ERMÜDUNG VON ENDLOSFASERVERSTÄRKTEN COMPOSITES
- ein neuer versagensmodus-basierter Ansatz -

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Abstrakt: Faserdominiert-ausgelegte Laminate werden häufig benutzt. Die Erfahrung zeigt: Aus UD-Schichten aufgebaute Laminate, die statisch auf eine Designgrenze von etwa 0.3% ausgelegt sind, erleiden praktisch keine Ermüdungsschädigung. Leichtbau verlangt aber eine höhere Werkstoffausnutzung. Der Verfasser präsentiert eine konsequent auf seinem erfolgreichen 'Versagensmoduskonzept'-basierte neue ingenieurpraktische Methode zur Abschätzung der ("Anriß"-) Lebensdauer mit: (1) Versagensmodusbasierter Modellierung der Betriebsbeanspruchung, (2) Messung der versagensmoduszugehörigen (Faserbruch) Master-Woehlerkurven, (3) Bestimmung anderer notwendiger Woehlerkurven mit Hilfe des Prinzips 'Gleichheit der Verzerrungsenergie', (4) Anwendung der Schädigungsakkumulationshypothese 'Miner'.

Stichwörter: Lebensdauervorhersage, Laminate, UD-Schicht, Bruchmodus-Vorgehensweise

FATIGUE OF ENDLESS FIBER-REINFORCED COMPOSITES
- a novel failure mode-based approach -

Abstract: Fiber-dominated designed laminates are often used. Experience shows: laminates composed of UD laminas, statically designed to a design strain limit $\varepsilon < 0.3\%$ do not fatigue. However, lightweight design requires a higher excertion of the material. The author presents a rigorous engineering-like method that is based on his successful 'Failure Mode Concept' for fatigue life estimation. It consists of : (1) failure mode-based modeling of the varying operating stress, (2) measurement of Master-S/N-curves for the activated failure mode (fiber fracture), (3) determination of necessary other S/N curves employing the principle of 'equality of strain energy', (4) application of the damaging accumulation hypothesis of Miner.

Keywords: Lifetime prediction, laminates, UD lamina, fracture mode approach

1 Introduction

Cyclic behavior

Design verification must be performed for static, cyclic and damage tolerance behaviour. In the case of cyclic behaviour the required lifetime is principally verified by a positive *margin of safety* $MoS_{Life} = \text{predicted lifetime} / (j_{Life} \cdot \text{design lifetime}) - 1 \geq 0$.

This should be executed by a practical lifetime prediction method. However a reliable general method, applicable to different laminas and laminates, is not yet available.

Therefore, the development of such a method is mandatory. Addressed here are: *Multi-directional laminates composed of uni-directional (UD) laminas (plies) and made from endless fiber-reinforced plastics*. The envisaged laminates are associated to plain undisturbed areas. Delamination-linked damage tolerance tasks are nor addressed.

Fatigue failure may occur if the structural material is cyclically loaded. The induced stress level at which this failure occurs is lower than that for static loading, see Fig.1. Initiation of fatigue failure is at discrete material spots and is affected by the environment such as by hygrothermal effects in the case of composites.

For polymer-matrix composites with many fiber orientations in the laminate experience proved that no fatigue certification is necessary if the static, 'fiber-dominated design' is performed for a limit design strain of about ε =0.3% for Fiber Failure FF. However, in order to better utilize the composite strength capacity, the design strain shall be increased to about 0.5%. Then, first weak filaments break (besides of an increasing damaging by Inter-Fiber-Failure IFF) and change the pleasant situation.

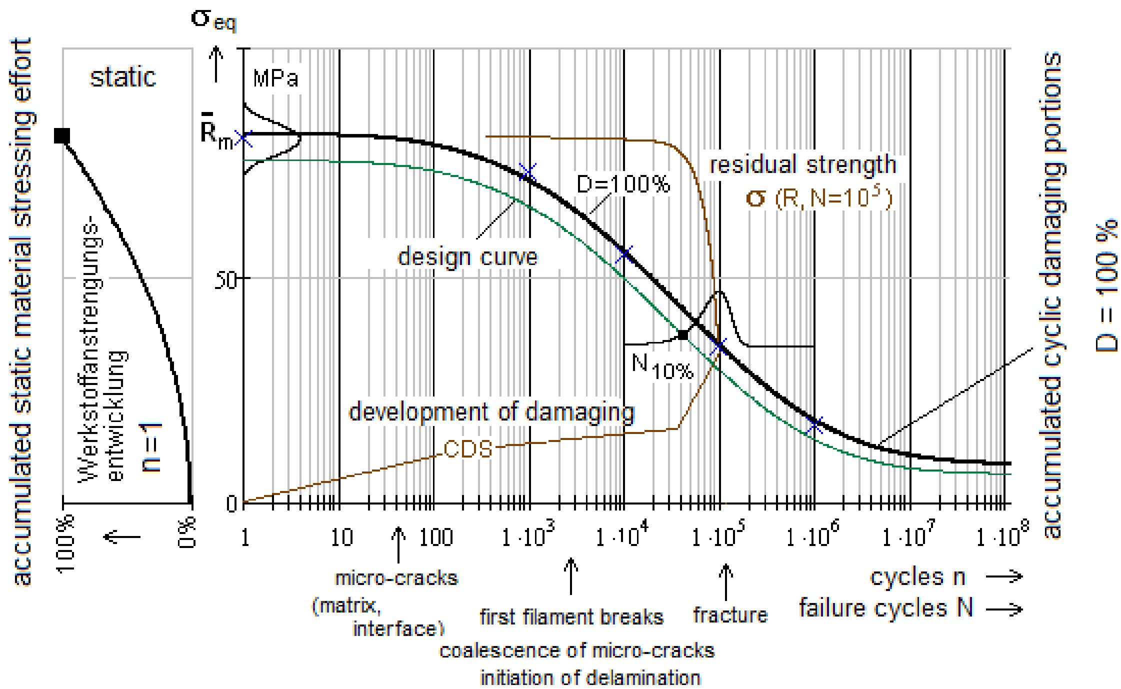


Fig. 1: Notions and development of damaging for a distinct fracture cycle number   N =10^5 and stress ratio R =$\sigma_{min} / \sigma_{max}$ of the envisaged fiber failure mode, D = sum of accumulated damaging portions, CDS= characteristic damaging state; material stressing effort ("Werkstoff-Anstrengung")

Cyclic fatigue life of composites consists of three phases: (I) Increasing diffuse damaging (in German "Schädigung") in the embedded laminas up to onset of discrete damaging. It is initiated after the end of elastic domain and dominated by diffuse micro-cracking with matrix yielding, and finally discrete micro-cracking with diffuse micro-delaminations); (II) Discrete damaging locally in the laminate up to onset of a discrete macro-delamination (stabile growth of dominating discrete micro-crack widths incl. micro-delaminations); (III) Final non-stabile fracture of the laminate initiated by failure of any lamina (FFs and IFFs) including a possible critical macro-delamination damage (in German "Schaden") size. Essential part in High Cycle Fatigue is the initiation life, phase

I and II, during which the diffuse damaging portions have summed up in the embedded *lamina* material to a critical damage size within the *laminate*, i.e. a macro-delamination.

Following Degrieck-Paepegem [Deg01] *fatigue life prediction models for composites* may be subdivided into three categories: (1) *Fatigue life models,* using S/N curves and a fatigue failure condition (this category is applied by the author, however failure mode-wise); (2) *Phenomenological models,* using residual stiffness or residual strength; and (3) *Progressive damage models,* using damage variables associated to measurable damaging quantities such as matrix cracks.

At present, presented lifetime prediction methods base on specific laminates and therefore cannot be generally applied. A generalization may be obtained by the application of an embedded lamina-oriented, in-situ behaviour considering procedure.

The here proposed lifetime prediction method is *physically-based* (uses fracture failure modes FFs and IFFs [Cun 12, 13] and material symmetry facts), is *lamina-oriented,* applies the well-known R stress ratio-dependent *S/N curve* but *failure mode linked*. The method considers plain material areas. Notches and delaminations are not regarded.

A *fatigue life model* that applies S/N curves needs many test data to establish these curves. In order to reduce the test effort, it would be very effective to perform a lifetime prediction for a laminate on basis of an in-situ *master* lamina S/N curve for each single failure mode. This enables the engineer in pre-dimensioning to build lifetime predictions for variable loading in tension, shear and compression on a cheaper basis. Above master S/N curve represents the basic R-ratio S/N curve of the addressed failure mode, which may be a normal fracture or a shear fracture mode. Further S/N curves for the same failure mode, required in fatigue analysis, can be predicted from the master curve by utilizing the classical strain energy equality principle [Sho06].

General procedure

It is assumed in this engineering approach that the S/N curve can be mapped by a straight line in the log-log diagram. The damaging portions are related to the material stress efforts (in German “Werkstoff-Anstrengung”) the material experiences under variable loading. They are determined via the author’s Failure Mode Concept. Its 3D multi-axial strength failure conditions are failure mode linked which enables to formulate an equivalent stress (analogous to the von Mises equivalent stress for the failure mode mechanism *yielding* in case of ductile behaving metals), because each single failure mode represents one failure mechanism and is governed by one strength, only. This equivalent stress is determined for each active failure mode. For the transversely-isotropic UD material there are five failure modes: two Fiber Failure (FF) and three Inter Fiber Failure (IFF) modes. [Cun04, 13].

In the approach, static strength failure conditions are employed with consideration of a degradation-caused shrinking of the failure surface (a failure condition is the mathematical formulation of such a failure surface). Thereby, the static strength in each strength failure condition is replaced by the residual strength which is dependent on actual cycle number.

In the case of FRP Composites, damaging may be described as:

- *Determination of damaging portions (from diffuse and discrete damaging)*
- *Accumulation of damaging portions (cycle-wise, or block-wise, or …)*

The author searched an engineering lifetime prediction method which employs:

- A full failure mode-linked thinking from loading modeling to accumulation of damaging
- A minimum number of failure-mode-linked representative '*mode S-N curves'* (from experiment) = master R stress-ratio curve of each mode, e.g. of R = 0.1, 10 (compression)
- The S-N curve can be mapped by a straight line in the log-log diagram and
- The Prediction of other necessary stress-ratio '*mode S-N curves'* on basis of the master curve which is typical for the envisaged mode (e.g. R=0.5 from R=0.1).

Basic components of Cuntze's 3D fatigue life model for fiber-dominated laminates are:

- A fully novel failure mode–wise apportionment of cyclic loading
- Use of a measured (available) master S/N curve, representative for one single failure mode (sub-laminate test specimens capture the in-situ effect)
- The derivation of necessary other S/N curves of a failure mode on basis of the mode master S/N curve by the use of the classical strain energy equality principle (a distinct strain energy level on the S/N curve R=0.5, for instance, will be reached at a larger lifetime than for R=0.1) and
- A failure mode-related accumulation of damaging portions (Miner : $D = \sum n_i/N_i$).

2 Fatigue Analysis Basics of Ductile Metals & Brittle UD Composites

The following chapter comprises some points essential for fatigue modeling:

* For brittle materials, degradation replaces yielding: Actual composites behave brittle but show quasi-ductile laminate behaviour effects mainly caused by micro-cracking in the laminas and not so much by the ductility of the matrix.

* Ratio between notched and un-notched composite endurance limit (n > 10^6) is about 1. Due to [Hwa86], fatigue failure will occur when fatigue strain reaches the static failure strain

* In contrast to ductile behaving metals, brittle behaving composites are very sensitive to a stress concentration which results in a local stiffness loss accompanied by a stress re-distribution onto the load-carrying fibers but then composites are relatively insensitive to further cyclic loading because the fibers rule the lifetime

* Composite fatigue curves are flatter, lifetime performance is better, but life scatter may be somewhat larger than for metals. That makes Design Verification some more difficult. In fatigue tests no kink point at high cycles is found (endurance?).

* In comparison with monotonic loading to failure, repeated loading at a lower load level until failure results in a greater number of instances of damaging matrix and fiber-matrix interface. The IFFs are more evenly (diffuse) distributed over the lamina

* Pulsating tensile stressing brings about notch effects in the adjacent UD laminas and thereby micro-cracks (IFFs) in the matrix transversely to the fiber direction and as a consequence some first FFs. All in all, the stressing in the 0° laminas which predominantly carry the load will increase such that at a specific load level more and more filaments reach their failure stress, until finally a 'sudden death' fracture of all remaining fibers in the critical 0° lamina results in a total failure of the full laminate

* Micro-cracks might not fully close after un-loading. Edge effects are normally more fatal in the test specimen than in the real structure. Residual stresses will decay with increasing degradation.

Traditional lifetime prediction methods for metals use:

- S-N curves, dependent on the stress ratio $R = \sigma_{\min} / \sigma_{\max}$ *with* *S := cyclic stress range =* $\Delta\sigma$*, N:= number of cycles to failure*
- Constant life Goodman Diagram to account for the mean stress effect.

In the case of ductile behaving metals slip band shear yielding occurs under cyclic tensile, compressive, and shear stress. Such a shear stress–caused yielding can be described by one yield failure condition! Shear stress is the damaging driver but the formulation may be in normal stresses (such as principal stresses). Semi-brittle and brittle behaving materials experience several failure modes (mechanisms). In consequence, several failure conditions are to be employed!

Fatigue Life Demonstration in case of semi-brittle and brittle behaviour (not just for semi-brittle behaving isotropic metals) is performed by *a 'uni-axial' stress amplitude method* with *mean stress correction* regardless whether the material behaves ductile or brittle. This method should be replaced by an advanced method where the mean stress is inherent, by *an equivalent (multi-axial) stress and failure mode reflecting method.*

If the laminas are initially free of essential flaws and are un-notched then experience with actual FRP composites shows up that the laminas in the laminate

- *behave brittle*
- *experience early fatigue damage*
- *show benign fatigue failure behaviour in case of 'well-designed', fiber-dominated laminates until final 'Sudden Death'* (fiber-dominated:= 0° plies in all significant loading directions, > 3 angles).

3 Brief Description of the author's Failure Mode Concept (FMC)

Failure Modes

This chapter is required to define those stress quantities which help to compute the damaging portions.

At first, the five well-known fracture failure modes of the transversely-isotropic UD material are shown in Fig. 2, see [Cun04] learnt from inspection results it can be concluded that:

* There are coincidences between brittle UD laminas and brittle isotropic materials

* Degradation begins with onset of diffuse damaging (hardening) until onset of IFF1 or IFF3

* Final fracture occurs with FF1, FF2, and IFF2 after onset of discrete damaging.

FMC-based UD failure conditions

Fig. 3 collects the mentioned equivalent stresses and material stressing efforts (in German "Werkstoffanstrengung") that must be employed in the determination of the damaging portions and the accumulation of them. Added is a typical data range for the friction value μ of the UD material, the determination of which is presented in [Cun 12, 13] and its relation to the friction parameter used in the development of the failure conditions. Further, a range for the modes' interaction exponent is presented.

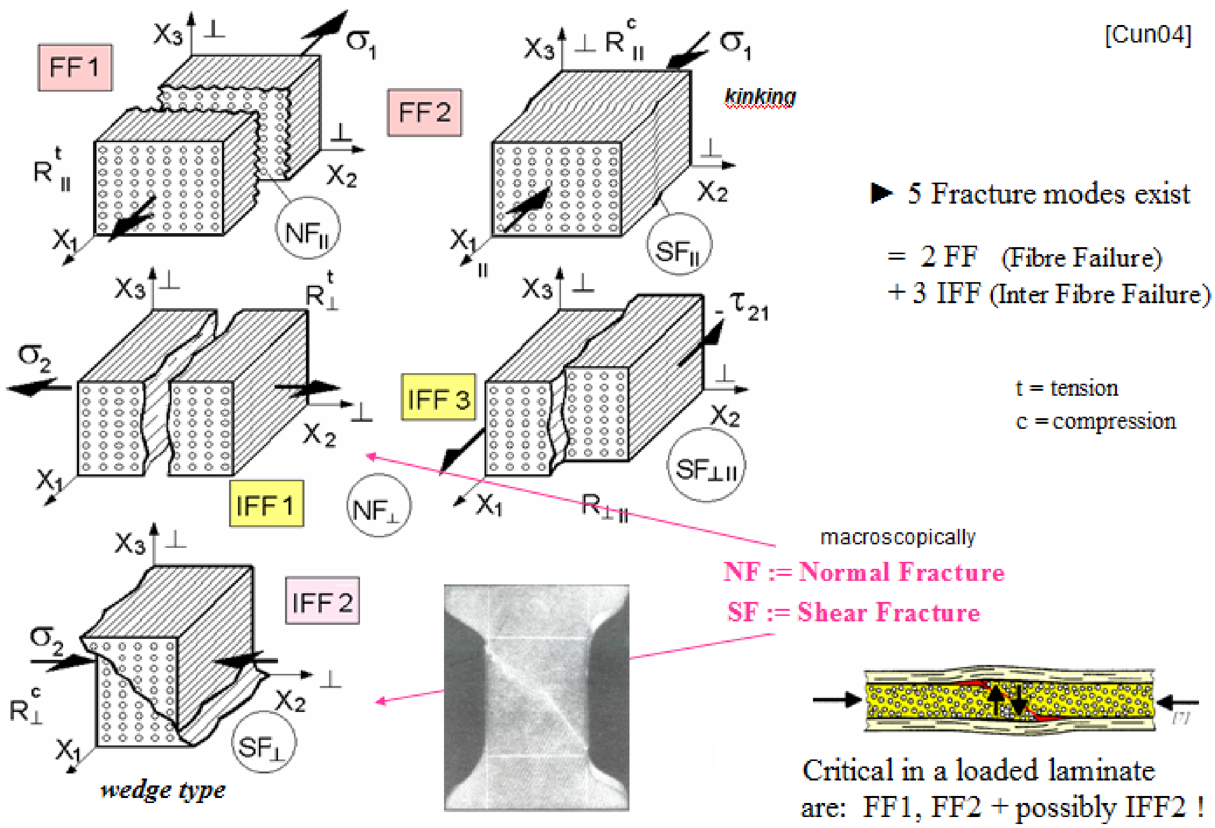


Fig. 2: Fracture failure modes of UD material

Keep in mind: FF cannot be described by a homogenized (smeared) macro-scopical stress $\sigma_1 = \sigma_\parallel$, because filaments do also break in case of bi-axial compression for zero external lamina stress ($\sigma_1 = 0$, Poisson effect). Hence, the macro-scopical modeling must be replaced by an accurate micro-scopical one. However, it can be approximately engineering-like formulated in macro-scopical quantities by the FEA-delivered macro-mechanical strain ε. No fiber properties are required.

The modes-interaction formula represents the static interaction of modes, $Eff^{\text{ modes}}$, are used linearly in the portions, whereas later in the cyclic interaction or damaging accumulation the squares are to be taken. As far as failure mode and failure mechanism remain, the 'static' knowledge above may be transferred from static to cyclic loading!

Only, if one basic strength governs a failure mode alone, then an equivalent stress can be formulated. For the UD material, 5 equivalent stresses can be defined and formalistically included in a vector $\{\sigma_{eq}^{mode}\} = (\sigma_{eq}^{\parallel\sigma},\ \sigma_{eq}^{\parallel\tau},\ \sigma_{eq}^{\perp\sigma},\ \sigma_{eq}^{\perp\tau},\ \sigma_{eq}^{\parallel\perp})^T$.
The indices τ, σ mark the inherent failure driving stress of the model. The failure surface (body), associated to the former strength conditions is depicted in Fig. 4. The

upper figure belongs to a 2D stress state. When changing the coordinates into equivalent stresses then the same failure surface can be used for a 3D stress state, too.

FF1 $Eff^{\|\sigma} = \sigma_1 / \bar{R}_{\|}^{t} = \sigma_{eq}^{\|\sigma} / \bar{R}_{\|}^{t}$, $\sigma_1 \cong \varepsilon_1^t \cdot E_{\|}$ (matrix negligible), *** filament ! modes**

FF2 $Eff^{\|\tau} = -\sigma_1 / \bar{R}_{\|}^{c} = +\sigma_{eq}^{\|\tau} / \bar{R}_{\|}^{c}$, $\sigma_1 \cong \varepsilon_1^c \cdot E_{\|}$

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / 2\bar{R}_{\perp}^{t} = \sigma_{eq}^{\perp\sigma} / \bar{R}_{\perp}^{t}$ **matrix**

IFF2 $Eff^{\perp\tau} = [(b_{\perp\perp} - 1)\cdot(\sigma_2 + \sigma_3) + b_{\perp\perp}\sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / \bar{R}_{\perp}^{c} = +\sigma_{eq}^{\perp\tau} / \bar{R}_{\perp}^{c}$ **modes**

IFF3 $Eff^{\perp\|} = \{[b_{\perp\|} \cdot I_{23-5} + (\sqrt{b_{\perp\|}{}^2 \cdot I_{23-5}{}^2 + 4 \cdot \bar{R}_{\perp\|}{}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,] / (2 \cdot \bar{R}_{\perp\|}{}^3)\}^{0.5} = \sigma_{eq}^{\perp\|} / \bar{R}_{\perp\|}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

Modes-Interaction: ***Eff* = 100% = 1**

$$Eff^{m} = (Eff_{\|}^{\tau})^m + (Eff_{\|}^{\sigma})^m + (Eff_{\perp}^{\sigma})^m + (Eff_{\perp}^{\tau})^m + (Eff_{\perp\|})^m = 1$$

Typical friction value data range: $b_{\perp\|} = \mu_{\perp\|},\ b_{\perp\perp} \cong 1/(1 - \mu_{\perp\perp})$

$0 < \mu_{\perp\|} < 0.3,\ 0 < \mu_{\perp\perp} < 0.2,\ 2.5 < m < 3.1$

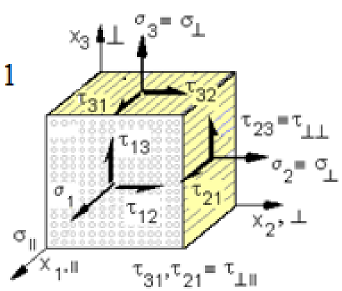


Fig. 3: FMC-based strength failure conditions [Cun12, 13]

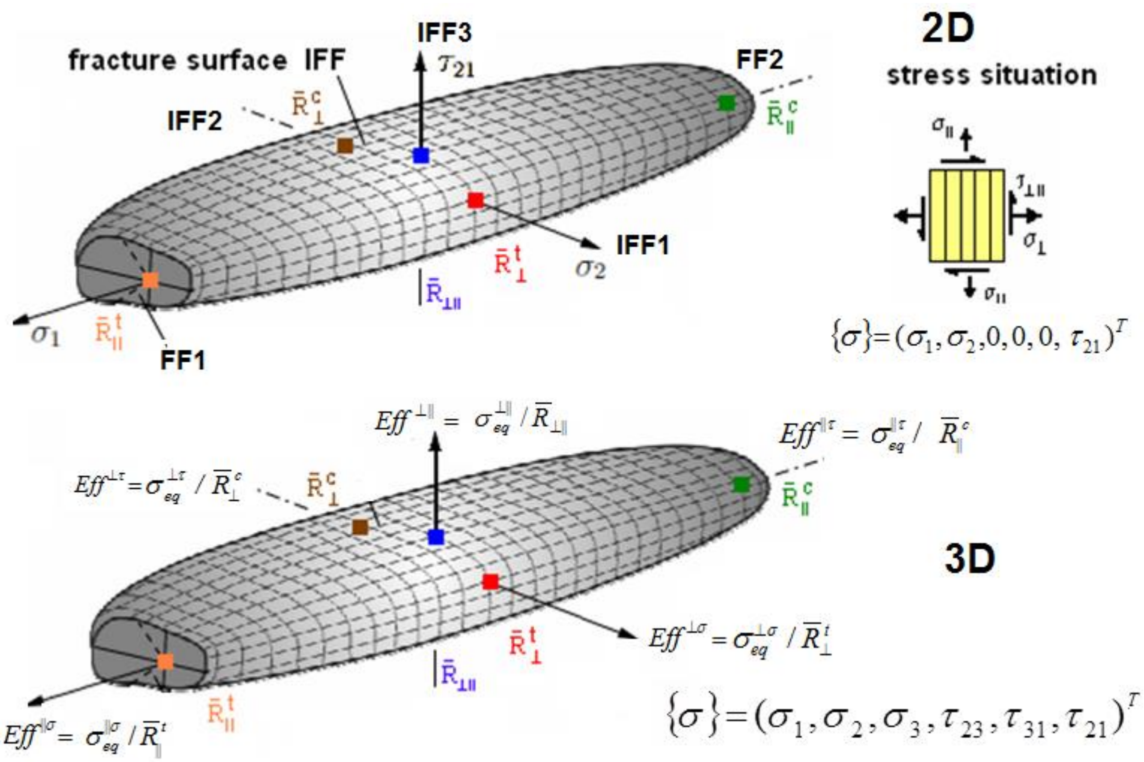


Fig. 4: 2D and 3D failure surface of the FMC-based strength failure conditions

4 Failure mode-based Lifetime Prediction Method (UD approach)

Ideas, the proposed method is based on, are depicted in the Figures 5 through 9.

Idea 1: Modeling of loading cycles

For the sake of simplicity for displaying the chosen load modeling idea an isotropic brittle material is taken with an R = -1 stress ratio, Fig. 5. This was still proposed in 1996 by the author, [Cun96]. The idea requires a *failure mode-linked apportionment of cyclic loading (better termed stressing).* Of course, this idea will finally also lead to a mode-related damaging accumulation.

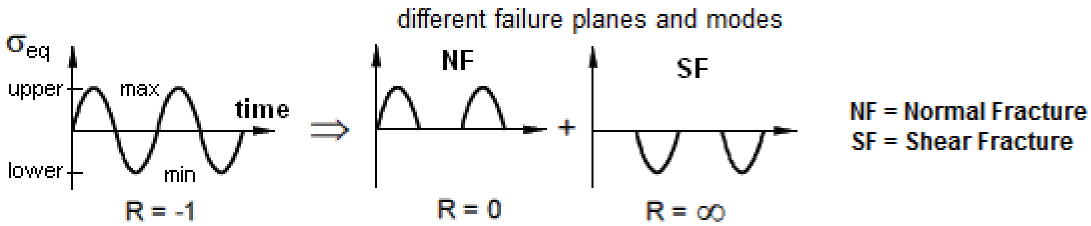


Fig. 5: Novel modeling of loading cycles

Idea 2: Modeling of the mode-representative master S-N curve

The next idea of the method can be formulated as the *S/N curve can be mapped by a straight line in the log-log diagram*, Fig. 6.

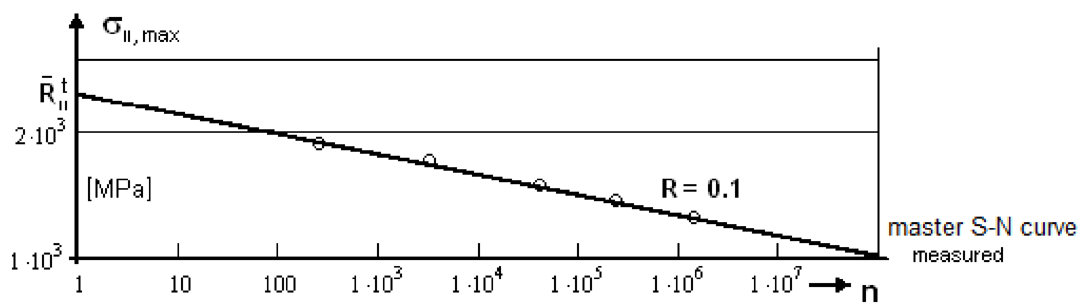


Fig. 6: Modeling of the mode-representative master S-N curve

Then, as mapping function for the S/N curve the simplest type can be taken, namely

$$\sigma^{repr}_{\|,\ \max}(n) \approx \bar{R}^{t}_{\|} \cdot n^{c_{repr}} .$$

Further possible are non-linear curves such as the Weibull-model and the Wearout-model, [VDI 2014].

In case of brittle behaving materials the strength values R_m may be advantageously used in the S/N curve as origins at n = 1. This possibility depends on the model used.

Idea 3: S/N curve prediction by using strain energy equivalence principle

The logic behind to use the strain energy equivalence is: Fatigue strain energy, required to generate a distinct damaging state is equal to the strain energy, which is necessary under monotonic loading to obtain the same damage state. Neglecting heat loss [Kad94] damaging is proportional to the supplied energy. *A distinct strain energy level, associated to a point at the given mode-representative S-N curve (here R = 0.1 as master R-ratio curve) will be reached for R = 0.5 at higher cycles*

This energy can be formulated as $\Delta W = \frac{1}{2} \cdot (\sigma_{max} \cdot \varepsilon_{max} - \sigma_{min} \cdot \varepsilon_{min})$.

Ignoring non-linearity and applying Hooke $\sigma = \varepsilon \cdot E$ the formulation takes the shape

$$\Delta W = (\sigma_{max}{}^2 - \sigma_{min}{}^2) / 2 \cdot E.$$

Of advantage, however not necessary, for the derivation of the present method, is a normalization of the strain energy [Sho06] $\Delta W \cdot \bar{R}^2 \propto \sigma_{max}{}^2 - \sigma_{min}{}^2 = \sigma_{max}{}^2 \cdot (1 - R^2)$.

Instead of an amplitude representation, for brittle behaving materials such as composites the maximum (upper) stress representation is more often used. It can be transferred to the stress range, see Fig. 7, according to

$$\sigma_{max} = \Delta\sigma / (1 - R) = 2 \cdot \sigma_a / (1 - R) \quad \text{with} \quad \Delta\sigma = \text{stress range}.$$

This is connected with the curve begins with the basic strength point at n=1.

(a) Demonstration of the idea: For the simple case of a fiber-dominated laminate under tension, with the change of strain energy between maximum and minimum loading, the FF1 strain energy portion reads

$$\Delta W^{\parallel\sigma} \cdot \bar{R}_{\parallel}^{t} \propto \Delta\sigma_{eq}^{\parallel\sigma 2} \Rightarrow \quad \Delta W^{\parallel\sigma} \cdot \bar{R}_{\parallel}^{t\,2} \propto \sigma_{\parallel,max}{}^2 \cdot (1 - R^2) \; = \sigma_{|eq}^{\parallel\sigma 2} \cdot (1 - R^2).$$

From experiment known are the following measured quantities: maximum stress, tensile strength, stress ratio R, and thereby also the fatigue strain energy *W*. The strain energy of all mode contributions (5 in the UD case) reads $\Delta W_{total} = \sum_1^5 \Delta W^{modes}$.

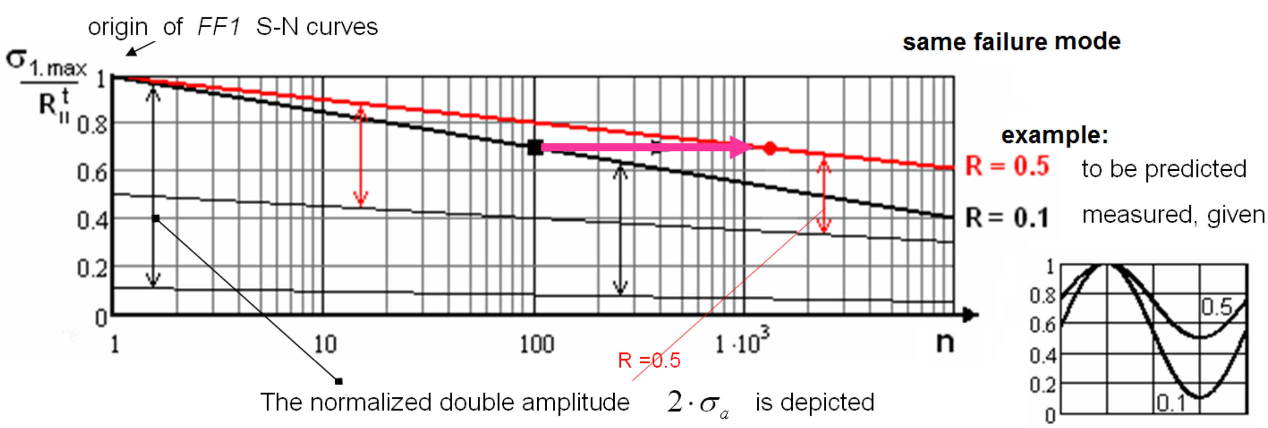


Fig. 7: Schematic S-N curve prediction by using the strain energy equivalence principle

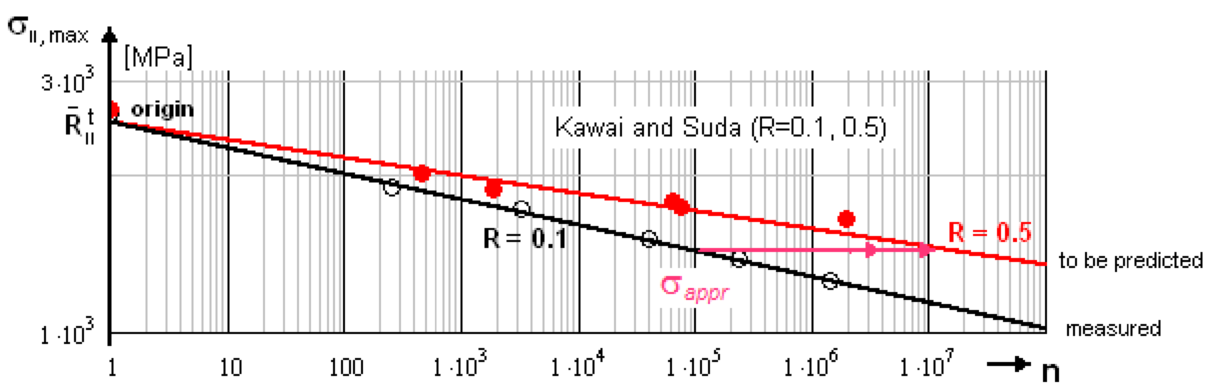


Fig. 8: Prediction of mode S-N curves from a mode-representative master curve

The application of the strain energy equivalence is demonstrated in Fig. 8.

(b) Example for predicting another mode-associated S/N curve, here R = 0.5:

Given: measured master S/N curve of R=0.1 $\sigma_{\parallel,\ \max}^{master}(n) \approx \overline{R}_{\parallel}^{t} \cdot n^{c_{master}}$.

Predicted: S/N curve of R=0.5

$$\Delta W_{R=0.1}^{\parallel\sigma}(n) = c_1 \cdot n^{-c_2} = 0.89 \cdot n^{-0.097}$$

$$\sigma_{1,\max}^{master}(n) = \overline{R}_{\parallel}^{t} \cdot \sqrt{\frac{\Delta W_{R=0.1}^{\parallel\sigma}}{1-R_{master}{}^2}} = \overline{R}_{\parallel}^{t} \cdot \sqrt{\frac{c_1 \cdot n^{-c_2}}{1-R_{master}{}^2}} = \overline{R}_{\parallel}^{t} \cdot n^{c_{master}(n)} \approx \overline{R}_{\parallel}^{t} \cdot n^{c_{master}},$$

$$\sigma_{1,\max}^{master}(n) \approx \overline{R}_{\parallel}^{t} \cdot n^{c_{master}} = \overline{R}_{\parallel}^{t} \cdot n^{-0.049}$$

To determine c_{pred} one anchor point is needed besides the strength point. As an appropriate anchor point for shift derivation is chosen $n_{appr} = 100000$ *cycles*

$$\sigma_{appr} = \overline{R}_{\parallel}^{t} \cdot \sqrt{\frac{c_1 \cdot (n_{appr} \cdot f_{pred})^{-c_2}}{1-R_{pred}{}^2}} \Rightarrow f_{pred} = \exp[-\ln(\frac{R_{pred}{}^2 - 1}{R_{master}{}^2 - 1}) \cdot \frac{1}{c_2}] = 210$$

as shift from the master curve to the to be predicted curve, here R=0.5, leading to

$$c_{pred} = -\ln(\overline{R}_{\parallel}^{t} / \sigma_{appr}) / \ln(n_{appr} \cdot f_{pre}) = -0.034 \quad \text{and} \quad \sigma_{1,\max}^{pred}(n) \approx \overline{R}_{\parallel}^{t} \cdot n^{c_{pred}}.$$

Idea 4: Accumulation of damaging portions by 'Miner Rule'

Eventually, in Fig. 9 a schematic application is presented. Of course, a maximum allowable damaging value D, the feasible D, is to be experimentally derived.

Example, see Fig. 9:

Static strengths $\{\overline{R}\} = (2560,\ 1590,\ 73,\ 185,\ 90)^T\ MPa$,

S/N curves $\sigma_{\parallel,\ \max}^{R=0.1}(n) \approx \overline{R}_{\parallel}^{t} \cdot n^{-0.049}$, $\sigma_{1,\max}^{R=0.5}(n) \approx \overline{R}_{\parallel}^{t} \cdot n^{-0.034}$, $\sigma_{1,\max}^{R=9}(n) \approx \overline{R}_{\parallel}^{t} \cdot n^{????}$,

To demonstrate the method, for simplicity reasons, the average S/N curves are used instead of the associated design curves, Fig.1.

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T, \quad \{\sigma_{eq}^{\mathrm{mode}}\} = \left(\sigma_{eq}^{\parallel\sigma},\ \sigma_{eq}^{\parallel\tau},\ \sigma_{eq}^{\perp\sigma},\ \sigma_{eq}^{\perp\tau},\ \sigma_{eq}^{\parallel\perp}\right)^T.$$

Loading $n_3(R = 0.1) = 0$

$n_1(R = 0.1) = 100000\ cycles,\quad \sigma_1^{(1)} = 12500\ MPa,\quad N_1(R = 0.1) = 2300000\ cycles,$

$n_2(R = 0.1) = 1600\ cycles,\quad \sigma_1^{(2)} = 1500\ MPa,\quad N_2(R = 0.1) = 55000\ cycles,$

$n_4(R = 10) = 6000\ cycles,\quad \sigma_1^{(4)} = -1150\ MPa,\quad N_4(R = 10) = 5000\ cycles,$

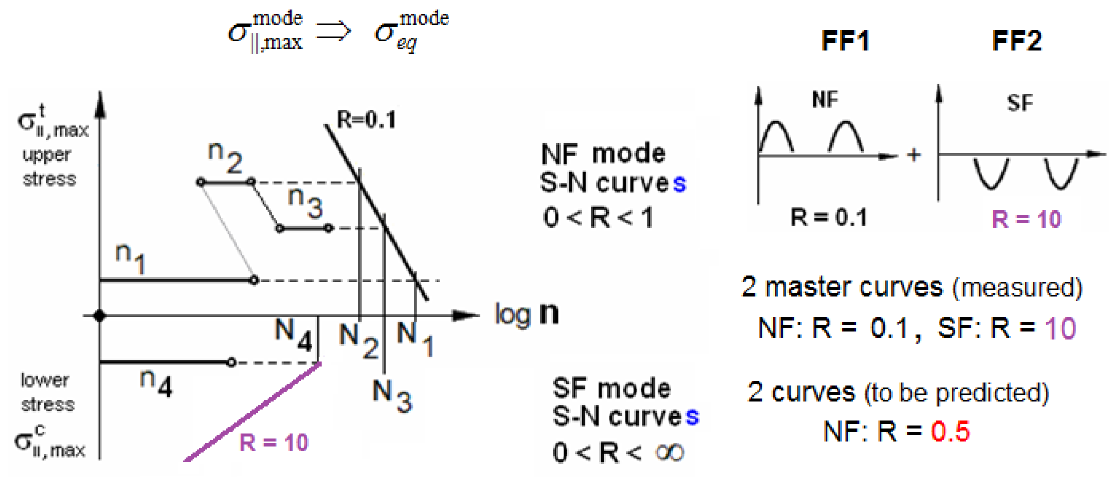
$n_5(R = 0.5) = 600000\ cycles,\quad \sigma_1^{(5)} = 1550\ MPa,\quad N_5(R = 0.5) = 2600000\ cycles.$

Miner application

$D = \sum n_i/N_i$) =100000/2300000 + 1600/55000 + 6000/5000 + 600000/2.600000 = 0.43

This set of loadings can be subjected several times to the laminate

$$\text{MoS} = \frac{\frac{D_{feasible}}{D}}{j_{life} \cdot D} - 1 = \frac{0.8/0.43}{3 \cdot 0.43} - 1 = 0.4 > 0.$$



FF1 **FF2**

$$D = NF:(n_1/N_1 + n_2/N_2 + n_3/N_3) + SF:(n_4/N_4) = D_{feasible}$$

Miner (Relative) application: $D_{feasible}$ - calibration from test experience

Fig. 9: Schematic application of the method for the two FF modes

What is still open: How essential mode interaction damaging really might be and how the effect of loading sequence accounts for?

5 Comments on Experiments

General Aspects

- Cyclic fatigue life, to be measured and monitored, consists of three phases: damaging initiation, damage growth and final fracture. Stable damaging growth is the main phase for predictions
- The embedded lamina is deformation-controlled. Therefore, a deformation-controlled fatigue testing is mandatory (thereby, in-situ-effect consideration), representative test specimens must be designed and investigated
- Fatigue testing is performed for lamina (ply) and laminate test specimens as well
- Laminate test specimens include the in-situ behaviour of the embedded (redundancy given) lamina, involving effects such as thin-layer effect, notching, smoothing
- Lamina data, used as input properties, are from so-called 'isolated' test spe (weakest link effect)
- Lamina on-axis coupons experience uni-axial stress states
- Lamina tube specimens may be uni-axially and multi-axially stressed
- Laminates test specimens primarily experience multi-axial stress states $(\sigma_1, \sigma_2, \tau_{21})$
- Off-axis coupons experience under uni-axial loading a (σ, τ)-stress state but suffer from the edge effect which impacts fatigue life
- Laminate and sub-laminate test specimens experience in its single laminas a deformation-controlled multi-axial stress state. This makes the use of several

failure conditions mandatory, at maximum 3 of the 5 UD failure modes for the present stress state.

Choice of test specimens, stress combinations and loading types

Demands on test specimens: Consideration of embedding of ply, ply-thickness effect, fiber volume fraction, stacking sequence, loadings

1 Flat coupon material *test specimens* (relatively cheap compared to tubes)

2 Tension/compression-torsion tube *test specimens* $(\sigma_1, \sigma_2, \tau_{21})$

3 Sub-laminate *test specimens (with internal proof ply and outer supporting plies)*

4 Flat off-axis coupons (the shortcomings 'free edge effect' and bi-axial stiffness loss cannot not accurately enough considered).

To be tested for fatigue model validation:

combinations of cyclic stresses (1D, 2D and 3D stress states)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T \Rightarrow \quad (\sigma_{\parallel}^t,\ \sigma_{\parallel}^c,\ \sigma_{\perp}^t,\ \sigma_{\perp}^c,\ \tau_{\perp\parallel})^T \quad \text{basic stresses.}$$

It is always to be controlled, whether the applied stresses are real lamina stresses or if they might have to be transformed due to test specimen torsion etc.[]Cun 12,13].

Loading types applied for the *operational lifetime estimation* are:

- *Constant-amplitude loading* : delivers S/N curves (Wöhler curve)
- *Block-loading* (if appropriate) : for a more realistic fatigue life estimation
- *Random spectrum loading* : fatigue life (Gaßner) curve.

Behavior: Diffuse and Discrete Damaging and finally damage

1 Growth of diffuse damaging (hardening) of the *material* lamina
until forming of discrete micro-cracks at Inter Fiber Failure (IFF)

2 Growth of micro-cracks (damage) softening of the material lamina
until characteristic damage state (CDS) incl. growth of micro-delaminations
and delamination onset through 3D stress concentrations including σ_3^t.

3 Growth of delamination within the *structural element* laminate or no-growth of delamination (crack propagation).

Assessment Tools: *Fracture Toughness, Damage Tolerance,*
mixed-mode Fracture Mechanics.

Conclusions and Outlook

A method is proposed for a *Failure mode-based lifetime prediction method for laminates* which employs the simple Relative Miner damaging accumulation rule. The proposed method looks like a 'macro-mechanical multiple mode damaging approach'. It avoids mean stress correction via a Goodman diagram, and gives some hope for a general treatment of laminates under variable loading.

It is an engineering, failure mode-linked lifetime prediction method for fiber-dominated laminates which employs:

1 Measurement of a minimum number of measured failure-mode-linked representative (master) S/N curves with one *master* R-ratio *curve for each mode.* This means the achievement of test cost reduction
2 Prediction of other necessary stress-ratio *mode S-N curves* on basis of an available representative one which is typical for the envisaged mode by using the strain energy equivalence: *strain energy being constant assumed* for this prediction.
3 Failure-mode-related damage accumulation (Miner).

Fatigue pre-dimensioning seems to be possible for so-called 'well-designed' fiber-dominated UD laminas-composed laminates by lamina dedicated FF- and IFF-linked mode-representative S-N curves derived from sub-laminate test specimens, which capture the in-situ effect. Initial failure depends on the cycles-dependent shrinking of the IFF body determined by the degrading residual strength.

Effects of the negative neighbour-lamina notching are to be regarded as well as its opposite, the positive embedding.

The still non-sufficient application of the presented novel approach looks promising.
Mind: The elaboration of the method was not funded!

A laminate is a random but not a deterministic failure system of its building blocks, the laminas. As with other static and cyclic methods for laminates, the treatment of this "failure system laminate" composed of the "sub-failure systems of the laminas" is not yet investigated. All the laminas of a laminate act together and cause in-situ effects as well as a joint failure behavior. For instance, an initiated IFF in one lamina will follow a load redistribution which might make another lamina critical. For a better understanding, the associated degradation behavior should be investigated by employing structural reliability as it has been simply performed by the author two decades before.
The method is to be transferred to not fully fiber-dominated lay-ups where all failure modes will contribute. Special situations such as non-proportional loading or not-in-phase loading can be tackled after having solved the basic task.

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