Gute Bemessung und Nachweis,
dass eine Festigkeits-Grenze noch nicht erreicht ist
verlangt die Anwendung
validierter Festigkeitsbedingungen.

Dazu gehören

Fließbedingungen für nicht-lineare Analyse und
Fließgrenzennachweis (duktiles Verhalten)

sowie

für den Bruchnachweis

= Festigkeitsbedingungen für Bruch

formuliert als ***F = 1 = 100%*.**

Zugehörige Bruch-Festigkeitsbedingungen und
deren Visualisierung als Bruchkörper
sind Gegenstand des Vortrags !

Wozu Bruch-Festigkeitsbedingungen ?

Mit 3D-validierten Bruch-Festigkeitsbedingungen

benötigt man später nur mehr

die immer auslegungs-notwendigen 1D-Festigkeitswerte,

als Stützpunkte für die Größenfestlegung

der Oberfläche des Bruchkörpers.

3D-Festigkeitsbedingungen für spröde Werkstoffe isotrop, transversal–isotrope UD-Schicht und orthotropes Gewebe

- ermittelt auf Basis des Failure-Mode-Concepts (FMC) von Cuntze -

1 General on Strength Failure Conditions (SFCs) ***criteria***
2 Some Notions, Global SFCs versus Modal SFCs
3 Basics of Cuntze's Failure-Mode-Concept (FMC)
4 Application: Isotropic Foam (Rohacell 71 G), *im Anhang Details*
5 Application: Transversely-isotropic UD-CFRP Lamina
6 Application: Orthotropic Ceramic Fabric
7 Anhang

Results of a time-consuming, never funded "hobby". ***Since 1970 in CFRP composite business.***

Prof. Dr.-Ing. habil. Ralf Cuntze VDI, formerly MAN-Technologie AG, linked to Carbon Composite e.V. (CCeV) Augsburg, heading the WGs "Engineering" (Mechanical Engineering, since 2009, "" Dimensioning and design verification of composite parts" in Civil Engineering' since 2011, and "Automated Manufacturing in Civil Engineering", since 2017.

Design Verification = Achievement of a Reserve against a Limit State

For each designed structural part

- for each distinct 'Load Case' with its single Failure Modes - must be computed:

<u>R</u>eserve <u>F</u>actor (is load-defined) **:** ***RF = Failure Load / applied Design Load***

Material Reserve Factor : f_{Res} = ***Strength / Applied Stress***

if linear analysis: $f_{Res} = RF = 1 / Eff$

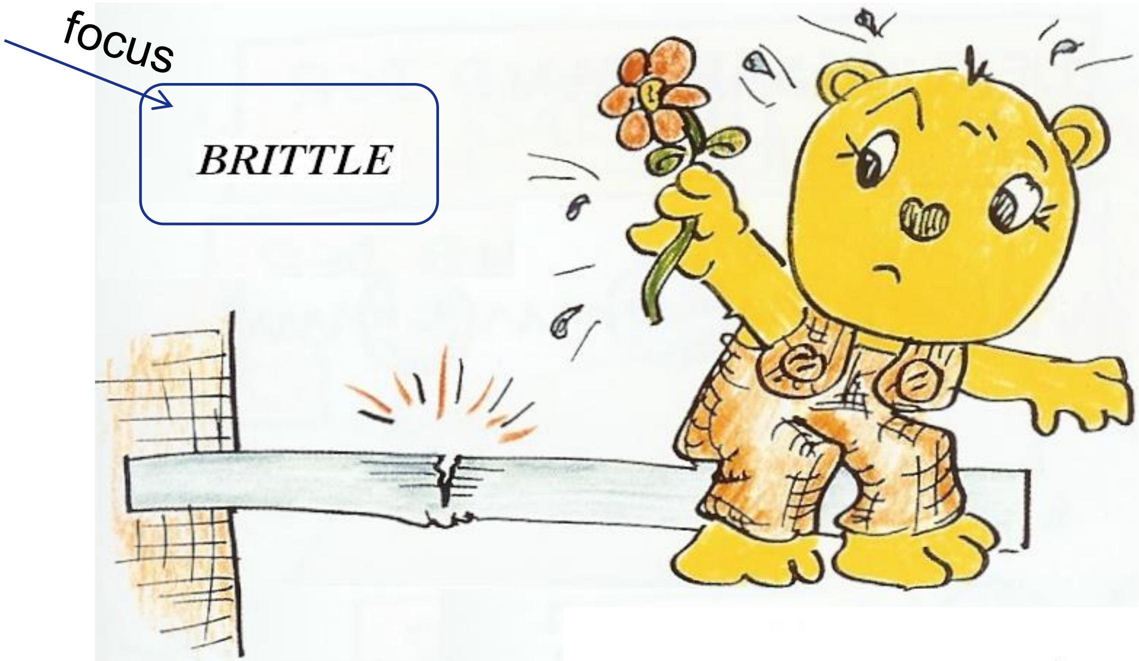
Material Stressing Effort : ***Eff = 100% if RF = 1***

(Kunstwort, entspricht Werkstoff-Anstrengung)

is applicable in linear and non-linear analysis.

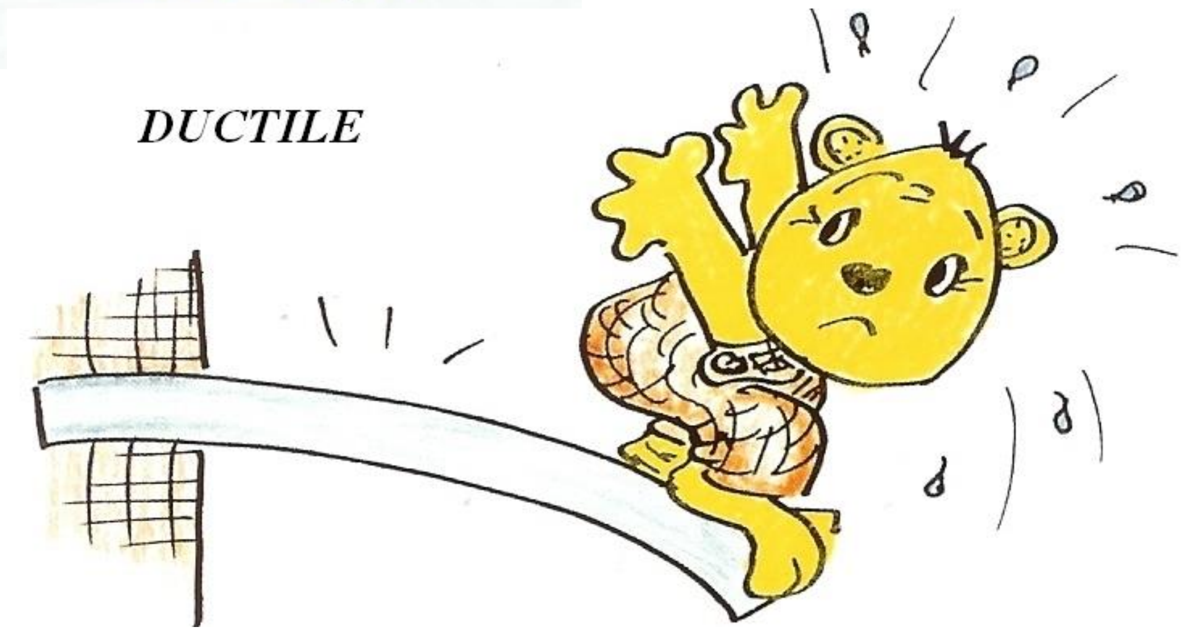
Eff : = accumulated static damaging portions under increased loading

How may one principally discriminate *Material Behaviour* ?



One feels good until sudden fracture occurs

Courtesy: Prof. C. Mattheck



Ductile Fracture = type of a failure mode in a material or structure generally preceded by a large amount of plastic deformation

Motivation: Erhalt einer Vergleichsspannung σ_{eq} !

Zwei Aspekte:

(1) Der Vergleichsspannungswert σ_{eq} beinhaltet die gemeinsame Wirkung =

Werkstoffanstrengung eines mehrachsigen Spannungszustands,

der bei einem bestimmten Versagensmodus aktiv ist

äquivalent* = gleichwertig dem Spannungszustand *wie in

**** Mises Vergleichsspannung: Modus ‘Schubspannungsfließen’***

**** Maximale Vergleichsspannung: spröd, Modus ‘Normalbruch ‘***

(2) Der einachsige Vergleichsspannungswert σ_{eq}

= vergleichbar mit Festigkeitswert *R*

des aktivierten Modus .

Motivation: Nutzung der Werkstoffanstrengung *Eff*

Werkstoffanstrengung Eff linear und nicht-linear anwendbar !

≡ *modal* material stressing effort *

$$Eff^{mode} = \sigma_{eq}^{mode} \,/\, \overline{R}^{mode}$$

equivalent mode stress

mode associated average strength

$$Eff^{fracture\ mode} = \sigma_{eq}^{fracture\ mode} / R_m$$

$$Eff^{Mises} = \sigma_{eq}^{Mises} / R_{p0.2}$$

z.B. v. Mises

Eff bei modalen Festigkeitsbedingungen immer klar definierbar

→

* **artificial technical term, created together with QinetiQ during the World-Wide-Failure-Exercise**

.. .. weitere Motivation: Art der Festigkeitsbedingungen

Globale verheiraten mathematisch alle Bruchmoden im Ansatz !

Nachteil: falls ein Festigkeitswert zu ändern ist, dann trifft es den ganzen Bruchkörper, wobei Teile des Bruchkörpers un-konservativ werden können, falls man den Verlauf aller Testdaten nicht wieder neu abbildet.

Drucker-Prager, Ottosen, Willam-Warnke, Tsai u.a.

1 Globale Festigkeitsbedingung : **F ({σ}, {R}) = 1 (übliche Formulierung)**

Satz von Modalen Festigkeitsbed. : **F ({σ}, R^{mode}) = 1 (im FMC gewählt)**

Mises, Cuntze

Interaktion notwendig

Isotrop: $\{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy})^T = (\sigma_I, \sigma_{II}, \sigma_{III})^T$

UD: $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$

Modale Festigkeitsbedingungen betrachten alle Modi getrennt:

Nachteil (klein): bedingt dann natürlich eine Interaktion aller Modi

Vorteile : Festigkeitswert-Änderung betrifft nur einen Modus,

Vergleichsspannungen σ_{eq} berechenbar !

Interaction of Single Strength Failure Modes in Modal FMC Applications

Interaction in the 'mode transition zones' of

adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

Summe der Bruchgefahranteile

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots} = 1 = 100\,\%, \text{ if failure}$$

with a mode-interaction exponent *$2.5 < m < 3$* **,** *from mapping experience*

It is assumed engineering-like : m takes the same value for all

mode transition zones captured by the interaction formula above

Later: example

Formulation of Strength Failure Conditions (SFC)

Heranziehung der 'Alten Meister'
mit dem Ergebnis

„Isn't it basically just *Beltrami* and *Mohr-Coulomb* ? " →

Hencky-
Mises-
Huber

Richard von Mises
1883-1953
Mathematician

Eugenio Beltrami
1835-1900
Mathematician

Otto Mohr
1835-1918
Civil Engineer

Charles de Coulomb
1736-1806
Physician

'Onset of Yielding'

'Onset of Cracking'

What were the driving ideas behind, when Generating the FMC?

Combined wishes:

analogously to **:**

- **failure mode-wise** *(shear yielding failure, etc.)* — **Mises, Hashin, Puck etc.**
- **stress invariant-based** *(J_2 etc.)* — **Mises, Tsai, Hashin, Christensen, etc.**
- **obtaining equivalent stresses** — **Mises for shear yielding, Rankine for fracture**

\+ Intensive Use of

- material symmetry requirements *and*
- physical content of invariants.

Failure mode-wise based **Features of the FMC,** 1995

- **Each failure mode represents 1 independent failure mechanism**

  and thereby 1 piece of the complete *failure surface*

- **Each failure mechanism is governed by 1 basic strength** (**is observed**!)

- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !!

…. according to the material symmetry requirements: that e.g
transversely-isotropic UD materials
exhibit a *'5-fold'* material symmetry characteristic
= 5 Strengths, 5 Failure Modes, 5 E-moduli etc.

12

Material Symmetry Requirements Aspects *(helpful, when generating SFCs)*

1 If a material element can be homogenized to an ideal (= frictionless) crystal,

then, material symmetry demands for the transversely-isotropic UD-material

- **5 e*lastic 'constants' E, ν ; 5 strengths R; 5 fracture toughnesses K_c*** and
- ***2 physical parameters (such as CTE, CME, material friction value μ etc.)***

(for isotropic materials the respective numbers are 2 and 1)

2 Mohr-Coulomb requires for the real crystal another inherent parameter,

- **the *physical parameter 'material friction' : UD*** $\mu_{\perp\|}, \mu_{\perp\perp}$ ***; isotropic*** μ

3 Fracture morphology witnesses:

- **Each strength corresponds to a distinct *failure mode***

and to a *fracture type* as Normal Fracture (NF) or Shear Fracture (SF).

more detailed

⇨ above Facts and Knowledge gave the reason why the FMC strictly employs single *independent* failure modes in its failure mode–wise concept.

Which physical content is hidden in a Stress Invariant?

Invariants (see Mises) are linked to a physical mechanism of the deforming solid !

Following Beltrami, Mises and Mohr-Coulomb *for isotropic materials*

- volume change : $\mathbf{I_1}^2$ … ***(dilatational energy)*** **relevant if porous**
- shape change : $\mathbf{J_2}$ (**Mises**) … ***(distortional energy)*** **relevant if material element shape changes**
- friction : $\mathbf{I_1}$ … ***(friction energy)*** **relevant if brittle**

Mohr-Coulomb

Example: isotropic invariants

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = \ f(\sigma) \ , \qquad 6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = \ f(\tau)$$

$$27\,J_3 = \ (2\sigma_I - \sigma_{II} - \sigma_{III}) \cdot (2\sigma_{II} - \sigma_I - \sigma_{III}) \cdot (2\sigma_{III} - \sigma_I - \sigma_{II})$$

Analogous for transversely-isotropic UD materials

Pre-requisites, when generating Strength Failure Conditions

A Failure Condition $F = 1$ is the mathematical formulation of the failure surface !

Pre-requisites for the establishment of failure conditions are:

- simply formulated, numerically robust,
- physically-based, and therefore, need only few information for pre-dimensioning
- shall allow for a simple determination of the design driving *RF* or *Eff*.

Assumptions for Material Modelling (Example: UD)

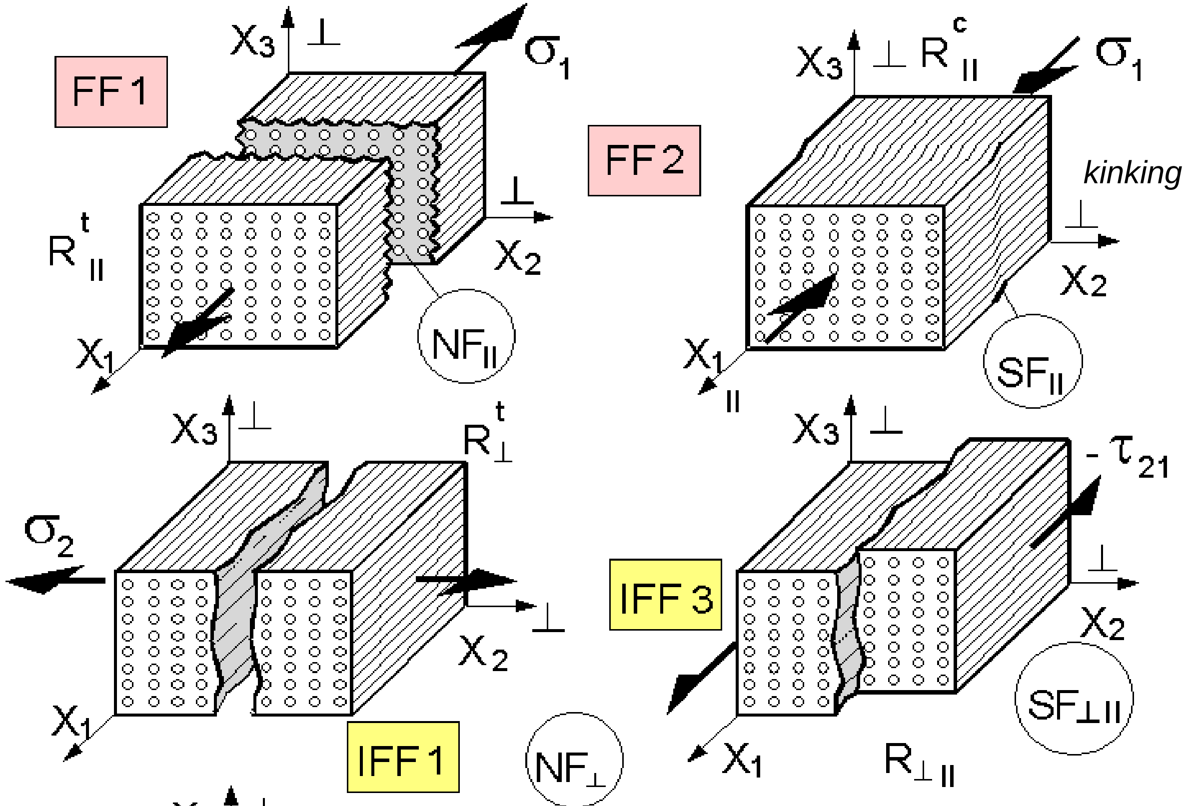
- The UD-lamina is homogenized to a macroscopically homogeneous solid. It is treated as a ‘smeared‘ material

- The UD-lamina is transversely-isotropic:
  On planes, parallel to the fiber direction it behaves orthotropic and on planes transverse to fiber direction isotropic (quasi-isotropic plane)

- For validation of the model a uniform stress state about the critical stress ‘point‘ (location) is necessary

Observed Strength Failure Modes of brittle UD Materials (example)

t = tension
c = compression

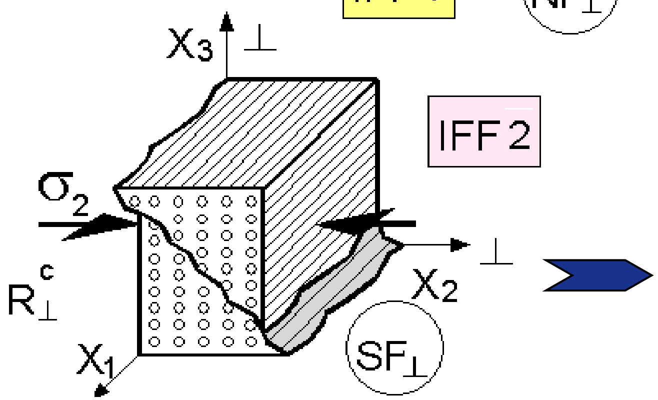


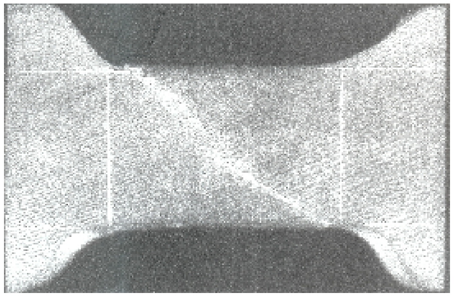
- ▶ 5 Fracture modes exist
  - = 2 FF (Fibre Failure)
  - \+ 3 IFF (Inter Fibre Failure)

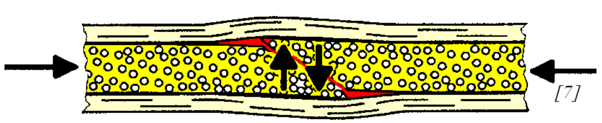
Fracture Types:

NF := Normal Fracture

SF := Shear Fracture







wedge failure type

Cuntze's Set of **Modal 3D UD Strength Failure Conditions** ('criteria')

Cuntze = 3D-'Mises' amongst the UD criteria

Invariants, replaced by their stress formulations

strains from FEA [Cun04, Cun11]

FF1 $Eff^{\|\sigma} = \breve{\sigma}_1 / \overline{R}_{\|}^{t} = \sigma_{eq}^{\|\sigma} / \overline{R}_{\|}^{t}$, $\breve{\sigma}_1 \cong \varepsilon_1^t \cdot E_{\|}$ *

FF2 $Eff^{\|\tau} = -\breve{\sigma}_1 / \overline{R}_{\|}^{c} = +\sigma_{eq}^{\|\tau} / \overline{R}_{\|}^{c}$, $\breve{\sigma}_1 \cong \varepsilon_1^c \cdot E_{\|}$

2 filament modes

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\ 2}}\,] / 2\overline{R}_{\perp}^{t} = \sigma_{eq}^{\perp\sigma} / \overline{R}_{\perp}^{t}$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\ 2}}\,] / \overline{R}_{\perp}^{c} = +\sigma_{eq}^{\perp\tau} / \overline{R}_{\perp}^{c}$

3 matrix modes

IFF3 $Eff^{\perp\|} = \{[\mu_{\perp\|} \cdot I_{23-5} + (\sqrt{\mu_{\perp\|}^{\ 2} \cdot I_{23-5}^{\ 2} + 4 \cdot \overline{R}_{\perp\|}^{\ 2} \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,] / (2 \cdot \overline{R}_{\perp\|}^{\ 3})\}^{0.5} = \sigma_{eq}^{\perp\|} / \overline{R}_{\perp\|}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

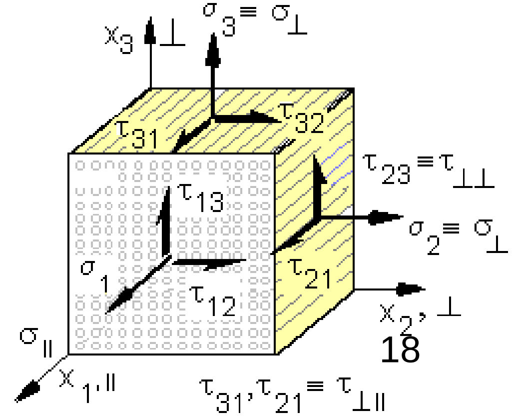
Interaction of modes:

$$Eff^{\,m} = (Eff^{\|\tau})^m + (Eff^{\|\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$

with mode-interaction exponent $2.5 < m < 3$ from mapping tests data

Typical friction value data range: $0.05 < \mu_{\perp\|} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$

see [Pet16] for measurement



Poisson effect * : bi-axial compression strains the filament without any σ_1

t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre

Cuntze's Pre-design Input for 3D UD SFCs

Test Data Mapping — Design Verification

- 5 strengths : $\{\bar{R}\} = (\bar{R}_{\|}^{t}, \bar{R}_{\|}^{c}, \bar{R}_{\perp}^{t}, \bar{R}_{\perp}^{c}, \bar{R}_{\perp\|})^{T}$ — $\{R\} = (R_{\|}^{t}, R_{\|}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\|})^{T}$

  average (typical) values — strength design allowables

- 2 friction values : for 2D $\mu_{\perp\|}$ for 3D $\mu_{\perp\|}, \ \mu_{\perp\perp}$

  $\mu_{\perp\|} = 0.1$ — $\mu_{\perp\perp} = 0.1$

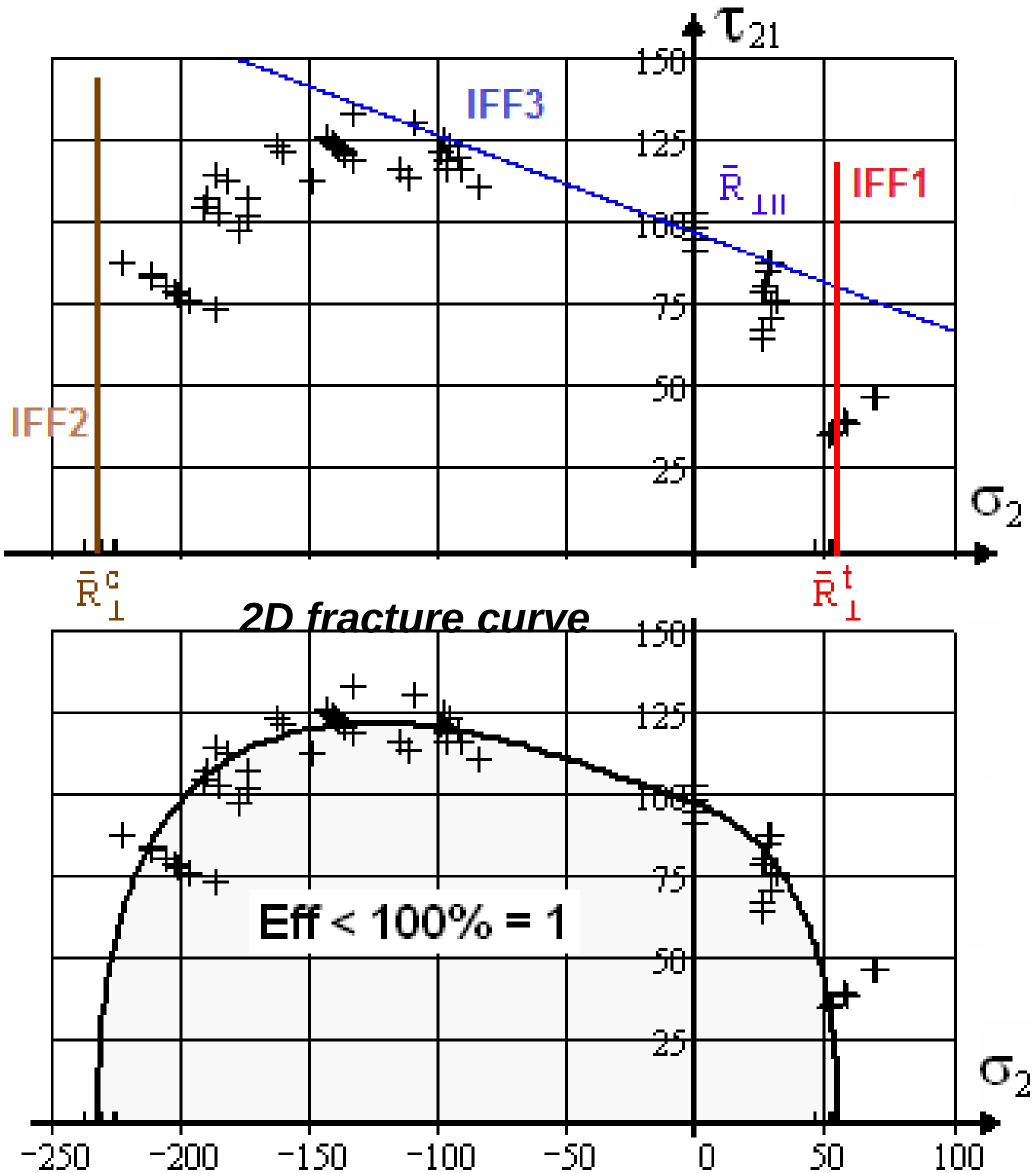
- 1 mode-interaction exponent : *m = 2.6 .*

values, recommended for pre-design

Visualization of Interaction of UD Failure Modes

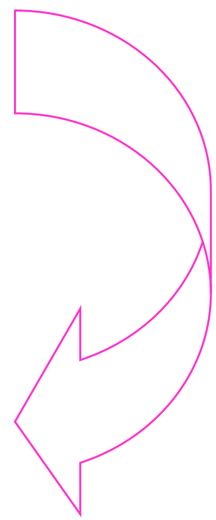
$\breve{\sigma}_1 = 0$

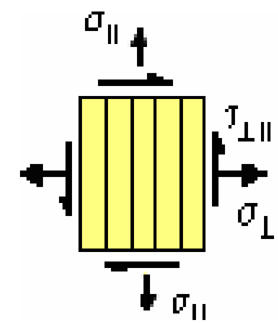
$\tau_{21}(\sigma_2)$ or $\{\sigma\} = (0, \sigma_2, 0, 0, 0, \tau_{21})^T$



Mapping of course of IFF test data in a pure mode domain by the *single Mode Failure Conditions.*

3 IFF pure modes = straight lines !.



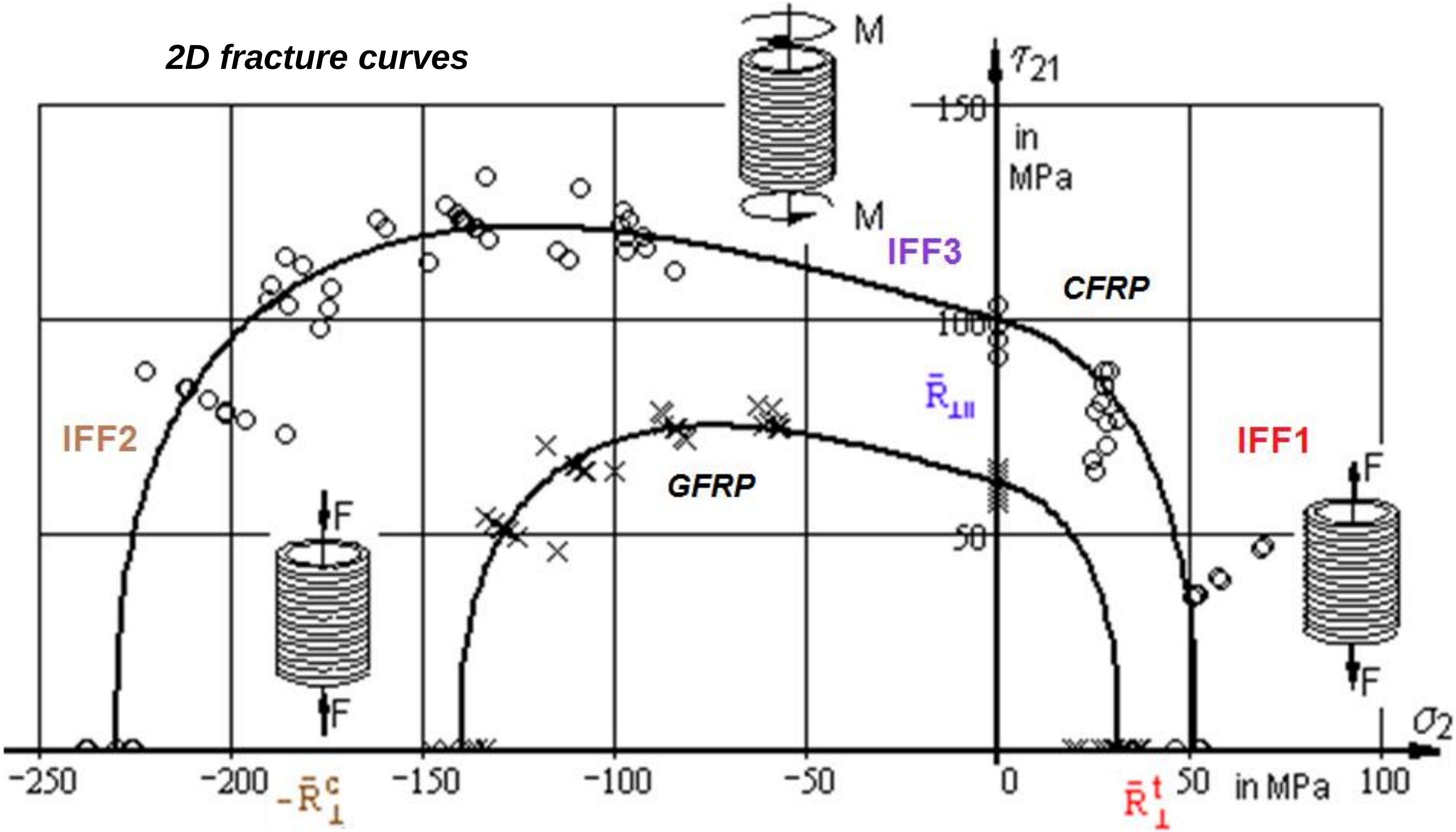


Mapping of course of test data by *the Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$

IFF 1-2-3 Cross–section of the Fracture Failure Surface (body)

2D fracture curves



* Above tested were so-called isolated test specimens.
* For the presented fatigue approach embedded laminas are to consider!

Isolated UD-material (generates hardening curve) and embedded (softening curve)

Isolated' lamina test specimens

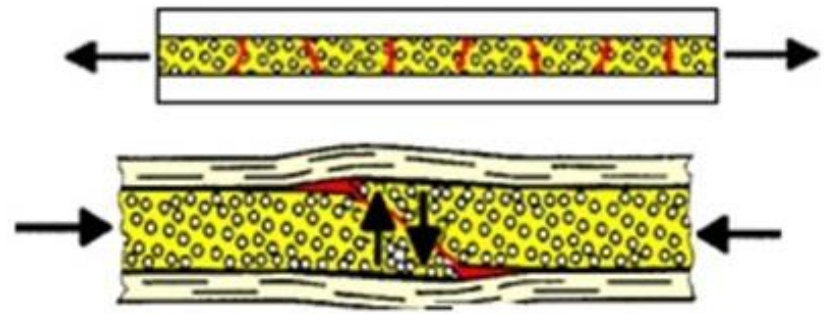
= weakest link results (series failure system)

'Embedded' laminas experience in-situ effects

= redundancy result (parallel failure system)

IFF 1 :

IFF 2 :



unconstrained lamina

delivers strength property, stress-strain curve

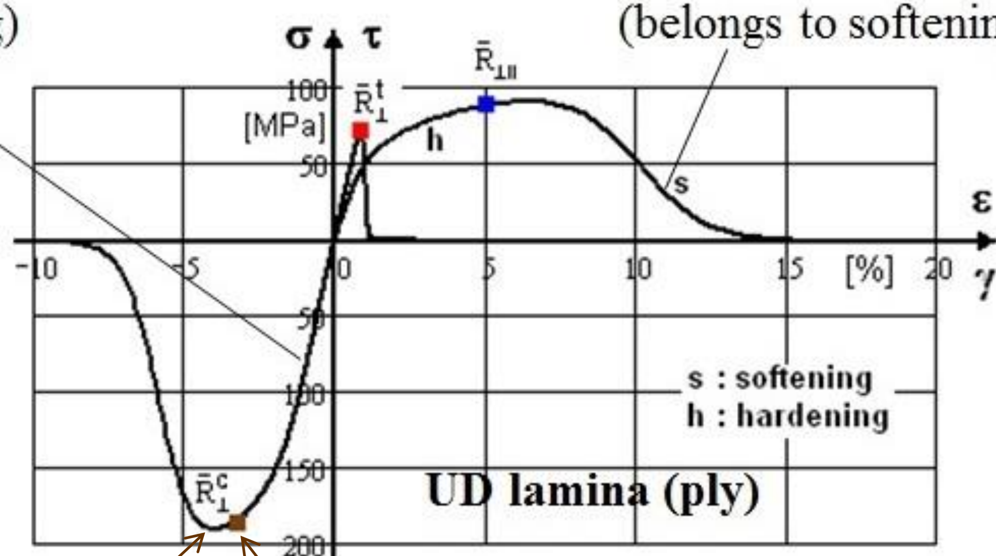
(belongs to hardening)

delivers **basic strength** as analysis input !

mutually constrained laminas, in laminates

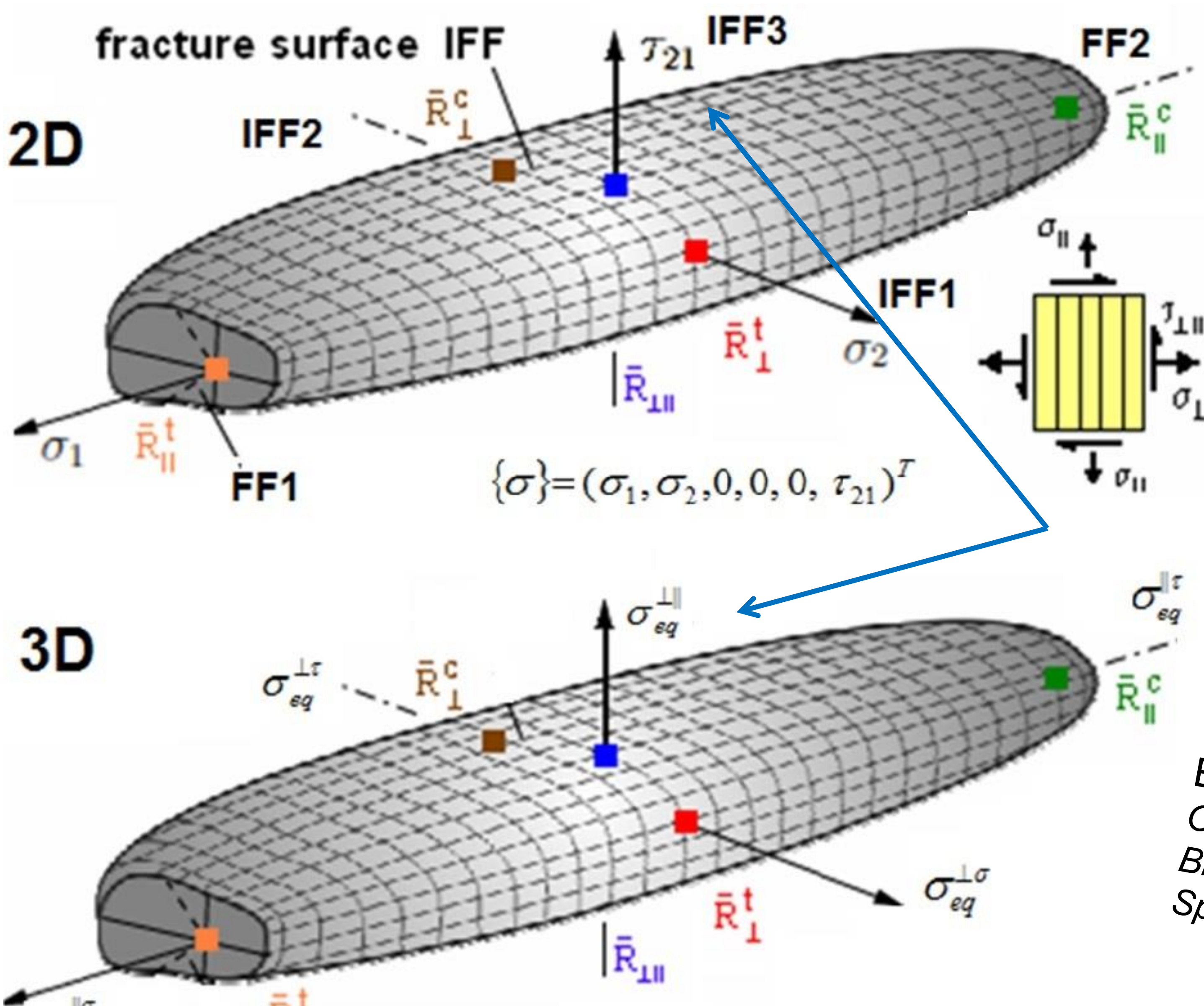
in <u>non-linear</u> laminate analysis

(belongs to softening)



in-situ strength *(basic)strength*

2D ➡ 3D Bruchkörper nach Ersetzen von σ durch σ_{eq}^{mode}



$$\{\sigma\}=(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{31},\tau_{21})^T$$

$\nu_{\perp\perp}=0.4,$
$\nu_{\perp\parallel}=0.3$

Delamination onset predictable

Bruchkörper = Oberfläche aller Bruchversagens-Spannungszustände

State of the Art for <u>Static</u> Strength Analysis *of UD laminas*

represented best by the results of the *World-Wide-Failure-Exercises*

on ***Static strength criteria for the <u>high-performance</u> UD composite parts***

Organizer : *QinetiQ , UK* *(Hinton, Kaddour, Soden, Smith, Shuguang Li)*

<u>Aim</u>: *'Testing Predictive Failure Theories including SFCs* *for* ***Fiber–Reinforced Polymer Composites to the full !'***

*(was for transversely-isotropic **UD materials**, only)*

Method of the World-Wide-Failure-Exercises-I and -II (1991-2013)**):**

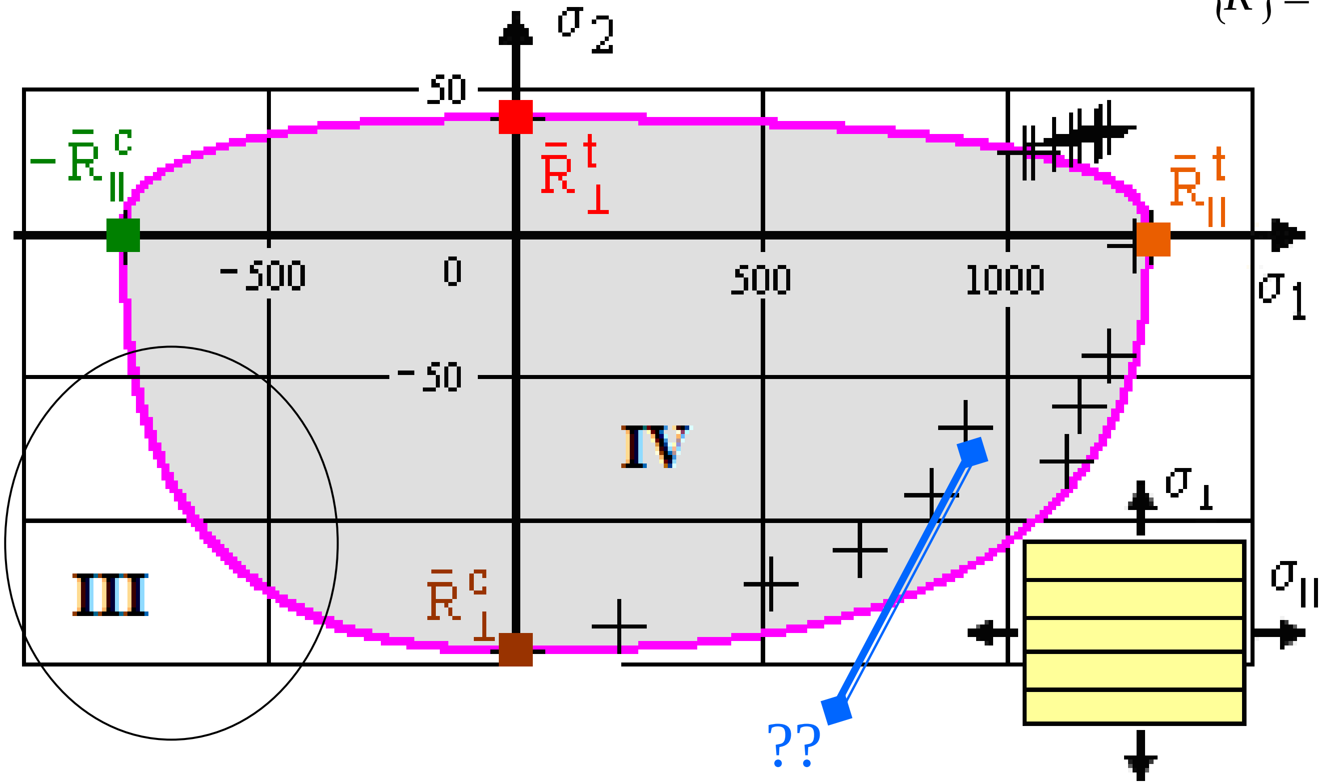
Part A of a WWFE: ***Blind Predictions on basic strength data***

Part B of a WWFE: ***Comparison Theory-Test*** *with 'not always correct'*
<u>Uni-axial</u> 'Failure Stress Test Data' (= <u>basic strength)</u> and
Multi-axial 'Failure Stress Test Data' (<u>plain</u> test specimens, no notch)

Cuntze's invariant-based strength criteria mapped the provided <u>accurate</u> test data sets best.
In WWFE-I (winner) and best in WWFE-II !

Test Case 3, WWFE-I $\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$



Hoop wound tube
UD-lamina.
E-glass/MY750epoxy

$\sigma_1 = \sigma_{hoop}$

$\sigma_2 = \sigma_{axial}$

Part A: Data of strength points were provided, only

Part B: Test data in quadrant IV show discrepancy, testing?

No data for quadrants II, III was provided !

REMARK

Remark on gaps between theory and experiment

- Experimental results can be far away from the reality

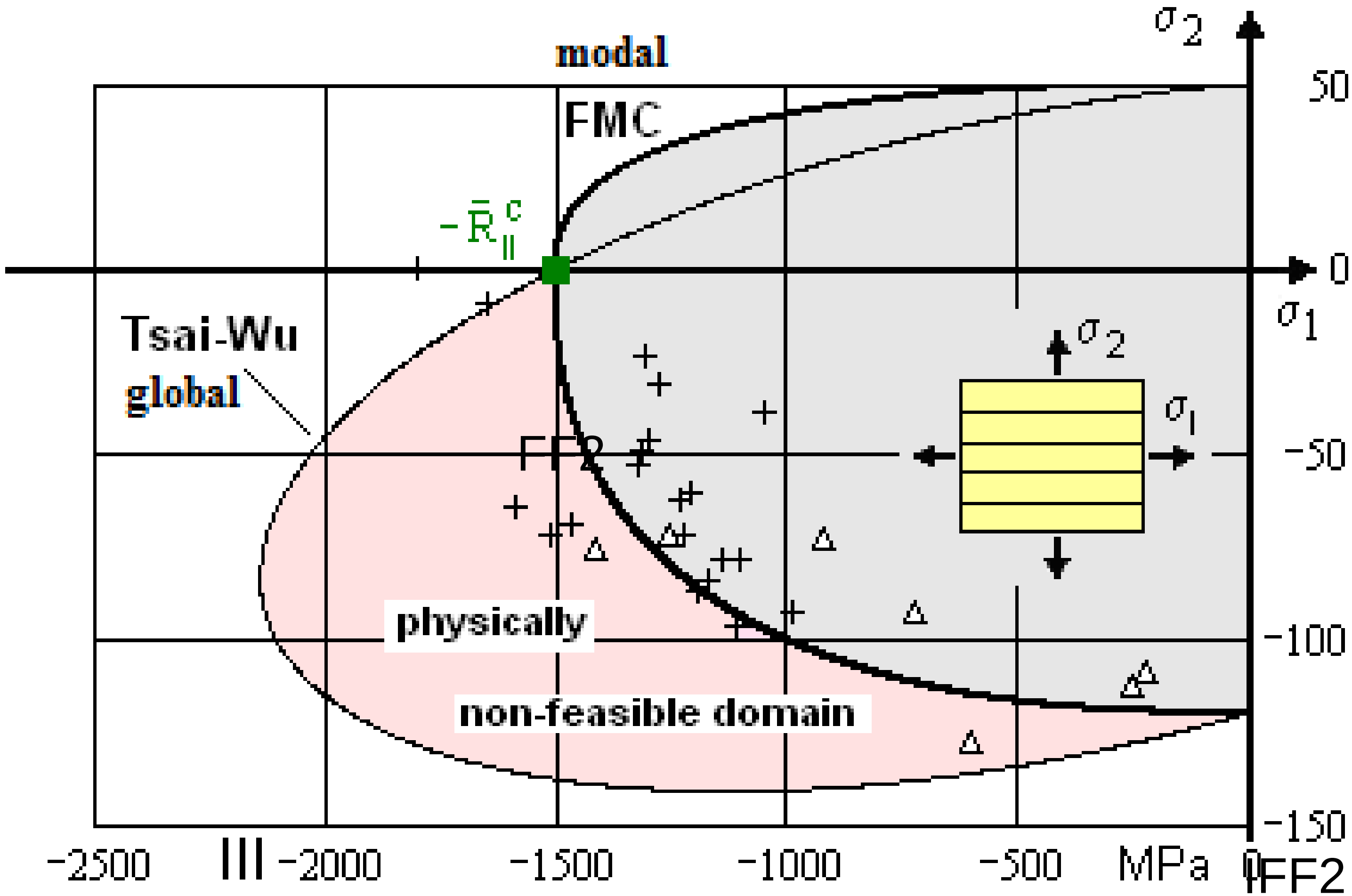
  like a bad theoretical strength model.

- Theory creates a model of the reality, ‘only’,

  and

  1 Experiment is ‘just’ 1 realisation of the reality.

Mapping in the 'Tsai-Wu non-feasible domain' (quadrant III)



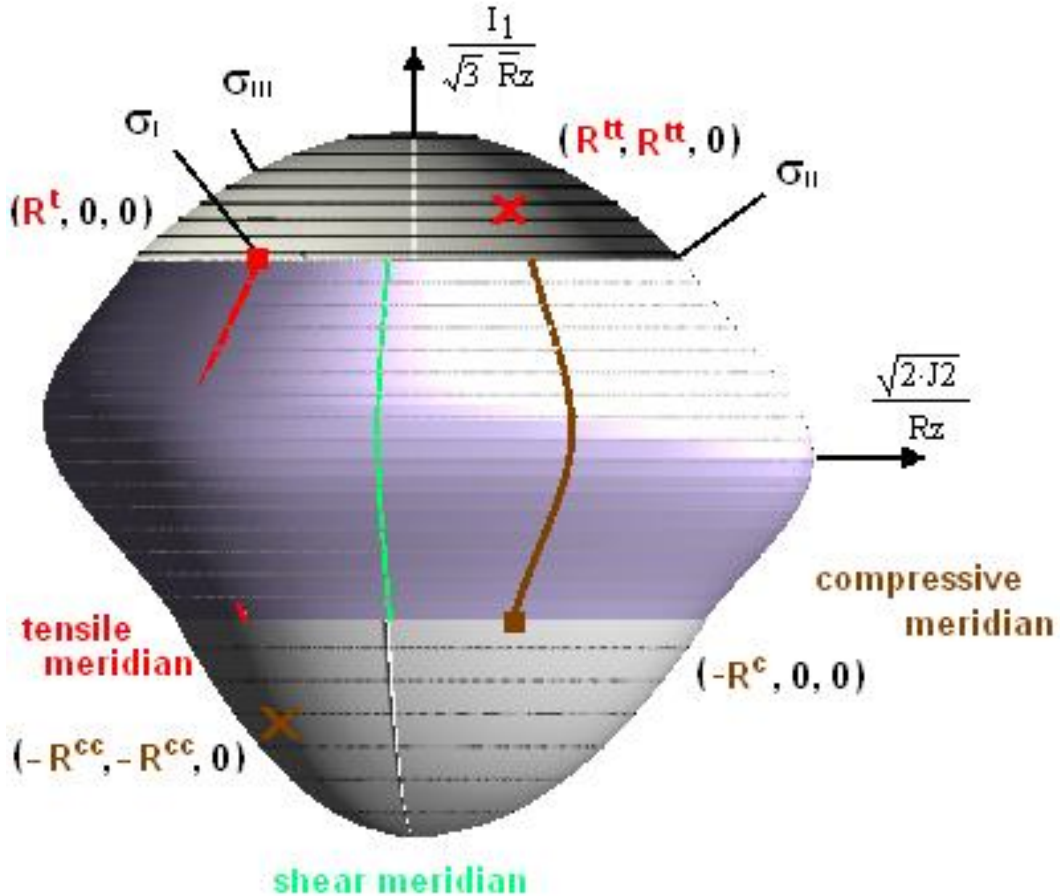
$\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

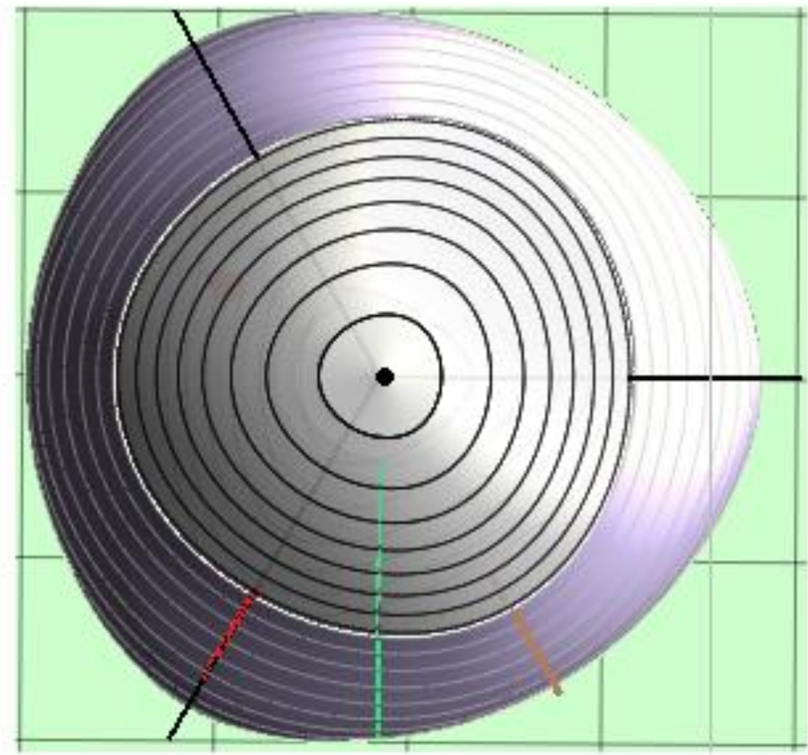
Data: courtesy IKV Aachen, Knops

Lesson Learnt: The modal FMC maps correctly, the global Tsai-Wu formulation predicts a non-feasible domain !

Fracture Failure Surface of Isotropic *Rohacell 71 IG*

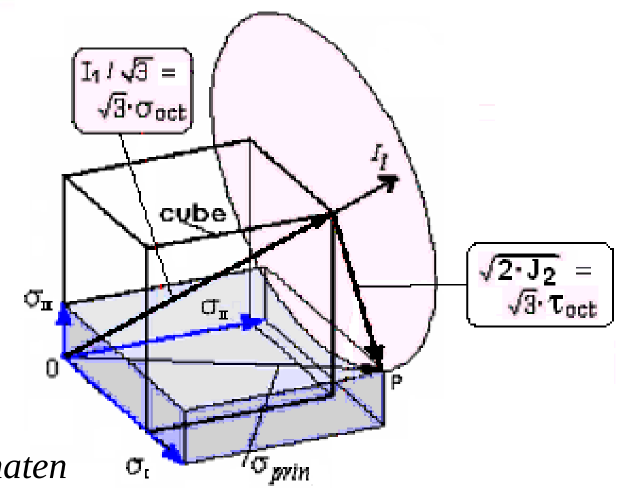
dent turns along axis !





Man kann die 3 Achsen austauschen wegen der 120°-Symmetrie isotroper Körper !

Schubmeridian gewählt als COS-Ursprung: $\Theta = \sqrt[3]{1 + D\cdot\sin(3\theta)}$



Diese Visualisierung erforderte eine 40-seitige MATHCAD-Berechnung

Visualisierung der Lode-(Haigh-Westergaard) Koordinaten

Conclusions wrt Isotropic Strength Failure Conditions (SFCs)

similar with UD etc.

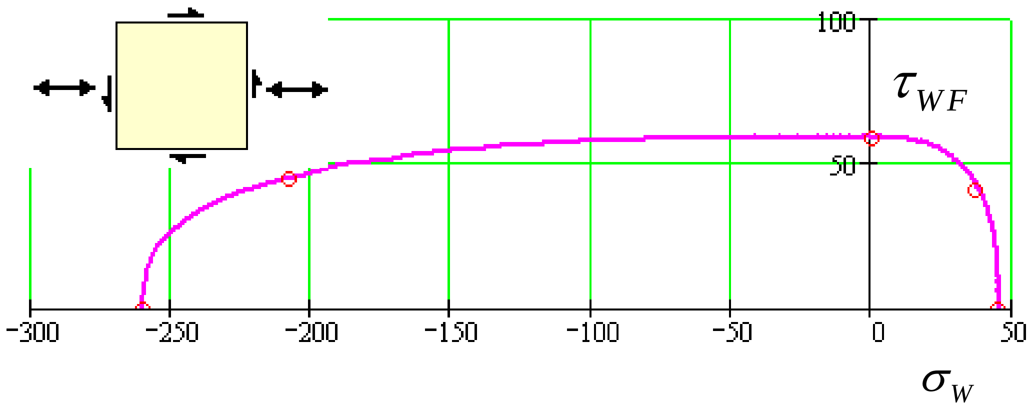
- **A SFC can only describe a 1-fold occurring failure mode.**
- **A multi-fold occurrence must be additionally considered in the formulas:**

  2-fold $\sigma_{II} = \sigma_I$ **(probabilistic effect) is elegantly solved with** J_3

  3-fold $\sigma_{II} = \sigma_I = \sigma_{III}$ **(prob. effect) hydrost. compression, by closing**

- **The 120°-located dents of the non-rotational failure body are the probabilistic result of a 2-fold acting of the same failure mode. This shape is usually described by replacing J_2 through $J_2 \cdot \Theta (J_3, J_2)$. They may be located in the domain $I_1 < 0$ oppositely to those in the domain $I_1 > 0$**
- **The Poisson effect, generated by a Poisson ratio *v*, may cause tensile failure under bi-axially compressive stressing (dense concrete; analogous to UD material, where filament tensile fracture may occur without any external tension loading σ_1)**
- **Hoop Planes = deviatoric planes = $\pi -$ planes: *convex***
- **Meridian Planes : *not convex !***

<u>Fabric</u> **Ceramic Fibre-Reinforced Ceramics (CFRC)** (brittle, porous)



C/C-SiC, T= 1600°C

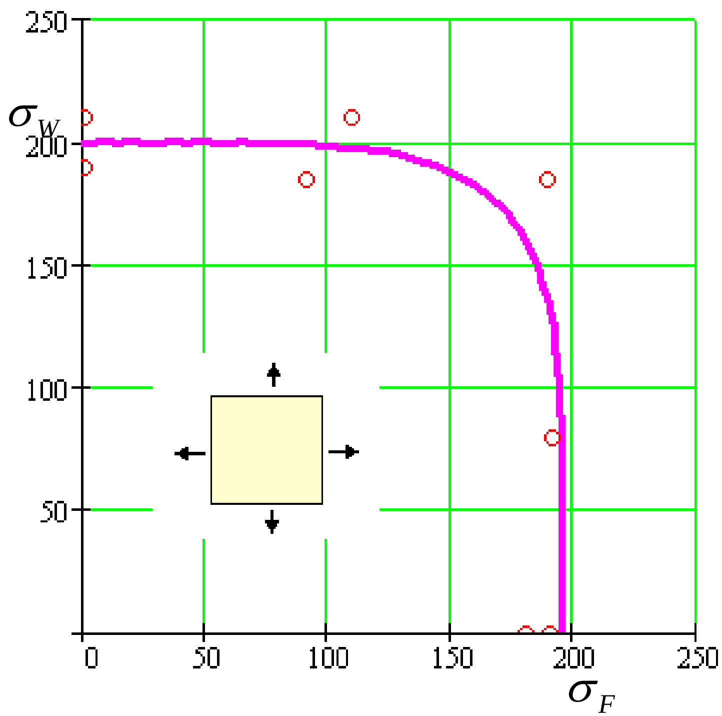
[Geiwitz/Theuer/Ahrendts 1997] ,

tension/compression-torsion, ***tube??***

$$\{\overline{R}\} = (\overline{R}_{\|}^{t}, \overline{R}_{\|}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp\|}) = (-, -, 45, 260, 59)^T$$

$m = 2.8$

$$\left(\frac{\sigma_W}{\overline{R}_W^t}\right)^m + \left(\frac{-\sigma_W}{\overline{R}_W^c}\right)^m + \left(\frac{\tau_{WF}^{\;2}}{\overline{R}_{WF}^{\;2}}\right)^m = 1$$



$$\{\overline{R}\} = (\overline{R}_W^t, \overline{R}_W^c, \overline{R}_F^t, \overline{R}_F^c, \overline{R}_{WF}, \overline{R}_3^t, \overline{R}_3^c, \overline{R}_{3F}, \overline{R}_{3W})^T$$

$$\{\overline{R}\} = \text{vector of mean strength values}$$

C/SiC, ambient temperature *[MAN-Technologie, 1996],*

tension/tension, tube

$$\{\overline{R}\} = (200, -, 195, -, -,,,,)^T, \; m = 5$$

$$\left(\frac{\sigma_W}{\overline{R}_W^t}\right)^m + \left(\frac{\sigma_F}{\overline{R}_F^t}\right)^m = 1$$

NOTE: For <u>woven fabrics</u> enough test information for a <u>real</u> validation is not yet available!

30

“Theory is the Quintessence of all Practical Experience”

A. Föppl

Dazu ergänzend meine Erfahrung:

„Die Erzeugung zuverlässiger 3D-Testdaten und Probekörper ist herausfordernder als die Aufstellung einer zugehörigen zuverlässigen*, auf physikalischen Überlegungen beruhenden Theorie“*

Dank fürs Zuhören und Zusehen.

Es wäre schön, falls ich Sie etwas für neue Ansätze Ihrerseits begeistern konnte.

Ihr Ralf Cuntze

Some Literature

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[Cun13b] Cuntze R.: *Fatigue of endless fiber-reinforced composites*. 40. Tagung DVM-Arbeitskreis Betriebsfestigkeit, Herzogenaurach 8. und 9. Oktober 2013, conference book
[Cun14] Cuntze R.: associated paper, see CCeV website **http://www.carbon-composites.eu/leistungsspektrum/fachinformationen/fachinformation-2**
[Cun15a] Cuntze, R.: *Static & Fatigue Failure of UD-Ply-laminated Parts – a personal view and more.* ESI Group, Composites Expert Seminar, Uni-Stuttgart, January 27-28, keynote presentation, see CCeV website)
[Cun15b] Cuntze, R.: *Reliable Strength Design Verification – fundamentals, requirements and some hints.* 3rd. Int. Conf. on Buckling and Postbuckling Behaviour of Composite Laminated Shell Structures, DESICOS 2015, Braunschweig, March 26 -27, extended abstract , conf. handbook, 8 pages (see CCeV website)
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Assessment of Strength Proofs

* Even in smooth stress regions a SFC can be only a necessary condition which may be not sufficient for the prediction of 'onset of fracture', i.e. for the *in-situ lateral strength in an embedded lamina*, see e.g. [Flaggs-Kural 1982], an energy-based second condition might be applied on top (in the past, this effect was often termed 'thin-layer effect').

* In case of discontinuities such as notches with steep stress decays only a *toughness* + *characteristic length-based energy balance condition* may form a sufficient fracture condition.

* Attempts to link 'onset of fracture/cracking' prediction methods for structural components are actually undergone, see e.g. [Leguillon 2002].

Test Data Mapping versus Design Verification

- Validation of SFC-Models with many Failure Test Data by

  mapping their course by an average Failure Curve (surface) based on very many experimental data

- Verification of the Design for the various Dimensiong Load Cases by

  calculation of a Margin of Safety or a (load) Reserve Factor

  $$MoS > 0 \quad oder \quad RF = MoS + 1 > 1$$

  on basis of a statistically reduced material failure curve

  and sometimes

  on one experiment *(in civil engineering usually not)*.

Fibre Failure Mode 1 (FF 1): Consideration of fracture without an acting $\sigma_1 \equiv \sigma_\parallel$

Limit of macro-homogenisation, if the micromechanical fibre stress is responsible:

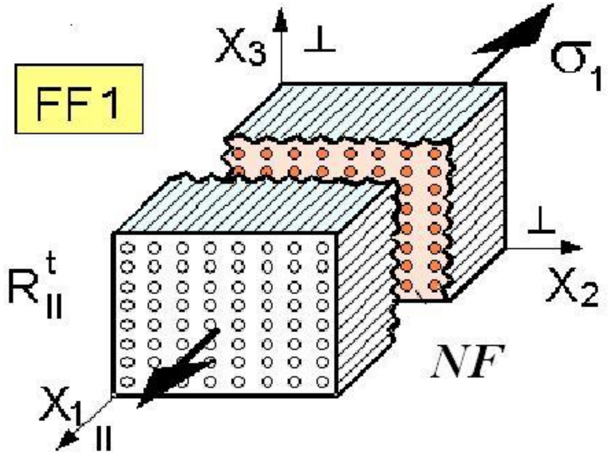
$$F_\parallel^\sigma = \frac{+\sigma_1}{\overline{R}_\parallel^t} = 1,$$

In case of transversal compression (2D or 3D) - due to Poisson's effect- tensile fibre failure (FF1) is possible without σ_1.

Problem by-passed by taking strains from FEA which considers the full stress-strain behaviour !

$$I_1 = \sigma_1$$

$$\cong \nu_f \cdot \sigma_{1f}^{\ t} = \nu_f \cdot \varepsilon_1^{\ t} \cdot E_{1f} = \varepsilon_1^{\ t} \cdot E_\parallel$$



$$F_\parallel^\sigma = \frac{\varepsilon_1 \cdot E_\parallel}{\overline{R}_\parallel^t}$$

Additionally Required Material Information

Example UD: 2 Material internal Friction Parameters (brittle behaviour)

$$\tau_{21} = R_{\perp\parallel} - b_{\perp\parallel} \cdot \sigma_2$$ *: FMC corresponds*

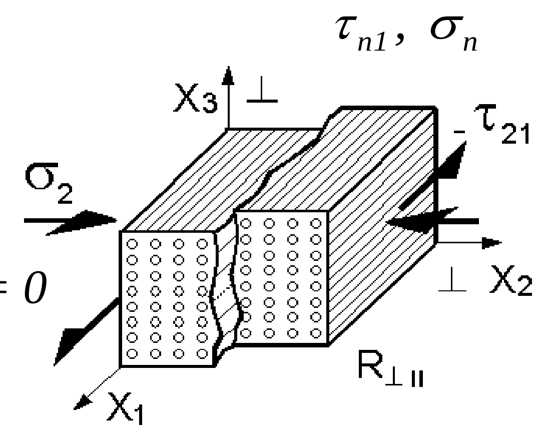
IFF 3 : $$\tau_{n1} = R_{\tau}^{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_n$$ *: Mohr*

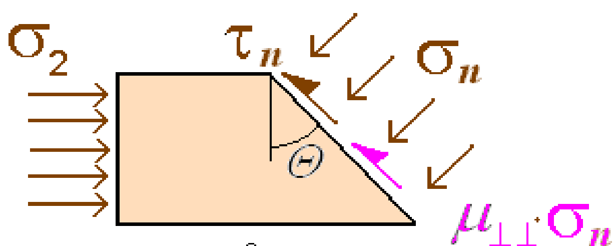
cohesion strength — *material internal friction coefficient*

Linear Mohr-Coulomb approach + denotation

IFF 2 : $$\tau_{nt} = R_{\tau}^{\perp\perp} - \mu_{\perp\perp} \cdot \sigma_n$$

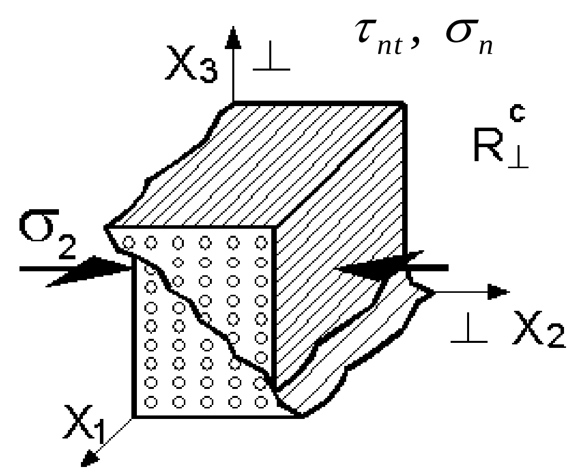
$\Theta_{fp} = 0$





$$\sigma_n = \cos^2\Theta \cdot \sigma_2$$
$$\tau_n = -\sin\Theta \cdot \cos\Theta \cdot \sigma_2$$

$\Theta_{fp} \geq 45°$



real material = crystal + friction

UD material: 2; isotropic material: 1

Additionally Required Material Information

Example UD: Micro-mechanical Properties

Some lamina analyses require a micro-mechanical input, but not all micro-mechanical properties can be measured :

Solution: *Micro-mechanical equations are calibrated by macro-mechanical test results (lamina level) = an inverse parameter identification*

Condition: *micro-mechanical properties can be used only together with the equations they have been determined with.*

Micro-mechanical formulas applied in:

Elasticity domain: may be helpful tools (new formulas)

Strength domain : attempted, but not yet successful.

Alle benötigten Werstoffkennwerte und Modellparameter sollten physikalisch erklärbar und eindeutig messbar sein.

Cuntzes 3D Festigkeitsbedingungen für isotrope poröse Werkstoffe

Ansätze: Zug $F^{NF} = \frac{\sqrt{4J_2 - I_1^2/3} + I_1}{2 \cdot \overline{R}_t} = 1$ Druck $F^{CrF} = \frac{\sqrt{4J_2 - I_1^2/3} - I_1}{2 \cdot \overline{R}_c} = 1$ (Schaum, Ytong)

Berücksichtigung bi-axialer Festigkeit (Versagensmodus zweifach): *in Effs*

$$Eff^{NF} = c_{NF} \cdot \frac{\sqrt{4J_2 \cdot (\Theta_{NF}) - I_1^2/3} + I_1}{2 \cdot \overline{R}_t} = \sigma_{eq}^{NF} / \overline{R}_t \qquad Eff^{CrF} = c_{CrF} \cdot \frac{\sqrt{4J_2 \cdot (\Theta_{CrF}) - I_1^2/3} - I_1}{2 \cdot \overline{R}_c}$$

Zweifache Versagenswahrscheinlichkeit mit der Invariante J_3 erfassbar,
D_{NF} und D_{CrF} sind die Nicht-Koaxialitätsparameter für die beiden Bruchmoden:

$$\Theta_{NF} = \sqrt[3]{1 + D_{NF} \cdot \sin(3\theta)} = \sqrt[3]{1 + D_{NF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}} \qquad \Theta_{CrF} = \sqrt[3]{1 + D_{CrF} \cdot \sin(3\theta)} = \sqrt[3]{1 + D_{CrF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}}$$

Interaktion der Versagensmoden: $Eff^{NF} = [(Eff^{NF})^m + (Eff^{CrF})^m]^{m^{-1}}$

Abschluß der Versagensoberfläche durch Paraboloid-Kappen oben und unten:

$$\frac{I_1}{\sqrt{3} \cdot R_t} = s_{cap} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta_{NF}}}{R_t}\right)^2 + \frac{\max I_1}{\sqrt{3} \cdot R_t}$$

auf die R_t-normierten Lodekoordinaten bezogen

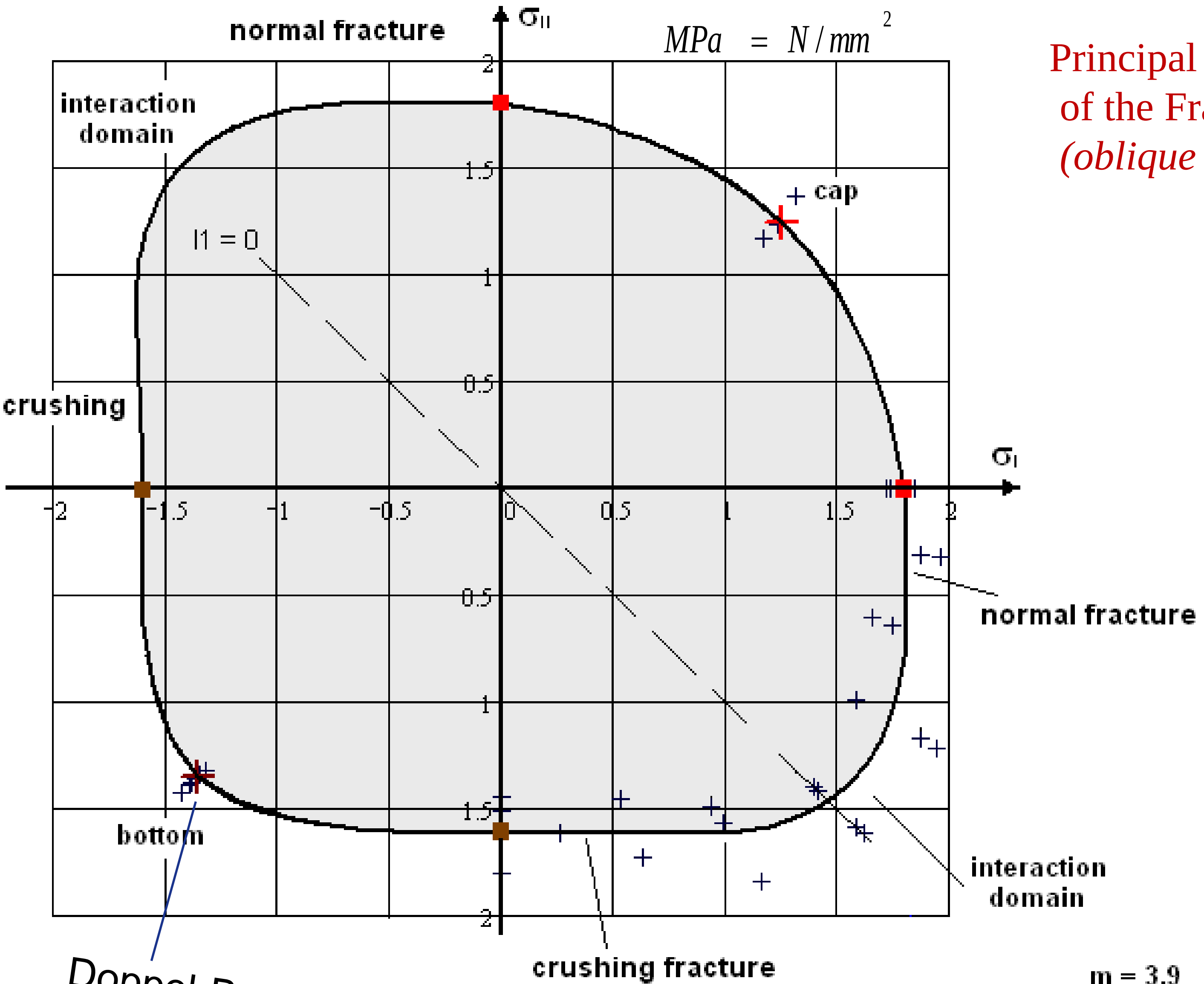
$$\frac{I_1}{\sqrt{3} \cdot R_t} = s_{bot} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta_{CrF}}}{R_t}\right)^2 + \frac{\min I_1}{\sqrt{3} \cdot R_t}$$

Zur Bestimmung der Steigungsparameter s müssen die hydrostatischen Werte bekannt sein: *maxI_1 kann nur abgeschätzt werden, minI_1* könnte gemessen werden.

Eff = material stressing effort = Werkstoff-Anstrengung (< 1 = 100%)

Normierung auf Zugfestigkeit

2D – Testdaten mit Abbildung in der Hauptspannungsebene *(brittle, porous)*



Doppel-Bruchmodus !
Erfassung durch $\Theta(J_3)$

Principal Plane Cross-section of the Fracture Body *(oblique cut)*

Rohacell 71 IG

Testdaten Courtesy: LBF-Darmstadt (DKI), Dr. Kolupaev

… als sich ähnlich verhaltendes Material

kompressibel

$\nu = 0$

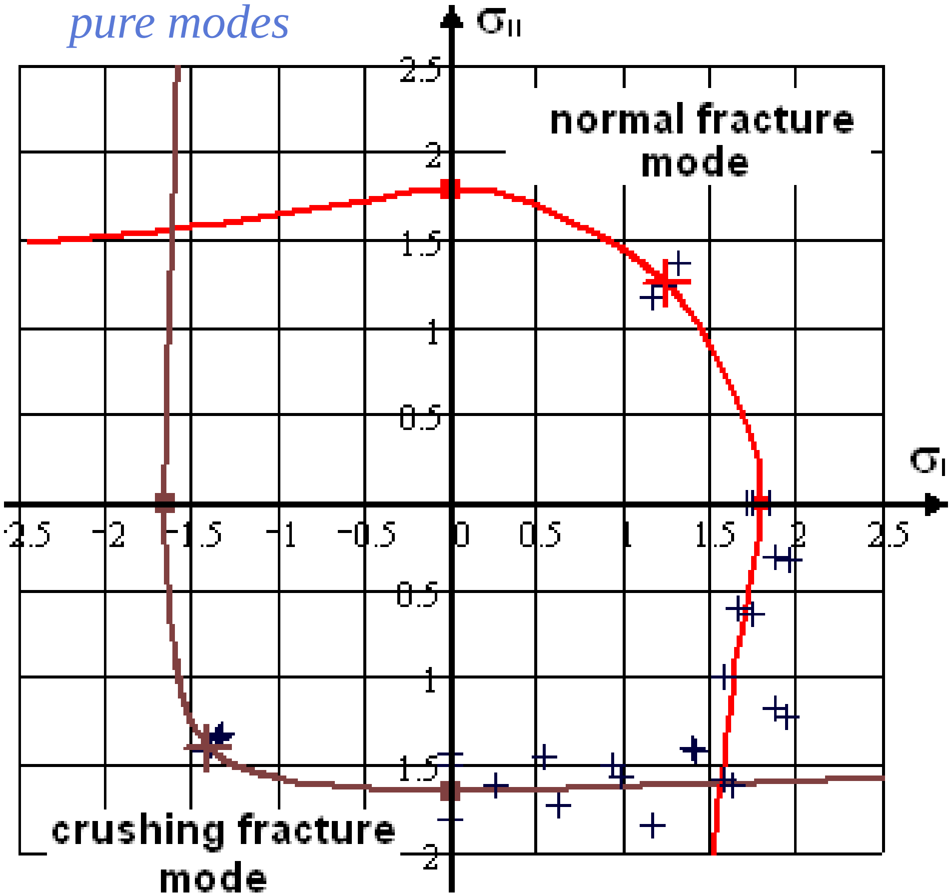
2D - Test Data Set and Mapping in the Principal Stress Plane

Rohacell 71 IG

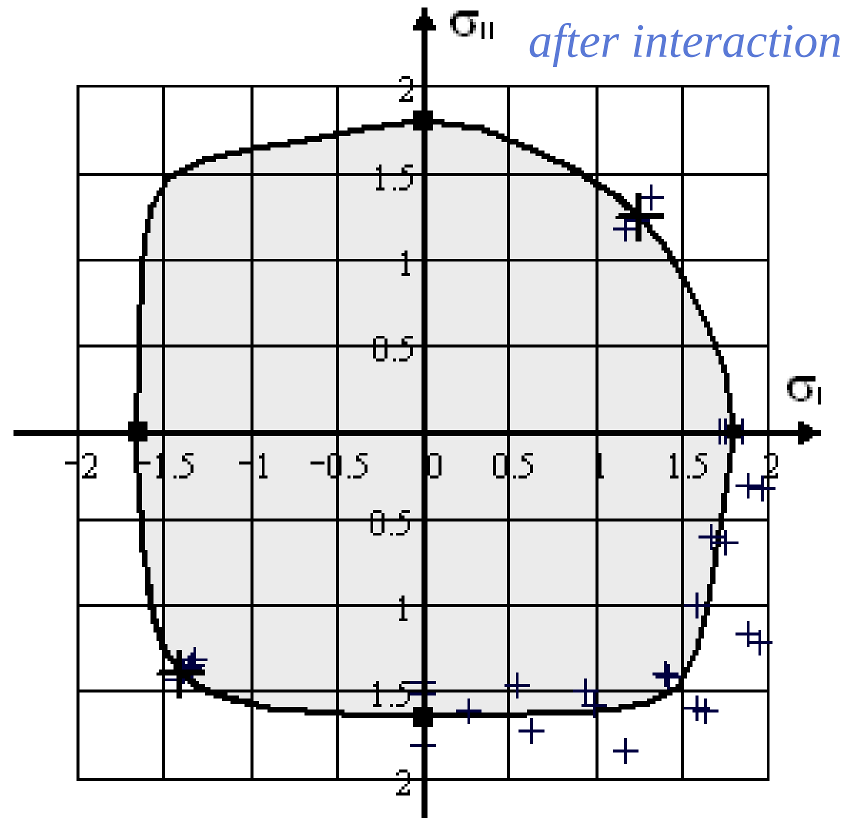
Principal Plane Cross-section of the Fracture Body (oblique cut)

as similarly behaving material

pure modes



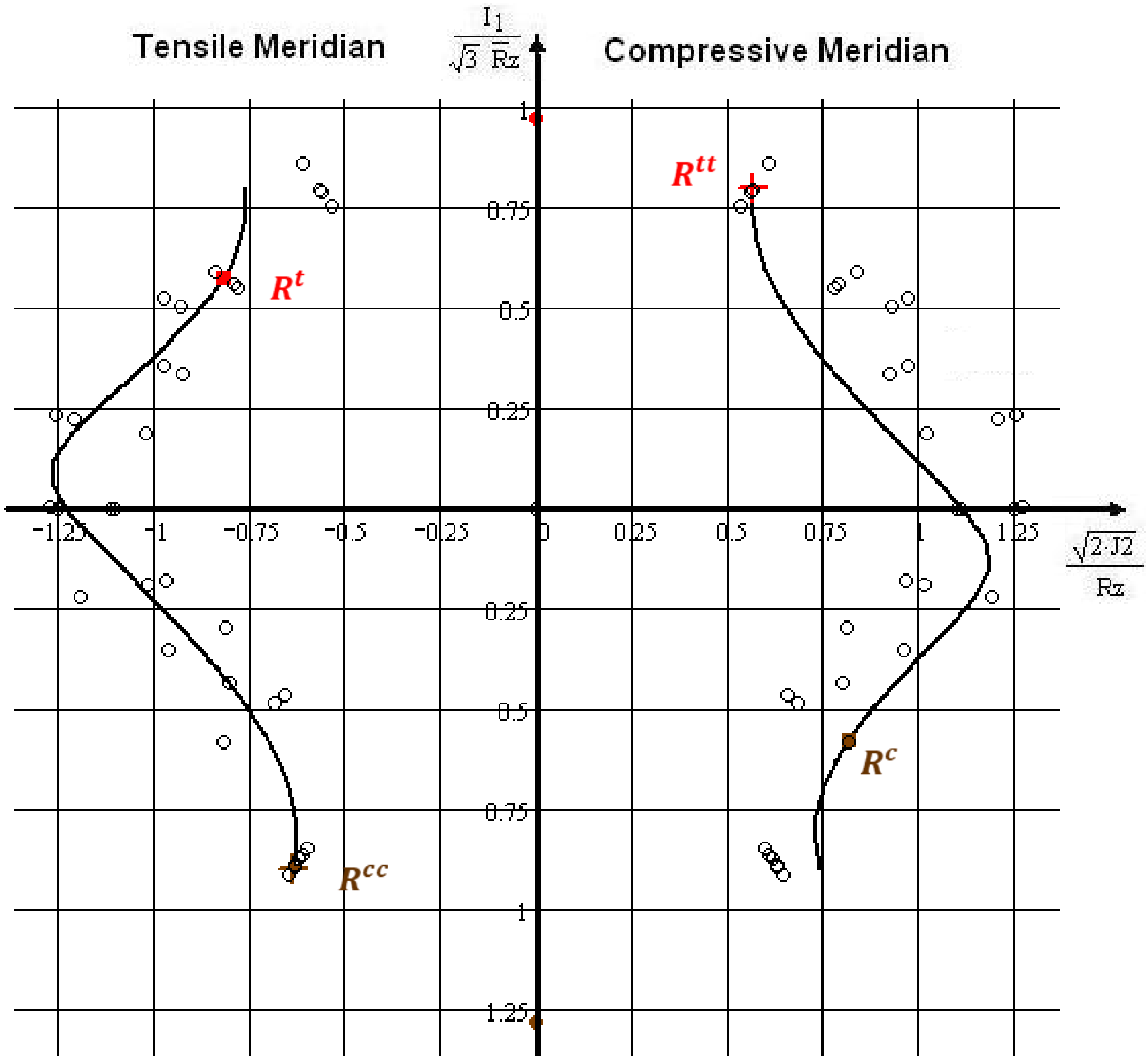
after interaction



- Mapping <u>must</u> be performed in the 2D-plane because fracture data set is given there
- The 2D-mapping uses the 2D-subsolution of the 3D-strength failure conditions
- The 3D-fracture failure surface (body) is based on the 2D-derived model parameters.

Courtesy: LBF-Darmstadt, Dr. Kolupaev

Generic Lines of Tensile and of Compressive Meridian



Meridional cross-sections of the Fracture Body

in **Lode-Haigh-Westergaard coordinates**

bi-axial = +

z = tensile,

d = compressive

The fracture test data are located at a distinct Lode angle of its associated ring o, 120°-symmetry of the isotropic failure surface (body) .

Cap and bottom are closed by a cone-ansatz, a shape being on the conservative side.

Messergebnisse sind ‘lediglich‘ das Ergebnis einer Prüfvereinbarung (Norm, Standard) und dienen der Vergleichbarkeit verschiedener Untersuchungen.

Die Prüfvereinbarung besteht aus Prüfeinrichtung, Prüfvorschrift, Probekörper und Auswerteverfahren.

Damit kann man nur von ‘exakten‘ Prüfergebnissen‘ im Sinne der Prüfvereinbarung reden.

Self-explaining, symbolic Notations for Strength Properties

prepared by the author for ESA - Material Handbook

		Fracture **Strength Properties**									
	loading	**tension**			**compression**			**shear**			
	direction or **plane**	1	2	3	1	2	3	12	23	13	
9	**general orthotropic**	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	**friction properties**
5	**UD**	$R_{\parallel}^t$ NF	$R_{\perp}^t$ NF	$R_{\perp}^t$ NF	$R_{\parallel}^c$ SF	$R_{\perp}^c$ SF	$R_{\perp}^c$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$\mu_{\perp\perp}$, $\mu_{\perp\parallel}$,
6	**fabrics**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	**fabrics general**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	μ_{W3}, μ_{F3}, μ_{WF}
5	**mat**	R_{1M}^t	R_{1M}^t	R_{3M}^t	R_M^c	R_{1M}^c	R_{3M}^c	R_M^τ	R_M^τ	R_M^τ	*(UD, turned direction)*
2	**isotropic matrix**	R_m SF	R_m SF	R_m SF	*deformation-limited*			R_M^τ	R_M^τ	R_M^τ	μ
		R_m NF	R_m NF	R_m NF	R_m^c SF	R_m^c SF	R_m^c SF	R_m^σ NF	R_m^σ NF	R_m^σ NF	μ

NOTE: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m:= 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

Elasticity Properties of the homogenised material

		Elasticity Properties									
	direction or **plane**	1	2	3	12	23	13	12	23	13	
9	***general orthotropic***	E_1	E_2	E_3	G_{12}	G_{23}	G_{13}	ν_{12}	ν_{23}	ν_{13}	**comments**
5	***UD, ≅ non-crimp fabrics***	$E_{\parallel}$	$E_{\perp}$	$E_{\perp}$	$G_{\parallel\perp}$	$G_{\perp\perp}$	$G_{\parallel\perp}$	$\nu_{\parallel\perp}$	$\nu_{\perp\perp}$	$\nu_{\parallel\perp}$	$G_{\perp\perp} = E_{\perp} / (2 + 2\nu_{\perp\perp})$ $\nu_{\perp\parallel} = \nu_{\parallel\perp} \cdot E_{\perp} / E_{\parallel}$ *quasi-isotropic 2-3-plane*
6	***fabrics***	E_W	E_F	E_3	G_{WF}	G_{W3}	$G_{\mid W3}$	ν_{WF}	ν_{W3}	ν_{W3}	*Warp = Fill*
9	***fabrics general***	E_W	E_F	E_3	G_{WF}	G_{W3}	G_{F3}	ν_{WF}	ν_{F3}	ν_{W3}	*Warp ≠ Fill*
5	***mat***	E_M	E_M	E_3	G_M	G_{M3}	G_{M3}	ν_M	ν_{M3}	ν_{M3}	$G_M = E_M/(2+2\nu_M)$ *1 is perpendicular to quasi-isotropic mat plane*
2	***isotropic*** *for comparison*	E	E	E	G	G	G	ν	ν	ν	$G=E/(2+2\nu)$

Lesson Learned: - *Unique, self-explaining denotations are mandatory*
- *Otherwise, expensively generated test data cannot be interpreted and go lost*

Hygrothermal Properties of homogenised materials

	direction	Hygro-thermal properties						
		1	2	3	1	2	3	
9	**general orthotropic**	α_{T1}	α_{T2}	α_{T3}	α_{M1}	α_{M2}	α_{M3}	**comments**
5	**UD, ≅ non-crimp fabrics**	$\alpha_{T\|\|}$	$\alpha_{T\perp}$	$\alpha_{T\perp}$	$\alpha_{M\|\|}$	$\alpha_{M\perp}$	$\alpha_{M\perp}$	
6	**fabrics**	α_{TW}	α_{TW}	α_{T3}	α_{MW}	α_{MW}	α_{M3}	$Warp = Fill$
9	**fabrics general**	E_W	E_F	E_3	α_{MW}	α_{MF}	α_{M3}	$Warp \neq Fill$
5	**mat**	α_{TM}	α_{TM}	α_{TM3}	α_{MM}	α_{MM}	α_{MM3}	
2	**isotropic** **for comparison**	α_T	α_T	α_T	α_M	α_M	α_M	

NOTE: Despite of annoying some people, I propose to rethink the use of α for the CTE and β for the CME.
Utilizing α_T and α_M automatically indicates that the computation procedure will be similar.

What do we speak about ? *Definitions*

Material: homogenized (macro-)model of the envisaged solid

Failure: structural part does not fulfil its functional requirements such as onset of yielding, brittle fracture, Fiber-Failure FF, Inter-Fiber-Failure IFF, leakage, deformation limit, delamination size limit, frequency bound

= *project-fixed* Limit State with *F* = Limit State Function

Failure Criterion: F >=< 1, Failure Condition : *F* = 1= 100%

Failure Theory: general tool to predict failure of a structural part

Strength Failure Condition: subset of a strength failure theory

tool for the assessment of a

'multi-axial failure stress state ' in a critical location of the material.

→ **Stresses** are judged by **Strengths !**

Some Definitions needed for Modeling *What is ?*

Material : homogenized (smeared) model of the envisaged complex material which might be a material combination

Failure : structural part does not fulfil its functional requirements such as FF = fiber failure, IFF = inter-fiber-failure (matrix failure), leakage, deformation limit (tube widening, delamination size limit, ..) ⟹ **= a project-defined 'defect'**

Fatigue : process, that degrades material properties

Fatigue Life Stages (1) accumulation of damaging until initiation of a critical damage size (classical fatigue life prediction domain), (2) damage growth until onset of final fracture (domain of damage tolerance concepts), (3) separation (not of interest)

Damaging (not also damage, as used in English literature) **:** process wherein the results, the damaging portions, finally accumulate to a damage size such as a macro-scopic delamination. Accumulation tool usually used is *Palmgren-Miner's Damaging Accumulation Rule (*= model)

Damage : sum of the accumulated damaging or an impact failure, that is judged to be critical. Then, *Damage Tolerance Analysis* is used to predict damage growth under further cyclic loading or static failure under Design Ultimate Load

Haigh Diagram : involves all S-N curves required for fatigue life prediction.

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- **1964, Dipl.-Ing.** **Civil Engineering** (structural eng., TU Hannover)
- **1968, Dr.-Ing.** **Structural Dynamics** (TU Hannover)
- **1968 – 1970, DLR** **FEA-programming**
- **1970 – 2004, MAN-Technologie: Development of Structures**
- **1978, Dr.-Ing. habil.** **Mechanics of Lightweight Structures** (TUM)
- **2004 - 2009** **working on multiple ESA/ESTEC Standards**
- **1972 – 2015** **contributor to the Aerospace Hdbk LTH-HSB, various ESA-Specs, VDI 2014 (editor of sheet 3)**
- **since 2009** **with Carbon Composites e.V.**