Invariant-based Theory of Composites

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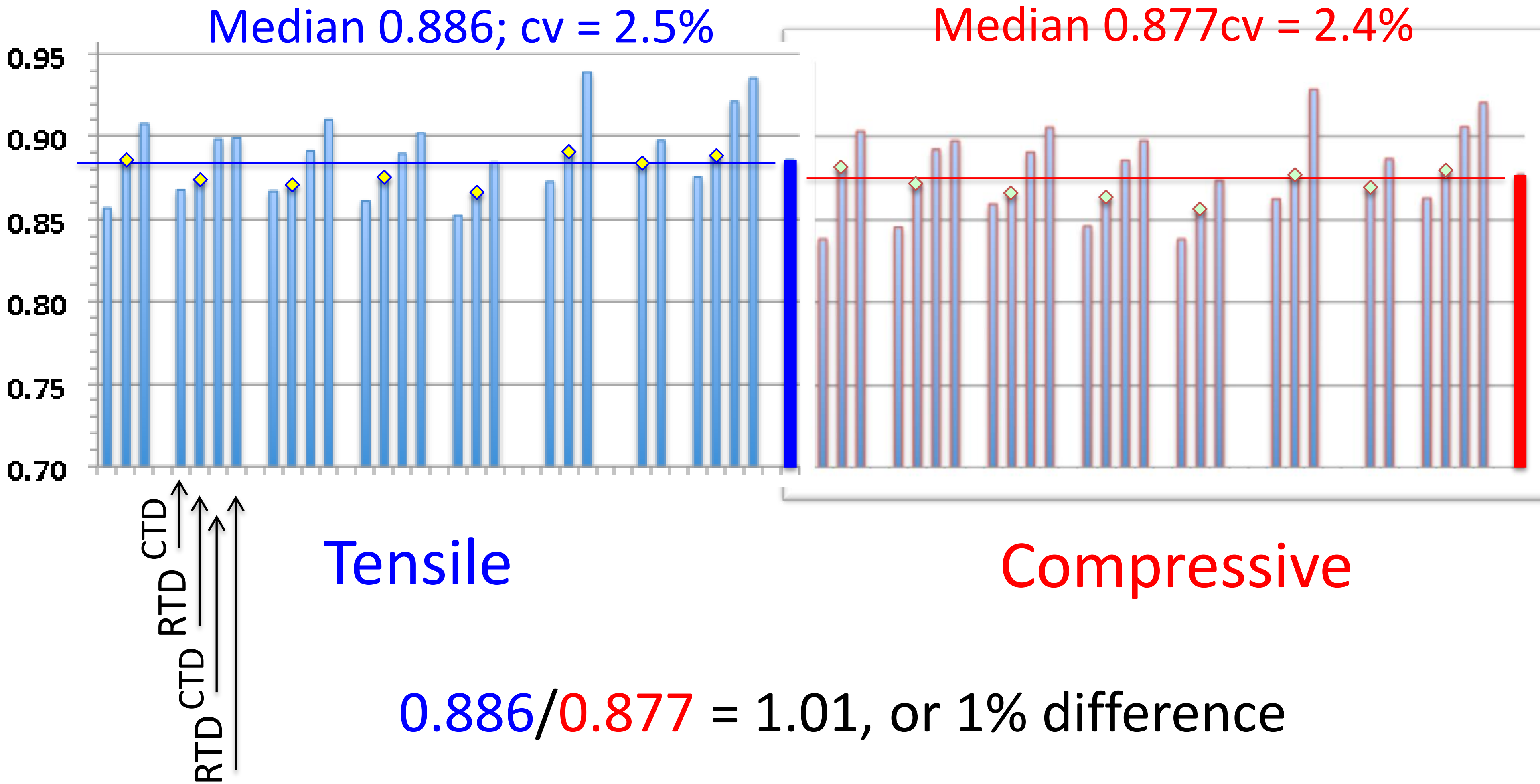
March 14, 2014

Tensile and Compressive $E_1°$/Trace

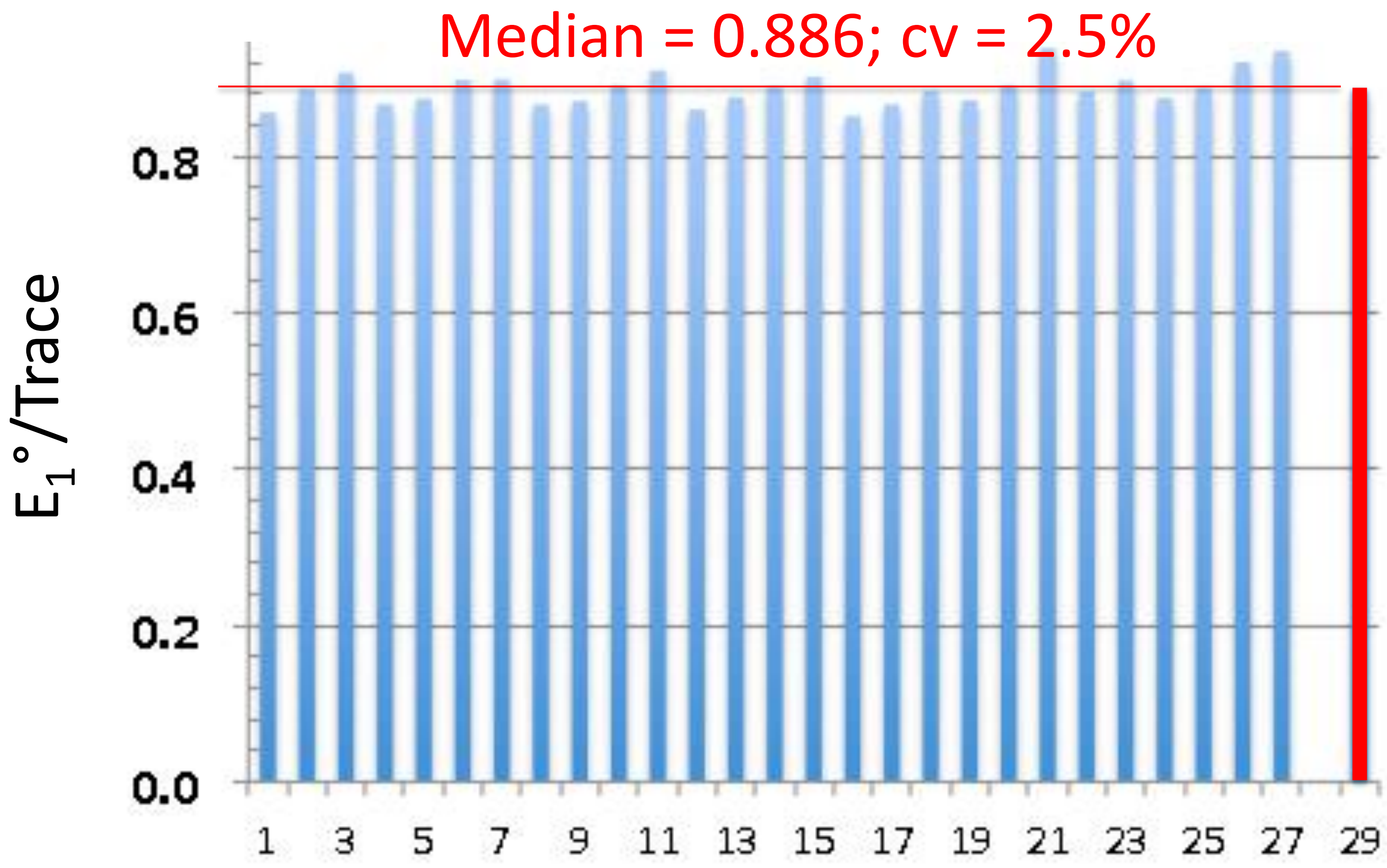
Room temp dry (with diamond), low temp dry, high temp dry and wet



0.886/0.877 = 1.01, or 1% difference

CFRP Fabric: E_1°/ Trace [A°]+

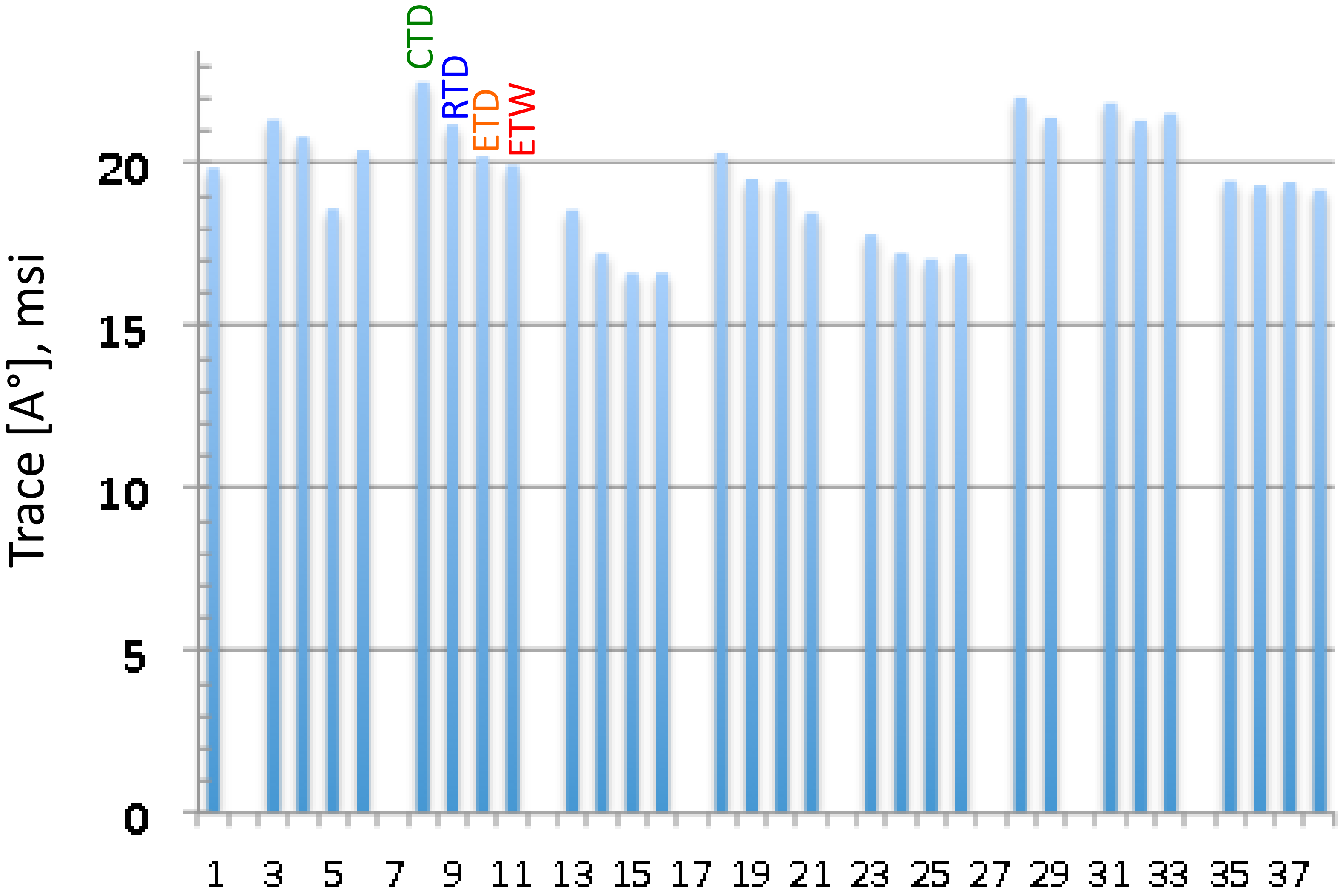
Room temp dry, low temp dry, high temp dry and wet



Median = 0.886; cv = 2.5%

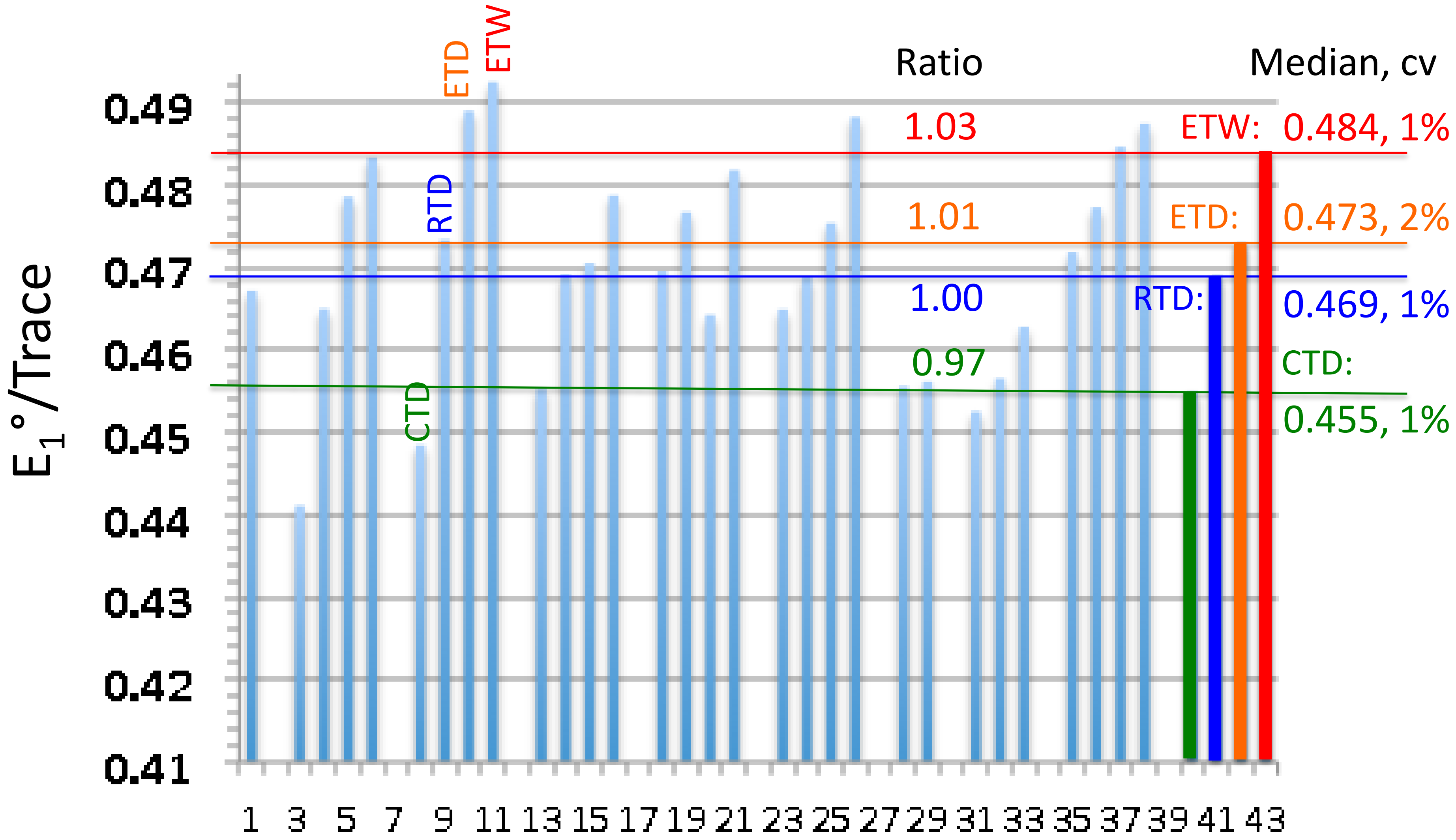
Absolute Value of Trace [A°] Tensile

Room temp dry, low temp dry, high temp dry and wet



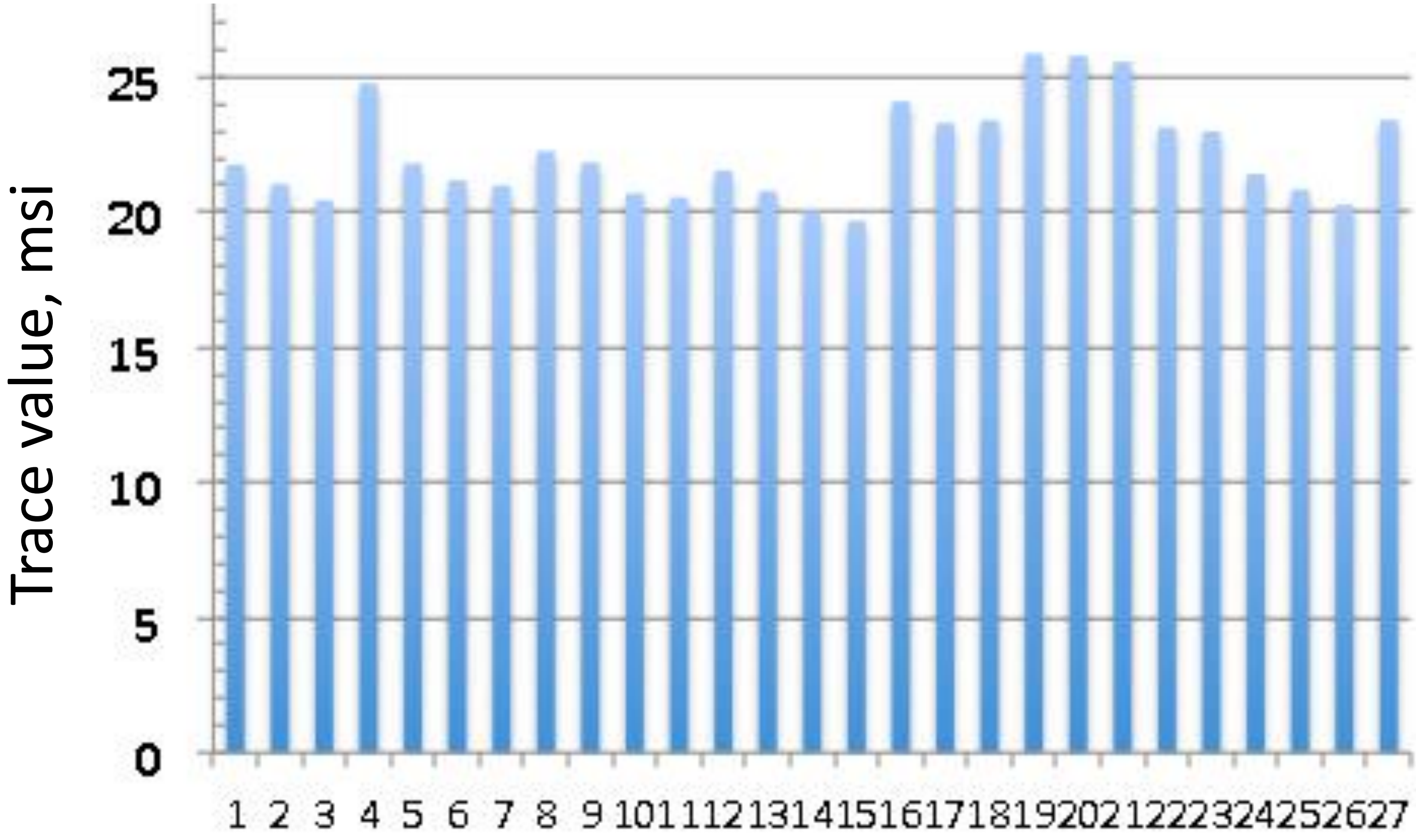
Normalized Fabric Stiffness: E_1°/Trace

Room temp dry, low temp dry, high temp dry and wet



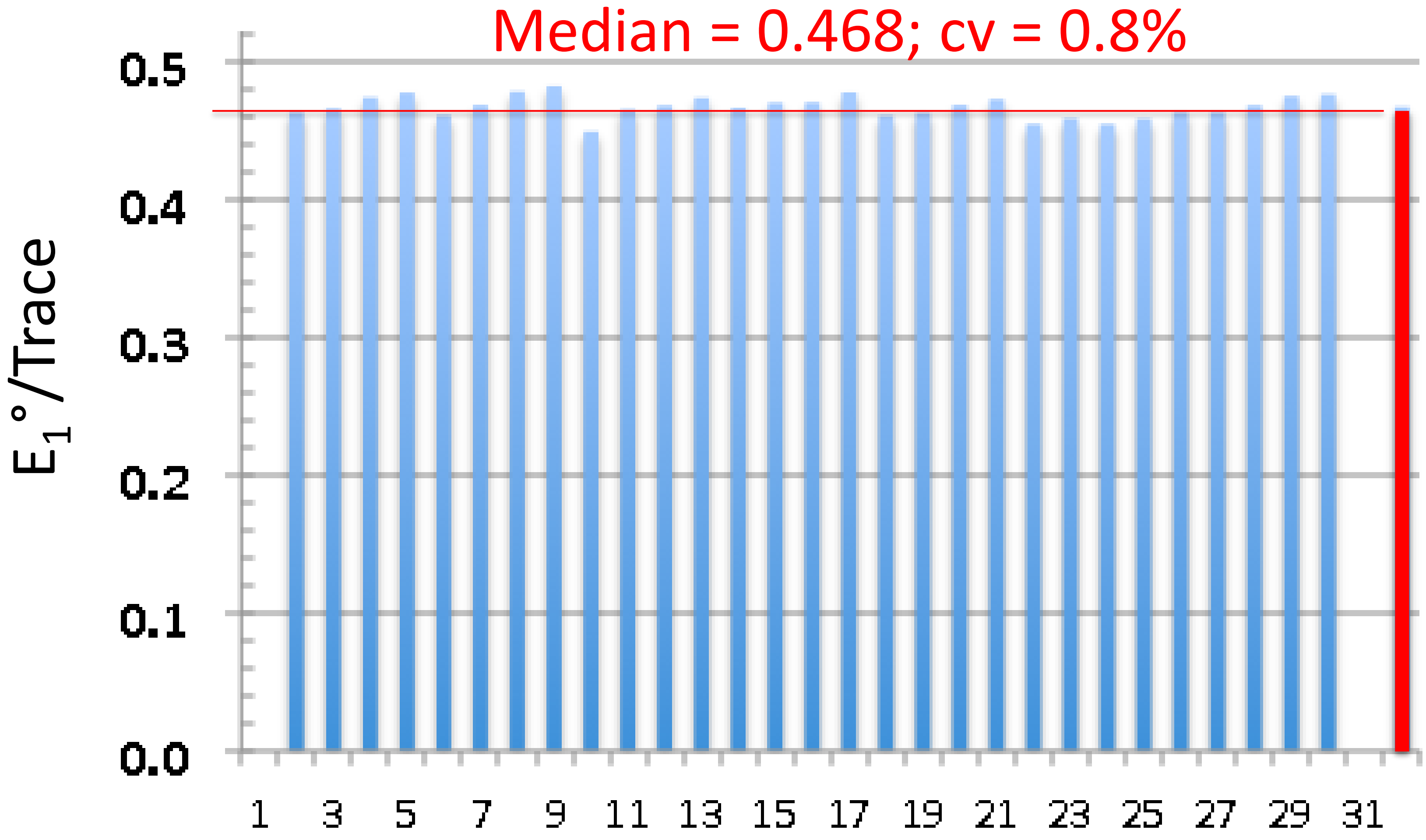
CFRP Tape: Trace Values +

Room temp dry, low temp dry, high temp dry and wet



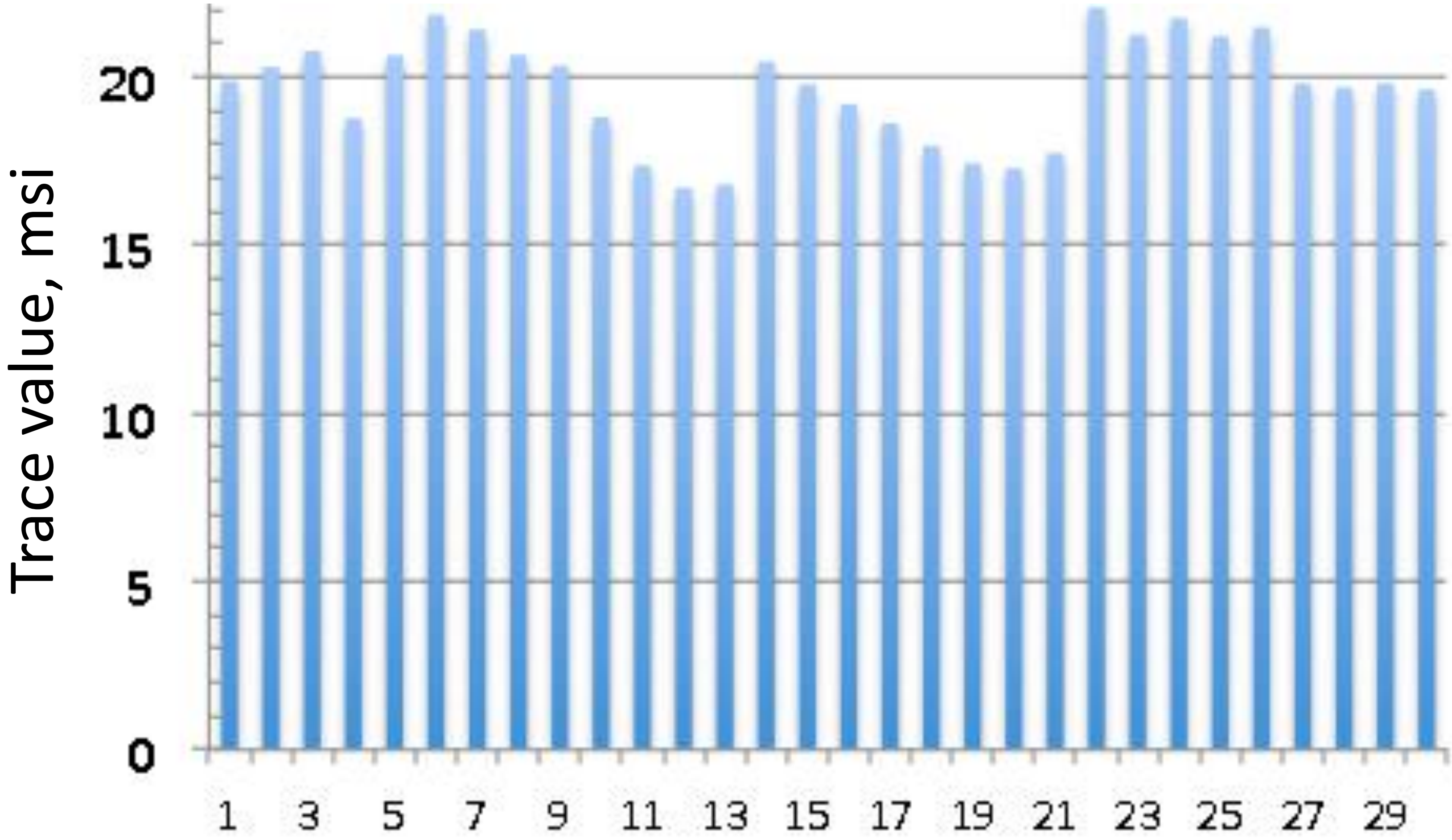
CFRP Fabric: $E_1°$/ Trace [A°]+

Room temp dry and wet, low temp dry, high temp wet



CFRP Fabric: Trace Values +

Room temp dry, low temp dry, high temp dry and wet



Some Practical Uses of Trace

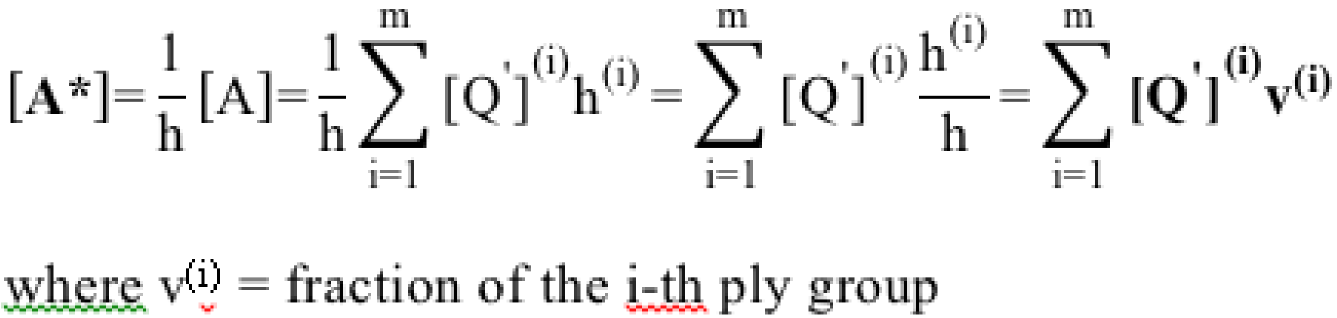
- Only invariant quantity that represents total stiffness potential of each composite material
- All stiffness components are fractions of trace
- One test can tell all about a given material
- Change in material is defined by their trace
- It can measure the quality of lamination; any defect or damage will lower trace value
- Test laminate: closer to real structure
- Track temperature effect by change in trace

Stiffness and Compliance Matrices

$$[Q] = \begin{bmatrix} \frac{E_x}{1-\nu_x\nu_y} & \frac{\nu_y E_x}{1-\nu_x\nu_y} & 0 \\ \frac{\nu_x E_y}{1-\nu_x\nu_y} & \frac{E_y}{1-\nu_x\nu_y} & 0 \\ 0 & 0 & E_s \end{bmatrix} \qquad [S] = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_y}{E_y} & 0 \\ -\frac{\nu_x}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & 1/E_s \end{bmatrix}$$

Reciprocal relation: $\nu_x E_y = \nu_y E_x$

Laminate in-plane stiffness in terms of ply stiffness [Q]:



$$[A^*] = \frac{1}{h}[A] = \frac{1}{h}\sum_{i=1}^{m}[Q']^{(i)}h^{(i)} = \sum_{i=1}^{m}[Q']^{(i)}\frac{h^{(i)}}{h} = \sum_{i=1}^{m}[\mathbf{Q'}]^{(\mathbf{i})}\mathbf{v}^{(\mathbf{i})}$$

where $v^{(i)}$ = fraction of the i-th ply group

Laminate Compliance Components

$$[\boldsymbol{a}] = [\boldsymbol{A}]^{-1}, |A|$$

$$= (A_{11}A_{22} - A_{12}^2)A_{66} + 2A_{12}A_{26}A_{16} - A_{11}A_{26}^2$$

$$- A_{22}A_{16}^2$$

$$a_{11} = \frac{(A_{22}A_{66} - A_{26}^2)}{|A|}, a_{22} = \frac{(A_{11}A_{66} - A_{16}^2)}{|A|}, a_{12}$$

$$= \frac{(A_{16}A_{26} - A_{12}A_{66})}{|A|}$$

$$a_{66} = \frac{(A_{11}A_{22} - A_{12}^2)}{|A|}, a_{16} = \frac{(A_{12}A_{26} - A_{22}A_{16})}{|A|}, a_{26} = \frac{(A_{12}A_{16} - A_{11}A_{26})}{|A|}$$

(3.4)

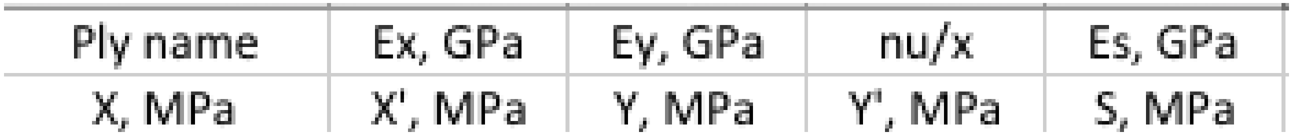
Laminate Engineering Constants

$$E_1^o = \frac{1}{a_{11}}^{*}, E_2^o = \frac{1}{a_{22}}^{*}, E_6^o = \frac{1}{a_{66}}^{*}$$

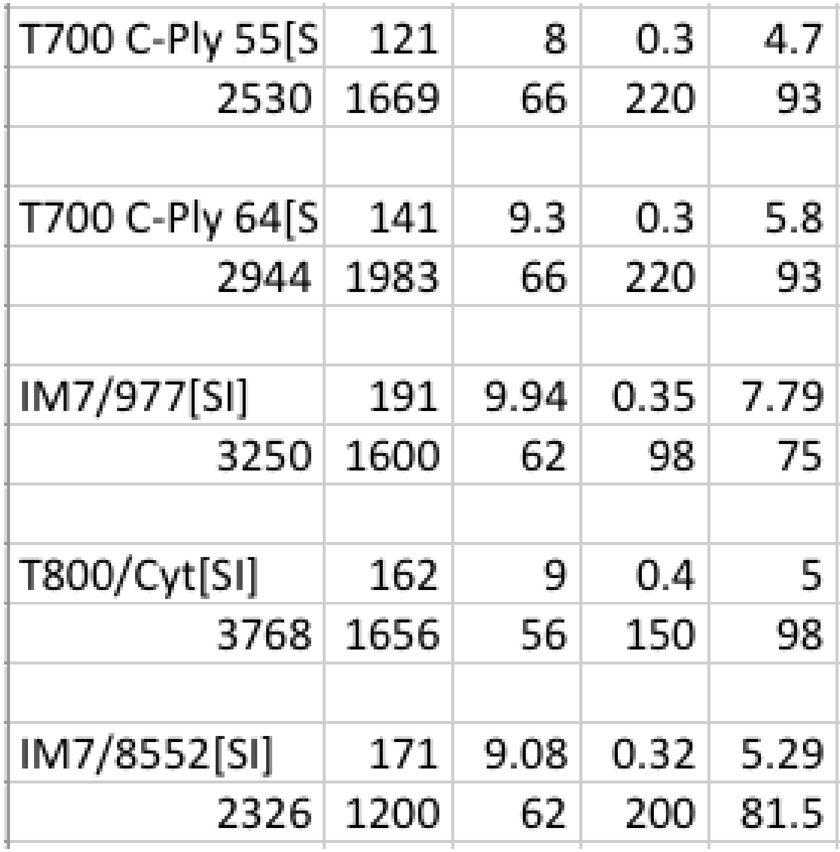
$$v_{21}^o = -\frac{a_{21}}{a_{11}}, v_{61}^o = \frac{a_{61}}{a_{11}}, v_{62}^o = \frac{a_{62}}{a_{11}}$$

$$v_{12}^o = -\frac{a_{12}}{a_{22}}, v_{16}^o = \frac{a_{16}}{a_{66}}, v_{26}^o = \frac{a_{26}}{a_{66}}$$

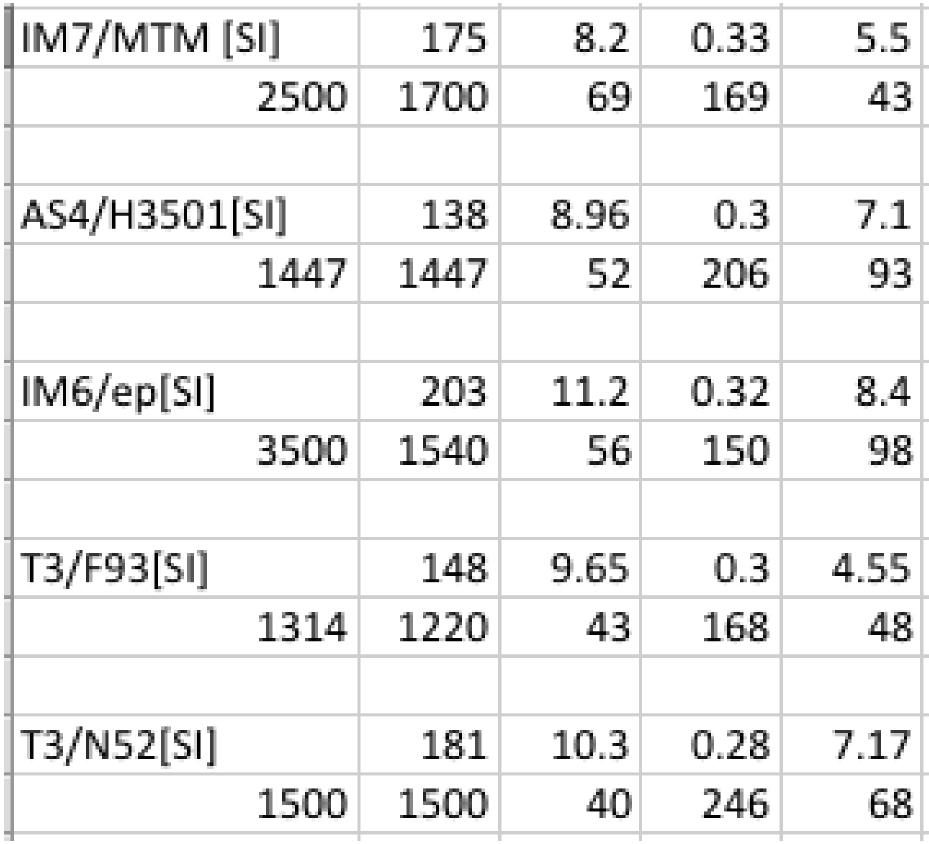
Input Data: Ply Stiffness and Strength



Ply name	Ex, GPa	Ey, GPa	nu/x	Es, GPa
X, MPa	X', MPa	Y, MPa	Y', MPa	S, MPa



T700 C-Ply 55[S	121	8	0.3	4.7
2530	1669	66	220	93
T700 C-Ply 64[S	141	9.3	0.3	5.8
2944	1983	66	220	93
IM7/977[SI]	191	9.94	0.35	7.79
3250	1600	62	98	75
T800/Cyt[SI]	162	9	0.4	5
3768	1656	56	150	98
IM7/8552[SI]	171	9.08	0.32	5.29
2326	1200	62	200	81.5



IM7/MTM [SI]	175	8.2	0.33	5.5
2500	1700	69	169	43
AS4/H3501[SI]	138	8.96	0.3	7.1
1447	1447	52	206	93
IM6/ep[SI]	203	11.2	0.32	8.4
3500	1540	56	150	98
T3/F93[SI]	148	9.65	0.3	4.55
1314	1220	43	168	48
T3/N52[SI]	181	10.3	0.28	7.17
1500	1500	40	246	68

Ply & Laminate Stiffness Matrix & Trace

Laminate

$$\left|\begin{array}{ccc} E_1^\circ & \nu_{12}^\circ & 0 \\ \nu_{21}^\circ & E_2^\circ & 0 \\ 0 & 0 & E_6^\circ \end{array}\right| \Leftrightarrow \left[\begin{array}{ccc} A_{11}^* & A_{12}^* & 0 \\ A_{21}^* & A_{22}^* & 0 \\ 0 & 0 & A_{66}^* \end{array}\right] \Leftrightarrow \left[\begin{array}{ccc} a_{11}^* & a_{12}^* & 0 \\ a_{21}^* & a_{22}^* & 0 \\ 0 & 0 & a_{66}^* \end{array}\right] \Leftrightarrow \left|\begin{array}{ccc} E_1^\circ & \nu_{12}^\circ & 0 \\ \nu_{21}^\circ & E_2^\circ & 0 \\ 0 & 0 & E_6^\circ \end{array}\right|$$

Eng'g constants array — In-plane stiffness matrix — Compliance matrix — Eng'g constants array

Ply data

$$\left|\begin{array}{ccc} E_x & \nu_y & 0 \\ \nu_x & E_y & 0 \\ 0 & 0 & E_s \end{array}\right| \Leftrightarrow \left[\begin{array}{ccc} Q_{xx} & Q_{xy} & 0 \\ Q_{yx} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{array}\right] \Leftrightarrow \left[\begin{array}{ccc} S_{xx} & S_{xy} & 0 \\ S_{yx} & S_{yy} & 0 \\ 0 & 0 & S_{ss} \end{array}\right] \Leftrightarrow \left|\begin{array}{ccc} E_x & \nu_y & 0 \\ \nu_x & E_y & 0 \\ 0 & 0 & E_s \end{array}\right|$$

Eng'g constants array — Ply stiffness matrix — Ply compliance matrix — Eng'g constants array

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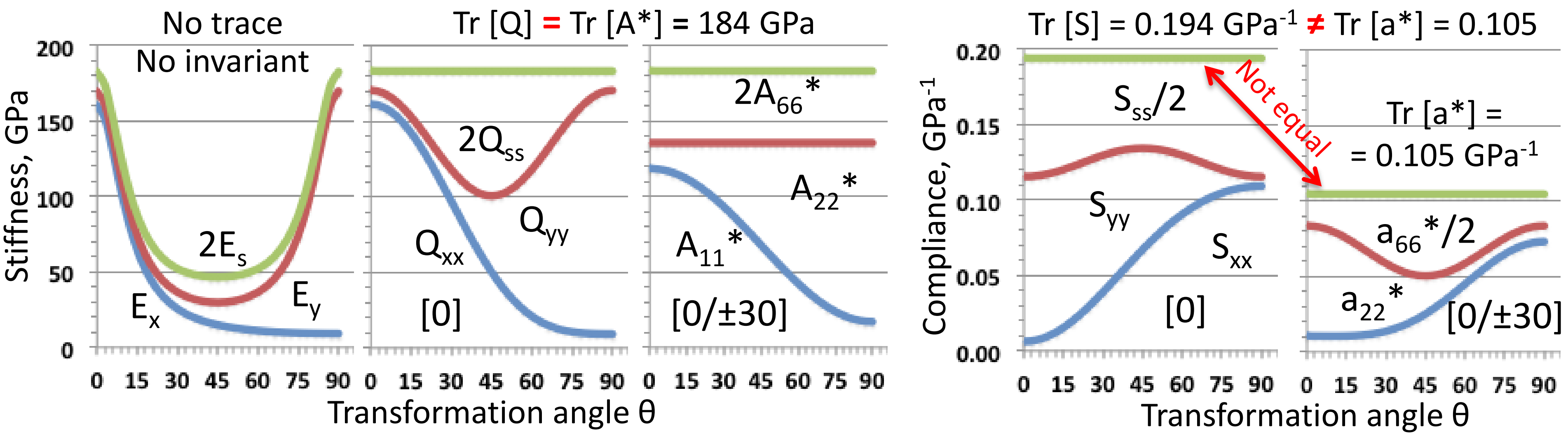


No trace

Tr [Q] = Tr [A*] = 184 GPa

Tr [S] = 0.194 GPa^{-1} ≠ Tr [a*] = 0.105

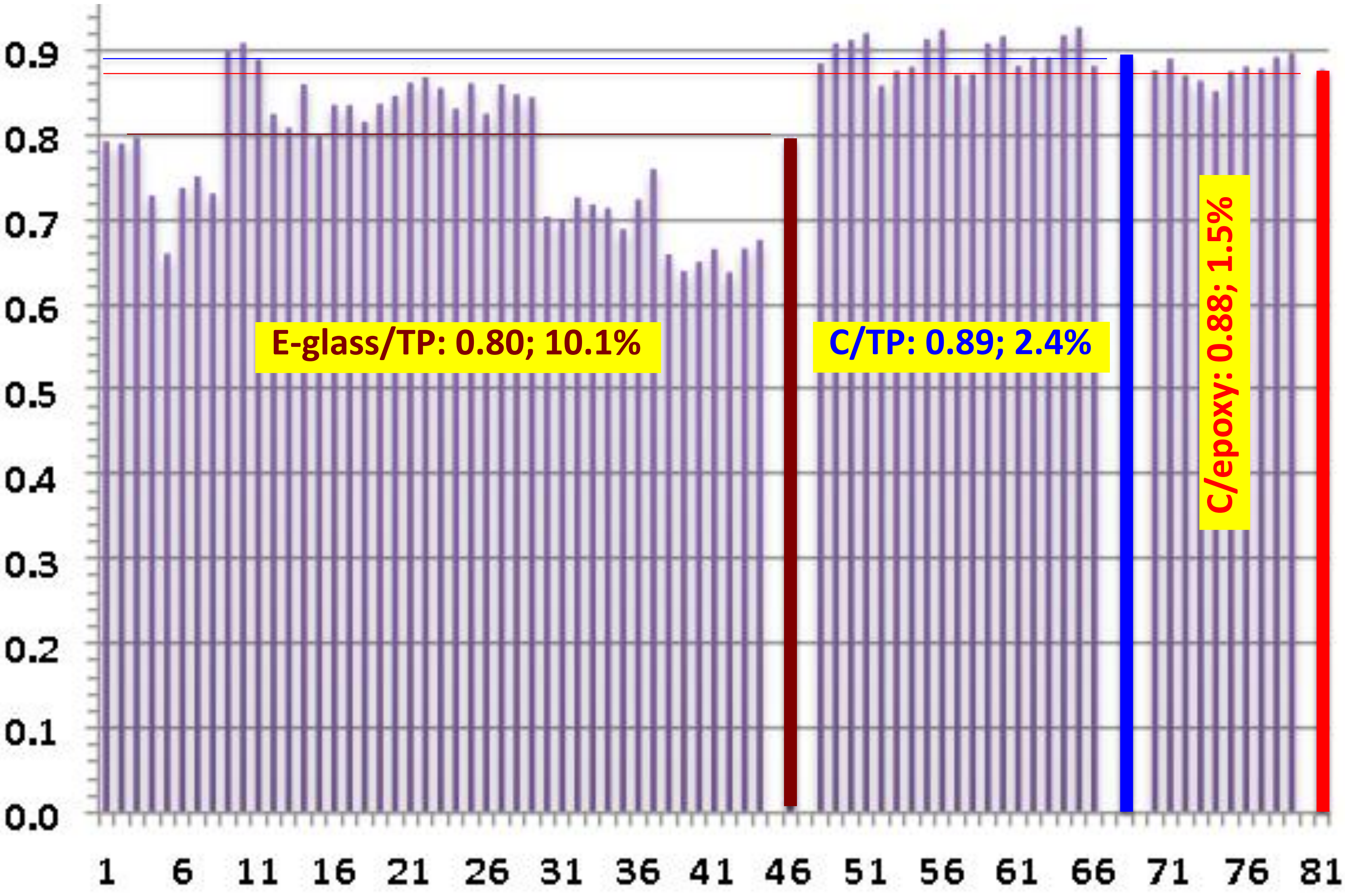
Master Ply Stiffness: Trace Normalized

Carbon/epoxy ply stiffness in trace normalized factors

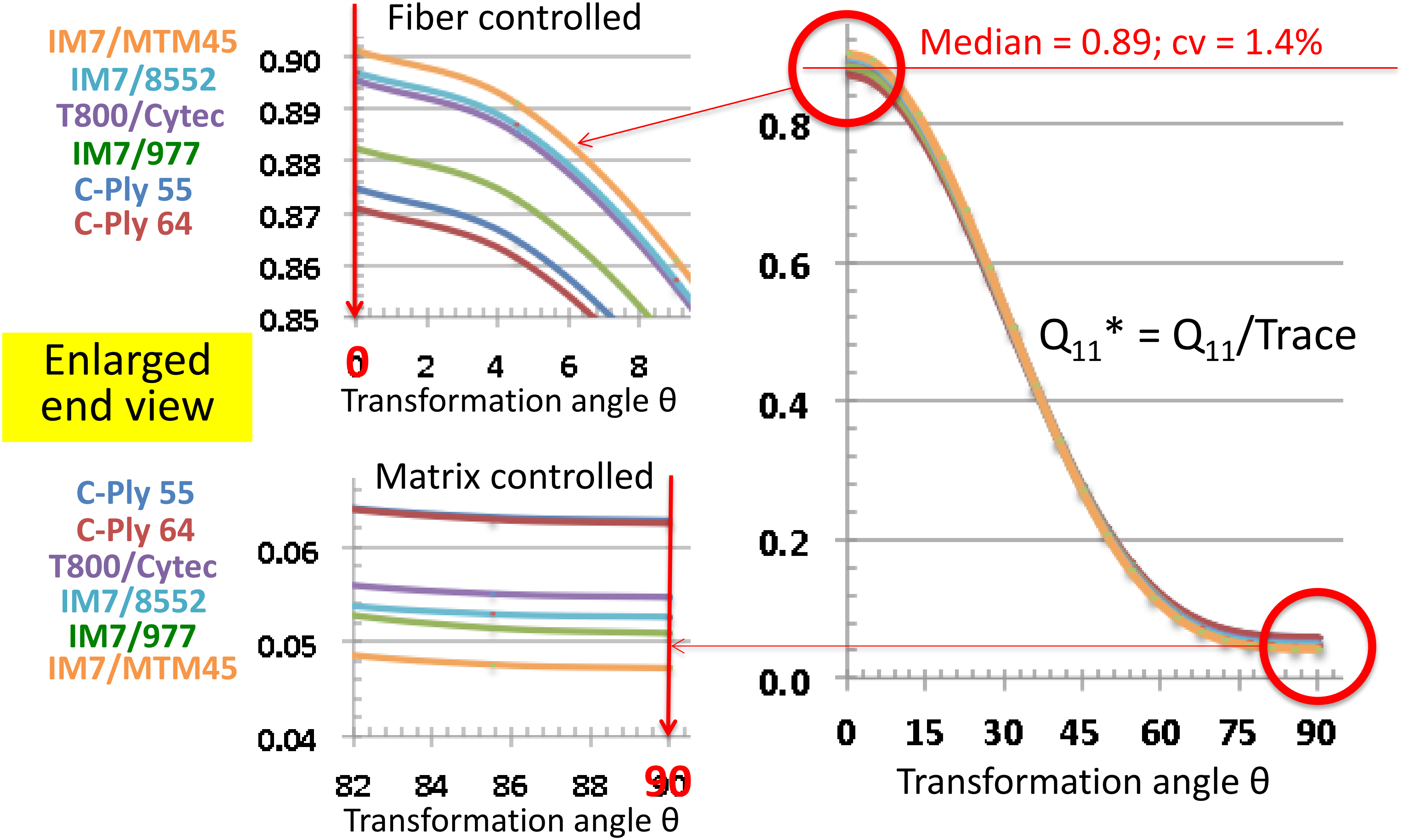
Material [0]	Qxx*	Qyy*	Qss*,Es	Tr, GPa	Trace*	Ex*	Ey*	nu/x*	Ey/Ex
IM7/977-3	0.88	0.046	0.036	218	1.00	0.88	0.046	0.35	0.052
T800/Cytec	0.90	0.050	0.027	183	1.00	0.89	0.049	0.40	0.056
T7 C-Ply 55	0.88	0.057	0.034	139	1.00	0.87	0.058	0.30	0.066
T7 C-Ply 64	0.87	0.057	0.036	163	1.00	0.86	0.057	0.30	0.066
AS4/3501	0.86	0.056	0.044	162	1.00	0.85	0.055	0.30	0.065
IM6/epoxy	0.88	0.049	0.036	232	1.00	0.88	0.048	0.32	0.055
AS4/F937	0.89	0.058	0.027	168	1.00	0.88	0.057	0.30	0.065
T300/N5208	0.88	0.050	0.035	206	1.00	0.88	0.050	0.28	0.057
IM7/8552	0.90	0.048	0.028	192	1.00	0.89	0.047	0.31	0.053
IM&/MTM45	0.90	0.042	0.028	195	1.00	0.90	0.042	0.33	0.047
Master ply	0.883	0.050	0.034	187	1.00	0.877	0.050	0.305	0.0609
Std dev	0.013	0.005	0.005	28.468	0.001	0.014	0.006	0.034	0.5%
CV	1.5%	10.9%	15.8%		0.1%	1.5%	11.1%	11.3%	9.0%
Master GPa	165	9.35	6.42		187	164	9.30	0.31	11.39

$Q_{xx} = Q_{xx}{}^{*} \times Tr = 0.883 \times 187 = 165$ GPa

Median and cv of E_x/Trace [Q]



Dispersion of Q_{11}^* at 0° and 90°

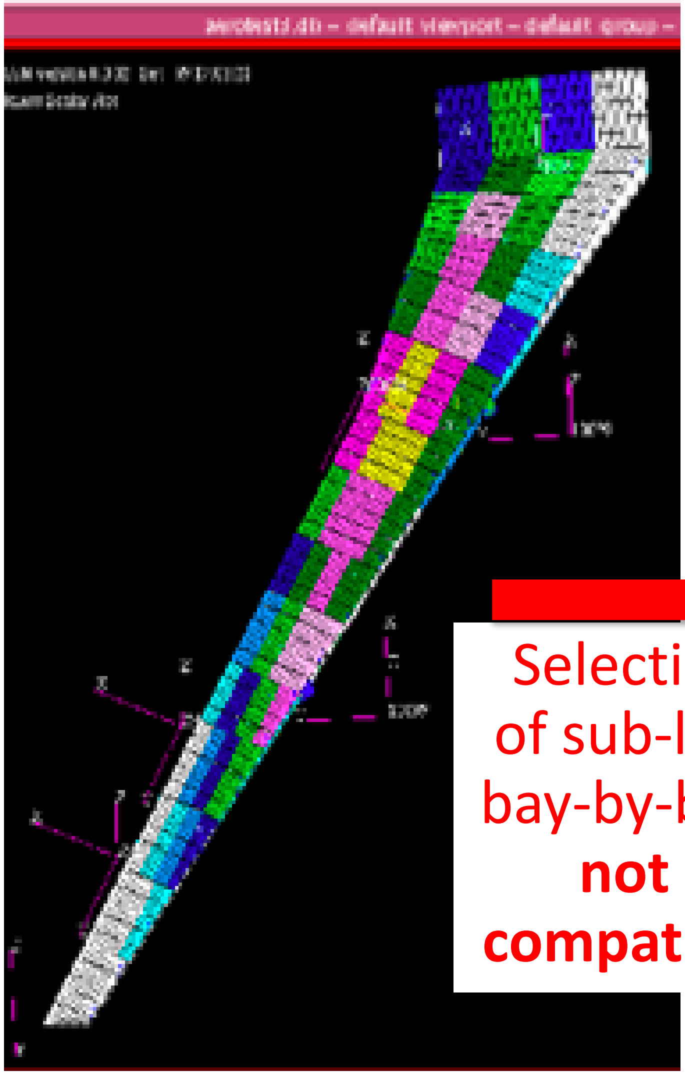


Enlarged end view

Median = 0.89; cv = 1.4%

$Q_{11}^* = Q_{11}/\text{Trace}$

Bay-by-bay not Optimized



Selection of sub-lam bay-by-bay: **not compatible**

Step 1: Ply combinatinon per bay:

$[0_3/\pm45_2/90]_{2S}$

(38/50/12)

[0]

[0]

[0]

[45]

[45]

[-45]

[-45]

[90]

Step 2: Sub-laminate Stacking permutation

Selection of stacking sequence: **not optimized**

45
0
-45
0
45
0
-45
90
45
0
-45
0
45
0
-45
90

Thousands of permutations

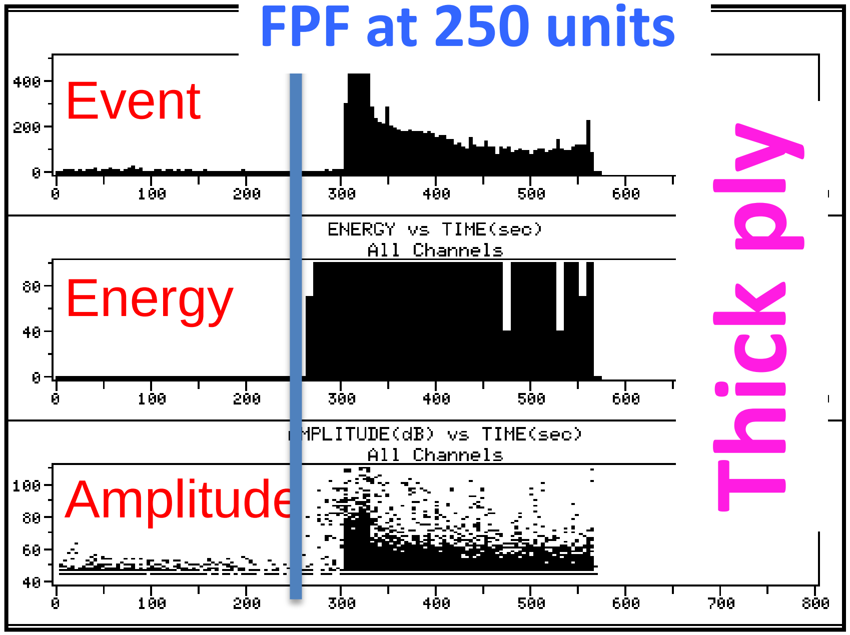
Acoustic Response of [±45/0/90]$_S$ Coupons

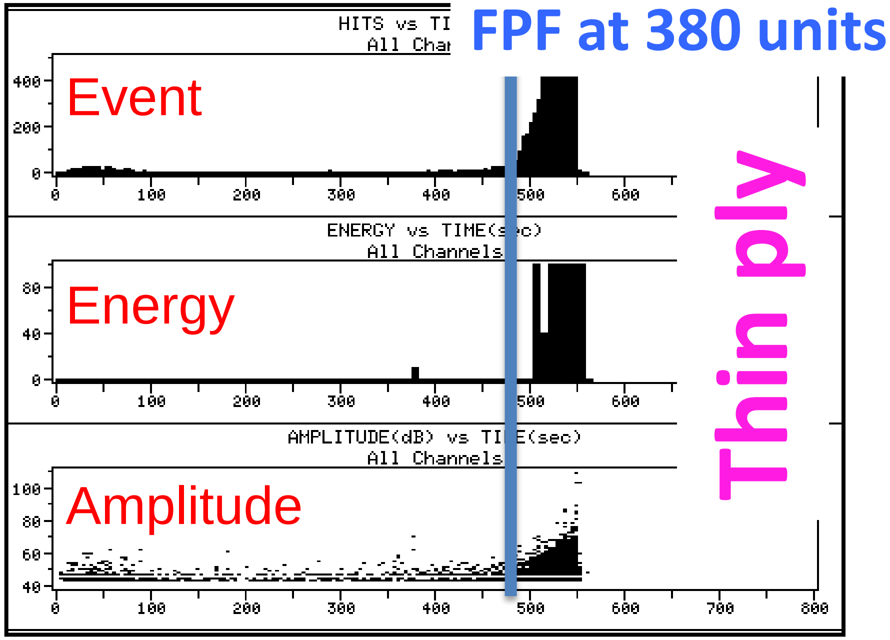
Normal ply thickness: 0.12 mm

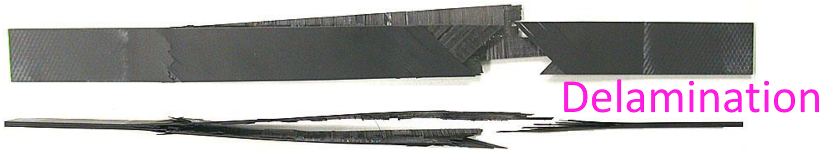
Note extensive signals after FPF

Thin ply: 0.04 mm

Less signals after much higher FPF







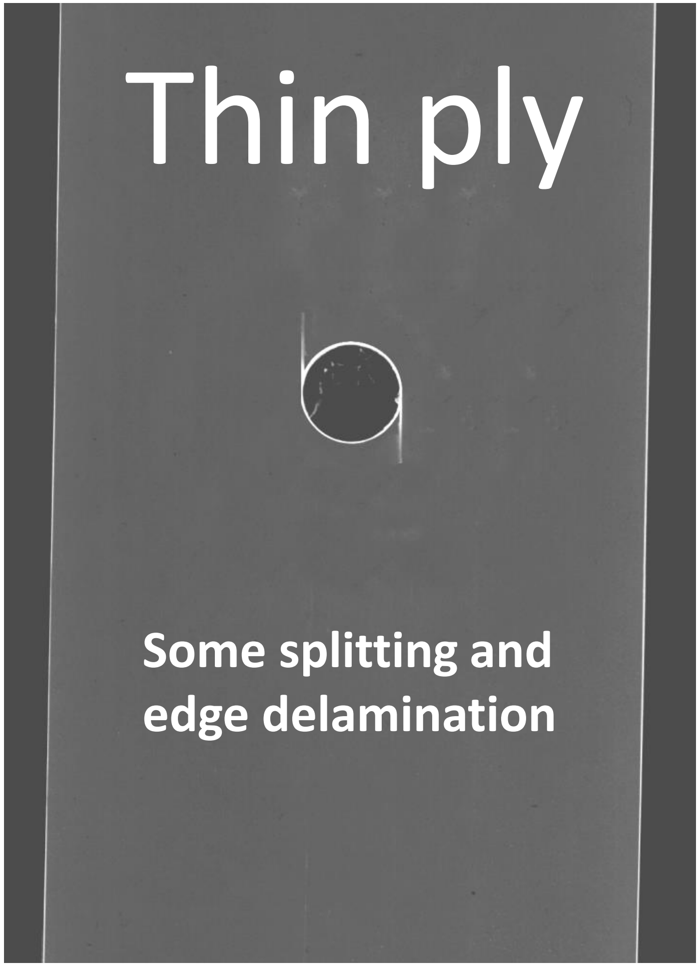
Top and side views of failed coupon, same total thickness

Note extensive delamination of thick ply coupon on the left

Tension Fatigue at RT - (50/40/10)

σ_{max} = 70 ksi (70% static), R = 0.1, f = 5 Hz, after 73,000 cycles
Ply thickness = 0.04 mm, Laminate thickness = 3.2 mm

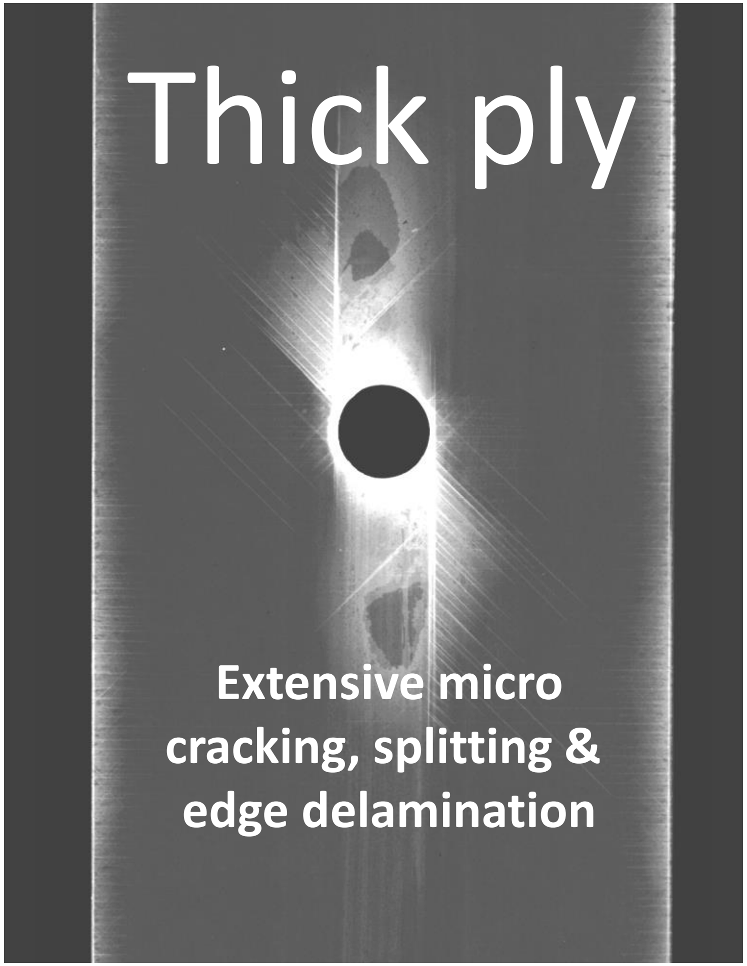
$[45/0_2/-45/90/45/0_2/45/0]_{5S}$



Thin ply

Some splitting and edge delamination

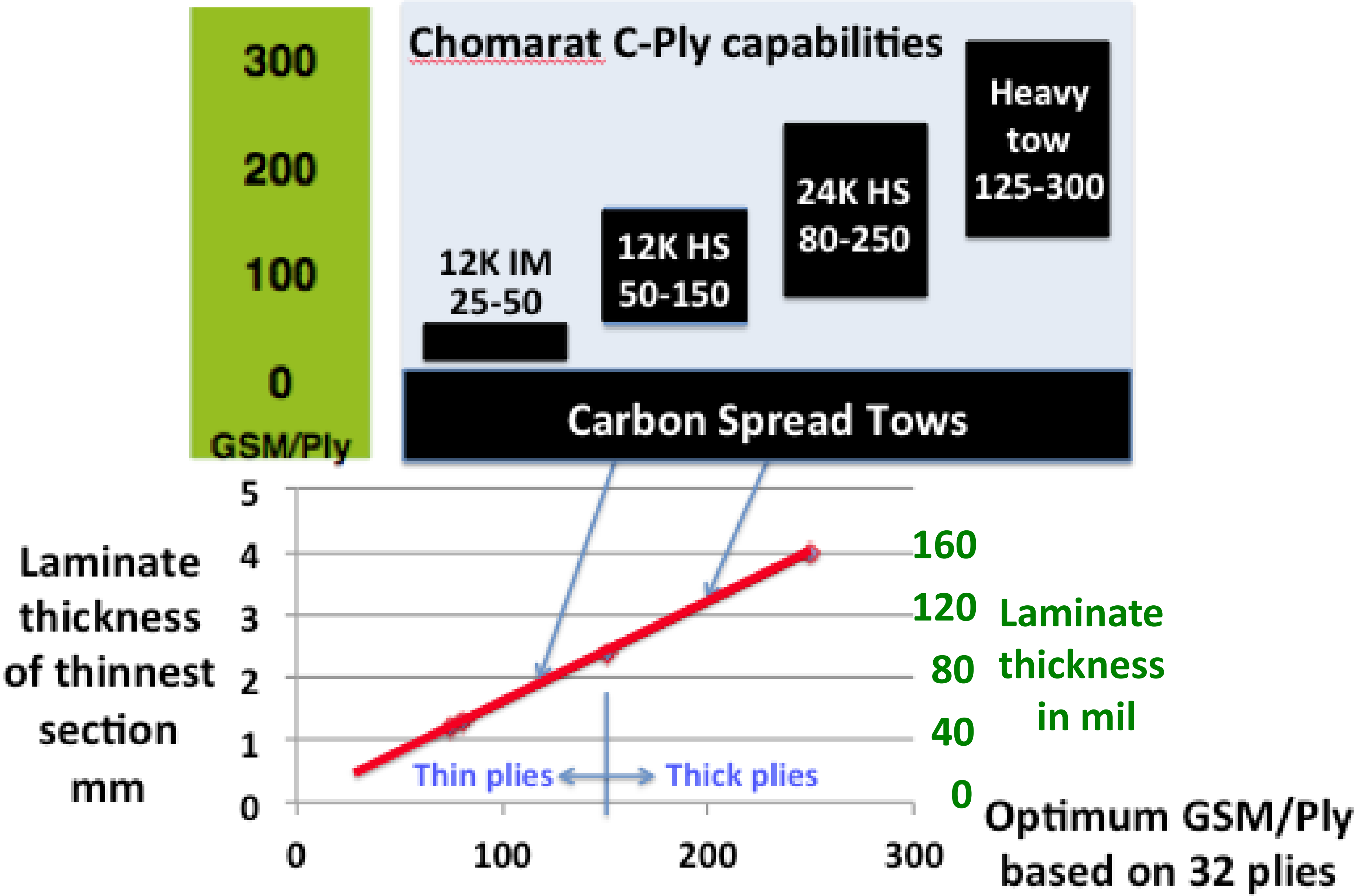
$[45_5/0_{10}/-45_5/90_5/45_5/0_{10}/45_5/0_5]_S$



Thick ply

Extensive micro cracking, splitting & edge delamination

Wide-range GSM to Meet Requirement



Advantages of Thin Plies

- Micro cracking and delamination suppressed
- Easy formation of bi-angle C-Ply to improve handling, and avoid layup of extra layers
- Good building block from bi- to tri-angle tape
- Provide design options for thin fuselage skins
- Increase layup speed with multi-angle tape
- Easy to reach homogenized laminates
- Once homogenized, options become possible: asymmetry, single ply drop, and optimization

Too Many Stacking Permutations

Jeremy Sanford, Spirit

$[0_p/\pm45_q/90_r], 1 \le [p,q,r] \le 3$

Permu-tations

4-ply sub-lam [1,2,1]

5-ply sub-lam [2,2,1], [1,2,2]

[90/0/-45/45] [90/45/0/-45] [0/90/45/-45] [0/90/-45/45] ...

24

120

6-ply sub-laminate → 180

8-ply sub-laminate → 2,520

10-ply sub-laminate → 25,250

[0] ply data

Material [0]	Qxx*	Qyy*	Qss*	Tr, GPa	Trace*	Ex*	Ey*	nu/x*
IM7/977-3	0.88	0.046	0.036	218	1.00	0.88	0.046	0.35
T800/Cytec	0.90	0.050	0.027	183	1.00	0.89	0.049	0.40
T700 C-Ply	0.88	0.058	0.034	139	1.00	0.87	0.058	0.30
AS4/3501	0.86	0.056	0.044	162	1.00	0.85	0.055	0.30
IM6/epoxy	0.88	0.049	0.036	232	1.00	0.88	0.048	0.32
AS4/F937	0.89	0.058	0.027	168	1.00	0.88	0.057	0.30
T300/N5208	0.88	0.050	0.035	206	1.00	0.88	0.050	0.28
Master ply	0.883	0.0502	0.0348	183	1.000	0.876	0.0500	0.300
Coeff var	1.1%	0.44%	0.53%			1.2%	0.5%	4.1%

Laminates have lower cv than plies

* normalized by Trace

[0/±30]

[0/±30]: 1:0	A11*	A22*	A66*	Tr, GPa	Trace*	E1*	E2*	nu/21*
IM7/977-3	0.65	0.091	0.13	218	1.00	0.52	0.072	1.2
T800/Cytec	0.66	0.091	0.13	183	1.00	0.50	0.069	1.3
T700 C-Ply	0.64	0.099	0.13	139	1.00	0.52	0.079	1.1
AS4/3501	0.64	0.101	0.13	162	1.00	0.53	0.084	1.0
IM6/epoxy	0.65	0.093	0.13	232	1.00	0.52	0.074	1.2
AS4/F937	0.65	0.096	0.13	168	1.00	0.50	0.074	1.2
T300/N5208	0.65	0.093	0.13	206	1.00	0.52	0.075	1.2
Master ply	0.647	0.0930	0.130	183	1.000	0.515	0.0745	1.18
Coeff var	0.57%	0.36%	0.16%			1.0%	0.5%	8.4%

$[0_2/\pm30_2/\pm60/90]$

[0/±30]; 2:1	A11*	A22*	A66*	Tr, GPa	Trace*	E1*	E2*	nu/21*
IM7/977-3	0.46	0.28	0.13	218	1.00	0.42	0.25	0.40
T800/Cytec	0.47	0.28	0.13	183	1.00	0.42	0.25	0.43
T700 C-Ply	0.46	0.28	0.13	139	1.00	0.42	0.25	0.40
AS4/3501	0.46	0.28	0.13	162	1.00	0.42	0.26	0.37
IM6/epoxy	0.46	0.28	0.13	232	1.00	0.42	0.25	0.39
AS4/F937	0.46	0.28	0.13	168	1.00	0.41	0.25	0.42
T300/N5208	0.46	0.28	0.13	206	1.00	0.42	0.25	0.39
Master ply	0.463	0.278	0.130	183	1.000	0.418	0.252	0.398
Coeff var	0.26%	0.12%	0.19%			0.18%	0.20%	1.75%

* normalized by Trace

$[0_2/\pm45_3/90]$

[0/±45]; 2:1	A11*	A22*	A66*	Tr, GPa	Trace*	E1*	E2*	nu/21*
IM7/977-3	0.38	0.29	0.16	218	1.00	0.32	0.24	0.48
T800/Cytec	0.39	0.29	0.16	183	1.00	0.31	0.23	0.52
T700 C-Ply	0.38	0.29	0.16	139	1.00	0.31	0.24	0.49
AS4/3501	0.38	0.29	0.16	162	1.00	0.32	0.25	0.46
IM6/epoxy	0.39	0.29	0.16	232	1.00	0.32	0.24	0.48
AS4/F937	0.38	0.29	0.16	168	1.00	0.31	0.23	0.52
T300/N5208	0.39	0.29	0.16	206	1.00	0.32	0.24	0.48
Master ply	0.385	0.293	0.161	183	1.000	0.316	0.240	0.485
Coeff var	0.08%	0.10%	0.09%			0.5%	0.5%	2.2%

Normalized Master Laminate Factors

Need only one test: $E_x/0.876 = \mathrm{Tr}\,[A°]$ >>> factors for $E_1°$, $E_2°$, ν_x, $E_6°$

Zero test: If you believe in rule of mixtures that $E_x = v_f E_f$

Or another single test of [π/4]: $E_1°/0.337 = \mathrm{Tr}\,[A°]$, …

Master Laminate	E1°/Tr	E2°/Tr	nu/x	E6°/Tr	Trace*
[0]	0.876	0.050	0.300	0.0343	1.000
[0/90]	0.468	0.468	0.036	0.031	0.999
[0/45/90/-45] = [π/4]	0.337	0.337	0.298	0.130	1.000
[0/±30]	0.515	0.0745	1.180	0.130	0.998
[(0/±30)2/±60/90]	0.418	0.252	0.398	0.130	1.000
[0/±30/±60/90] = [π/6]	0.338	0.338	0.297	0.130	1.002
[0/±45]	0.377	0.158	0.709	0.161	1.000
[(0)2/(±45)3/90]	0.316	0.240	0.485	0.161	0.999
[0/(±45)2/90]	0.280	0.280	0.419	0.161	1.001

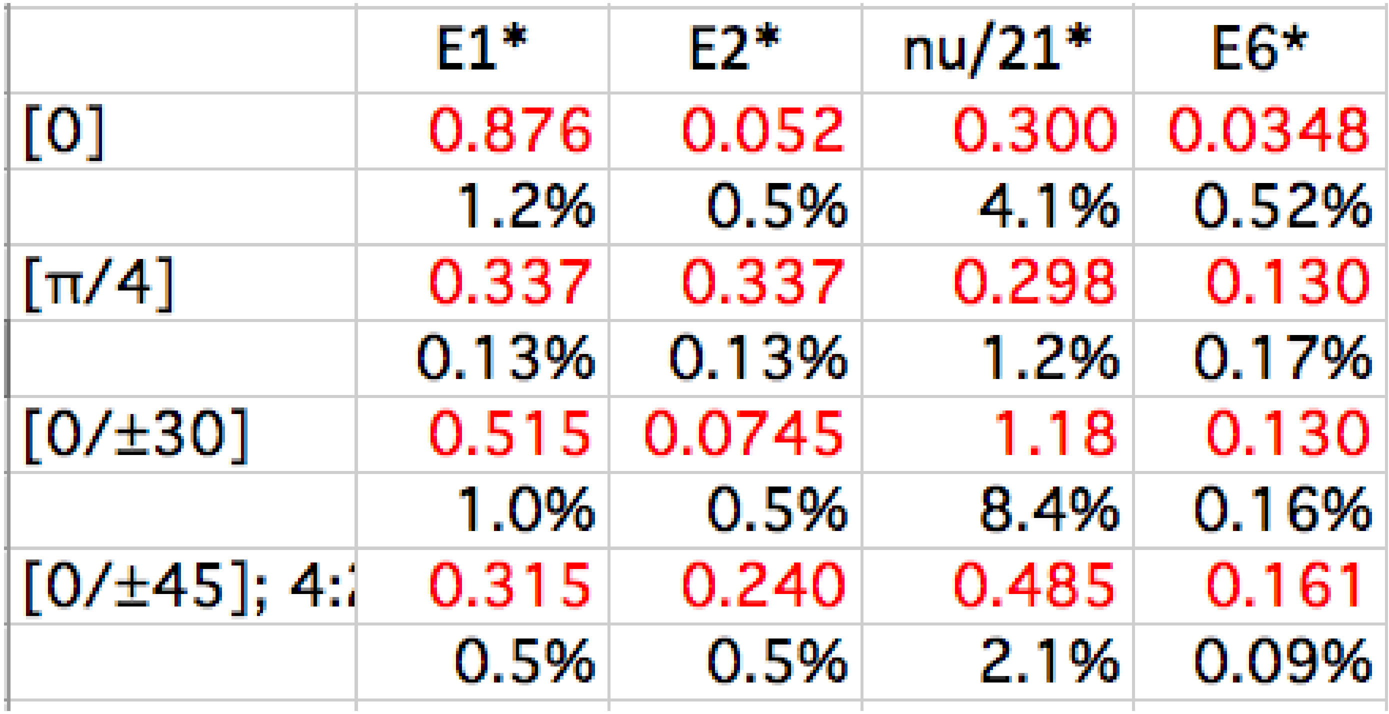
Examples: For [0/±45], $E_1° = 0.377$ Tr; $E_6° = 0.161$ Tr (shear test can be avoided)

For C-Ply 55, Tr = 139 GPa, $E_1° = 0.377 \times 139 = 52.4$ GPa; $E_6° = 0.161 \times 139 = 22.4$ GPa

For T800/Cytec, Tr = 183 GPa, , $E_1° = 0.377 \times 183 = 69.0$; $E_6° = 0.161 \times 183 = 29.4$ GPa

How Many Specimens: 1 or 0

$E_f \overset{v_f}{>>>>>>>>} E_x \overset{0.88}{>>>>>>>>}$ Trace [Q] >>>>> Laminate stiffness:



	E1*	E2*	nu/21*	E6*
[0]	0.876	0.052	0.300	0.0348
	1.2%	0.5%	4.1%	0.52%
[π/4]	0.337	0.337	0.298	0.130
	0.13%	0.13%	1.2%	0.17%
[0/±30]	0.515	0.0745	1.18	0.130
	1.0%	0.5%	8.4%	0.16%
[0/±45]; 4:	0.315	0.240	0.485	0.161
	0.5%	0.5%	2.1%	0.09%

Lowest Cost Layup of Thick-thin C-Ply

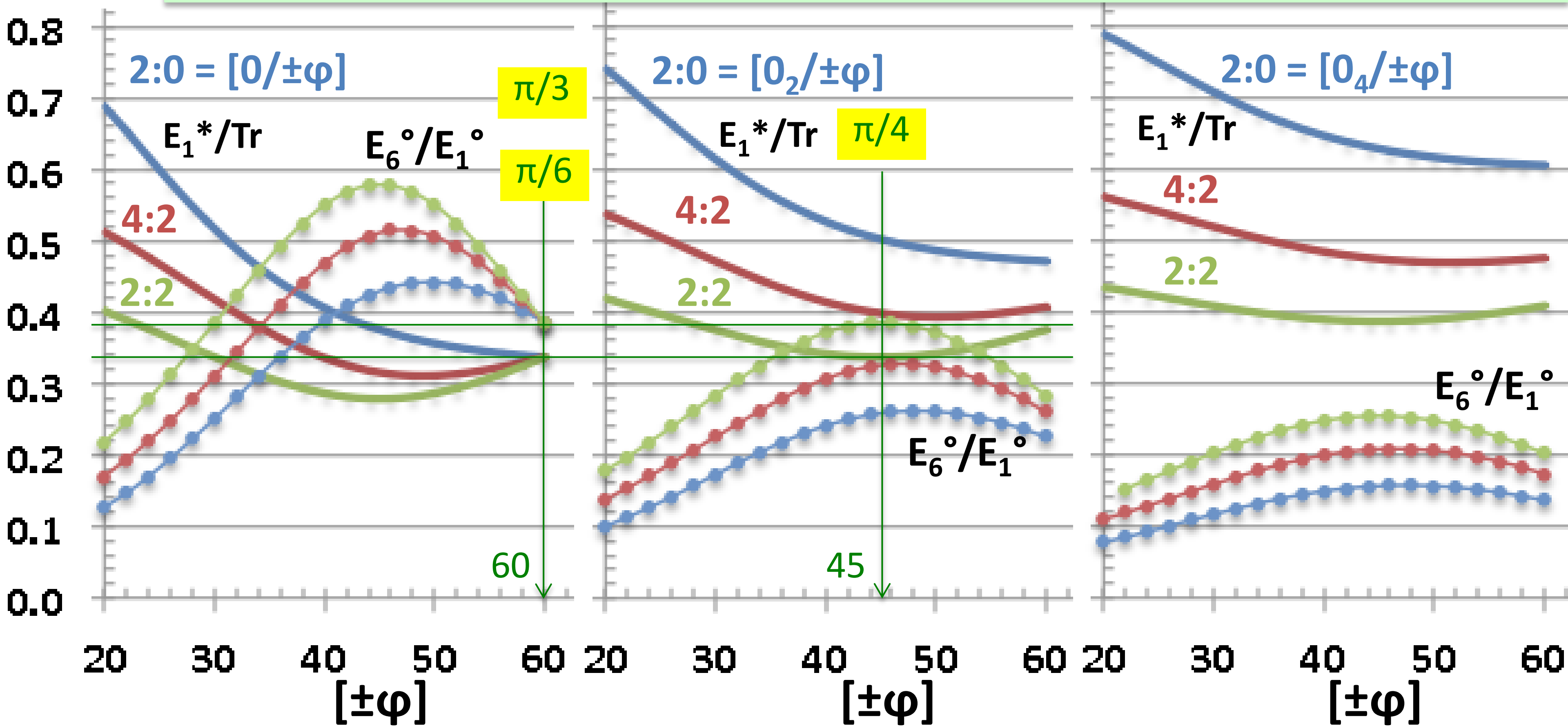
Starting C-Ply	$[0/\varphi_2]$ - Thin-Thick (33/67/0) – 150 gsm	$[0/\varphi]$ - Thin-Thin (50/50/0) – 100 gsm	$[0_2/\varphi]$ - Thick-Thin (67/33/0) – 150 gsm
1-axis 2:0	$[0/\pm\varphi]_2$ $= [\pi/3]_2$ for $\varphi = 60$ (33/67/0)	$[0_2/\pm\varphi]$ (50/50/0)	$[0_4/\pm\varphi]$ (67/33/0)
2-axis 4:2	$[(0/\pm\varphi)_2/(\pm\psi/90)]_2$ $\Psi = 90 - \varphi$ (22/67/11)	$[(0_2/\pm\varphi)_2/\pm\psi_2/90_2]$ $\Psi = 90 - \varphi$ (33/50/17)	$[(0_4/\pm\varphi)_2/\pm\psi/90_4]$ $\Psi = 90 - \varphi$ (44/33/22)
2-axis 2:2	$[0/\pm\varphi/\pm\psi/90]_2$ $= [\pi/6]_2$ for $\varphi = 30$ (17/66/17)	$[0_2/\pm\varphi/\pm\psi/90_2]$ $= [\pi/4]_2$ for $\varphi = 45$ (25/50/25)	$[0_4/\pm\varphi/\pm\psi/90_4]$ $\Psi = 90 - \varphi$ (33/33/33)

A Master Laminate Design Chart

Smooth lines = trace normalized = E_1^*, GPa ; Dots = E_6/E_1

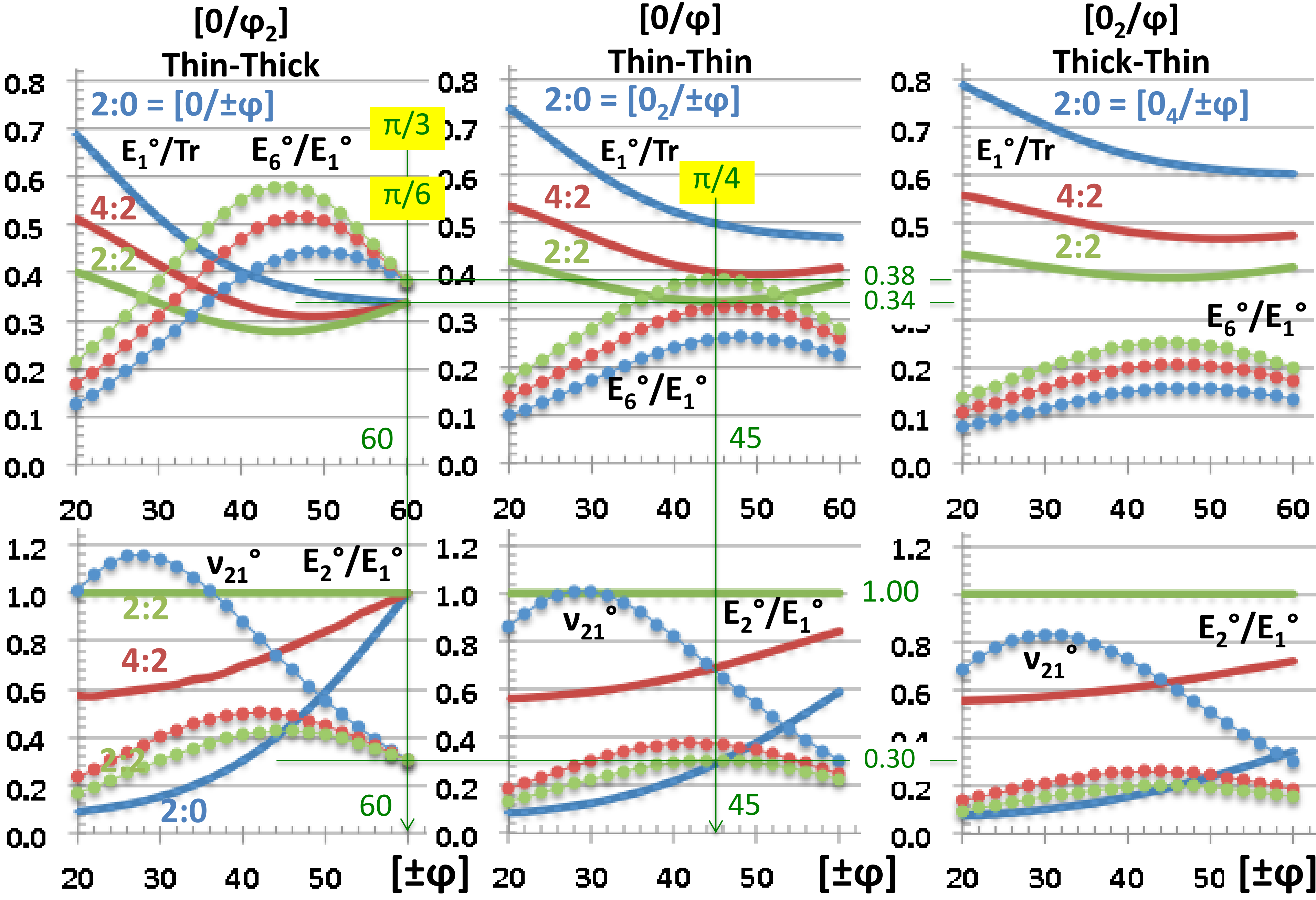
Bi-angle C-Ply:

[0/φ$_2$] **Thin-Thick**	**[0/φ]** **Thin-Thin**	**[0$_2$/φ]** **Thick-Thin**



Master Ply Stiffness Chart

Bi-angle C-Ply:



$[0/\varphi_2]$ Thin-Thick

$[0/\varphi]$ Thin-Thin

$[0_2/\varphi]$ Thick-Thin

Plane Elasticity & Bending Equations

Plane elasticity: $$a_{22}^{*}\frac{\partial^4 F}{\partial x^4}+(2a_{12}^{*}+a_{66}^{*})\frac{\partial^4 F}{\partial x^2\partial y^2}+a_{11}^{*}\frac{\partial^4 F}{\partial y^4}=0.$$

Plate bending: $$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}=0$$

Homogeneity: [D*] = [A*]; [B] = 0

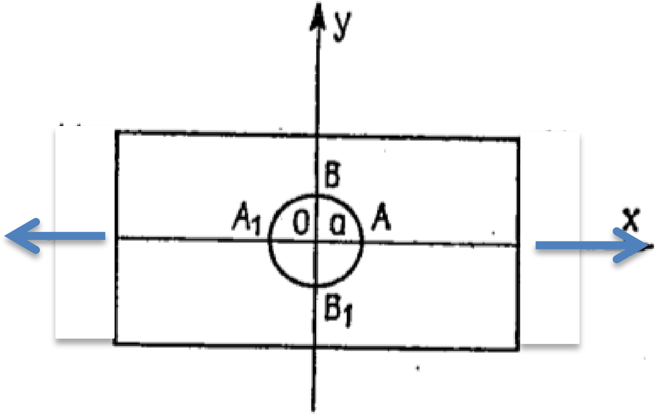
[0/±30]; 2:0	$\frac{2a_{12}+a_{66}}{a_{22}}$	$\frac{a_{11}}{a_{22}}$	$\frac{2(A_{12}+2A_{66})}{A_{11}}$	$\frac{A_{22}}{A_{11}}$	**Trace, GPa**
IM7/977-3	2.23	0.58	1.20	0.58	218
T800/Cytec	2.23	0.58	1.20	0.58	183
T7 C-Ply 55	2.23	0.58	1.20	0.58	139
T7 C-Ply 64	2.23	0.58	1.20	0.58	163
AS4/3501	2.18	0.59	1.23	0.59	162
IM6/epoxy	2.23	0.58	1.20	0.58	232
AS4/F937	2.33	0.59	1.19	0.59	168
T300/N5208	2.24	0.58	1.20	0.58	206
Master ply	**2.23**	**0.58**	**1.20**	**0.58**	**175**
Std dev	0.044	0.004	0.013	0.004	
Coeff var	2.0%	0.7%	1.1%	0.7%	

Lekhnitskii’s Elasticity Solutions

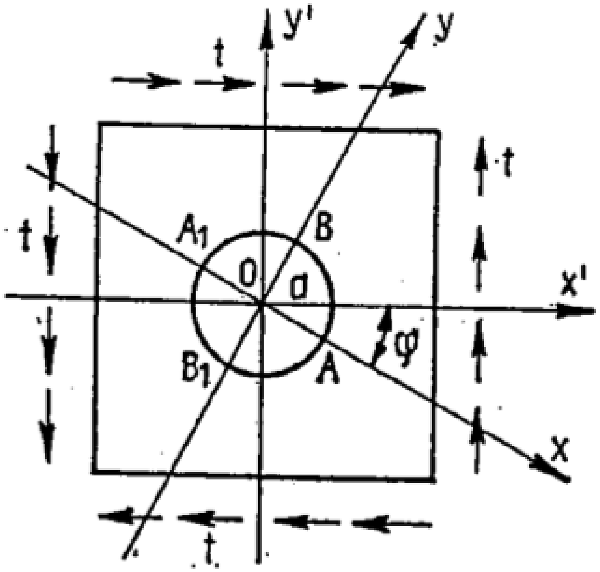
$$k = -\mu_1\mu_2 = \sqrt{\frac{E_1}{E_2}}$$

$$n = -i(\mu_1 + \mu_2) = \sqrt{2\left(\frac{E_1}{E_2} - \nu_1\right) + \frac{E_1}{G}}$$

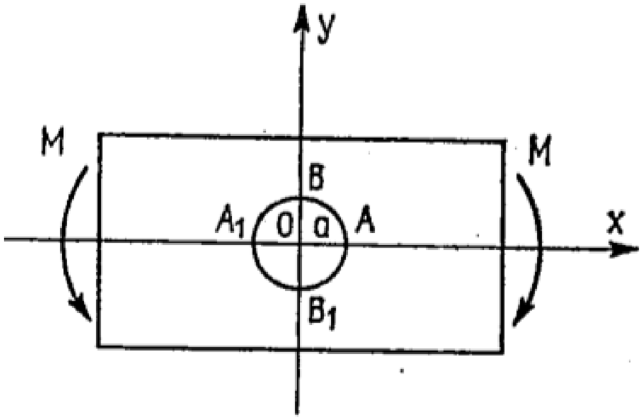
Key parameters: *k, n*



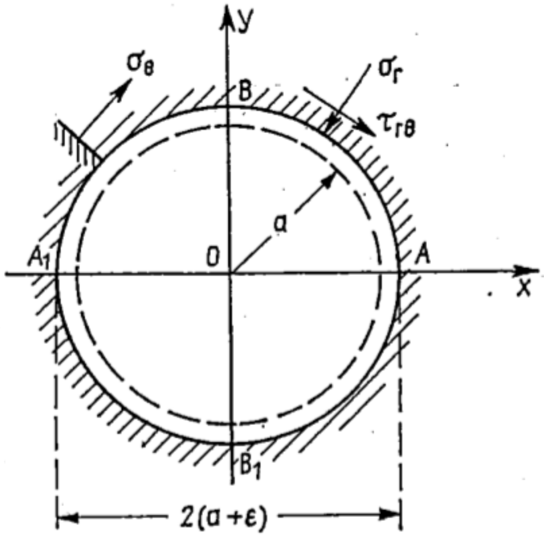
Open hole tension



Shear

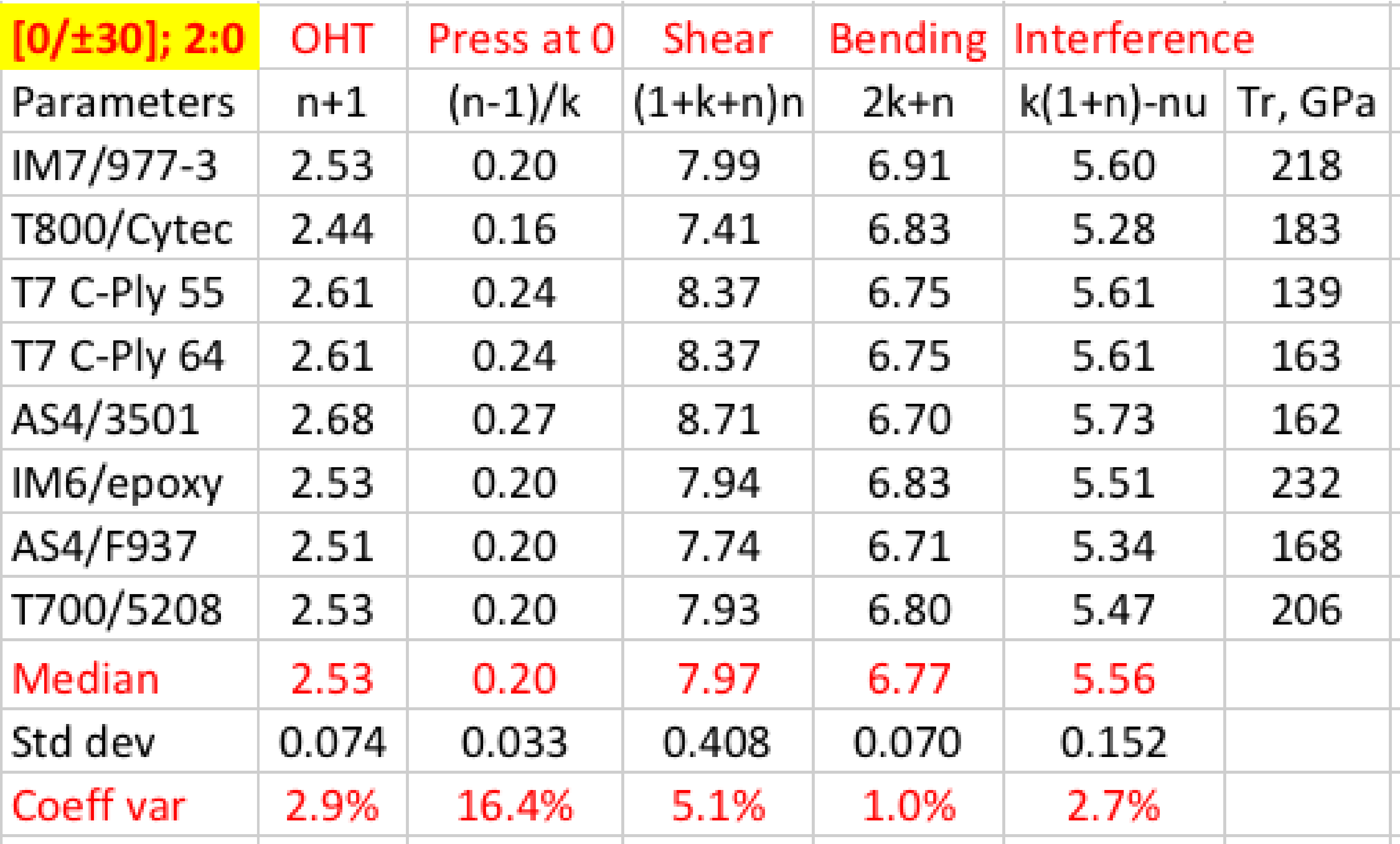


Bending



Interference

Same Solutions for 8 CFRP’s for [0/±30]

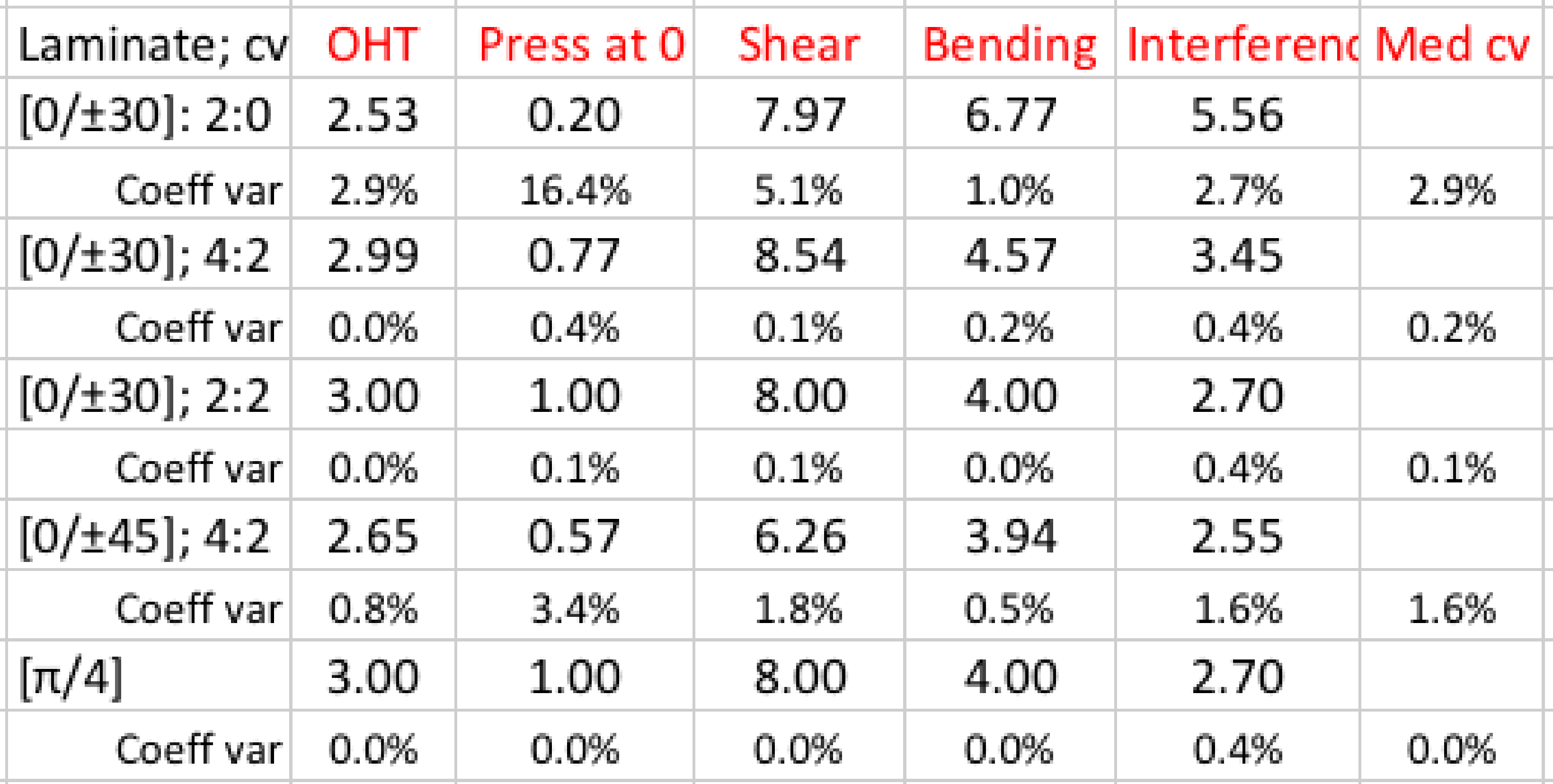


[0/±30]; 2:0	OHT	Press at 0	Shear	Bending	Interference	
Parameters	n+1	(n-1)/k	(1+k+n)n	2k+n	k(1+n)-nu	Tr, GPa
IM7/977-3	2.53	0.20	7.99	6.91	5.60	218
T800/Cytec	2.44	0.16	7.41	6.83	5.28	183
T7 C-Ply 55	2.61	0.24	8.37	6.75	5.61	139
T7 C-Ply 64	2.61	0.24	8.37	6.75	5.61	163
AS4/3501	2.68	0.27	8.71	6.70	5.73	162
IM6/epoxy	2.53	0.20	7.94	6.83	5.51	232
AS4/F937	2.51	0.20	7.74	6.71	5.34	168
T700/5208	2.53	0.20	7.93	6.80	5.47	206
Median	2.53	0.20	7.97	6.77	5.56	
Std dev	0.074	0.033	0.408	0.070	0.152	
Coeff var	2.9%	16.4%	5.1%	1.0%	2.7%	

Median values can be used for most cases with error less than experimental

Exact solutions from Lekhnitskii’s *Anisotropic Plates*

Solutions for Different Laminates



Laminate; cv	OHT	Press at 0	Shear	Bending	Interferenc	Med cv
[0/±30]: 2:0	2.53	0.20	7.97	6.77	5.56	
Coeff var	2.9%	16.4%	5.1%	1.0%	2.7%	2.9%
[0/±30]; 4:2	2.99	0.77	8.54	4.57	3.45	
Coeff var	0.0%	0.4%	0.1%	0.2%	0.4%	0.2%
[0/±30]; 2:2	3.00	1.00	8.00	4.00	2.70	
Coeff var	0.0%	0.1%	0.1%	0.0%	0.4%	0.1%
[0/±45]; 4:2	2.65	0.57	6.26	3.94	2.55	
Coeff var	0.8%	3.4%	1.8%	0.5%	1.6%	1.6%
[π/4]	3.00	1.00	8.00	4.00	2.70	
Coeff var	0.0%	0.0%	0.0%	0.0%	0.4%	0.0%

Median values can be used for different laminates with error less than experimental

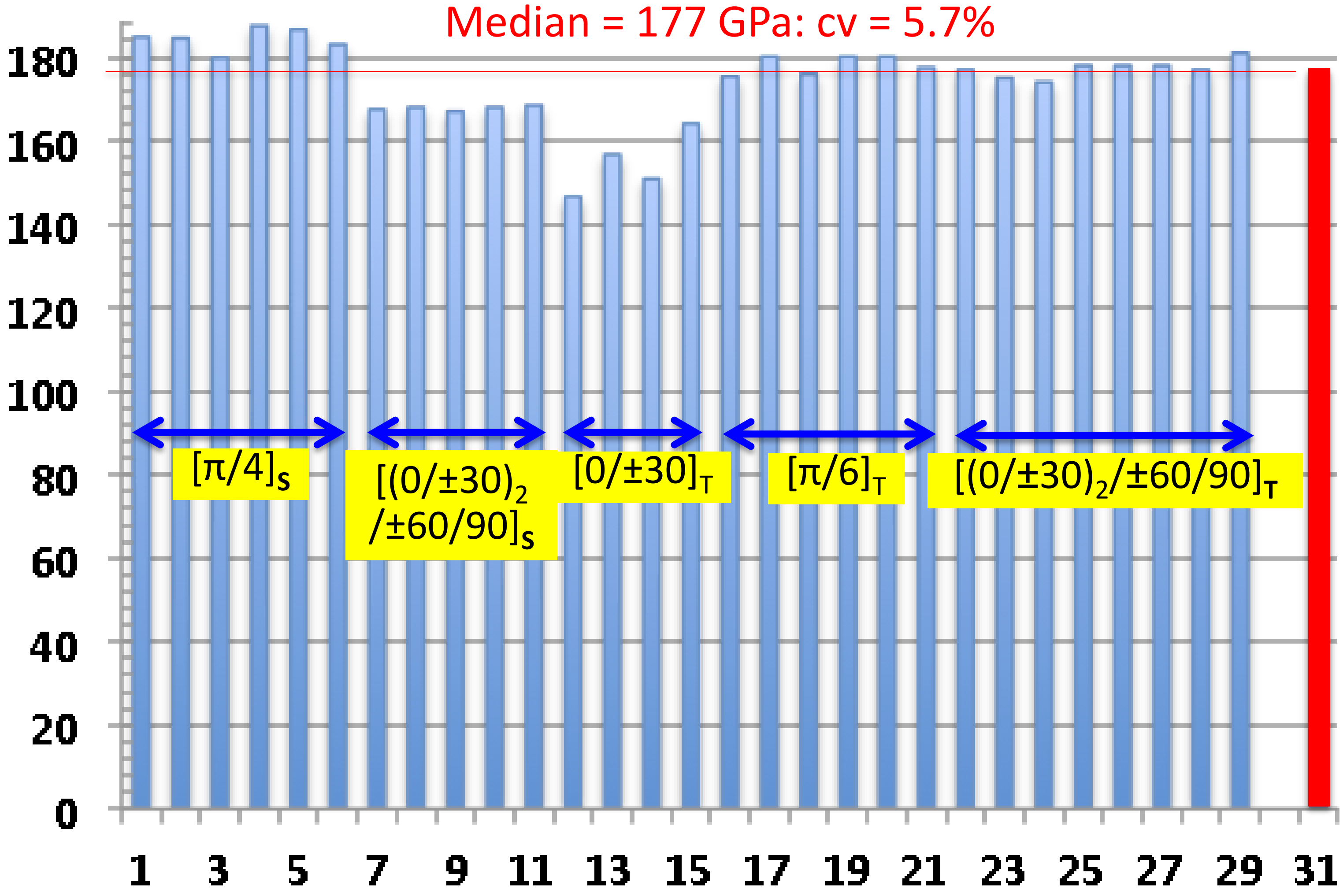
One Test for Trace = Multiple Solutions

Trace

Lam; Ratio	OHT	Press at 0	Shear	Bending	Interference
[0/±30]: 2:0	✓	✓	✓	✓	✓
[0/±30]: 4:2	✓	✓	✓	✓	✓
[0/±30]; 2:2	✓	✓	✓	✓	✓
[0/±45]; 2:0	✓	✓	✓	✓	✓
[0/±45]; 4:2	✓	✓	✓	✓	✓
[0/±45]; 2:2	✓	✓	✓	✓	✓
[0/±phi]; 2:0	✓	✓	✓	✓	✓
[0/±phi]; 4:2	✓	✓	✓	✓	✓
[0/±phi]; 2:2	✓	✓	✓	✓	✓

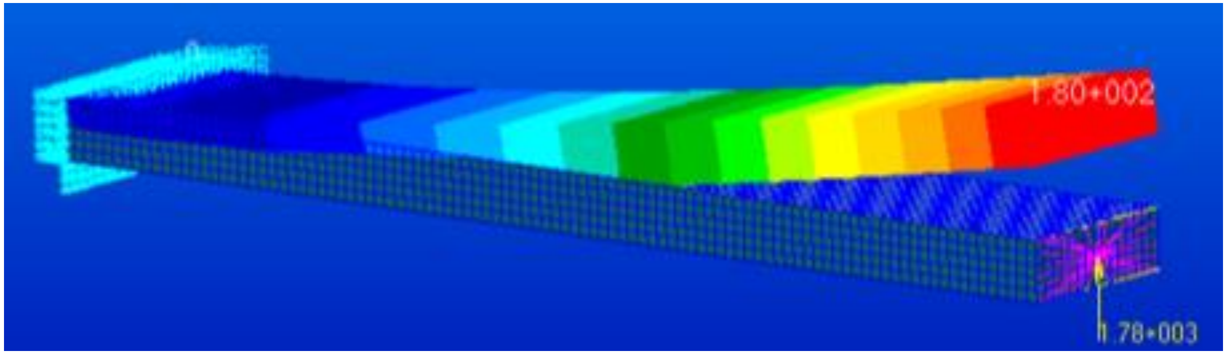
Measurement of Trace from $E_1°$

Material: T800/AR250



Scaling by Trace for Material/Laminate

Giulio Romeo



Scale materials: same [0/±45/90]$_{8S}$

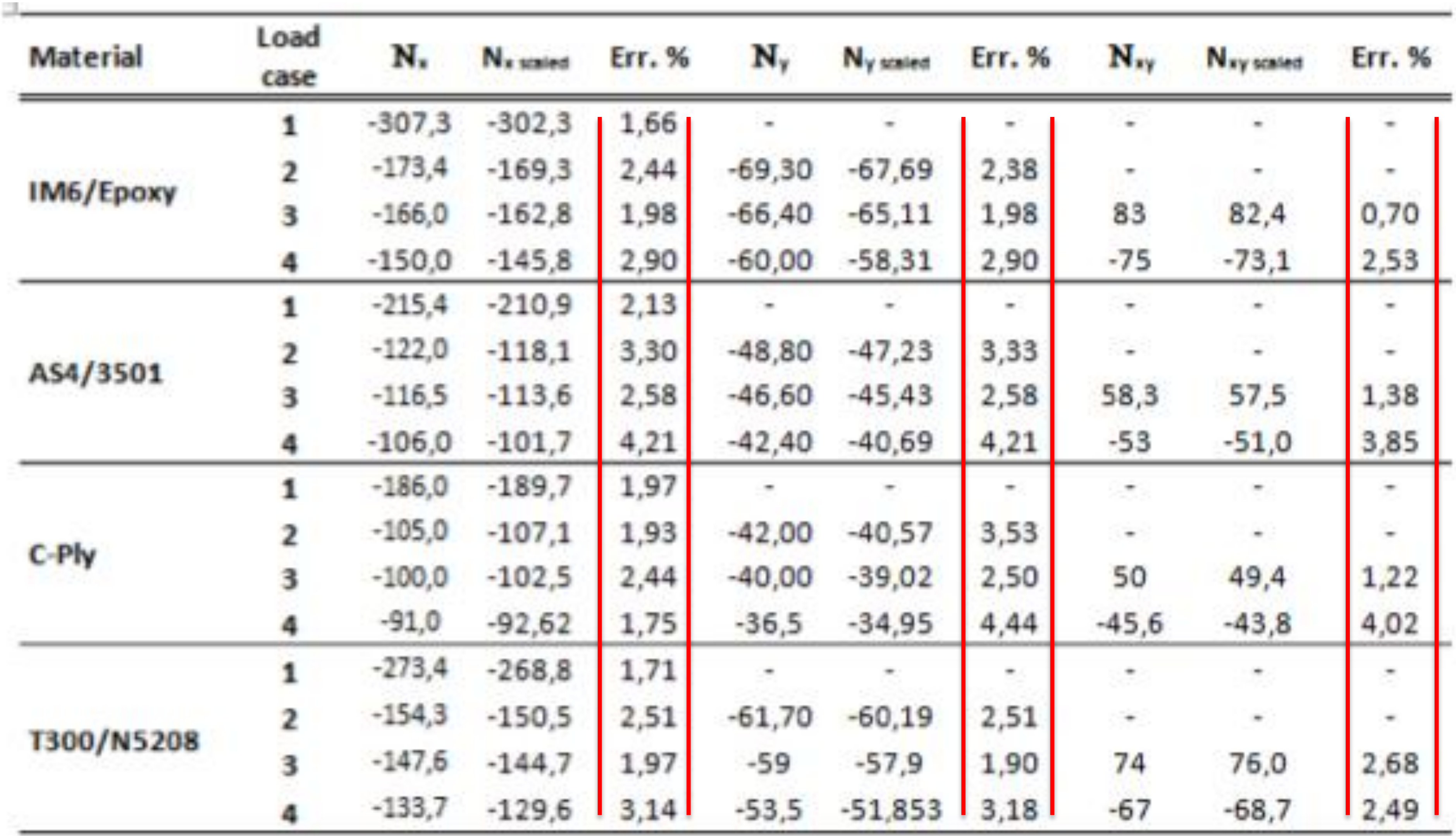
Material	η	η_{scaled}	Err. %
IM6/Epoxy	180,27	180,27	0,00
AS4/3501	257,60	258,36	-0,30
C-Ply	301,42	300,79	0,21
T300/N5208	202,72	202,72	0,00

Scale laminates: same T300/N5208

Material	θ	η	η_{scaled}	Err. %
[0/±θ/90]$_{8S}$	15	120,77	120,58	0,16
	30	147,98	150,68	1,79
	60	249,34	259,48	3,91
	75	270,35	286,56	5,66
[0/±θ/90]$_{16T}$	15	121,059	121,061	0,00
	30	148,84	151,284	1,62
	60	250,63	260,519	3,80
	75	271,87	287,706	5,50

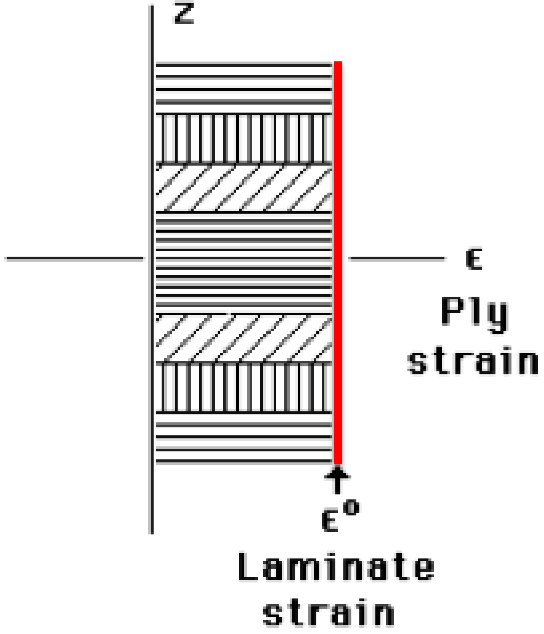
Scaling by Trace for Panel Buckling

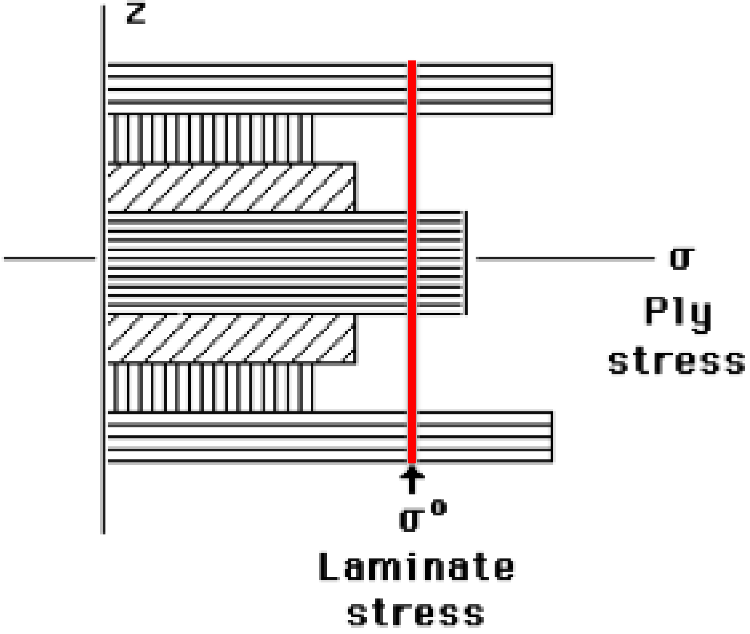
Giulio Romeo



Material	Load case	N_x	$N_{x\ scaled}$	Err. %	N_y	$N_{y\ scaled}$	Err. %	N_{xy}	$N_{xy\ scaled}$	Err. %
IM6/Epoxy	1	-307,3	-302,3	1,66	-	-	-	-	-	-
	2	-173,4	-169,3	2,44	-69,30	-67,69	2,38	-	-	-
	3	-166,0	-162,8	1,98	-66,40	-65,11	1,98	83	82,4	0,70
	4	-150,0	-145,8	2,90	-60,00	-58,31	2,90	-75	-73,1	2,53
AS4/3501	1	-215,4	-210,9	2,13	-	-	-	-	-	-
	2	-122,0	-118,1	3,30	-48,80	-47,23	3,33	-	-	-
	3	-116,5	-113,6	2,58	-46,60	-45,43	2,58	58,3	57,5	1,38
	4	-106,0	-101,7	4,21	-42,40	-40,69	4,21	-53	-51,0	3,85
C-Ply	1	-186,0	-189,7	1,97	-	-	-	-	-	-
	2	-105,0	-107,1	1,93	-42,00	-40,57	3,53	-	-	-
	3	-100,0	-102,5	2,44	-40,00	-39,02	2,50	50	49,4	1,22
	4	-91,0	-92,62	1,75	-36,5	-34,95	4,44	-45,6	-43,8	4,02
T300/N5208	1	-273,4	-268,8	1,71	-	-	-	-	-	-
	2	-154,3	-150,5	2,51	-61,70	-60,19	2,51	-	-	-
	3	-147,6	-144,7	1,97	-59	-57,9	1,90	74	76,0	2,68
	4	-133,7	-129,6	3,14	-53,5	-51,853	3,18	-67	-68,7	2,49

Ply Strain and Stress of a Laminate



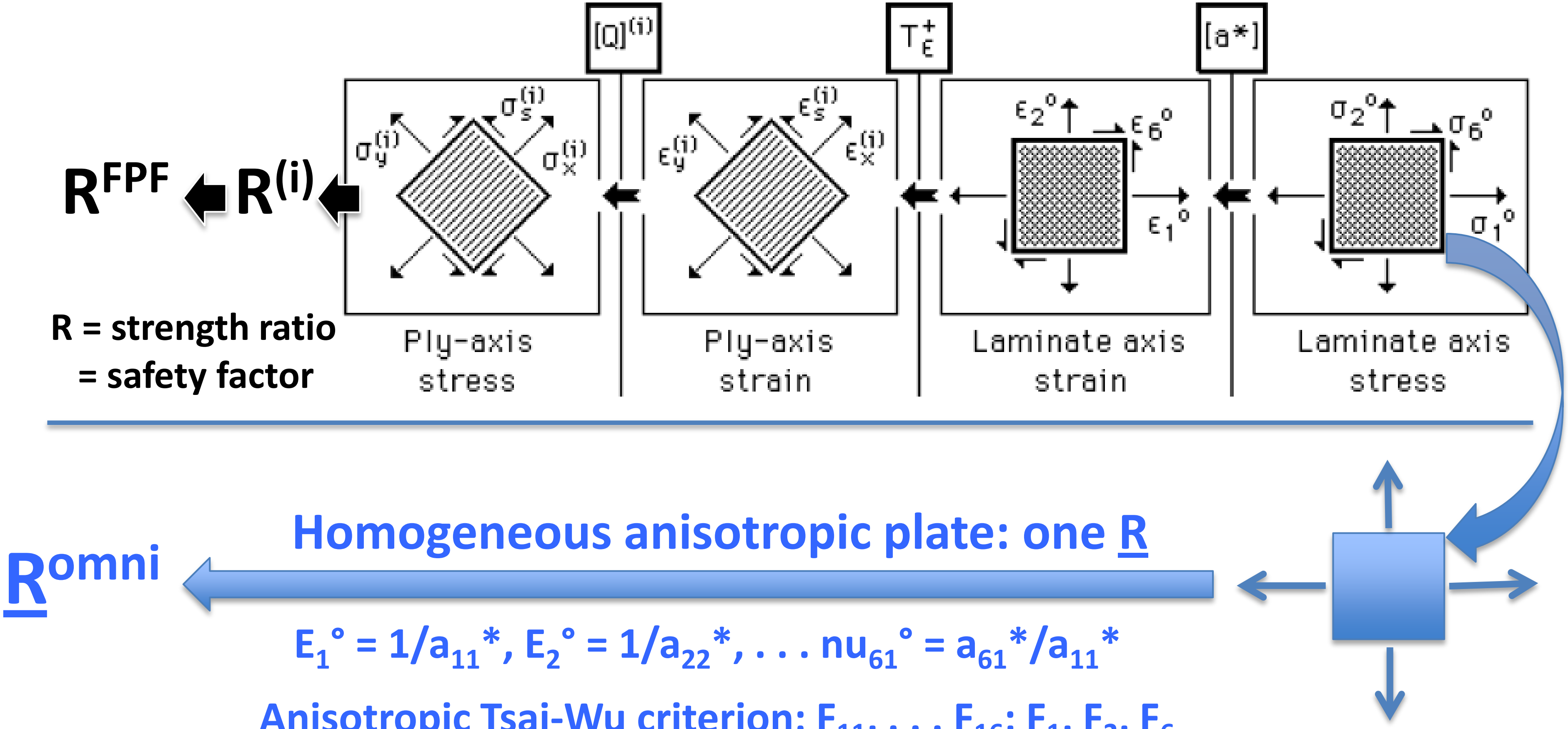


Since ply and laminate strains are equal, strain-based failure criteria are functions of ply angles only, independent of ply composition of the laminate. So a strain-based criterion is the same for all laminates

Ply stress various from ply to ply depending on the ply angles. The stiffer ply will have higher ply stress. Unlike strain-based failure, stress-based failure tensors [F] and {F} are functions of not only each ply angle but also ply composition of the laminate. Thus each laminate has its own failure envelope.

Ply-by-Ply vs Homogenized Plate

Ply-by-ply $R^{(i)}$ of a laminated anisotropic or orthotropic plate



$R^{FPF} \leftarrow R^{(i)} \leftarrow$

R = strength ratio
= safety factor

$\underline{R}^{omni}$

Homogeneous anisotropic plate: one $\underline{R}$

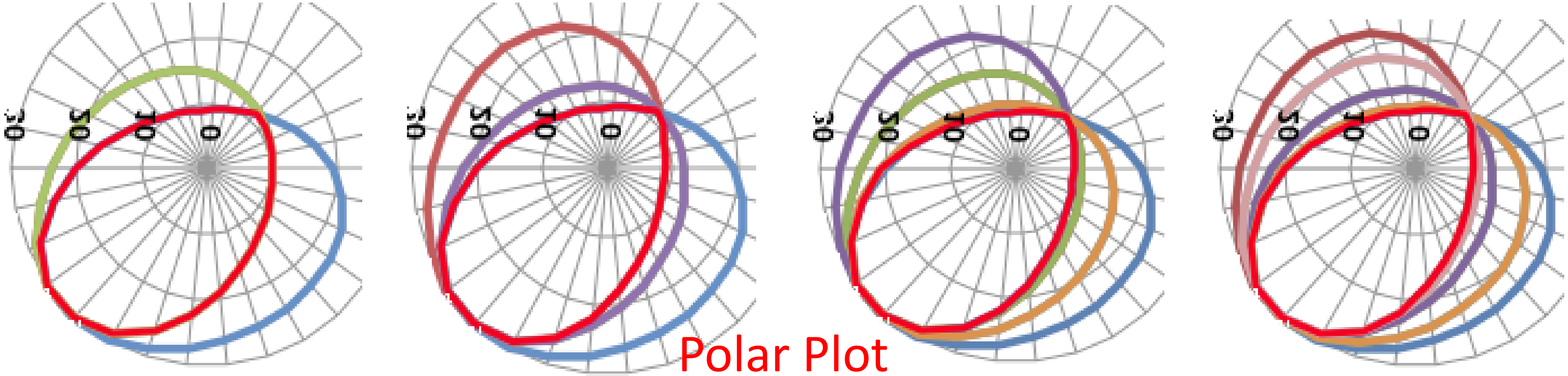
$E_1^{\circ} = 1/a_{11}^{*}$, $E_2^{\circ} = 1/a_{22}^{*}$, . . . $nu_{61}^{\circ} = a_{61}^{*}/a_{11}^{*}$

Anisotropic Tsai-Wu criterion: $\underline{F}_{11}$, . . . $\underline{F}_{16}$; $\underline{F}_{1}$, $\underline{F}_{2}$, $\underline{F}_{6}$

Back to the basics: many closed-form and FEM solutions easily applied;
speed increases by n (number of plies) in model formation and stress recovery

Successive Increase in Ply Angles

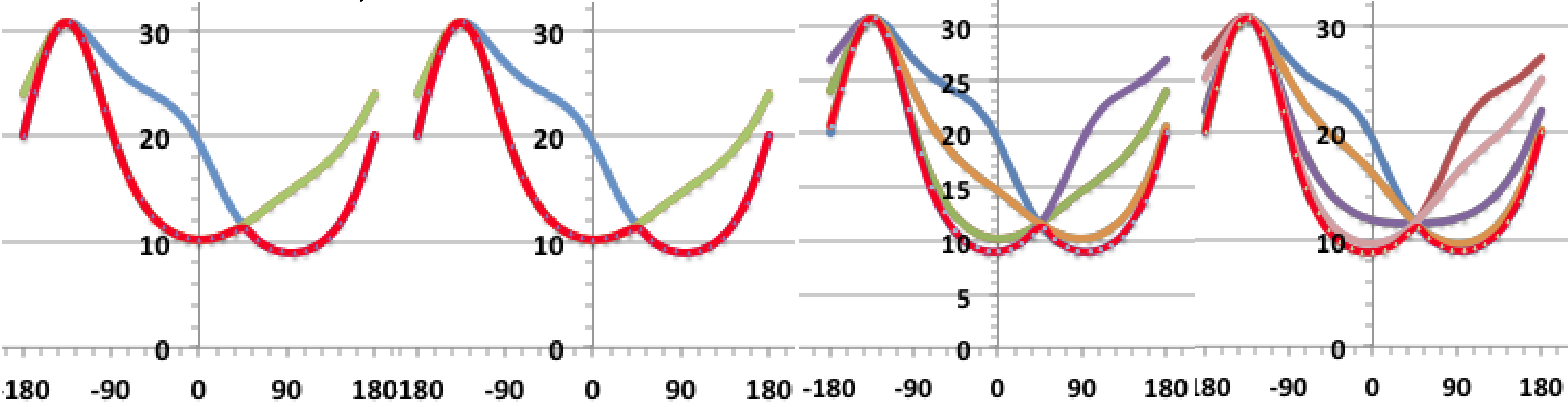
C-Ply 55 [π/3]; 8.94 C-Ply 55 [π/4]; 8.85 C-Ply 55 [π/6]; 8.94 C-Ply 55 [π/8]; 8.85



Polar Plot

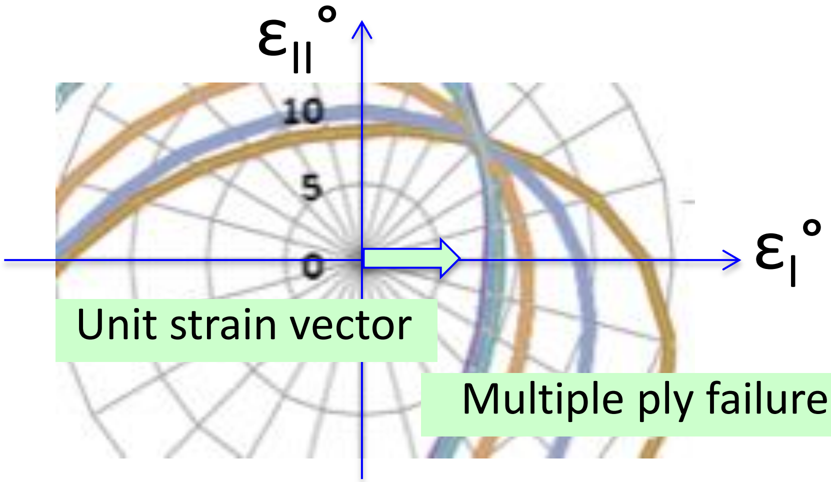
Cartesian Plot

Radial strain, 10^{-3}



Transformation angle of strain envelope radii θ

Omni Strain FPF Envelopes: C-Ply 64



Unit strain vector

Multiple ply failures

Polar angle of radial strain vector: 0 to 2π @15° increments➔➔➔

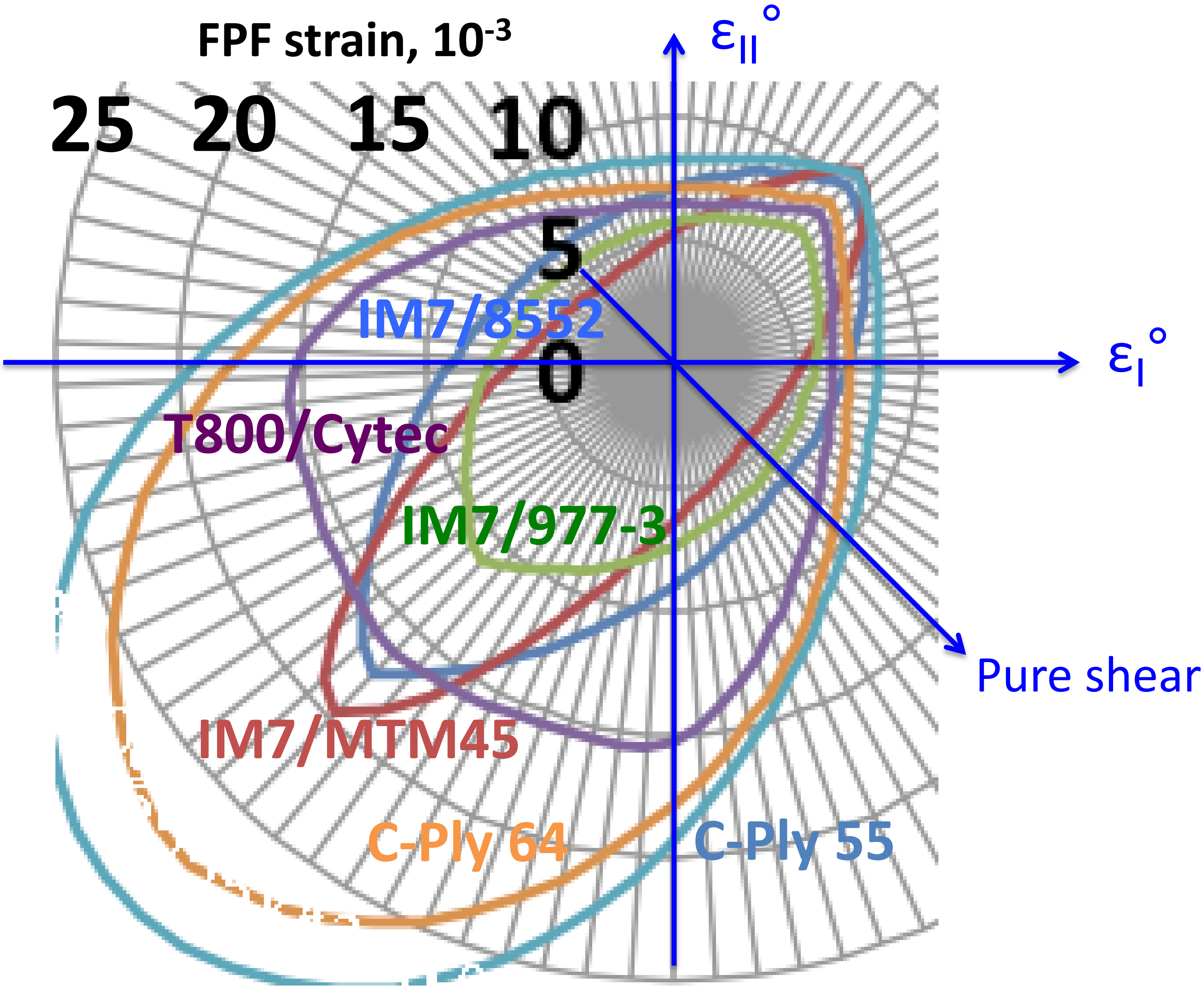
Ply angle: 0 to 2π @15° delta

➔
➔
➔

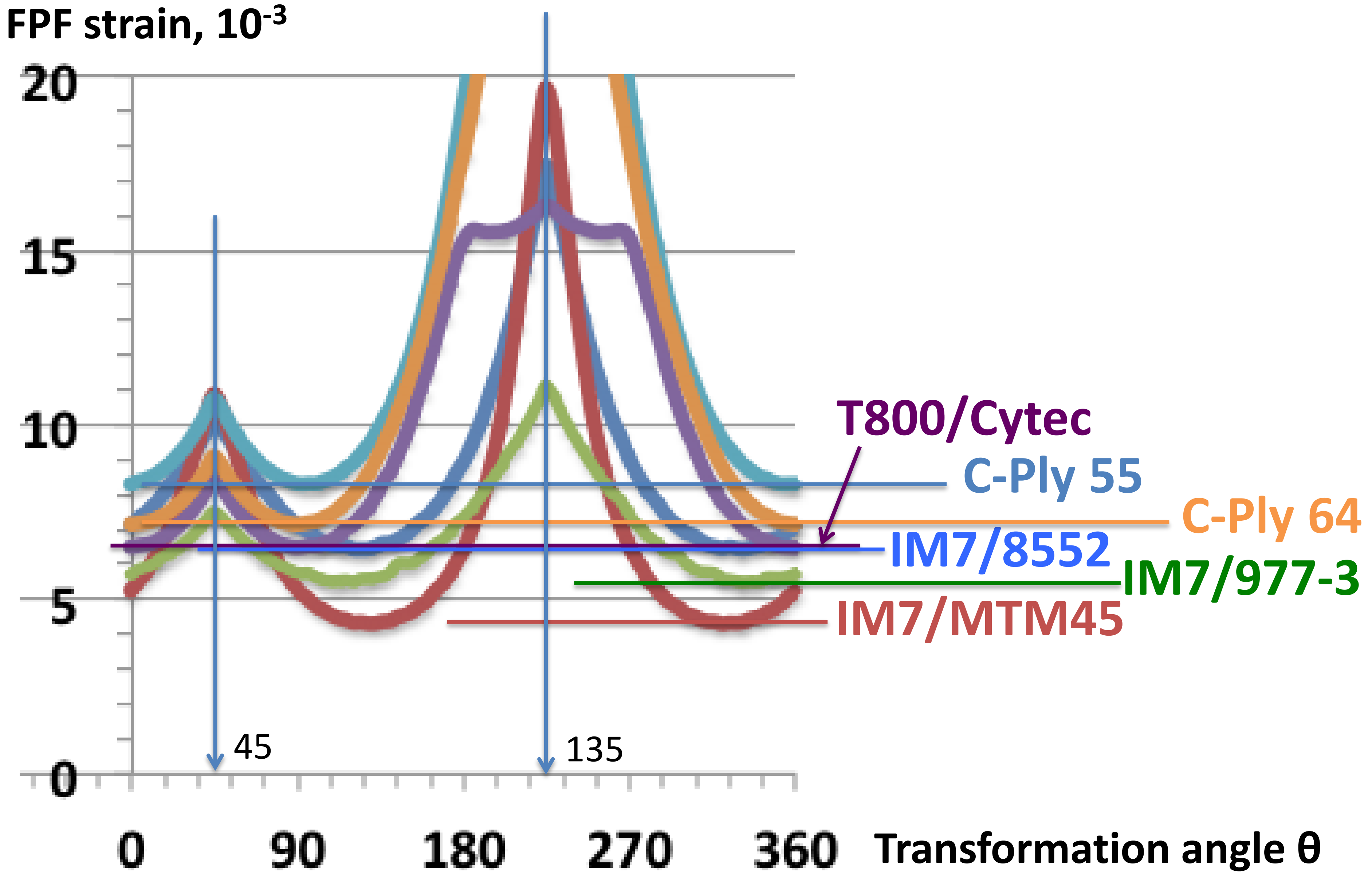
+/- Teta	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
0	18.36	14.37	11.18	9.12	7.91	7.29	7.10	7.30	7.95	9.22	11.45	15.23	20.83	26.46	28.98	28.38	26.60	24.90	23.73	23.13	23.01	23.14	22.97	21.58	18.365
15	15.76	13.17	10.82	9.12	8.05	7.47	7.29	7.49	8.12	9.31	11.36	14.72	19.68	25.17	28.43	28.38	26.49	24.29	22.46	21.15	20.28	19.66	18.99	17.80	15.764
30	11.88	10.97	10.00	9.12	8.45	8.06	7.96	8.18	8.77	9.84	11.59	14.32	18.29	23.21	27.33	28.38	26.34	23.10	20.04	17.61	15.81	14.50	13.52	12.70	11.876
45	9.345	9.18	9.13	9.12	9.13	9.18	9.35	9.70	10.32	11.34	12.89	15.17	18.38	22.49	26.55	28.38	26.55	22.49	18.38	15.17	12.89	11.34	10.32	9.70	9.3451
60	7.963	8.06	8.45	9.12	10.00	10.97	11.88	12.70	13.52	14.50	15.81	17.61	20.04	23.10	26.34	28.38	27.33	23.21	18.29	14.32	11.59	9.84	8.77	8.18	7.9628
75	7.294	7.47	8.05	9.12	10.82									24.29	26.49	28.38	28.43	25.17	19.68	14.72	11.36	9.31	8.12	7.49	7.2942
90	7.097	7.29	7.91	9.12	11.18									24.90	26.60	28.38	28.98	26.46	20.83	15.23	11.45	9.22	7.95	7.30	7.0966
105	7.294	7.47	8.05	9.12	10.82									24.29	26.49	28.38	28.43	25.17	19.68	14.72	11.36	9.31	8.12	7.49	7.2942
120	7.963	8.06	8.45	9.12	10.00	10.97	11.88	12.70	13.52	14.50	15.81	17.61	20.04	23.10	26.34	28.38	27.33	23.21	18.29	14.32	11.59	9.84	8.77	8.18	7.9628
135	9.345	9.18	9.13	9.12	9.13	9.18	9.35	9.70	10.32	11.34	12.89	15.17	18.38	22.49	26.55	28.38	26.55	22.49	18.38	15.17	12.89	11.34	10.32	9.70	9.3451
150	11.88	10.97	10.00	9.12	8.45	8.06	7.96	8.18	8.77	9.84	11.59	14.32	18.29	23.21	27.33	28.38	26.34	23.10	20.04	17.61	15.81	14.50	13.52	12.70	11.876
165	15.76	13.17	10.82	9.12	8.05	7.47	7.29	7.49	8.12	9.31	11.36	14.72	19.68	25.17	28.43	28.38	26.49	24.29	22.46	21.15	20.28	19.66	18.99	17.80	15.764
180	18.36	14.37	11.18	9.12	7.91	7.29	7.10	7.30	7.95	9.22	11.45	15.23	20.83	26.46	28.98	28.38	26.60	24.90	23.73	23.13	23.01	23.14	22.97	21.58	18.365
195	15.76	13.17	10.82	9.12	8.05	7.47	7.29	7.49	8.12	9.31	11.36	14.72	19.68	25.17	28.43	28.38	26.49	24.29	22.46	21.15	20.28	19.66	18.99	17.80	15.764
210	11.88	10.97	10.00	9.12	8.45	8.06	7.96	8.18	8.77	9.84	11.59	14.32	18.29	23.21	27.33	28.38	26.34	23.10	20.04	17.61	15.81	14.50	13.52	12.70	11.876
225	9.345	9.18	9.13	9.12	9.13	9.18	9.35	9.70	10.32	11.34	12.89	15.17	18.38	22.49	26.55	28.38	26.55	22.49	18.38	15.17	12.89	11.34	10.32	9.70	9.3451
240	7.963	8.06	8.45	9.12	10.00	10.97	11.88	12.70	13.52	14.50	15.81	17.61	20.04	23.10	26.34	28.38	27.33	23.21	18.29	14.32	11.59	9.84	8.77	8.18	7.9628
255	7.294	7.47	8.05	9.12	10.82	13.17	15.76	17.80	18.99	19.66	20.28	21.15	22.46	24.29	26.49	28.38	28.43	25.17	19.68	14.72	11.36	9.31	8.12	7.49	7.2942
270	7.097	7.29	7.91	9.12	11.18	14.37	18.37	21.58	22.97	23.14	23.01	23.13	23.73	24.90	26.60	28.38	28.98	26.46	20.83	15.23	11.45	9.22	7.95	7.30	7.0966
285	7.294	7.47	8.05	9.12	10.82	13.17	15.76	17.80	18.99	19.66	20.28	21.15	22.46	24.29	26.49	28.38	28.43	25.17	19.68	14.72	11.36	9.31	8.12	7.49	7.2942
300	7.963	8.06	8.45	9.12	10.00	10.97	11.88	12.70	13.52	14.50	15.81	17.61	20.04	23.10	26.34	28.38	27.33	23.21	18.29	14.32	11.59	9.84	8.77	8.18	7.9628
315	9.345	9.18	9.13	9.12	9.13	9.18	9.35	9.70	10.32	11.34	12.89	15.17	18.38	22.49	26.55	28.38	26.55	22.49	18.38	15.17	12.89	11.34	10.32	9.70	9.3451
330	11.88	10.97	10.00	9.12	8.45	8.06	7.96	8.18	8.77	9.84	11.59	14.32	18.29	23.21	27.33	28.38	26.34	23.10	20.04	17.61	15.81	14.50	13.52	12.70	11.876
345	15.76	13.17	10.82	9.12	8.05	7.47	7.29	7.49	8.12	9.31	11.36	14.72	19.68	25.17	28.43	28.38	26.49	24.29	22.46	21.15	20.28	19.66	18.99	17.80	15.764
360	18.36	14.37	11.18	9.12	7.91	7.29	7.10	7.30	7.95	9.22	11.45	15.23	20.83	26.46	28.98	28.38	26.60	24.90	23.73	23.13	23.01	23.14	22.97	21.58	18.365

Controlling ply angles

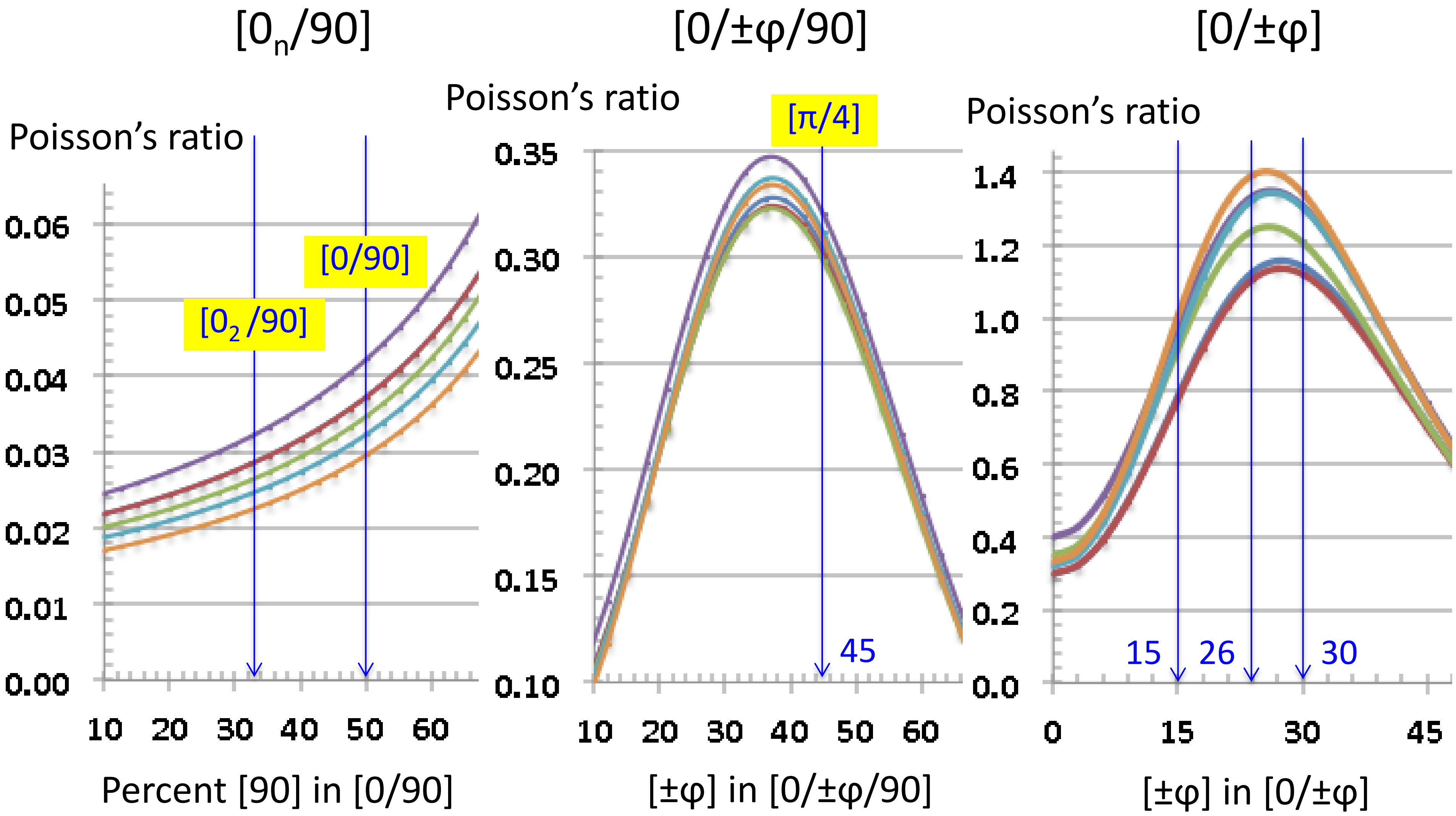
Omni Envelope in Polar Plot



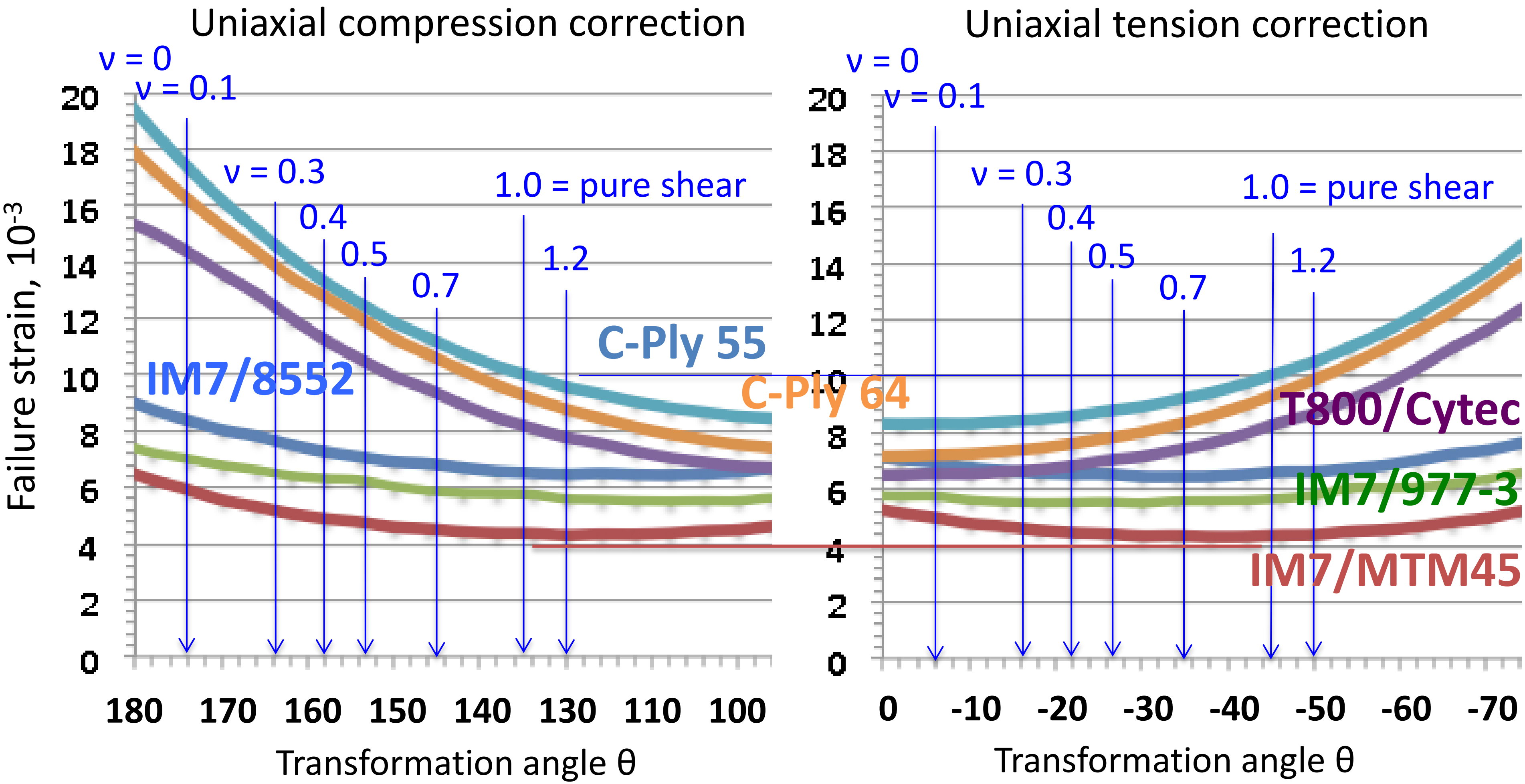
Omni Envelope in Cartesian



Poisson’s Ratio of CFRP Laminates



Poisson's Correction for Omni strain



2^{nd} quadrant of strain envelope

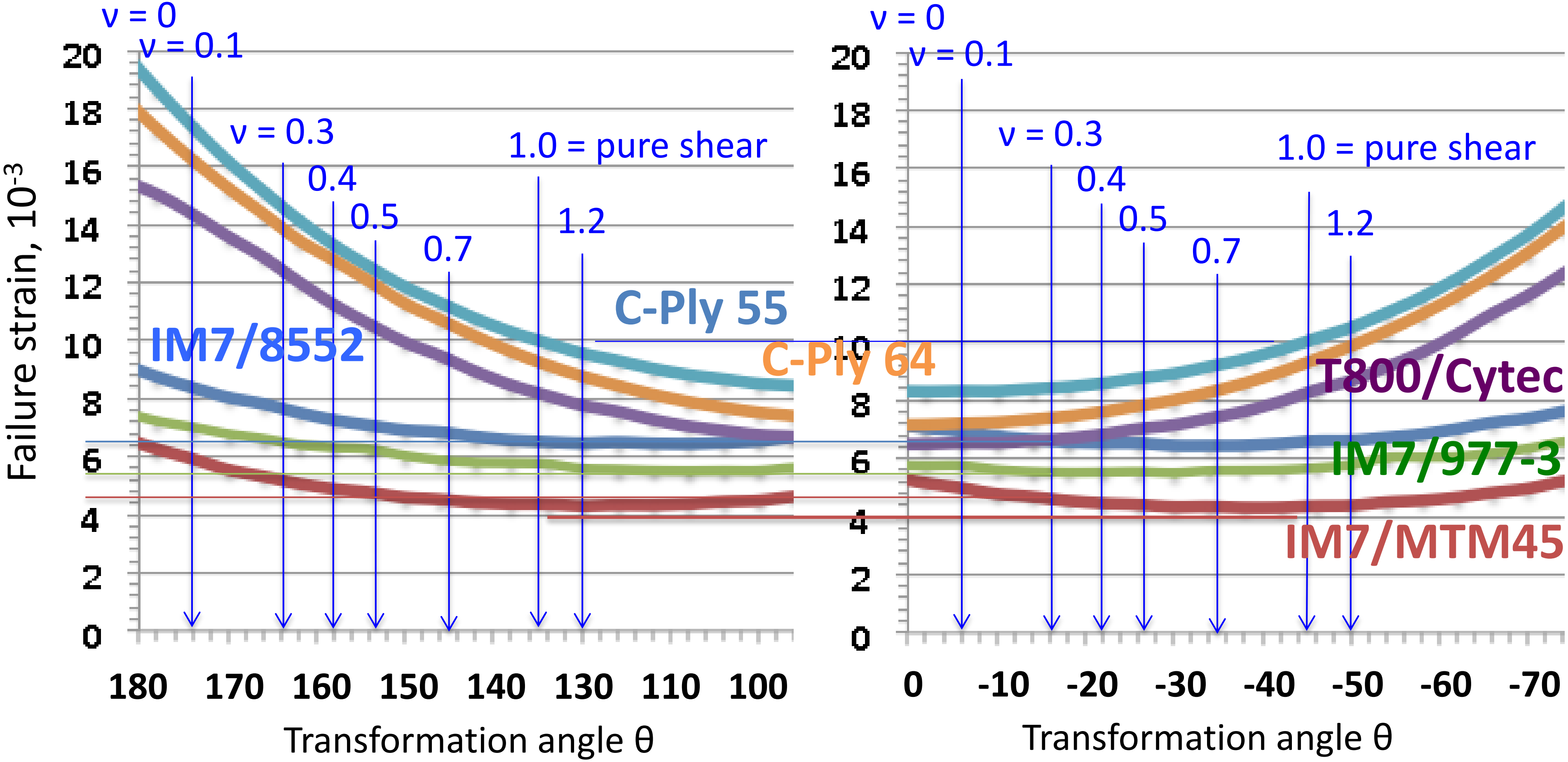
4th quadrant of strain envelope

Angle measured clockwise from 0 degree along x-axis; 90 degree, y-axis

Poisson's Correction for Omni strain

Uniaxial compression correction

Uniaxial tension correction

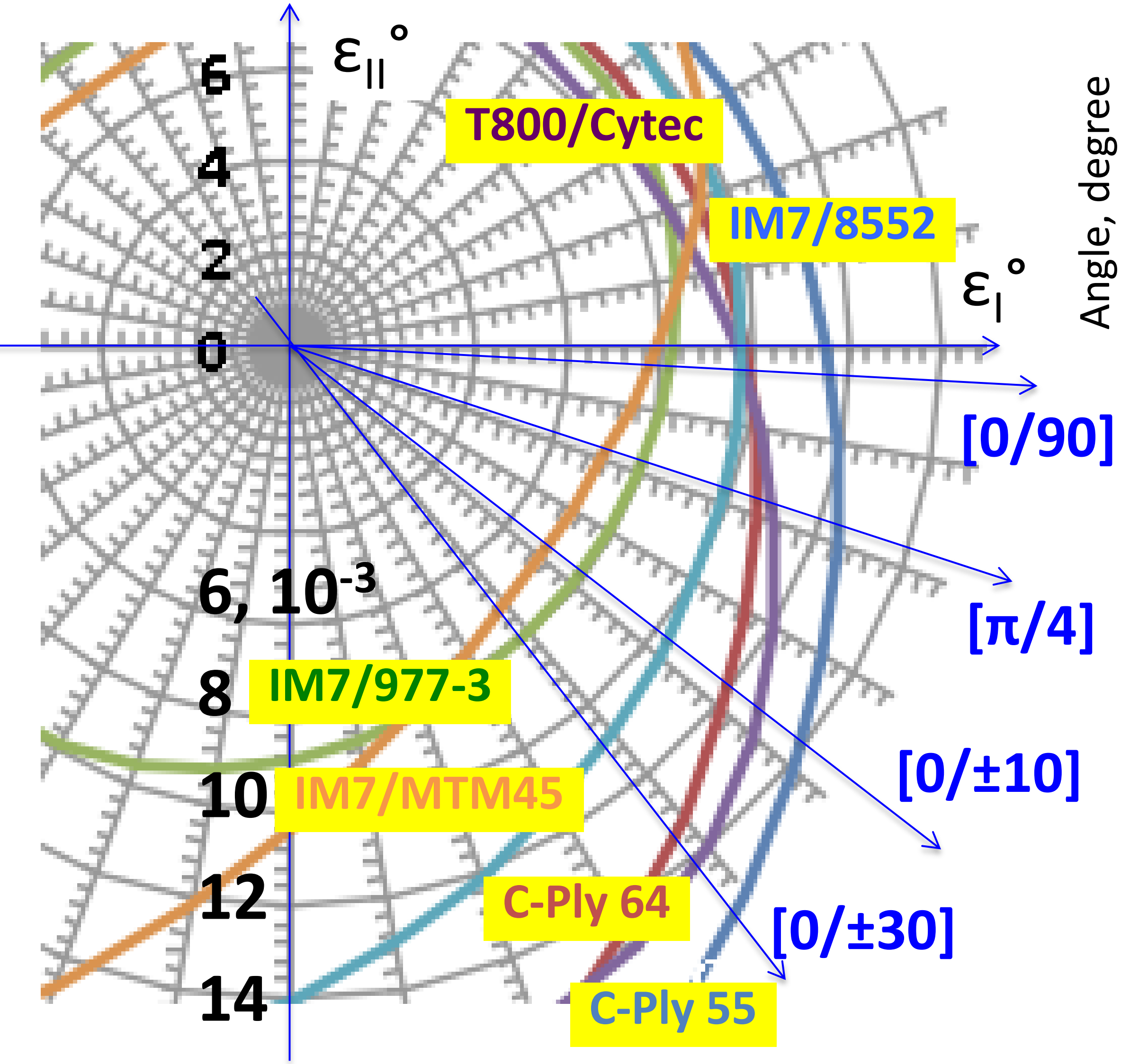


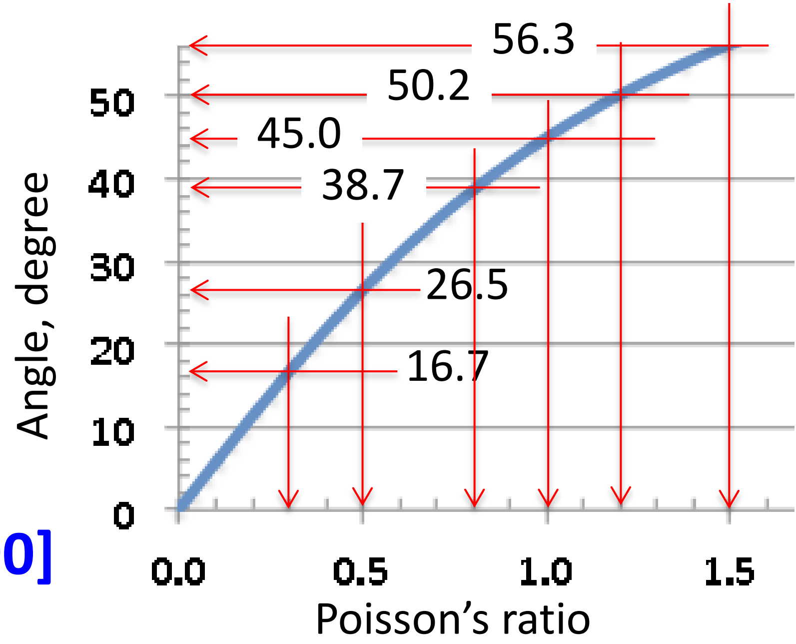
2^{nd} quadrant of strain envelope

4th quadrant of strain envelope

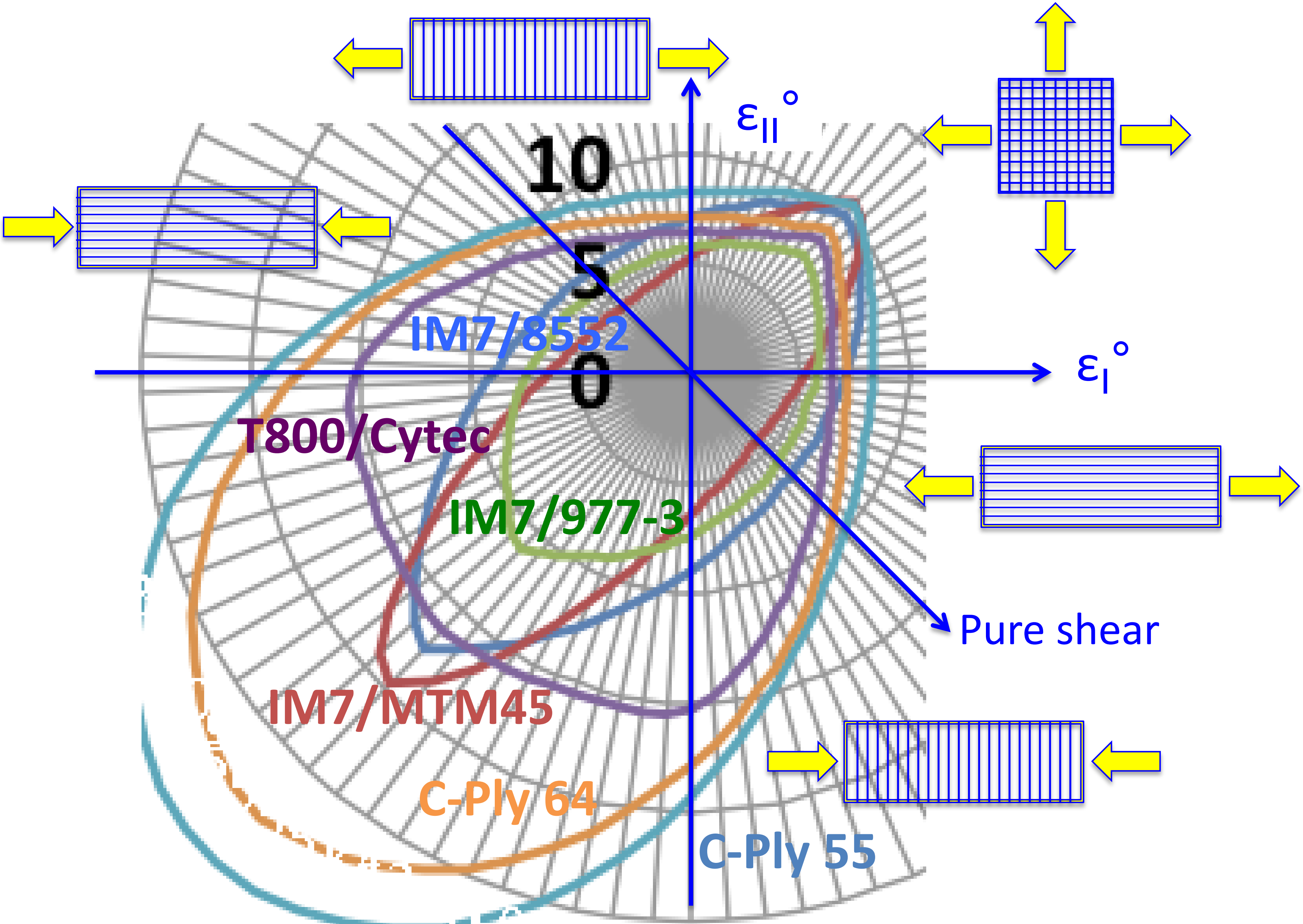
Angle measured clockwise from 0 degree along x-axis; 90 degree, y-axis

Preferred Coupons for Master Envelopes



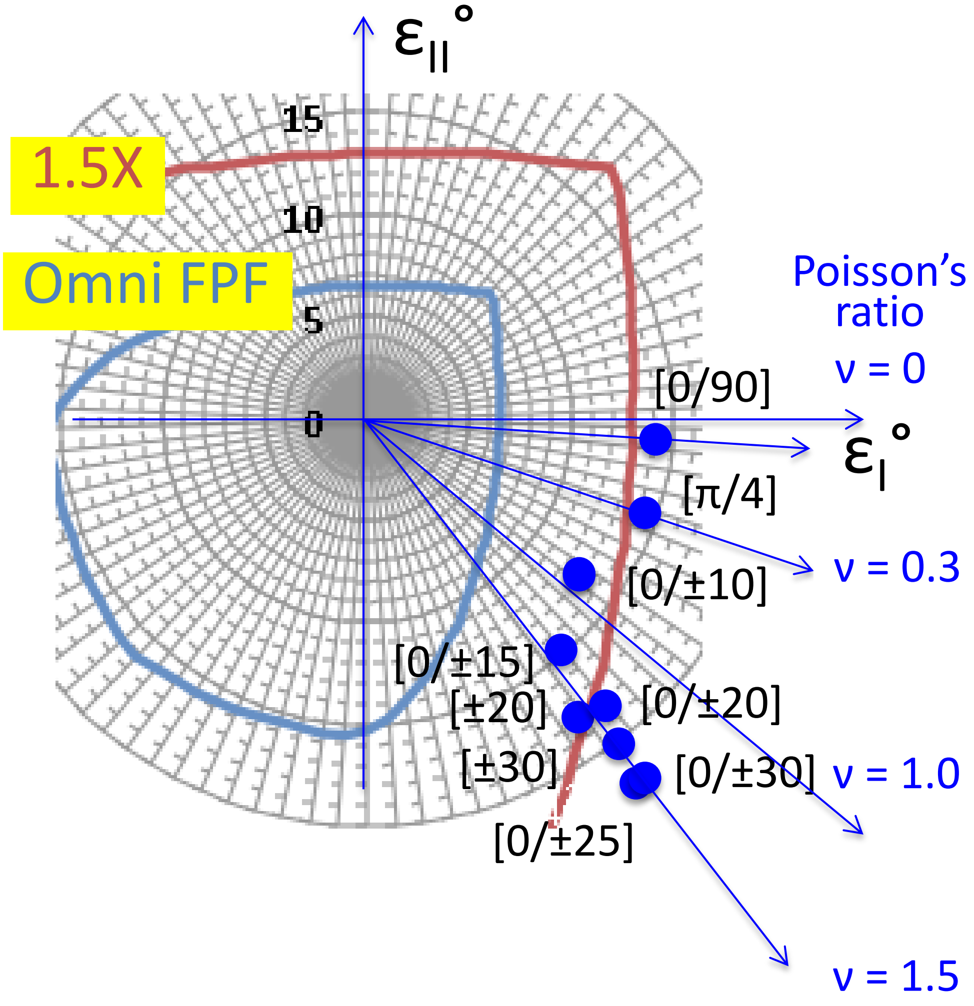


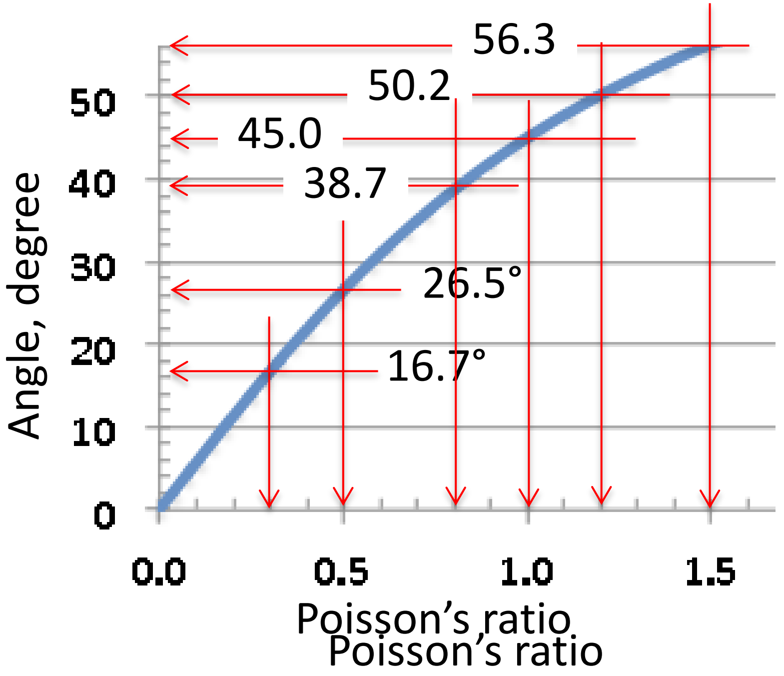
Uniaxial Data validating Omni Envelope



Omni Strain Envelope for T800/Cytec

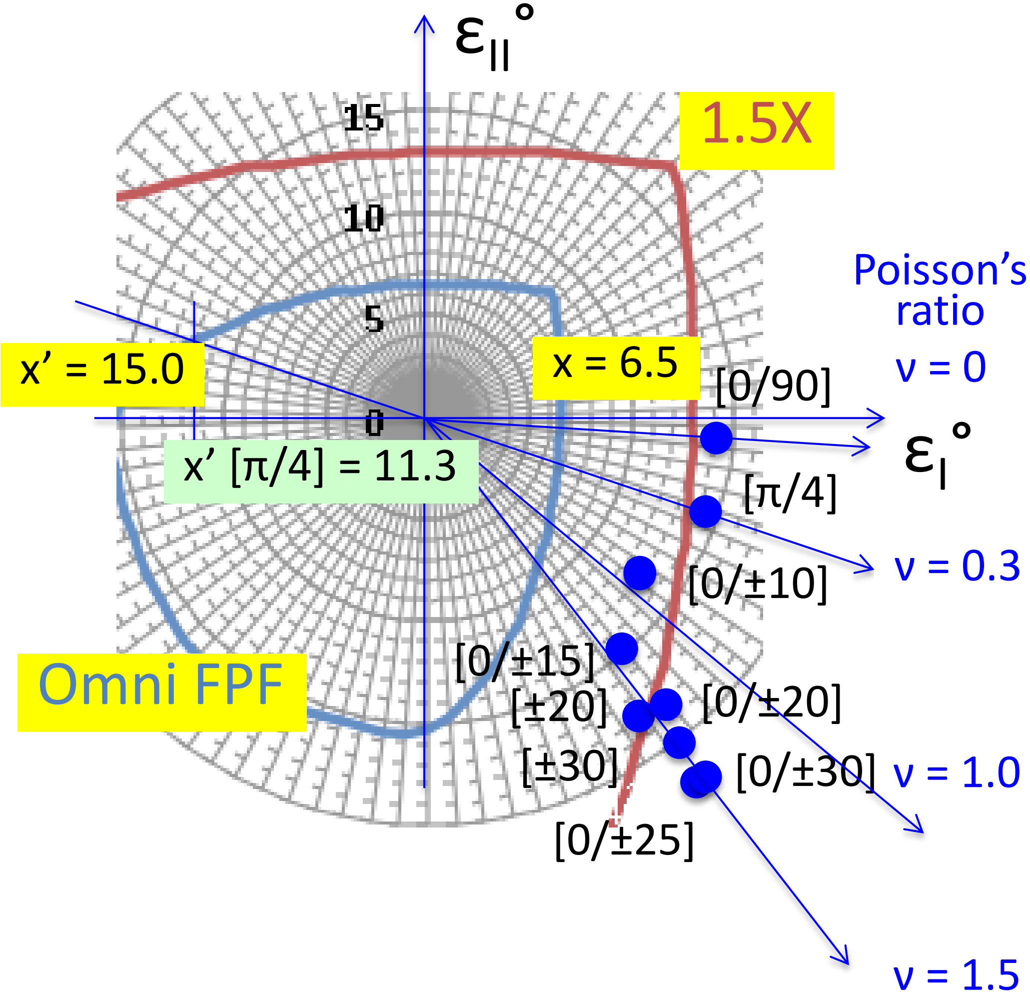
All uniaxial tensile data can be placed on this principal strain plane

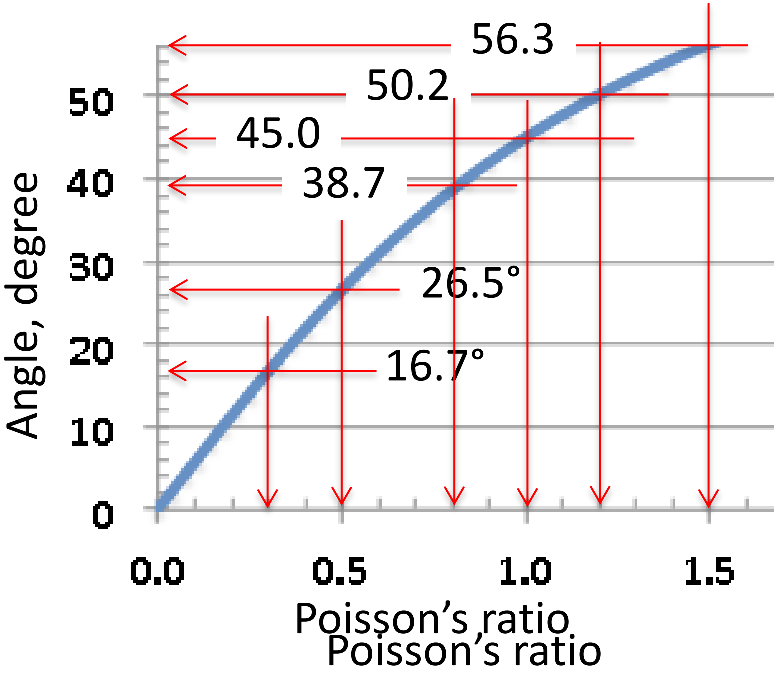




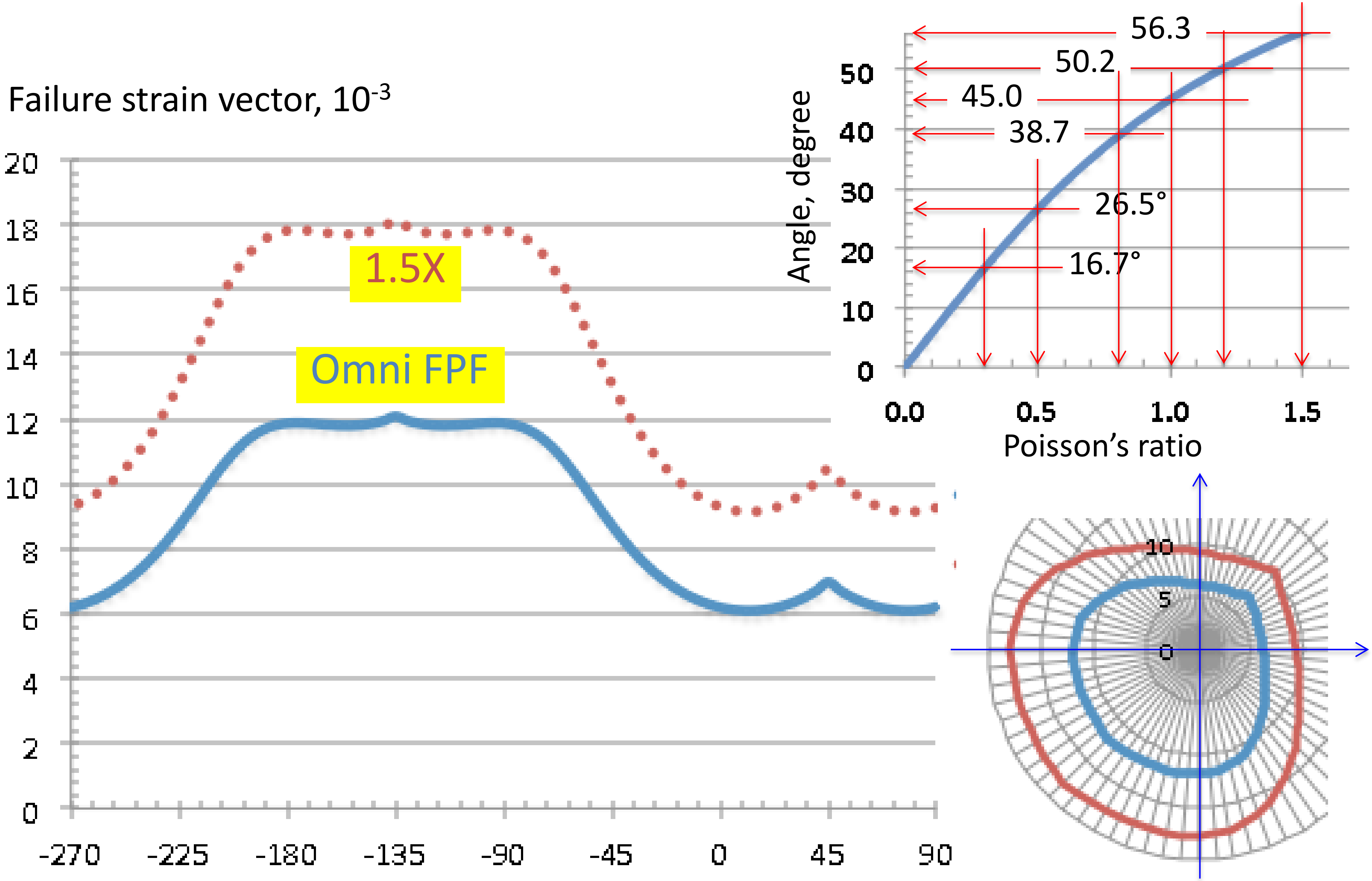
Omni Strain Envelope for T800/Cytec

All uniaxial tensile data can be placed on this principal strain plane

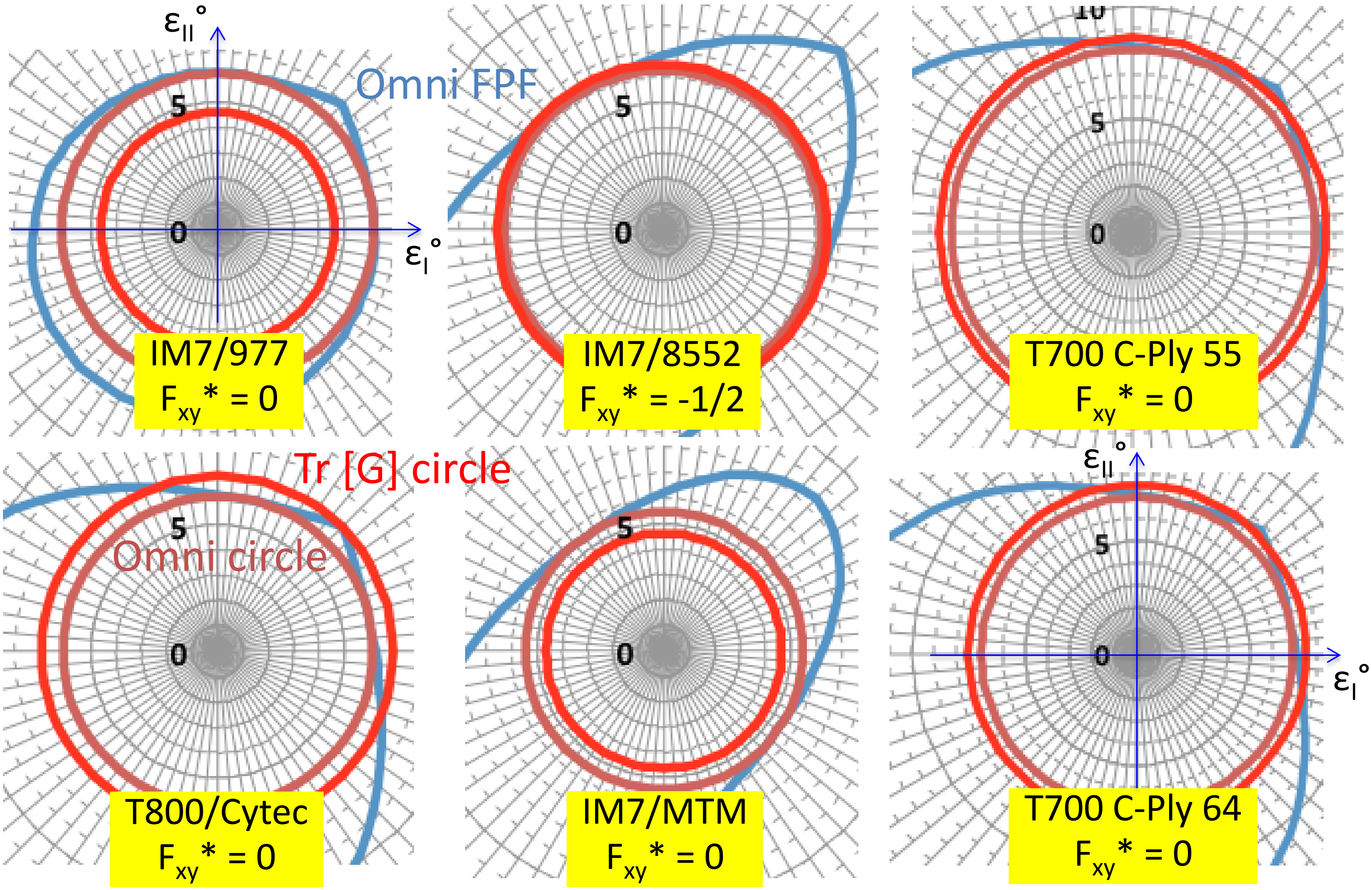


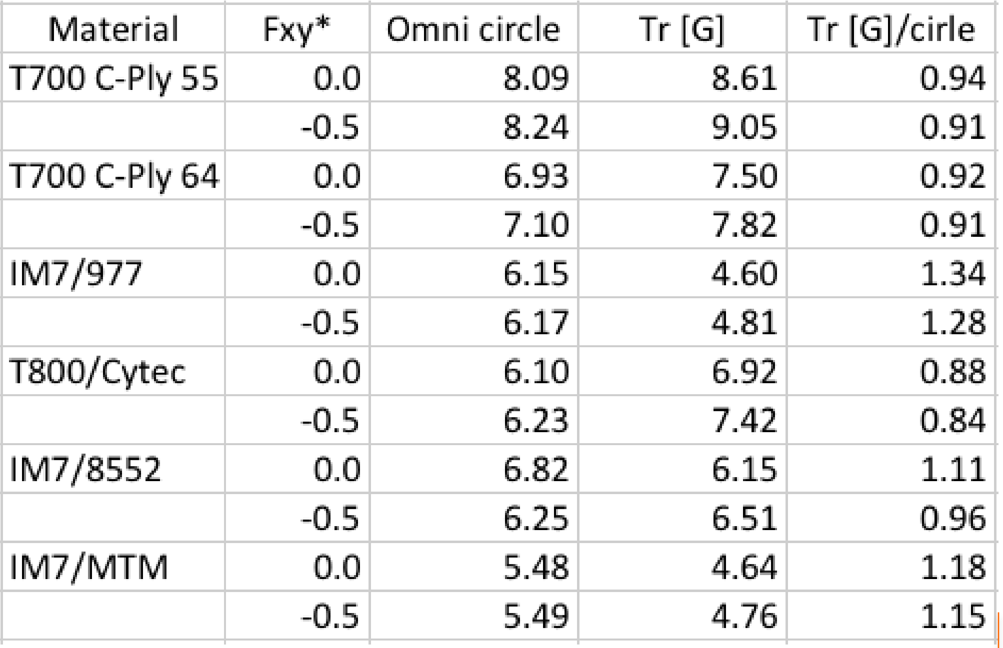


Cartesian Plot of Omni Strain: T800/Cytec

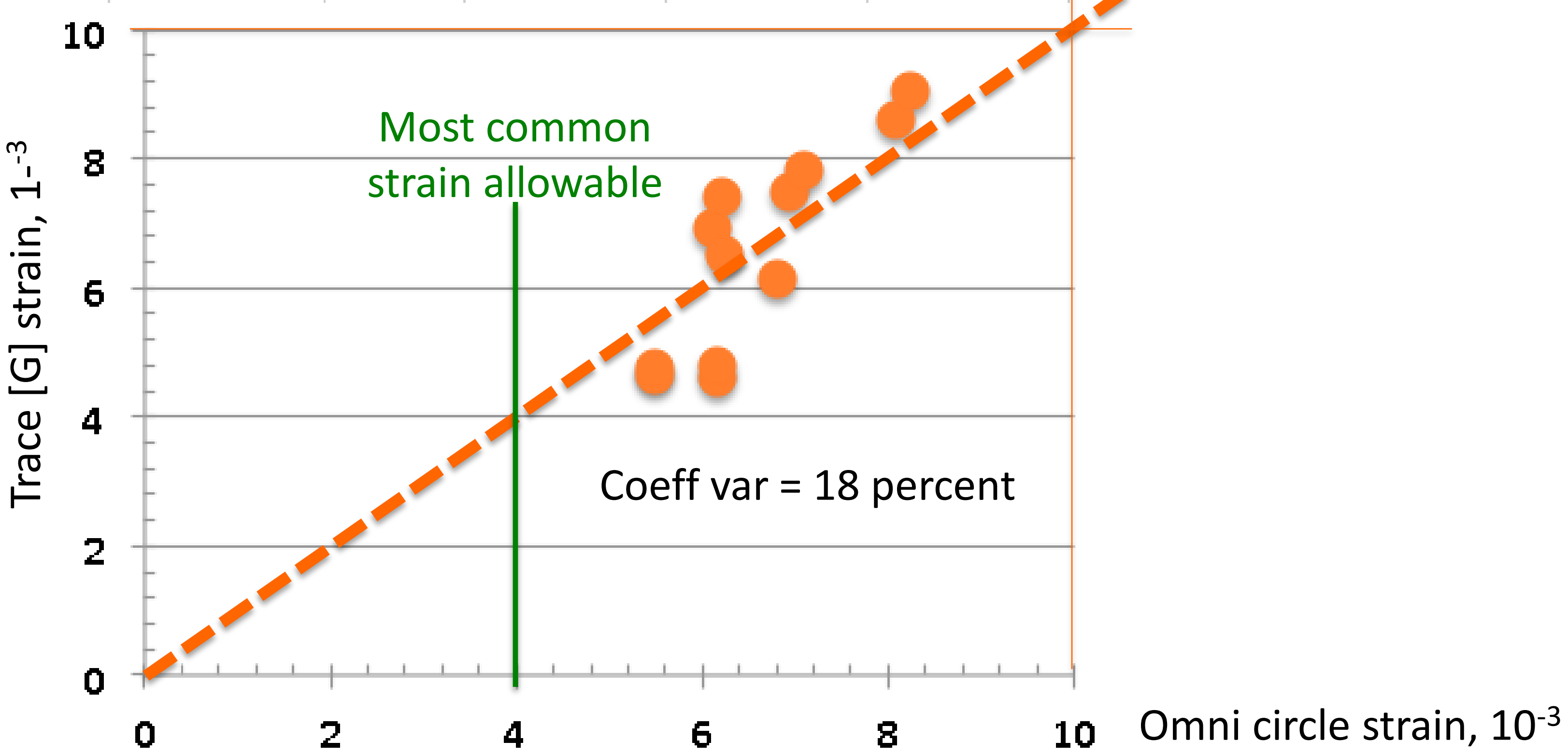


Omni Strain FPF and Circle, and Tr [G]





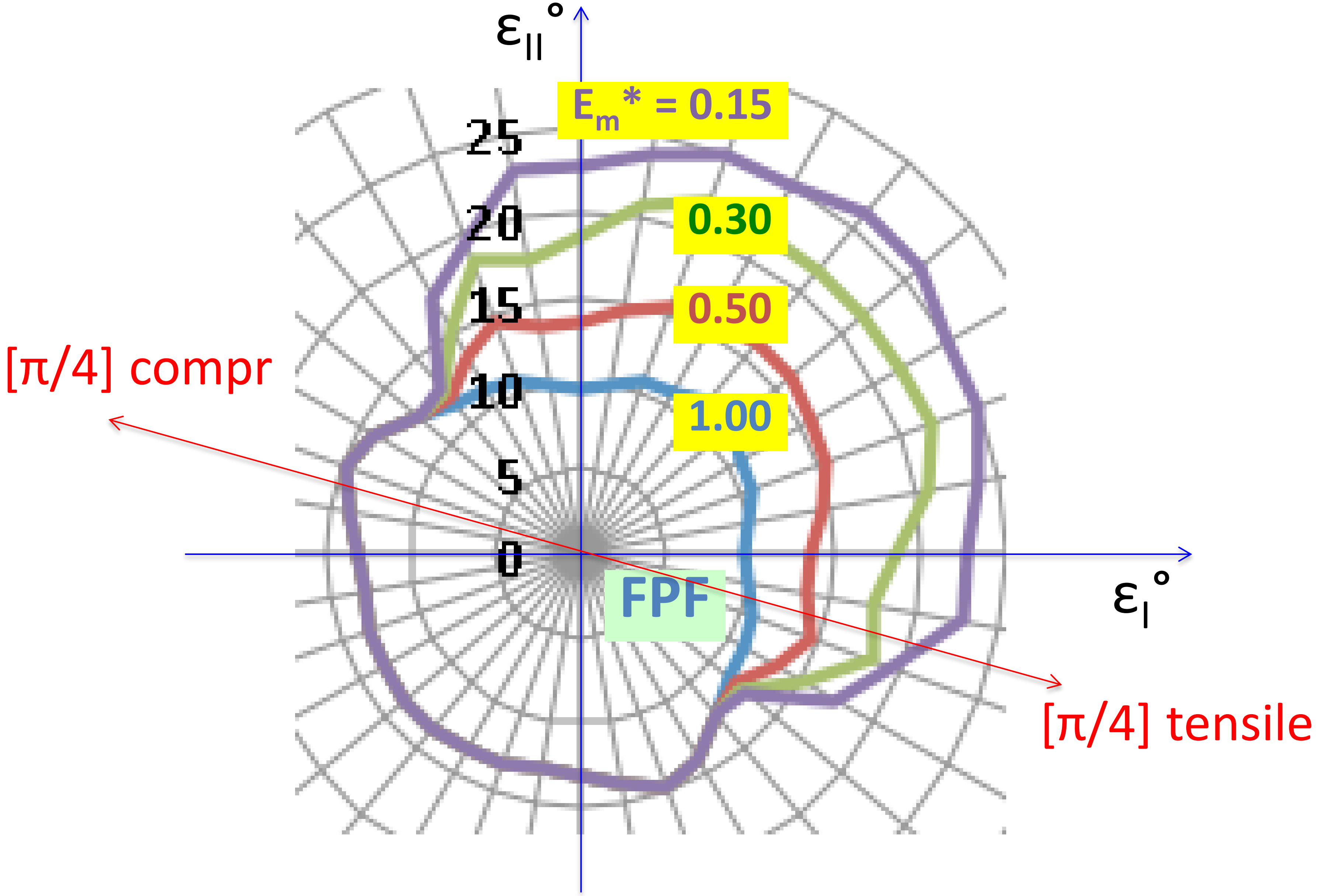
Material	Fxy*	Omni circle	Tr [G]	Tr [G]/cirle
T700 C-Ply 55	0.0	8.09	8.61	0.94
	-0.5	8.24	9.05	0.91
T700 C-Ply 64	0.0	6.93	7.50	0.92
	-0.5	7.10	7.82	0.91
IM7/977	0.0	6.15	4.60	1.34
	-0.5	6.17	4.81	1.28
T800/Cytec	0.0	6.10	6.92	0.88
	-0.5	6.23	7.42	0.84
IM7/8552	0.0	6.82	6.15	1.11
	-0.5	6.25	6.51	0.96
IM7/MTM	0.0	5.48	4.64	1.18
	-0.5	5.49	4.76	1.15



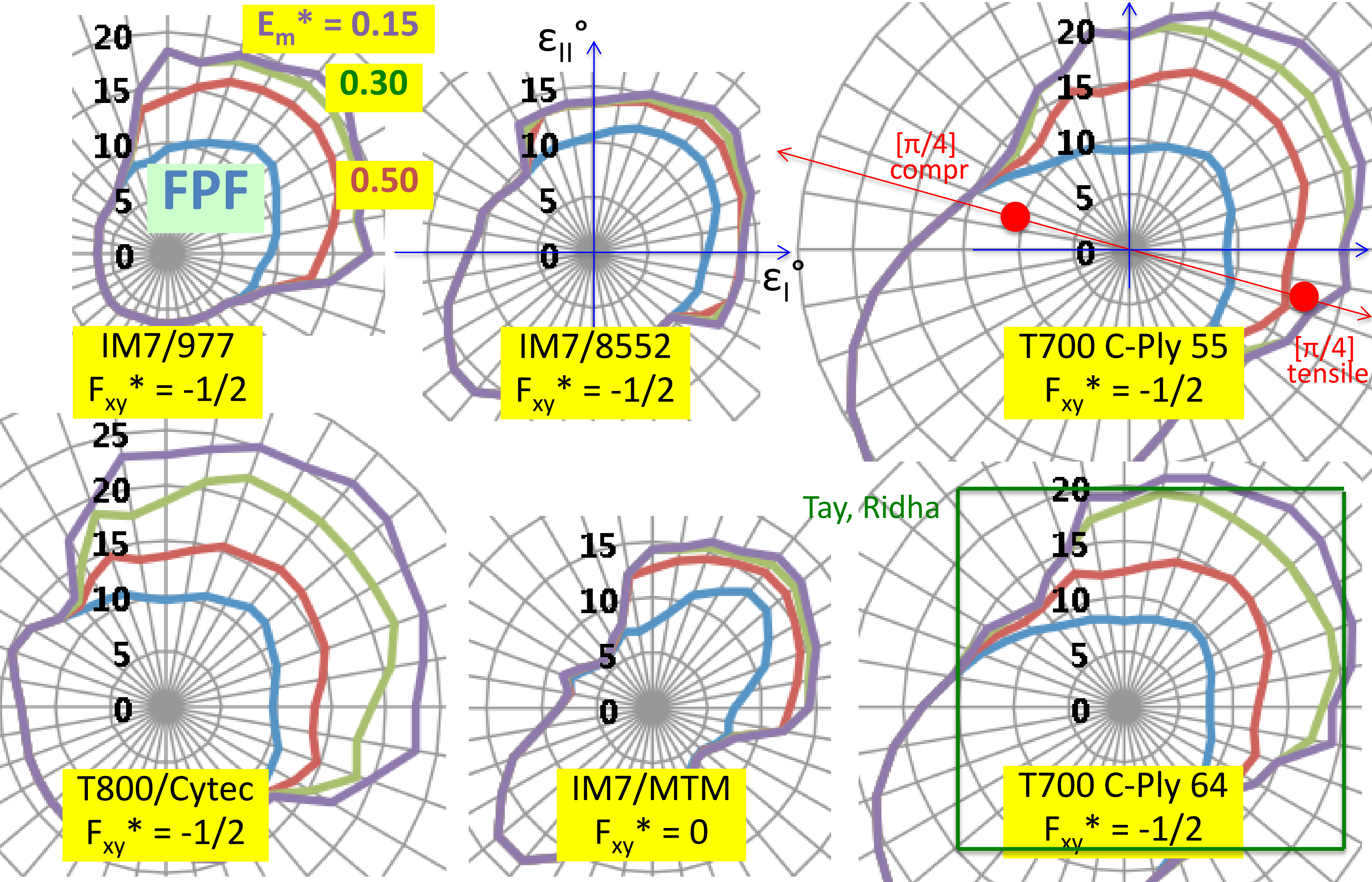
Most common strain allowable

Coeff var = 18 percent

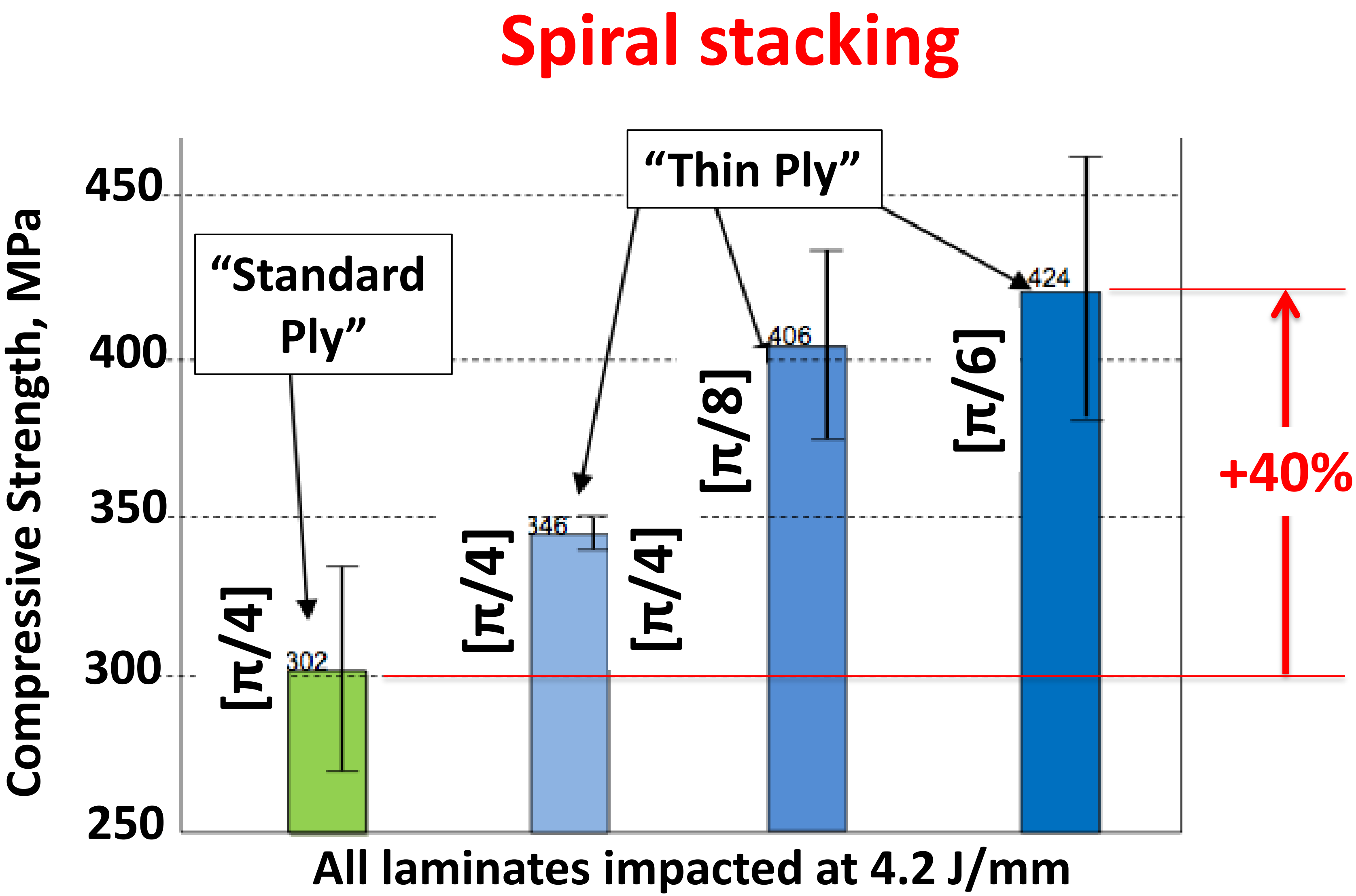
T800/Cytec: FPF and LPF



FPF and LPF: $E_m^* = 0.5, 0.3, 0.15$

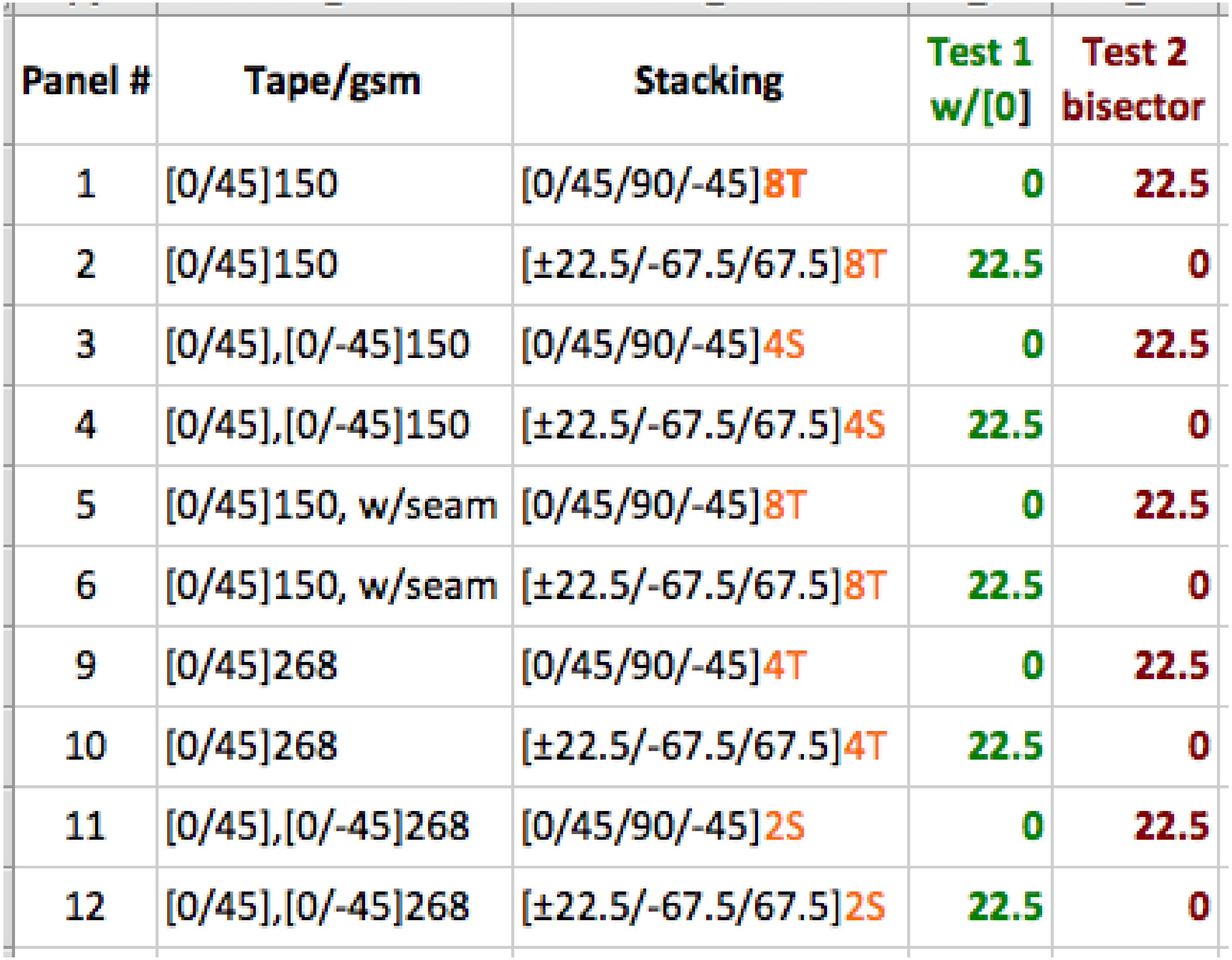


Impact Resistance of [π/6] Laminates

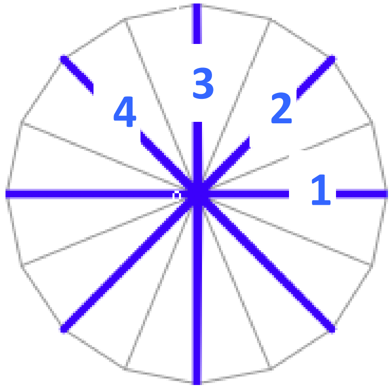


Stacking Options of [π/4] C-Ply

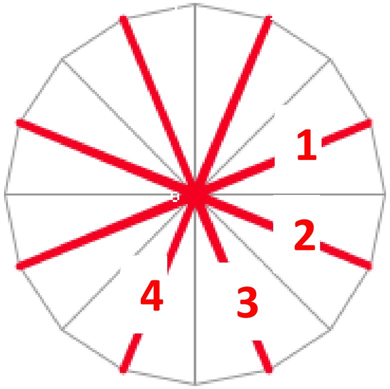
Asymm (T) vs symm (S); w vs w/o seams; 150 vs 268 gsm; test 1 vs 2



Panel #	Tape/gsm	Stacking	Test 1 w/[0]	Test 2 bisector
1	[0/45]150	[0/45/90/-45]8T	0	22.5
2	[0/45]150	[±22.5/-67.5/67.5]8T	22.5	0
3	[0/45],[0/-45]150	[0/45/90/-45]4S	0	22.5
4	[0/45],[0/-45]150	[±22.5/-67.5/67.5]4S	22.5	0
5	[0/45]150, w/seam	[0/45/90/-45]8T	0	22.5
6	[0/45]150, w/seam	[±22.5/-67.5/67.5]8T	22.5	0
9	[0/45]268	[0/45/90/-45]4T	0	22.5
10	[0/45]268	[±22.5/-67.5/67.5]4T	22.5	0
11	[0/45],[0/-45]268	[0/45/90/-45]2S	0	22.5
12	[0/45],[0/-45]268	[±22.5/-67.5/67.5]2S	22.5	0



[(0/45)/(90/-45)]
Right handed spiral

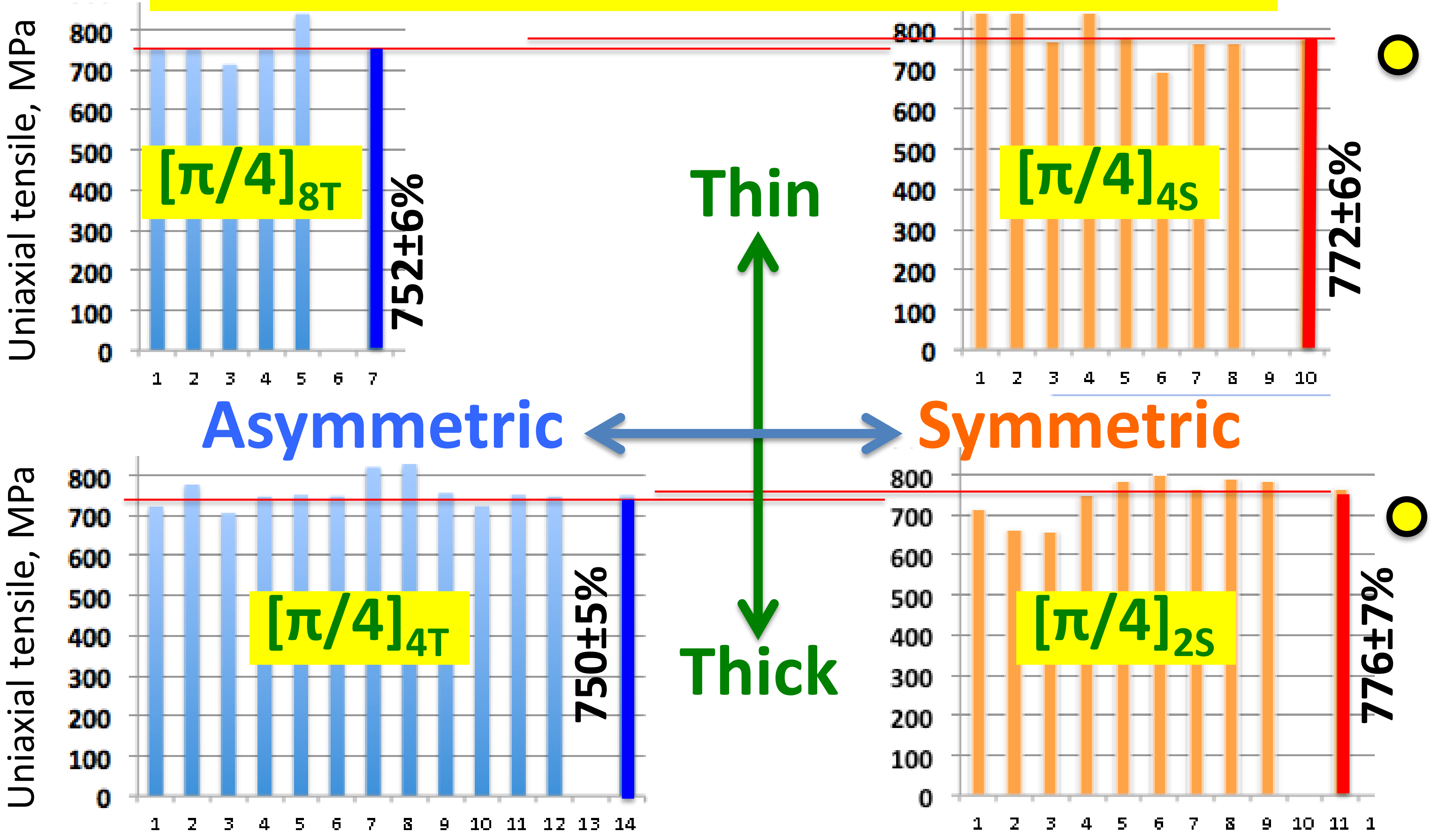


[(±22.5)/(-67.5/67.5)]
Left handed spiral

Homogeneity: Symmetry; 150 vs 268

Smooth coupon with load applied along a [0] ply

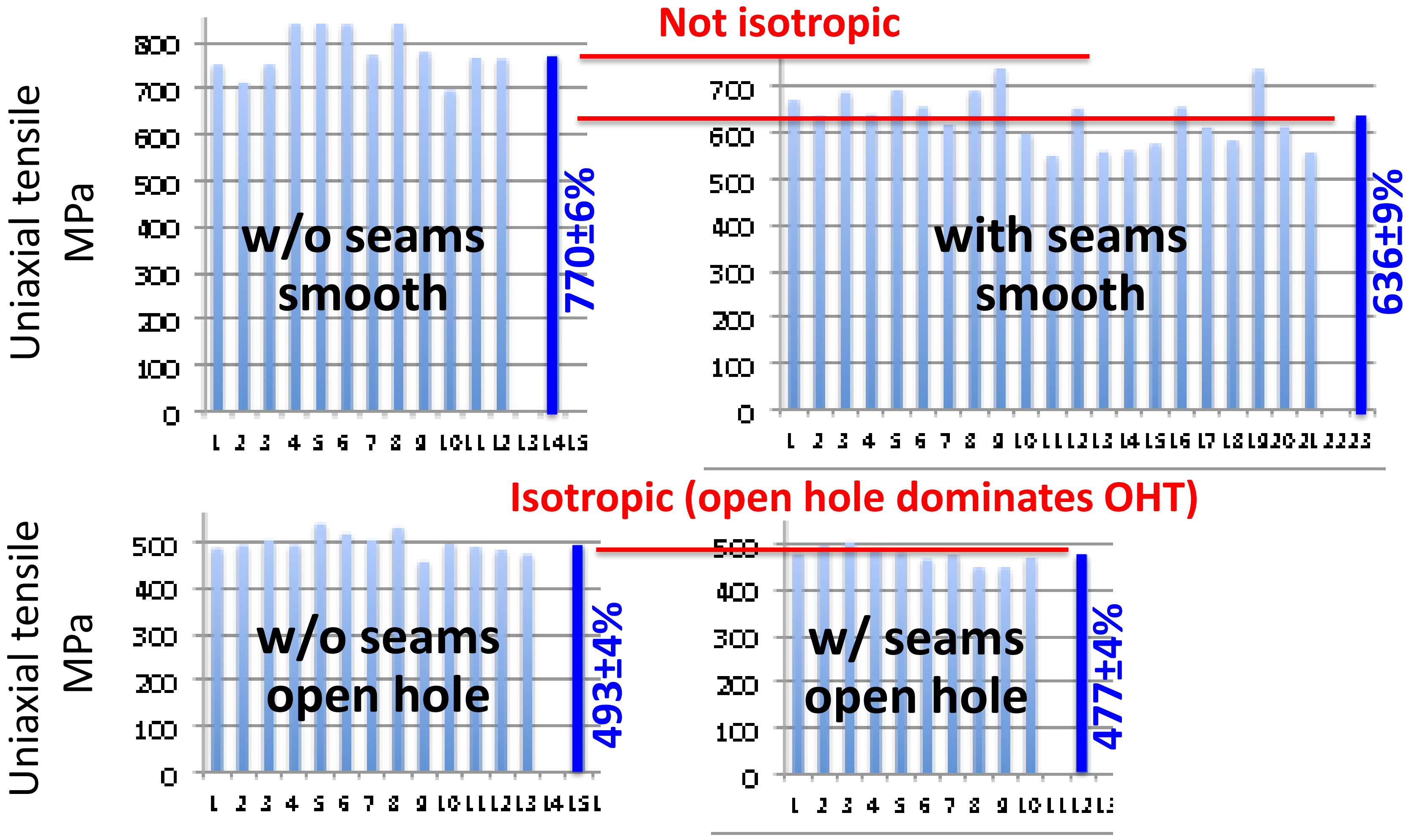
Medium 762±6% MPa: Insensitive to symmetry and thin-thick



Laminates With or Without Seams

Laminates w/o seams:
w and w/o symmetry

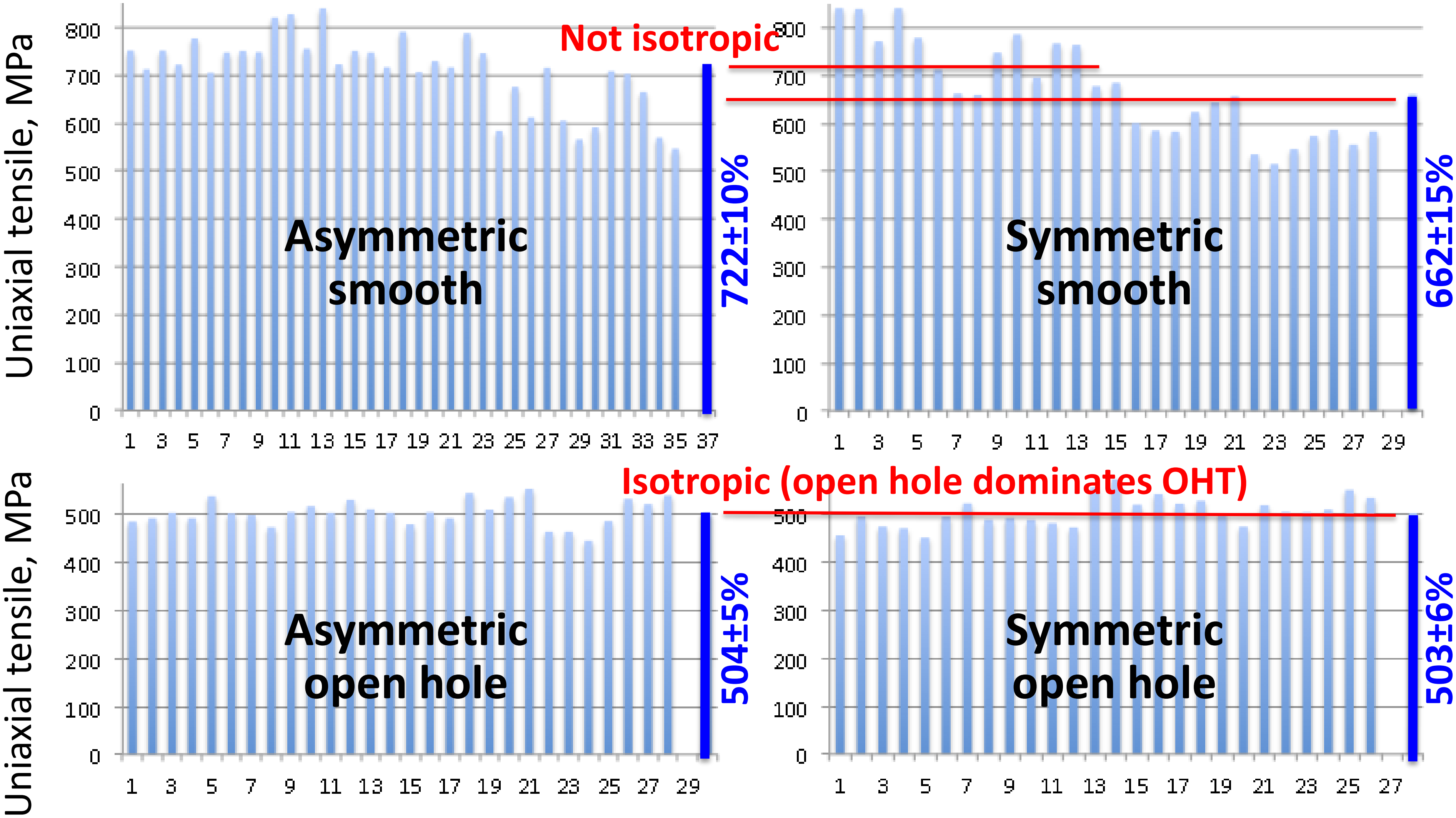
Laminates with seams:
with symmetry only



Asymmetric vs Symmetric Laminates

Mid-plane asymmetry: Mid-plane symmetry:

Tensile load along [0] and bisector; 150 and 268 gsm

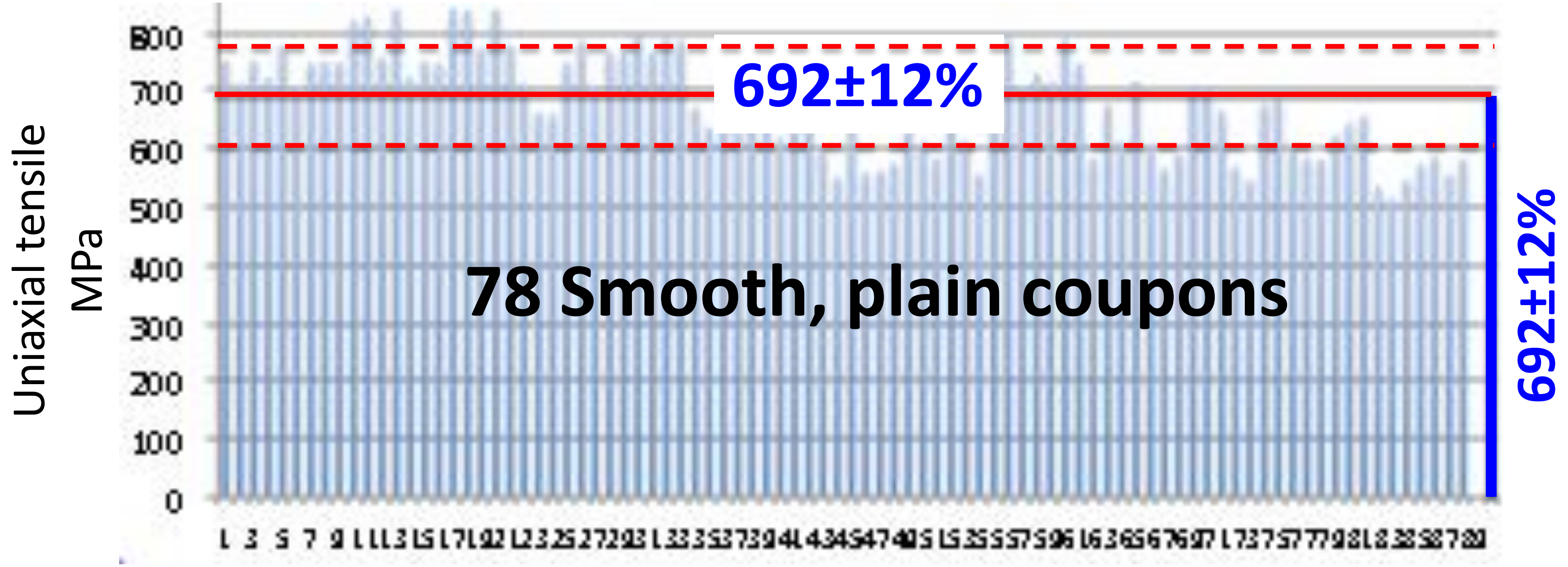


Not isotropic

Isotropic (open hole dominates OHT)

Smooth vs OHT Coupons from [π/4]

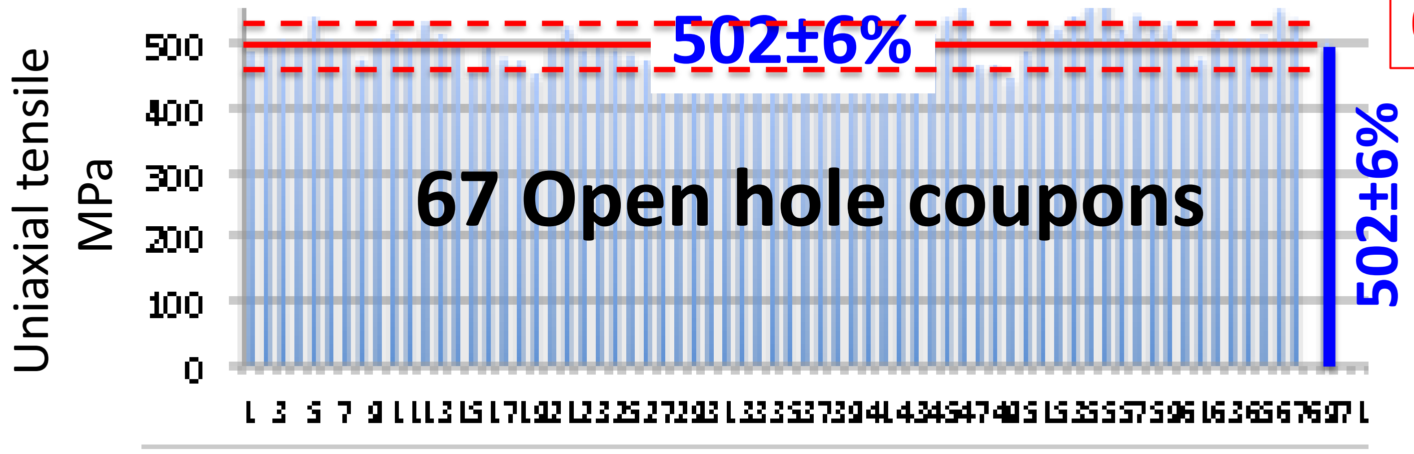
With and without: symmetry and seams; thick-thin plies, load along [0] and bisector



692±12%

78 Smooth, plain coupons

692±12%



502±6%

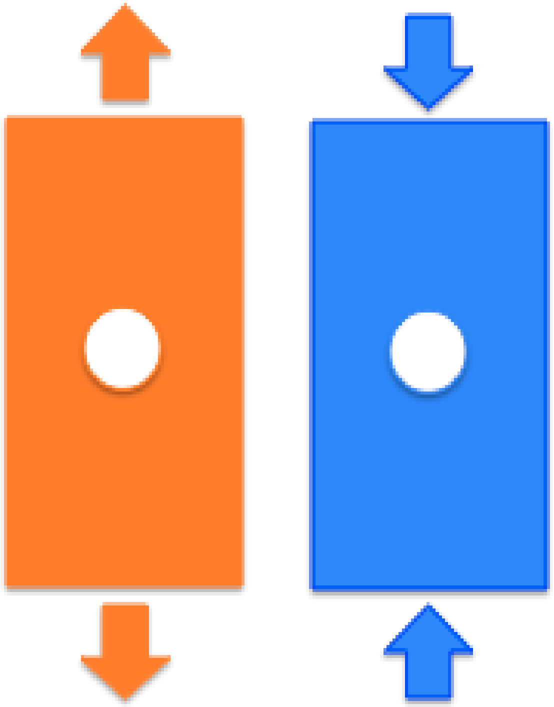
67 Open hole coupons

502±6%

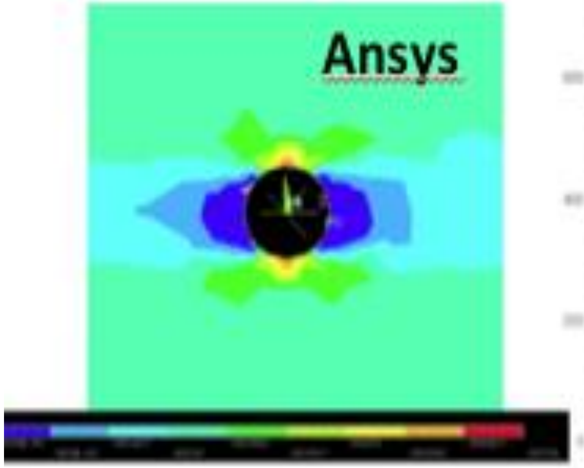
692/502 = 1.38

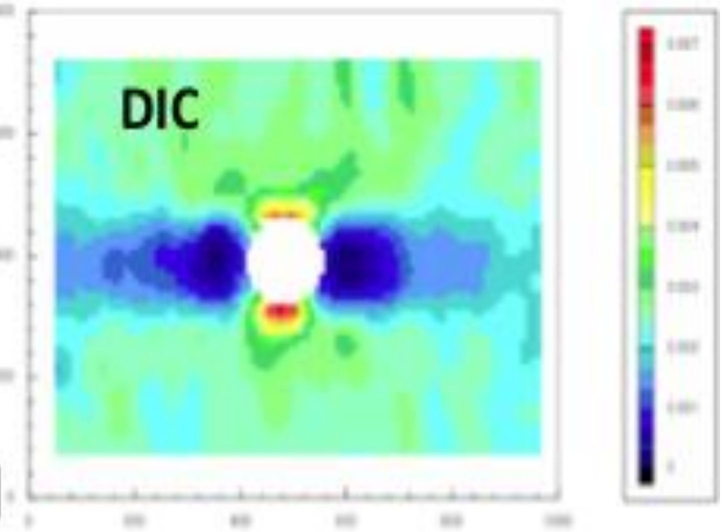
Open Hole: a safe & simple approach

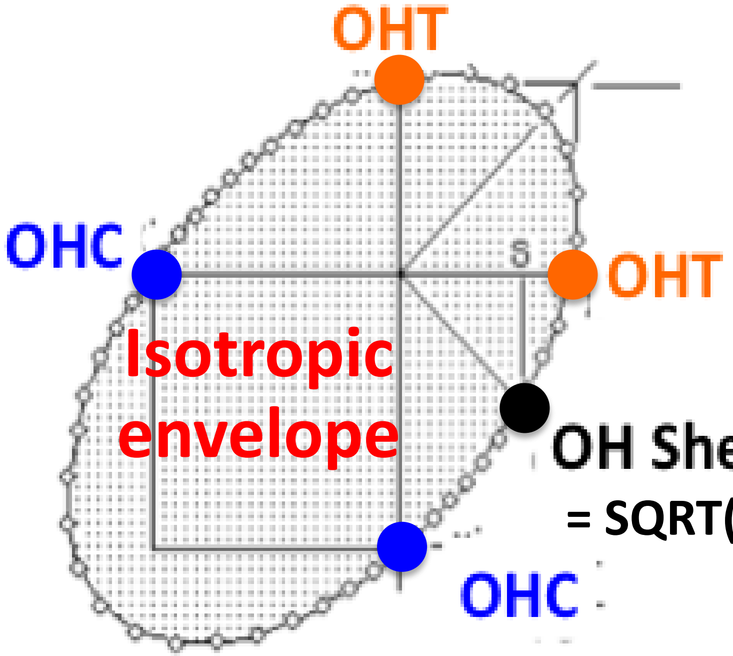
OHT and OHC



[π/3], [π/4], [π/6], …



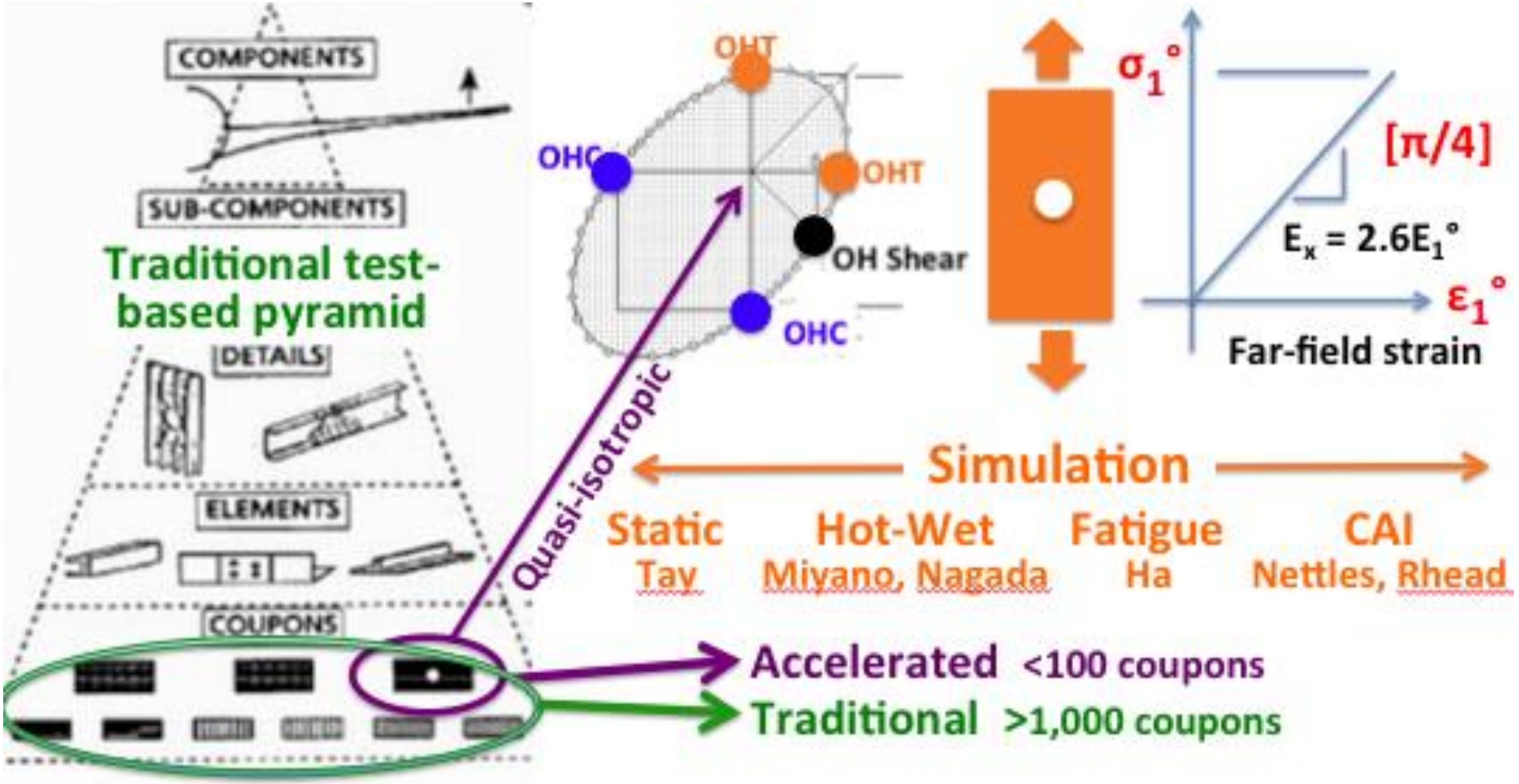


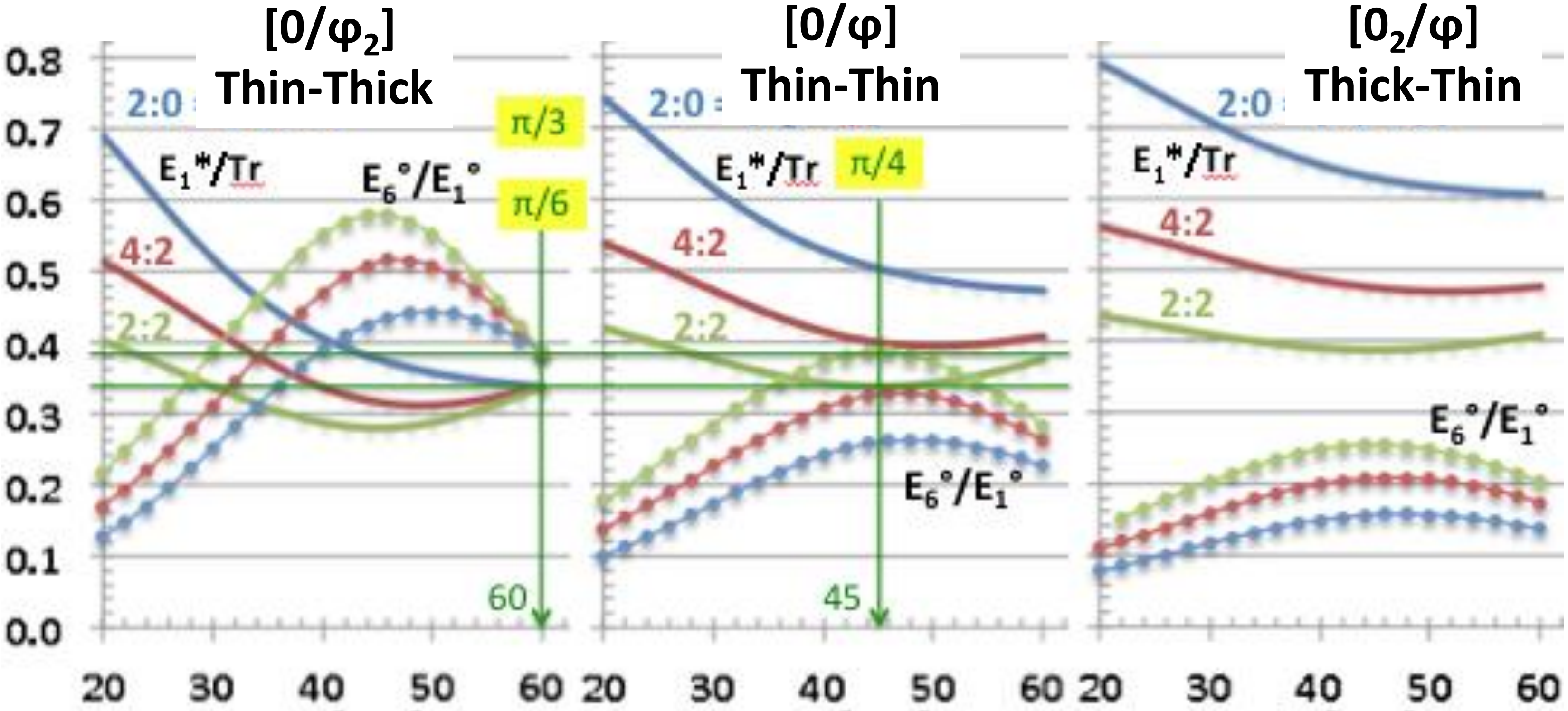


OH Shear

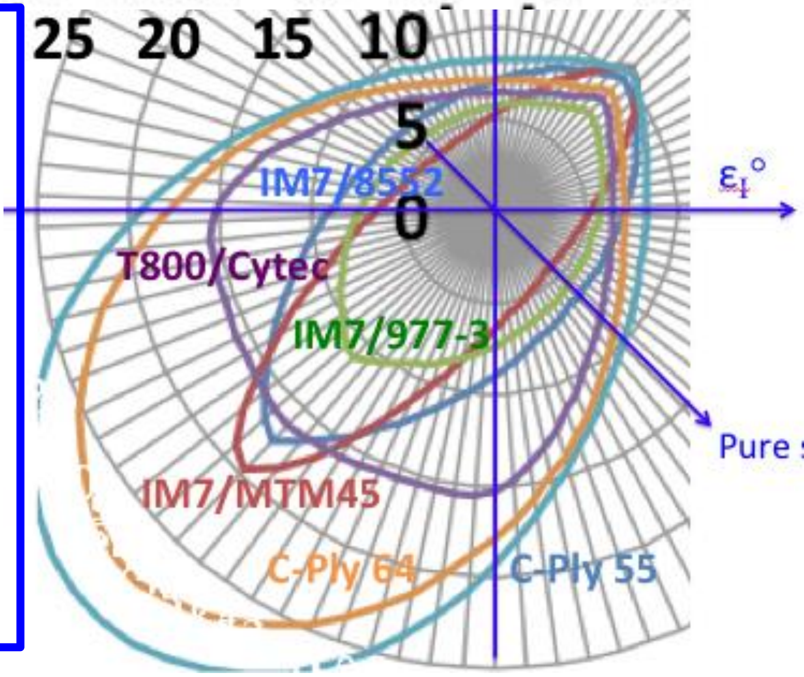
= SQRT(OHT*OHC/3)

Accelerated Allowable Generation





Need only 2 charts on
1 page to define all CFRP
and all laminates:
1) Trace [Q] for stiffness
2) Omni strain for strength



Master Ply and its Laminates

- Plane stress stiffness [Q] is better represented by its invariant trace: $Q_{xx} + Q_{yy} + 2Q_{ss}$ - - a linear scaling factor
- When normalize by trace [Q*] plies and laminates are insensitive among many composite plies justifying a master ply
- The same invariance holds from ply to in-plane, and to flexure (not shown here) – to scale design is made easy
- Power of bi- and tri-angle tapes can save cost through 1- or 2-axis; increase CAI through 6-angle laminates
- Certification of asymmetric layup and homogenization of composite laminates can be accelerated with fewer coupons, and more simulation guided by invariants
- Recommend laminates with holes as test coupons

Opportunities in Composites Design

- Master fundamental theories, like invariants for Master ply, a one parameter for design
- Multi-angle tape layup can achieve >2X in speed and 6-angle laminates for increased CAI while limited to 1- or 2-axis layup, no more 4-axis
- Thin plies can increase toughness and homogenization - amenable to optimization, and ply angle used as a continuous variable
- Simulation will guide tests for hot-wet, fatigue, CAI, damage tolerance, and micromechanics
- Design allowable and certification can be simplified by testing laminates with open hole replacing smooth coupons of plies and laminates