Unlocking Mystery of Composites

Stephen W. Tsai & Daniel Melo
Stanford University
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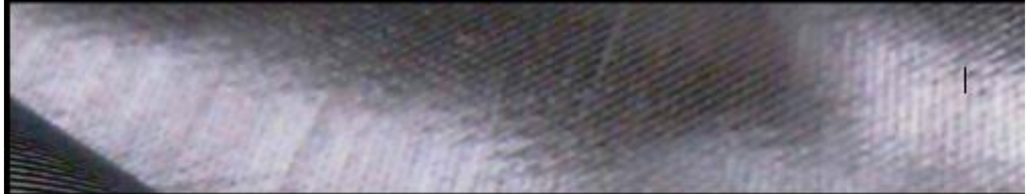
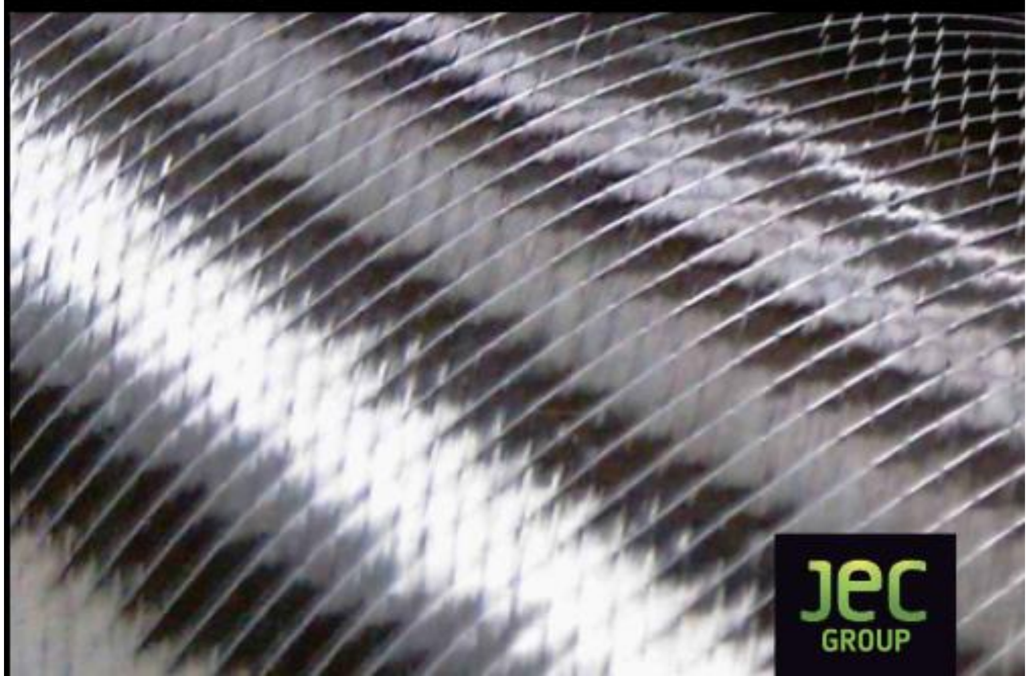


Composite Materials Design and Testing

Unlocking mystery with invariants

Stephen W. Tsai
José Daniel D. Melo



An innovative invariant-based approach developed makes trace of the plane stress stiffness matrix as the heart and soul of composite materials. It can put order in both stiffness and strength of composite laminates. Testing unidirectional [0] specimens should provide enough information for the prediction of the failure envelope of laminates. Then, laminates can be tested "as-built" and data compared to the predicted properties using a master stress-strain template, which greatly reduces the number of tests required. This method can be much more effective than the traditional building block approach. With fewer specimens to be tested, emphasis can be placed on the quality of laminates as reflected by the conversion of stiffness and strength from processing and its associated defects. Environmental effects such as hot-wet conditions can also be accounted for through their respective trace values. This new approach unlocks the mystery of composites design and opens up many opportunities in laminate design and manufacturing.

" ... very impressed with the underlying logic and ingenuity ... ... offers an expedient process for introducing, evaluating and adopting new candidate material systems without the need for extensive and expensive testing ..."

Isaac Daniel, Northwestern University

"... a major finding in the mechanics of composites ... "

Peter Linde, Airbus

Tensors and Traces for Composites

- Stress components σ_i
- Strain components ε_i
- Stiffness: Q_{ij}, A_{ij}, D_{ij}, B_{ij},…
- Failure criterion in stress space: F_{ij}, F_i

  where $F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$
- Failure criterion in strain space: G_{ij}, G_i

  where $G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i = 1$

  $G_{ij} = Q_{ik}Q_{jl}F_{kl}$. $G_i = Q_{ij}F_j$

Tr [σ] $= \sigma_1 + \sigma_2 + \sigma_3$ = pressure

Tr [ε] $= \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \Delta$ volume

Tr [Q] $= Q_{11} + Q_{22} + 2Q_{66}$

= Tr [A*] $= A_{11}^* + A_{22}^* + 2A_{66}^*$

= Tr [D*] $= D_{11}^* + D_{22}^* + 2D_{66}^*$

Tr [B*] = 0

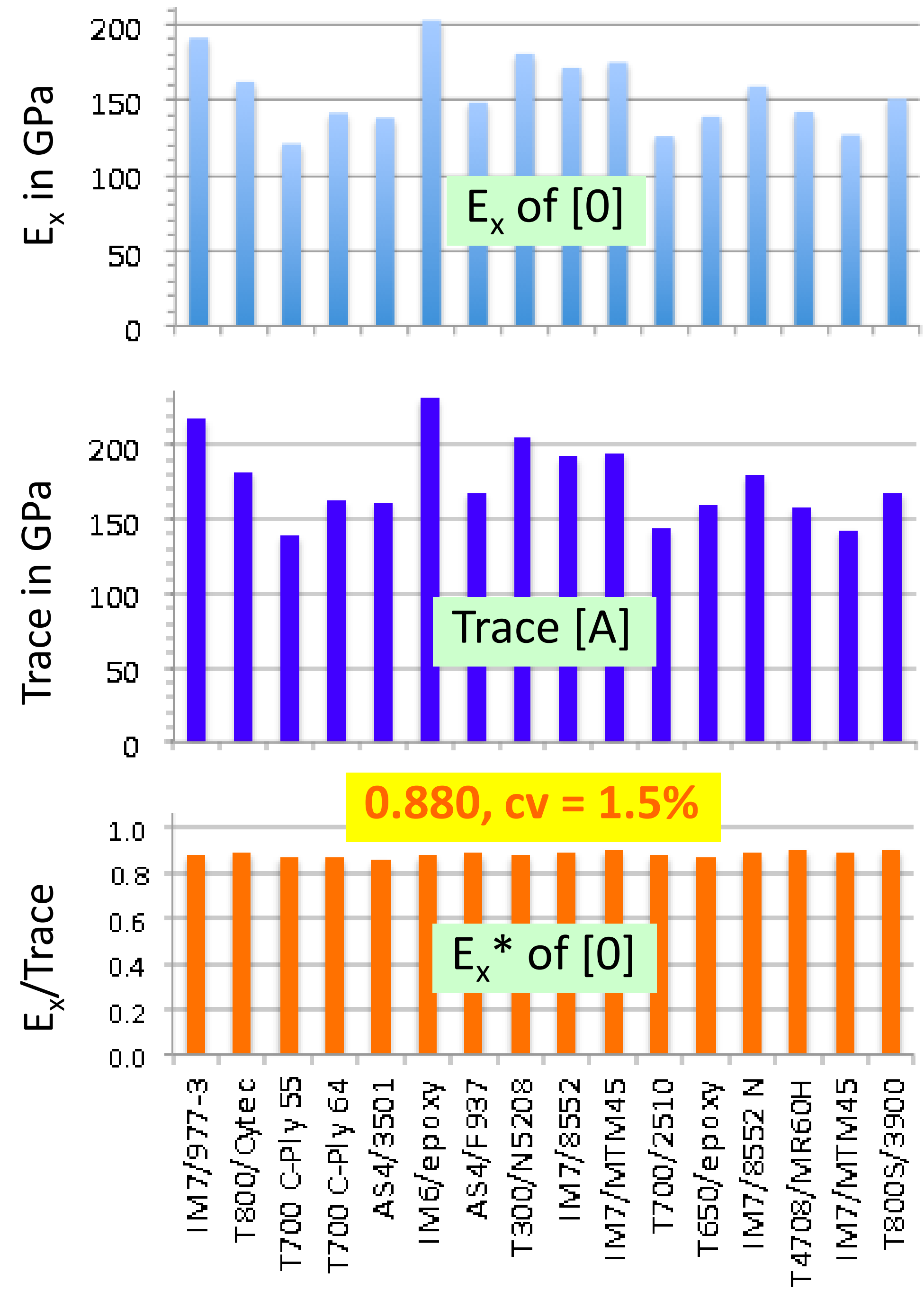
Tr [F]$= F_{11} + F_{22} + F_{66}/2$

Tr {F} $= F_1 + F_2$

Tr [G] $= G_{11} + G_{22} + 2G_{66}$

Tr {G} $= G_1 + G_2$

Material	Ex	Trace	Ex*
IM7/977-3	191	218	0.88
T800/Cytec	162	183	0.89
T700 C-Ply 55	121	139	0.87
T700 C-Ply 64	141	163	0.87
AS4/3501	138	162	0.85
IM6/epoxy	203	232	0.87
AS4/F937	148	168	0.88
T300/N5208	181	206	0.88
IM7/8552	171	192	0.89
IM7/MTM45	175	195	0.90
T700/2510	126	144	0.88
T650/epoxy	139	160	0.87
IM7/8552 N	159	180	0.88
T4708/MR60H	142	158	0.90
AS4/MTM45	127	143	0.89
T800S/3900	151	168	0.90
		Median	0.88
		cv, %	1.3



CFRP: Trace and Lamination Factors

Laminate	Trace	[0]	[0/90]	[π/4]	Soft1	Hard1	Soft2	Hard2
Percent [0]		100	50	25	10	50	9	55
Percent [±45]		0	0	50	80	40	73	36
Percent [90]	GPa	0	50	25	10	10	18	9
IM6/epoxy	232	0.874	0.463	0.337	0.226	0.518	0.226	0.550
IM7/977-3	218	0.877	0.464	0.337	0.225	0.519	0.225	0.551
T300/N5208	206	0.877	0.465	0.337	0.224	0.518	0.225	0.551
IM7/MTM45	195	0.897	0.471	0.337	0.213	0.523	0.217	0.557
T800/Cytec	183	0.888	0.472	0.335	0.211	0.518	0.216	0.552
IM7/8552	180	0.884	0.469	0.336	0.217	0.519	0.220	0.552
T300/F937	168	0.883	0.472	0.335	0.211	0.517	0.216	0.550
T800S/3900	168	0.898	0.476	0.335	0.206	0.521	0.212	0.555
T700 C-Ply 64	163	0.866	0.464	0.337	0.225	0.514	0.226	0.546
AS4/3501	162	0.852	0.456	0.338	0.237	0.512	0.234	0.543
T650/epoxy	160	0.866	0.465	0.337	0.222	0.514	0.224	0.546
T4708/MR60H	158	0.897	0.475	0.335	0.206	0.521	0.212	0.555
T700/2510	144	0.877	0.470	0.336	0.215	0.515	0.219	0.548
AS4/MTM45	143	0.889	0.474	0.335	0.206	0.518	0.214	0.552
T700 C-Ply 55	139	0.869	0.466	0.337	0.222	0.515	0.224	0.547
Mean (master ply)		0.880	0.468	0.336	0.218	0.517	0.221	0.550
Std deviation		0.013	0.005	0.001	0.009	0.003	0.006	0.004
Coeff variation		1.50%	1.17%	0.31%	4.17%	0.58%	2.84%	0.70%

Plane Elasticity & Bending Equations

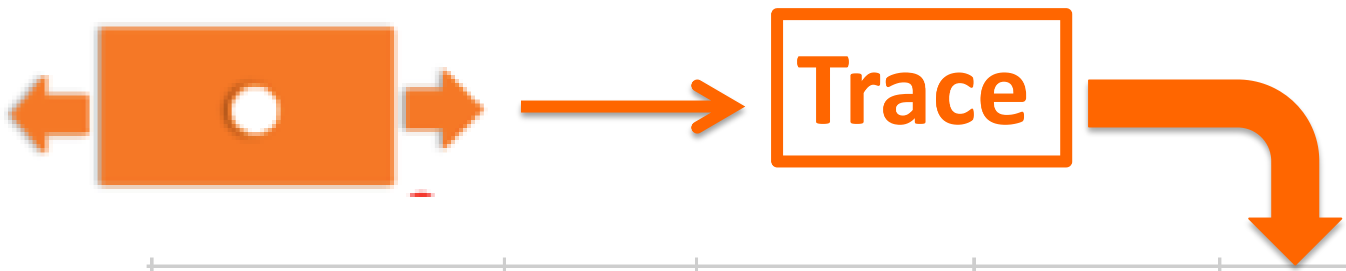
Plane elasticity: $a_{22}^{*}\frac{\partial^4 F}{\partial x^4}+(2a_{12}^{*}+a_{66}^{*})\frac{\partial^4 F}{\partial x^2\partial y^2}+a_{11}^{*}\frac{\partial^4 F}{\partial y^4}=0.$

Plate bending: $D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}=0$

Homogeneity: [D*] = [A*]; [B] = 0

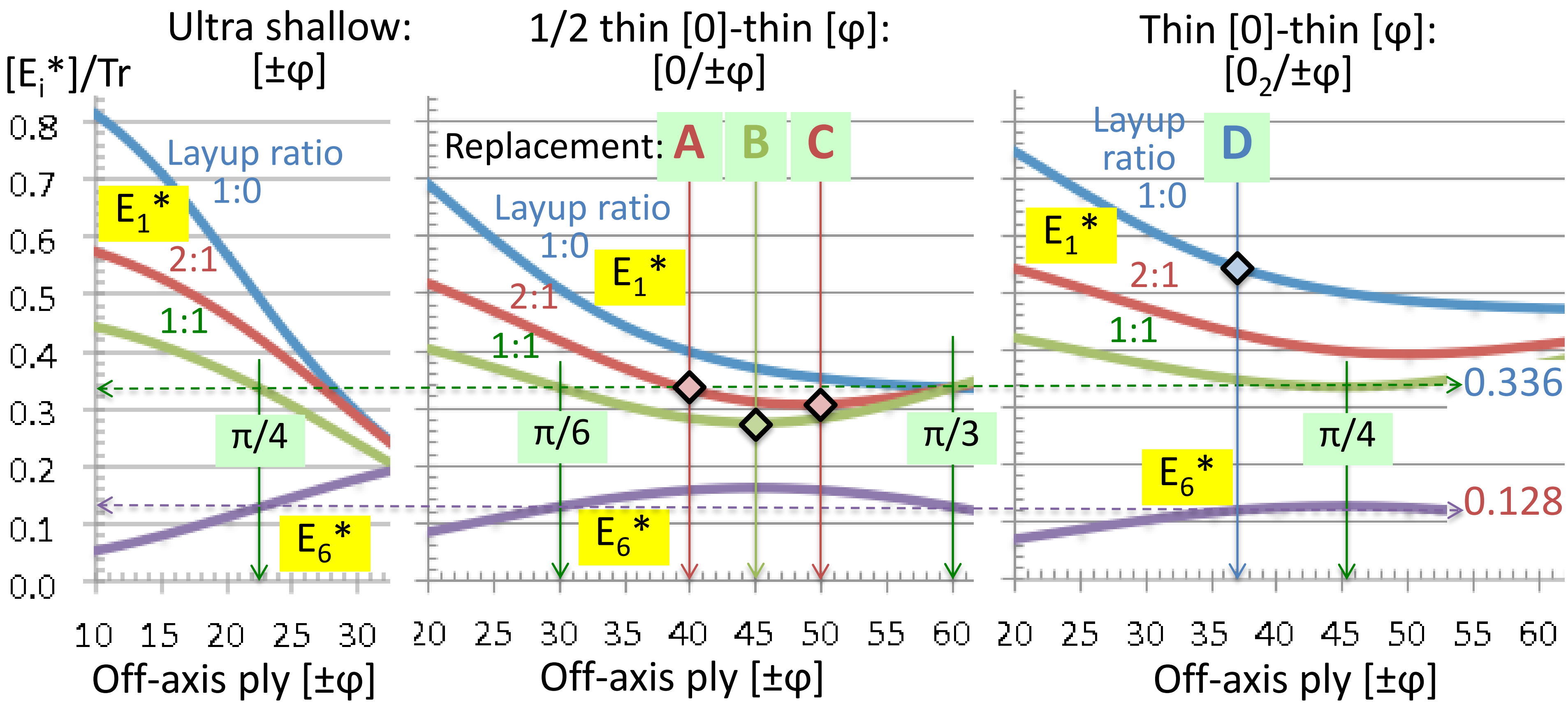
[0/±30]; 2:0	$\frac{2a_{12}+a_{66}}{a_{22}}$	$\frac{a_{11}}{a_{22}}$	$\frac{2(A_{12}+2A_{66})}{A_{11}}$	$\frac{A_{22}}{A_{11}}$	Trace, GPa
IM7/977-3	2.23	0.58	1.20	0.58	218
T800/Cytec	2.23	0.58	1.20	0.58	183
T7 C-Ply 55	2.23	0.58	1.20	0.58	139
T7 C-Ply 64	2.23	0.58	1.20	0.58	163
AS4/3501	2.18	0.59	1.23	0.59	162
IM6/epoxy	2.23	0.58	1.20	0.58	232
AS4/F937	2.33	0.59	1.19	0.59	168
T300/N5208	2.24	0.58	1.20	0.58	206
Master ply	**2.23**	**0.58**	**1.20**	**0.58**	**175**
Std dev	0.044	0.004	0.013	0.004	
Coeff var	2.0%	0.7%	1.1%	0.7%	

One Test for Trace = Multiple Solutions



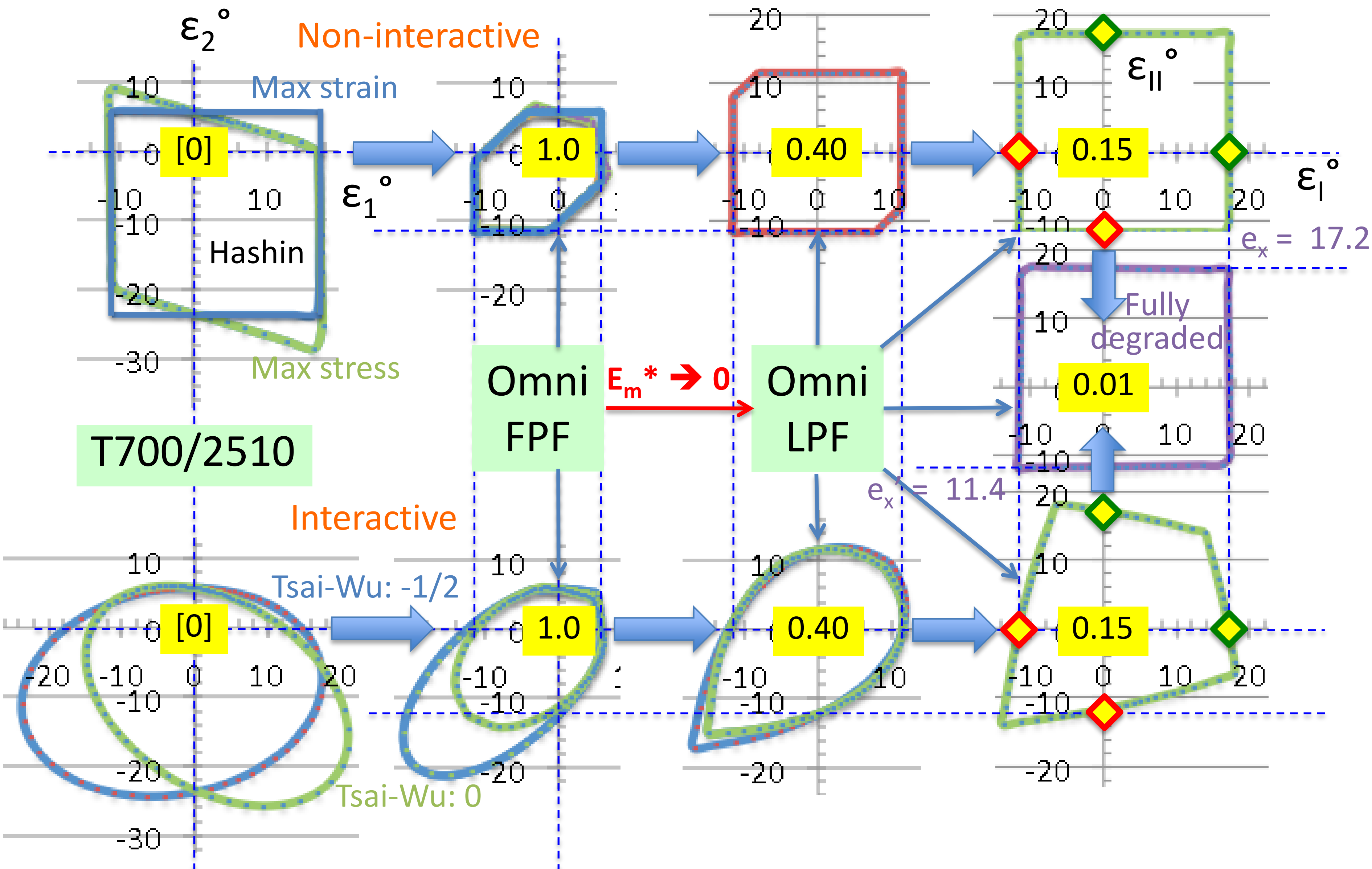
Lam; Ratio	OHT	Press at 0	Shear	Bending	Interference
[0/±30]: 2:0	✔	✔	✔	✔	✔
[0/±30]: 4:2	✔	✔	✔	✔	✔
[0/±30]; 2:2	✔	✔	✔	✔	✔
[0/±45]; 2:0	✔	✔	✔	✔	✔
[0/±45]; 4:2	✔	✔	✔	✔	✔
[0/±45]; 2:2	✔	✔	✔	✔	✔
[0/±phi]; 2:0	✔	✔	✔	✔	✔
[0/±phi]; 4:2	✔	✔	✔	✔	✔
[0/±phi]; 2:2	✔	✔	✔	✔	✔

C-Ply Replacement of Old Laminates



Old C-Ply E_1° E_6°	**A**: $[0_{1.5}/\pm45_2/90]$ $[(0/\pm40)_2/\pm50/90]$ 45.0 vs 46.5 GPa 21.0 vs 21.9 GPa	**B**: $[0/\pm45_4/90]$ $[(0/\pm45_2/90]$ 26.3 vs 30.8 GPa 25.3 vs 25.9 GPa	**C**: $[0_3/\pm45_2/90]$ $[(0/\pm50)_2/\pm40/90]$ 43.0 vs 43.6 GPa 21.8 vs 22.4 GPa	**D**: $[0_5/\pm45_2/90]$ $[0_2/\pm37]$ 75.7 vs 71.6 GPa 17.0 vs 15.3 GPa

Convergence of Failure Criteria



Ply orientation →

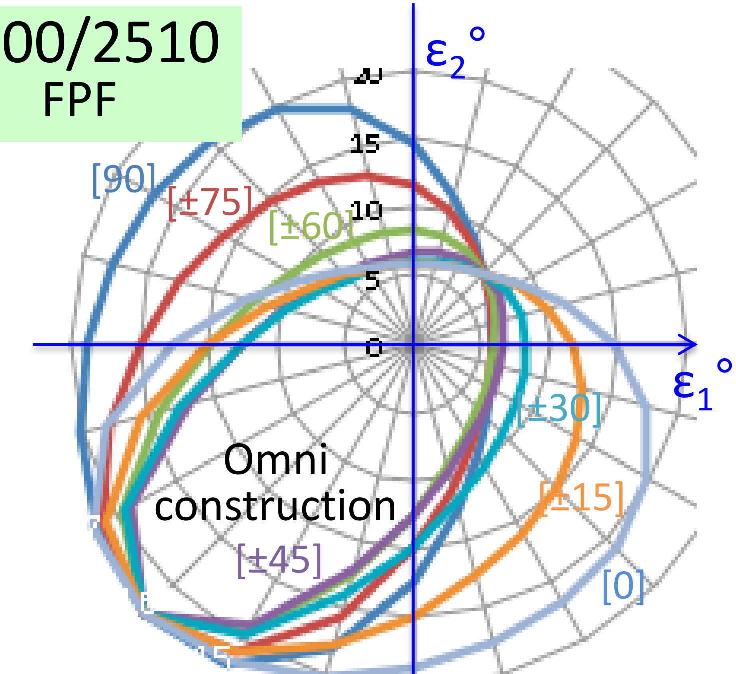
FPF strain vector ↓

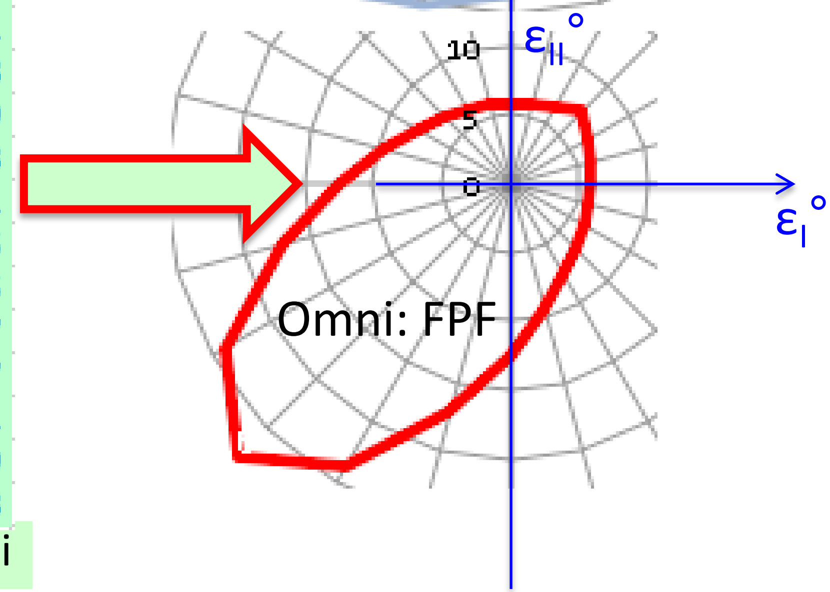
Angle	0	±15	±30	±45	±60	±75	90	Omni
0	14.6	11.6	8.3	6.6	6.0	5.8	5.8	5.8
15	11.4	10.1	8.2	6.9	6.2	6.0	5.9	5.9
30	8.9	8.6	7.9	7.2	6.7	6.5	6.4	6.4
45	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4
60	6.4	6.5	6.7	7.2	7.9	8.6	8.9	6.4
75	5.9	6.0	6.2	6.9	8.2	10.1	11.4	5.9
90	5.8	5.8	6.0	6.6	8.3	11.6	14.6	5.8
105	6.0	5.9	5.9	6.5	8.4	12.8	17.7	5.9
120	6.5	6.3	6.1	6.7	8.7	13.8	19.8	6.1
135	7.6	7.1	6.7	7.2	9.3	14.7	21.0	6.7
150	9.4	8.5	7.7	8.1	10.4	16.0	21.8	7.7
165	12.6	11.0	9.5	9.7	12.1	17.6	22.7	9.5
180	17.5	14.9	12.5	12.5	14.8	19.9	23.8	12.5
195	23.2	20.6	17.6	17.1	19.1	22.9	25.3	17.1
210	27.1	26.0	24.4	23.8	24.6	26.1	26.9	23.8
225	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9
240	26.9	26.1	24.6	23.8	24.4	26.0	27.1	23.8
255	25.3	22.9	19.1	17.1	17.6	20.6	23.2	17.1
270	23.8	19.9	14.8	12.5	12.5	14.9	17.5	12.5
285	22.7	17.6	12.1	9.7	9.5	11.0	12.6	9.5
300	21.8	16.0	10.4	8.1	7.7	8.5	9.4	7.7
315	21.0	14.7	9.3	7.2	6.7	7.1	7.6	6.7
330	19.8	13.8	8.7	6.7	6.1	6.3	6.5	6.1
345	17.7	12.8	8.4	6.5	5.9	5.9	6.0	5.9
360	14.6	11.6	8.3	6.6	6.0	5.8	5.8	5.8

Controlling plies

Omni

T700/2510
FPF





T700/2510

$E_m^* = 0.15$

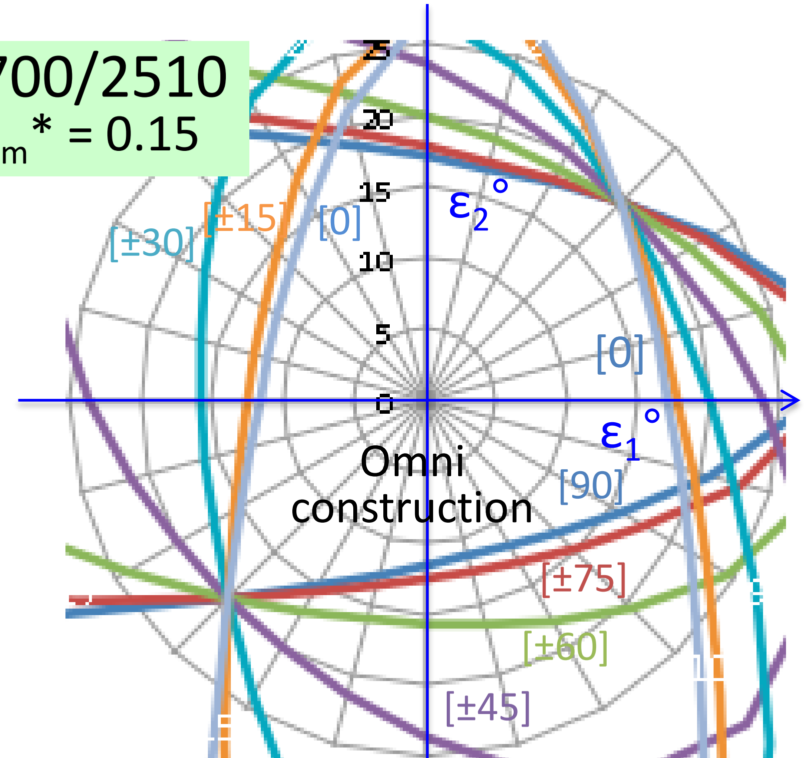
Ply orientation →

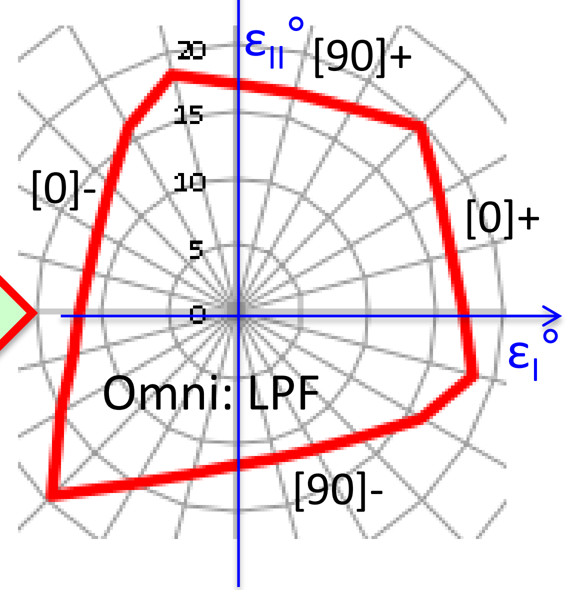
LPF strain vector ↓

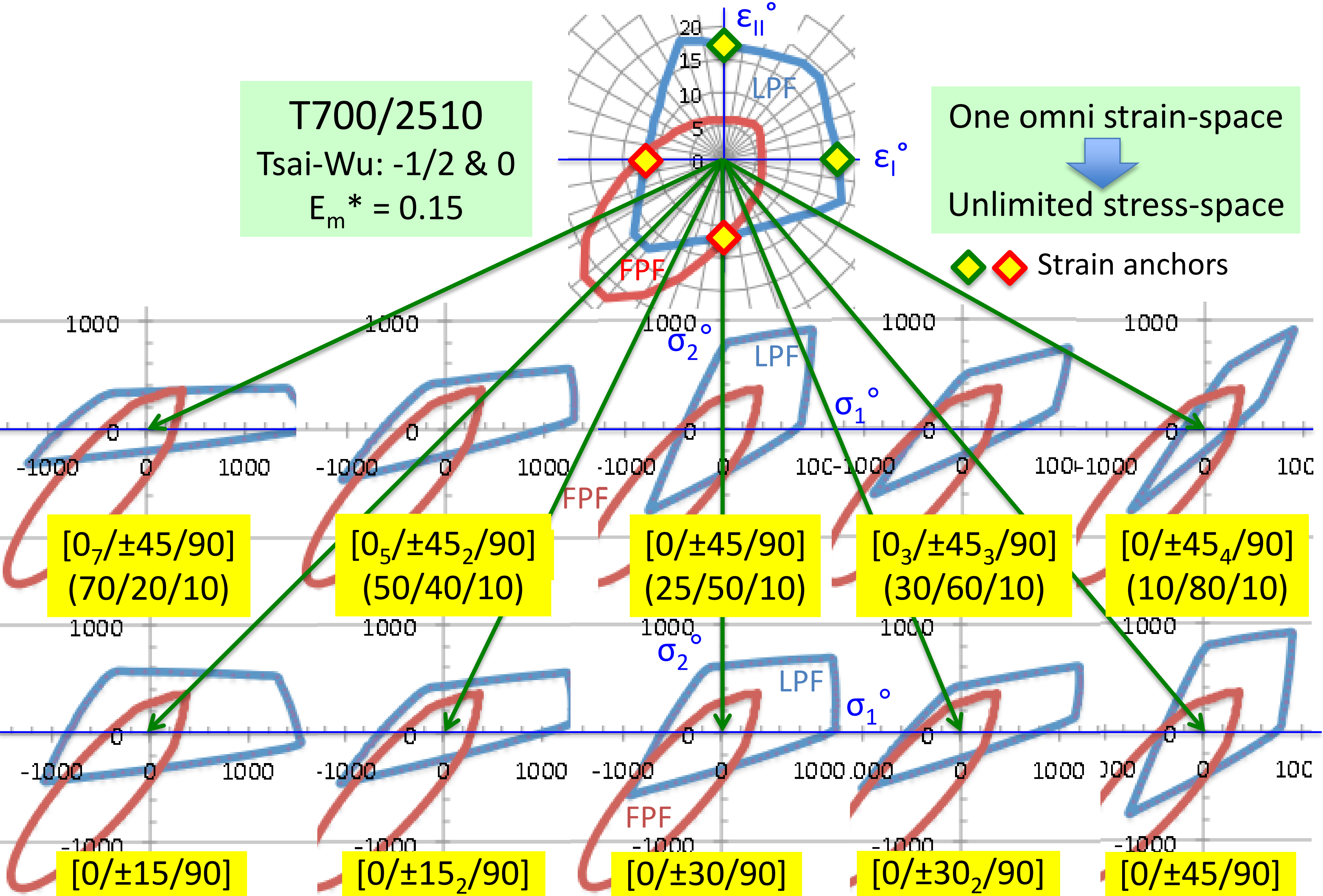
Angle	0	±15	±30	±45	±60	±75	90	Omni
0	17.1	17.8	20.0	23.7	27.4	28.1	27.4	17.1
15	16.9	17.4	18.9	21.4	24.4	26.8	27.7	16.9
30	17.6	17.9	18.8	20.0	21.5	22.7	23.1	17.6
45	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6
60	23.1	22.7	21.5	20.0	18.8	17.9	17.6	17.6
75	27.7	26.8	24.4	21.4	18.9	17.4	16.9	16.9
90	27.4	28.1	27.4	23.7	20.0	17.8	17.1	17.1
105	20.9	23.0	27.7	26.9	22.5	19.4	18.5	18.5
120	15.9	17.9	24.4	30.4	26.9	22.8	21.5	15.9
135	13.2	14.7	20.6	32.1	34.3	29.4	27.4	13.2
150	11.9	13.1	17.8	30.4	45.1	42.7	39.9	11.9
165	11.4	12.4	16.3	26.9	51.9	70.2	70.4	11.4
180	11.6	12.5	15.7	23.6	43.9	85.0	111.7	11.6
195	12.7	13.4	15.9	21.3	31.9	48.8	59.9	12.7
210	15.0	15.5	17.1	20.0	24.1	28.2	30.1	15.0
225	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6
240	30.1	28.2	24.1	20.0	17.1	15.5	15.0	15.0
255	59.9	48.8	31.9	21.3	15.9	13.4	12.7	12.7
270	111.7	85.0	43.9	23.6	15.7	12.5	11.6	11.6
285	70.4	70.2	51.9	26.9	16.3	12.4	11.4	11.4
300	39.9	42.7	45.1	30.4	17.8	13.1	11.9	11.9
315	27.4	29.4	34.3	32.1	20.6	14.7	13.2	13.2
330	21.5	22.8	26.9	30.4	24.4	17.9	15.9	15.9
345	18.5	19.4	22.5	26.9	27.7	23.0	20.9	18.5
360	17.1	17.8	20.0	23.7	27.4	28.1	27.4	17.1

Controlling plies

Omni

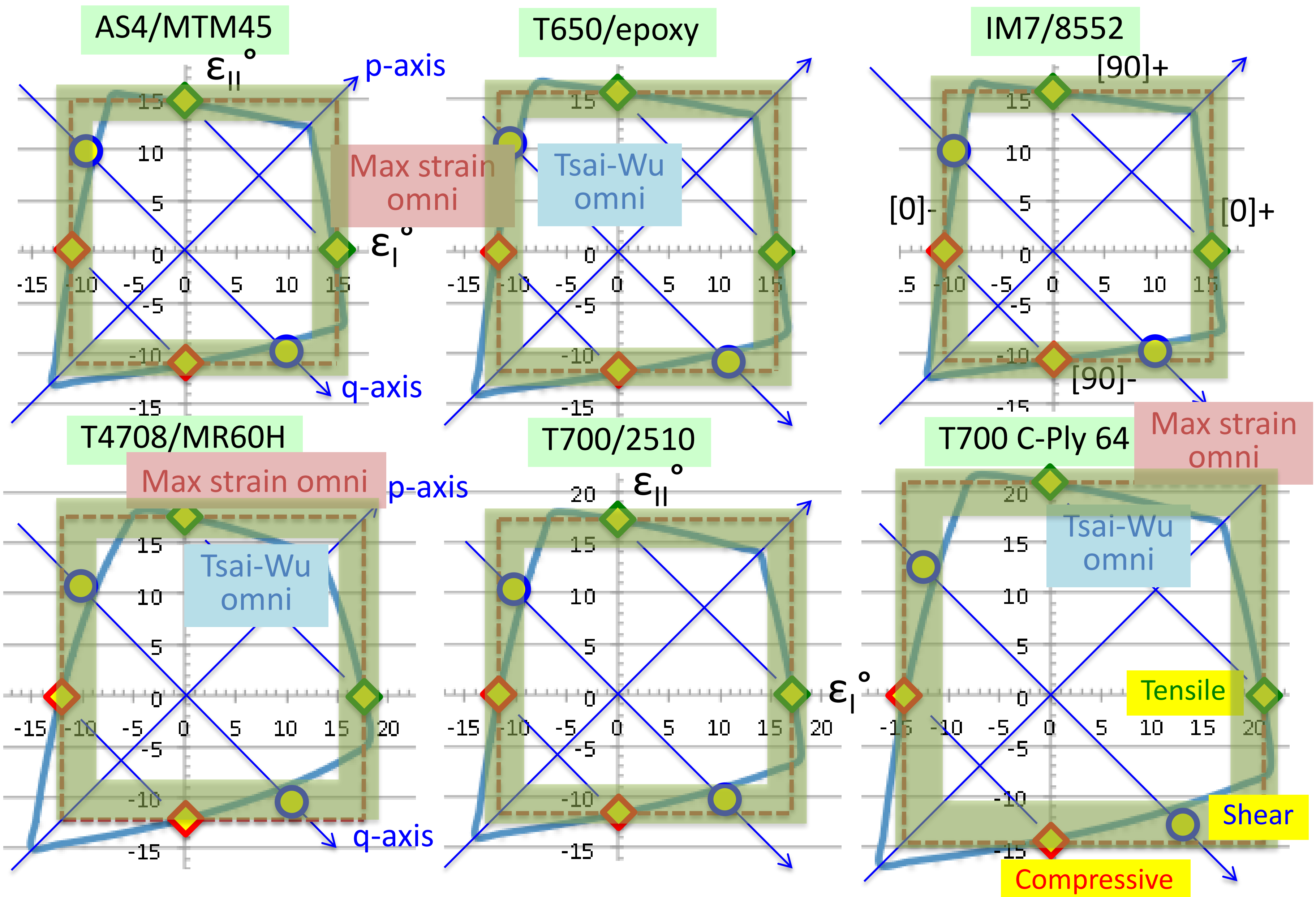






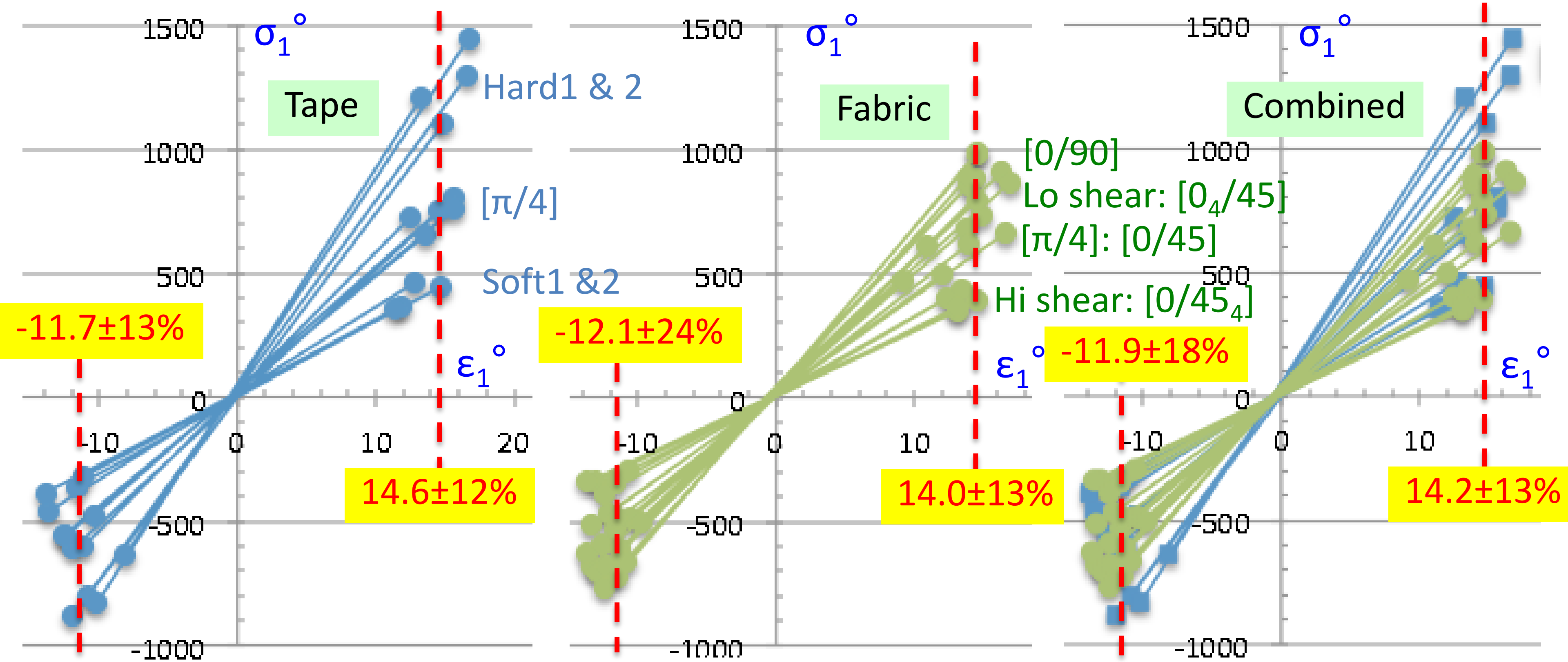
T700/2510
Tsai-Wu: -1/2 & 0
E_m^* = 0.15
One omni strain-space
Unlimited stress-space
Strain anchors
ε_{II}°
ε_I°
LPF
FPF
σ_2°
σ_1°
[0_7/±45/90]
(70/20/10)
[0_5/±45_2/90]
(50/40/10)
[0/±45/90]
(25/50/10)
[0_3/±45_3/90]
(30/60/10)
[0/±45_4/90]
(10/80/10)
[0/±15/90]
[0/±15_2/90]
[0/±30/90]
[0/±30_2/90]
[0/±45/90]

Omni LPF Strain Envelopes

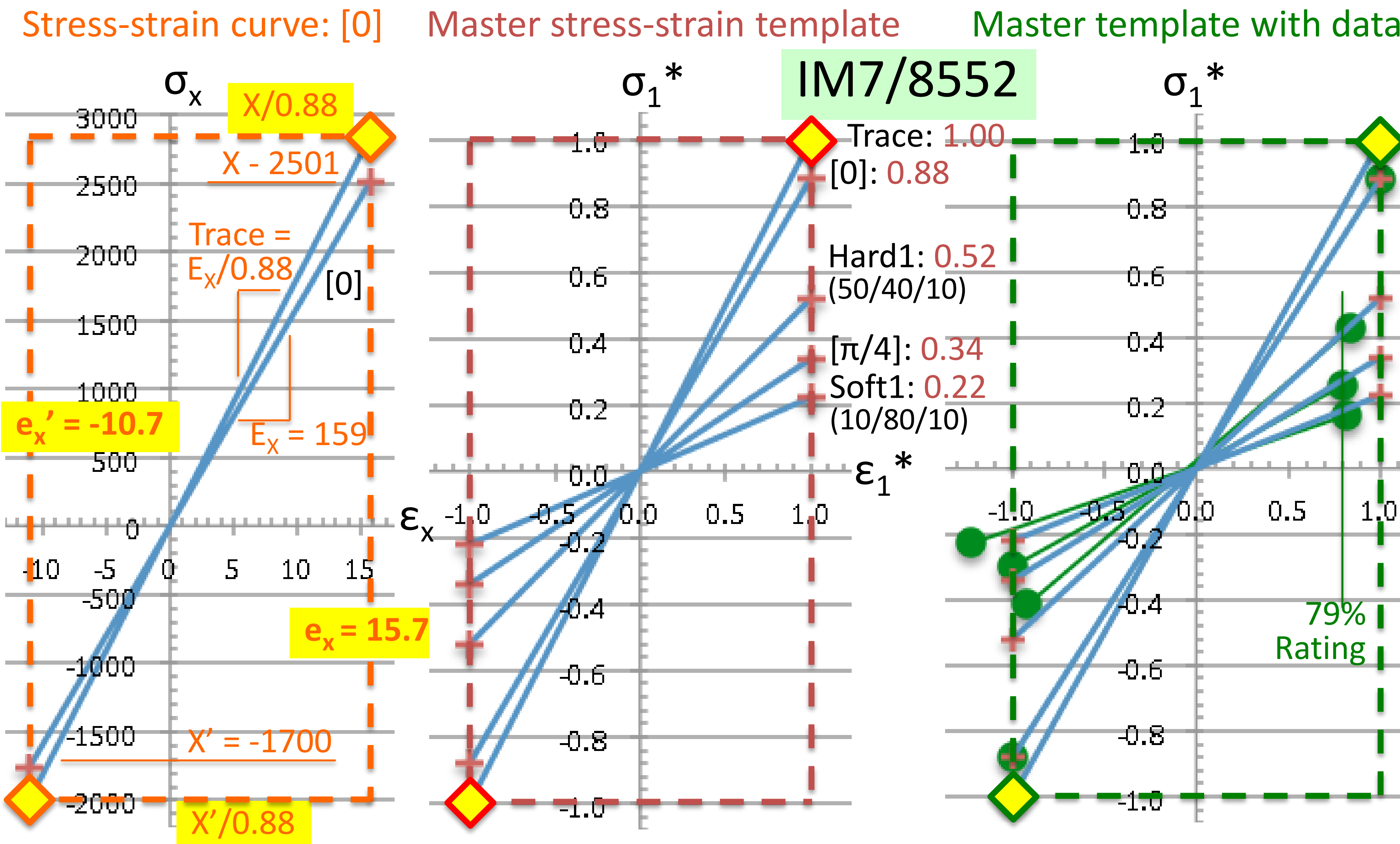


Tensile & Compressive Strength

CFRP unitape and fabric laminates

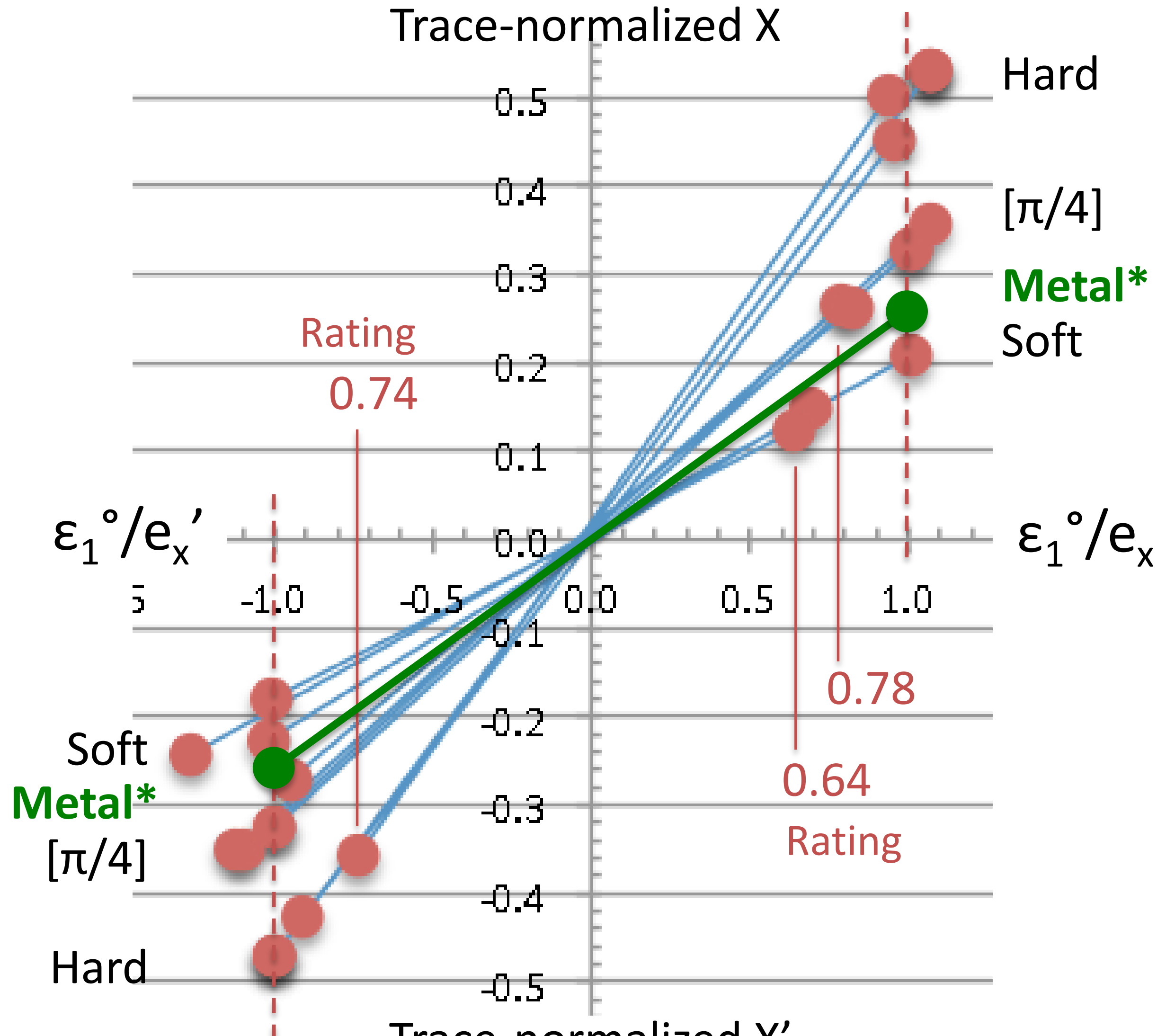


Generation of Master Stress-Strain Template



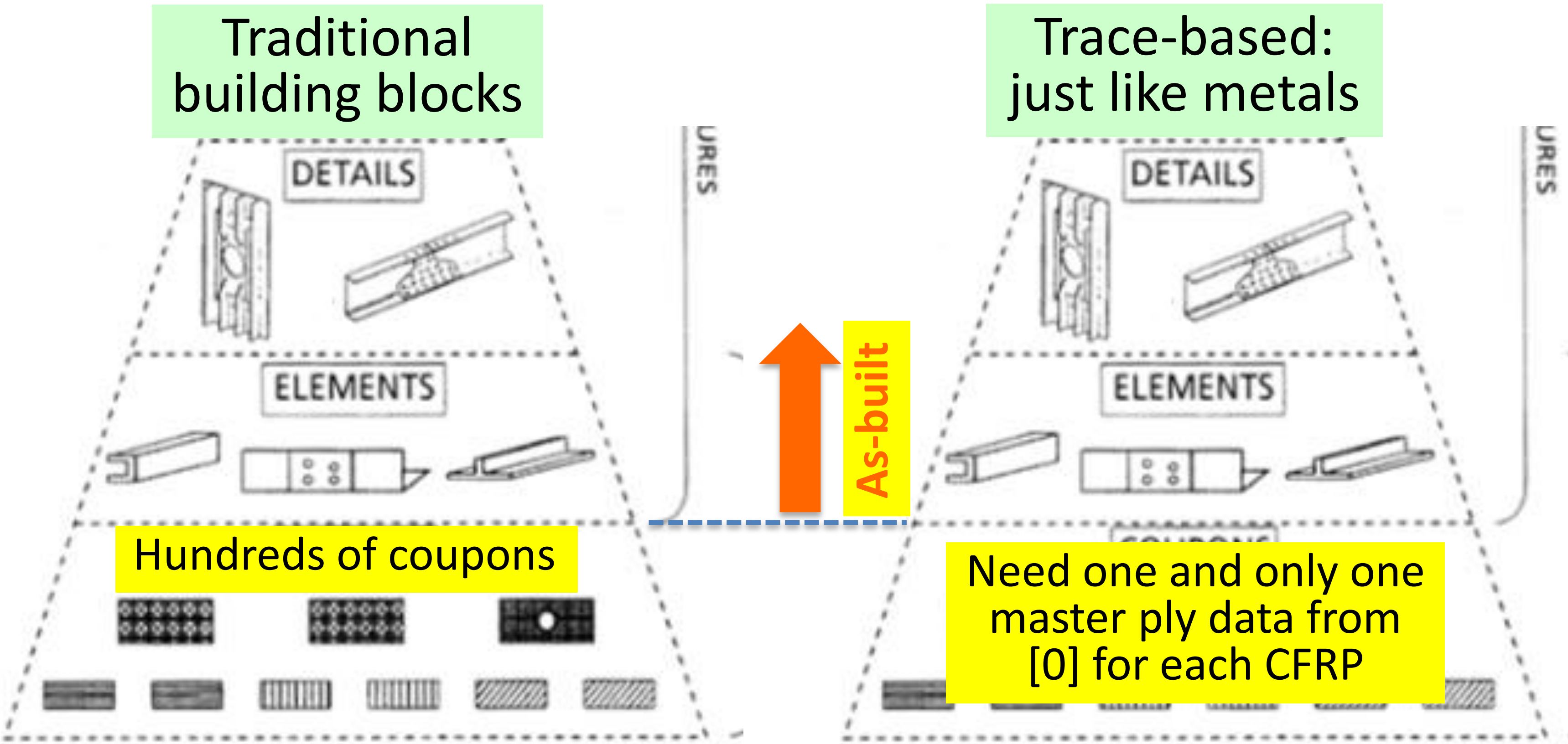
Summary Master Stress-Strain Curves for CFRP

	Laminate	Strain*	Stress*
T700/2510	[π/4]	0.79	0.27
		-1.10	-0.35
	Soft1	0.69	0.15
		-1.02	-0.23
	Hard1	1.07	0.53
		-1.00	-0.47
T650/epoxy	[π/4]	1.01	0.33
		-1.00	-0.33
	Hard1	1.07	0.53
		-1.00	-0.47
IM7/8552	[π/4]	1.01	0.33
		-1.00	-0.33
	Soft1	1.07	0.53
		-1.00	-0.47
	Hard1	0.96	0.45
		-0.91	-0.43
T4807/MR6(	[π/4]	0.82	0.26
		-0.95	-0.27
	Soft2	0.64	0.12
		-1.01	-0.18
	Hard2	0.94	0.50
		-0.74	-0.36
IM7/MTM45	[π/4]	1.07	0.36
		-1.13	-0.35
	Hard1	1.01	0.21
		-1.26	-0.24
	Median+	1.01	
	Median-	-1.00	



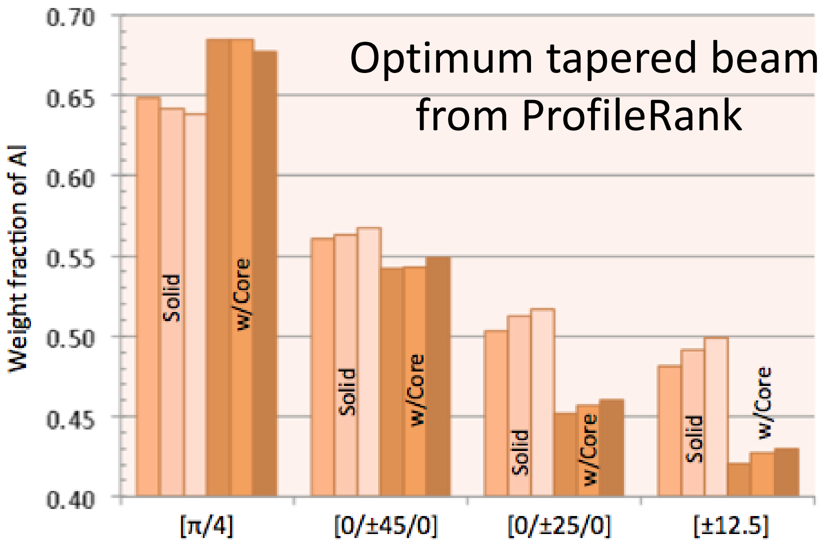
Metal*: Specific stiffness of Al, Ti and Fe (normalized by density)

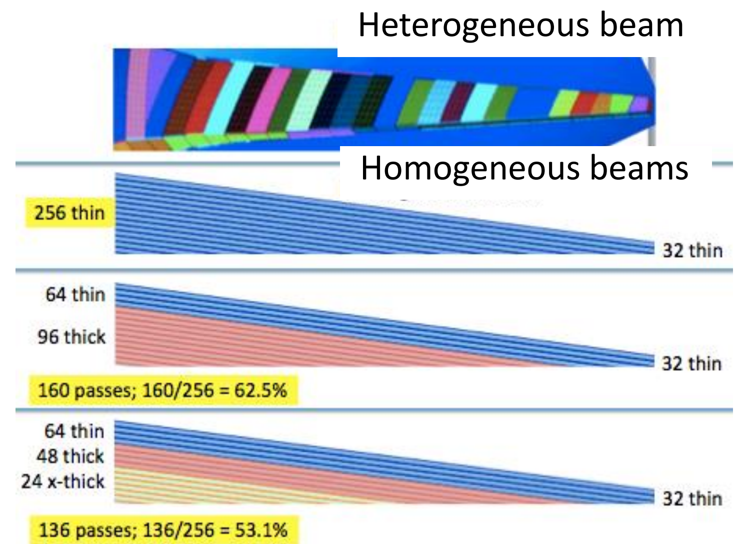
Traditional vs Trace-based Pyramids

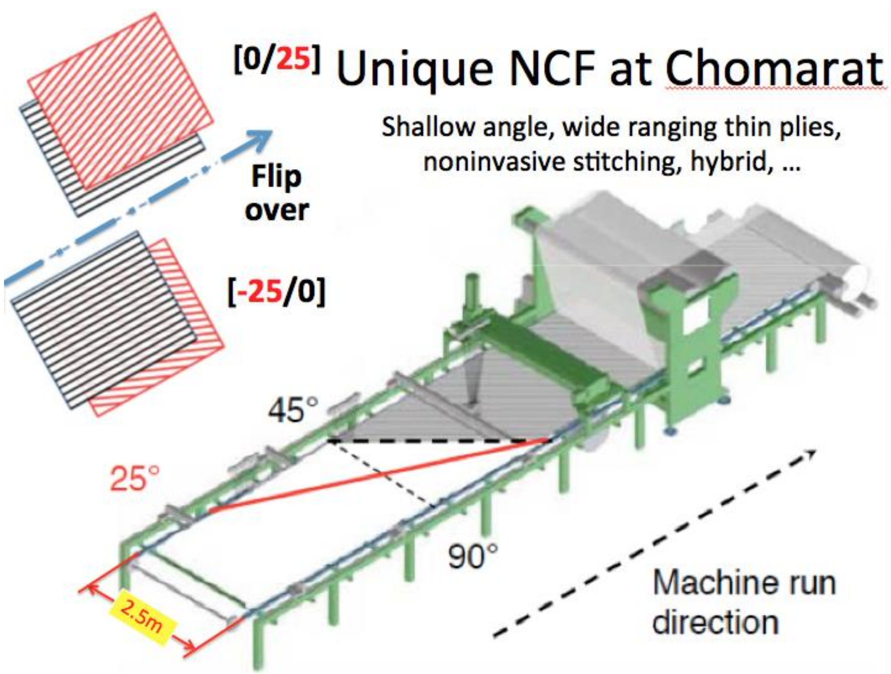


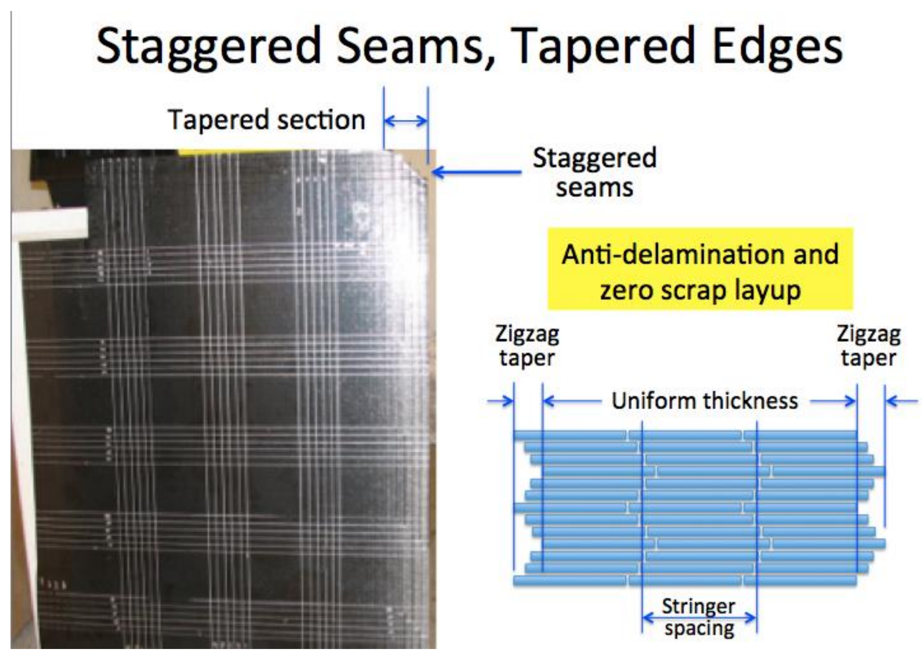
[0] >>> Trace >>> Master template >>> As-built rated >>> Structure

Opportunities in Homogenization









Comparisons between the Old and New

The old	The new
[0/±45/0] & 10 percent	[0/±φ/±ψ/90]
Uniform thickness [0]	Thick-thin [0/φ]
4-axis layup	1- or 2-axis layup
Heterogenous laminate	Homogeneous laminate
Optimum not possible	Optimum strategy possible
Symmetric ply drop	Asymmetric, single ply drop
Square edges (delam)	Tapered (reduced delam)
Hundreds of coupons	Like metals, [0] only
Multiple laminates	As-built laminate(s) only
No basis for rating	Rating based on max strain