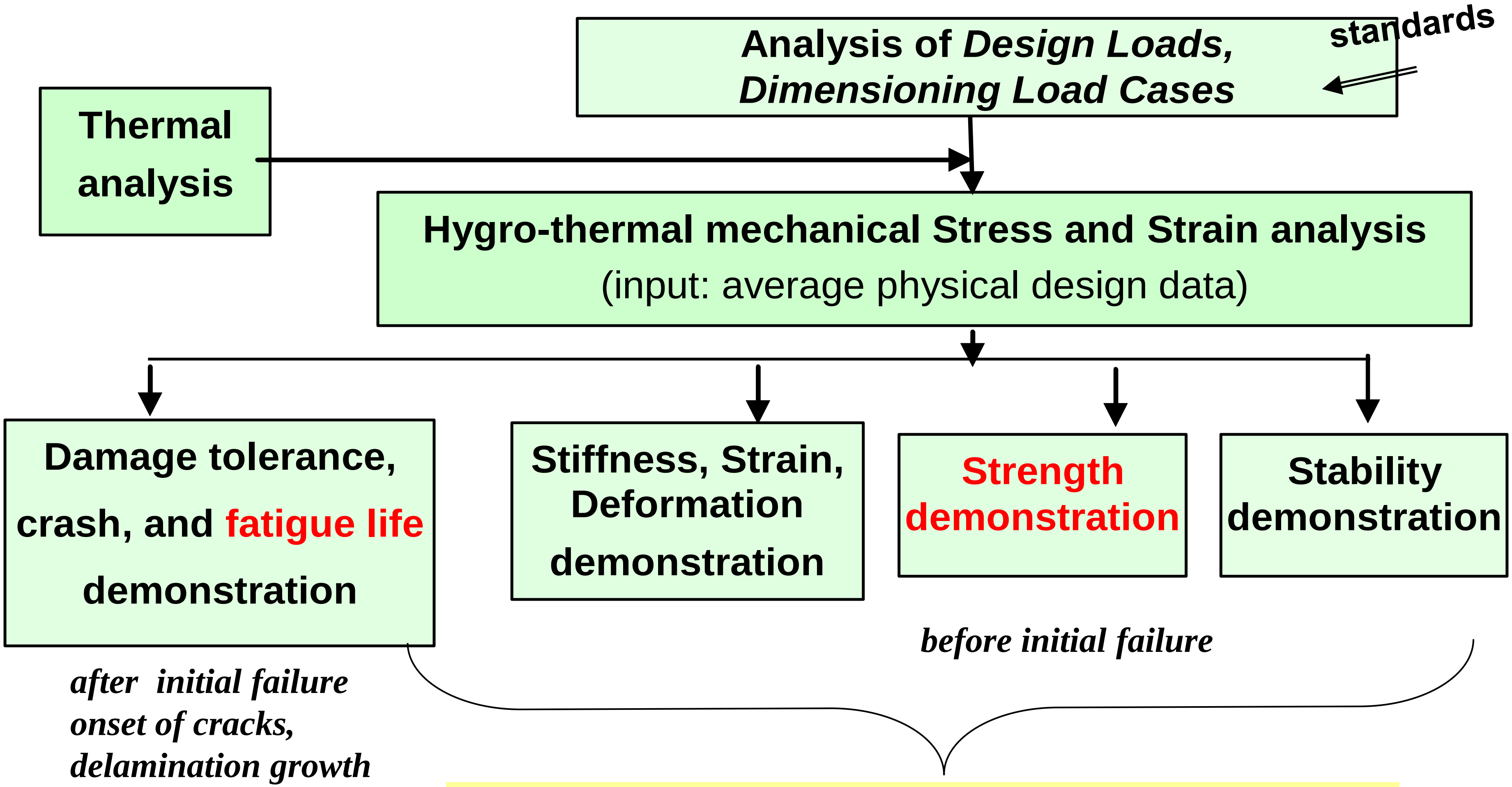
Which Design Verifications are mandatory in Structural Design ?



standards

Analysis of *Design Loads, Dimensioning Load Cases*

Thermal analysis

Hygro-thermal mechanical Stress and Strain analysis
(input: average physical design data)

Damage tolerance, crash, and fatigue life demonstration

Stiffness, Strain, Deformation demonstration

Strength demonstration

Stability demonstration

before initial failure

after initial failure
onset of cracks,
delamination growth

TASKS thereby:

Demonstration of Structural Resistances *by a positive* **Margin of Safety *(MoS)*** *or* ***RF >1,***
to proof ***Design Verification***
for achieving **Structural Integrity**

CONSTRAINTS in Design Development Process : *Cost and Time Reduction*

Industry looks for robust & reliable analysis procedures
in order to replace the expensive ‘Make and Test Method‘
as far as reasonable.

Virtual tests *shall reduce the amount of* physical tests.

In this context:

Structural Design Development

can be only effective and offer high fidelity

if

qualified analysis tools and necessary test data input are available

for Design Dimensioning and for Manufacturing as well.

The presentation plus further literature may be downloaded from http://www.carbon-composites.eu/leistungsspektrum/fachinformationen/fachinformation-2

Some definitions first for a Common Understanding

Material: homogenized (macro-)model of the envisaged complex solid

Failure: structural part does not fulfil its functional requirements such as onset of yielding, brittle fracture, Fiber-Failure FF, Inter-Fiber-Failure IFF, leakage, deformation limit, delamination size limit, frequency bound

= project-fixed Limit State with F = Limit State Function (here: strength failure function)

Failure Criterion: F >=< 1

Failure Condition : $F = 1 = 100\%$

Failure Theory: tool to predict failure of a structural part

Fracture Failure Surface (body): surface of all uni-/multi-axial fracture failure stresses

Strength Failure Condition (SFC): subset of a strength failure theory

tool for the assessment of a

‘multi-axial failure stress state ‘ in a critical location of the material.

→ **Stress states** are judged by **Strengths !**

Static & Fatigue Failure of High-Strength Laminates

- the World-Wide-Failure-Exercise and more

Some Introductory slides

1 State of the Art of Static & Cyclic Failure Conditions (FC)

2 Fundamentals when modelling Static & Cyclic Failure

3 Global Strength FCs versus Modal ones (strength criteria)

4 Cuntze's Failure-Mode-Concept (FMC) *applied to* obtain
Strength Failure Conditions (SFCs) for *UD Materials*

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an example Design Verification by a Static Reserve Factor *RF*

6 Lifetime Prediction Model for a cyclically-loaded UD-CFRP with
a numerical example

Report on a time-consuming, never funded "hobby" of an engineer, retired from industry,
Prof. Dr.-Ing. habil. Ralf Cuntze VDI, now linked to Carbon Composite e.V. (CCeV) Augsburg,

Structural Testing of GROWIAN, GFRP shell, 1980 at IABG

GROWIAN-Flügel

L = 50 m

Dieter Muser

Industrial Requirements to Improve *Designing Composite Parts*

Static loading:

- **Validated 3D strength failure conditions for isotropic (foam), transversely-isotropic UD materials, and orthotropic materials (e.g. textiles) to determine ‘Onset of fracture‘ and ‘Final fracture‘**
- **Standardisation of material test procedures, test specimens, test rigs, and test data evaluation for the structural analysis input**
- **Consideration of manufacturing imperfections (tolerance width of uncertain design variables) in order to achieve a production cost minimum by „Design to Imperfections“, includes defects**

Cyclic (dynamic) loading : fatigue

- **Development of practical, physically-based lifetime-prediction methods**
- **Generation of S-N curve test data for the verification of prediction models**
- **Delamination growth models: for duroplastic and thermoplastic matrices**
- **Consideration of media, temperature, creeping, aging**
- **Provision of more damping because parts become more monolithic.**

Dimensioning Load Cases and Boundary Conditions

From the numerous *Load Cases*

the design driving ***Dimensioning Load Cases*** ***(DimLC)*** are to be sorted out:

- for ductile behaviour the : Yielding-related Load Cases,
- for brittle behaviour the : Ultimate-related Load Cases (i.e. CFRP).

A minimum set of DimLCs is searched in order to:

- support fast engineering decisions in cases of 'input' changes

- avoid analysis and analysis data evaluation overkill and

- better understand structural behaviour (as hidden aspect).

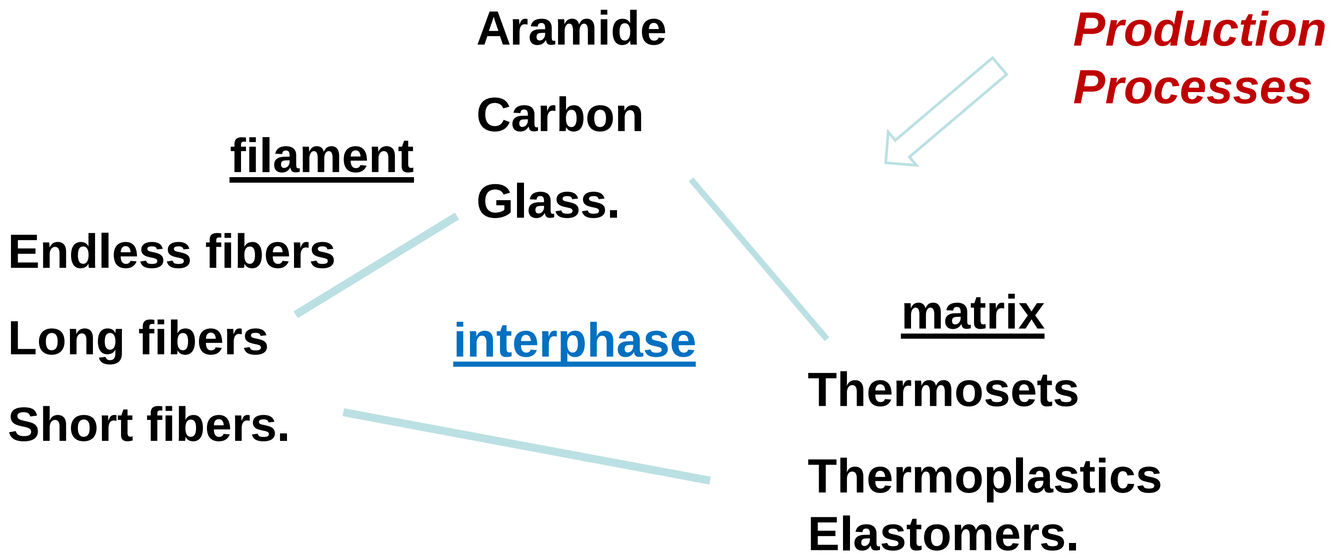
+ Boundary Conditions,
two examples

Peak Design Loads for GROWIAN (Große Windenergie-Anlage)
have been too low.

GROWIAN Measurement Campaign in 1981

generated the basis for the follower wind mills !

Materials: Plenty combinations of different Composite Constituents



Aramide

Carbon

Glass.

filament

Endless fibers

Long fibers

Short fibers.

Production Processes

interphase

matrix

Thermosets

Thermoplastics

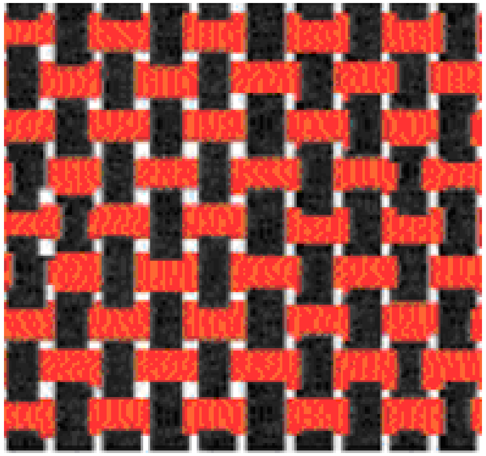
Elastomers.

All these combinations

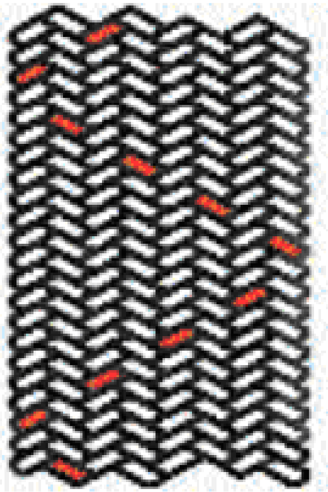
- **need a different treatment and**
- **afford an associated understanding of its internal material behaviour.**

… and - coming up more and more – an increasing variety of 2D- and 3D-fabrics

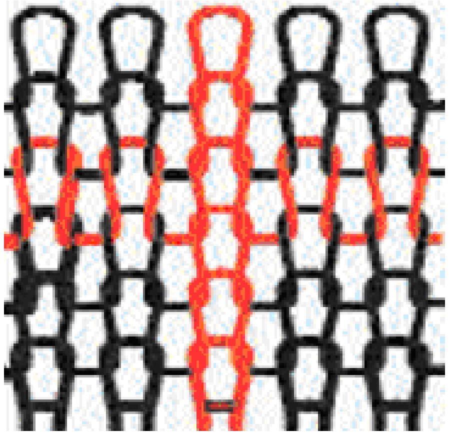
Coming up: The Textile Challenge to achieve Certification



plain weave

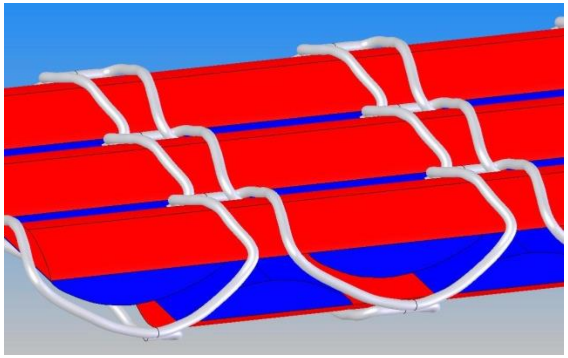


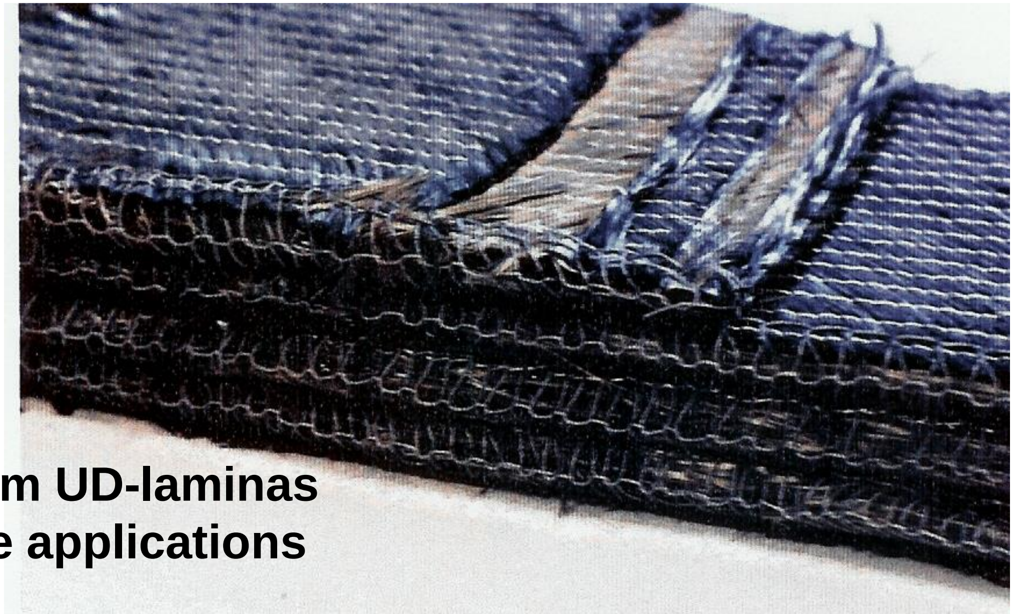
braid



weft knit

warp knit





non-crimp fabrics from UD-laminas
for high-performance applications

Design Verification: Achievement of a Reserve against a Design Limit State

For each distinct Load Case with its single Failure Modes must be computed:

Reserve Factor (load-defined !) **:**

determinisitic or semi-probabilistic

$$RF = \frac{\textit{Failure Load} \ \ at\ \ Eff = 100\%}{\textit{applied Design Load}}$$

valid in linear and non-linear analysis

Material Reserve Factor :

f_{Res} = *Strength Design Allowable / Applied Stress*

$f_{Res} = RF = 1 / Eff$, valid in linear analysis

Material Stressing Effort :

(Werkstoff-Anstrengung)

$Eff = 100\%$ *if* $RF = 1$

material exhausted

applied Design Load = Factor of Safety j x Design Limit Load

State-of- the-Art in <u>Static</u> Strength Analysis of UD laminas

---

best* represented: by the results of the *World-Wide-Failure-Exercises

Organizer : *QinetiQ , UK (Hinton, Kaddour, Soden, Smith, Shuguang Li)*

Aim: *'Testing Predictive Failure Theories for*

Fiber–Reinforced Polymer Composites to the full !'

(for high-performance UD materials , only !)

Procedure of the World-Wide-Failure-Exercises-I, -II (1992-2013):

Part A of a WWFE: *Blind Predictions on basic strengths, only*

Part B of a WWFE: *Comparison Theory-Test using provided*
***Uni-axial 'Failure Stress Test Data' (= basic strength)* and**
Multi-axial 'Failure Stress Test Data'

What were the contents of these Exercises ?

(plain test specimens, no notch)

WWFE-I: *2D (in-plane) loading ,Test Data for 14 Test Cases (2003)*

WWFE-II: *3D loading, Test Data Packs for 12 Test Cases (2013)*

WWFE-III: ***Application of advanced failure models based on Damage and Fracture Mechanics Models***

Deals with validating and benchmarking failure theories that are capable of predicting damage, regarding

- **matrix crack initiation and development,**
- **delamination initiation triggered by transverse cracks,**
- **deformation up to final fracture.**

Cuntze did not contribute to WWFE-III

<u>Task</u> was : for endless fiber-reinforced polymers the

Mapping of courses of test data by the contributor‘s

specific Strength Failure conditions *SFCs* (criteria),.

State-of-the-Art in Cyclic Strength Analysis of UD Laminas (plies), Laminates

- **Procedures base on specific laminates and therefore cannot be generally applied.**
  Hence, no generally applicable Lifetime Prediction Method is available !

- **Procedures base – as with metals – on stress amplitudes and mean stress correction.**
  Is this correct? Can one neglect that the damaging portions are linked to the various fracture failure modes in the case of brittle behaving materials?

- **Present: Engineering Approach:** ***Static Design Limit Strain* of < 0.3% ,**
  negligible matrix-microcracking.
  Design experience proved: No fatigue danger is given for multi-angle laminates

- **Future : *Design Limit Strain* shall be increased for better material exploitation**
  (EU-project: MAAXIMUS)
  Above ε = 0.5% level: *first filament breaks , diffuse matrix-microcracking*
  occurs in usually *fiber-dominated laminates,* used in high-stress applications.

To tackle this, much effort must be put on this in future !

German Research, considering Fatigue Lifetime Modelling

- **Germanischer Lloyd :** originally for the GROWIAN (1980) windmill, to be reworked
- **VDI 2014, sheet 3**: (released by Cuntze, as convenor, in 2006. Fatigue to be reworked)
- **University activities: BeNa group, (*“Betriebsfestigkeits-Nachweis“)***

  for High-Performance Structures (founded by Cuntze in 2010)

  BeNa members-agreed conditions for Lifetime modeling are:

  * *physically-based (on failure modes),*

  * *ply-oriented in order to obtain a generalisation for any*
  UD lamina-composed laminate

Objective of BeNa group:
Release of a VDI-Guideline

- **CCeV (Carbon Composites e.V.**.) *Augsburg:* **Practiced in my working group and symposia**
- **Company activities: partly issued models and software**

Existing Software: As far as (suitable) lifetime prediction models are available

From industry and Software houses

- **Company-owned programs: AUDI (diss. Hahne), AIRBUS?, BMW, …**
  - **HBM GMbH nCode products: Dr. Vervoort**
  - **Magna Powertrain: Mr. Spindelberger**
  - **Safe Technology Ltd: Dr. Sobczak**
  - **LMS, Dr. Hack**
  - **Firehole Composites: (multi-level model)**
  - **….**

From the German BeNa group (university efforts) for instance:

- **ILK, TU-Dresden (UD, textile attempts)**
- **IVW-Kaiserslautern (thermoset and -plastic UD)**
- **ISD, TU-Hannover (multi-level model)**
- **…..**

Personal Activities

1. Foundation of the German Academic Research Group (BeNa)
"*Betriebsfestigkeits-Nachweis*"
for High-Performance Structures (2010)

* *physically-based (on failure modes),*
* *ply-oriented in order to obtain a generalisation for any UD lamina-composed laminate*

Objective:
Release of a VDI-Guideline

2. Foundation of sub-group of my CCeV-working group 'Engineering'
"*Composite Fatigue*"
together with the CCeV member company CADCON (2012).

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Assumptions for Material Modelling (example: UD material) and **Test**

1 Lamina (ply) = Layer of a Laminate, e.g. UD-laminas = “Bricks“

- **The UD-lamina is macroscopically homogeneous. It can be treated as a homogenized (‘smeared‘ material).** ***Homogenisation of a solid to a material brings benefits. Then, knowledge of Material Symmetry applicable : number of required material properties are minimal, test-costs too***
- **The UD-lamina is transversely-isotropic. On planes, parallel with the fiber direction it behaves orthotropically and on planes transverse to fiber direction isotropically** (quasi-isotropic plane)
- **Mapping: Uniform stress state about the critical stress location**
- **Pore-free material, specimen surfaces polished, well sealed** (WWFE-II) **, fiber volume is constant, tube specimens show no warping and do not bulge, perfect bonding, no layer waviness, edge effects do not exist, …**
- **From engineerring point of view Macro-mechanical SFCs are desired. However, the SFCs should consider that failure starts in constituents**

Material Characterisation f (Temp, Moisture, time, etc.)

Specifica for the UD-lamina-based High Performance Laminates

Specific Pre-requisites for the establishment of 3D-UD-SFCs:

- simply formulated from engineering point of view, numerically robust,
- physically-based, and therefore need only few information for pre-dimensioning
- shall allow for a simple determination of the design driving reserve factor
- shall capture failure of the constituents matrix (cohesive), interphase (adhesive), filament
- consider residual stresses

Compliant with John Hart-Smith

- consider micro-mechanical stress concentration of the matrix around the filaments under transversal stress (a means: using matrices showing > 6% fracture strain which heps to capture a stress concentration factor of about 6 up to 1% applied transversal strain
- consider FF, if taking place under bi-axial compression with no external axial stress

$$\{\sigma\} = (\sigma_1 = 0,\ \sigma_2,\ \sigma_3, 0, 0, 0)^T$$

Features of Modelling laminated, high-performance Composites

* ***Lamina-based, sub-laminate-based*** *(e.g. for non-crimp fabrics)* ***or laminate-based !***

* **Is performed, if applicable, *according to the distinct symmetry of envisaged material***

* **For the chosen material model, if material symmetry-based, *the number of the measured inherent Strengths and Elasticity Properties is the same as the observed number of Failure Modes !!*** *Test costs reduction*

* **Achievement of equivalent stresses for each failure mode to obtain information where the lamina design screw must be turned !**

Lesson-Learned: As far as the failure mode or failure mechanism remains,

Static Strength Criteria can be used for Cyclic Loading , too !

Very essential !

Cyclic design: Questions an engineer poses, hoping to get answers from failure models

1. When does damaging start?
2. How can one consider the single (micro-)damaging portion?
3. How are the single damaging portions accumulated?
4. When do the accumulated damageing portions become a damage?
5. When becomes such a damage (delamination, impact) critical?
6. How is the damage growth in the 3rd or final phase of fatigue life (fixation of part replacement time, inspection intervals)?

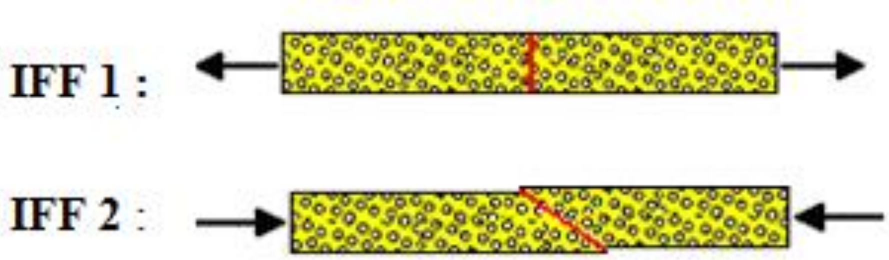
Mind the difference in analysis : Isolated and embedded properties, behaviour

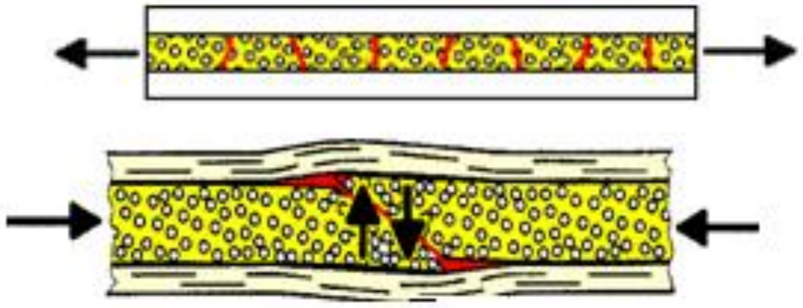
‘Isolated‘ lamina test specimens

= weakest link results (series failure system)

‘Embedded‘ laminas experience in-situ effects

= redundancy result (parallel failure system)





unconstrained lamina

mutually constrained laminas, in laminates

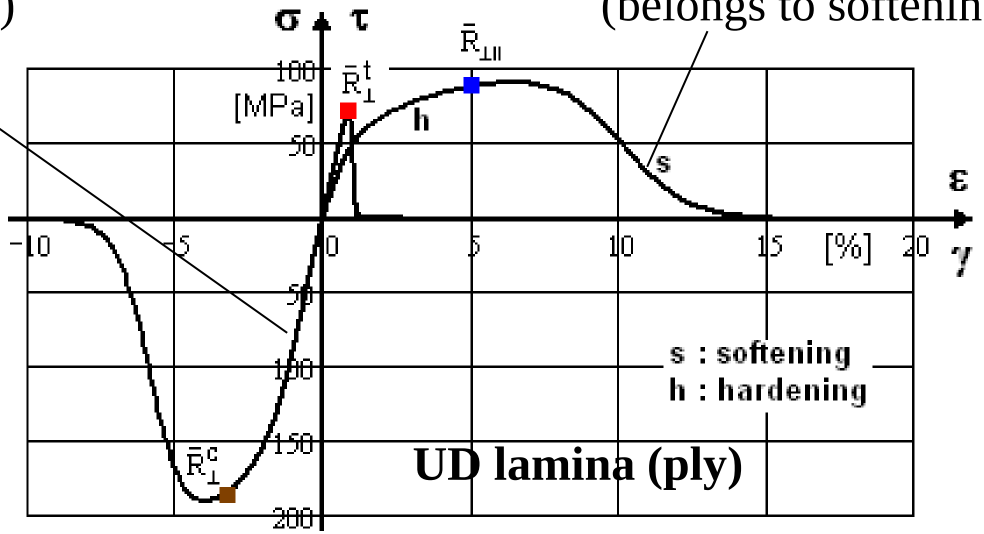
delivers strength property, stress-strain curve

(belongs to hardening)

in <u>non-linear</u> laminate analysis

(belongs to softening)

delivers **basic strength** as analysis input !



UD lamina (ply)

Self-explaining Notations for Strength Properties (homogenised material)

		Fracture Strength Properties									***required by material symmetry***
	loading	**tension**			**compression**			**shear**			
	direction or **plane**	1	2	3	1	2	3	12	23	13	
9	**general orthotropic**	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	**comments**
5	**UD, $\cong$ non-crimp fabrics**	$R_{\parallel}^{t}$ NF	$R_{\perp}^{t}$ NF	$R_{\perp}^{t}$ NF	$R_{\parallel}^{c}$ SF	$R_{\perp}^{c}$ SF	$R_{\perp}^{c}$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$R_{\perp\perp} = R_{\perp}^{t} / \sqrt{2}$ (compare Puck's modelling)
6	**fabrics**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	**fabrics general**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp ≠ Fill*
5	**mat**	R_{1M}^t	R_{1M}^t	R_{3M}^t	R_M^c	R_{1M}^c	R_{3M}^c	R_M^τ	R_M^τ	R_M^τ	$R_M^\tau\,(R_M^t\,)$
2	**isotropic**	R_m SF	R_m SF	R_m SF	*deformation-limited*			R_M^τ	R_M^τ	R_M^τ	*ductile, dense* $R_M^\tau = R_m / \sqrt{2}$
		R_m NF	R_m NF	R_m NF	R_m^c SF	R_m^c SF	R_m^c SF	R_m^σ NF	R_m^σ NF	R_m^σ NF	*brittle, dense* $R_M^\sigma = R_m^t / \sqrt{2}$

NOTE: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m:= 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

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Global and Modal Strength Failure Conditions *General View*

Drucker-Prager, Tsai-Wu

1 Global ***strength*** **failure condition** : F ({σ}, {R}) = 1 (usual formulation)

Set of Modal **strength failure conditions**: F ({σ}, R^{mode}) = 1 (addressed in FMC)

Mises, Puck, Cuntze

Example: UD

vector of 6 stresses (general)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of 5 strengths

$$\{R\} = (R_\parallel^t, R_\parallel^c, R_\perp^t, R_\perp^c, R_{\perp\parallel})^T$$

needs an Interaction of Failure Modes: performed by a

probabilistic-based 'rounding-off' approach (series failure system model)

directly delivering the (material) reserve factor in linear analysis

---

Note: In the quasi-isotropic plane of the UD material just 5 stresses are active:

$$\{\sigma\}^{quasi\text{-}isotropic\ plane}_{principal} = (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$$

By-the-way: Experience with Failure Prediction prove

A Strength Failure Condition (SFC) is a necessary but not a sufficient condition to predict Strength Failure (example: thin-layer problem).

On top, an energy condition may be to fulfill.

Facts of Global and Modal Strength Failure Conditions (SFCs)

Global (one surface) **SFCs:**

- **Combine all failure modes in one single mathematical formulation. This might even capture**
  - a twofold acting failure mode (e.g. if $\sigma_I = \sigma_{II}$ (isotropic) or if $\sigma_2 = \sigma_3$ (transversely-isotropic UD material) and
  - a threefold acting failure mode under hydrostatic loading
- **Re-calculation of all model parameters by new a data course mapping if a test data is to be replaced in one failure mode domain. Then all Reserve Factors have to be determined again!**
- **Some simple global SFCs just use strengths as model parameters. In this case, a change in one failure domain deforms the failure surface in all other (physically independent) failure domains. There is a big chance that a Reserve Factor in such a domain is not on the safe side!**

Modal (multi-surface) **SFCs:**

- **Describe one single failure mode in one single mathematical formulation** (part of failure surface)**.**
  - determine all model parameters in the respective failure mode
  - capture a twofold acting failure mode (e.g. if $\sigma_I = \sigma_{II}$ (isotropic) or if $\sigma_2 = \sigma_3$ (transversely-isotropic UD material) separately, modal-wise by one additional Ansatz (J_3)
  - capture a threefold acting failure mode under hydrostatic loading alike
- **Re-calculation of the model parameters just in the modal domain if a test data is to be replaced. One Reserve Factor must be freshly determined.**

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MIND: The production process ‘bakes‘ the composite material !

Driver for my research work on Strength Failure Conditions (criteria)

Achievement of practical, physically-based criteria under some *pre-requisites* :

- ***physically convincing***
- ***simple, as much as possible***
- ***allow to compute an equivalent stress*** *(helpful, where to turn the design screw)*
- ***rigorous independent treatment of each single failure mode (2 FF + 3 IFF)***
- ***using a material behaviour-linked thinking and not a material-linked one***
- ***engineering approach where all model parameters can be measured***
- ***invariant-based fracture failure conditions for brittle behaving materials, analogous to the 'Mises' yield failure condition for ductile beh. materials*****.**

Note on Pucks and Cuntzes UD strength failure conditions:

Puck's action plane approach involves some basic differences to Cuntzes Failure-mode-concept-based approach: (1) is not invariant-based, (2) interacts the 3 Inter-Fiber-Failure modes (IFF) by a Mohr-Coulomb-based equation, (3) post-corrects the IFF- influence on FF.

Cuntze provides for each failure mode an equivalent stress, that captures the influence of IFF on FF by his interaction equation, uses less model parameters**.**

Basic Features of the author‘s Failure-Mode-Concept (FMC)

- **Each failure mode represents 1 independent failure mechanism**

  and thereby 1 piece of the complete *failure surface*

- **Each failure mechanism is governed by 1 basic strength** (is observed !)

- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !

- **In consequence, this separation requires :**

  Interaction of the Modal Failure Modes !

Basic Features of the author‘s Failure-Mode-Concept

- **Each failure mode represents 1 independent failure mechanism and thereby 1 piece of the complete *failure surface***
- **Each failure mechanism is governed by 1 basic strength** (is observed !)
- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !!

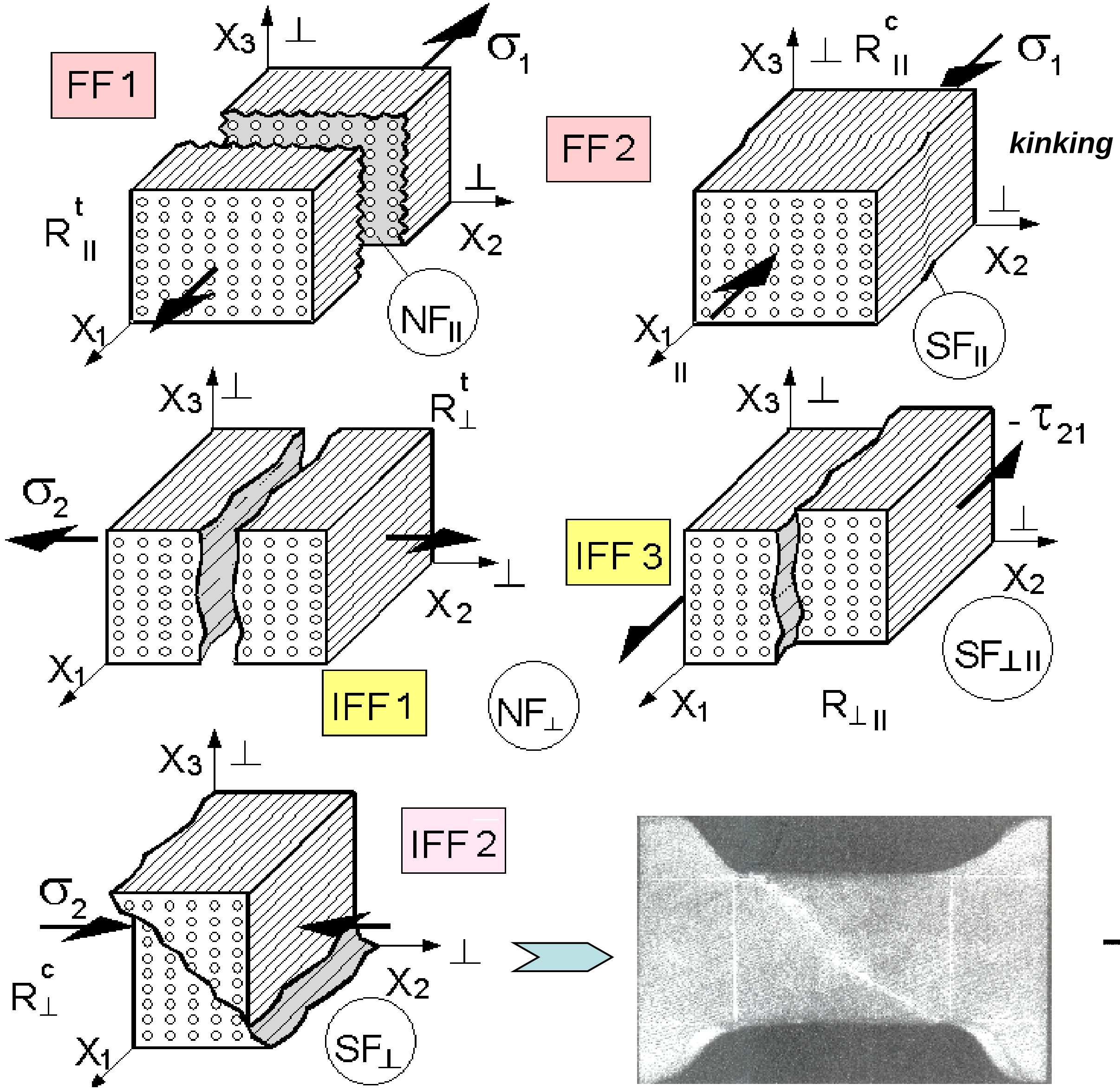
- **In consequence: Interaction of the Failure Modes is needed in the case of modal Strength Failure Conditions (SFCs)!**

- **The Formulation of the SFCs for the homogenized material is :**

  - invariant-based: the choice of the used invariants is linked to the fact, whether the material element experiences a *volume* change, a *shape change* and *friction*

  - material symmetry –based: fixes the number of modes, strengths, …

Observed Strength Failure Modes with Strengths of brittle UD Materials

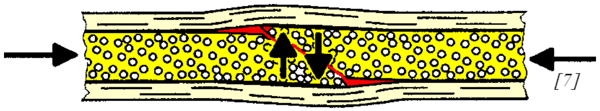


wedge failure type

t = tension
c = compression

► **5 Fracture modes exist**

= 2 FF (Fibre Failure)
+ 3 IFF (Inter Fibre Failure)



Fracture Types:

NF := Normal Fracture

SF := Shear Fracture

Cuntzes <u>3</u>D Modal SFCs (criteria) for Transversely-Isotropic UD-materials

Invariants replaced by their stress formulations

---

strains from FEA

[Cun04, Cun11]

FF1 $Eff^{\parallel\sigma} = \breve{\sigma}_1 / \bar{R}^t_{\parallel} = \sigma^{\parallel\sigma}_{eq} / \bar{R}^t_{\parallel}\,,$ $\quad \breve{\sigma}_1 \cong \varepsilon^t_1 \cdot E_{\parallel}$ *

FF2 $Eff^{\parallel\tau} = -\breve{\sigma}_1 / \bar{R}^c_{\parallel} = +\sigma^{\parallel\tau}_{eq} / \bar{R}^c_{\parallel}\,,$ $\quad \breve{\sigma}_1 \cong \varepsilon^c_1 \cdot E_{\parallel}$

2 <u>filament</u> modes

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,]/\,2\bar{R}^t_{\perp} = \underline{\sigma^{\perp\sigma}_{eq}} / \bar{R}^t_{\perp}$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / \bar{R}^c_{\perp} = +\sigma^{\perp\tau}_{eq} / \bar{R}^c_{\perp}$

3 matrix modes

IFF3 $Eff^{\perp\parallel} = \{[b_{\perp\parallel} \cdot I_{23-5} + (\sqrt{b_{\perp\parallel}{}^2 \cdot I_{23-5}{}^2 + 4 \cdot \bar{R}_{\perp\parallel}{}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,]/(2 \cdot \bar{R}_{\perp\parallel}{}^3)\}^{0.5} = \sigma^{\perp\parallel}_{eq} / \bar{R}_{\perp\parallel}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

Modes-Interaction :

$$Eff^{m} = (Eff^{\parallel\tau})^m + (Eff^{\parallel\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

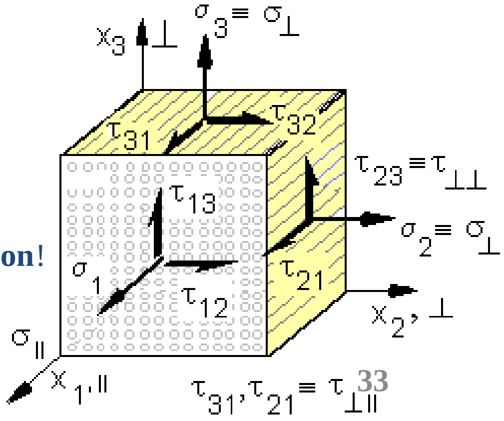
with mode-interaction exponent $2.5 < m < 3$ from mapping tests data

Typical friction value data range: $0.05 < \mu_{\perp\parallel} = 0.5 \cdot b_{\perp\parallel} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$

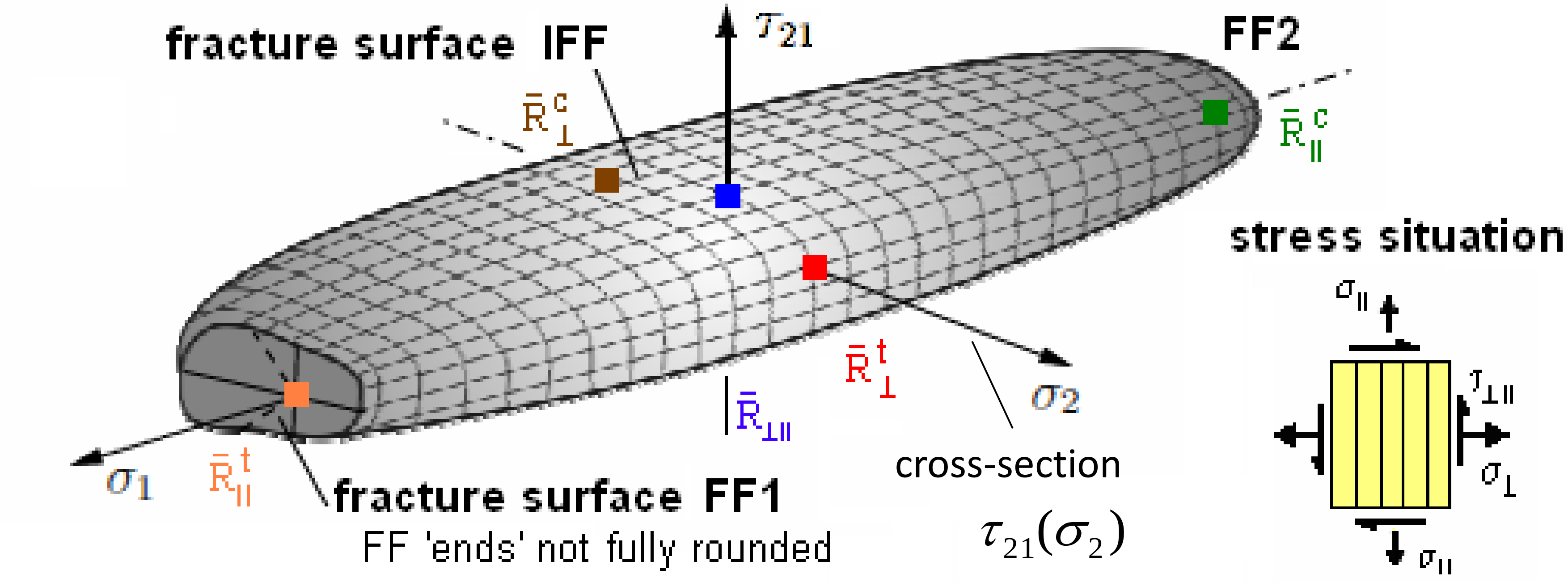
The friction value, as a model parameter, can be only applied together with the associated SFC equation!

Poisson effect * : bi-axial compression strains the filament without any σ_1

t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre



Visualization of 2D-UD-SFCs as Fracture Failure Surface (Body)



$$\{\sigma\}=(\sigma_1,\sigma_2,0,0,0,\ \tau_{21})^T$$

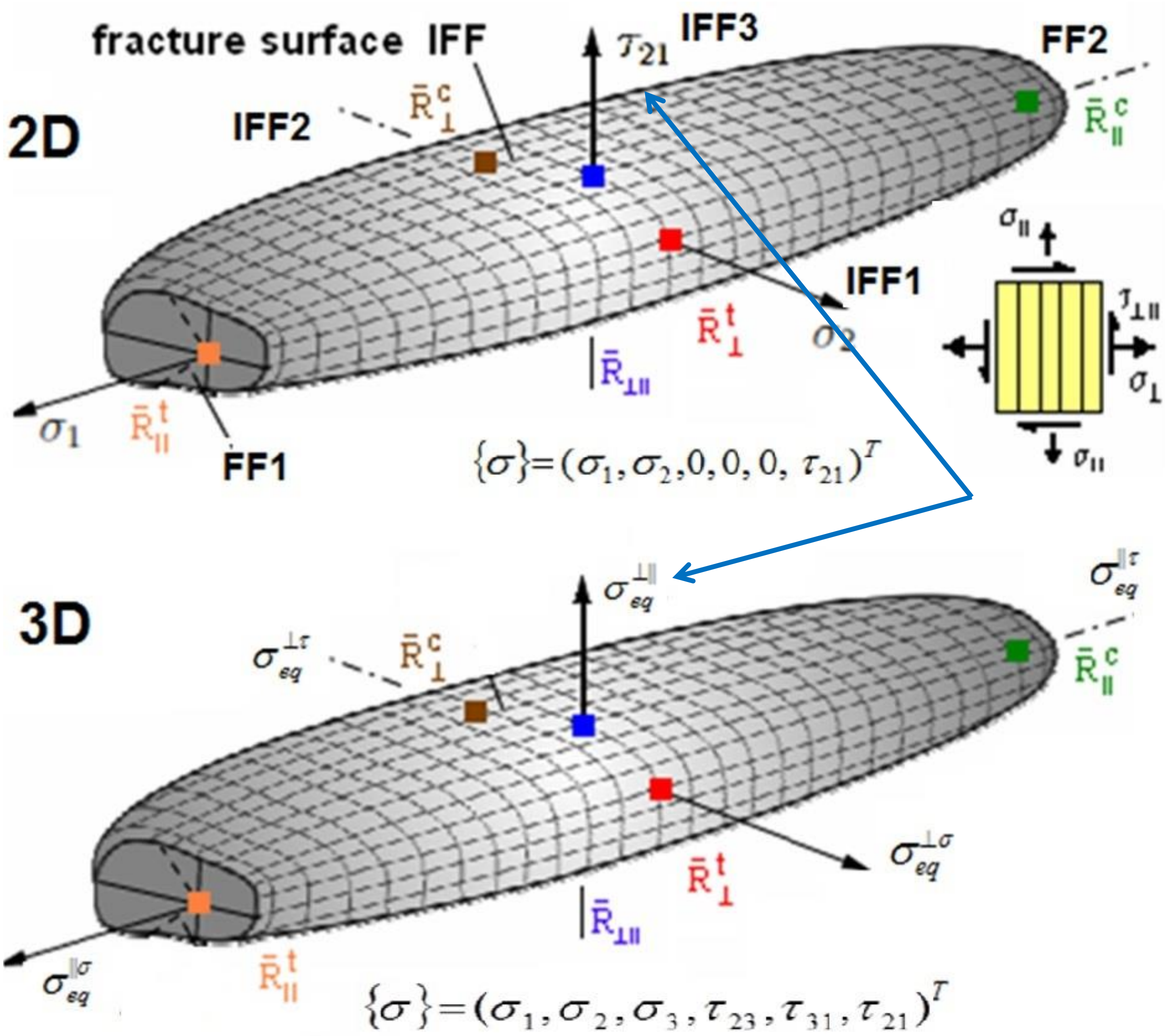
Mode interaction fracture failure surface *of FRP UD lamina*

$$Eff^{\,m} = (Eff^{\,\parallel\tau})^m + (Eff^{\,\parallel\sigma})^m + (Eff^{\,\perp\sigma})^m + (Eff^{\,\perp\tau})^m + (Eff^{\,\perp\parallel})^m = 1$$

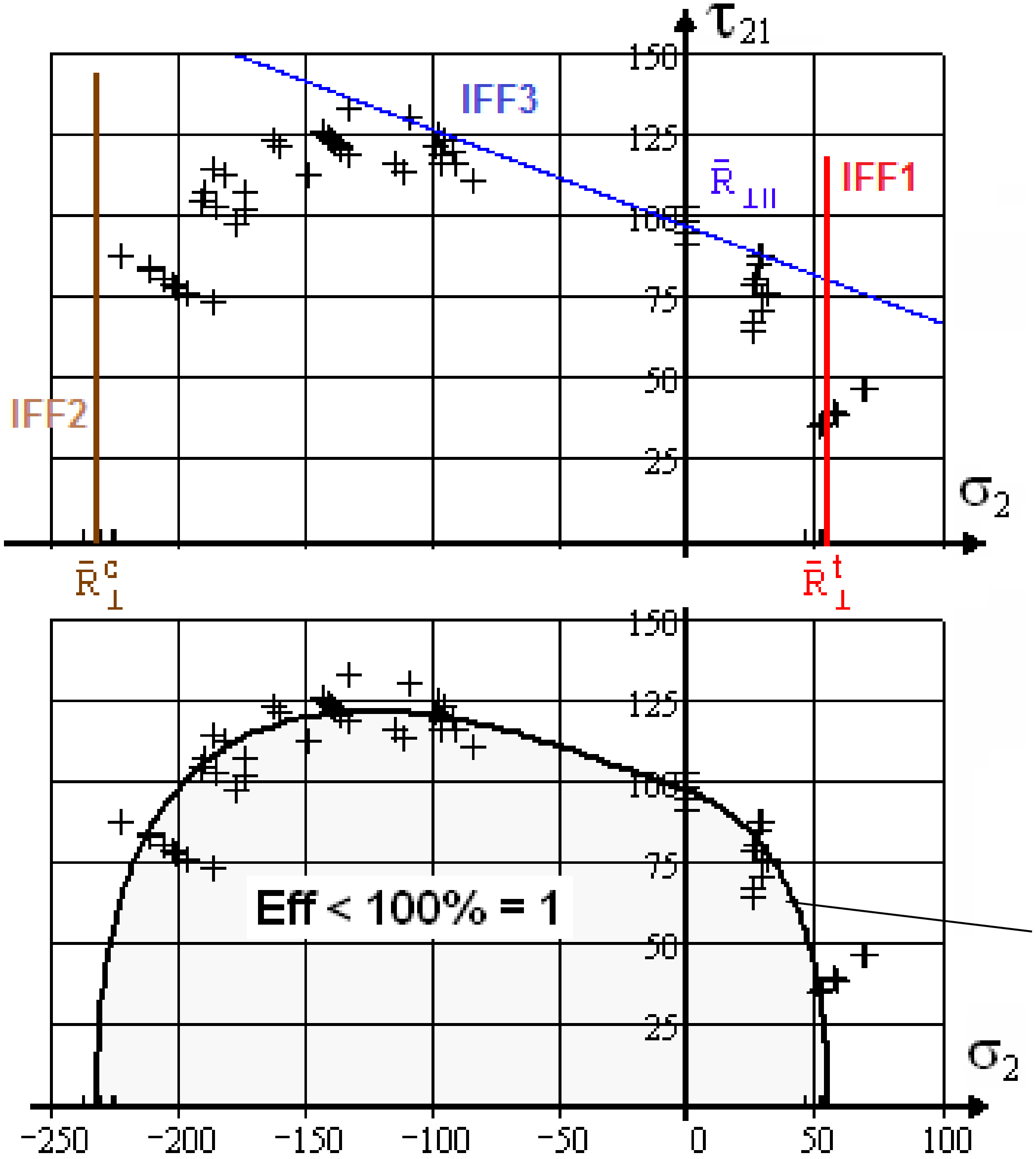
(courtesy W. Becker) .

Mapping: Average strengths indicated by a bar over

2D = 3D Fracture surface if replacing *stresses* by *equivalent stresses*

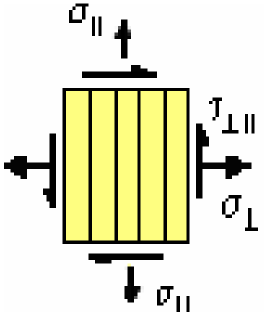


2D-Demonstration: Interaction of UD Failure Modes for $\tau_{21}(\sigma_2)$, $\breve{\sigma}_1 = 0$



Mapping of course of IFF test data in a pure mode domain by the associated *Mode Failure Condition.*

3 IFF pure modes = straight *lines !.*



IFF 1 : $\frac{\sigma_2}{\bar{R}_{\perp}^{t}} = 1$

IFF 2 : $\frac{-\sigma_2}{\bar{R}_{\perp}^{c}} = 1$

IFF 3 *(2D-simplified)* : $\frac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 1$

Mapping of course of test data by *Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

$$m = 2.5, \quad \mu_{\perp\parallel} = 0.3$$

Test Data Mapping *versus* Design Verification

***Validation of SFCs with Failure Test Data** by

mapping the course by an average failure curve (surface)

using average (typical**) strengths** $\bar{R}$ (from resistance)

*** Delivery of a reliable Design Verification** by

calculation of a Margin of Safety or a (load) Reserve Factor

MoS > 0 or RF = MoS – 1

on basis of a statistically reduced failure curve (surface)

= use of strength design allowables R (no bar over).

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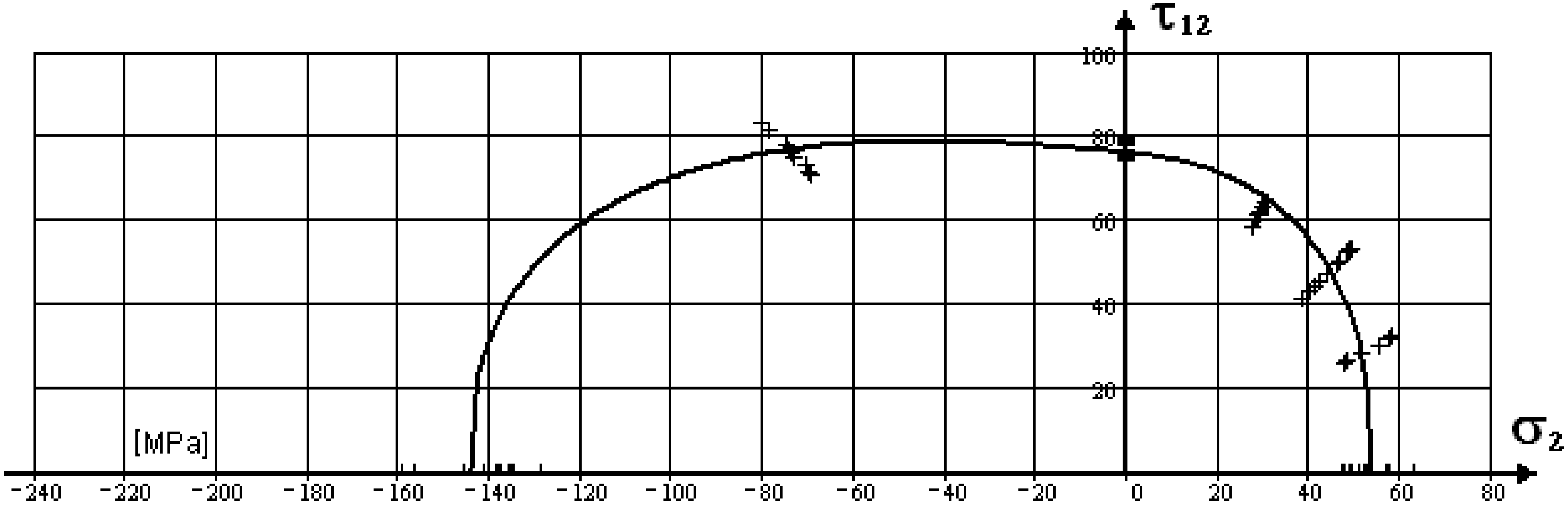
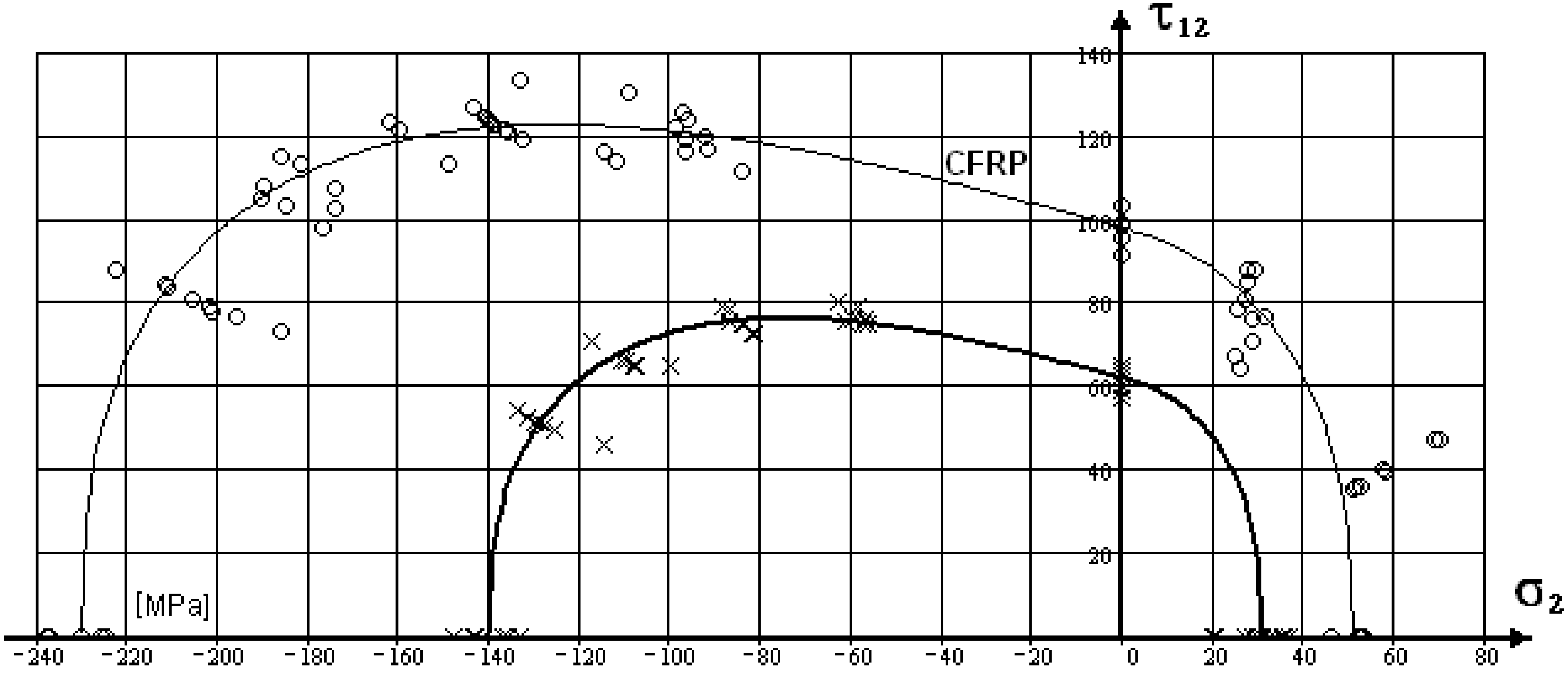
a numerical example

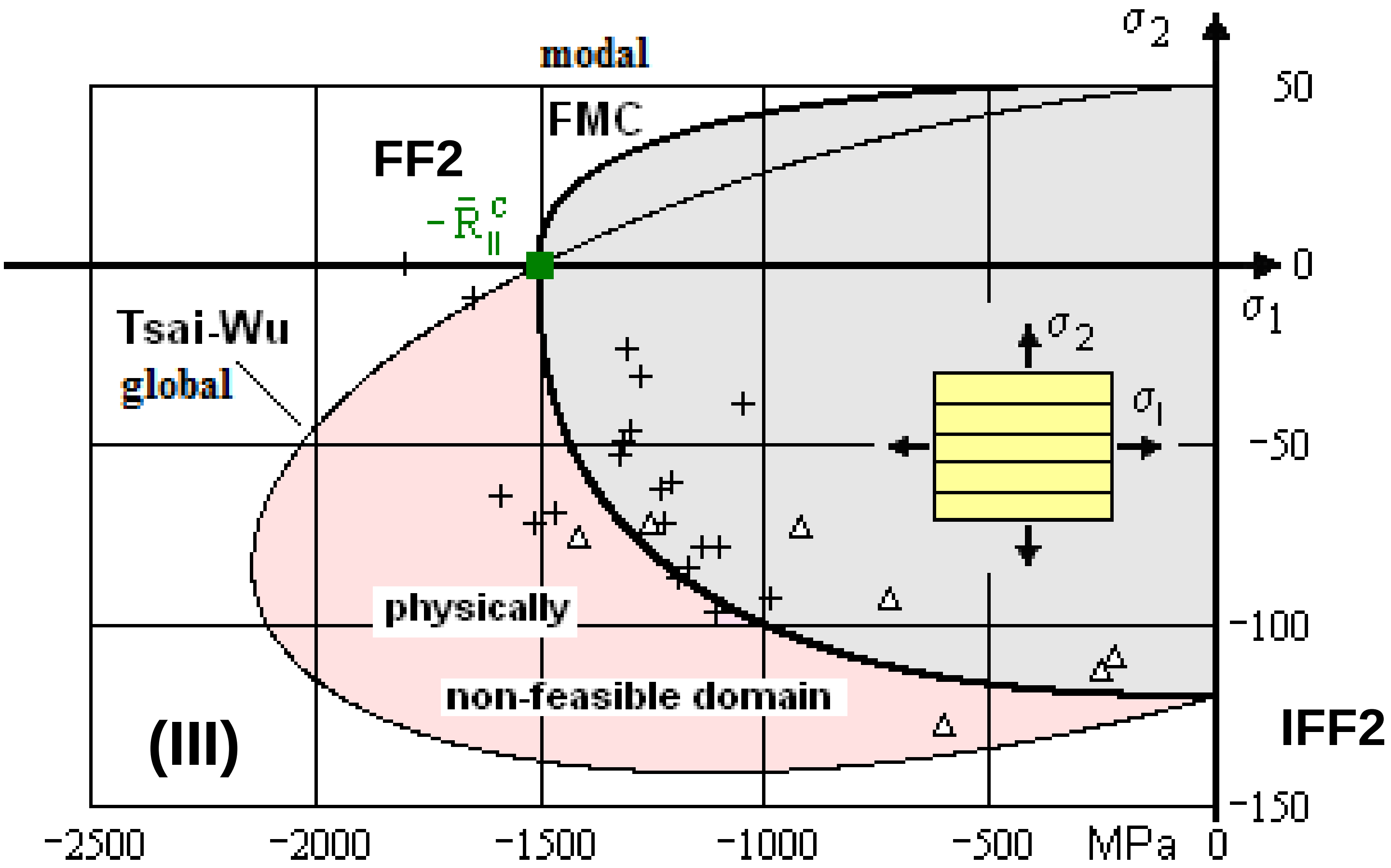
GFRP, CFRP examples, mapped by FMC–based UD SCF, 2D stress state

IFF





Mapping in the ‘Tsai-Wu non-feasible domain‘ (quadrant III)



$$\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$$

Data: ***courtesy IKV Aachen, Dr. Knops***

Lesson Learnt: The modal FMC maps correctly, the *global* Tsai-Wu formulation predicts a non-feasible domain !

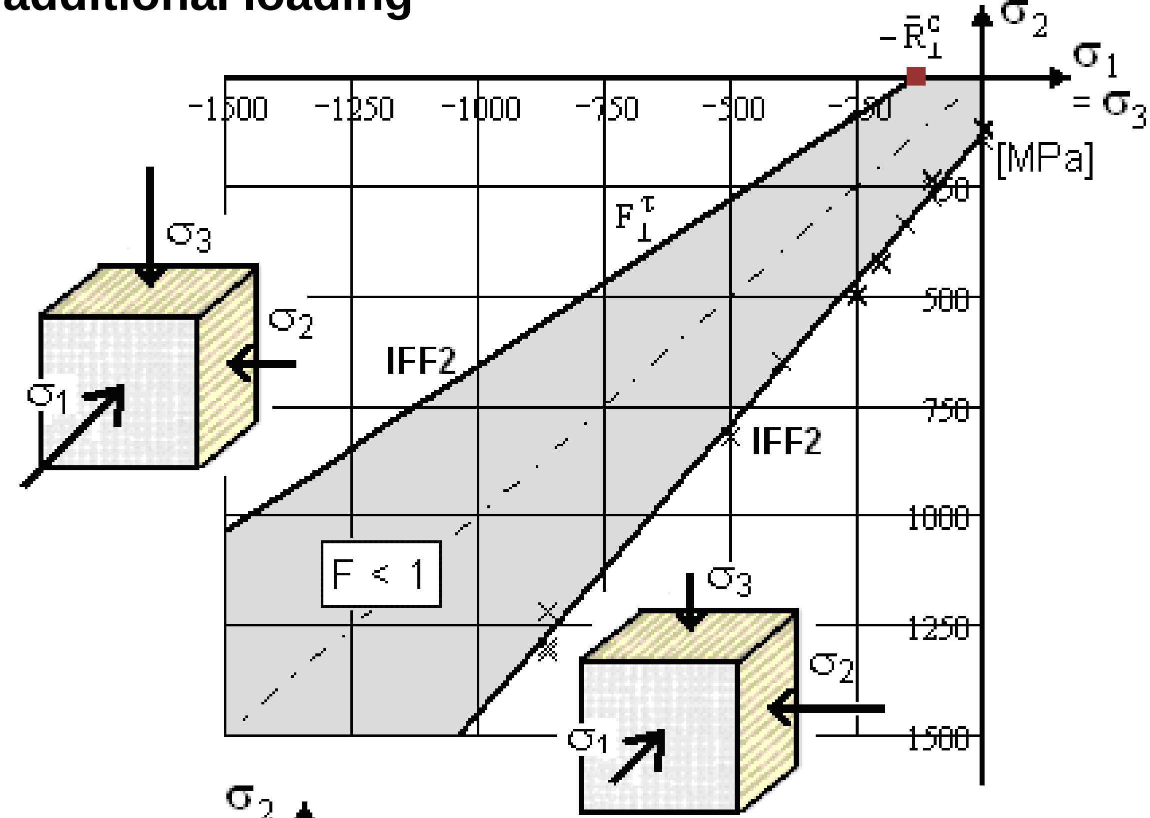
Test Case 5, WWFE-II, UD test specimen, 3D stress state $\sigma_2(\sigma_1 = \sigma_3)$

= hydrostatic pressure with additional loading

UD E-glass/MY750epoxy.

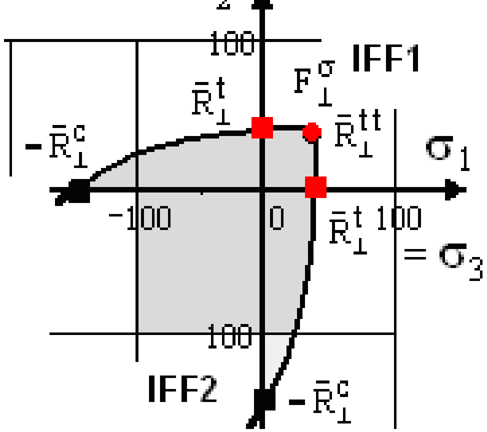
$\nu_{\perp\parallel} = 0.28, \quad \mu_{\perp\perp} = 1.16, \quad m = 2.8\,,$

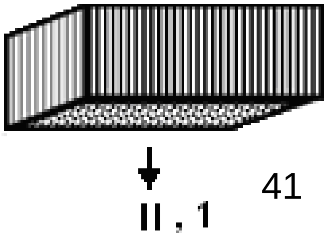
$\{\bar{R}\} = (1280,\ 800,\ 40,\ 132,\ 73)^T\ MPa$



Good Mapping, <u>after</u> QinetiQ re-evaluation of the lower branch test data Then, the upper branch was fitting other test data, too !

Result: ***Both branches were then reliable and could be used for model validation***





Numerical Determination of the load-defined Reserve Factor *RF*

Linear elastic problem for the envisaged brittle behaving CFRP

then simplified $\boldsymbol{RF} = \boldsymbol{f_{Res}}$ **(material reserve factor)** $= \boldsymbol{Eff}^{-1}$

Residual stresses : 0 (effect vanishes with increasing micro-cracking) in $MPa = N/mm^2$

Stress state vector : $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T = (0,\ -60,\ 0,\ 0,\ 0,\ 50)^T$

Strengths vector: $\{R\} = (R_\parallel^t, R_\parallel^c, R_\perp^t, R_\perp^c, R_{\perp\parallel})^T = (1200,\ 850,\ 35,\ 100,\ 80)^T$

Roughly estimated from average values

$\{\bar{R}\} = (1378,\ 950,\ 40,\ 125,\ 97)^T$

WWFE-I: UD T300/PR319EP

Mode interaction exponent: $m = 2.7$

Friction value: $\mu_{\perp\parallel} = 0.3$

Calculation: negative *Effs* are nonsense and are to be bypassed

$$Eff^{\perp\sigma} = \frac{\sigma_2 - |\sigma_2|}{\bar{R}_\perp^t} = 0 \qquad Eff^{\perp\tau} = \frac{-\sigma_2 + |\sigma_2|}{\bar{R}_\perp^c} = 0.60 \qquad Eff^{\perp\parallel} = \frac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 0.51$$

$$Eff^m = (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m$$

$$Eff = 0.72,\ \boldsymbol{RF = 1/Eff = 1.39},\ MoS = RF - 1 = 0.39$$

Loading may be increased by the factor *RF* until obtaining fracture limit state $\boldsymbol{Eff = 100\% \equiv RF = 1}$ **.**

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What is what, in fatigue ?

Fatigue : process, that degrades material properties. 3 fatigue phases exist

Damaging (= Schädigung, but not damage (Schaden), as it is used in English, too): a process wherein the results, the damaging portions, finally accumulate to a damage size such as a macro-scopic delamination (onset of 3rd phase).

Used as means: the Palmgren-*Miner Damaging Accumulation* model

Damage : damage size that is judged to be critical. *Then Damage Tolerance Analysis is used to predict the damage growth under further cyclic loading.*

Material : homogenized (smeared) model of the envisaged complex material which might be a material combination.

Engineer‘s Desires for a Composite Fatigue Lifetime Prediction Model

to

- **capture multi-axial, variable loadings**
- **be physically-based**
- **deal on the simpler homogenized composite material level** (numerical efficiency) **but account for failure of the composite material constituents matrix, fiber and interphase**
- **be applicable to any laminate**
- **set up a fatigue model with clearly measurable parameters**
- **have it implemented into a standard commercial software.**

Quantification of the Damaging Portions in the damaging progress, *by Static Strength Failure Conditions possible?*

Experience-proven Assumption:

if damaging mechanisms (failure modes) in static and cyclic case are equal,
then

- failure parameters that drive cyclic damaging are equal, too, and

- transferability from static failure to cyclic failure is permitted !!

However, static strength must be replaced by the
fatigue strength = residual strength of the shrinking failure body,
which is associated to the respective lifetime !

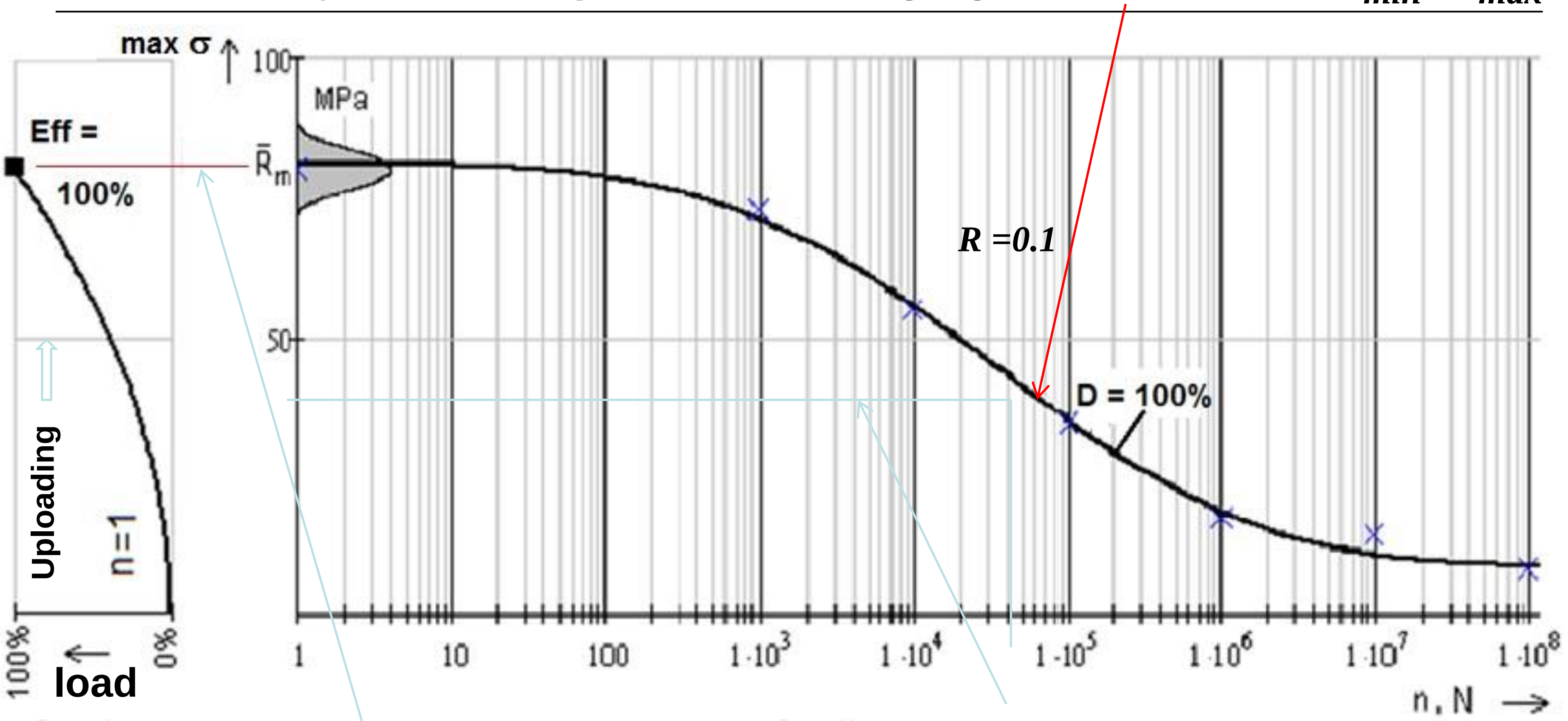
Therefore, to obtain quantified damaging portions

my FMC-based **Static Failure Conditions (criteria) can be used,**

Measurable quantities to describe damaging:

Micro-crack density, Residual strength, Residual stiffness.

Static and cyclic development of damaging, S-N-curve $R = \sigma_{min} / \sigma_{max}$



Static

$\bar{R}_m$ basic strength

Cyclic

residual strength $\sigma(R,N)$

Analogous limits of the material capacities :

- Static : material stressing effort *Eff* = 100 %

- Cyclic : material damaging sum *D* = 100 %

The static material stressing effort *Eff* (Werkstoffanstrengung) is replaced by the cyclic *D* !

For brittle behaving materials it is advantageous to use $max\sigma \equiv R_m$ instead of $\Delta\boldsymbol{\sigma}$

Question: Failure mode-linked Master S-N-curves dterminabl to save test costs?

For lifetime estimation usually several S-N-curves are needed.

(constant amplitude loading is a seldom case but variable loading)

Idea: Measurement for each mode one associated '*mode master S-N-curve*'

- for a fixed stress ratio *R*
- prediction of additionally necessary S-N-curves within a mode on basis of the mode master S-N-curve and on the '*principle of equivalent strain energy*'!

Then, for the often used

all possible load orientations capturing fiber-dominatedly designed, multidirectional laminates, composed of UD plies,

an engineering-like model for plain laminates is derivable !

It's *characteristical steps are:*

4 steps

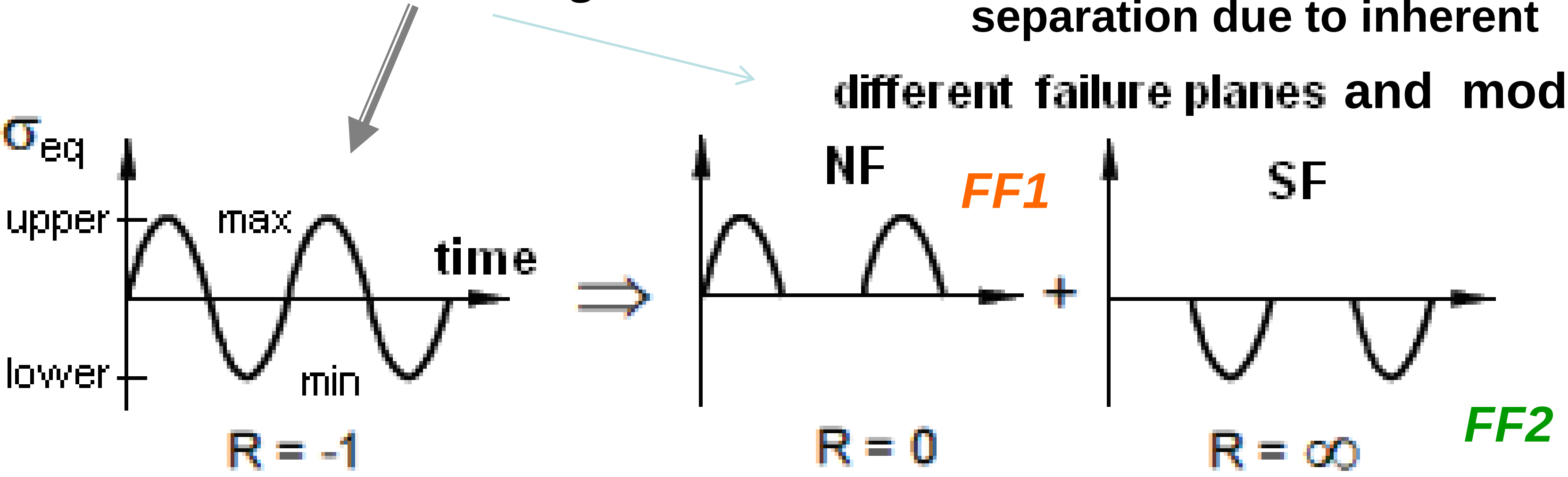
S := cyclic stress range $\Delta\sigma$, N := number of cycles to failure, stress ratio R := $min\sigma/max\sigma$

Failure mode-wise Modelling of Loading Cycles for high-performance 'fiber-dominated designed', UD laminas-composed laminates

For simply displaying the approach *it is chosen* :

- **the usually 'fiber-dominated' laminate and**
- ***R = -1 loading***

separation due to inherent different failure planes and modes



$\sigma_{mean} = 0$

NF := Normal Fracture, SF := Shear Fracture

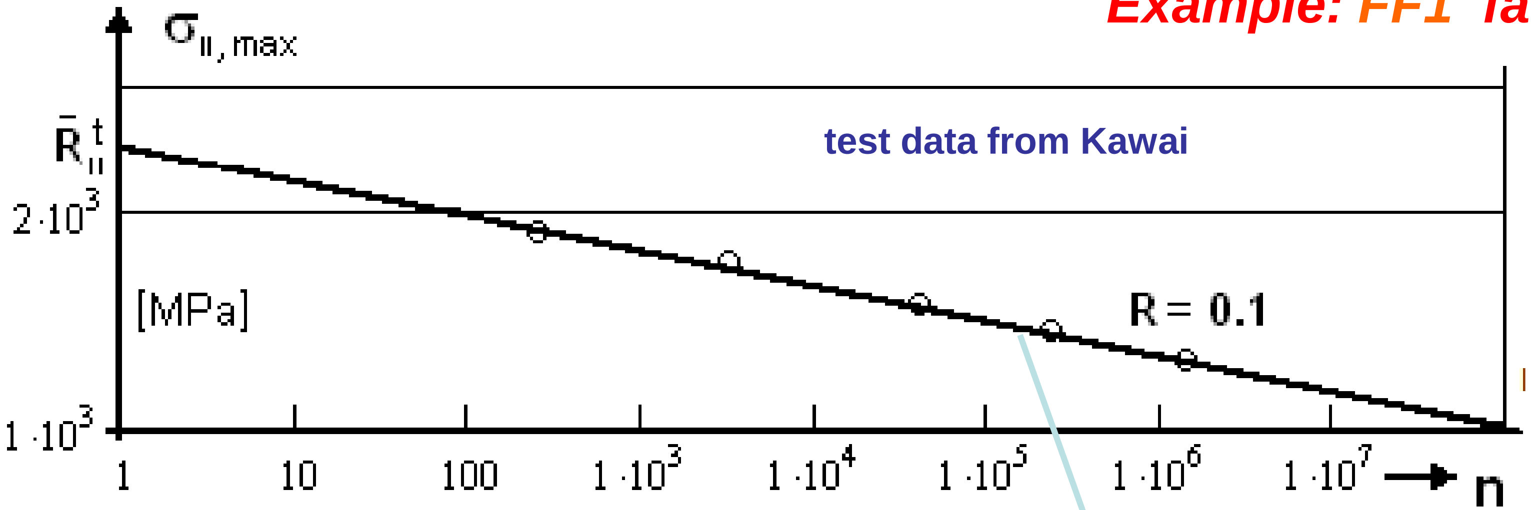
Step 1 : <u>Failure mode-wise</u> apportionment of cyclic loading (novelty 1!)

Specific rain-fall procedure to be applied,

***FF1*:= fiber tensile fracture; *FF2*:= fiber compressive failure**

Mapping of Mode S-N data by a representative *Master curve*

Example: FF1 failure mode



Step 2 **: S-N curve can be mapped by a straight line in a *log-log* graph**

Measured test data mapped by

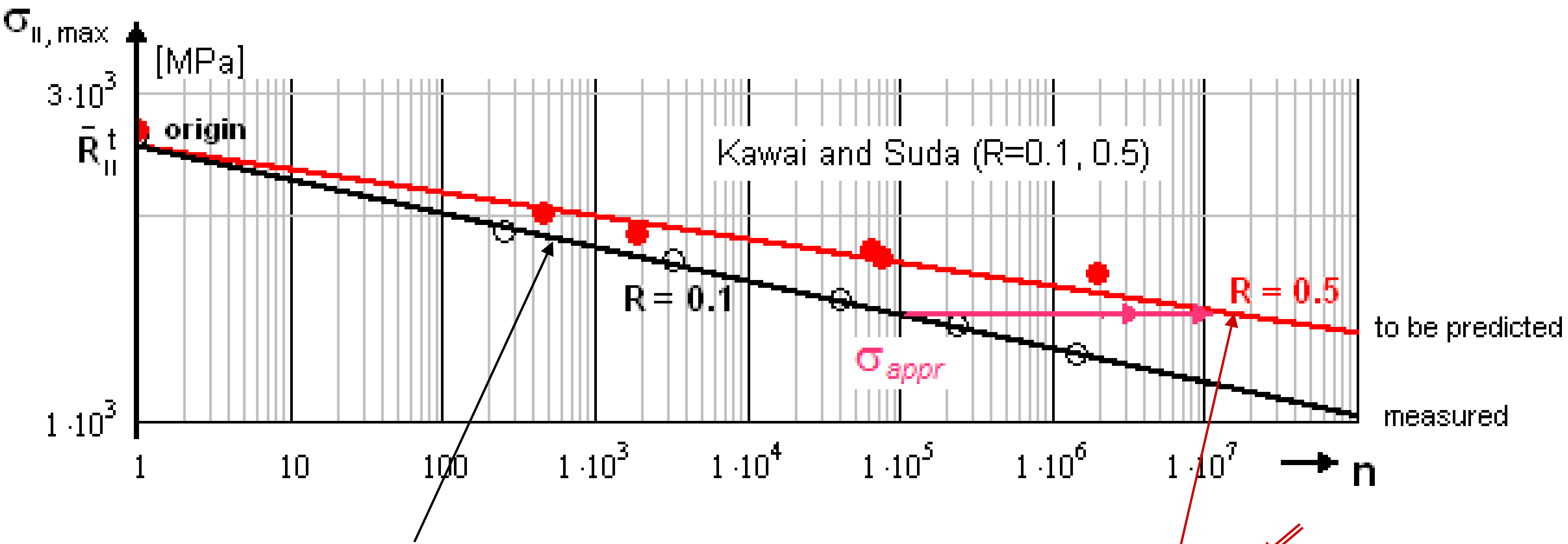
$$\sigma^{Master}_{\|,\ \max}(n) \approx \overline{R}^{t}_{\|} \cdot n^{c_{Master}}$$

FF1 **strength**

as mode-representative *Master S-N curve* for *FF1* .

For the general case *Variable Loading*, several S-N-curves are needed !

Prediction of needed other *FF1* S-N curves from Master *FF1* Curve



Given : $\sigma_{\parallel,\ \max}^{Master}(n) \approx \overline{R}_{\parallel}^{t} \cdot n^{c_{Master}}$ **Searched :** $\sigma_{\parallel,\max}^{pred} = \overline{R}_{\parallel}^{t} \cdot n^{c_{pred}}$

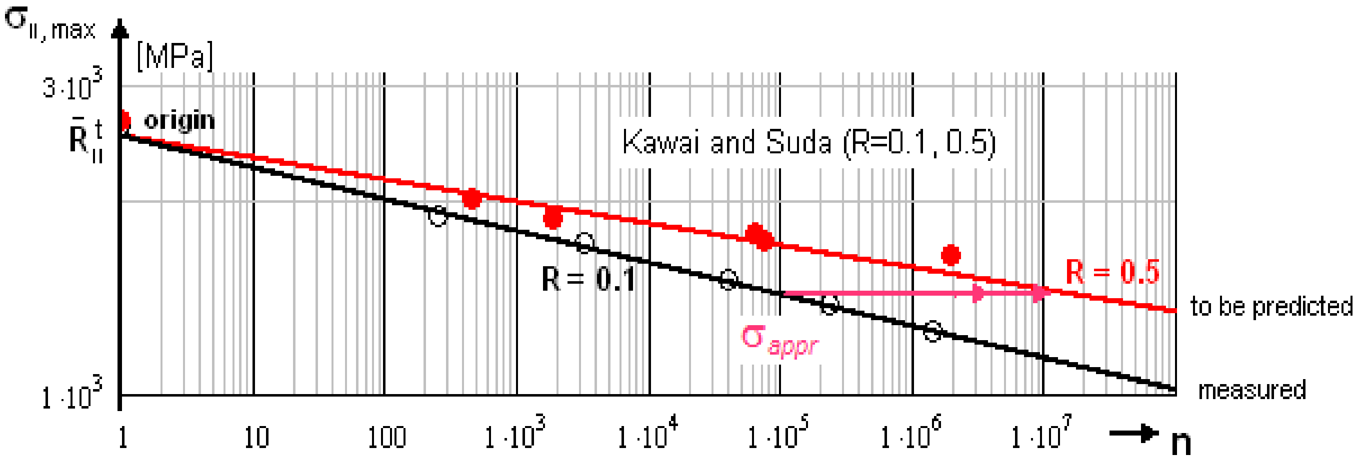
Slope of R = 0.5 ?

Step 3: Application of the principle of constant strain energy

A distinct strain energy level will be reached for R > 0.1 at higher cycles.

S *:= cyclic stress range =* $\Delta\sigma$*, N:= number of cycles to failure, n:= cycle number*

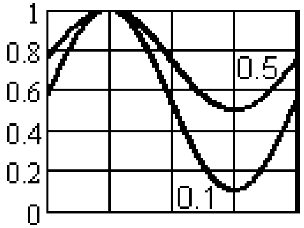
How does the method work for a UD lamina? Numerical example: $R_{0.5}$ from $R_{0.1}$



$n_{appr} = 10000$ cycles

$R_{master} = 0.1, \quad R_{pred} = 0.5$

$D_{feasible} = 0.8$



$$f_{pred} = \exp[-\ln(\frac{R_{pred}^{\ 2}-1}{R_{master}^{\ 2}-1})\cdot\frac{1}{c_2}] \ = 210$$

$$\sigma_{\|,\ \max}^{master}(n) \ \approx \bar{R}_{\|}^{t}\cdot n^{c_{master}} = \bar{R}_{\|}^{t}\cdot n^{-0.049} \quad \Rightarrow \quad \sigma_{1,\max}^{pred}\ (n) \approx \bar{R}_{\|}^{t}\cdot n^{c_{pred}}.$$

$$c_{pred} = -\ln(\bar{R}_{\|}^{t}/\sigma_{appr})/\ln(n_{appr}\cdot f_{pre}) = -0.034$$

Loading $n_3(R = 0.1) = 0$

$n_1(R = 0.1) = 100000\ cycles, \quad \sigma_1^{(1)} = 12500\ MPa, \quad N_1(R = 0.1) = 2300000\ cycles,$

$n_2(R = 0.1) = 1600\ cycles, \quad \sigma_1^{(2)} = 1500\ MPa, \quad N_2(R = 0.1) = 55000\ cycles,$

$n_4(R = 10) = 6000\ cycles, \quad \sigma_1^{(4)} = -1150\ MPa, \quad N_4(R = 10) = 5000\ cycles,$

$n_5(R = 0.5) = 600000\ cycles, \quad \sigma_1^{(5)} = 1550\ MPa, \quad N_5(R = 0.5) = 2600000\ cycles.$

Miner application

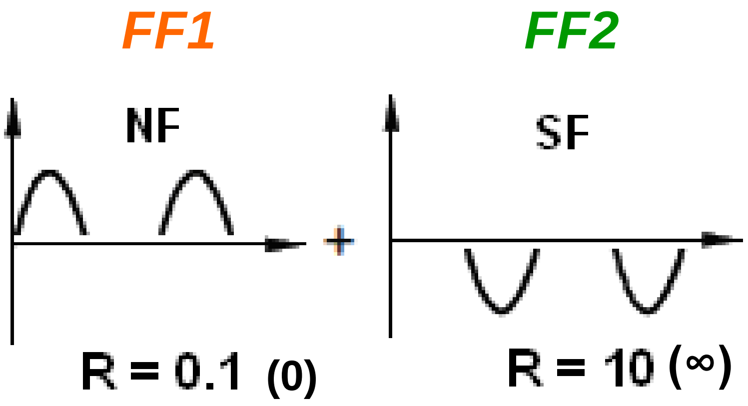
$D = \sum n_i/N_i\ ) = 100000/2300000 + 1600/55000 + 6000/5000 + 600000/2.600000 = 0.43$

$$\mathrm{MoS} = \frac{D_{feasible}/D}{j_{life}\cdot D_{\square}} - 1 = \frac{0.8/0.43}{3\cdot 0.43} - 1 = 0.4 > 0.$$

$\{\bar{R}\} = (2560,\ 1590,\ 73,\ 185,\ 90)^T$ MPa

$\sigma_{\max} = 2\cdot\sigma_a/(1-R) = \Delta\sigma/(1-R)$ **with Δσ := stress range**

Application of Relative Miner-‘Rule‘



Simple Example: again

R = - 1

← practically measured curves

$$D\ (FF1, FF2) = NF : (n_1 / N_1 + n_2 / N_2 + n_3 / N_3) + SF : (n_4 / N_4)$$

$$+ \quad D\ (IFF1, IFF2, IFF3) \ = \ D \leq \ D_{feasible} < 100\%$$

$D_{feasible}$ value from test experience

Step 4: Mode-wise Accumulation of Damaging Portions (novelty 2 !)

Calulation, from [Cun13b], see Annex

FF = Fiber Fracture, IFF = Inter Fiber Fracture

General Conclusions on lifetime prediction models

- Generally applicable, practical lifetime prediction models are not available
- For UD-materials the model situation is promising
- For ‘higher‘ textiles the model situation is not satisfying
- The implementation of available models into Software is in progress.

Therefore:

Conclusions on Cuntze's FMC-based Static UD Strength Fail. Conditions

Lessons Learnt ***from the WWFEs:***

1. **General Prediction is not possible with Basic Strength data only, if physically necessary friction values must be considered** (for shear fracture prediction of brittle behaving materials: consideration of friction is mandatory). **Global SFCs do not directly consider friction; therefore have shortcomings.**
2. **Validation of failure conditions requires a uniform stress field in the critical domain.** This was not always given for the WWFE test cases.
   2D stress case: Test data mapping was successful, validation achieved.
   3D stress case: Successful, if reliable 3D test data were available.
   Unfortunately, this was just partly the case.
3. **The FMC delivers a combined formulation of *independent modal failure modes*, without the well-known drawbacks of global SFC formulations** (which *mathematically combine in-dependent failure modes)*
4. **The FMC-based 3D UD Strength Failure Conditions are simple but describe physics of each single failure mechanism pretty well**
5. **FMC** may be termed **the 'anisotropic Mises' .**

Some Final Comments

- Properties are ‘agreed’ values to achieve a common and *comparable* design basis
- Properties must be provided with average value and coefficient of variation
- Changing a certified material is economically seldom possible in a final phase of project
- Sources of uncertainty should be investigated
- Model parameters should be measurable and physically self-explaining
- Variety of Composites: Many properties for design and manufacturing not yet available
- For brittle behaving materials, multi-axial stress assessment is not possible on basis of the uni-axial strength values alone. Knowledge of material internal friction values, *following Mohr-Coulomb*, is mandatory
- Theory creates a model of the reality ‘only’, an Experiment is ‘just’ one realisation of the reality.
  Experimental results can be far away from reality like an inaccurate theoretical model.

Therefore, put sufficient effort into both, analysis and test,
to achieve the desired FIDELITY.

DANKE

Keep in mind !

All is difficult prior to becoming simple!

[Moslik Saadi]

Literature

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[Deg01] Degrieck J. and van Paepegem W.V.: *Fatigue Damage of Fiber-reinforced Composite Materials – Review*. Appl. Mech. Rev., Vol .54 (2001), No 4, 279-300

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[Sho06] Shokrieh M.M. and Tahery-Behroz F.: *A unified fatigue model based on energy method.* Composite Structures 75 (2006), 444-450

[VDI2014] VDI 2014: German Guideline, Sheet 3 *"Development of Fiber-Reinforced Plastic Components, Analysis".* Beuth Verlag, 2006. (in German and English).

Literature may be downloaded from http://www.carbon-composites.eu/leistungsspektrum/fachinformationen/fachinformation-2

ANNEX

Strength Failure Conditions are for homogeneous materials

Prediction of ***Onset of Yielding*** + ***Onset of Fracture*** for non-cracked materials

Assessment of multi-axial stress states in a critical material location,

by **utilizing the uniaxial strength values R** and an
equivalent stress σ_{eq}, representing a distinct actual multi-axial stress state.

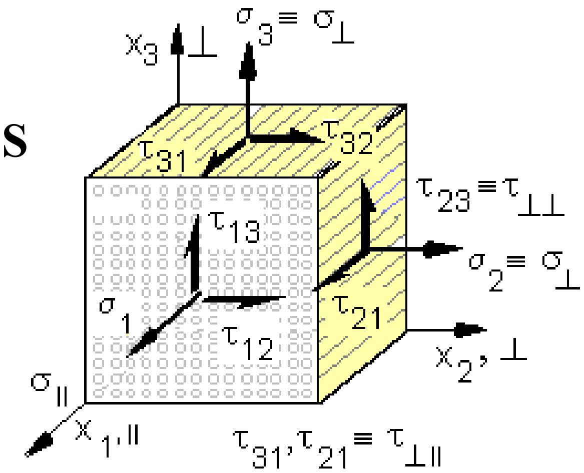
for *** dense & porous,**
*** ductile & brittle behaving materials,**
ductile : $R_{p0.2} \cong R_{c0.2}$ (Mises) brittle, dense : $R_m^{\,c} \geq 3R_m^{\,t} ; R_{c0.2} > R_{p0.2}$

for *** isotropic material**
*** transversally-isotropic material (UD := uni-directional material)**
*** rhombically-anisotropic material (fabrics) + ‘higher‘ textiles etc.**

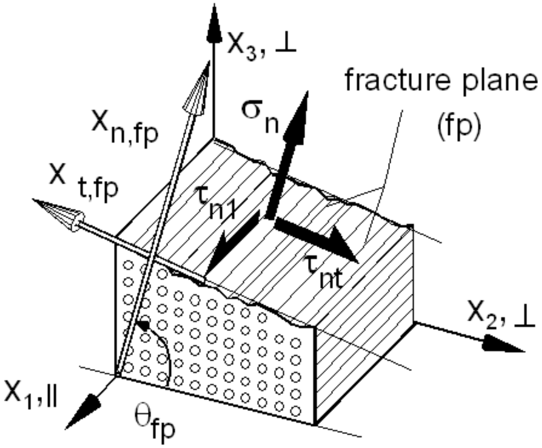
Shall allow for inserting stresses from the utilized various coordinate systems into stress-formulated failure conditions, -and if possible- invariant-based.

Transversely-Isotropic Material (UD composite): Stresses & Invariants

lamina +
'principal' COS



Transformation into the quasi-isotropic plane



Mohr, Puck, Hashin: Fracture is determined by the (Mohr) stresses in the fracture plane !

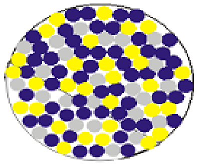
$\{\sigma\}^{quasi\text{–}isotropic\ plane}_{principal} =$	$\{\sigma\}_{lamina} =$	$\{\sigma\}_{Mohr} =$
$= (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$	$= (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$	$(\sigma_\ell,\ \sigma_n,\ \sigma_t,\ \tau_{nt},\ \tau_{t\ell},\ \tau_{\ell n})^T$
$I_1 = \sigma_1$, $I_2 = \sigma_2^p + \sigma_3^p$	$I_1 = \sigma_1,\quad I_2 = \sigma_2 + \sigma_3$	$I_1 = \sigma_1,\quad I_2 = \sigma_n + \sigma_t$
$I_3 = \tau_{31}^{p\ 2} + \tau_{21}^{p\ 2}$	$I_3 = \tau_{31}^2 + \tau_{21}^2$ **'UD invariants'!** [Boehler]	$I_3 = \tau_{t\ell}^2 + \tau_{n\ell}^2$
$I_4 = (\sigma_2^p - \sigma_3^p)^2 + 0$	$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2$	$I_4 = (\sigma_n - \sigma_t)^2 + 4\tau_{nt}^2$
$I_5 = (\sigma_2^p - \sigma_3^p)(\tau_{31}^{p\ 2} - \tau_{21}^{p\ 2}) + 0$	$I_5 = (\sigma_2 - \sigma_3)(\tau_{31}^2 - \tau_{21}^2) - 4\tau_{23}\tau_{31}\tau_{21}$	$I_5 = (\sigma_n - \sigma_t)(\tau_{t\ell}^2 - \tau_{n\ell}^2) - 4\tau_{nt}\tau_{t\ell}\tau_{n\ell}$

Invariant := Combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system. Good for an optimum formulation of *scalar Failure Conditions*

Variety of possible Composites Types

Filaments: **glass, aramide, carbon, ceramics, ..** (short, long fibers → endless fibers)

Fiber preforms (+ sizing) **from** ***roving, tape, weave, braid, knit, stitch***
dry or wet ***(2D and 3D),*** **or mixed as in a *preform hybrid***
non-crimp fabric laminates



PP/glass/aramidePEEK/
glass –filament-yarn

Matrices (resin + hardener): **polymers, thermoplastics, ceramics, concrete, ..**

Polymers (crystalline and amorphous)		
Plastics		Elastomers
thermo-plastics	thermo-sets	
Acrylic, polycarbonat, polyimide, polypropylene	epoxy, phenolic, polyurethane, silicon	natural rubber, polyurethan, thermoplastic elastomer

polymers
are also
bonding materials
(3D understanding)

Manufacturing processes : **pre-pregging, wet winding, RTM, fiber placement, ..**

Rovings: 2k through > 48 k

… and - coming up more and more – an increasing variety of 2D- and 3D-fabrics

Analyses needs Provided Properties and Manufacturing Process Information

Analytical, semi-analytical and numerical procedures for

- **Process-Simulation (CAD, FEM, CFD, etc.)**

  (draping, flow front, fusion weld, fiber orientation, curing, Tg value, curing stresses etc.) and the intensively linked

- **Structure-Analysis (FEM, BEM, pre- and post-processing)**

Thereby, epistemic Uncertainties to achieve a Robust Design must be tackled:

- Certification must focus an uncertainty quantification.
- Reduction of the Coefficient of Variation is of higher importance than increasing the average value a bit
- Design to Imperfections in manufacturing
- Provide ease-of-use and ease-of-interpretation of the results.

Aleatoric Uncertainty: play at dice (Würfel), number by chance, cannot be influenced !

Epistemic Uncertainty: reduced knowledge from too few tests etc.

Failure Analysis Flow Path (multi-level 2-scale approach)

isotropic or a
quasi-isotropic continuum

Constituent materials: fiber, matrix, interphase of the interface
build up a **layered continuum**

failure usually occurs at micro-level

Isotropic Matrix

Micro-mechanical (scale) Model

Transversely-isotropic Lamina (ply)

calibration by macromechanical properties *elasticity, strength*

Meso-Model (RVEs, Voxel)

maufacturing 'significances'
flaws, waviness, matrix-nests,

calibration by macromechanical properties

Ply-by-Ply analysis of the Laminate

merely non-degraded material **strain-hardening**

degrading material **strain-softening + non-linear analysis beyond IFF**

Failure Conditions for the Homogenized Solid = Material

Strength Failure Conditions for the Material

validation of strength failure conditions by strength test series on material level

Macro-mechanical (scale) Model

Orthotropic Laminate

Fracture Mechanics Failure Conditions for the Structural Part

final failure: FF in an lamina or delamination in a laminate

arbitray crack in an isotropic or a
quasi-isotropic continuum

delamination crack in a layered
continuum (laminate)

Meso-level is no scale, per definitionem !

RVE:Representative Volume Element, voxel : volumetric pixel

Which is the Work-Flow of a Fatigue Lifetime Prediction?

1 Input

- **Operational loadings: Load-time curves (modeling by rain flow, ..)**
- **Safety concept: Design to Life $j_{Life} = 3 – 4$, inspection interval**

Consideration of the operational (service) loading:

- **Time domain: Cycle-by-cycle or collective-by-collective (less computational effort)**
- **Frequency domain: Load spectra (loss of load sequence) or block loadings, etc**

2 Transfer of operational loading into stresses by a Structural Analysis

3 Output for several S-N regions

- **Low Cycle Fatigue LCF: high stressing,**
- **High Cycle Fatigue HCF: intermediate stressing**
- **Very High Cycle Fatigue VHCF: low stressing and strains**

(DFG Research Program SPP1466, started 2010).

Interaction of Single Strength Failure Modes in the modal FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= ‘Accumulation’ of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots.} \quad \mathbf{= 1 = 100\%,\ \textit{if failure}}$$

with mode-interaction exponent *2.5 < m < 3 from mapping experience*

***as modal* material stressing effort * (in German Werkstoffanstrengung)**

and

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / \overline{R}^{\text{mode}}$$

equivalent mode stress

mode associated average strength

later ⇒ example

*** artificial technical term created together with QinetiQ**

Cuntze‘s Pre-design Input for 3D UD SFCs

Test Data Mapping **Design Verification**

- **5 strengths :** $\{\overline{R}\} = (\overline{R}_{\parallel}^{t}, \overline{R}_{\parallel}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp\parallel})^{T}$ $\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^{T}$

  average (typical) values **strength design allowables**

- **2 friction values : for 2D** $\mu_{\perp\parallel}$ **, for 3D** $\mu_{\perp\parallel}, \ \mu_{\perp\perp}$

  $\mu_{\perp\parallel} = 0.1$ $\mu_{\perp\perp} = 0.1$

- **1 mode-interaction exponent :** ***m* = *2.6* .**

values, recommended for pre-design

Material Symmetry Requirements *(helpful, when generating SFCs)*

1 If a material element can be homogenized to an ideal (= frictionless) crystal,

then, material symmetry demands for the transversely-isotropic UD-material

- **5 *elastic ‘constants’ , 5 strengths, 5 fracture toughnesses*** **and**
- ***2 physical parameters (such as CTE, CME, material friction, etc.)***

(for isotropic materials the respective numbers are 2 and 1)

2 Mohr-Coulomb requires for the real crystal another inherent parameter,

- **the *physical parameter ’material friction’ : UD*** $\mu_{\perp\parallel}$, ;$\mu_{\perp\perp}$ ***, Isotropic*** μ

3 Fracture morphology witnesses:

- **Each strength corresponds to a distinct *failure mode***

and to a *fracture type* as Normal Fracture (NF) or Shear Fracture (SF).

Above Facts and Knowledge gave reason

why the FMC strictly employs single *independent* failure modes

by its failure mode–wise concept.

Mechanisms of Interest when considering Property Measurement

Yielding versus quasi-yielding:
In ductile behaving materials the failure mechanism *yielding* is active for the loadings tension, compression and shear whereas in case of brittle behaving composites the diffuse damaging as *quasi-yielding* belongs to different macroscopic failure mechanisms in tension (NF) and shear (SF)..

Diffuse Damaging:
damaging, occurring fro onset of micro-cracking until onset of discrete local macro-cracks, often indicated by whitening (for ductile thermoplastics it is connected to void intiation and void growth)

Discrete Damaging:
localization of diffuse damaging which sometimes ends with CDS (characteristic damage state)

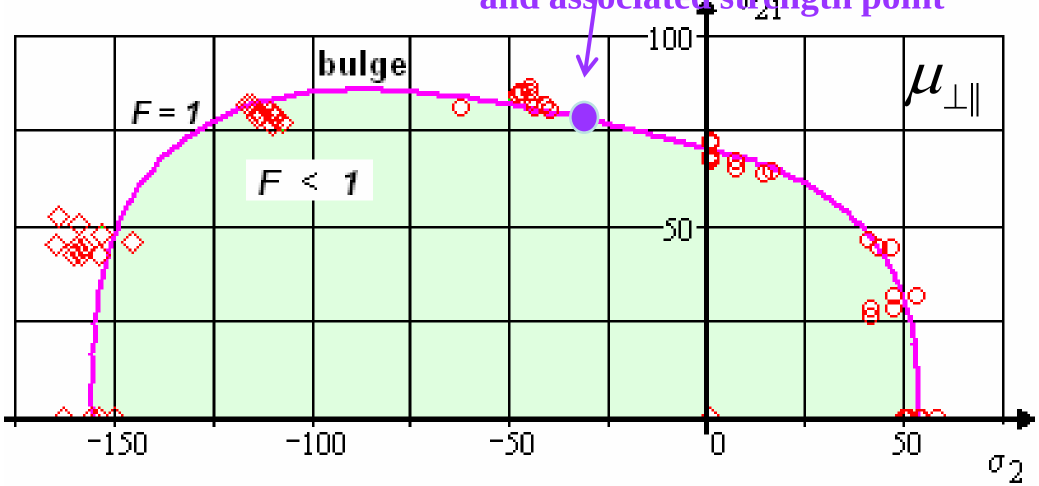
Micro-mechanical 'notching':
- onset of micro-cracks degrade the matrix in a transversely stressed lamina the more the thicker the lamina is ('thin-layer effect' ; energy release rate becomes larger)
- onset of filament breaks causes 3D stress states resulting in growth of lateral micro-cracks and lamina-parallel micro-delaminations (more critical in general)

Guess of Friction Values from slopes (bi-axial test points) $\mu_{\perp\|}$, $\mu_{\perp\perp}$

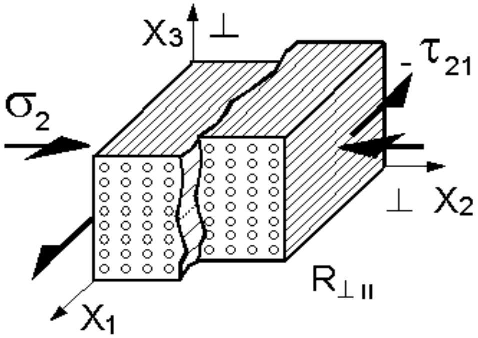
$$|\tau_{21}| = \overline{R}_{\perp\|} - \mu_{\perp\|} \cdot \sigma_2$$

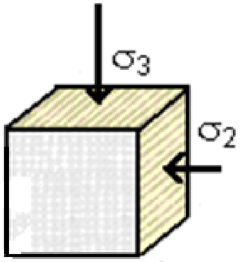
Estimation:

Straight line through magenta point and associated strength point

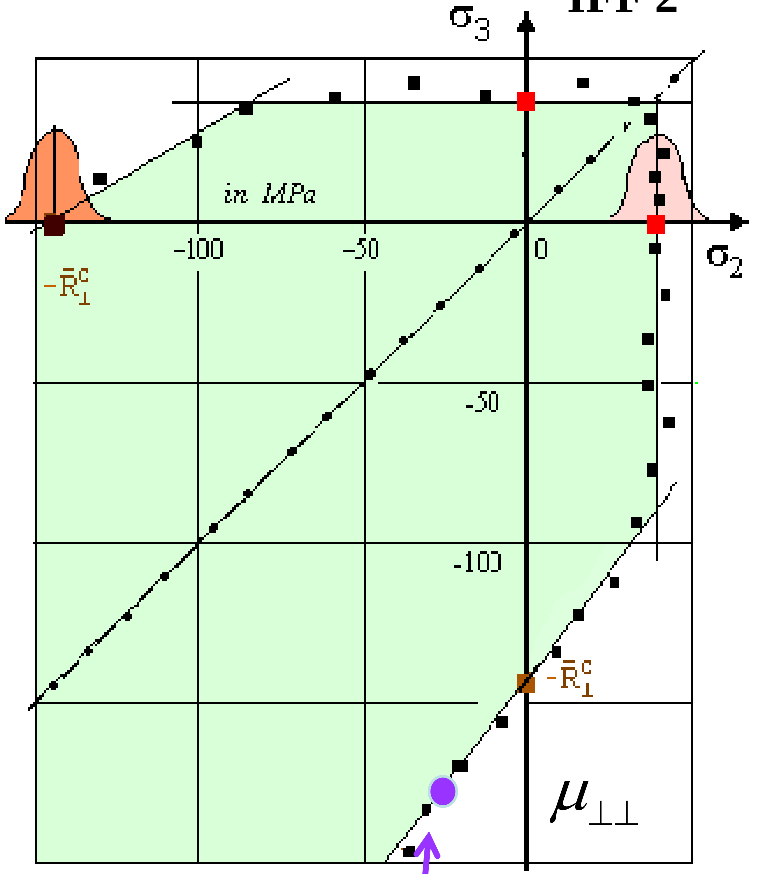


IFF 3





IFF 2



1. **Fitting of course of test data (min error square) in ‘pure‘ failure mode domains**
2. **Estimation with one strength value and one multi-axial failure stress point**
3. **For $\mu_{\perp\perp}$ in addition : derivation from fracture plane measurements possible.**

minimum error square

Verification Levels of the Structural Part with

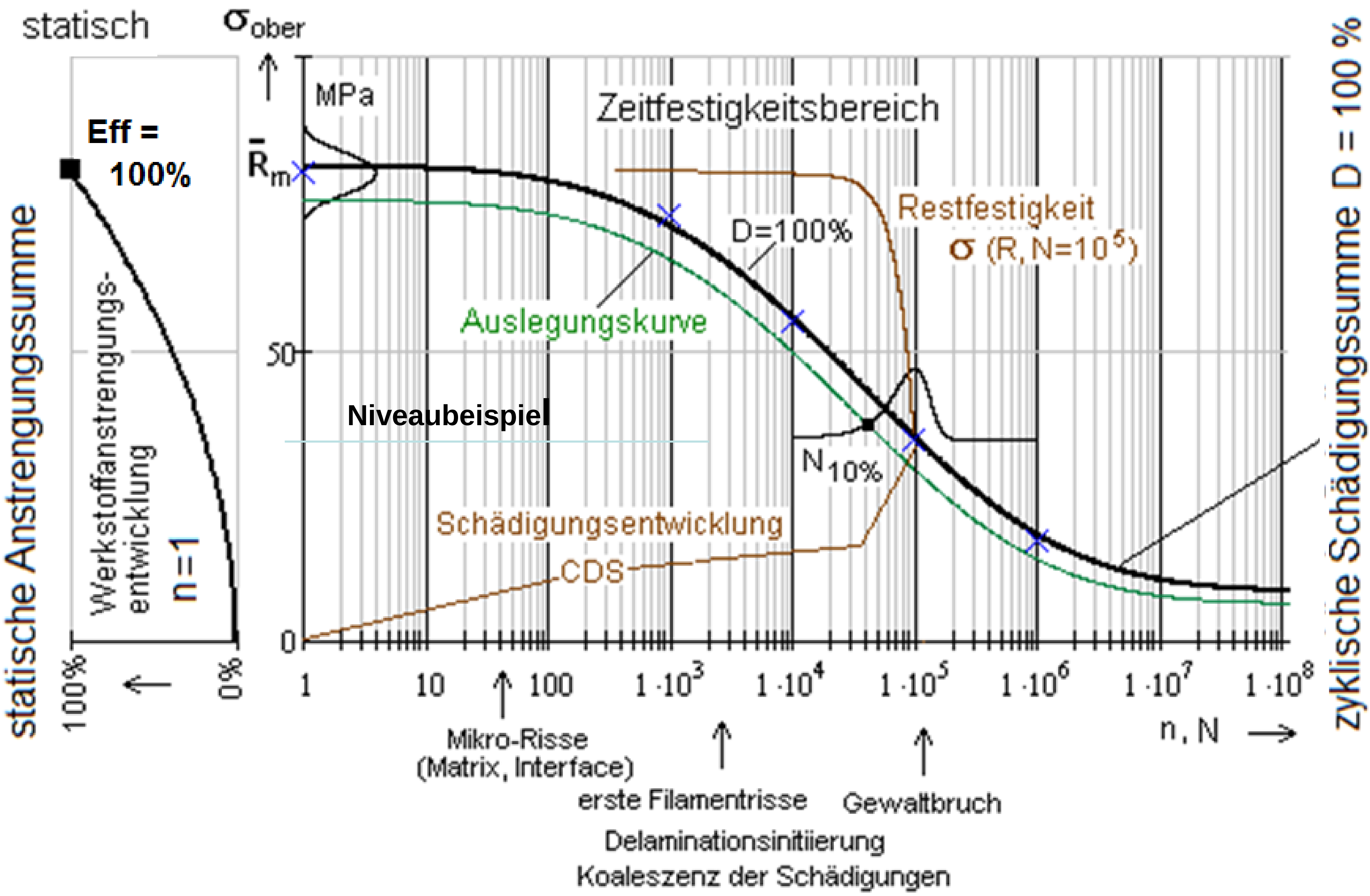
- **Local Stress at a critical material 'point': continuumsmechanics, strength criteria**
  verification by a *basic strength* or a *multi-axial failure stress state*
  Applied stresses are local stresses

- **Stress concentration at a notch (stress peak at a joint): notch mechanics**
  verification by a *notch strength (usually Neuber-like, Nuismer, etc..)*
  'Far'-field stresses are acting and are not directly used in the notch strength analysis

- **Stress intensity (delamination = crack): fracture mechanics**
  verification by a *fracture toughness (energy –related)*
  ***Applied stresses are 'far'-field stresses.*(far from the crack-tip)**

is valid, statically and cyclic.

Statische und zyklische Schädigungsentwicklung, sowie Wöhlerkurve *R = 0*



CDS:= characteristic damage state at the end of diffuse damaging

<u>Brittle</u> **Behaving Composites**

lessons learnt from other brittle materials

Cyclic fatigue life consists of three phases:

Phase I: Increasing damaging in embedded Lamin<u>as</u> up to discrete damage onset
(determination of accumulating damaging portions (= Schädigungen), initiated at end of elastic domain and dominated by diffuse micro-cracking + matrix yielding, and finally micro-delaminations)

Phase II: Stabile local growth of discrete damaging in Lamin<u>ate</u> up to delamination
(growth of dominating discrete micro-crack widths incl. micro-delaminations)

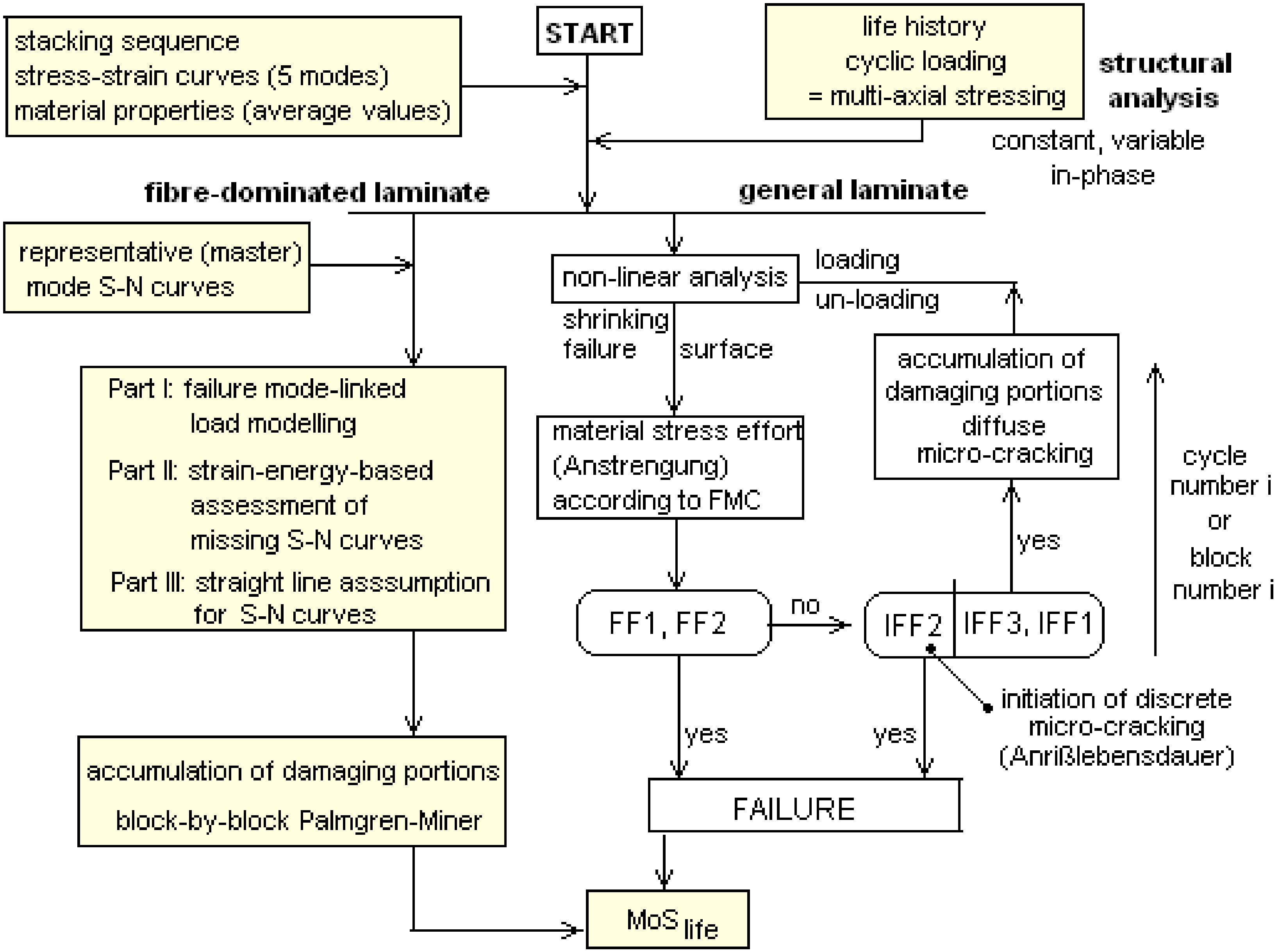
Phase III: Final in-stabile fracture of Lamin<u>ate</u> initiated by FFs, IFF2 of any lamina
+ possible delamination (= Schaden) criticality of the loaded laminate

FF:= fibre failure. IFF:= Inter Fibre Failure

CDS:= characteristic damage state at the end of diffuse damaging

- ***Determination of damaging portions (from diffuse and later discrete damaging)***
- ***Accumulation of damaging portions (cycle-wise, block-wise, or otherwise ?)***

Failure-Mode-Concept-based Lifetime Prediction



stacking sequence
stress-strain curves (5 modes)
material properties (average values)

START

life history
cyclic loading
= multi-axial stressing

structural analysis

constant, variable
in-phase

fibre-dominated laminate

general laminate

representative (master)
mode S-N curves

non-linear analysis

loading

un-loading

shrinking
failure

surface

Part I: failure mode-linked
load modelling

Part II: strain-energy-based
assessment of
missing S-N curves

Part III: straight line asssumption
for S-N curves

material stress effort
(Anstrengung)
according to FMC

accumulation of
damaging portions
diffuse
micro-cracking

cycle
number i
or
block
number i

FF1, FF2

no

IFF2

IFF3, IFF1

yes

yes

yes

initiation of discrete
micro-cracking
(Anrißlebensdauer)

accumulation of damaging portions
block-by-block Palmgren-Miner

FAILURE

MoS$_{life}$

Failure mode-based Lifetime Prediction Method

Approach incl. Accumulation of Damaging Portions

Logic behind: Fatigue strain energy, required to generate a distinct damage state is equal to the strain energy, which is necessary under monotonic loading to obtain the same damage state.

$$\Delta W = \sum_{1}^{5} \Delta W^{\text{modes}}$$

strain energy of all mode contributions (5 in the UD case)

Idea demonstrated for simple case of ‘well-designed, laminates under tension, where the change of strain energy between maximum and minimum loading for FF1 reads:

$$\Delta W^{\parallel\sigma} = \Delta(\sigma_{eq}^{\parallel\sigma} / \bar{R}_{\parallel}^{t})^{2} \Rightarrow \qquad \Delta W^{\parallel\sigma} \cdot \bar{R}_{\parallel}^{t\,2} = \sigma_{1,\max}{}^{2} - \sigma_{1.\min}{}^{2} = \sigma_{1,\max}{}^{2} \cdot (1 - R^{2})$$

Solving for the maximum stress delivers:

$$\sigma_{1,\max}(n) = \bar{R}_{\parallel}^{t} \cdot \sqrt{\Delta W^{\parallel\sigma}(n) / /(1 - R^{2})}.$$

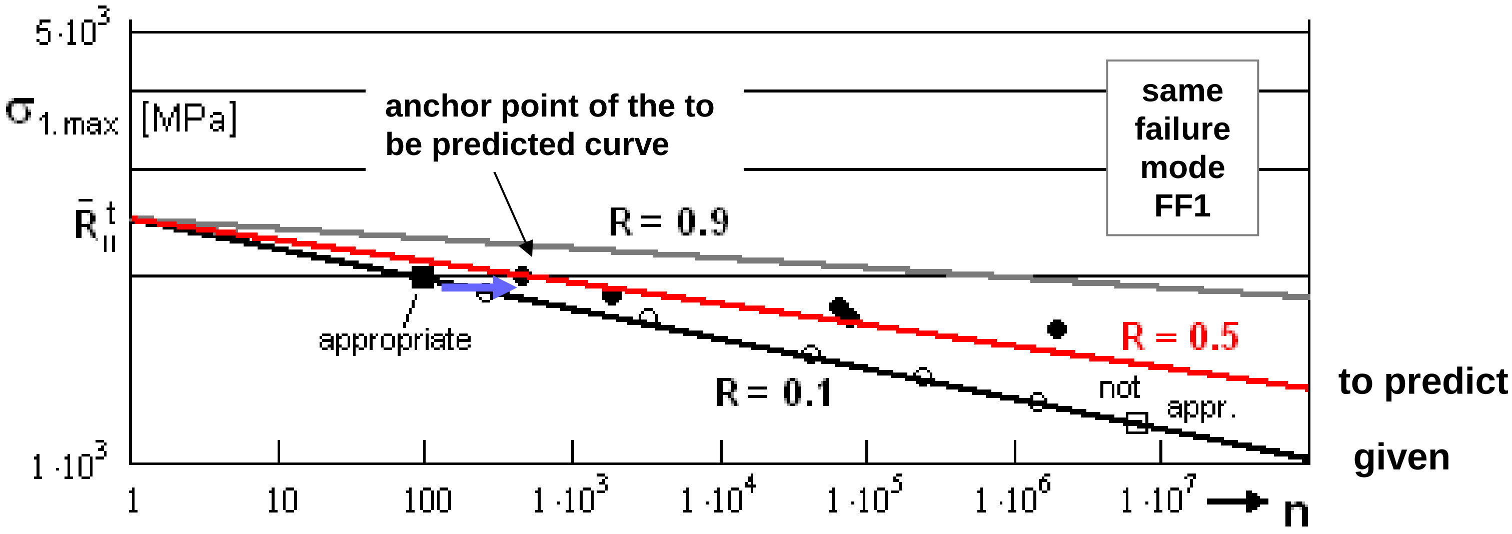
From experiment known:

- Max stress + tensile strength + stress ratio *R;* and thereby the *fatigue strain energy*.
- Course of strain energy can be described by a simple power law function, forming a straight line in a log-log diagram:

$$\Delta W^{\parallel\,\sigma}(n) = c_1 \cdot n^{-c_2}$$ [Hwang] .

Failure mode-based Lifetime Prediction Method

Procedure for the Prediction of S-N curves (test-based Example)



Given:

Test points +

$$\Delta W^{\parallel\sigma}_{R=0.1}(n)$$
$$= c_1 \cdot n^{-c_2}$$
$$= 0.89 \cdot n^{-0.097}$$
$$n_{appr} = 100 cycles$$

III : S-N curve may be mapped by a straight line in a log-log graph (safe side)

Given: normalized mode-representative curve (R = 0.1); to be predicted curve: (R > 0.1)

$$\sigma_{1,\max\ repr}(n) = \bar{R}_{\parallel}^{t} \cdot \sqrt{\frac{\Delta W^{\parallel\sigma}_{R=0.1}}{1-R_{repr}^{2}}} = \bar{R}_{\parallel}^{t} \cdot \sqrt{\frac{c_1 \cdot n^{-c_2}}{1-R_{repr}^{2}}} = \bar{R}_{\parallel}^{t} \cdot n^{c_{repr}(n)} \approx \bar{R}_{\parallel}^{t} \cdot n^{c_{repr}},$$

$$\sigma_{1,\max\ pred}(n) \approx \bar{R}_{\parallel}^{t} \cdot n^{c_{pred}}$$

Example *R=0.5* : Procedure to determine c$_{pred}$ *(one anchor point needed besides the strength point)* is depicted below:

$$\sigma_{1,\max\ repr}(n_{appr}) = \bar{R}_{\parallel}^{t} \cdot \sqrt{\frac{c_1 \cdot n_{appr}^{-c_2}}{1-R_{repr}^{2}}} = \sigma_{appr}$$

shift from representative curve to predicted curve →

$$\sigma_{appr} = \bar{R}_{\parallel}^{t} \cdot \sqrt{\frac{c_1 \cdot (n_{appr} \cdot f_{pred})^{-c_2}}{1-R_{pred}^{2}}}$$

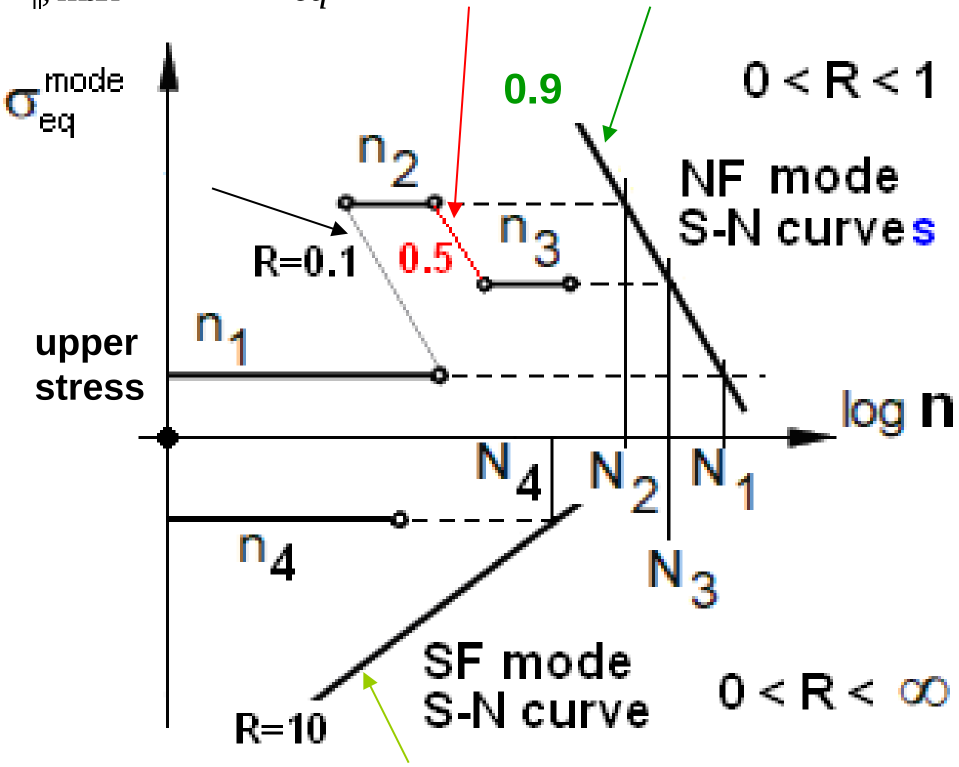
$$c_{pre} = \ln(\bar{R}_{\parallel}^{t} / \sigma_{appr}) / \ln(n_{appr} \cdot f_{pre}) = -0.034 \quad \Leftarrow \quad f_{pred} = \exp[-\ln(\frac{R_{pred}^{2}-1}{R_{repr}^{2}-1}) \cdot \frac{1}{c_2}] = 17.5 \quad \leftarrow$$

R = 0.5

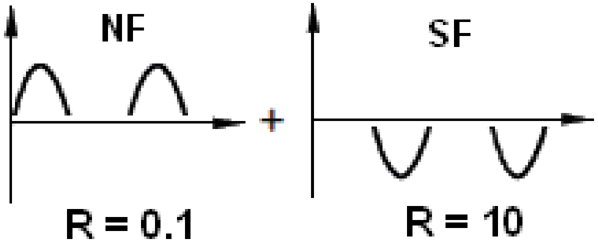
Failure mode-based Lifetime Prediction Method

Schematic Application (principle: for simple isotropic case as example, 4 block

$$\sigma_{\|,\max}^{mode} \Rightarrow \sigma_{eq}^{mode}$$



remind



here:

2 master curves

NF: R = 0.1

SF: R = 10

2 predicted curves

NF: R = 0.5, 0.9

Miner application:

$$D = n_1 / N_1 + n_2 / N_2 + n_3 / N_3 + n_4 / N_4$$