applications

- **FMC is an efficient concept, that improves prediction + simplifies design verification**

  is applicable to brittle+ductile, dense+porous, isotropic → orthotropic material

  - if clear failure modes can be identified and
  - if the homogenized material element experiences a *volume* or *shape change* or *friction*

- **Delivers a global formulation of *'individually' combined independent failure modes,***
  without the well-known drawbacks of global failure conditions
  which *mathematically combine in-dependent failure modes* .

- **Failure conditions are simple but describe physics of each failure mechanism pretty well**

- **Material behaviour Links have been outlined:**

  Paradigm*:* Basically, a compressed brittle *porous* concrete can be described like
  a tensioned ductile *porous* metal ('Gurson' domain)

The man years of development of the FMC were never funded !

Conclusions II: UD lamina-composed Laminates

FMC-based Static Strength Failure Conditions :

1) 2D stress case: Test data mapping was successful , Validation achieved
2) 3D stress case: Looks promising as far as reliable 3D test data was available.
   To be done: Generation of missing 3D strength test data.

- Prediction is not possible if physically necessary friction values must be considered.
Global conditions do not consider them, therefore have shortcomings

- Validation of failure conditions requires a uniform stress field in the critical domain.
This was be not always given for the WWFE test cases.

Lesson Learnt:

Generating reliable 3D test data is a bigger challenge than generating a theory !

Conclusions II: Novel Lifetime Prediction method

Engineering, failure mode-linked lifetime prediction method which employs**:**

1.) Failure-mode-related damage accumulation (Miner)
2.) Measurement of a minimum number of
failure-mode-linked representative S-N curves
(= *master* R-ratio *curve for each mode*) → test cost reduction
3.) Prediction of other necessary stress-ratio *mode S-N curves* on
basis of an available representative Master curve, typical
for the envisaged mode
4.) Use of *strain energy equivalence*

Outlook

* The application of the idea looks promising
* The procedure is to be transferred to not fibre-dominated lay-ups
where the other failure modes will be significant, too
* In-situ-effect consideration by deformation controlled testing .
* As sufficient test data are not available experiments are required

.