Formulas for Trace Calculation

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Some Practical Uses of Trace

- Only invariant quantity that represents total stiffness potential of each composite material
- All stiffness components are fractions of trace
- One test can tell all about a given material
- Linear scaling for change in material/laminate
- It can measure the quality of lamination; any defect or damage will lower trace value
- Test laminate: closer to real structure
- Track temperature effect by change in trace

Stiffness and Compliance Matrices

$$[Q] = \begin{bmatrix} \frac{E_x}{1-\nu_x\nu_y} & \frac{\nu_y E_x}{1-\nu_x\nu_y} & 0 \\ \frac{\nu_x E_y}{1-\nu_x\nu_y} & \frac{E_y}{1-\nu_x\nu_y} & 0 \\ 0 & 0 & E_s \end{bmatrix} \qquad [S] = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_y}{E_y} & 0 \\ -\frac{\nu_x}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & 1/E_s \end{bmatrix}$$

Reciprocal relation: $\nu_x E_y = \nu_y E_x$

Laminate in-plane stiffness in terms of ply stiffness [Q]:

$$[\mathbf{A}^*] = \frac{1}{h}[A] = \frac{1}{h}\sum_{i=1}^{m}[Q']^{(i)}h^{(i)} = \sum_{i=1}^{m}[Q']^{(i)}\frac{h^{(i)}}{h} = \sum_{i=1}^{m}[\mathbf{Q}']^{(\mathbf{i})}\mathbf{v}^{(\mathbf{i})}$$

where $v^{(i)}$ = fraction of the i-th ply group

Laminate Compliance Components

Trace [A] = $A_{11} + A_{22} + 2A_{66}$

$$[\boldsymbol{a}] = [\boldsymbol{A}]^{-1}, |A|$$

$$= (A_{11}A_{22} - A_{12}^2)A_{66} + 2A_{12}A_{26}A_{16} - A_{11}A_{26}^2$$

$$- A_{22}A_{16}^2$$

$$a_{11} = \frac{(A_{22}A_{66} - A_{26}^2)}{|A|}, a_{22} = \frac{(A_{11}A_{66} - A_{16}^2)}{|A|}, a_{12}$$

$$= \frac{(A_{16}A_{26} - A_{12}A_{66})}{|A|}$$

$$a_{66} = \frac{(A_{11}A_{22} - A_{12}^2)}{|A|}, a_{16} = \frac{(A_{12}A_{26} - A_{22}A_{16})}{|A|}, a_{26} = \frac{(A_{12}A_{16} - A_{11}A_{26})}{|A|}$$

(3.4)

Laminate Engineering Constants

$$E_1^o = \frac{1}{a_{11}}^* , E_2^o = \frac{1}{a_{22}}^* , E_6^o = \frac{1}{a_{66}}^*$$

$$v_{21}^o = -\frac{a_{21}}{a_{11}}, v_{61}^o = \frac{a_{61}}{a_{11}}, v_{62}^o = \frac{a_{62}}{a_{11}}$$

$$v_{12}^o = -\frac{a_{12}}{a_{22}}, v_{16}^o = \frac{a_{16}}{a_{66}}, v_{26}^o = \frac{a_{26}}{a_{66}}$$

Input Data: Ply Stiffness and Strength

Ply name	Ex, GPa	Ey, GPa	nu/x	Es, GPa
X, MPa	X', MPa	Y, MPa	Y', MPa	S, MPa

T700 C-Ply 55[S	121	8	0.3	4.7
2530	1669	66	220	93
T700 C-Ply 64[S	141	9.3	0.3	5.8
2944	1983	66	220	93
IM7/977[SI]	191	9.94	0.35	7.79
3250	1600	62	98	75
T800/Cyt[SI]	162	9	0.4	5
3768	1656	56	150	98
IM7/8552[SI]	171	9.08	0.32	5.29
2326	1200	62	200	81.5

IM7/MTM [SI]	175	8.2	0.33	5.5
2500	1700	69	169	43
AS4/H3501[SI]	138	8.96	0.3	7.1
1447	1447	52	206	93
IM6/ep[SI]	203	11.2	0.32	8.4
3500	1540	56	150	98
T3/F93[SI]	148	9.65	0.3	4.55
1314	1220	43	168	48
T3/N52[SI]	181	10.3	0.28	7.17
1500	1500	40	246	68

Normalized Master Laminate Factors

Need only one test: E_x /0.876 = Tr [A°] >>> factors for E_1°, E_2°, ν_x, E_6°

Zero test: If you believe in rule of mixtures that $E_x = v_f E_f$

Or another single test of [π/4]: E_1°/0.337 = Tr [A°], …

Master Laminate	E1°/Tr	E2°/Tr	nu/x	E6°/Tr	Trace*
[0]	0.876	0.050	0.300	0.0343	1.000
[0/90]	0.468	0.468	0.036	0.031	0.999
[0/45/90/-45] = [π/4]	0.337	0.337	0.298	0.130	1.000
[0/±30]	0.515	0.0745	1.180	0.130	0.998
[(0/±30)2/±60/90]	0.418	0.252	0.398	0.130	1.000
[0/±30/±60/90] = [π/6]	0.338	0.338	0.297	0.130	1.002
[0/±45]	0.377	0.158	0.709	0.161	1.000
[(0)2/(±45)3/90]	0.316	0.240	0.485	0.161	0.999
[0/(±45)2/90]	0.280	0.280	0.419	0.161	1.001

Examples: For [0/±45], E_1° = 0.377 Tr; E_6° = 0.161 Tr (shear test can be avoided)

For C-Ply 55, Tr = 139 GPa, E_1° = 0.377 x 139 = 52.4 GPa; E_6° = 0.161 x 139 = 22.4 GPa

For T800/Cytec, Tr = 183 GPa, , E_1° = 0.377 x 183 = 69.0; E_6° = 0.161 x 183 = 29.4 GPa