Fundamentals in the Development of *Reliable Structural Components?*

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Abstract

Modern light-weight structures are the result of an optimisation compromise between all the product's functional requirements (e.g. stiffness, strength) and all the operational requirements (e.g. lifetime).

Very often the strength requirement determines the mass. Design driving hereby are the material properties and the failure conditions for fracture and yielding (functional design verification).

Responsible for the quality of a structural component achieved under the minimum mass requirement are a qualified analysis procedure, a reliable data input including the dimensioning load cases, and the safety concept.

Special task of the designer is the development of a so-called robust structure which does not essentially change its behaviour under the usual scatter of the stochastic parameters with shortcomings for the fulfilment of its functions.

Due to above aspects the following elements will be dealt with

- *1. Problem Description*
- *2. Introduction to Design Dimensioning and Design Verification (Nachweis)*
- *3. Safety Concept Applied*
- *4. Input of Appropriate Properties for Linear and Non-linear Analysis*
- *5. Design Limit Loads, Dimensioning Load Cases, Load Interaction Failure*
- *6. Link Deterministic to Probabilistic Design.*

Also, the definitions of some notions such as Design Limit Load (Sichere Last) and Dimensioning Load Case are presented.

Finally, an application of the reliability analysis to the ARIANE 5 Boosters shall demonstrate the usefulness.

Keywords: Robust Design, Optimization, Stochastic

General

Robustness and reliability are cross-linked to some extent. Non-robustness with respect to the requirements in the Technical Specification may cause heavy changes of the reliability level. The *risk*, defined as *costs in case of failure times the probability the failure may occur* is higher than in case of a robust design optimisation.

Stochastic design parameters are termed uncertain basic variables, the size of which is uncertain (before realization of them) and random (after realization). Uncertainty and randomness can be described by a distribution law with its distribution parameters, e.g. in case of a parent normal distribution (Gauß) the statistical parameters mean μ (test sample \bar{x}) and standard deviation σ (test sample *s*).

Usual optimisation procedure for a structural component is optimisation in respect of the different actual failure modes. A distinct set of design parameters was optimized in the design space with respect for an optimal state such as for the failure modes buckling, fracture, limited strains or a natural frequency.

All the possible (failure) limit states are not met by the deterministic set of optimal design parameters by a certain distance due to the required FoS which are usually used as Design Limit Load-increasing factors. Unfortunately, this distance is not quantifiable.

However, the probabilistic optimisation provides the designer with a measure for the distance by giving him a number for a *reliability = 1- failure probability*. Of course this number is a fictitious one because it depends on the quality of the used model.

A deterministic-optimal set of design parameters does not consider the sometimes heavily different coefficients of variation $CoV = \sigma / \mu$ of the driving design parameters and the probability of their combined appearance (joint probability of failure). This means the values, the stochastic design parameters may take in the design space. Therefore normally, the set of deterministically derived optimal design parameters will be different to that of a probabilistically derived one with the consequence, and the nominal values in the drawing will be different.

In probabilistics-based optimisation no factors of safety (FoS) are utilized but the distributions of the stochastic design parameters are applied (for loads usually an extreme value distribution is assumed). Essential aspect of this type of optimisation is the sensitivity this means the influence of a design parameter on the objective function such as a collapse load or a mass value. The lower the change of the sensitivity measures is -in case of a change of the scatter of a design parameter- the more robust the design is. The knowledge of such sensitivity measures helps with management decisions, e.g. Which of the geometrical tolerances can be met simpler and therefore more pricy but keeping the same reliability?

Fundamentals in the Development of Reliable Structural Components

Motivation:

Industry looks for *robust & reliable analysis procedures* **in order to replace the expensive 'Make and Test Method' as far as reasonable.**

TASK:

► Sort out weakest link in design process which involves highest uncertainty. → Just then: a Qualified Prediction Method is achievable.

Fundamentals in the Development of *Reliable Structural Components*

Contents of Presentation: (35 min + 5 discussion time)

- **1. Problem Description**
- **2. Introduction to** *Design + Analysis* **and** *Design Verification*
- **3. Safety Concept Applied**
- **4. Input of Appropriate Properties for Linear and Non-linear Analysis**
- **5. Design Limit Loads, Dimensioning Load Cases, Load Interaction Failure**
- **6. Link Deterministic to Probabilistic Design**

Conclusions + some Comments

1 Problem Description

1.1 Definitions: Robust Design and Structural Reliability

*Robust Design***:**

Entwurf einer Struktur, der bei den normalen Streuungen der Entwurfsparameter keine so große Veränderung des Tragverhaltens nach sich zieht, so dass lediglich nur eine tolerierbare Gefahr für die Nicht-Erfüllung der Funktionsanforderungen vorhanden ist .

Struktur-Zuverlässigkeit:

Zuverlässigkeit eines Struktur-Bauteils ist die Fähigkeit während einer vorgegebenen Zeit (mit einer bestimmten Wahrscheinlichkeit P) vorgegebene Funktionsanforderungen zu erfüllen.

1 Problem Description

1.2 Development Phases and Associated Topics

Tight schedule constraints lead to a so -called 'Success oriented Development' Logic with its (theoretically) *'One Phase Design Development'* idea

certification of product

Topics which are mandatory to be considered when aiming at a *'SoD' Logic* are :

- *excellent Technical Specifications of the to be developed product*

- a consistent design philosophy incl. safety concept, margins, nonlinear analyses, …

- *- Simultaneous Engineering*
- *-* a *practical risk judging*
- *- accurate modelling* incl. design allowables, mean (σ,ε)-curves, failure hypothesis, mean geometry, ...
- *-* a *back-up design solution* or realization of Design Target "Robustness"…

1 Problem Description

1.3. The Cost Pyramide in Development of a Structural Design

2 Introduction to Design Dimensioning and Design Verification

2.1 Design Requirements

Design must fulfill all *design requirements:*

- *- mass, production cost and life cycle cost, geometry*
- *- loads, temperature, moisture, chemical environment*
- *- limits of deformation, lifetime, leakage, eigenfrequency,*
- *- strength , stiffness , dimensional stability , buckling…*

topic

2 Introduction to Design Dimensioning and Design Verification

2.2 C*losed* **Design analysis and Design Verification** *procedure*

EXAMPLE: Shell buckling:

- 1. Determination of buckling load and buckling modes for the *idealized perfect* shell
- 2. Investigation of realistic imperfections for area pressure and local single forces
- 3. Fixation of most unfavourable imperfections
- 4. Prediction of buckling load and buckling modes for *imperfect* shell
- 5. Measurement of imperfect shell geometry
- 6. Simulation and model improvement
- *7. Improved design concept* **(buckling load determination).**

► Consistent procedures help to minimize uncertainty

2 Introduction to Design and Design Verification

2.3 Static Structural Analysis *Flow Chart (isotropic case)*

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2 Introduction to Design Dimensioning and Design Verification

2. 4 Demonstration of Static Strength Capability

The Design's strength is demonstrated if

- no relevant strength failure (= limit state of a failure mode) is met and - all dimensioning load cases are respected , by a

positive Margin of Safety *MS > 0* or a **Reserve Factor** $f_{Res} = MS + I$ for the prevailing failure modes.

2 Introduction to Design and Design Verification

2.5 Demonstration of Static Strength Capability

Strength Demonstrations are a subset of all *Design Verifications ('Nachweise')*

1) onset of detrimental deformation (**global** yielding, *yield failure*) equivalent stress at *DYL* smaller yield strength 2) onset of fracture *(final failure)* equivalent stress at *DUL* smaller fracture strength $\sigma_{_{eq}} \leq R_{_{m}}$ $\sigma_{eq} \leq R_{p0.2}$ **fracture limit functional limit** Normal Fracture or Shear Fracture

> $R_{p0.2}$ = a practical engineering fixation, stands for a remaining plastic strain of *εpl = 0.2% or a maximum detrimental deformation!* σ_{eq}^{\prime} = *equivalent stress*

3.1 The Uncertainties in the Design incl. Design Load Derivation

Uncertainties can be found

- *- in the area of data input*
- *- in the analysis of the whole modelling process*
- *in manufacture.*

Stages with uncertainties comprise

- ** load analysis, testing and test data evaluation,*
- ** choice of non-linear stress-strain curve and safety concept,*
- ** choice of yield condition and fracture conditions,*
- ** structural analysis procedure, and finally the*
- ** determination of the MS value itself.*

(loads, strength properties, geometry, elasticity properties, to lerances, imperfections, etc.)

NOTE: Here, uncertainties include inaccuracies as well as any si mplifications in the design.

3 Safety Concept Applied

3.2 Tpes of Uncertainties

The nature of Uncertainty of *scattering design parameters might be of*

- mechanical type but also of

- statistical type (e.g. the way measurements are performed,

- ** lack of accurate information due to*
	- *insufficient sample size in measurements of a specific design parameter,*
- * limited observations or tests used for estimating the statistical distributions *,*
- ** some uncertainty in the calculation model (e.g. solution procedure, mesh, ..) as well as in the*
- ** results provided by testing , evaluation of 'raw test data'.*

3.3 The Uncertainties in the Design incl. Design Load Derivation

All these uncertainty sources contribute to the *overall Structural Risk* defined here arbitrarily as

Amount of costs (incurred in the case of later failure)

x the probability that the distinct failure occurs in the structural part.

3 Safety Concept Applied

3.4 General Concepts

Safety Concept:

concept, that implements structural reliability in design (safety is actually a wrong term but used).

Two formats are available for considering design uncertainties:

- *- The deterministic format accounts for design uncertainties in a lumped manner by enlarging the design limit loads by multiplication with FOS.*
- *- The probabilistic format maps each single design parameter's uncertainty into a probability density function. Thereby, the joint probability of failure caused by a combination of design parameters can be considered.*

NOTE: The *joint probability of failure* respects the

combination of all scatter-caused varying design parameters

= **Simplest form of the so-called** *Partial Safety Factor concept,* **being** *the simplest probabilistic safety factor concept !*

Which is the actual safety concept in aerospace ?

3.5 Concept applied in Aerospace

Actual safety concept in aerospace = an *improved deterministic format*

- *- enlarges the deterministic loads (or stresses, if linear analysis is permitted) and*
- *- causes a distance to the load resistance (or strengths).*
	- *This distance represents the required positive margin of safety (MS).*
- *-* discriminates load uncertainties considering factors (K_M , K_P) from *design uncertainties considering factors (FoS) !*

3 Safety Concept Applied 3.6 General on (Design) Factors of Safety FoS

Purpose of the Design FoS:

Guaranty quality of the design and of the test in order to achieve a certain level of Structural Reliability for the (flight) hardware!

Mind:

•FoS are used to decrease the chance of failure by covering the un certainties

(affecting the risk of structural failure) of all the given vari ables outside the control of the designer which are primarily uncertainties in the statistical distribution of loads, uncertainties in manufacturing process, material strength properties .

•Missing accuracy in modelling, computing, or test data determina tion cannot be covered by the FoS !

•Assumption: Spacecraft standard is reached by contractor by validaded design methods, qualified manufacturing processes etc.!

•Values for the FoS are different for cases such as :

Manned, un-manned spacecraft and 'Design verification by Analysis only'.

NOTE: The design risk is counteracted by the FoS. Different industry, however, has different risk acceptance attit udes and apply differently high FoS values !

Mit Arthurs HSB JAR Blatt checken

3. 7 Additional Factors in Design

Additionally utilized in design, is taken when the **sizing approach** is **complex.**

Such a factor accounts for

specific uncertainties **linked to analysis difficulties**.

Such factors are *Fitting Factor, Welding Factor, Casting Factor*,…etc .

• FoS values are based on long term experience with

• structural testing (Composite experience is shorter)

3 Safety Concept Applied

3.8 Example for a *Factors of Safety (FOS) Table Draft*

Experience won, shows up higher risk than usual

thermal loading

Term *j p0.2* **does not so much fit to actual (relatively brittle) composites!**

4 Input of Appropriate Properties for Linear and Non-linear Analysis

4.1 Self-explaining Notations for Strength Properties (homogenised material)

NOTE: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m := 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

4 Input of Appropriate Properties for Linear and Non-linear Analysis

4.2 Utilization of which Statistical Properties ?

expected to fall, with a 95% confidence (on test data achievement) level, see MIL-Hdbk 17.

5 Design Limit Loads and Dimensioning Load Cases

5.1 Load Cases for the example High Pressure Vessel

ESA/ESTEC requirement standard ECSS-30-2 *on* pressurised hardware *says:*

As a minimum, any item of a pressurised hardware shall possess, throughout the respective service life of the hardware in the expected operating environments, a strength such to withstand the:

- *1. PP (proof pressure) without detrimental deformation;*
- *2. DBP (design burst pressure) without experiencing rupture or fibre failure;*
- *3. DYL and simultaneous internal pressure multiplied by j yield without detrimental deformation;*
- *4. MDP multiplied by j yield and simultaneous loads multiplied by j yield, without detrimen. deformation;*
- *5. DUL and simultaneous internal pressure multiplied by j ult without experiencing rupture or fibre failure;*
- *6. DLP (* \equiv *MDP)* multiplied by j_{ult} and simultaneous loads multiplied by j_{ult} without experiencing *rupture or fibre failure;*
- *7.DUL and simultaneous external pressure multiplied by j ult, without experiencing rupture or fibre failure when pressurised to the minimum anticipated operating pressure.*

NOTE: j yield (respectively jult) to be applied on pressure loads and j yield (respectively jult) to be applied on external or thermal loads can be different. Fos for pressure vessels are higher than for mechanical loading. Positive margins of safety shall be demonstrated by analysis or test or both.

This indicates how one can come from single DLL loadings to high number of load cases (may be hundreds or more). From them, finally, the designer has to sort out a limited number of Dimensioning Load Cases (*DLC)*.

5 Design Limit Loads and Dimensioning Load Cases

5.2 Dimensioning Load Cases for the example High Pressure Vessel

In the case of a metallic High Pressure Vessel from the previously given *Load* are derived the following *Dimensioning Load Cases (DimLC)*:

- for ductile behaviour the : Yielding-related Load Cases
- for brittle behaviour the : Ultimate-related Load Cases.

6 Link *Deterministic to Probabilistic Design* **6.1 Procedures**

The usual way to "verify a design" is to show by computation, $MS > 0$,

the resistance of a structure is higher than the loading (for critical cross section loads or stress combinations or ...). \mathfrak{R} and \mathfrak{R}

In the reliability analysis, this way is more complex.

→

It's objective is the evaluation of a probability of occurence of a given failure state p_f < **admissible** p_f ,

or **of a survival probability** = **reliability** \mathbb{R} > **required .**

$$
\Re = 1 - p_f
$$

6 Link *Deterministic to Probabilistic Design* **6.2 Analysis Goal and Optimisation**

Goals in *Structural Analysis*

- **in order to achieve** *Structural Integrity :*
- **Mass minimisation of the structure (mathematical formulation)**

in the prescibed design space wrt *side constraints* **such as** *cost, project deadlines, manufacturing* **and** *NDI needs , risk* **(amount of failure cost • probability this failure occurs)**

 Prediction of structural behaviour and strength analysis (*Design Verification* **)** → **result : a** set of nominal (mean) design parameters.

6 Link *Deterministic to Probabilistic Design*

6.3 Analysis Goal and Optimisation

Goals of Structural Analysis

- **Mass minimisation of the structure (mathematical formulation) in the prescibed design space wrt** *side constraints* **such as** *cost, project deadlines, manufacturing and NDI needs , risk* **(amount of** *failure cost* **•** *probability* **this failure occurs)**
- **Prediction of structural behaviour and strength analysis (** *Design Verification* **) result : a set of nominal (mean) design parameters.**

Deterministic optimisation of a structural model

 \rightarrow set of mean design parameters

with which the (failure) limit states are not reached by a not quantifiable distance in the frame of the scatter covering deterministic FoS

Stochastic (probabilistic) optimisation (dream becomes slowly true)

 \rightarrow set of mean design parameters (*'most probable failure point'*) however with p_f as a measure for this distance and directly considering the scatter of all design parameters.

6.3 Modelling Types definition of structure (system)/component/part definition of failure modes: e.g. collapse, burst due to FF, weeping, due to IFF stiffness limit, deformation limit etc. to be modelled on topdefinition of suitable failure criteria: e.g. inter-fibre failure (IFF), fiber failure (FF), plastic hinge (ductile material), max. stress at a surface (brittle material), buckling, creep deformation * Logical Modelling of Failure formulation of corresponding limit state functions $g_i(X_i)$: System • Analysis of the log. structure of generation of system equations, structural the failure modes on the levels analysis (considering nonlinearities) "system", laminate, layer by • Mechanical Modelling • applying series, parallel or combined system behaviour · Stochastic Modelling of UBV · distribution type, parameters (μ, σ) and correlation of uncertain basic variables (UBV) Choice of Method for the **Results**: $p_{f,i} = P(g_i(X_i) \le 0)$; $\alpha_{i,i}$; $p_f \le \sum p_{f,i}$ probabilistic analysis

6 Link *Deterministic to Probabilistic Design*

 $UBV =$ uncertain basic variable

Utilization for lay-out and design verification

6.4 Example: Pressure Vessel (e.g. ARIANE 5 Booster)

6.4 Example : Tension Rod

vertically cut off by $r = s$ **or** $g(X) = 0$ **. NOTE: Failure probability corresponds to percentage of the volume of the probability hill**

Example: Pressure Vessel

3 Effect of different Strength Distribution on Failure Probability

O.K.

NOTE: Despite lowering the mean strength value the failure probability p_f is reduced **due to the reduction of the Coefficient of Variation !**

Further application: Ariane 5 Launcher, Booster 4 Influence of Reduction of Manufacturing Tolerance

Thickness tolerance

Reduction in scatter permits

– keeping the same theoretical reliability value $\mathcal{R} = I - p_f = I - 5 \cdot 10^{-6}$ -New nominal thickness : $t = 8.1 + 0.05$ mm

> \blacktriangleright mass reduction of ? kg at the same reliability level!

> > + Fuel savings ???

Summary of experience with Deterministic + Probabilistic analyses

- 1. A *robust reliable design* or robustness to later changes of the design parameters with identification of the most sensitive design parameters is a need in order to save costs
- 2. Deterministic optimization may deliver fully non-robust, even non-stabile designs and probably not the real optimum. Counteraction means more effort for the designer: Execution of several common deterministic optimization runs regarding the scatter of the design parameters
- 3. Stochastic optimization means a little more input effort for the designer and a high effort for the computer. However, just a stochastic robust design informs about the risk
- 4. Failure probability p_f does not dramatically increase if a Margin of Safety turns slightly negative. A local safety measure of e.g. $MS = -1\%$ should be no problem in *design development.*

The MS value does not outline the risk or the failure probability. Therefore, do not overreact by re-designing but apply a 'Think (about) Uncertainties' attitude by recognizing the main driving design parameters and by reducing the scatter (uncertainty) of them. This highly pays off!

- 5. Essential question wrt all uncertainties is whether these increase the risk to an unacceptable level or not
- 6. Physics have to be modelled accurately in the analysis part *Mechanical Modelling.* All Dimensioning Load Cases have to be accounted for. The choice of the task-corresponding (σ,ε) curve has to be carefully performed (min or mean or max). The choice of an engineering or a true stress-strain curve depends on the output of the utilized code
- 7. Both, an increasing mean value and a decreasing standard deviation lower p_f
- 8. Final comment: *Theory* 'only' creates a model of the reality, and *Experiment* is 'just' one realisation of the reality. Experimental results can be far away from the reality like an inaccurate theoretical model.

So, find a compromise to achieve an improved *Test Verification* of the design.

ANNEXES:

Risk = amount of cost incurred in the case of later failure x probability of its occurrence

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