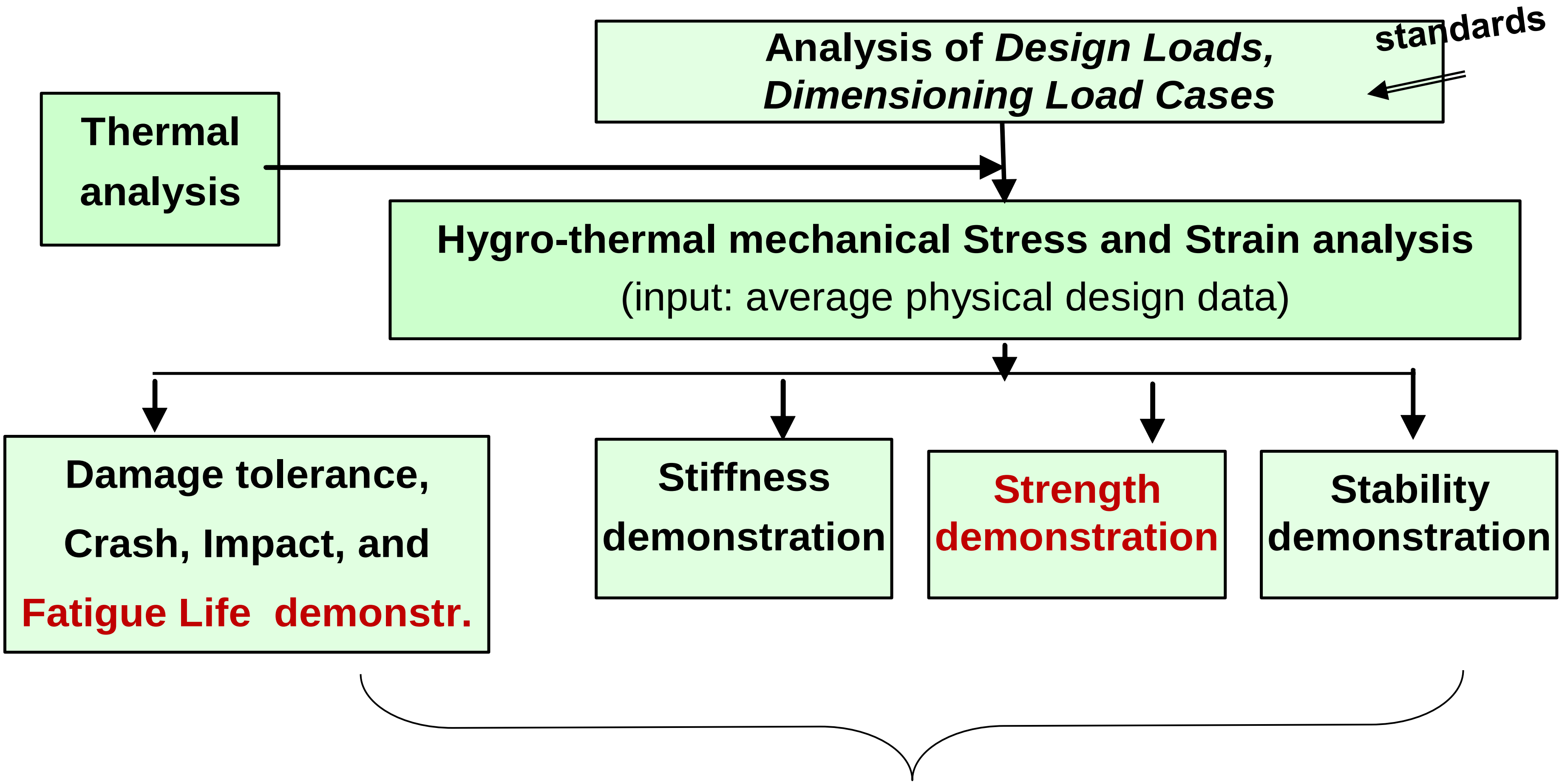
Which Verifications are mandatory in Structural Design ?



Thereby: Structural Resistances of the designed part must be demonstrated by a positive Margin of Safety (MoS) in order to verify Structural Integrity !

CONSTRAINT in Design Development Process *: Cost with Time Reduction*

Industry looks for robust & reliable analysis procedures in order to replace the expensive 'Make and Test Method' as far as reasonable.

Virtual tests *shall reduce the amount of* physical tests.

In this context: **Structural Design Development**

can be only effective and offer fidelity if

realistic analysis procedures and input information are available

for Design Dimensioning and for Manufacturing as well.

Outline of my talk

An extended presentation plus further literature may be downloaded from
http://www.carbon-composites.eu/leistungsspektrum/fachinformationen/fachinformation-2

CARBON COMPOSITES

Static & Fatigue Failure of UD Ply-laminated Parts ?

- a personal view and more -

1 State of the Art of Static & Cyclic Failure Conditions (FC)

2 Fundamentals when modelling Static & Cyclic Failure

3 Global Strength FCs versus Modal ones (strength criteria)

4 Cuntze's Failure-Mode-Concept (FMC) applied to obtain Strength Failure Conditions (SFCs) for UD Material

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Results of a time-consuming "hobby" of an engineer, retired from industry

Prof. Dr.-Ing. habil. ***Ralf Georg Cuntze VDI, linked to Carbon Composite e.V. (CCeV) Augsburg***

Motivation for my non-funded **Investigations on Static and Cyclic Failure**

An interesting fact in the Mechanical Behaviour inspired me:

Different structural materials can

- ***possess a similar <u>material behaviour</u> or can***
- ***belong to the same class of material symmetry.***

Welcomed Consequence, I found:

The same strength failure function F can be used for different materials.

In other words: ***Look at the material itself***

whether it possesses brittle or ductile behaviour or dense or porous consistency

and not at the material family

whether it is a steel, a CFRP, a concrete or a foam

Author's Background:

Experience with structural material applications in the range 4 K - 2000 K.

State-of- the-Art in Static Strength Analysis of UD laminas

best* represented: by the results of the *World-Wide-Failure-Exercises

Organizer : *QinetiQ , UK (Hinton, Kaddour, Soden, Smith, Shuguang Li)*

Aim: *'Testing Predictive Failure Theories for*

Fiber–Reinforced Polymer Composites to the full !'

(for high-performance UD materials , only !)

Procedure of the World-Wide-Failure-Exercises-I, -II (1992-2013):

Part A of a WWFE: *Blind Predictions on basic strengths, only*

Part B of a WWFE: *Comparison Theory-Test using provided*
***Uni-axial 'Failure Stress Test Data' (= basic strength)* and**
Multi-axial 'Failure Stress Test Data'

What were the contents of these Exercises ?

(plain test specimens, no notch)

WWFE-I: ***2D (in-plane) loading ,Test Data for 14 Test Cases (2003)***

WWFE-II: ***3D loading, Test Data Packs for 12 Test Cases (2013)***

WWFE-III: ***Application of advanced failure models based on Damage and Fracture Mechanics Models***

Deals with validating and benchmarking failure theories that are capable of predicting damage, regarding

- **matrix crack initiation and development,**
- **delamination initiation triggered by transverse cracks,**
- **deformation up to final fracture.**

Cuntze did not contribute to WWFE-III

Task was : for endless fiber-reinforced polymers the

Mapping of courses of test data by the contributor's

specific strength failure conditions (criteria), *SFCs.*

State-of-the-Art in Cyclic Strength Analysis of UD Laminas (plies), Laminates

- **Procedures base on specific laminates and therefore cannot be generally applied.**
  Hence, no generally applicable Lifetime Prediction Method is available !
- **Procedures base – as with metals – on stress amplitudes and mean stress correction.**
  Is this correct? Can one neglect that the damaging portions are linked to the various fracture failure modes in the case of brittle behaving materials?
- **Present: Engineering Approach: *Static Design Limit Strain* of < 0.3% ,**
  negligible matrix-microcracking.
  Design experience proved: No fatigue danger is given for multi-angle laminates
- **Future : *Design Limit Strain* shall be increased for better material exploitation**

  (EU-project: MAAXIMUS)

  Above $\varepsilon = 0.5\%$ level: *first filament breaks , diffuse matrix-microcracking* occurs in usually *fiber-dominated laminates,* used in high-stress applications.

To tackle this, much effort must be put on this in future !

German Research State-of-the-Art of Fatigue Lifetime Modelling

- **Germanischer Lloyd :** originally for the GROWIAN (1980) windmill, to be reworked
- **VDI 2014, sheet 3**: (released by Cuntze, as convenor, in 2006. Fatigue to be reworked)
- **University activities: BeNa group, ("*<u>Be</u>triebsfestigkeits-<u>Na</u>chweis"*)**

  for High-Performance Structures (founded by Cuntze in 2010)

  BeNa members-agreed conditions for Lifetime modeling are:

  * *<u>physically-based</u> (on failure modes),*

  * *<u>ply-oriented</u> in order to obtain a generalisation for any*

  UD lamina-composed laminate

Objective of BeNa group: Release of a VDI-Guideline

- **CCeV (Carbon Composites e.V**.*) Augsburg:* **Practiced in my working group and symposia**
- **Company activities: partly issued models and software**

Existing Software: As far as (suitable) lifetime prediction models are available

From industry and Software houses

- **Company-owned programs: AUDI (diss. Hahne), AIRBUS?, BMW, …**
  - **HBM GMbH nCode products: Dr. Vervoort**
  - **Magna Powertrain: Mr. Spindelberger**
  - **Safe Technology Ltd: Dr. Sobczak**
  - **LMS, Dr. Hack**
  - **Firehole Composites: (multi-level model)**
  - **….**

From the German BeNa group (university efforts) for instance:

- **ILK, TU-Dresden (UD, textile attempts)**
- **IVW, TU-Kaiserslautern (thermoset and -plastic UD)**
- **ISD, TU-Hannover (multi-level model)**
- **…..**

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Some definitions first for a Common Understanding

Material: homogenized (macro-)model of the envisaged complex solid

Failure: structural part does not fulfil its functional requirements such as onset of yielding, brittle fracture, Fiber-Failure FF, Inter-Fiber-Failure IFF, leakage, deformation limit, delamination size limit, frequency bound

= project-fixed Limit State with *F* = Limit State Function (here: strength failure function)

Failure Criterion: *F* >=< 1 ,

Failure Condition : *F* = 1= 100%

Failure Theory: tool to predict failure of a structural part

Fracture Failure Surface (body): surface of all uni-/multi-axial fracture failure stresses

Strength Failure Condition (SFC): subset of a strength failure theory

tool for the assessment of a

'multi-axial failure stress state ' in a critical location of the material.

→ **Stress states** are judged by **Strengths !**

Industrial Requirements for Improved Designing of Composite Parts

Static loading:

- **Validated 3D strength failure conditions for isotropic (foam), transversely-isotropic UD materials, and orthotropic materials (e.g. textiles) to determine 'Onset of fracture' and 'Final fracture'**
- **Standardisation of material test procedures, test specimens, test rigs, and test data evaluation for the structural analysis input**

Cyclic (dynamic) loading : fatigue

- **Development of practical, physically-based lifetime-prediction methods**
- **Generation of S-N curve test data for the verification of prediction models**
- **Consideration of manufacturing imperfections (tolerance width of uncertain design variables) in order to achieve a production cost minimum by „Design to Imperfections" includes defects**
- **Delamination growth models: for duroplastic and thermoplastic matrices**
- **Consideration of media, temperature, creeping, aging**
- **Provision of more damping because parts become more monolithic.**

Features of Modelling laminated, high-performance Composites

* ***Lamina-based, sub-laminate-based*** *(e.g. for non-crimp fabrics)* ***or laminate-based !***

* **Is performed, if applicable, *according to the distinct symmetry of envisaged material***

* **For the chosen material model, if material symmetry-based, *the number of the measured inherent Strengths and Elasticity Properties is the same as the observed number of Failure Modes !!*** *Test costs reduction*

* **Achievement of equivalent stresses for each failure mode to obtain information where the lamina design screw must be turned !**

Lesson-Learned: As far as the failure mode or failure mechanism remains, Static Strength Criteria can be used for Cyclic Loading , too !

Very essential !

Plenty combinations of different Constituents of polymeric Composites

Aramide

filament **Carbon**

Glass

Production Processes

Endless fibers

Long fibers

Short fibers.

matrix

Thermosets

Thermoplastics
Elastomers

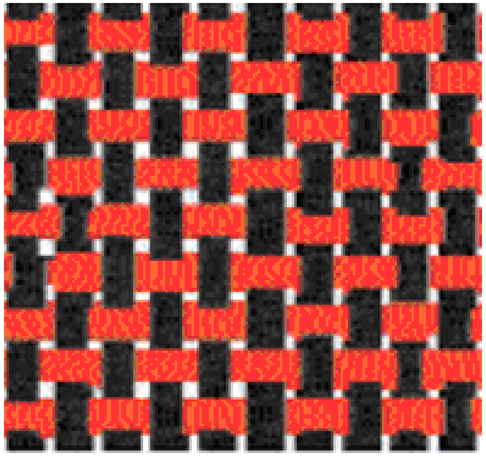
interphase

All these combinations

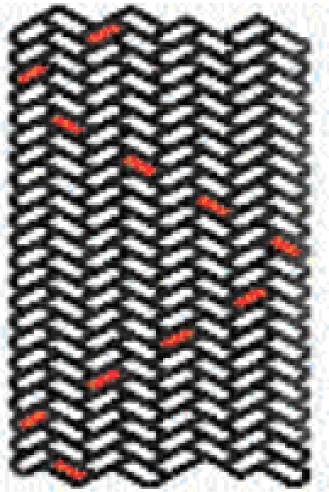
- **need a different treatment and**
- **afford an associated understanding of its internal material behaviour.**

… and - coming up more and more – an increasing variety of 2D- and 3D-fabrics

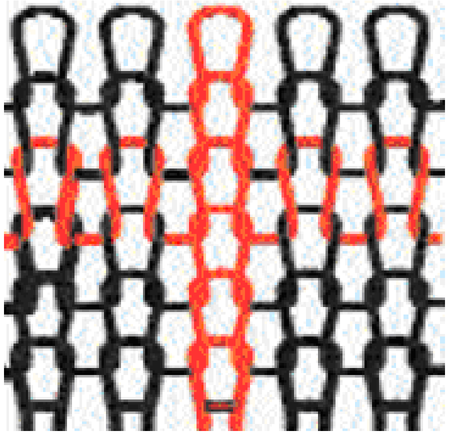
Coming up: The Textile Challenge to achieve Certification



plain weave

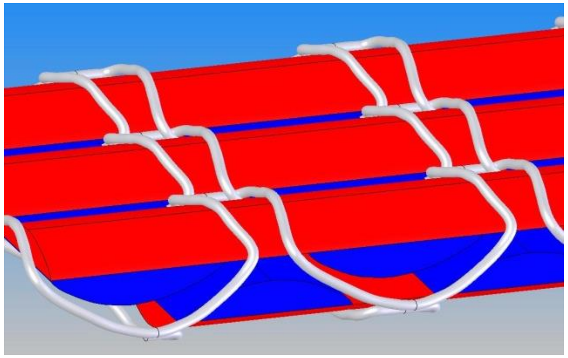


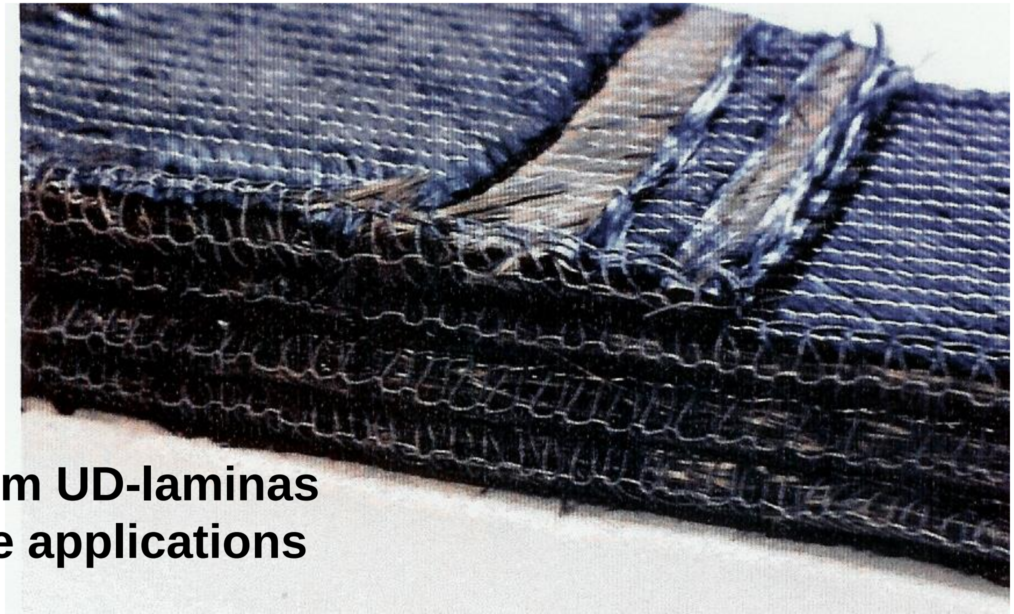
braid



weft knit

warp knit





non-crimp fabrics from UD-laminas for high-performance applications

Modelling: 'Simple' UD material = Lamina (ply)

1 Lamina = Layer of a Laminate, e.g. UD-laminas = "Bricks"

- ***Homogenisation of a solid to a material brings benefits*.**
- ***Then Knowledge of Material Symmetry applicable : number of required material properties are minimal, test-costs too***

UD-lamina, modelled a homogenised ('smeared') material requires in:

Material Characterisation f (Temp, Moisture, time, etc.)

Assumptions for UD Modelling and Mapping of Failure Stress data

- **The UD-lamina is macroscopically homogeneous.**

  It can be treated as a homogenized (‘smeared‘) material

  Homogenisation of a solid to a material brings benefits.

  Then Knowledge of Material Symmetry applicable : number of required material properties are minimal, test-costs too

1 Lamina (ply) = Layer of a Laminate, e.g. UD-laminas = “Bricks“

- The UD-lamina is transversely-isotropic:
On planes, parallel to the fiber direction it behaves orthotropic and on planes transverse to fiber direction isotropic (quasi-isotropic plane)

- **Mapping: Uniform stress states are about the critical stress location !**

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Global and Modal Strength Failure Conditions *General View*

Drucker-Prager, Tsai-Wu

1 Global ***strength*** **failure condition** : F ({σ}, {*R*}) = 1 (usual formulation)

Set of Modal strength failure conditions: F ({σ}, R^{mode}) = 1 (addressed in FMC)

Mises, Puck, Cuntze

Example: UD

vector of 6 stresses (general)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of 5 strengths

$$\{R\} = (R_{\parallel}^t, R_{\parallel}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\parallel})^T$$

needs an Interaction of Failure Modes: performed by a

probabilistic-based 'rounding-off' approach (series failure system model)

directly delivering the (material) reserve factor in linear analysis

Note: In the quasi-isotropic plane of the UD material just 5 stresses are active:

$$\{\sigma\}_{principal}^{quasi-isotropic\ plane} = (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$$

By-the-way: Experience with Failure Prediction prove

A Strength Failure Condition (SFC) is a necessary but not a sufficient condition to predict Strength Failure (example: thin-layer problem).

On top, an energy condition may be to fulfill.

Facts of Global and Modal Strength Failure Conditions (SFCs)

Global (one surface) **SFCs:**

- **Combine all failure modes in one single mathematical formulation. This might even capture**
  - a twofold acting failure mode (e.g. if $\sigma_I = \sigma_{II}$ (isotropic) or if $\sigma_2 = \sigma_3$ (transversely-isotropic UD material) and
  - a threefold acting failure mode under hydrostatic loading
- **Re-calculation of all model parameters by new a data course mapping if a test data is to be replaced in one failure mode domain. Then all Reserve Factors have to be determined again!**
- **Some simple global SFCs just use strengths as model parameters. In this case, a change in one failure domain deforms the failure surface in all other (physically independent) failure domains. There is a big chance that a Reserve Factor in such a domain is not on the safe side!**

Modal (multi-surface) **SFCs:**

- **Describe one single failure mode in one single mathematical formulation** (part of failure surface)**.**
  - determine all model parameters in the respective failure mode
  - capture a twofold acting failure mode (e.g. if $\sigma_I = \sigma_{II}$ (isotropic) or if $\sigma_2 = \sigma_3$ (transversely-isotropic UD material) separately, modal-wise by one additional Ansatz (J_3)
  - capture a threefold acting failure mode under hydrostatic loading alike
- **Re-calculation of the model parameters just in the modal domain if a test data is to be replaced. One Reserve Factor must be freshly determined.**

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MIND: The production process 'bakes' the composite material !

Driver for my research work on Strength Failure Conditions (criteria)

Achievement of practical, physically-based criteria under some *pre-requisites* :

- ***physically convincing***
- ***simple, as much as possible***
- ***invariant-based** (like the Mises yield condition)*
- ***allow to compute an equivalent stress** (very helpful for a distinct failure mode)*
- ***rigorous indepent treatment of each single failure mode (2 FF + 3 IFF)***
- ***using a material <u>behaviour</u>-linked thinking and not a material-linked one***
- ***engineering approach where all model parameters can be measured*.**

<u>Note on UD strength failure conditions</u>:

Puck's action plane approach involves some basic differences to Cuntzes Failure-mode-concept-based approach: (1) is not invariant-based, (2) interacts the 3 Inter-Fiber-Failure modes (IFF) by a Mohr-Coulomb-based equation, (3) post-corrects the IFF- influence on FF.

Cuntze provides for each failure mode an equivalent stress, that captures the influence of IFF on FF by his interaction equation, uses less model parameters.

Basic Features of the author's Failure-Mode-Concept

- **Each failure mode represents 1 independent failure mechanism**

  and thereby 1 piece of the complete *failure surface*

- **Each failure mechanism is governed by 1 basic strength** (is observed !)
- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !!

- **In consequence: Interaction of the Failure Modes is needed**

  in the case of modal Strength Failure Conditions (SFCs)!

- **The Formulation of the SFCs for the homogenized material is :**

  - invariant-based: the choice of the used invariants is linked to the

  fact, whether the material element experiences

  a *volume* change, a *shape change* and *friction*

  - material symmetry –based: fixes the number of modes, strengths, …

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Cuntzes 3D Strength Failure Conditions (criteria) for UD-material

(top-ranked in the World-Wide-Failure-Exercises, invariants replaced by their stress formulations)

FF1 $Eff^{\parallel\sigma} = \breve{\sigma}_1 / \bar{R}_\parallel^t = \sigma_{eq}^{\parallel\sigma} / \bar{R}_\parallel^t ,$ $\breve{\sigma}_1{}^* \cong \varepsilon_1^t \cdot E_\parallel$ **filament strains from FEA**

FF2 $Eff^{\parallel\tau} = -\breve{\sigma}_1 / \bar{R}_\parallel^c = +\sigma_{eq}^{\parallel\tau} / \bar{R}_\parallel^c ,$ $\breve{\sigma}_1 \cong \varepsilon_1^c \cdot E_\parallel$ **2 filament modes**

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2}] / 2\bar{R}_\perp^t = \sigma_{eq}^{\perp\sigma} / \bar{R}_\perp^t$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2}] / \bar{R}_\perp^c = +\sigma_{eq}^{\perp\tau} / \bar{R}_\perp^c$

3 'matrix' modes

IFF3 $Eff^{\perp\parallel} = \{[\mu_{\perp\parallel} \cdot I_{23-5} + (\sqrt{\mu_{\perp\parallel}^2 \cdot I_{23-5}^2 + 4 \cdot \bar{R}_{\perp\parallel}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2}] / (2 \cdot \bar{R}_{\perp\parallel}^3)\}^{0.5} = \sigma_{eq}^{\perp\parallel} / \bar{R}_{\perp\parallel}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

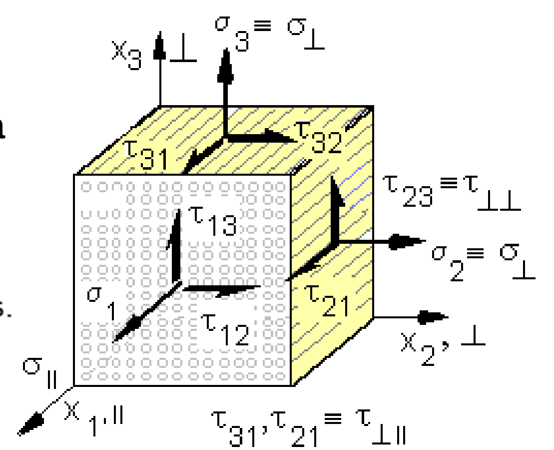
[Cun04, Cun11]

Modes-Interaction $Eff^m = (Eff^{\parallel\tau})^m + (Eff^{\parallel\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m$

with influence IFF on FF : $= 1 = 100\%$ is 'onset of failure'

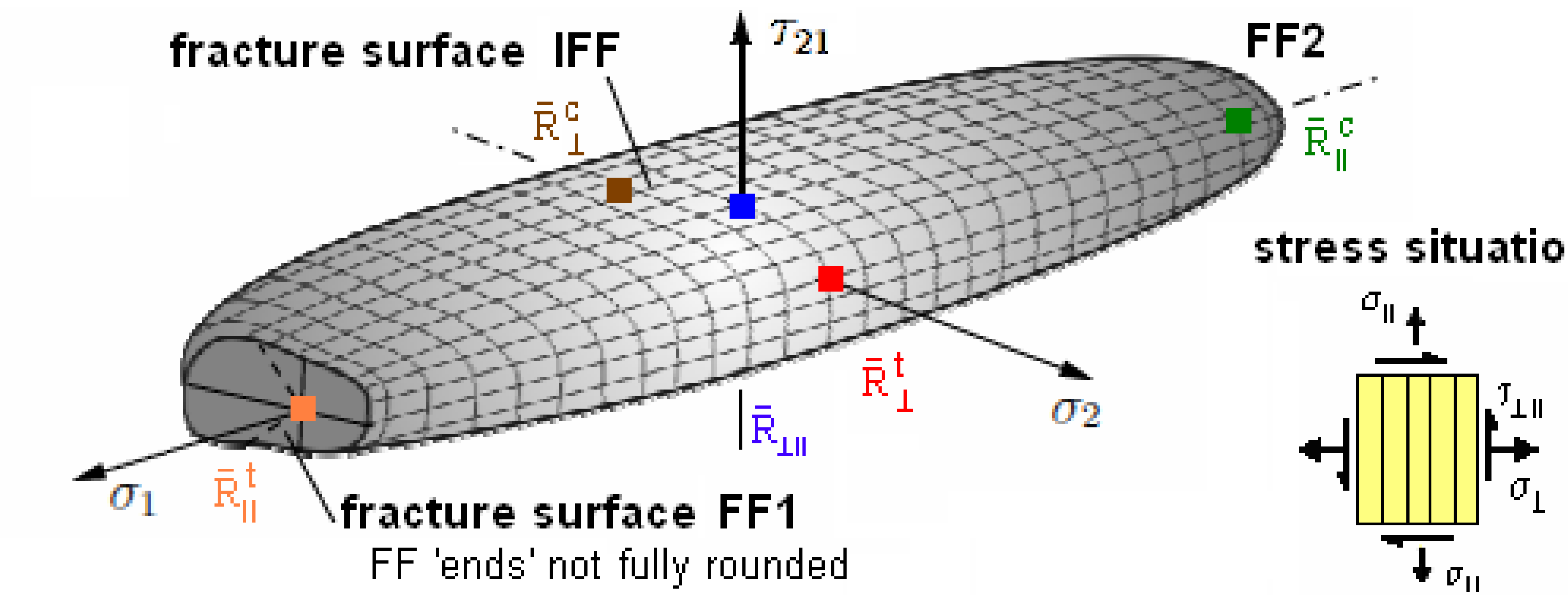
with mode-interaction exponent $2.5 < m < 3$ **from mapping test data**

Typical friction value data range: $0.05 < \mu_{\perp\parallel} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$



Eff:= material stressing effort (Werkstoffanstrengung), *R*:= UD strength, σ_{eq}:= equivalent stress.
Eff:= artificial word, fixed with QinetiQ in 2011, to have an equivalent English term.
Poisson effect considered*: bi-axial compression strains a filament without any σ_1
t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre

Visualization of 2D UD SFCs as Fracture Failure Surface (Body)



(courtesy W. Becker) .

$$\{\sigma\}=(\sigma_1,\sigma_2,0,0,0,\ \tau_{21})^T$$

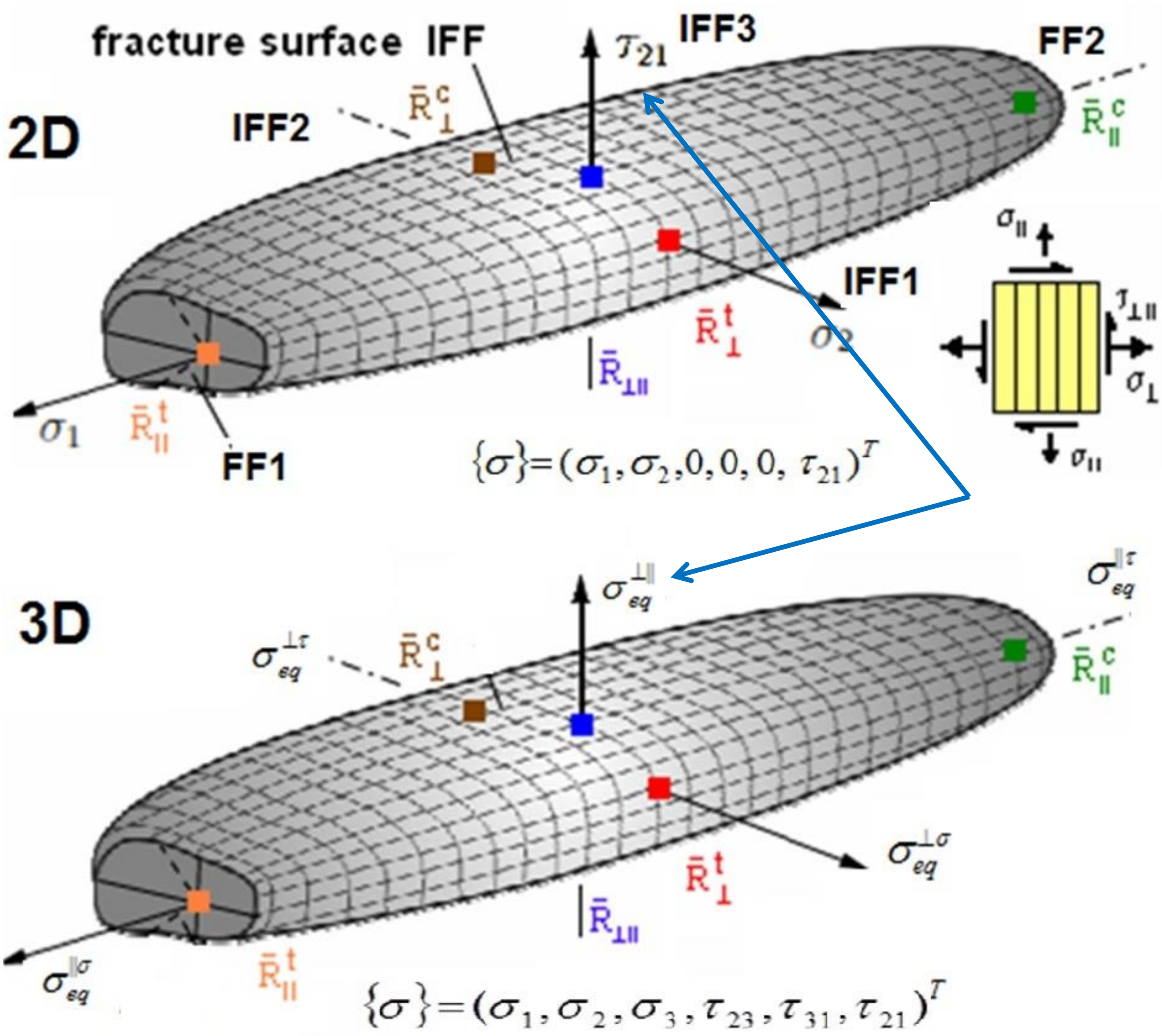
Mode interaction fracture failure surface *of FRP UD lamina*

(series failure system model) Eff = material stressing effort = Werkstoffanstrengung

$$Eff^{\,m} = (Eff^{\parallel\tau})^m + (Eff^{\parallel\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

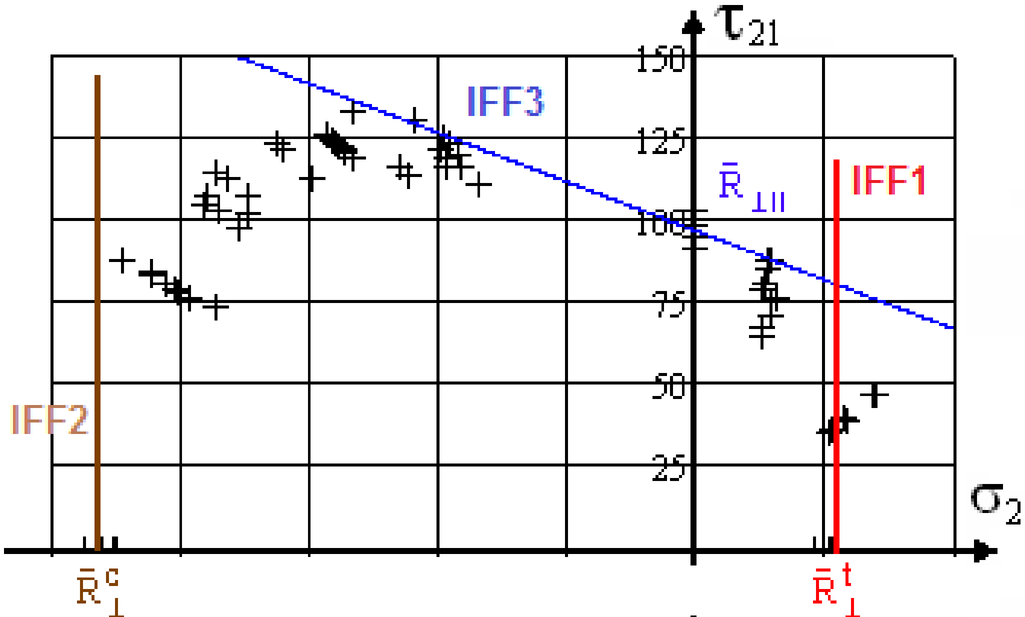
Mapping: Average strengths indicated by an upper bar

2D ➡ 3D Fracture surface by replacing the stress by the equiv. stress



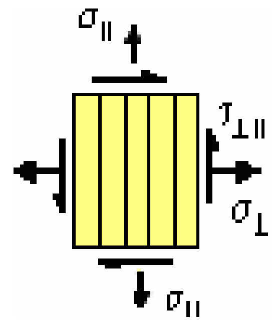
Demonstration: Interaction of UD Failure Modes

$\tau_{21}(\sigma_2)\ ,\ \breve{\sigma}_1 = 0$



Mapping of course of IFF test data in a pure mode domain by the associated *Mode Failure Condition.*

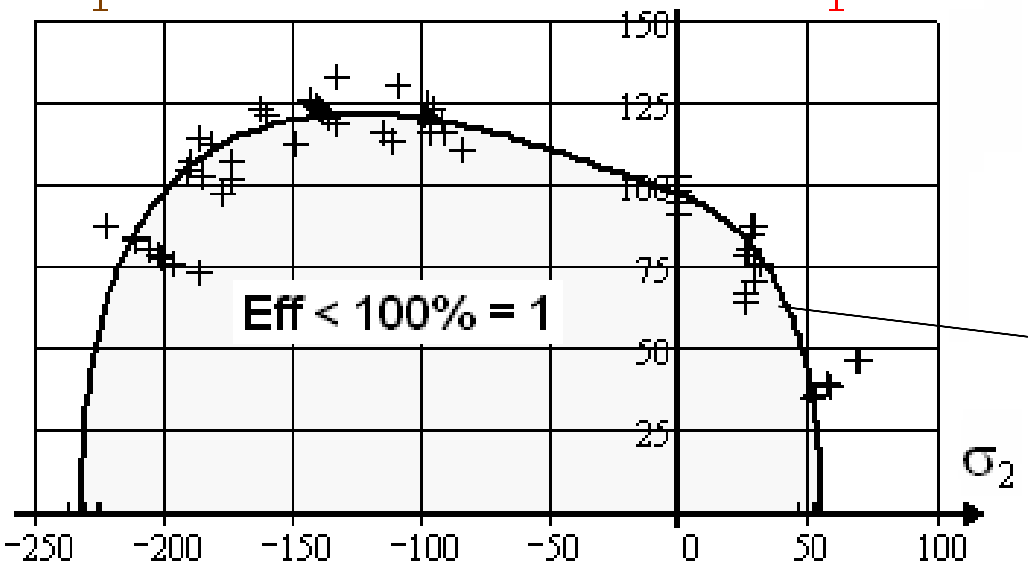
3 IFF pure modes = straight lines !.



IFF 1 : $\dfrac{\sigma_2}{\bar{R}_\perp^t} = 1$

IFF 2 : $\dfrac{-\sigma_2}{\bar{R}_\perp^c} = 1$

IFF 3 ***(2D simplified)*** : $\dfrac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 1$



Mapping of course of test data by *Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

$$m = 2.5,\ \ \mu_{\perp\parallel} = 0.3$$

Test Data Mapping versus Design Verification

• **Validation of SFCs with Failure Test Data** by

mapping their course by an average Failure Curve (surface)

use of average (typical) strengths $\bar{R}$ *(from resistance)*

•**Delivery of a reliable Design Verification** by

calculation of a Margin of Safety or a (load) Reserve Factor

$$MoS > 0 \quad oder \quad RF = MoS + 1 > 1$$

on basis of a statistically reduced failure curve (surface)

use of strength design allowables R *(no bar over)*.

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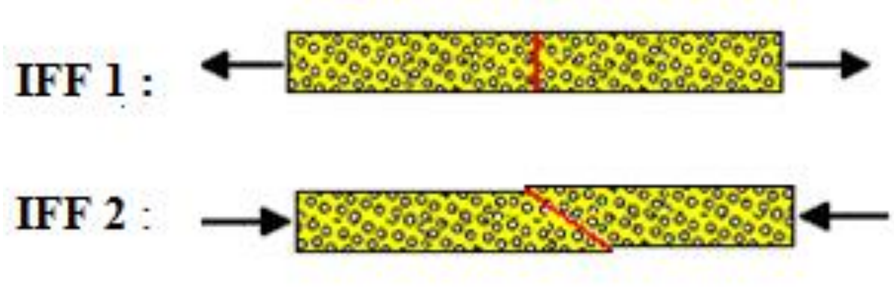
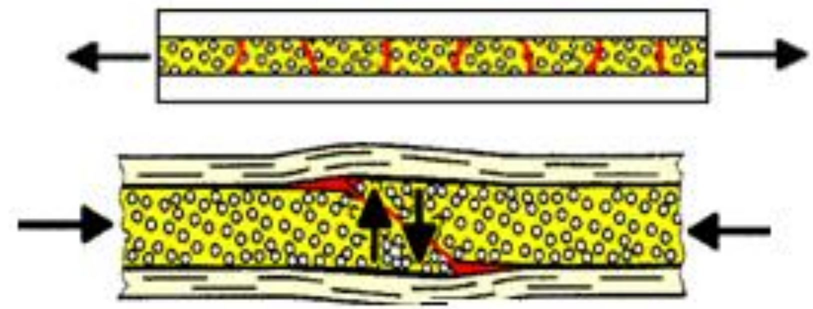
'Isolated' lamina test specimens

= weakest link results (series failure system)

'Embedded' laminas experience in-situ effects

= redundancy result (parallel failure system)





unconstrained lamina

delivers strength property, stress-strain curve

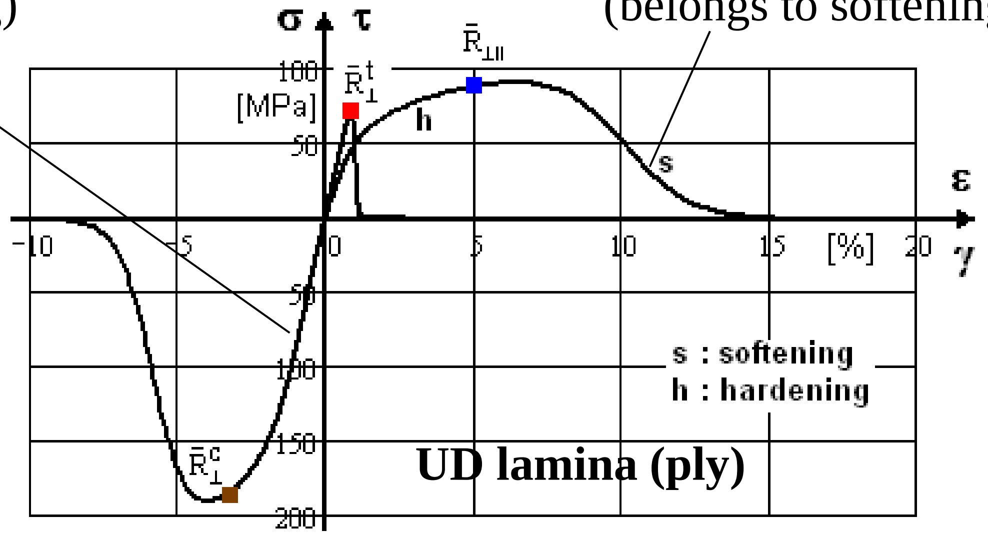
(belongs to hardening)

delivers **basic strength** as analysis input !

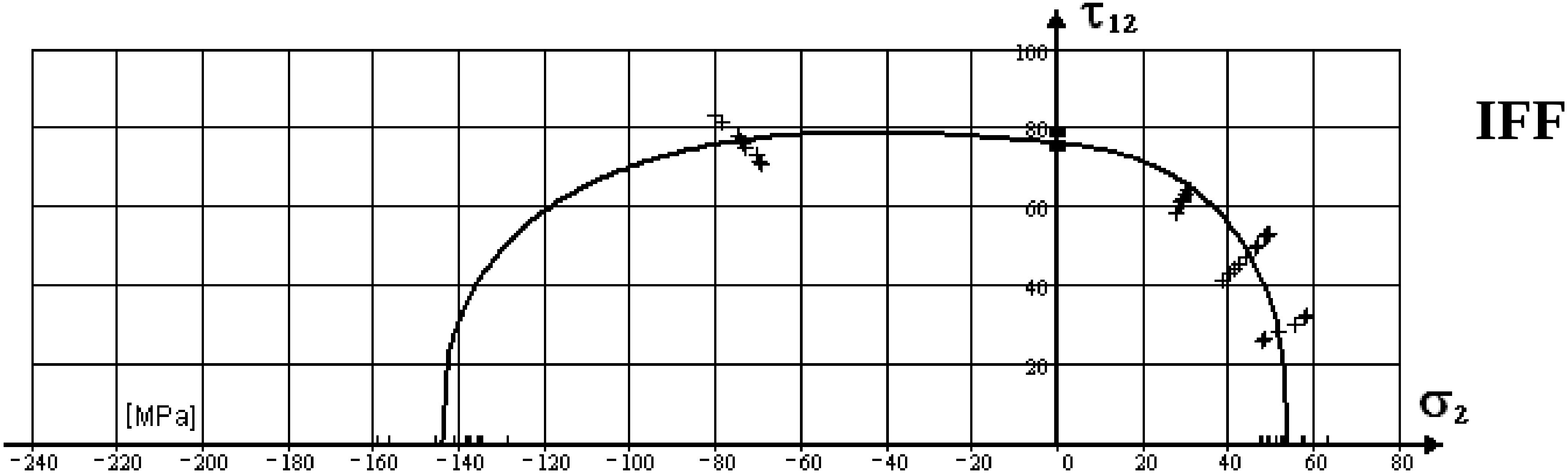
mutually constrained laminas, in laminates

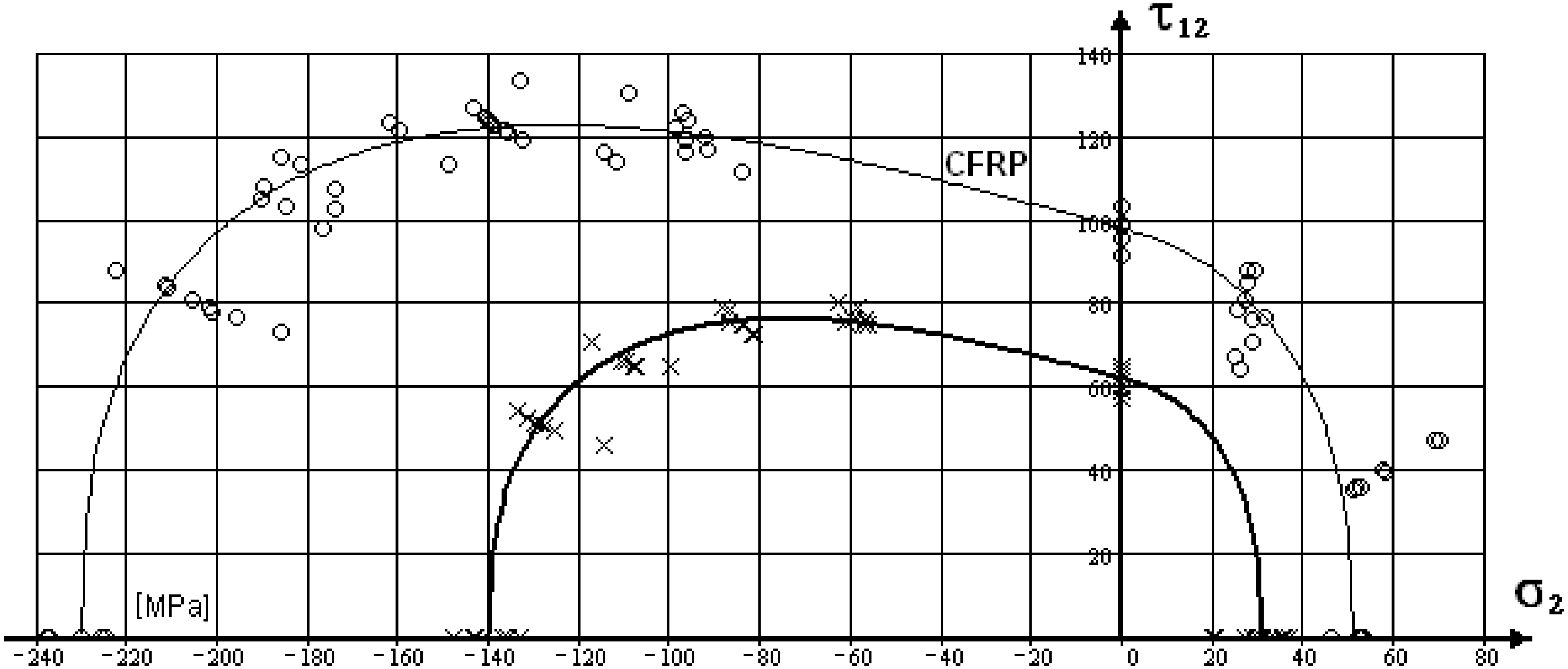
in <u>non-linear</u> laminate analysis

(belongs to softening)



GFRP, CFRP examples, mapped by FMC–based UD SCF, 2D stress state





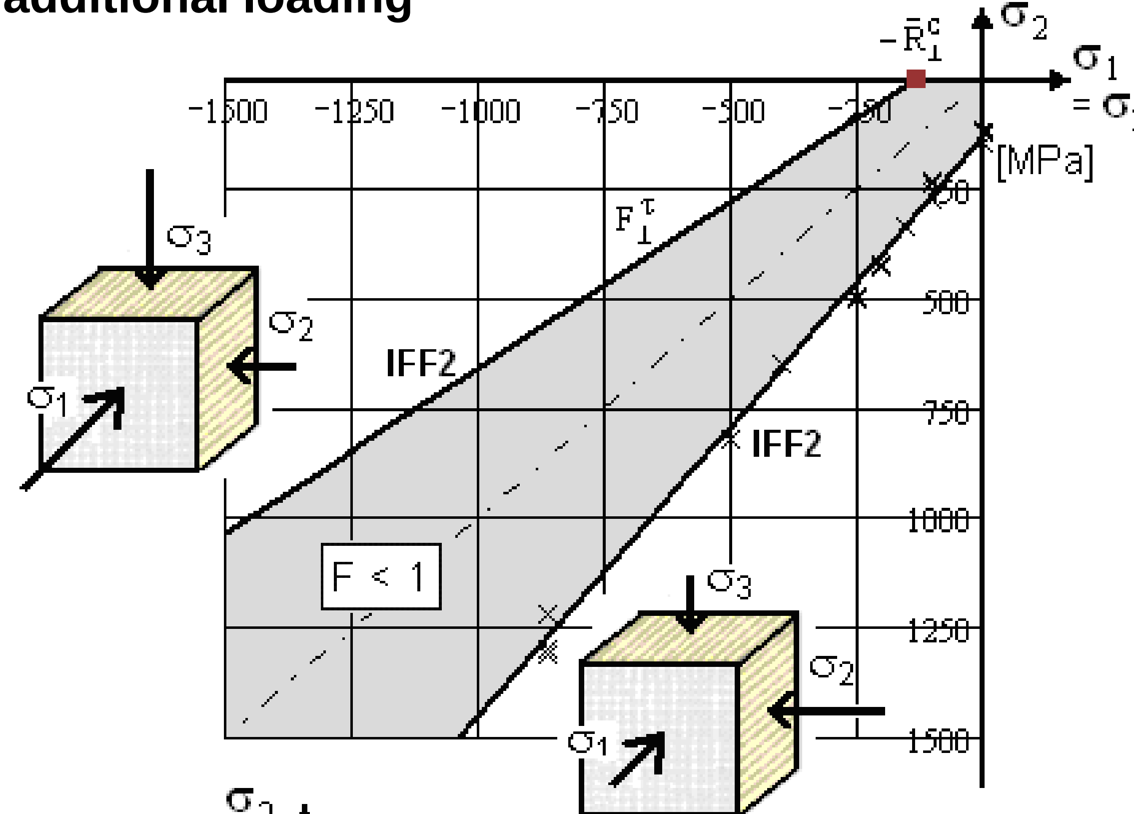
Test Case 5, WWFE-II, UD test specimen, 3D stress state $\sigma_2(\sigma_1 = \sigma_3)$

= hydrostatic pressure with additional loading

UD E-glass/MY750epoxy.

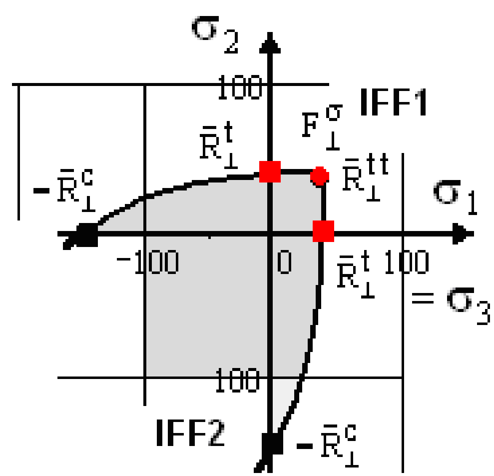
$\nu_{\perp\parallel} = 0.28$, $\mu_{\perp\perp} = 1.1??6$, $m = 2.8$,

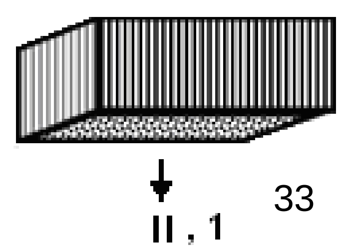
$\{\bar{R}\} = (1280,\ 800,\ 40,\ 132,\ 73)^T\ MPa$



Good Mapping, <u>after</u> QinetiQ re-evaluation of the lower branch test data
Then, the upper branch was fitting other test data, too !

Result: ***Both branches were then reliable and could be used for model validation***





Design Verification: Achievement of a Reserve against a Design Limit State

For each distinct Load Case with its single Failure Modes must be computed:

Reserve Factor (load-defined !) **:**

determinisitic or semi-probabilistic

$$RF = \frac{\textbf{Failure Load} \text{ at } Eff = 100\%}{\textbf{applied Design Load}}$$

valid in linear and non-linear analysis

Material Reserve Factor :

f_{Res} = ***Strength Design Allowable / Applied Stress***

$f_{Res} = RF = 1 / Eff$, valid in linear analysis

Material Stressing Effort :

(Werkstoff-Anstrengung)

$Eff = 100\%$ ***if*** $RF = 1$ ***material exhausted***

applied Design Load = Factor of Safety *j* x Design Limit Load

Determination of the load-defined Reserve Factor *RF*

Linear elastic problem for the envisaged brittle behaving CFRP

then simplified $\boldsymbol{RF = f_{Res}}$ **(material reserve factor)** $= \boldsymbol{Eff^{-1}}$

Residual stresses : 0 (effect vanishes with increasing micro-cracking) in $MPa = N/mm^2$

Stress state vector : $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T = (0,\ -60,\ 0,\ 0,\ 0,\ 50)^T$

Strengths vector: $\{R\} = (R_{\parallel}^t, R_{\parallel}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\parallel})^T = (1200,\ 850,\ 35,\ 100,\ 80)^T$

Mode interaction exponent: $m = 2.7$

Friction value: $\mu_{\perp\parallel} = 0.3$

Roughly estimated from average values

$\{\overline{R}\} = (1378,\ 950,\ 40,\ 125,\ 97)^T$

WWFE-I: UD T300/PR319EP

Calculation: negative *Effs* are nonsense and are to be bypassed

$$Eff^{\perp\sigma} = \frac{\sigma_2 - |\sigma_2|}{\overline{R}_{\perp}^t} = 0 \qquad Eff^{\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{\overline{R}_{\perp}^c} = 0.60 \qquad Eff^{\perp\parallel} = \frac{|\tau_{21}|}{\overline{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 0.51$$

$$Eff^{m} = (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m$$

$$Eff = 0.72,\ \boldsymbol{RF = 1/Eff = 1.39},\quad MoS = RF - 1 = 0.39$$

Loading may be increased by the factor *RF* until obtaining fracture limit state $\boldsymbol{Eff = 100\% \equiv RF = 1}$ **.**

1 State of the Art of Static & Cyclic Failure Conditions (FC)

2 Fundamentals when modelling Static & Cyclic Failure

3 Global Strength FCs versus Modal ones (strength criteria)

4 Cuntze's Failure-Mode-Concept (FMC) applied to obtain Strength Failure Conditions (SFCs) for UD Material

5 Static Failure Modelling of Transversely-isotropic UD-CFRP with an example Design Verification by a Static Reserve Factor *RF*

6 Lifetime Prediction Model for a cyclically-loaded UD-CFRP with a numerical example

Fatigue : process, that degrades material properties. 3 fatigue phases exist

Damaging (= Schädigung, but not damage (Schaden), as it is used in English, too): a process wherein the results, the damaging portions, finally accumulate to a damage size such as a macro-scopic delamination (onset of 3rd phase).

Used as means: the Palmgren-*Miner Damaging Accumulation* model

Damage : damage size that is judged to be critical. *Then Damage Tolerance Analysis is used to predict the damage growth under further cyclic loading.*

Material : homogenized (smeared) model of the envisaged complex material which might be a material combination

.

Which questions does an engineer pose in the case of cyclic design?

1. When does damaging start? 1st phase of fatigue life
2. How can one quantify the single (micro-)damaging portions?
3. How can the single damaging portions be accumulated?
4. When do the accumulated damaging portions represent a real damage?
5. When does such a damage (delamination, impact) become critical?
6. How is the damage growth in the 3rd phase (final) of fatigue life ?

   (fixation of part replacement time, inspection intervals)

Engineer‘s Desires for a Fatigue Lifetime Prediction Model

to

- **capture multi-axial, variable loadings**
- **be physically-based**
- **deal on the simpler homogenized composite material level (numerical efficiency) but account for failure of the composite material constituents matrix, fiber and interphase**
- **be applicable to any laminate**
- **set up a fatigue model with clearly measurable parameters**
- **have it implemented into a standard commercial software.**

Quantifying the damaging portions in the damaging progress: *By Static Strength Failure Conditions possible?*

Experience-proven Assumption:

if damaging mechanisms (failure modes) in static and cyclic case are equal,
then

- failure parameters that drive cyclic damaging are equal, too, and

- transferability from static failure to cyclic failure is permitted !!

However, static strength must be replaced by the
fatigue strength = residual strength of the shrinking failure body,
which is associated to the respective lifetime !

Therefore, to obtain quantified damaging portions

my FMC-based **Static Failure Conditions (criteria) can be used,**

Measurable quantities within damaging:

Micro-crack density, Residual strength, Residual stiffness.

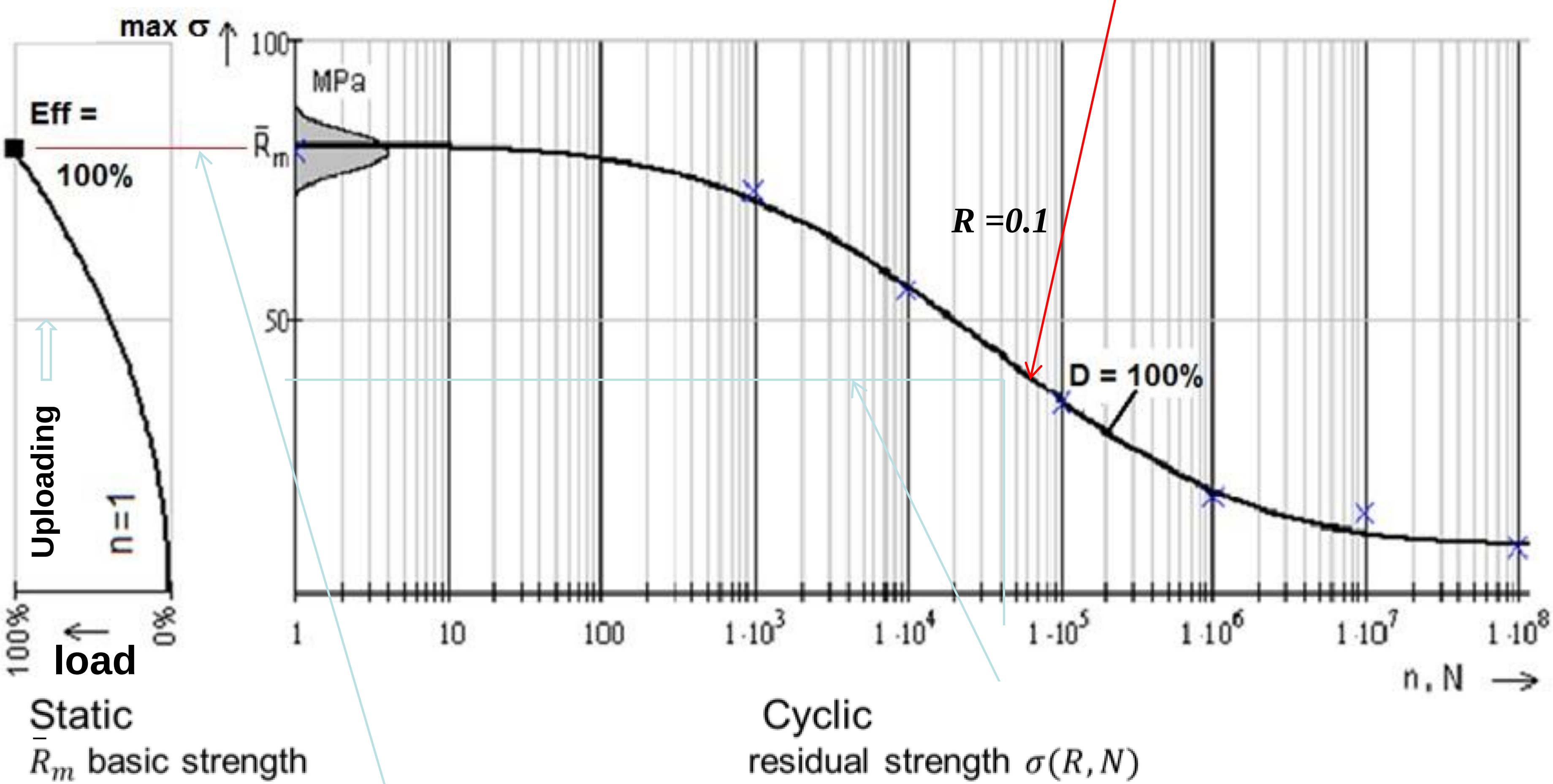
Quantification of Damaging Portions: What is needed?

- **S-N curves** $R = const = \sigma_{unter} / \sigma_{ober}$
- **Hypothesis to accumulate the damaging portions (rel. Miner most often)**
- **Model to quantify the damaging portions under cyclic loading**

Experience proved for brittle behaving composites that above static strength failure conditions can be used.

From above one can conclude:
The treatment of composites needs more effort.

Static and cyclic development of damaging, S-N-curve $R = \sigma_{min} / \sigma_{max}$



Analogous limits of the material capacities :

- Static : material stressing effort *Eff* = 100 %

- Cyclic : material damaging sum *D* = 100 %

The static material stressing effort *Eff* (Werkstoffanstrengung) is replaced by the cyclic *D* !

For brittle behaving materials it is advantageous to use $max\sigma \equiv R_m$ instead of $\Delta\boldsymbol{\sigma}$

Question: Failure mode-linked Master S-N-curves dterminabl to save test costs?

For lifetime estimation usually several S-N-curves are needed.

(constant amplitude loading is a seldom case)

<u>Idea:</u> Measurement for each failure mode: just one modal Master S-N-curve

- for a fixed stress ratio *R*
- prediction of additionally necessary S-N-curves of a mode on basis of the master curve and on the
'principle of equivalent strain energy'!

Then, for the often used

all possible load orientations capturing fiber-dominatedly designed, multidirectional laminates, composed of UD plies,

an engineering-like model for plain laminates is derivable !

The model's characteristical steps are:

4 steps

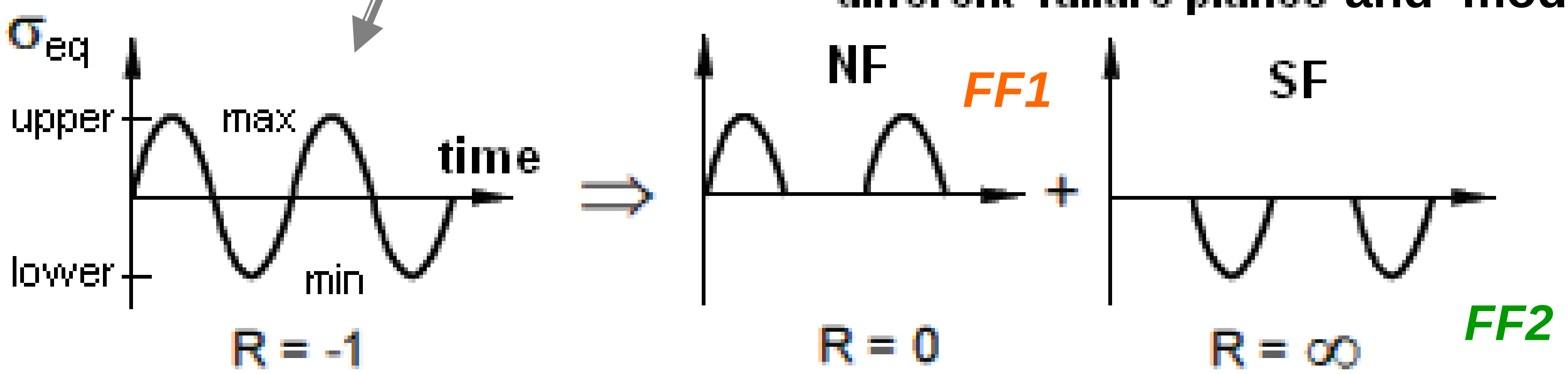
S := cyclic stress range $\Delta\sigma$, N := number of cycles to failure, stress ratio R := $min\sigma / max\sigma$

Failure mode-wise Modelling of Loading Cycles for high-performance ‘fiber-dominated designed’, UD laminas-composed laminates

For simply displaying the approach *it is chosen* :

- **the usually ‘fiber-dominated‘ laminate and**
- ***R = -1 loading***

separation due to inherent different failure planes and modes



$\sigma_{mean} = 0$

NF := Normal Fracture, SF := Shear Fracture

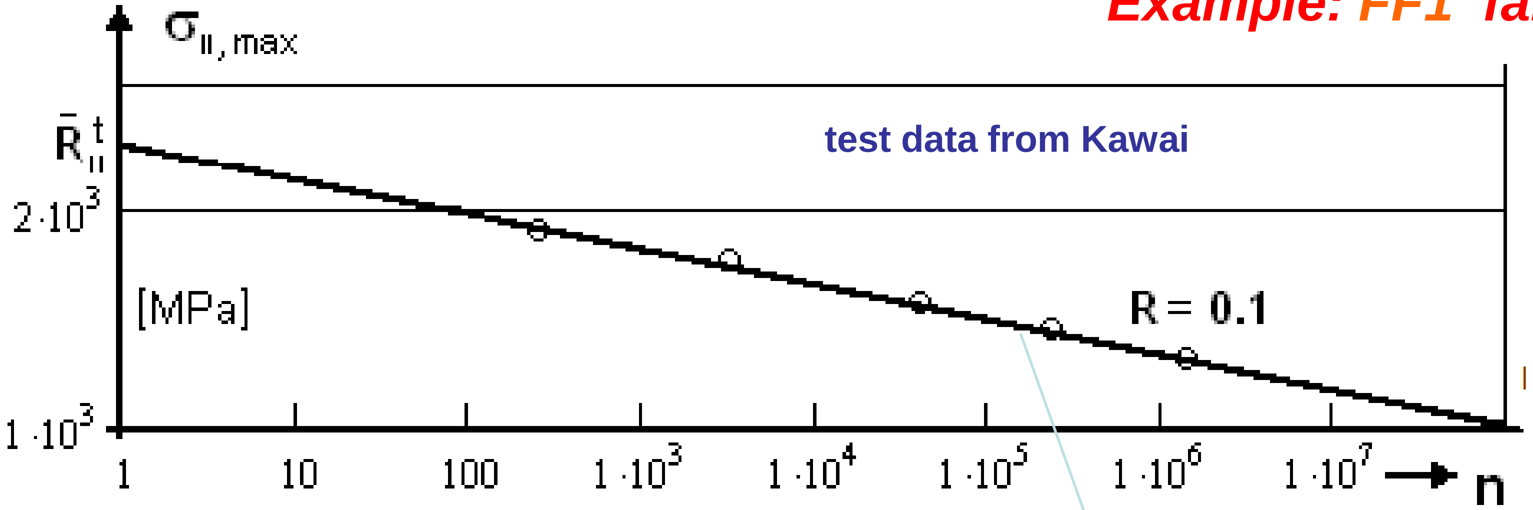
Step 1 : <u>Failure mode-wise</u> apportionment of cyclic loading (novelty 1!)

Specific rain-fall procedure to be applied,

***FF1*:= fiber tensile fracture; *FF2*:= fiber compressive failure**

Mapping of Mode S-N data by a representative *Master curve*

Example: FF1 failure mode



Step 2 **: S-N curve can be mapped by a straight line in a *log-log* graph**

$$\sigma_{\parallel,\ \max}^{Master}(n) \approx \overline{R}_{\parallel}^{t} \cdot n^{c_{Master}}$$

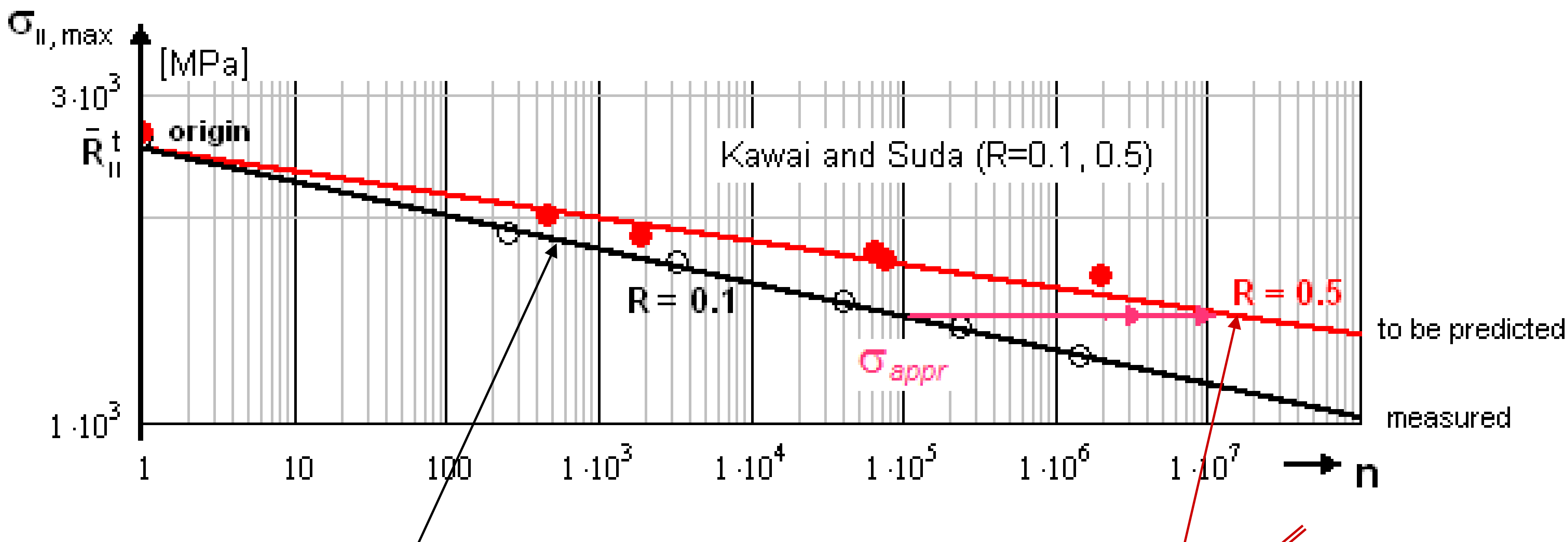
FF1 **strength**

Measured test data mapped by

as mode-representative *Master S-N curve* for *FF1 .*

In the general case of variable loading, several S-N-curves are needed !

Prediction of needed other *FF1* S-N curves from Master *FF1* Curve



Given : $\sigma_{\parallel,\ max}^{Master}(n) \approx \bar{R}_{\parallel}^{t} \cdot n^{c_{Master}}$ **Searched :** $\sigma_{\parallel,max}^{pred} = \bar{R}_{\parallel}^{t} \cdot n^{c_{pred}}$

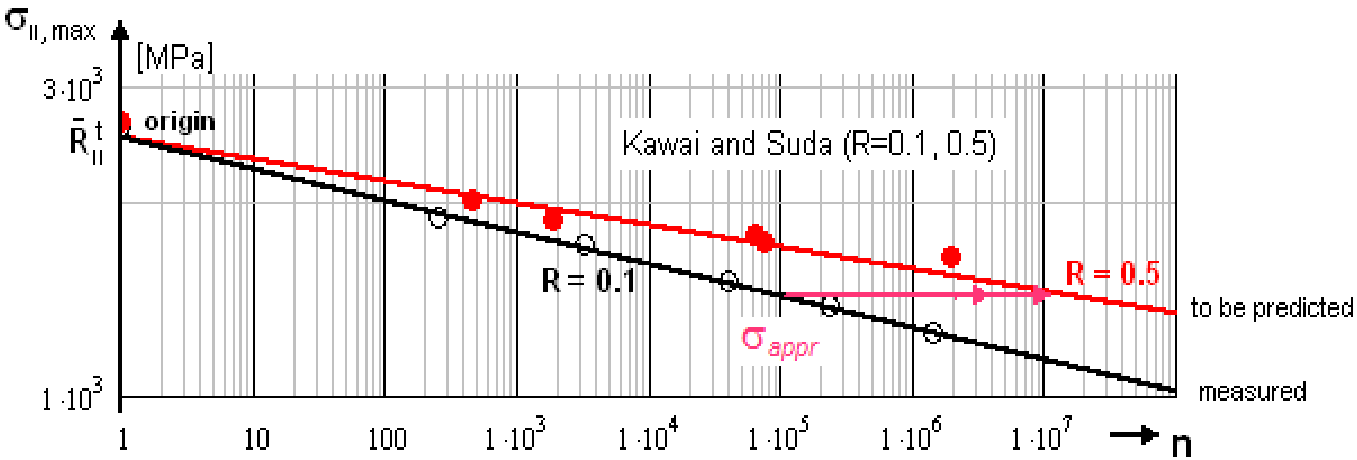
Slope of R = 0.5 ?

Step 3: Application of the principle of constant strain energy

A distinct strain energy level will be reached for R > 0.1 at higher cycles.

S := cyclic stress range = Δσ, N:= number of cycles to failure, n:= cycle number

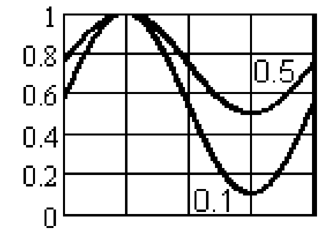
How does the method work for a UD lamina? Numerical example: $R_{0.5}$ from $R_{0.1}$



$$n_{appr} = 10000 \text{ cycles}$$

$$R_{master} = 0.1, \quad R_{pred} = 0.5$$

$$D_{feasible} = 0.8$$



$$f_{pred} = \exp[-\ln(\frac{R_{pred}^{\ 2} - 1}{R_{master}^{\ 2} - 1}) \cdot \frac{1}{c_2}] \ = 210$$

$$\sigma_{\|,\ \max}^{master}(n) \ \approx \overline{R}_{\|}^{t} \cdot n^{c_{master}} = \overline{R}_{\|}^{t} \cdot n^{-0.049} \quad \Rightarrow \quad \sigma_{1,\max}^{pred}\ (n) \approx \overline{R}_{\|}^{t} \cdot n^{c_{pred}}.$$

$$c_{pred} = -\ln(\overline{R}_{\|}^{t} / \sigma_{appr}) / \ln(n_{appr} \cdot f_{pre}) = -0.034$$

Loading $n_3(R = 0.1) = 0$

$n_1(R = 0.1) = 100000\ cycles,\quad \sigma_1^{(1)} = 12500\ MPa,\quad N_1(R = 0.1) = 2300000\ cycles,$

$n_2(R = 0.1) = 1600\ cycles,\quad \sigma_1^{(2)} = 1500\ MPa,\quad N_2(R = 0.1) = 55000\ cycles,$

$n_4(R = 10) = 6000\ cycles,\quad \sigma_1^{(4)} = -1150\ MPa,\quad N_4(R = 10) = 5000\ cycles,$

$n_5(R = 0.5) = 600000\ cycles,\quad \sigma_1^{(5)} = 1550\ MPa,\quad N_5(R = 0.5) = 2600000\ cycles.$

Miner application

$D = \sum n_i/N_i\ ) = 100000/2300000 + 1600/55000 + 6000/5000 + 600000/2.600000 = 0.43$

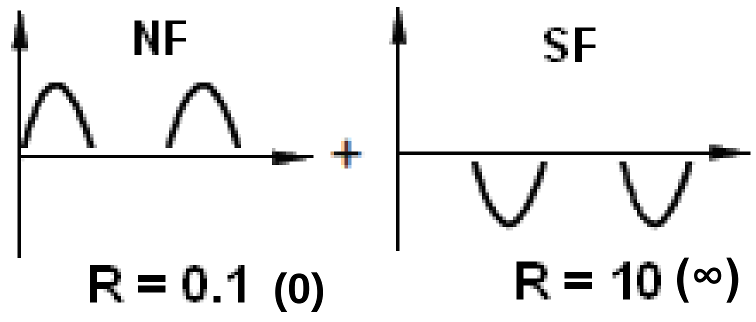
$$\text{MoS} = \frac{\frac{D_{feasible}}{D}}{j_{life} \cdot D} - 1 = \frac{0.8/0.43}{3 \cdot 0.43} - 1 = 0.4 > 0.$$

$$\{\overline{R}\} = (2560,\ 1590,\ 73,\ 185,\ 90)^T \text{ MPa}$$

$\sigma_{\max} = 2 \cdot \sigma_a / (1 - R) = \Delta\sigma / (1 - R)$ **with $\Delta\sigma$:= stress range**

Application of Relative Miner-'Rule'

FF1 FF2



Simple Example: again

R = - 1

← practically measured curves

$$D\ (FF1, FF2) = NF : (n_1 / N_1 + n_2 / N_2 + n_3 / N_3) + SF : (n_4 / N_4)$$

$$+ \quad D\ (IFF1, IFF2, IFF3) \ = \ D \leq \ D_{feasible} < 100\%$$

$D_{feasible}$ value from test experience

Step 4: Mode-wise Accumulation of Damaging Portions (novelty 2 !)

Calulation, from [Cun13b], see Annex

FF = Fiber Fracture, IFF = Inter Fiber Fracture

Conclusions for the presented UD Lifetime Prediction Method

Idea recalled: ***It employs for the usually fibre-dominated designed laminates***

1) **Failure mode-linked load modelling and damaging accumulation (Miner)**

2) **Measurement of a minimum number of Master S-N curves**

3) **Prediction of other necessary *mode S-N curves* is performed on basis of the Master Curve by employing the *strain energy equivalence***

4) **Accumulation of damaging portions.** These depend on cycles-linked shrinking of the static failure surface. In-situ-effect consideration by deformation-controlled testing that captures the embedding (in-situ) effects

5) **No 'mean stress correction' necessary? Probably**

To be further done:

Deeper investigation of the novel idea and of probable additional damaging caused by mode changes (FF, IFF, mixed).

General Conclusions on lifetime prediction models and Outlook

- **Generally applicable, practical lifetime prediction models are not available**
- **For UD-materials the model situation is promising**
- **For ‘higher‘ textiles the model situation is not satisfying**
- **The implementation of available models into Software is in progress.**

Conclusions on Cuntze‘s FMC-based Static UD Strength Fail. Conditions

Lessons Learnt ***from the WWFEs:***

1. **General Prediction is not possible with Basic Strength data only, if physically necessary friction values must be considered** (for shear fracture prediction of brittle behaving materials: consideration of friction is mandatory).

2. **Global SFCs do not directly consider friction; therefore have shortcomings.**
3. **Validation of failure conditions requires a uniform stress field in the critical domain.** This was not always given for the WWFE test cases.

4. **2D stress case: Test data mapping was successful, validation achieved**
5. **3D stress case: Successful, if reliable 3D test data were available.** Unfortunately, this was just partly the case.
6. **The FMC delivers a combined formulation of independent delivers a combined formulation of** ***independent modal failure modes*****, without the well-known drawbacks of global SFC formulations** (which *mathematically combine in-dependent failure modes)*
7. **The FMC-based 3D UD Strength Failure Conditions are simple but describe physics of each single failure mechanism pretty well.**

Keep in mind !

All is difficult prior to becoming simple!

[Moslik Saadi]

Literature

[Cun04] Cuntze R.: *The Predictive Capability of Failure Mode Concept-based Strength Criteria for Multidirectional Laminates*. WWFE-I, Part B, Comp. Science and Technology 64 (2004), 487-516

[Cun12] Cuntze R.: *The predictive capability of Failure Mode Concept-based Strength Conditions for Laminates composed of UD Laminas under Static Tri-axial Stress States. - Part A of the WWFE-II*. Journal of Composite Materials 46 (2012), 2563-2594

[Cun13] Cuntze R.: *Comparison between Experimental and Theoretical Results using Cuntze's 'Failure Mode Concept' model for Composites under Triaxial Loadings - Part B of the WWFE-II*. Journal of Composite Materials, Vol.47 (2013), 893-924

[Cun13b] Cuntze R.: *Fatigue of endless fiber-reinforced composites*. 40. Tagung DVM-Arbeitskreis Betriebsfestigkeit, Herzogenaurach, 8. und 9. Oktober 2013, conference handbook

[Cun14] Cuntze R.: *The WWFEs I and II for UD-materials – valuable attempts to validate failure theories on basis of more or less applicable test data.* SSME 2014 Braunschweig, 1-3 April, conference handbook

[Deg01] Degrieck J. and van Paepegem W.V.: *Fatigue Damage of Fiber-reinforced Composite Materials – Review*. Appl. Mech. Rev., Vol .54 (2001), No 4, 279-300

[Hwa86] Hwang W. and Han K.S.: *Fatigue of composites – fatigue modulus concept and life prediction*. J. Compos. Mater. 1986; 20, 54-65

[Kaw04] Kawai M.: *A phenomenological model for for off-axis fatigue behaviour of uni-directional polymer matrix composites under different stress ratios*. Composites Part A 35 (2004), 955-963

[Sho06] Shokrieh M.M. and Tahery-Behroz F.: *A unified fatigue model based on energy method.* Composite Structures 75 (2006), 444-450

[VDI2014] VDI 2014: German Guideline, Sheet 3 *"Development of Fiber-Reinforced Plastic Components, Analysis".* Beuth Verlag, 2006. (in German and English).

Literature may be downloaded from http://www.carbon-composites.eu/leistungsspektrum/fachinformationen/fachinformation-2

ANNEX

Which property value is mandatory as Input in Structural Analysis ?

- The best prediction of the <u>typical</u> behaviour of the structure is performed with typical values = avarage values

- In the design verification – *dependent on the requirements* - the average, the upper or the lower value of the property is used.

Keep in mind:

Be similarly certain/reliable in the design with applied equations, properties, etc. !!

Strength Failure Conditions are for homogeneous materials

Prediction of ***Onset of Yielding*** + ***Onset of Fracture*** for non-cracked materials

Assessment of multi-axial stress states in a critical material location,

by **utilizing the uniaxial strength values R** and an **equivalent stress σ_{eq}, representing a distinct actual multi-axial stress state.**

for *** dense & porous,**

*** ductile & brittle behaving materials,**

ductile : $R_{p0.2} \cong R_{c0.2}$ (Mises) brittle, dense : $R_m^{\ c} \geq 3R_m^{\ t}; R_{c0.2} > R_{p0.2}$

for *** isotropic material**

*** transversally-isotropic material (UD := uni-directional material)**

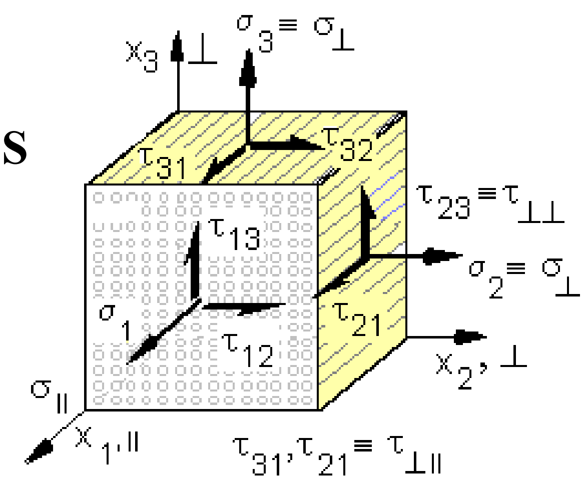
*** rhombically-anisotropic material (fabrics) + ‘higher‘ textiles etc.**

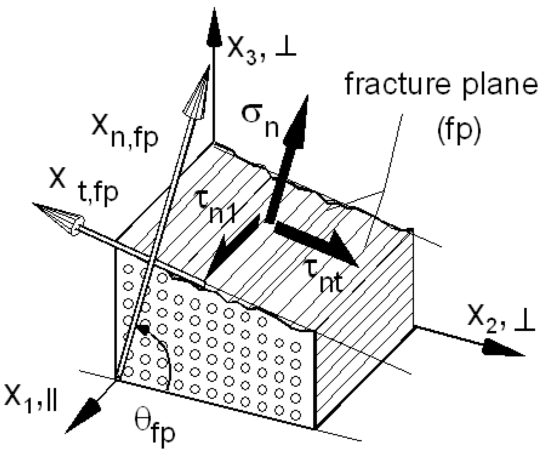
Shall allow for inserting stresses from the utilized various coordinate systems into stress-formulated failure conditions, -and if possible- invariant-based.

Transversely-Isotropic Material (UD composite): Stresses & Invariants

lamina +
'principal' COS

Transformation into the quasi-isotropic plane





Mohr, Puck, Hashin: Fracture is determined by the (Mohr) stresses in the fracture plane !

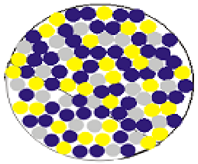
$\{\sigma\}^{quasi-isotropic\ plane}_{principal} =$ $= (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$	$\{\sigma\}_{lamina} =$ $= (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$	$\{\sigma\}_{Mohr} =$ $(\sigma_\ell,\ \sigma_n,\ \sigma_t,\ \tau_{nt},\ \tau_{t\ell},\ \tau_{\ell n})^T$
$I_1 = \sigma_1$, $I_2 = \sigma_2^p + \sigma_3^p$ $I_3 = \tau_{31}^{p\ 2} + \tau_{21}^{p\ 2}$	$I_1 = \sigma_1$, $I_2 = \sigma_2 + \sigma_3$ $I_3 = \tau_{31}^2 + \tau_{21}^2$ **'UD invariants'!** [Boehler]	$I_1 = \sigma_1$, $I_2 = \sigma_n + \sigma_t$ $I_3 = \tau_{t\ell}^2 + \tau_{n\ell}^2$
$I_4 = (\sigma_2^p - \sigma_3^p)^2 + 0$ $I_5 = (\sigma_2^p - \sigma_3^p)(\tau_{31}^{p\ 2} - \tau_{21}^{p\ 2}) + 0$	$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2$ $I_5 = (\sigma_2 - \sigma_3)(\tau_{31}^2 - \tau_{21}^2) - 4\tau_{23}\tau_{31}\tau_{21}$	$I_4 = (\sigma_n - \sigma_t)^2 + 4\tau_{nt}^2$ $I_5 = (\sigma_n - \sigma_t)(\tau_{t\ell}^2 - \tau_{n\ell}^2) - 4\tau_{nt}\tau_{t\ell}\tau_{n\ell}$

Invariant := Combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system. Good for an optimum formulation of *scalar Failure Conditions*

Variety of possible Composites Types

Filaments: glass, aramide, carbon, ceramics, .. (short, long fibers ⇨ endless fibers)

Fiber preforms **(+ sizing) from** ***roving, tape, weave, braid, knit, stitch***
dry or wet ***(2D and 3D),*** **or mixed as in a** ***preform hybrid***
non-crimp fabric laminates



PP/glass/aramidePEEK/ glass –filament-yarn

Matrices **(resin + hardener): polymers, thermoplastics, ceramics, concrete, ..**

Polymers (crystalline and amorphous)		
Plastics		Elastomers
thermo-plastics	thermo-sets	
Acrylic, polycarbonat, polyimide, polypropylene	epoxy, phenolic, polyurethane, silicon	natural rubber, polyurethan, thermoplastic elastomer

⇦ *polymers are also bonding materials (3D understanding)*

Manufacturing processes : pre-pregging, wet winding, RTM, fiber placement, ..

Rovings: 2k through > 48 k

… and - coming up more and more – an increasing variety of 2D- and 3D-fabrics

Analyses needs Provided Properties and Manufacturing Process Information

Analytical, semi-analytical and numerical procedures for

- **Process-Simulation (CAD, FEM, CFD, etc.)**

  (draping, flow front, fusion weld, fiber orientation, curing, Tg value, curing stresses etc.) and the intensively linked

- **Structure-Analysis (FEM, BEM, pre- and post-processing)**

Thereby, epistemic Uncertainties to achieve a Robust Design must be tackled:

- Certification must focus an uncertainty quantification.
- Reduction of the Coefficient of Variation is of higher importance than increasing the average value a bit
- Design to Imperfections in manufacturing
- Provide ease-of-use and ease-of-interpretation of the results.

Aleatoric Uncertainty: play at dice (Würfel), number by chance, cannot be influenced !

Epistemic Uncertainty: reduced knowledge from too few tests etc.

Comments

- Properties are ‘agreed’ values to achieve a common and *comparable* design basis
- Properties must be provided with average value and coefficient of variation
- Changing a certified material is economically seldom possible
- Sources of uncertainty should be investigated
- Model parameters should be measurable and physically self-explaining
- Variety of Composites: Many properties for design and manufacturing not yet available
- For brittle behaving materials, multi-axial stress assessment is not possible on basis of the uni-axial strength values alone. Knowledge of material internal friction values, *following Mohr-Coulomb*, is mandatory
- Theory ‘only’ creates a model of the reality, an Experiment is ‘just’ one realisation of the reality.
  Experimental results can be far away from reality like an inaccurate theoretical model.
  Therefore, put sufficient effort into both, analysis and test,
  to achieve the desired FIDELITY.

DANKE

Which is the Work-Flow of a Fatigue Lifetime Prediction?

1 Input

- **Operational loadings:** Load-time curves (modeling by rain flow, ..)
- **Safety concept:** Design to Life $j_{Life} = 3 – 4$, inspection interval

Consideration of the operational (service) loading:

- **Time domain:** Cycle-by-cycle or collective-by-collective (less computational effort)
- **Frequency domain:** Load spectra (loss of load sequence) or block loadings, etc

2 Transfer of operational loading into stresses by a Structural Analysis

3 Output for several S-N regions

- Low Cycle Fatigue LCF: high stressing,
- High Cycle Fatigue HCF: intermediate stressing
- Very High Cycle Fatigue VHCF: low stressing and strains

(DFG Research Program SPP1466, started 2010).

Drivers of damaging within materials

• **Ductile material behaviour (example: isotropic metal)**

1 Mechanism = "shear stress sliding"

occurs under all cyclic loadings under:

tensile stresses, compressive stresses, shear and torsion stresses !

Therefore this single mechanismus 'shear stress sliding' can be described by a 1 (yield) failure condition !

•**Brittle material behaviour , isotropic material**

2 Damaging creating Mechanisms (normal fracture and shear fracture types)

•**Brittle material behaviour, UD- material** ***5***

Damaging creating Mechanisms.

Assumptions for Material Modelling (<u>example</u>: UD material) and **Test**

- **The UD-lamina is macroscopically homogeneous.**
  It can be treated as a homogenized (‘smeared‘ material)
- **The UD-lamina is transversely-isotropic.**
  On planes, parallel with the fiber direction it behaves orthotropically and on planes transverse to fiber direction isotropically (quasi-isotropic plane)
- **Uniform stress state about the critical stress ‘point‘** (location)
- **Pore-free material, specimen surfaces polished, well sealed** (WWFE-II) **, fiber volume is constant, tube specimens show no warping and do not bulge, perfect bonding, no layer waviness, edge effects do not exist, …**
- **From engineerring point of view Macro-mechanical SFCs are desired.**
  However, the SFCs should consider that failure starts in constituents

Failure Analysis Flow Path (multi-level 2-scale approach)

isotropic or a quasi-isotropic continuum

Constituent materials: fiber, matrix, interphase of the interface

build up a **layered continuum**

failure usually occurs at micro-level

Isotropic Matrix

Micro-mechanical (scale) Model

Transversely-isotropic Lamina (ply)

calibration by macromechanical properties *elasticity, strength*

Meso-Model (RVEs, Voxel)

maufacturing 'significances'
flaws, waviness, matrix-nests,

calibration by macromechanical properties

Ply-by-Ply analysis of the Laminate

merely non-degraded material **strain-hardening**

degrading material **strain-softening + non-linear analysis beyond IFF**

Failure Conditions for the Homogenized Solid = Material

Strength Failure Conditions for the Material

validation of strength failure conditions by strength test series on material level

Macro-mechanical (scale) Model

Orthotropic Laminate

Fracture Mechanics Failure Conditions for the Structural Part

final failure: FF in an lamina or delamination in a laminate

arbitray crack in an isotropic or a quasi-isotropic continuum

delamination crack in a layered continuum (laminate)

Meso-level is no scale, per definitionem !

RVE:Representative Volume Element, voxel : volumetric pixel

Interaction of Single Strength Failure Modes in the modal FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots.} \quad = 1 = 100\%, \text{ if failure}$$

with mode-interaction exponent *2.5 < m < 3 from mapping experience*

***as modal* material stressing effort * (in German Werkstoffanstrengung)**

and

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / \bar{R}^{\text{mode}}$$

equivalent mode stress

mode associated average strength

later example

*** artificial technical term created together with QinetiQ**

Cuntze's Pre-design Input for 3D UD SFCs

Test Data Mapping | **Design Verification**

- **5 strengths :** $\{\bar{R}\} = (\bar{R}_{\parallel}^{t}, \bar{R}_{\parallel}^{c}, \bar{R}_{\perp}^{t}, \bar{R}_{\perp}^{c}, \bar{R}_{\perp\parallel})^{T}$ $\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^{T}$

  average (typical) values | **strength design allowables**

- **2 friction values :** **for 2D** $\mu_{\perp\parallel}$ **, for 3D** $\mu_{\perp\parallel}, \ \mu_{\perp\perp}$

  $\mu_{\perp\parallel} = 0.1$ $\mu_{\perp\perp} = 0.1$

- **1 mode-interaction exponent :** ***m = 2.6 .***

values, recommended for pre-design

Material Symmetry Requirements *(helpful, when generating SFCs)*

1 If a material element can be homogenized to an ideal (= frictionless) crystal,

then, material symmetry demands for the transversely-isotropic UD-material

- 5 *elastic 'constants' , 5 strengths, 5 fracture toughnesses* **and**

- *2 physical parameters (such as CTE, CME, material friction, etc.)*

(for isotropic materials the respective numbers are 2 and 1)

2 Mohr-Coulomb requires for the real crystal another inherent parameter,

- the *physical parameter 'material friction' : UD* $\mu_{\perp\parallel}$, ;$\mu_{\perp\perp}$ **, *Isotropic*** μ

3 Fracture morphology witnesses:

- Each strength corresponds to a distinct *failure mode*

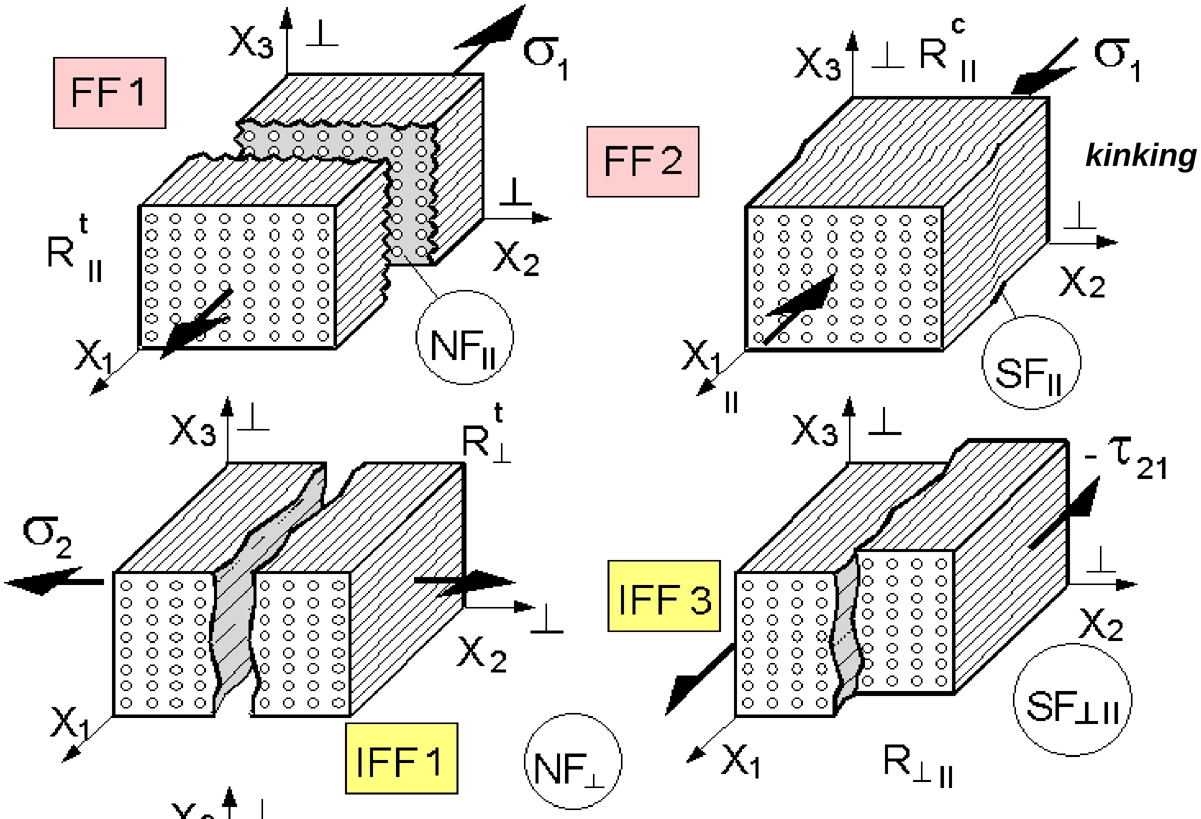
and to a *fracture type* as Normal Fracture (NF) or Shear Fracture (SF).

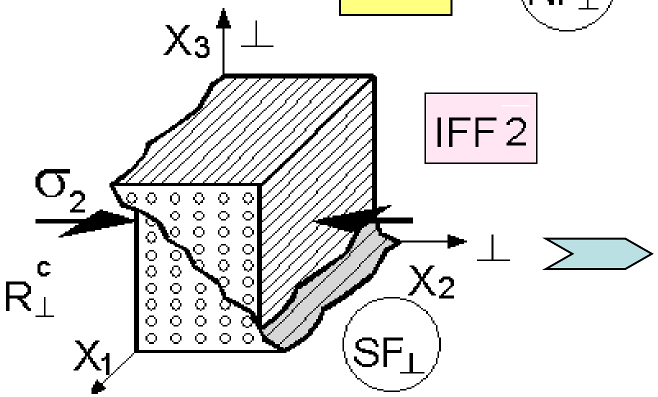
Above Facts and Knowledge gave reason

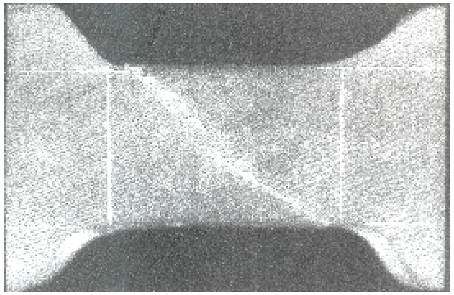
why the FMC strictly employs single *independent* failure modes

by its failure mode–wise concept.

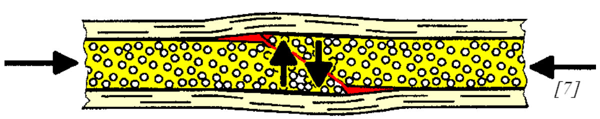
Observed Strength Failure Modes with Strengths of brittle UD Materials







wedge failure type



t = tension

c = compression

- **5 Fracture modes exist**

  = 2 FF (Fibre Failure)

  + 3 IFF (Inter Fibre Failure)

Fracture Types:

NF := Normal Fracture

SF := Shear Fracture

Mechanisms of Interest when considering Property Measurement

Yielding versus quasi-yielding:
In ductile behaving materials the failure mechanism *yielding* is active for the loadings tension, compression and shear whereas in case of brittle behaving composites the diffuse damaging as *quasi-yielding* belongs to different macroscopic failure mechanisms in tension (NF) and shear (SF)..

Diffuse Damaging:
damaging, occurring fro onset of micro-cracking until onset of discrete local macro-cracks, often indicated by whitening (for ductile thermoplastics it is connected to void intiation and void growth)

Discrete Damaging:
localization of diffuse damaging which sometimes ends with CDS (characteristic damage state)

Micro-mechanical 'notching':
- onset of micro-cracks degrade the matrix in a transversely stressed lamina the more the thicker the lamina is ('thin-layer effect' ; energy release rate becomes larger)
- onset of filament breaks causes 3D stress states resulting in growth of lateral micro-cracks and lamina-parallel micro-delaminations (more critical in general)

Failure Theory and Failure Conditions

A **3D Failure Theory** has to include:

1. Failure Conditions to ***assess multi-axial states of stress***
2. Non-linear Stress-strain Curves of a material as input
3. Non-linear Coding for structural analysis

A Failure Condition is the mathematical formulation of the failure surface !

Pre-requisites for the establishment of failure conditions are:

- **simply formulated, numerically robust,**
- **physically-based, and therefore, need only few information for pre-dimensioning**
- **shall allow for a simple determination of the design driving reserve factor.**

Fundamentals of the FMC (*example*: UD material)

Remember:

- **Each of the observed fracture failure modes was linked to one strength**
- **Symmetry of a material showed :** ***Number of strengths =*** $R_{||}^{t}, R_{||}^{c}, R_{\perp||}, R_{\perp}^{t}, R_{\perp}^{c}$

  number of elasticity properties ! $E_{\|}, E_{\perp}, G_{\|\perp}, \nu_{\perp\|}, \nu_{\perp\perp}$

Due to the facts above the

FMC postulates in its *'Phenomenological Engineering Approach'* :

► Number of failure modes = number of strengths, too !

e.g.: isotropic = 2 or above transversely-isotropic (UD) = 5

t:= tensile, c: = compression, || : = parallel to fibre, := transversal to fibre

Interaction of Single Strength Failure Modes in the FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* $Eff^{\,\mathrm{mode}}$

$$Eff = \sqrt[m]{(Eff^{\,\mathrm{mode}\ 1})^{m} + (Eff^{\,\mathrm{mode}\ 2})^{m} + \ldots.} \quad \textbf{= 1 = 100\%, if failure}$$

with mode-interaction exponent *m* *from mapping experience*

and

$$Eff^{\,\mathrm{mode}} = \sigma_{eq}^{\,\mathrm{mode}} / \overline{R}^{\,\mathrm{mode}}$$

***modal* material stressing effort**

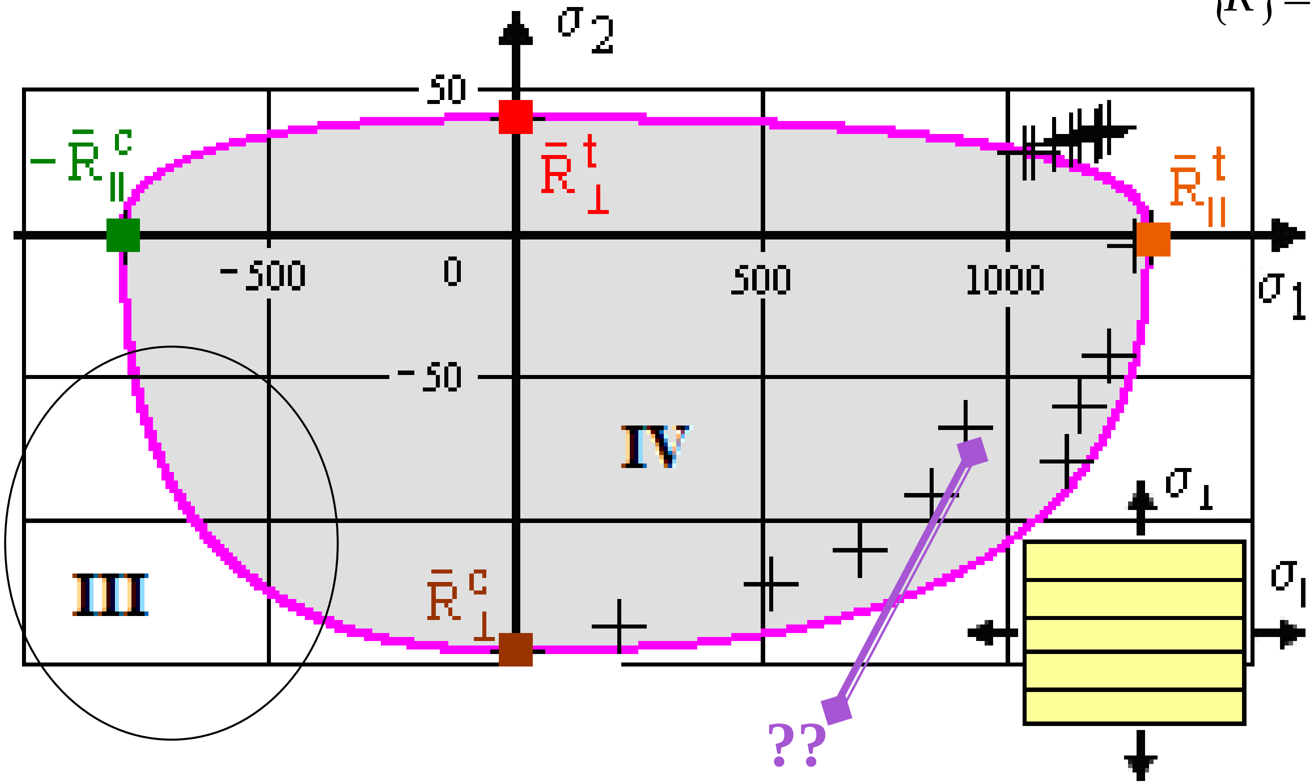
(Werkstoffanstrengung)

equivalent mode stress

mode associated average strength

Test Case 3, WWFE-I $\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$



Hoop wound tube UD-lamina. E-glass/MY750epoxy +

$\sigma_1 = \sigma_{hoop}$

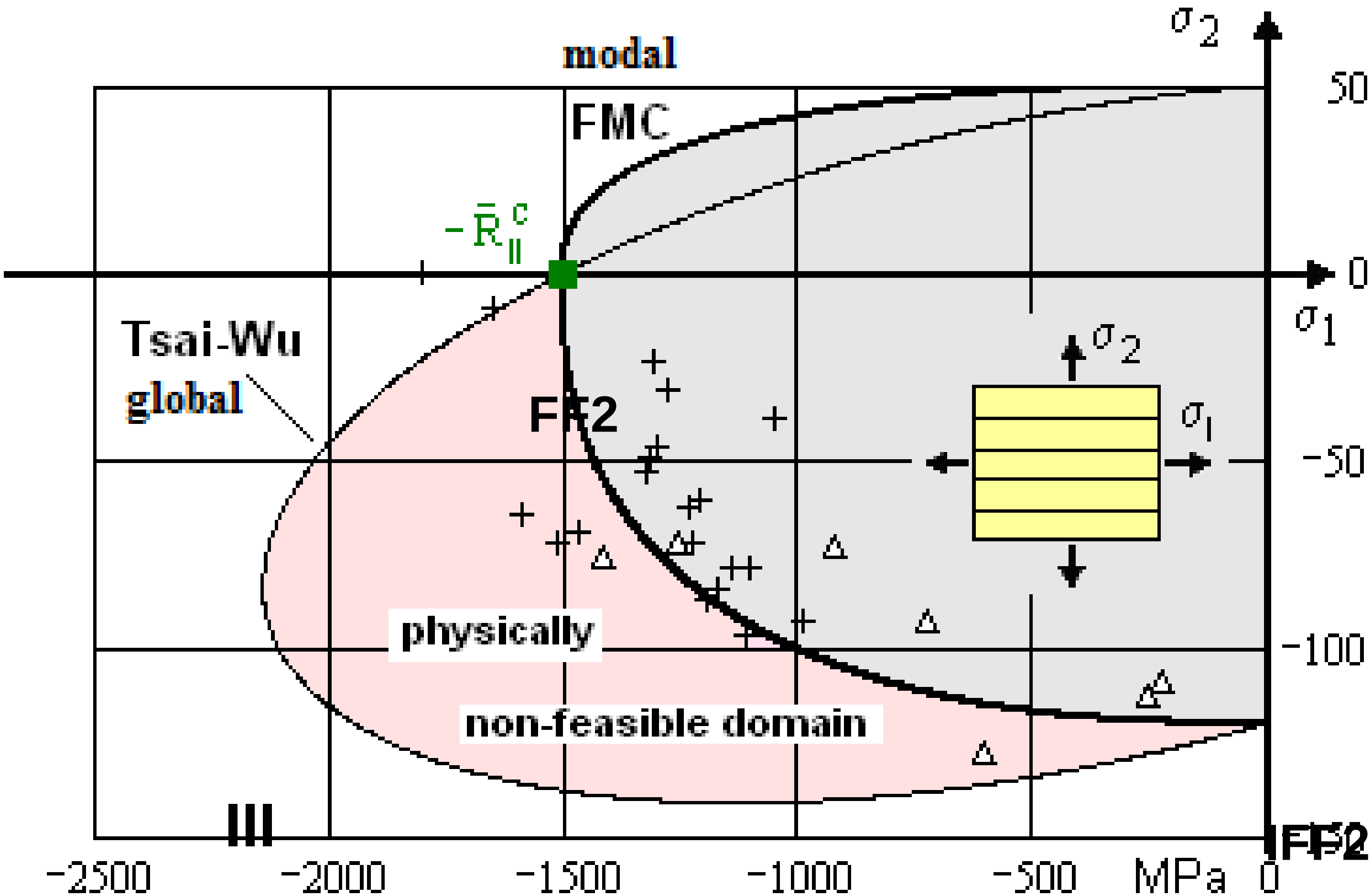
$\sigma_2 = \sigma_{axial}$

Part A: Data of strength points were provided, only

Part B: Test data in quadrant IV show discrepancy , testing?

No data for quadrants II, III was provided ! But, ..

Mapping in the 'Tsai-Wu non-feasible domain' (quadrant III)



$\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

Data: courtesy IKV Aachen, Knops

Lesson Learnt: The modal FMC maps correctly, the *global* Tsai-Wu formulation predicts a non-feasible domain !

Self-explaining Notations for Strength Properties (homogenised material) neu !!!!

		Fracture Strength Properties									
	loading	**tension**			**compression**			**shear**			***required by material symmetry***
	direction or **plane**	1	2	3	1	2	3	12	23	13	
9	**general orthotropic**	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	**comments**
5	**UD, ≅ non-crimp fabrics**	$R_{\parallel}^{t}$ NF	$R_{\perp}^{t}$ NF	$R_{\perp}^{t}$ NF	$R_{\parallel}^{c}$ SF	$R_{\perp}^{c}$ SF	$R_{\perp}^{c}$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$R_{\perp\perp} = R_{\perp}^{t} / \sqrt{2}$ (compare Puck's modelling)
6	**fabrics**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	**fabrics general**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp ≠ Fill*
5	**mat**	R_{1M}^t	R_{1M}^t	R_{3M}^t	R_M^c	R_{1M}^c	R_{3M}^c	R_M^τ	R_M^τ	R_M^τ	$R_M^\tau (R_M^t)$
2	**isotropic**	R_m SF	R_m SF	R_m SF	*deformation-limited*			R_M^τ	R_M^τ	R_M^τ	*ductile, dense* $R_M^\tau = R_m / \sqrt{2}$
		R_m NF	R_m NF	R_m NF	R_m^c SF	R_m^c SF	R_m^c SF	R_m^σ NF	R_m^σ NF	R_m^σ NF	*brittle, dense* $R_M^\sigma = R_m^t / \sqrt{2}$

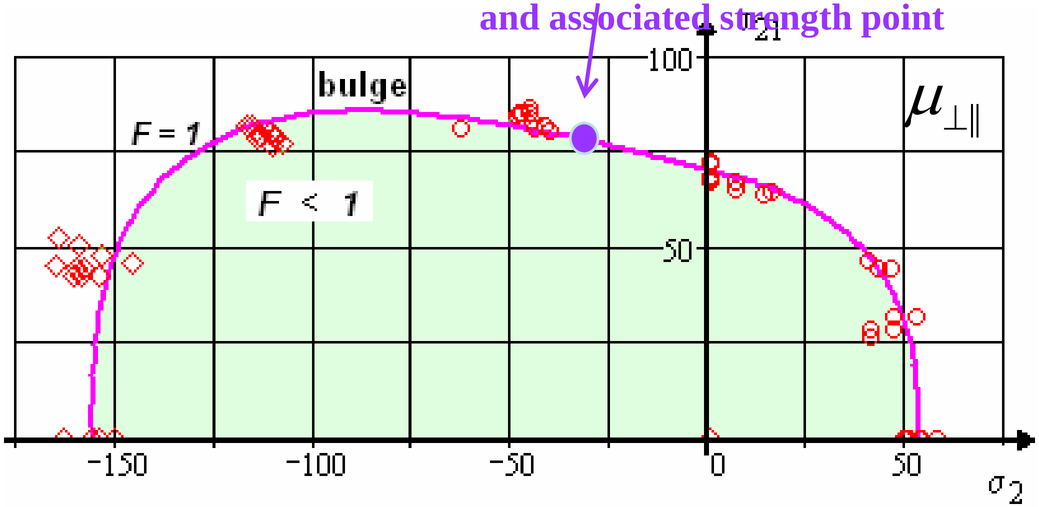
NOTE: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m := 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

Guess of Friction Values from slopes (bi-axial test points) $\mu_{\perp\|}$, $\mu_{\perp\perp}$

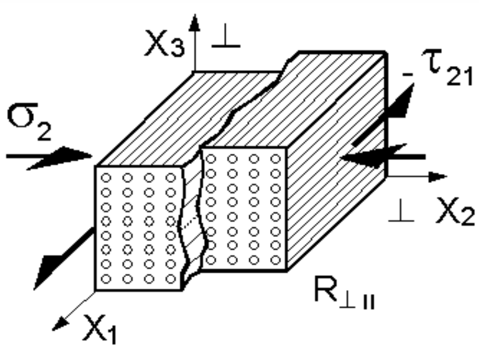
$$|\tau_{21}| = \bar{R}_{\perp\|} - \mu_{\perp\|} \cdot \sigma_2$$

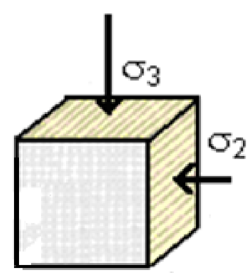
Estimation:

Straight line through magenta point and associated strength point

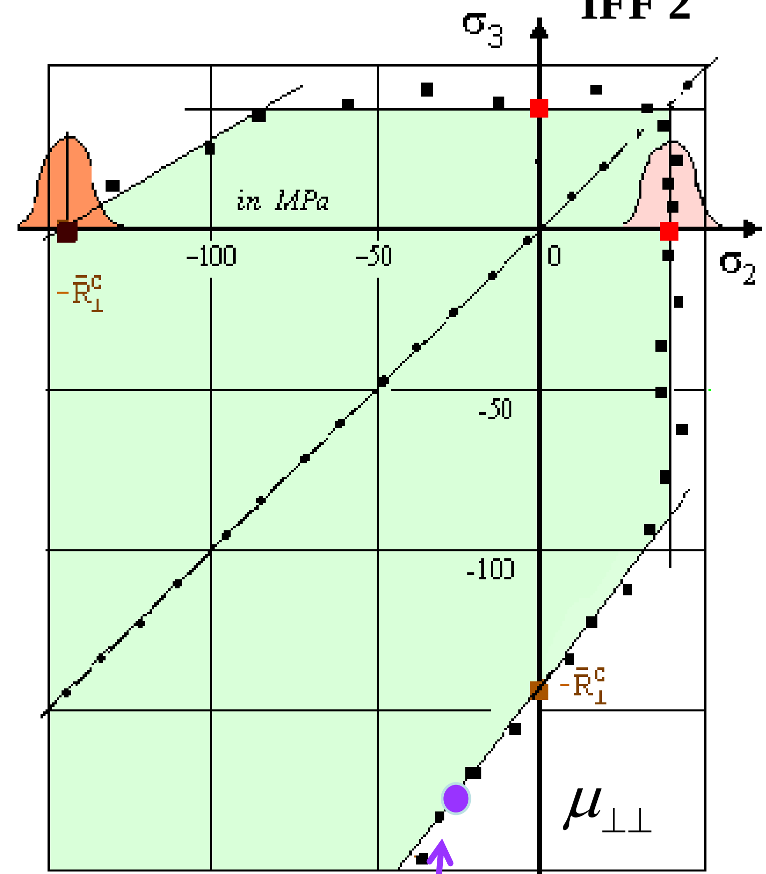


IFF 3





IFF 2



1. **Fitting of course of test data (min error square) in ‘pure‘ failure mode domains**
2. **Estimation with one strength value and one multi-axial failure stress point**
3. **For $\mu_{\perp\perp}$ in addition : derivation from fracture plane measurements possible.**

minimum error square

Verification Levels of the Structural Part with

- **Local Stress at a critical material 'point': continuumsmechanics, strength criteria**

  verification by a *basic strength* or a *multi-axial failure stress state*

  Applied stresses are local stresses

- **Stress concentration at a notch (stress peak at a joint): notch mechanics**

  verification by a *notch strength* *(usually Neuber-like, Nuismer, etc..)*

  'Far'-field stresses are acting and are not directly used in the notch strength analysis

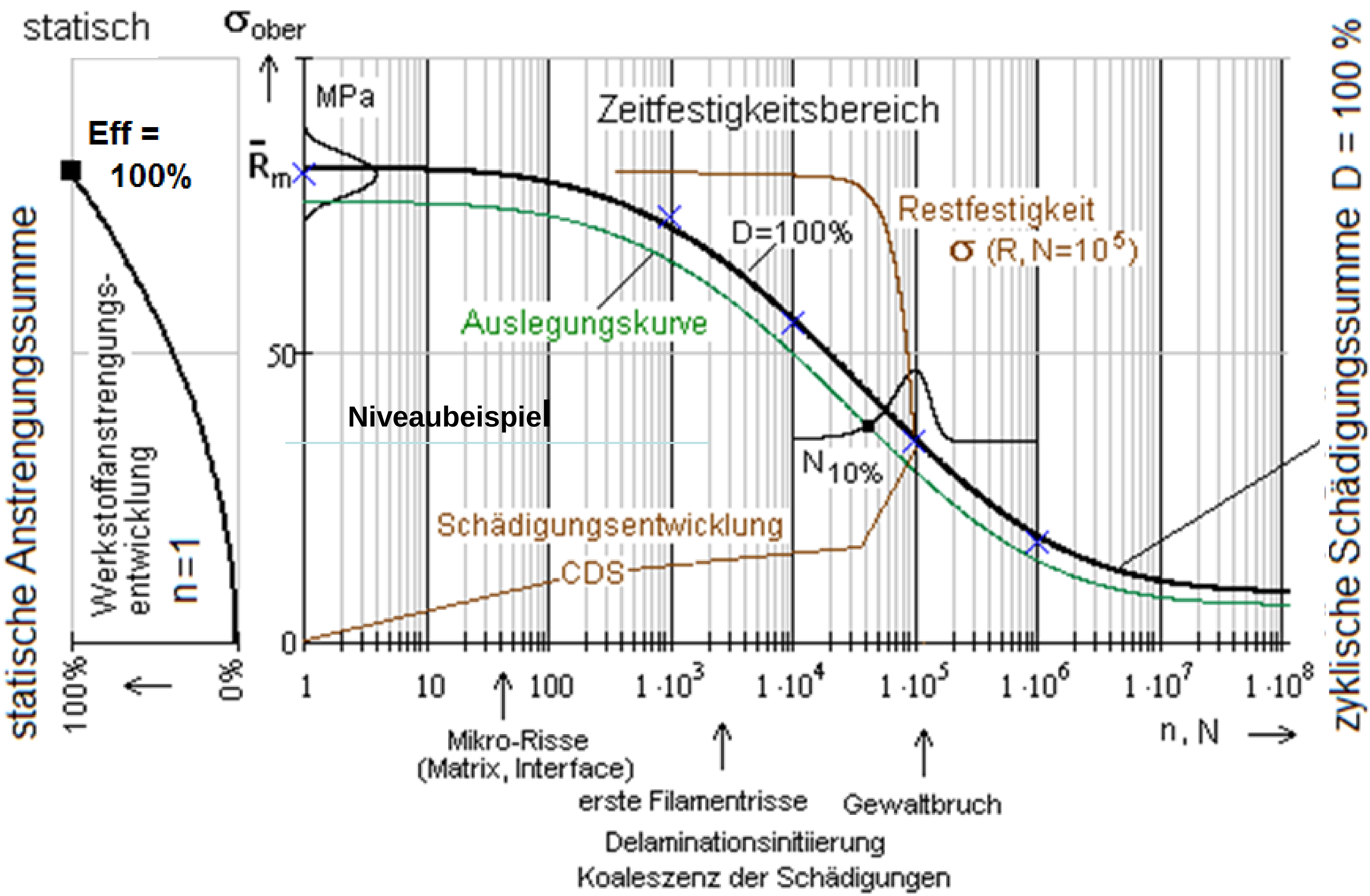
- **Stress intensity (delamination = crack): fracture mechanics**

  verification by a *fracture toughness (energy –related)*

  ***Applied stresses are 'far'-field stresses.*(far from the crack-tip)**

is valid, statically and cyclic.

Statische und zyklische Schädigungsentwicklung, sowie Wöhlerkurve *R = 0*



CDS:= characteristic damage state at the end of diffuse damaging

Cyclic fatigue life consists of three phases:

Phase I: Increasing damaging in embedded Lamin<u>as</u> up to discrete damage onset
(determination of accumulating damaging portions (= Schädigungen), initiated at end of elastic domain and dominated by diffuse micro-cracking + matrix yielding, and finally micro-delaminations)

Phase II: Stabile local growth of discrete damaging in Lamin<u>ate</u> up to delamination
(growth of dominating discrete micro-crack widths incl. micro-delaminations)

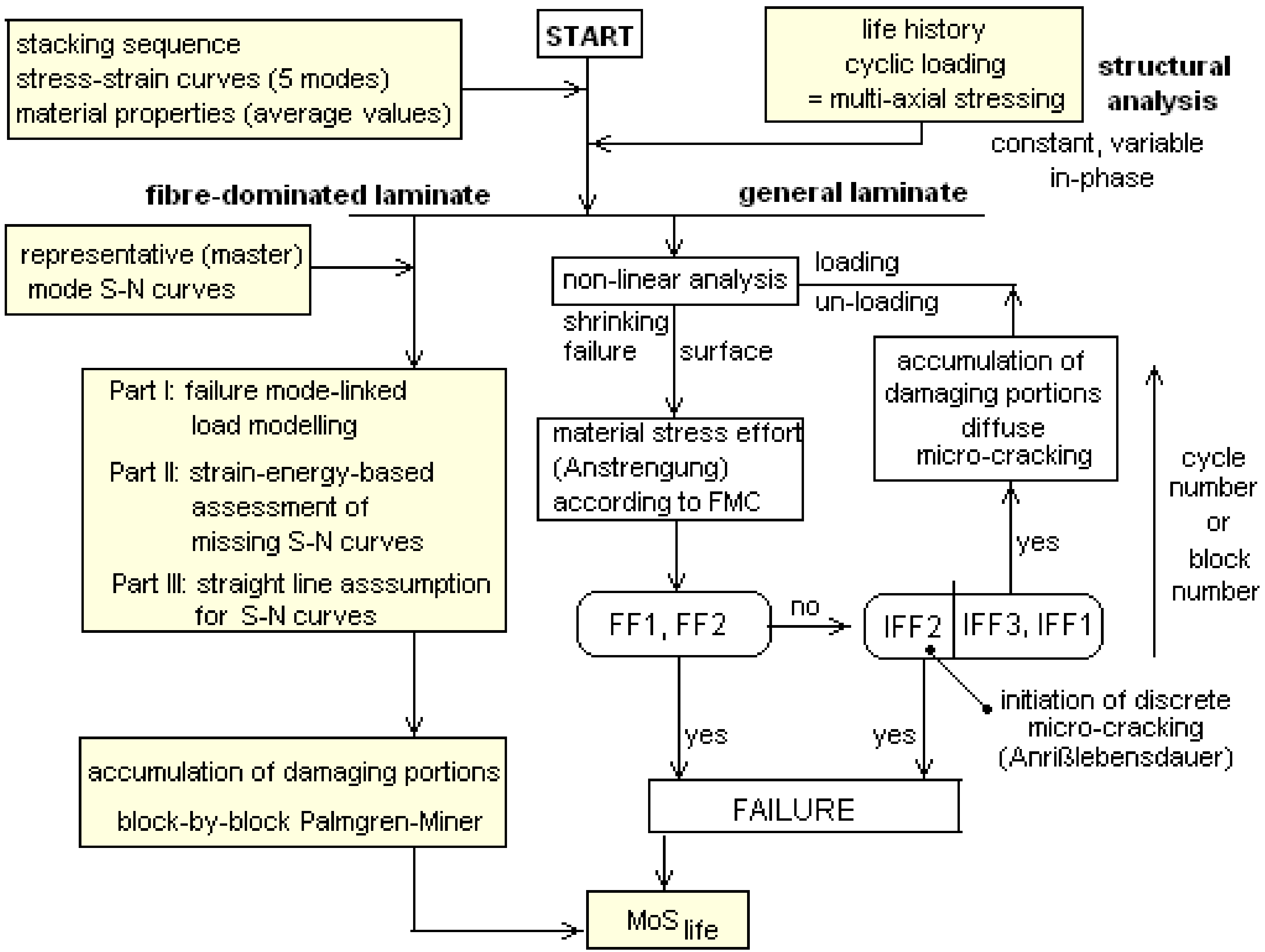
Phase III: Final in-stabile fracture of Lamin<u>ate</u> initiated by FFs, IFF2 of any lamina
+ possible delamination (= Schaden) criticality of the loaded laminate

FF:= fibre failure. IFF:= Inter Fibre Failure

CDS:= characteristic damage state at the end of diffuse damaging

- ***Determination of damaging portions (from diffuse and later discrete damaging)***
- ***Accumulation of damaging portions (cycle-wise, block-wise, or otherwise ?)***

Failure-Mode-Concept-based Lifetime Prediction



Failure mode-based Lifetime Prediction Method

Approach incl. Accumulation of Damaging Portions

Logic behind: Fatigue strain energy, required to generate a distinct damage state is equal to the strain energy, which is necessary under monotonic loading to obtain the same damage state.

$$\Delta W = \sum_1^5 \Delta W^{\text{modes}}$$

strain energy of all mode contributions (5 in the UD case)

Idea demonstrated for simple case of 'well-designed, laminates under tension, where the change of strain energy between maximum and minimum loading for FF1 reads:

$$\Delta W^{\|\sigma} = \Delta(\sigma_{eq}^{\|\sigma} / \overline{R}_{\|}^{t})^2 \Rightarrow \quad \Delta W^{\|\sigma} \cdot \overline{R}_{\|}^{t\,2} = \sigma_{1,\max}{}^2 - \sigma_{1.\min}{}^2 = \sigma_{1,\max}{}^2 \cdot (1 - R^2)$$

Solving for the maximum stress delivers:

$$\sigma_{1,\max}(n) = \overline{R}_{\|}^{t} \cdot \sqrt{\Delta W^{\|\sigma}(n) / /(1 - R^2)}.$$

From experiment known:

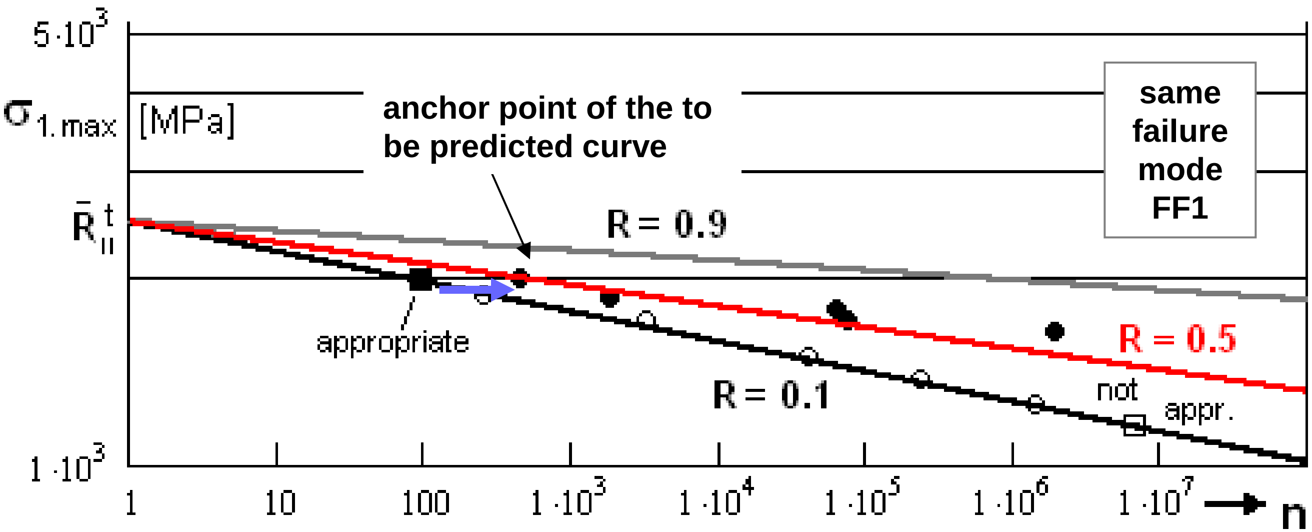
- Max stress + tensile strength + stress ratio *R;* and thereby the *fatigue strain energy*.

- Course of strain energy can be described by a simple power law function, forming a straight line in a log-log diagram:

$$\Delta W^{\|\,\sigma}(n) = c_1 \cdot n^{-c_2}$$ **[Hwang] .**

Failure mode-based Lifetime Prediction Method

Procedure for the Prediction of S-N curves (test-based Example)



to predict

given

Given:

Test points +

$$\Delta W^{\parallel\sigma}_{R=0.1}(n) = c_1 \cdot n^{-c_2} = 0.89 \cdot n^{-0.097}$$

$$n_{appr} = 100 cycles$$

·

III : S-N curve may be mapped by a straight line in a log-log graph (safe side)

Given: normalized mode-representative curve (R = 0.1); to be predicted curve: (R > 0.1)

$$\sigma_{1,\max\ repr}(n) = \bar{R}^t_\parallel \cdot \sqrt{\frac{\Delta W^{\parallel\sigma}_{R=0.1}}{1-R_{repr}^{\ 2}}} = \bar{R}^t_\parallel \cdot \sqrt{\frac{c_1 \cdot n^{-c_2}}{1-R_{repr}^{\ 2}}} = \bar{R}^t_\parallel \cdot n^{c_{repr}(n)} \approx \bar{R}^t_\parallel \cdot n^{c_{repr}},$$

$$\sigma_{1,\max\ pred}(n) \approx \bar{R}^t_\parallel \cdot n^{c_{pred}}$$

Example *R=0.5* : Procedure to determine c_{pred} *(one anchor point needed besides the strength point)* is depicted below:

$$\sigma_{1,\max\ repr}(n_{appr}) = \bar{R}^t_\parallel \cdot \sqrt{\frac{c_1 \cdot n_{appr}^{\ -c_2}}{1-R_{repr}^{\ 2}}} = \sigma_{appr}$$

shift from representative curve to predicted curve →

$$\sigma_{appr} = \bar{R}^t_\parallel \cdot \sqrt{\frac{c_1 \cdot (n_{appr} \cdot f_{pred})^{-c_2}}{1-R_{pred}^{\ 2}}}$$

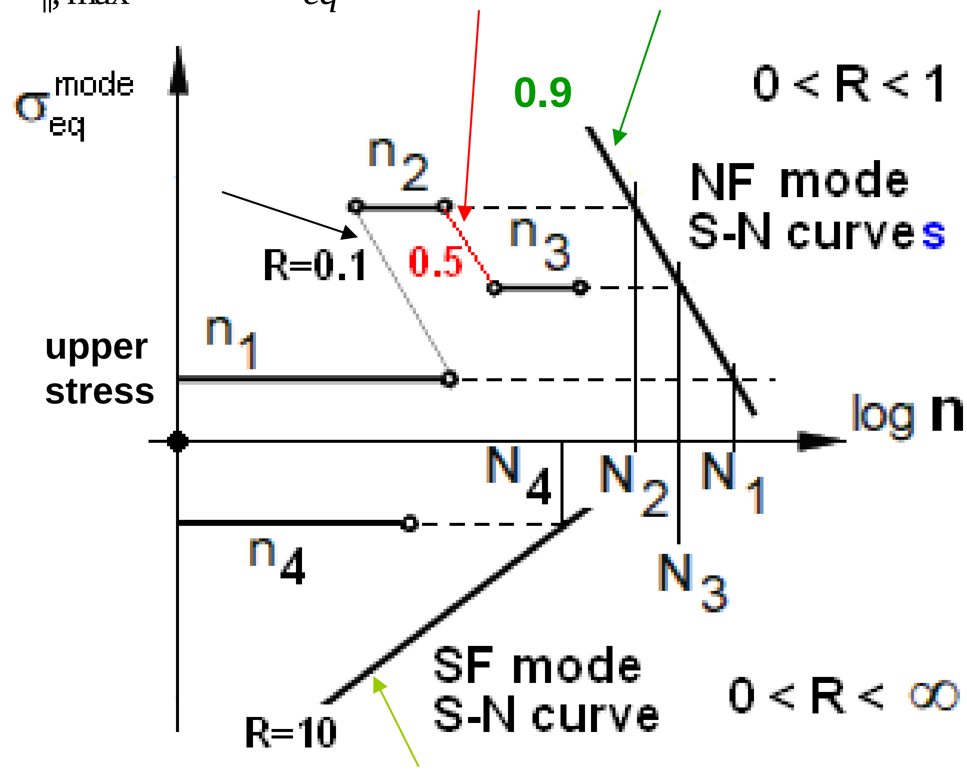
$$c_{pre} = \ln(\bar{R}^t_\parallel / \sigma_{appr}) / \ln(n_{appr} \cdot f_{pre}) = -0.034 \quad \Leftarrow \quad f_{pred} = \exp[-\ln(\frac{R_{pred}^{\ 2}-1}{R_{repr}^{\ 2}-1}) \cdot \frac{1}{c_2}] = 17.5$$

◄ **R = 0.5**

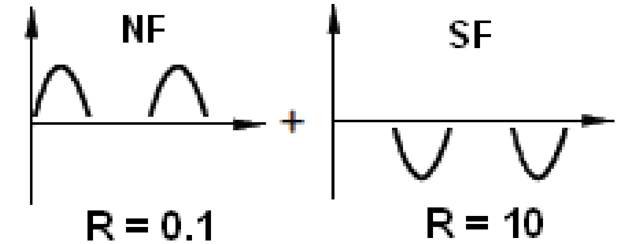
Failure mode-based Lifetime Prediction Method

Schematic Application (principle: for simple isotropic case as example, 4 blocks

$$\sigma_{\parallel,\max}^{mode} \Rightarrow \sigma_{eq}^{mode}$$



remind



here:

2 master curves

NF: R = 0.1

SF: R = 10

2 predicted curves

NF: R = 0.5, 0.9

Miner application:

$$D = n_1 / N_1 + n_2 / N_2 + n_3 / N_3 + n_4 / N_4$$