Minisymposium *bei* ***Carbon Composites e.V.***
IHK Schwaben, Augsburg, März 4, 2010, ??.00 Uhr
Abstimmung: *Composite Fatigue Strategie*

Lifetime prediction of
UD Lamina-composed Laminates
(such as non-crimp fabrics)
- a lamina-based engineering approach for fibre-dominated laminates -

results of a private research

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Objectives of the approach

An Engineering, failure modes-linked lifetime prediction method for *plain* laminates which employs**:**

1.) Failure-mode-related damage accumulation

2.) A minimum number of failure-mode-linked representative *mode S-N curves* (from experiment) = master R-curve of each mode

3.) Prediction of other necessary stress-ratio *'mode S-N curves'* on basis of an available representative one which is typical for the envisaged mode (e.g. R=0.5 from R=0.1)

GOAL: Margin of safety for Lifetime

$$MoS_{Life} = (\text{predicted lifetime})/(j_{Life} \cdot \text{design lifetime}) - 1 > 0$$

Contents of Presentation

1. Fatigue Analysis Basics of Ductile Behaving Metals and Brittle Composites

1.1 Ductile Behaving Metals

•**Standard Lifetime Prediction Method for metals:** uses

- **S-N curves,** dependent on the stress ratio $R = \sigma_{min} / \sigma_{max}$

  S *:= cyclic stress range =* $\Delta\sigma$ *, N:= number of cycles to failure*

- **Constant life Goodman Diagram** to account for the mean stress effect

• **In case of ductile behaving metals**

**** Slip band shear yielding occurs under cyclic tensile, compressive, and shear stress***

**** This shear stress–caused yielding can be described by one yield failure condition !***

But, *semi-brittle, brittle* behav. materials experience several failure modes (mechanisms)

$\longrightarrow$ *Consequence: Several failure conditions are to be employed !*

1. Fatigue Analysis Basics of Ductile Behaving Metals and Brittle Composites

1.2 Brittle Behaving Composites ⇐ lessons learnt from other brittle materials

Cyclic fatigue life consists of three phases:

Phase I: Increasing damaging in embedded Laminas up to discrete damage onset
(determination of accumulating damaging portions (= Schädigungen), initiated at end of elastic domain and dominated by diffuse micro-cracking + matrix yielding, and finally micro-delaminations)

Phase II: Stabile local discrete damage growth in Laminate up to delamination

(growth of dominating discrete micro-crack widths incl. micro-delaminations)

Phase III: Final in-stabile fracture of Laminate initiated by FFs, IFF2 of any lamina

\+ possible **delamination** (= Schaden) **criticality of the loaded laminate**

FF:= fibre failure. IFF:= Inter Fibre Failure

CDS:= characteristic damage state at the end of diffuse damaging

1.3 Fatigue Life Demonstration in case of Brittle Behaviour

Traditional fatigue life demonstration

(not just for semi-brittle behaving isotropic metals applied !)**:**

"Uni-axial" Stress amplitude procedure **with** ***mean stress correction***

to be replaced by an advanced procedure
where mean stress correction is inherent, namely

Equivalent (multi-axial) stress state procedure **with** ***failure mode reflection***

1.4 Experience with Composites

further lessons, learnt from composites

Actual composites from Fibre Reinforced Plastics (FRP):

- behave brittle

- experience early fatigue damage

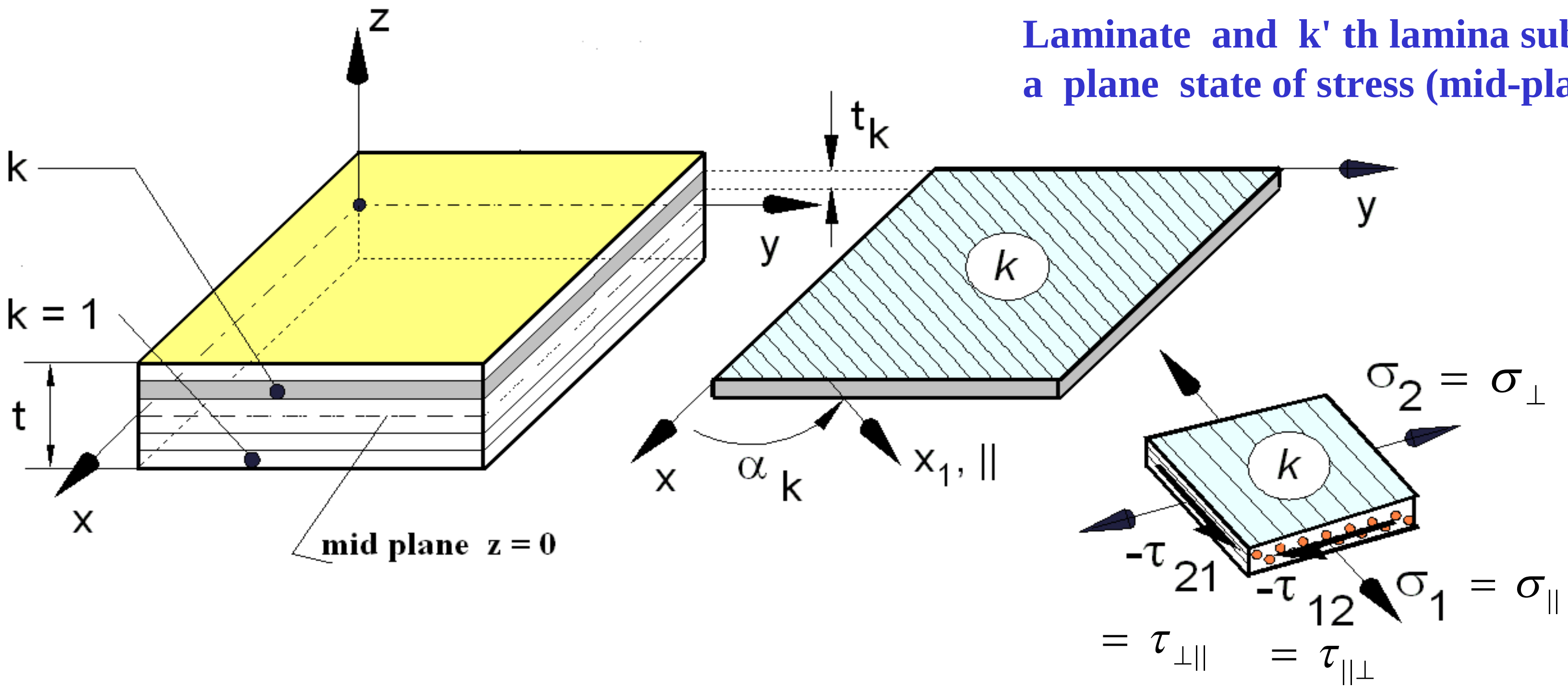
- show benign fatigue failure behaviour in case of 'well-designed' and especially of fibre-dominated laminates

(fibre-dominated: 0° plies in all significant loading directions, > 3 angles)

2. Sress State, Strengths, and Elasticity Properties
2D Uni-Directional lamina

Lamina (ply) = homogenized (smeared) material = building block of the Laminate !

Laminate and k' th lamina subjected to a plane state of stress (mid-plane z = 0)



Due to material symmetry of the transversely-isotropic UD lamina,

necessary for 2D analysis :

5 strengths : $R_\parallel^t\,(=X^t),\ R_\parallel^c\,(=X^c),\ R_\perp^t\,(=Y^t),\ R_\perp^c\,(=Y^c),\ R_{\perp\parallel}\,(=S)$

US notation in ( )

4 (5) elasticity properties : $E_\parallel, E_\perp, G_{\parallel\perp}, \nu_{\perp\parallel},\ (\nu_{\perp\perp},\ \ just\ if\ 3D\,)$

t, c := tensile, compression . Symbolic notation is of advantage for a clear definition of properties

3. Attempt for a Strength Failure Mode Systematization

3.1 Lessons learned from inspection

* There are coincidences between brittle UD <u>laminae</u> and brittle isotropic materials

* Degradation begins with onset of diffuse damaging (hardening) until IFF1, IFF3

* Final fracture failure occurs with FF1. FF2, and IFF2 after onset of discrete damaging

3 Attempt for a Strength Failure Mode Systematization

3.2 Scheme of Strength Failures of *the brittle UD lamina (ply) material*

Stability | **Strength** | Deformation

strength *failure modes* on lamina macro level

Onset of quasi-Yielding
initiated by
Diffuse Damaging (pores, flaws, crazing, ..)

Onset of Fracture

defined by Discrete IFF Damage of a *lamina*

Normal Fracture	Shear Fracture	Crushing Fracture
NF	SF	CrF
dense or porous	*dense*	*porous*

+ delamination failure of *laminate*

according to material symmetry :

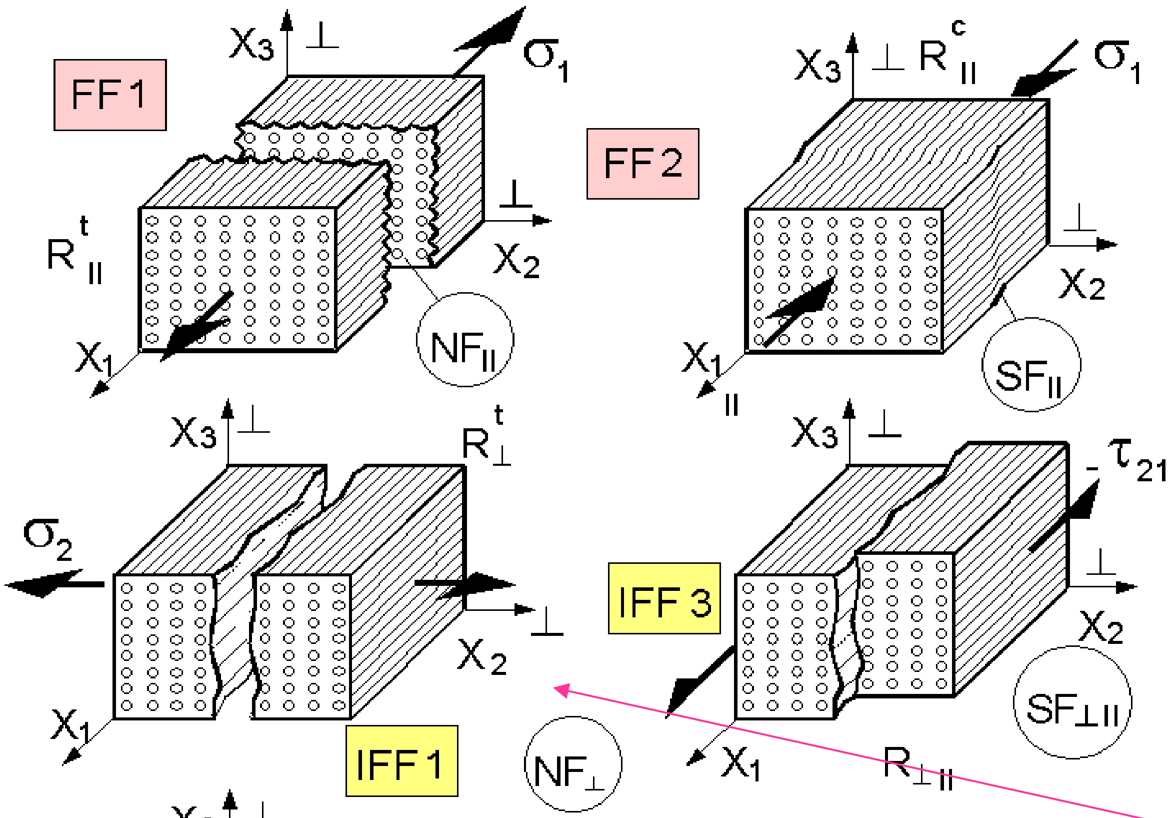
$F_{\parallel}^{\sigma}, F_{\perp}^{\sigma}, F_{\perp\parallel}$ $F_{\parallel}^{\tau}, F_{\perp}^{\tau}$ or $(F_{\perp}^{cr})$: 5 failure conditions F

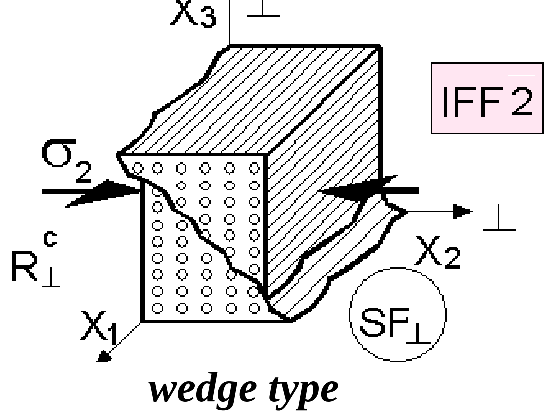
FF1, IFF2, IFF3   FF2, IFF2 : 5 failure modes (2 FF + 3 IFF)

3. Attempt for a Strength Failure Mode Systematization

3.3 Fracture Failure Modes of UD material *(known from fractography of UD specimens)*

t = tension
c = compression



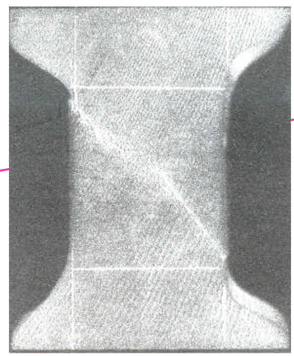


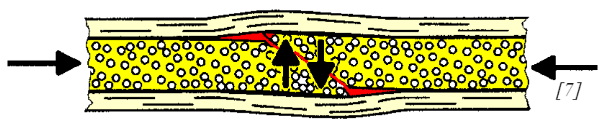
► 5 Fracture modes exist

= 2 FF (Fibre Failure)
+ 3 IFF (Inter Fibre Failure)

NF := Normal Fracture

macroscopically: **SF := Shear Fracture**





Critical in a loaded laminate are: FF1, FF2 + possibly IFF2 !

4. Brief Description of author's Failure Mode Concept (FMC)

4.1 FMC-based Failure Conditions $F = 1$ for UD material

A failure condition is the mathematical formulation, F = 1, of the failure surface:.

* **1 global failure condition : $F(\{\sigma\}, \{\bar{R}\}) = 1$ (usual formulation !) ;**

= 'fully interactive conditions' which include several modes

$\{\bar{R}\} = (\bar{R}_{\parallel}^{t}, \bar{R}_{\parallel}^{c}, \bar{R}_{\perp}^{t}, \bar{R}_{\perp}^{c}, \bar{R}_{\perp\parallel})^T$ ***strength vector***

* **Set of mode failure conditions : $F(\{\sigma^{mode}\}, \bar{R}^{mode}) = 1$ (used in Cuntze's FMC)**

vector of stresses being active in a mode

mode-associated strength, only

<u>Notes</u>:

$F >=< 1$ is a failure criterion. $\bar{R}$ marks mean (average) strength in the physical (mapping) model.

Global failure conditions are often applied in case of brittle behaving materials, despite of the fact that many modes may become active under operational loading

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

4. Brief Description of author‘s Failure Mode Concept (FMC)

4.2 Introduction of Failure Mode-related *Equivalent Stress* and *Strength*

•**Stress vector** (6 stresses)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

•**Equivalent stress vector** (5 equ. stresses)

needed later

$$\{\sigma_{eq}^{mode}\} = \left(\sigma_{eq}^{||\sigma},\ \sigma_{eq}^{||\tau},\ \sigma_{eq}^{\perp\sigma},\ \sigma_{eq}^{\perp\tau},\ \sigma_{eq}^{||\perp}\right)^T$$

of the 5 fracture failure modes

Known examples for equiv. stresses:

-‘Yielding‘ , **Mises** for comparison

$$\sigma_{eq}^{Mises} = \sqrt{3J_2} = \overline{R}_{p0.2}$$

- ‘Normal Fracture‘

$$\sigma_{eq}^{NF} = \sigma_I = \overline{R}_m$$

•**Strength vector** (5 strengths, modeling, 50% expectance value)

$$\{\overline{R}\} = (\overline{R}_{\parallel}^{t}, \overline{R}_{\parallel}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp\parallel})^T$$

$$\overline{R}\ \text{(modeling)} \rightarrow R\ \text{(strength verification)}$$

= mean value

= strength design allowable

= statistical minimum value according to MIL-HDBK 5

4. Brief Description of author's Failure Mode Concept (FMC)

4.3 FMC-based 3D Static Failure Conditions for plain UD material

Equivalent stress formulations of failure conditions

The indices σ, τ mark the failure mode driving stress ! * Limit of homogenization (smearing)

FF1: $$1 = \sigma_1 / \bar{R}_{\|}^t = \sigma_{eq}^{\|\sigma} / \bar{R}_{\|}^t \quad \text{with} \quad \sigma_1 \cong \varepsilon_1^t \cdot E_{\|} ,$$

FF2: $$1 = -\sigma_1 / \bar{R}_{\|}^c = +\sigma_{eq}^{\|\tau} / \bar{R}_{\|}^c \quad \text{with} \quad \sigma_1 \cong \varepsilon_1^c \cdot E_{\|} ,$$

filament ! Modes

IFF1: $$1 = [(\sigma_2+\sigma_3) + \sqrt{\sigma_2^{\,2} - 2\sigma_2\cdot\sigma_3 + \sigma_3^{\,2} + 4\tau_{23}^{\,2}}\,] / 2\bar{R}_{\perp}^t = \sigma_{eq}^{\perp\sigma} / \bar{R}_{\perp}^t ,$$

IFF2: $$1 = [(b_{\perp}^{\tau}-1)\cdot(\sigma_2+\sigma_3) + b_{\perp}^{\tau}\sqrt{\sigma_2^{\,2} - 2\sigma_2\sigma_3 + \sigma_3^{\,2} + 4\tau_{23}^{\,2}}\,] / \bar{R}_{\perp}^c = \sigma_{eq}^{\perp\tau} / \bar{R}_{\perp}^c$$

IFF3: $$1 = \{[b_{\perp\|}\cdot I_{23-5} + (\sqrt{b_{\perp\|}^{\,2}\cdot I_{23-5}^{\,2} + 4\cdot\bar{R}_{\perp\|}^{\,2}\cdot(\tau_{31}^2+\tau_{21}^2)^2}\,] / (2\cdot\bar{R}_{\perp\|}^{\,3})\}^{0.5}$$

$$= \sigma_{eq}^{\perp\|} / \bar{R}_{\perp}^{\perp\|} \quad \text{with} \quad I_{23-5} = 2\sigma_2\cdot\tau_{21}^2 + 2\sigma_3\cdot\tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21} .$$

matrix modes

Note: material stress effort of the mode = 100% = 1. The two *bs are material friction parameters*

5. Fatigue Models and Damage Determination & Accumulation

* ***Fatigue models*** for composites

may be subdivided into three categories [Degrieck-Paepegem], termed:

1.) ***Fatigue life models***

using S-N curves and fatigue failure condition (applied by author, but failure mode-wise)

2.) ***Phenomenological models***

using residual stiffness or residual strength

3.) ***Progressive damage models***

using damage variables associated to measurable damage quantities such as matrix cracks; and further mixed versions.

- ***Damage determination & Accumulation in case of Composites***
  - ***Determination of damaging portions*** *(from diffuse and discrete damaging)*
  - ***Accumulation of damaging portions*** *(cycle-wise, or block-wise, or ?)*

6. Failure mode-based Lifetime Prediction Method

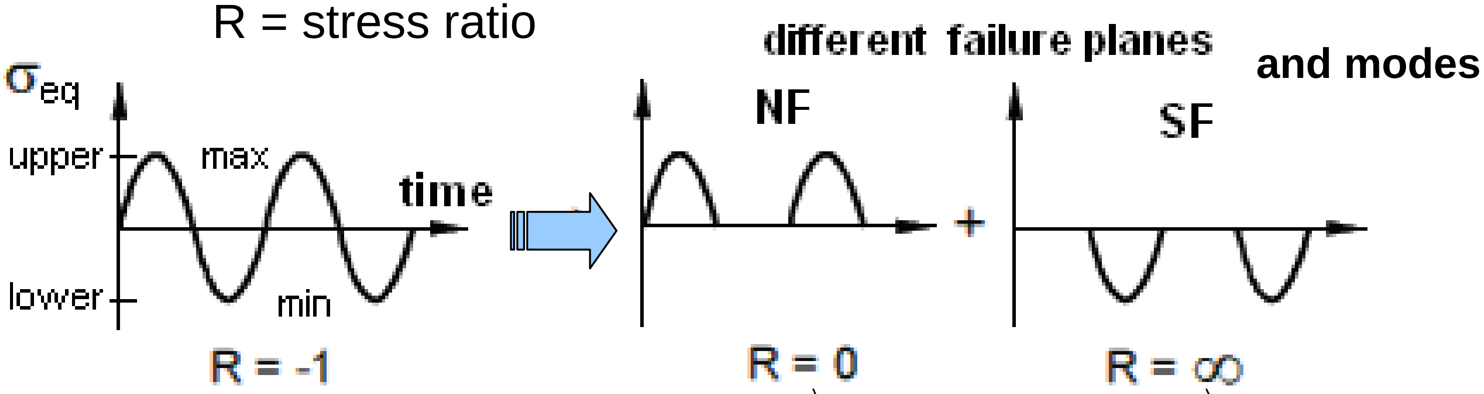
6.1 Modeling of Loading Cycles *failure mode-linked*

IDEA: *PART I*

For the sake of simplicity for displaying the principle :
Isotropic brittle material is chosen and R = -1 loading!

NF:= Normal Frature
SF:= Shear Fracture

R = stress ratio



R = 0.1 and **R = 10** : usually tested master curves

I : <u>Failure mode-linked</u> apportionment of the cyclic loading *(better stressing)*

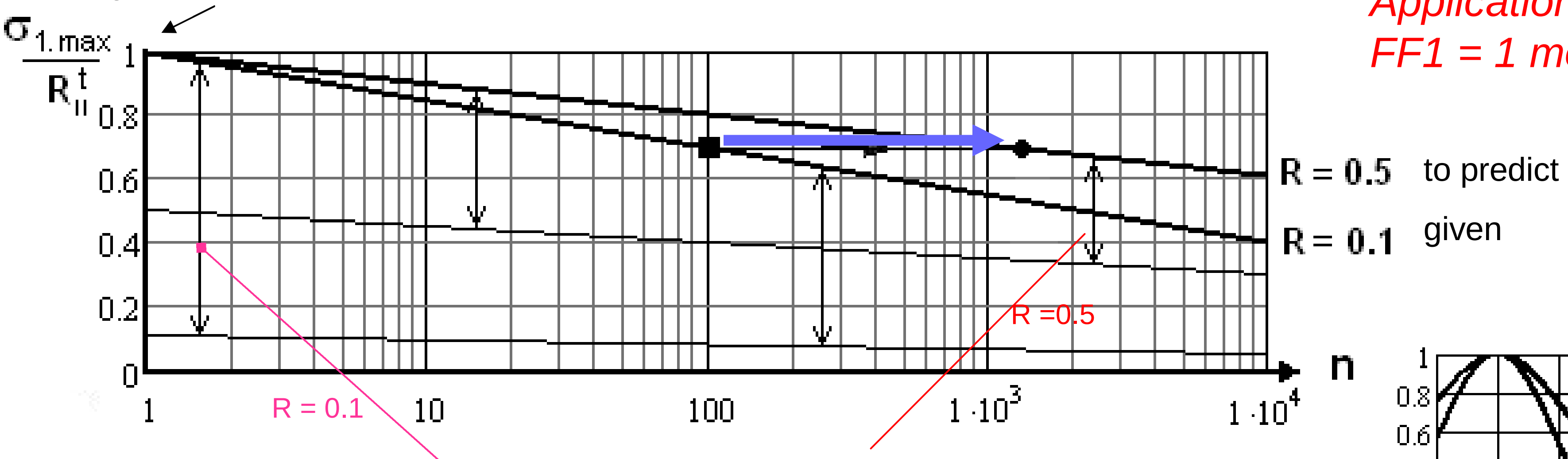
Miner damage accumulation: $D = \Sigma(n_i / N_i) \longrightarrow D = f(NF : \Sigma n_i / N_i + SF : \Sigma n_j / N_j) = 1$

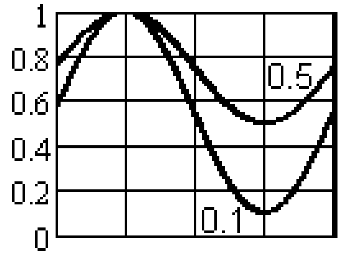
6. Failure mode-based Lifetime Prediction Method

6.2 Prediction of a S-N curve from an available mode-representative one

origin of *FF1* (bunch of) S-N curves

Application:
FF1 = 1 mode !





The normalized double amplitude $2 \cdot \sigma_a$ is depicted

Curve R = 0.5 is to be predicted using the ***approach***:

II : The strain energy level, associated to a point at the <u>given</u> mode-representative S-N curve (here R = 0.1 as 'master' R-ratio curve) will be reached for R = 0.5 at higher cycles.

<u>Note</u>: In case of brittle behaving materials the strength values can be used in the S-N curve as origins at n=1. Therefore, with brittle materials such as composites the maximum (upper) stress representation is more often used. It can be transferred to the stress range according:

$$\sigma_{\max} = 2 \cdot \sigma_a / (1 - R) = \Delta\sigma / (1 - R) \qquad \text{with } \Delta\sigma := \text{stress range}$$

6. Failure mode-based Lifetime Prediction Method

6.3 Approach incl. Damage Accumulation

Logic behind: Fatigue strain energy, required to generate a distinct damage state is equal to the strain energy, which is necessary under monotonic loading to obtain the same damage state.

$$\Delta W = \sum_1^5 \Delta W^{\,modes}$$

strain energy of all mode contributions (5 in UD case)

Idea demonstrated for simple case of 'well-designed, laminates under tension, where the change of strain energy between maximum and minimum loading **for FF1** reads:

$$\Delta W^{\|\sigma} = \Delta(\sigma_{eq}^{\|\sigma} / \bar{R}_{\|}^{t})^2 \Rightarrow \qquad \Delta W^{\|\sigma} \cdot \bar{R}_{\|}^{t\,2} = \sigma_{1,\max}{}^2 - \sigma_{1.\min}{}^2 = \sigma_{1,\max}{}^2 \cdot (1 - R^2)$$

Solving for the maximum stress delivers:

$$\sigma_{1,\max}(n) = \bar{R}_{\|}^{t} \cdot \sqrt{\Delta W^{\|\sigma}(n) / /(1 - R^2)}.$$

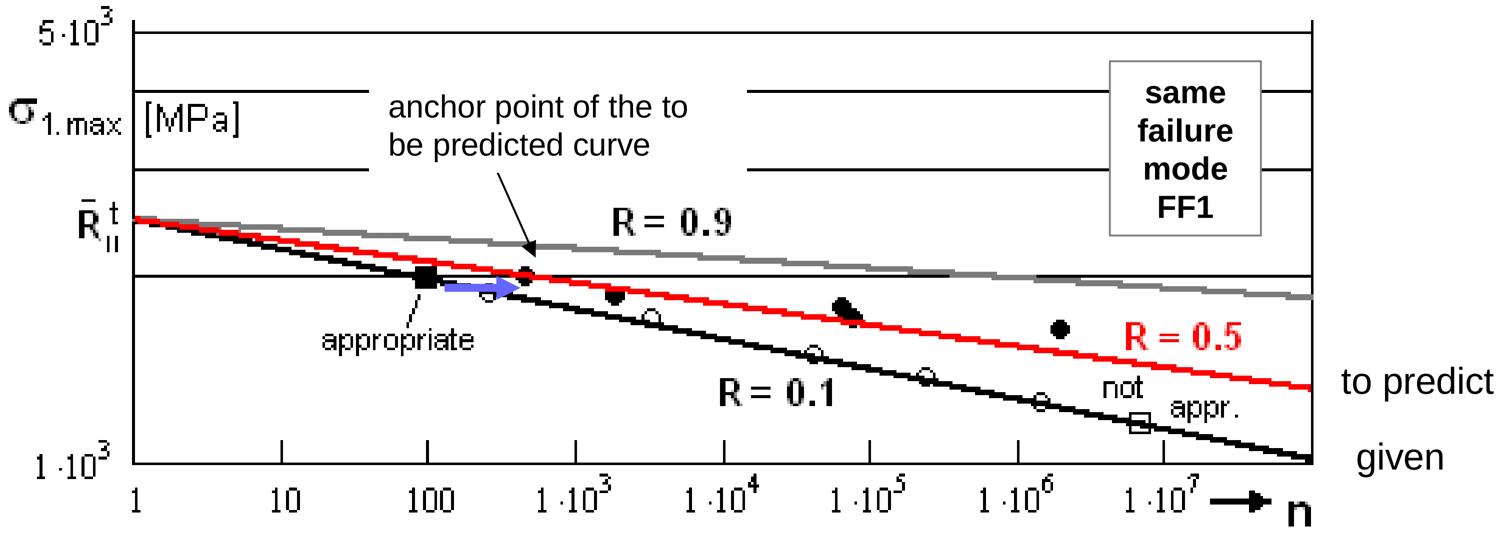
From experiment known:

- Max stress + tensile strength + stress ratio *R;* and thereby the *fatigue strain energy*.
- Course of strain energy can be described by a simple power law function, forming a straight line in a log-log diagram:

$$\Delta W^{\|\,\sigma}(n) = c_1 \cdot n^{-c_2} \text{ [Hwang] }.$$

6. Failure mode-based Lifetime Prediction Method

6.4 Procedure for the Prediction of S-N curves (test-based Example)



Given:

Test points +

$$\Delta W_{R=0.1}^{\parallel\sigma}(n) = c_1 \cdot n^{-c_2} = 0.89 \cdot n^{-0.097}$$

$n_{appr} = 100 cycles$

III : S-N curve may be mapped by a straight line in a log-log graph (safe side)

Given: normalized mode-representative curve (R = 0.1); to be predicted curve: (R > 0.1)

$$\sigma_{1,\max\ repr}(n) = \bar{R}_{\parallel}^{t} \cdot \sqrt{\frac{\Delta W_{R=0.1}^{\parallel\sigma}}{1-R_{repr}^{\ 2}}} = \bar{R}_{\parallel}^{t} \cdot \sqrt{\frac{c_1 \cdot n^{-c_2}}{1-R_{repr}^{\ 2}}} = \bar{R}_{\parallel}^{t} \cdot n^{c_{repr}(n)} \approx \bar{R}_{\parallel}^{t} \cdot n^{c_{repr}},$$

$$\sigma_{1,\max\ pred}(n) \approx \bar{R}_{\parallel}^{t} \cdot n^{c_{pred}}$$

?

Example ***R=0.5*** **: Procedure to determine c_{pred}** *(one anchor point needed besides the strength point)* is depicted below:

$$\sigma_{1,\max\ repr}(n_{appr}) = \bar{R}_{\parallel}^{t} \cdot \sqrt{\frac{c_1 \cdot n_{appr}^{\ -c_2}}{1-R_{repr}^{\ 2}}} = \sigma_{appr}$$

shift from representative curve to predicted curve $\rightarrow$

$$\sigma_{appr} = \bar{R}_{\parallel}^{t} \cdot \sqrt{\frac{c_1 \cdot (n_{appr} \cdot f_{pred})^{-c_2}}{1-R_{pred}^{\ 2}}}$$

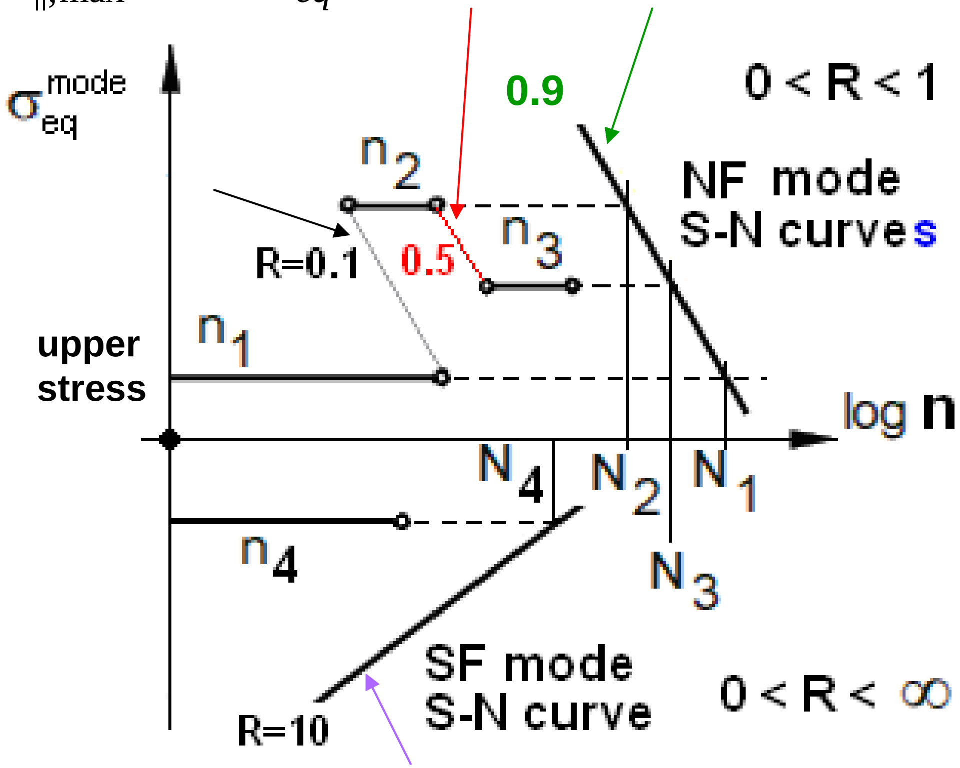
$$c_{pre} = -\ln(\bar{R}_{\parallel}^{t} / \sigma_{appr}) / \ln(n_{appr} \cdot f_{pre}) = -0.034 \quad \Leftarrow \quad f_{pred} = \exp[-\ln(\frac{R_{pred}^{\ 2} - 1}{R_{repr}^{\ 2} - 1}) \cdot \frac{1}{c_2}] = 17.5$$

R = 0.5

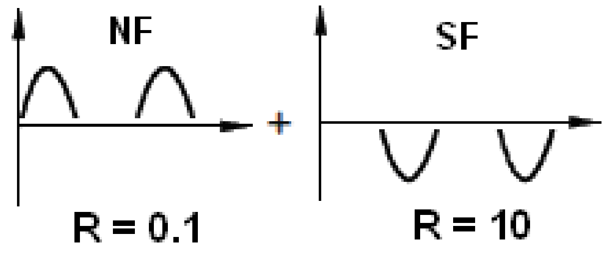
6. Failure mode-based Lifetime Prediction Method

6.5 Schematic Application (principle: for simple isotropic case as example, 4 blocks)

$$\sigma_{\parallel,\max}^{\text{mode}} \Rightarrow \sigma_{eq}^{\text{mode}}$$



remind



here:

2 master curves

NF: R = 0.1

SF: R = 10

2 predicted curves

NF: R = 0.5, 0.9

Miner application:

$$D = n_1 / N_1 + n_2 / N_2 + n_3 / N_3 + n_4 / N_4$$

7. Ideas for Experimental Proof

Choice of Test Specimens, Stress Combinations and Loading Types

Demands on test specimens: Consideration of
embedding of ply, ply-thickness effect, fibre volume fraction, stacking sequence, loadings

1 : Flat coupon material *test specimens* (relatively cheap compared to tubes)

2 : Tension/compression-torsion tube *test specimens* $(\sigma_1, \sigma_2, \tau_{21})$

3 : Sub-laminate *test specimens* *(with internal proof ply and outer supporting plies)*

4 : Flat off-axis coupons (shortcomings ‘free edge effect‘ + bi-axial stiffness loss not accurately considered)

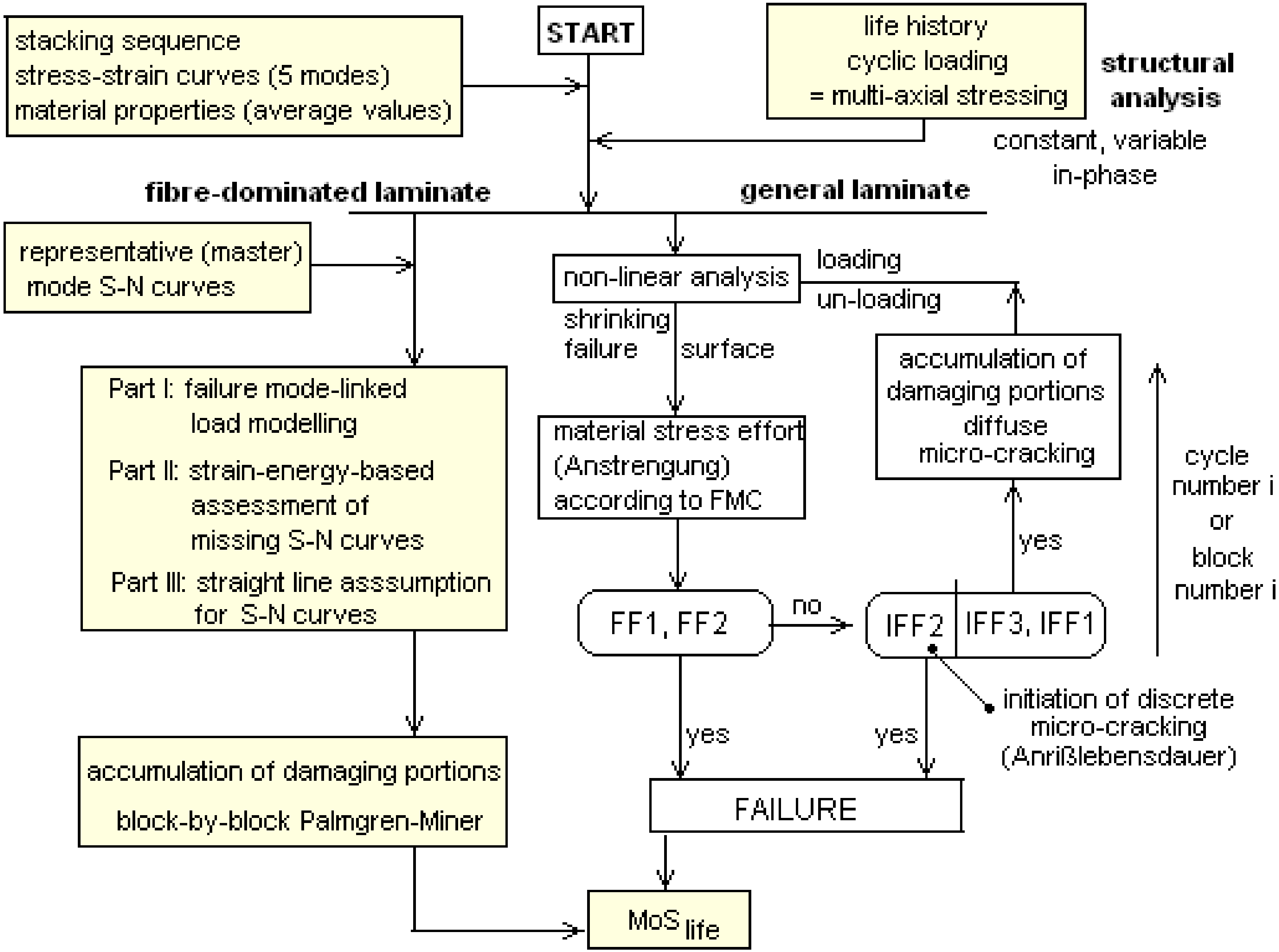
To be tested: Combinations of stresses (3D or 2D state of stresses)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T \quad \Rightarrow \quad \sigma_{\parallel}^t, \; \sigma_{\parallel}^c, \; \sigma_{\perp}^t, \; \sigma_{\perp}^c, \; \tau_{\perp\parallel}$$ **basic stresses**

Model VALIDATION: Loading types applied for the *operational lifetime estimation* are

- ***Constant-amplitude loading*** : delivers S-N curves (Wöhler curve)
- ***Block-loading*** : (if appropriate) for a more realistic Fatigue Life estimation
- ***Random spectrum loading*** : Fatigue Life (Gaßner) curve

Flußdiagramm FMC-basierte Lebensdauerabschätzung



Conclusions and Outlook

It is an **Engineering, failure mode-linked lifetime prediction method for fibre-dominated laminates**

which employs**:**

1.) Failure-mode-related damage accumulation (Miner)

2.) Measurement of a minimum number of failure-mode-linked representative S-N curves

(= *master* R-ratio *curve for each mode*) → test cost reduction

3.) Prediction of other necessary stress-ratio *mode S-N curves* on basis of an available representative one which is typical for the envisaged mode

4.) Use of *strain energy = constant* for this prediction.

Outlook

* The application of the idea looks promising
* The procedure is to be transferred to not fibre-dominated lay-ups where the other failure modes will be significant.

from Cuntze, R.: *Lifetime Prediction for Structural Components made from Composite Materials – industrial view and one idea*. NAFEMS World Congress 2009, Conference publication