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Is a Costly Re-Design Really Justified if Slightly Negative Safety Margins are Encountered? (Part 2)

Teure Umkonstruktion bei leicht negativen Sicherheitsmargen? (Teil 2)

Abstract *The in KONSTRUKTION 3-2005 previously published part of the article gave some hints why a costly re-design in case of a slightly negative safety margin is not mandatory. Scattering design parameters such as load or strength were addressed as well as the terms risk, reliability, factor of safety and margin of safety (MS) were discussed. In this second part the deterministic MS is assessed by a probabilistically computed MS. This assessment is substantiated by a parameter investigation. The main conclusion outlines: In case of slightly negative MS values one just should try –instead of stopping the development- to increase as a counteracting means the reliability of the input parameters. This conclusion primarily aims at aerospace engineering, but may also be applicable to other engineering disciplines where risk analysis and margins of safety are of importance.*

Inhalt *Der in KONSTRUKTION 3-2005 zuvor veröffentlichte Teil dieses Fachaufsatzes gab einige Hinweise, warum eine teure Umkonstruktion im Fall kleiner negativer Sicherheitsmargen nicht zwingend notwendig ist. Dazu werden sowohl streuende Konstruktionsparameter wie Last oder Festigkeit angesprochen als auch werden die Begriffe Risiko, Zuverlässigkeit, Sicherheitsfaktor und Sicherheitsmarge (MS) diskutiert. Im zweiten Teil wird die deterministische MS mittels einer probabilistisch berechneten MS bewertet. Diese Bewertung wird untermauert durch eine Parameteruntersuchung. Hauptschlussfolgerung ist: Im Falle kleiner negativer MS-Werte soll man als Gegenmittel –anstatt die Entwicklung zu stoppen- versuchen, die Zuverlässigkeit der Eingabe-Parameter zu erhöhen. Diese Schlussfolgerung zielt hauptsächlich auf den Luft- und Raumfahrtbereich, ist aber ebenso anwendbar auf andere Ingenieurdisziplinen, wo Risikoanalyse und Sicherheitsmargen von Bedeutung sind.*

Probabilistic Solution:

a) Solution with 4 stochastic parameters

With the code COMREL [Com87] the four-dimensional convolution integral with the stochastic design parameters $X_j = R_{p0.2}$, p_{int}, d, t can be solved according to Eq (1)

$$p_f = P(R \le S) = P(R - S \le 0) = \\ = P\left(g(X_1, X_2, X_3, X_4) \le 0 = \\ = \int_{g(X)\le 0} f_{\underline{x}}(x)\,dx. \qquad (12)$$

Mind: Results are x_j, α_j and p_f.

These results are the coordinates x_j of the most likely failure point x^* and the sensitivity measures α_j. Both are depicted in *Table 2*. All other combinations of x_j have lower failure probabilities than the depicted $p_f = 1.8 \cdot 10^{-7}$.

b) Solution with 2 stochastic parameters

If the probabilistic task with four stochastic design parameters $X_j = R_{p0.2}$, p_{int}, d_m, t is reduced to a two-parameter '*R-S* problem', the difference in *MS* between the deterministic and the probabilistic treatment can be simply quantified and graphically illustrated by two distributions, one stands for *R* and one for *S*. This is done by splitting the problem into a load resistance part, which is identical to the strength part *R* (represents $R_{p0.2}$) and a stress part *S* (represents here σ_{eq} (p_{int}, d, t)), which depends on three of the four parameters. This stress part reads

$$\sigma_{eq} = p_{int} \cdot \frac{d}{4t} \cdot \sqrt{3} \qquad (13)$$

and describes the distribution of the Mises equivalent stress.

With the code COMREL [Com87] and the limit state function (failure criterion)

$$g(X_1, X_2) = R_{p0.2} - \sigma_{eq} = 0, \qquad (14)$$

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j	X_j	μ_j	σ_j	f_j	CoV_j
1	σ_{eq}^{Mises}	295 MPa	18.6 MPa	ND	6.3 %
2	$R_{p0.2}$	442 MPa	22.1 MPa	ND	5 %

input

α_j	x_j of x^*	p_f
-0.66	**356** MPa	$1.8 \cdot 10^{-7}$
+0.76	**356** MPa	

results from Annex 1

Table 3

Input and solution for the 2 parameter (COMREL-reduced) problem

the integral in its various versions (see [Tho82, p. 73])

$$p_f = P\left(g\left(R_{p0.2} - \sigma_{eq}\right)\right) = \qquad (15a)$$

$$= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{r=s} f_R(r)\cdot dr\right) f_s(s)\, ds =$$

$$= \int_{-\infty}^{+\infty} F_R(x) f_s(x)\, dx = \int_{-\infty}^{+\infty}\int_{g\le 0}^{+\infty} f_{R,S}(r.s)\, dr\, ds$$

can be solved fully utilizing the distribution function F_R and density function f_S (data, see *Table 3*). Just a simple 'R-S problem' remains to be tackled (see *Table 1*, too) by the integration.

Also, the above mentioned analytical solution, Eq. 3, can be applied

$$p_f = \Phi(-\beta) \quad \text{with}$$

$$\beta = (\mu_R - \mu_S) / \sqrt{\sigma_R^2 + \sigma_S^2}. \qquad (15b)$$

In usual application of COMREL or of another similar code, just a p_f value is calculated for the given probability density functions f_j of all stochastic design parameters (X_1, X_2, X_3, X_4). However here, for the purpose of a comparison, a *MS* value has to be computed in addition because both numbers MS_{prob} and MS_{det} are to be based on the same failure probability p_f. This needs the determination of the p_f-associated design stress value σ_{prob}, which belongs to the chosen value $p_f = 1.8\ 10^{-7}$. As this is a distinct value on the probability distribution function curve $F_S(\sigma_{eq})$ one has to generate this function. The problem consists in the determination of point estimates. As limit state function serves, analogous to Eq. 14,

$$g(X_1, X_3, X_4) = \sigma_{eq} - p_{int}\cdot\frac{d}{4t}\cdot\sqrt{3} = 0. \qquad (16)$$

Figure 6 delineates the procedure to determine the σ_{eq} function: 1) Increase continuously $\sigma_{eq,k}$; 2) Compute increasing

$$p_{f,k} = P\left(g\left(\sigma_{eq} - p_{\text{int}}\cdot\frac{d}{4t}\cdot\sqrt{3}\right)\right),$$

(COMREL was utilized), each representing a single point on $F_S(\sigma_{eq})$; 3) From the fully derived F_S (normal distribution assumed) estimate its statistical parameters $\mu_{\sigma eq}$, $\sigma_{\sigma eq}$, and an approximate density distribution $f_{\sigma_{eq}} = f_{\sigma_{eq}}^{Mises}$ will be achieved. The accuracy depends on the number *k* of the chosen computation points. Further *Fig. 6* depicts, how the density function $f_S(\sigma_{eq})$ is linked to the distribution function $F_S(\sigma_{eq})$.

Now, with the knowledge of $\mu_{\sigma_{eq}}$, $\sigma_{\sigma_{eq}}$, the final *R-S* convolution integral reduces to the desired manually determinable Gauss type integral (see Eq. 15b), if it can be assumed that all stochastic design parameters are normally distributed. Applying above COMREL results allows the simplified *Eqs (3)* and (*4*) to be applied in the (R-S)-analysis. As margin of safety

$$MS_{prob} = \frac{\text{strength design allowable}}{\text{stress at design load, chosen } p_f} -$$

$$-1 = \frac{R_{p0.2}}{\sigma_{prob}^*} - 1 = \frac{376}{356} - 1 \equiv +5.9\,\%$$

is yielded, obviously a very satisfying positive value. The results are collected in *Table 3*.

It is documented that the sensitivity measures practically have not changed by the intermediate probabilistic computation step.

Mind: Results are $\Rightarrow \alpha_j$, x_j, and σ_{prob}^*, MS_{prob}^* at chosen $p_f = 1.8\ 10^{-7}$.

Note: Much practise in the past -with a distinct type of structures- helped and helps to produce such types of structural parts which are 'safe enough'. The similar results from FoS-based or reliability-based treatment underline this [Rac04].

Discussion of results of the case study:

- The most likely failure point is obtained for the combination of x_j values shown in the output section of *Table 2*, related to a chosen failure probability $p_f = 1.8 \cdot 10^{-7}$ ($\le p_f^{admissible} = 2 \cdot 10^{-7}$) that corresponds to a reliability index $\beta = 5.1$.
- The probabilistic calculation leads to $MS_{p0.2} = +5.9\,\%$ (*Table 4*) while the conventional method results in *–2.6 %*, thus, a remarkable *9 %* difference between the deterministic-based and the probabilistic-based $MS_{p0.2}$ (at *DYL* level) is given for above p_f which is marginally smaller than the assigned value $p_f^{admissible}$.
- Structural reliability analysis reveals the influence of each stochastic design parameter on the distinct failure mode through the values of the design sensitivity measures α_j. The code COMREL provides such values (see *Table 2*) that denote the relative importance in the actual failure mode

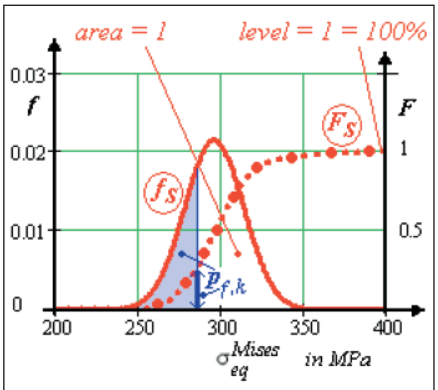

Figure 6

Estimation of the probability density function f_S for 'onset of yielding' from point-wise determined probability (cumulative) distribution function F_S. [Cun88]

The probability $p_{f,k}$ is identical to the area under f_S and to the ordinate of the distribution function F_S. The filled circles are the k computed points $p_{f,k} = P(g_k(X_j) \le 0$.

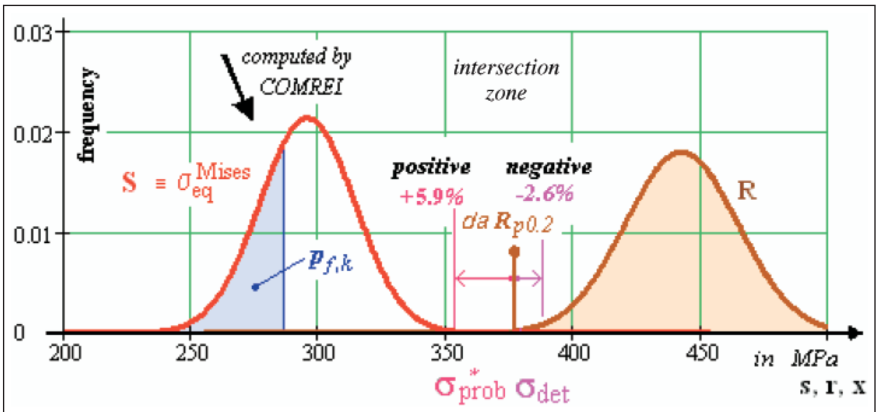

Figure 7

Probabilistic and deterministic strength Design Verification on stress level. σ_{prob}^* corresponds to the most likely failure situation ($p_f = 1.8\ 10^{-7}$) the structure may experience.

σ_{prob}^* = 355 MPa, σ_{det} = 386 MPa, $daR_{p0.2}$ = 376 MPa

stress level in MPa	$daR_{p0.2}$ = 376	$\sigma_{prob^*}(p_f)$ = 355	$\sigma_{det}(j_{po2})$ = 386	P_f = 1.8 10^{-7}	β = 5.1	MS_{prob^*} = +5.9% !	MS_{det} = -2.6% !!
				probabilistically			deterministic.

Table 4

Comparison of deterministic and probabilistic results of the vessel example.

$DYL = DLL \bullet j_{po.2} = 6.83 \bullet 1.1 = 7.51$ bar = 0.751 MPa; $R_{p0.2} = \mu_R - k_R\sigma_R = 376$ MPa

(here yielding). The uncertain yield strength $R_{p0.2}$ has the dominating influence (α = +0.76) on the reliability $\Re$, followed by internal pressure p_{int} (α = –0.62).

Fig. 7 illustrates the different results obtained in the deterministic and the probabilistic strength analysis. Given is a load pressure *distribution* not only a *fixed number*. The conventional procedure is to compute 'worst σ' and compare it with the design allowable $daR_{p0.2}$, which in the case study considered here results in a negative *MS*. In order to better understand these values, and to get a comparative feeling about the failure risk, *Table 4* demonstrates the effect of a consequent probabilistic thinking and its advantages.

Structural reliability thinking, this means 'thinking in uncertainties of the driving design parameters', helps to judge the deterministic design procedure. Understanding how the various design parameters influence the *MS* value, provides a feeling about the failure risk, which an *MS* value alone cannot do. This is best illustrated by the given simple probabilistic example.

An improved data base for the stochastic load model almost always will result in a more economic design. Further, reducing the scatter of important (see sensitivity measures) geometrical tolerances will improve the design. Truncated distributions, to be achieved by some more effort in measurement and in controlling, will lead to an improved reliability, too. Reducing the fluctuation of the structural response (stress field) provides increased confidence in the analytical results. Both, an increasing mean value and a decreasing standard deviation will lower p_f.

<u>Note</u>: Probabilistic thinking highly pays off.

4.2.3 Sensitivity of MS and p_f to Distribution Parameters (μ, σ)

Fig. 8a presents the interrelation of a fixed load pressure distribution, the so-called density distribution f_S and four hypothetical strength distributions f_R with the associated *MS* values and the failure probability $p_f = 1 - \Re$. *Fig. 8a* depicts how far – to gain safety distance – the strength distribution f_{R1} has to be shifted to reach *MS* (≡ MS_{prob}) = +10% and +20%. Further, the effect of lowering the scatter, by utilizing f_{R2} instead of f_{R1}, is outlined.

Fig. 8b zooms the intersection zone of the fixed load distribution with two different load resistance distributions the difference of which is just caused by the different coefficients of variation, whereas the mean value remains the same. As the interference (overlapping) zone is much bigger in case of f_{R1} it becomes $p_{f1} > p_{f2}$. Depicted in the intersection zone is the distribution of the failure probability fp_{f2}. The area below this curve corresponds to the value of p_{f2}. The zoom b indicates the different size of the interference area, and the curves in c indicate the resulting effect of the different size. The distribution function f_{R1} leads to a higher failure probability than f_{R2}.

Fig. 8c depicts how the resistance curve f_{R1} (*CoV* remains constant) has to be shifted to achieve higher *MS* or p_f values: 1.) *MS* = 0%, $p_f = 5\ 10^{-5}$; 2.) *MS* = 10%, $p_f = 2 \cdot 10^{-7}$; 3.) *MS* = 20%, $p_f = 1 \cdot 10^{-9}$. The squares on the p_{f1} curve indicate which reduced failure probability or increased structural reliability is reached in the example for *MS* is 0%, 10%, 20%. Reducing the scatter, when taking f_{R2} instead of f_{R1}, increases reliability (reduces failure probability) and *MS*.

For the given conditions (f_S, f_R), according to *Fig. 8* the following values are computed: $p_f = 1.8 \cdot 10^{-7}$ and *MS* = +10%. Probabilistically, this theoretically means failure if as target value e.g. 10^{-8} is assigned.

<u>Notes</u>:
- *The larger the distance between the stress and the strength distribution the larger become MS and the survival probability = reliability $\Re = 1 - p_f$.*
- *Goal of the design is the achievement of a reliable structure. One might reach this goal by the achievement of a positive MS.*
- *A design is not reliably verified if an assigned target value $\Re = 1 - p_f$ is not met or, a positive MS verifies a design reliably only if an assigned target value for $\Re$ is met.*

4.2.4 Effect of Variation of Strength Density Distribution on MS and p_f

A variation of the strength density distribution shall highlight the impact on the reliability and on the margin of safety. Due to the achievement of a qualified production process and some more experimental data the mean value decreased a little, $\mu_{Rp0.2}$ = 440 MPa, but the standard deviation of $\sigma_{Rp0.2}$ was reduced from 22.1 MPa to 19.5 MPa. Thereby the design allowable changes to $daR_{p0.2} = 440 - 3 \cdot 19.5 = 381.5$ MPa which causes the better values for p_f and *MS* in *Table 5*. The difference between the deterministic and the probabilistic *MS* becomes about 6% for a $p_f = 3.2 \cdot 10^{-8}$.

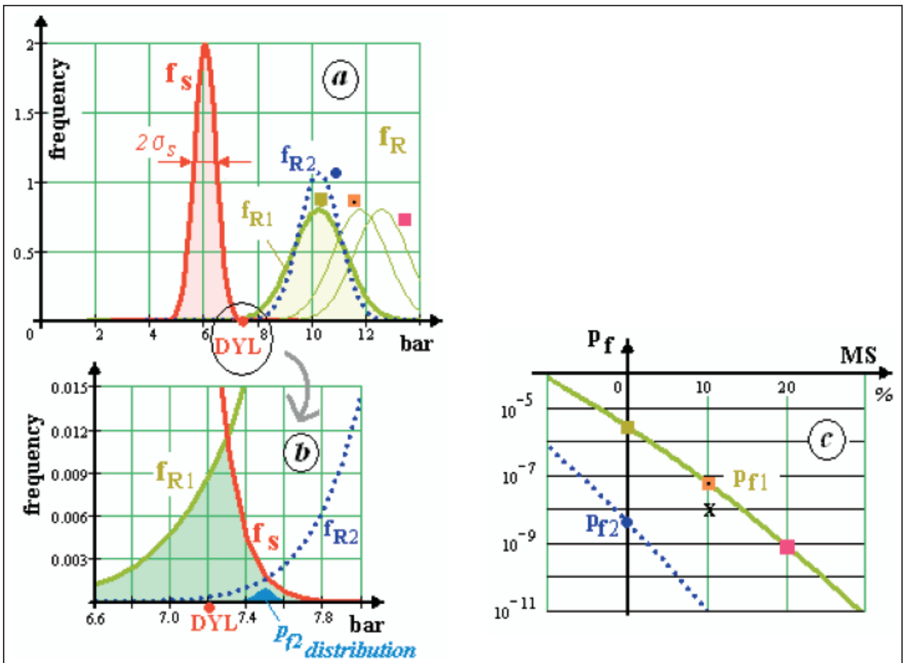

Figure 8

Case study for visualization (at 'onset of yielding') of distribution characteristics: Sensitivity of *MS* and p_f to density distributions f_S of load *S* and f_{Ri} of load resistance *R*, and the inter-relation of *MS* with p_f. chosen: $k_R = 3$, $k_S = 2.3$.

j	X_j	μ_j	σ_j	f_j	CoV_j
1	σ_{eq}^{Mises}	295 MPa	18.6 MPa	ND	6.3 %
2	$R_{p0.2}$	**440** MPa	**19.5** MPa	ND	4.4 %

input

α_j	x^*_j	p_f
-0.69	**363** MPa	$3.2 \cdot 10^{-8}$
0.73	**363** MPa	

results from Annex 1

$daR_{p0.2}$ = **381.5**	$\mu_S = 295$	$\sigma^*_{prob}(p^*_f)$ = **363MPa**	$\sigma_{det}(DYL)$ =386 MPa	p^*_f = **3.2 10^{-8}**	β = 5.4	MS_{prob}* = **+5.1%**	MS_{det} = **- 1.1%**
	$\mu_S = 302.9$	$\sigma_{prob}(p_f)$ > **368MPa**	$\sigma_{det}(DYL)$ =396MPa	p_f = 1.8 10^{-7}	β = 5.1	MS_{prob} =+ **3.7%**	MS_{det} = **- 3.7%**
stress level *in MPa*				**probabilistically**			**determin.**

Table 5

Effect of variation of strength density distribution. Input, solution and display of deterministic and probabilistic results

If the design situation allows for the previously taken $p_f = 1.8 \cdot 10^{-7}$, then, the stress situation may be sharpened (means load increase) by shifting f_S from $\mu_S = 295$ *MPa* to $\mu_S = 302{,}9$ *MPa* and $\sigma^*_{prob} = 363$ *MPa* can be even replaced by a value σ_{prob}. = 368 *MPa*. Of course, this leads to a margin loss, however the p_f requirement is met.

Note: Design Verification benefits from an improved strength situation. Scatter plays a big role.
A probable project requirement for a survival probability or reliability $\Re = 1 - p_f$ decides on the acceptance of a design.

Conclusions

The conclusions drawn from the case study considered here are primarily aimed at aerospace engineering, but may also be applicable to other engineering disciplines where risk analysis and safety margins are of importance and where loads get close to the point of failure. Engineering disciplines where large *FoS* are a prerequisite, e.g. nuclear engineering, are less likely to benefit as any variation in *MS* value between the two concepts will be small compared to the overall *FoS* required there.

Risk comes in two varieties: a) a Technical Risk, associated with the complexity of failure potentials and hazardous characteristics of the actual structural system, and b) a Programmatic Risk, associated with the failures to meet the project goals cost and schedule.

Of course, the essential findings, presented now, cannot be taken as fully generally valid conclusions. However, they will make us sensitive to think about how one can lower risk and improve risk management. This is delineated by the following bullets:

- A slightly negative *MS* value must not necessarily be critical as demonstrated in the previous case study.
- MS values should not be interpreted as absolute and final safety measures. They cannot provide a 'feeling' for the risk of failure. The simple example has demonstrated for an assigned p_f a remarkable difference between the two procedures.
- An appropriate '*Think (about) uncertainties*' attitude has to be developed in order to identify the main sources of uncertainty and to reduce the scatter of the *driving design parameters*. This could result in shorter product development and in cost reductions. Such attitude should be adopted by all parties involved in the design verification and certification process, i.e. design office at the supplier, customers and licensing authorities. The important message is that concentrating efforts on reducing the uncertainties can be far more productive than lengthy discussions over slightly negative margins.
- Of all uncertain design parameters it is in many cases the *load* uncertainty, which has the highest scatter and affects analyses and tests most. Therefore, reducing the uncertainties of the stochastic design parameters, especially of the loads, as the project progresses along will be most effective and economic. Further, one has to eliminate all so-called 'pocket factors' (i.e. the load engineer's reserve in the uncertainty considering K-value or, when the design engineer utilizes a too low strength design allowable) along with the maturity of the design and reserve the *MS* for the still remaining unknown effects that should not be hidden by not officially known pocket factors. A distinct uncertainty has to be covered by clearly dedicated, risk-dependent *FoS*, only! One should improve the quality of the design input and increase the sample size or improve test information for establishing better strength design allowables and design loads with the maturing design.
- In the late phase of development, supplier and customer (and licensing authorities, if applicable) should jointly decide whether the risk is acceptable, whether more structural testing is necessary, or whether really a re-design is needed, [Sar97]. In this context, slightly negative *MS* values have to be interpreted with care applying best engineering judgement.

Final remarks:

– The goal of any design engineer should be to end up with a robust design. To achieve this, the driving stochastic design parameters have to be used to outline the robustness of the design against the envisaged actual failure mode by firstly computing the sensitivity measures α_j and then investigating the reduction of the

load *in bar*		**load resistance at** yield initiation *in bar*					
μ_j	σ_j	μ_j	σ_j	*da LR*	p_f	β	MS_{det}
6	0.36	10	1.0	*7.00*	$8.4 \cdot 10^{-5}$	*3.76*	*-6.8%*
		10	0.75	*7.75*	$7.6 \cdot 10^{-7}$	*4.81*	*+3.2%*
		11	1.0	*8.00*	$1.3 \cdot 10^{-6}$	*4.70*	*+6.5%*
		12	1.0	*9.00*	$8.2 \cdot 10^{-9}$	*5.65*	*19.8%*
		'evaluated' test results					

Table 6

Comparison of deterministically derived *MS* values and failure probabilities p_f due to different load resistance distributions. $DLL = \mu_S + k_S\,\sigma_S$, $DYL = DLL \bullet j_{p0.2}$, $da\,LR\,(R_{p0.2}) = \mu_R - k_R\sigma_R$, chosen: $k_R = 3$, $k_S = 2.3$; $MS_{det} = daLR/DYL$

design's sensitivity to changes of X_j *while* keeping pf at the prescribed level.

- Verderaime [Ver92] calls the application of the FOS concept with its superposition of worst case assumptions on load, environment, damage size etc. an 'Inherent violation of the error propagation law incurred when reducing statistical data to deterministic values'. A probabilistic method tackles the 'combined uncertainties' and respects the probability of occurrence.
- A probabilistic method adds technical information being not attainable by the FOS concept. It enables to disclose the risk characteristics determined by the design-driving parameters. Efficient numerical solution procedures such like FORM deliver the desired numbers.
- The applicability of a probabilistic method is mandatory when a reliability target or a failure probability p_f has to be met for the full failure system (e.g. launcher). This system failure probability p_f consists of the *failure rate* dependent failure probabilities (valves, batteries, ..) and the *failure state* dependent failure probabilities from the structural sub-failure system.

Note: Experimental results can be far away from the reality like an in-accurate theoretical model. Theory 'only' creates a model of the reality and experiment is 'just' one realisation of the reality.

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Annex 1: Derivation of Data for Table 1, probability part

Input data *in MPa*: $\mu_R = 442$, $\sigma_R = 22.1$, $\mu_S = 295$, $\sigma_S = 18.6$.

1) Vector of stochastic variables:
$\underline{Y} = (X_1, X_2)^T = (R, S)^T$

2) Hasofer-Lind Transformation:

$$\underline{Y} = \begin{Bmatrix} R \\ S \end{Bmatrix} = \begin{bmatrix} \sigma_R & 0 \\ 0 & \sigma_S \end{bmatrix} \begin{Bmatrix} U_R \\ U_S \end{Bmatrix} + \begin{Bmatrix} \mu_R \\ \mu_S \end{Bmatrix} = [\underline{T}\{\underline{U}\}]$$

3) Limit state function:
$g(\underline{Y}) = g(R, S) = (R - S)$
$= (\sigma_R U_R + \mu_R) - (\sigma_S U_S + \mu_S)$

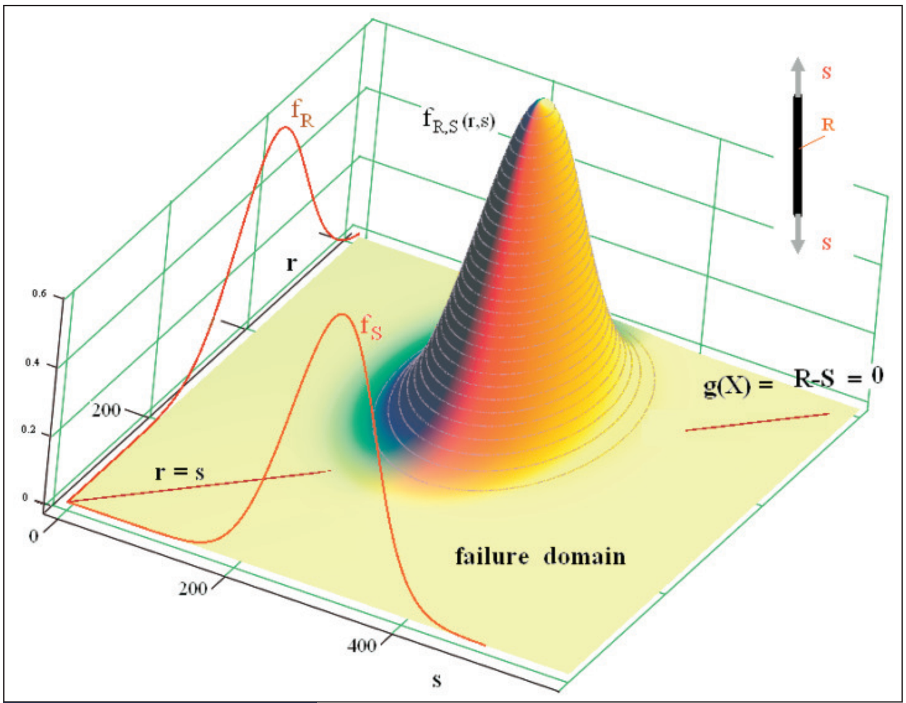


Figure A2-1
Probability hill (joint distribution function $f_{R,S}(r,s)$) with lines of constant probability.
Data: $\mu_R = 440$, $\sigma_R = 70$, $\mu_S = 300$, $\sigma_S = 50$; $p_f = 5.2 \cdot 10^{-2}$. Example: Tension rod

4) Standardization:
$\sigma_Z = (\sigma_R^2 + \sigma_S^2)^{0.5}$

5) Insertion of 2) into Limit state:
$g(\underline{Y})/\sigma_Z = (\sigma_R/\sigma_Z)U_R - (\sigma_S/\sigma_Z)U_S + (\mu_R - \mu_S)/\sigma_Z = 0$
$= \alpha_R U_R - \alpha_S U_S + \beta = 0,$
$= \alpha^T \underline{U} - \alpha^T \underline{u}^* = 0;$
$x_j^* = u_j^* \sigma_j + \mu_j.$

6) Equations for failure probability and reliability index β
$p_f = \Phi(-\beta)$ and
$\beta = (\mu_R - \mu_S)/(\sigma_R^2 + \sigma_S^2)^{0.5} = (\mu_R - \mu_S)/\sigma_Z$

7) Numerical example from Table 1b:
$\sigma_Z = (\sigma_R^2 + \sigma_S^2)^{0.5} = (22.1^2 + 18.6^2)^{0.5} = 28.9$ MPa
$\mu_Z = \mu_R - \mu_S = 442 - 295 = 147$ MPa,
$\beta = \mu_Z/\sigma_Z = 147/28.9 = 5.1$
$\alpha_R = \sigma_R/\sigma_Z = 22.1/28.9 = 0.765,$
$\alpha_S = -\sigma_S/\sigma_Z = -18.6/28.9 = -0.644$
$\sum\alpha_j^2 = (0.765^2 + 0.644^2) = 1.00$
$u_R^* = -\beta \cdot \alpha_R = -5.1 \cdot 0.765 = -3.88,$
$u_S^* = -\beta \cdot \alpha_S = -5.1 \cdot (-0.644) = +3.28$
$x_R^* = -3.88\ 22.1 + 442 = 356$ MPa $\cong x_S^*,$
$x_S^* = +3.28\ 18.6 + 295 = 356$ MPa
$p_f = \Phi(-\beta) = 1.8 \cdot 10^{-7}$, $\Re = 1 - p_f$.

Annex 2: Visualization of the Characteristic Quantities of the Probabilistic Solution Procedure

For a specific design case the following fictitious data set with very high standard deviations is chosen for achieving a good visualization of all characteristic quantities except of the sensitivity measures α_j (see [Tho82, Sch81]).

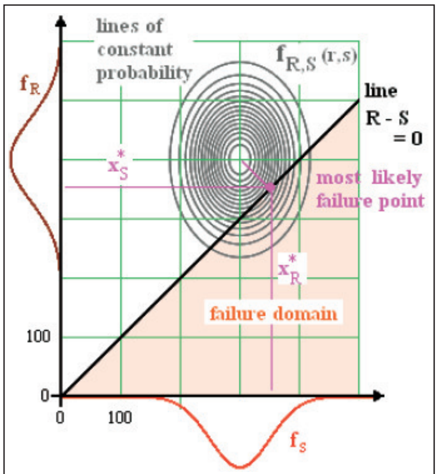


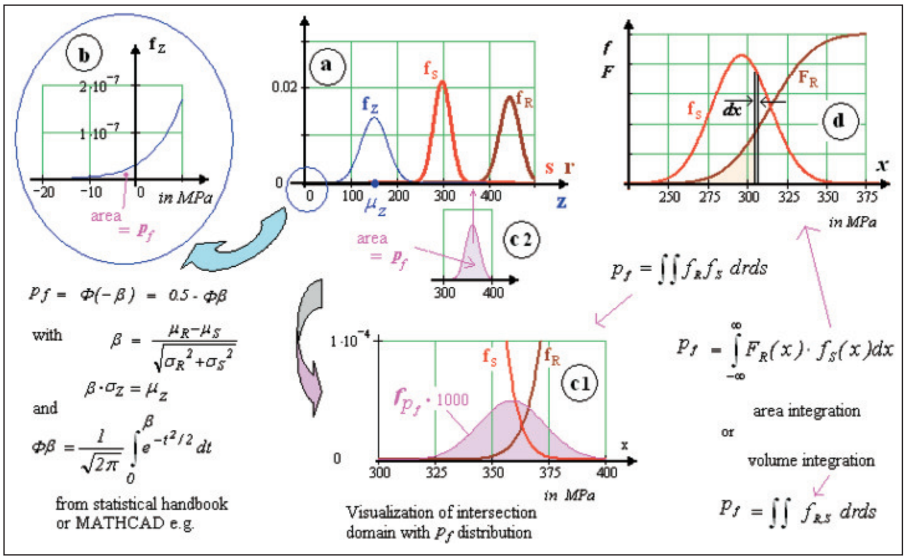
Figure A2-2
Visualization of the most probable failure point and the horizontally projected lines of constant probability on the probability hill

– **Input data** (*in MPa*)
$\mu_R = 440$, $\sigma_R = 70$, $\mu_S = 300$, $\sigma_S = 50$.

– **Characteristic quantities**
$\sigma_Z = 86.0$ MPa, $\mu_Z = 140.0$ MPa,
$\alpha_R = 0.814$, $\alpha_S = -0.581$, $\sum\alpha_j^2 = 1.00$,
$\beta = 1.627$, $p_f = 5.2 \cdot 10^{-2}$,
$x_R^* \cong x_S^* = 347$ MPa, $S = R$.

Attributed to the design case above is a failure probability of $p_f = 5.2 \cdot 10^{-2}$. This corresponds to 5.2 % of the volume of the probability hill $f_{R,S}(r,s)$ vertically cut off by $r = s$ or $g(X) = 0$.

Annex 3: Visualizations of types of the determination of failure probability p_f. Assumption: Normal Distributions



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