The Predictive Capability of Failure Mode Concept-based Strength Criteria for Multidirectional Laminates - *Part B* -

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Abstract

This paper represents the author's contribution to Part B of the world-wide failure exercise. An assessment is made of the correlation between the predictions of Part A, based on Failure Mode Concept (FMC)-derived 3D strength criteria for UD laminae, and experimental results provided for biaxial initial and final failure envelopes and stress strain curves of various unidirectional and multidirectional laminates.

Some simplifying refinements of the failure conditions and some improvements of the computer code used are presented. Special emphasis has been put on the difference between an isolated and an embedded lamina.

The predictions for the $[+55/-55]_s$ laminate tube showed the highest discrepancy but could be partly interpreted? For this extremely non-linear test case still the improved code needs an upgrading that tackles bulging in addition.

UD strength input and associated laminate test results seem not to match in some test cases. Therefore, some tests have to be carefully re-evaluated and others repeated. The information about the stress-strain-deformation behaviour was not sufficient to fully explain the test data.

Finally, areas which require further work are depicted with the essential result: Too sophisticated applications of IFF conditions based on tests with isolated UD specimens (weakest link problem) needs to be re-examined, as embedded laminae in a laminate possess redundancy and are believed not to exhibit the scatter of isolated laminae. This should be investigated targeting at an engineering method for the Ultimate *Proof of Design* which accounts for the diminishing influence of the IFF conditions with increasing laminate strain level.

Keywords: multi-axial stressing, non-linear behaviour, multidirectional laminates

NOTATION

Unidirectional lamina

a_s, b_s: Ramberg/Osgood parameters in softening regime

$b_\perp^\tau, b_{\perp\parallel}, b_{\perp\parallel}^\tau$: Curve parameters

$E_1 = E_\parallel$, $E_2 = E_3 = E_\perp$: Elastic moduli of a UD lamina in the directions x_1, x_2, x_3

$E_{1(tan)}$, $E_{3(sec)}$: A tangent and a secant elastic modulus

E_{1f}: Elastic fibre modulus in x_1 direction

$Eff^{(res)}$: Resultant stress effort of all interacting failure modes. Corresponds to Puck's exposure factor f_E

$Eff^{(mode)}$: Stress Effort of a UD-lamina in a failure mode, eg $\sigma_{eq}^{\|\tau} / R_{\|}^{c} = Eff^{\|\tau}$. Corresponds to $1/f_{Res}^{\|\tau}$ if linear behaviour

$maxEFF^{(mode)}$: Stress Effort of the maximum stressed failure mode

$e_{\|}^{t}, e_{\|}^{c}$: Tensile and compressive failure strain of a UD-lamina in x_1 direction

$F_{\|}^{\sigma}, F_{\|}^{\tau}, F_{\perp}^{\sigma}, F_{\perp}^{\tau}, F_{\perp\|}$: Failure functions for FF and IFF

$f_{Res}^{(mode)}$: Reserve factor = stretching factor, if linear analysis applicable, for the applied stress state necessary to achieve the failure stress state of the mode, eg $f_{Res}^{\perp\sigma} = R_{\perp}^{t} / \sigma_{eq}^{\perp\sigma}$ =1. That means the ratio: multi-axial *strength* / multi-axial design *stress* applied

$f_{Res}^{(res)}$: Resultant reserve factor of all interacting failure modes. In general for linear and non-linear cases the ratio: max. *load resistance* achieved in test or analysis / design *load* applied

G_{21}, $G_{21(sec)}$: Shear modulus of a UD lamina in the x_2, x_1 direction; secant shear modulus

I_1, I_2, I_3, I_4, I_5: Invariants of the transversally-isotropic UD-material

$\dot{m}$: Mode interaction coefficient (rounding-off exponent)

$\overline{R}, R$: Mean strength, design allowable

$R_{p0.2}$: Stress value at 0.2 % plastic strain = yield strength

$R_{\|}^{t} \equiv X^{t}, R_{\|}^{c} \equiv X^{c}$: UD tensile and compressive (basic) strength parallel to the fibre direction

$R_{\perp}^{t} \equiv Y^{t}, R_{\perp}^{c} \equiv Y^{c}$: UD tensile and compressive strength transverse to the fibre direction

$R_{\perp\|} \equiv S$: Shear strength of a UD lamina transverse/parallel to the fibre direction

v_f : volume fraction

x_1, x_2, x_3: Coordinate system of a unidirectional (UD-)lamina (x_1 = fibre direction, x_2 = direction transverse to the fibre, x_3 = thickness direction)

ε_1, ε_2, ε_3: Normal strains of a unidirectional lamina

ν_{12}: Major Poisson's ratio in 'failure exercise' (corresponds to $\nu_{\perp\|}$ in German VDI guideline)

σ_1, σ_2, σ_3: Normal stresses in a unidirectional layer

$\sigma_2^{c}, \sigma_1^{t}$: Compressive stress across and tensile stress along the fibre direction

$\sigma_{\|}$, $\sigma_{\perp}$: Stresses parallel and transverse to the fibre direction

$\sigma_{eq}^{(mode)}$: Equivalent stresses of a mode ($\sigma_{eq}^{\|\sigma}, \sigma_{eq}^{\|\tau}, \sigma_{eq}^{\perp\sigma}, \sigma_{eq}^{\perp\tau}, \sigma_{eq}^{\perp\|}$)include load-induced mechanical stresses and residual stresses

σ_{1f}, σ_{2f}: Stress in x_1 direction; stress in x_2 direction. (is this for the fibres?)

$\hat{\sigma}$: mean (average) stress of the laminate

$\tau_{12} = \tau_{21}$, $\tau_{13} = \tau_{31}$, $\tau_{23} = \tau_{32}$: Shear stresses of a unidirectional lamina in the elastic symmetry directions. The first subscript locates the direction normal to the plane on which the shear stress is acting; the second subscript indicates the direction of the shear force

$\tau_{\perp\parallel}$, $\tau_{\perp\perp}$: Shear stressing transverse/parallel and transverse/transverse to the fibre direction

$\gamma_{12} = \gamma_{21}$; $\gamma_{13} = \gamma_{31}$; $\gamma_{23} = \gamma_{32}$: Shear strains of a unidirectional layer.

Abbreviations

CLT : Classical Laminate Theory

F : Failure function

FF : Fibre Failure

FMC : Failure Mode Concept

FRP : Fibre-Reinforced Plastic

IFF : Inter-Fibre Failure

MS : Margin of Safety

UD : Uni-directional.

Indices, signs

c, t : compression, tension (German Guideline VDI 2014)

f, m : fibre, matrix

n :repetitions in stack

Res : Reserve

(res) : resultant

s :symmetric lay-up, softening

- :statistical mean.

τ, σ :indicate the failure induced by the normal or shear Mohr stress.

INTRODUCTION

Recently, and as a part of a world-wide failure exercise aimed at highlighting the current capability of failure prediction, Cuntze presented a methodology[14], based on what is the so-called Failure Mode Concept (FMC), for the prediction of failure in composite laminates.

The exercise was carried out in two stages, referred to as Part A and Part B. The predictions, representing Part A contribution, were made for fourteen test cases and no experimental data were used. In the second part (Part B), the author was requested to submit a paper describing the correlation between those Part A predictions and a set of experimental results[8] provided by the organisers of the failure exercise.

This paper represents the author’s submission for Part B.

In order to ensure that the paper is self-contained, the author opted first for providing some of the background to the FMC-based criteria and the associated non-linear modelling of UD laminae and stacked laminates. Correlation between Part A prediction and test data was then described. Based on the available test data, the paper then describes some modifications which were introduced in order to obtain a better fit between the predictions and the experiments. Areas of potential problems in the test data and in the methodology are highlighted.

1.1 Designer's Demand 'Design to Failure Modes'

There are a number of fundamental issues worth highlighting here:

- According to the standards, in static design a designer in general has to dimension a laminate against two main types of failure: inter-fibre-failure (IFF) of the *laminae* and fibre-failure (FF) which is to be dedicated to the *laminate*. Some standards require to prove to have no IFF occurring if the laminate is subjected to the so-called Design Limit Load which to some extent corresponds to the Yield *Proof of Design* in case of isotropic materials. A correct Ultimate *Proof of Design* with its loads at fracture level demands for an analysis beyond IFF.

  An IFF mode normally indicates the *onset of failure* in a laminate whereas the appearance of FF in a single lamina embedded in a laminate usually marks *final failure* of that laminate. In the case of brittle FRP composites, failure coincides with fracture. Fracture is defined in this article as a separation of material, which was initially free of damage such as technical cracks (size of a mm) and delaminations but not free of tiny defects/flaws (size of microns) prior to loading
- The IFF modes incorporate cohesive fracture of the matrix and adhesive fracture of the fibre-matrix interface. Both fracture types are often termed 'matrix failure'
- Loading a composite by a $\sigma_{\parallel}$ stress will always induce a matrix stress acting in fibre direction. The matrix stress is normally obtained from use of micromechanics equations, eg Ref[9annex]. However, as long as the fracture strain of the matrix is multi-fold that of the fibre (eg fibre 2%, matrix 6%) one may neglect matrix stresses in fibre direction of a 0° layer because their magnitude does not practically affect the failure of the fibres
- The 'explosive' effect of the so-called *wedge shape failure* (a $\sigma_{\perp}^{c}$-caused IFF) of a lamina in a laminate, described in Ref[1, 9], may also directly lead to *final failure*, as for example in the case of Puck's torsion spring[1], or, via local delaminations, to buckling of the adjacent laminae and therefore to final failure of the laminate. This

IFF, where parts of a lamina move in thickness direction may also initiate a catastrophic failure like FF (*Figure 1*).

1.2 FMC-based failure conditions for UD laminae

The characterisation of the strength of transversally-isotropic composites requires –according to the FMC- the measurement of five independent basic strengths: $R_{\parallel}^{t}$, $R_{\parallel}^{c}$ (fibre parallel tensile and compressive strength), $R_{\perp}^{t}$, $R_{\perp}^{c}$ (tensile, compressive strength transversal to the fibre direction) and $R_{\perp\parallel}$ (fibre parallel shear strength). See ***Figures 2***[5].

The subsequently growing *yield* surface is confined by a *fracture* surface (yield capacity then exhausted). Here, it is a fracture surface which consists of failure mode-related partial fracture surfaces which piecewise confine an *an-isotropic yield* surface. This yield surface possesses a shape different to the fracture surface.

Above partial fracture surfaces are essentially described by those fracture conditions for the UD lamina which are matrix-dominated.

The lamina is defined here to be the material or the building block a laminate is made of.

The FMC generates a phenomenological three-dimensional, lamina stress-based *engineering approach*. As failures generally to be addressed in design are yielding and fracture. Here, fracture consists of various types (mechanisms) and yielding of one type, only. Main focus in case of the brittle FRP materials are the fracture types. of course. It shall be just addressed that some yielding is generated (initial degradations), and, that the fracture point reached will depend to some extent on the loading path in the yielding zone.

Yielding: In isotropic materials, and within the context of plasticity theory, yielding is normally described by a single function describing a global condition for a single failure mechanism. In anisotropic material, the situation is a little more complicated as will be described later in the FMC-based derivation of the an-isotropic yield condition.

Fracture: Cuntze's method of applying the FMC methodology (see ***Table 1***) is to strictly propose a set of equations for a number of failure modes in the individual lamina (ply) and then combine these equations in a suitable manner to predict failure in a laminate. Each failure mode is described by a distinct equation containing terms which show an ***interaction between*** the various acting ***stresses***. The method can be easily adopted to finite element methods (FE). This can be achieved, for instance, by using the output stresses as input values in those equations. The total number of failure mode[14]s is five; two FF modes and three IFF modes.

The choice of the invariants in Cuntze's FMC is based on physical considerations as outlined by Beltrami[2] . The appropriate choice of selecting the basic invariant is affected by the type of deformation of the material element, i.e whether the element is subjected to a volume change or to a shape change, or to both. An invariant can be dedicated to a volume change or to a shape change.

The ***interaction between*** the FF and IFF ***modes*** as well as between the various IFF modes is considered probabilistically. In order to account for this interaction between failure modes (referred to here as a Mixed Failure Domain MiFD), a rounding-off process in the interaction domains is employed, utilizing the probabilistics-based 'series spring model' approach for describing the combined effect of this system of failures.

1.3 Non-linear Analysis of laminates composed of UD laminae

1.3.1 Input

A full 3D *stress analysis of a unidirectional lamina* normally requires 5 elastic constants and 5 strength values. In the FMC method, five equations are required to describe the five fracture modes. It is worth noting that for a 2D analysis, the input consists in 4 elastic properties and 5 strength properties.

Applying a non-linear analysis a maximum of 5 non-linear stress-strain curves are required. However, for most conventional FRP materials, two non-linear curves $\sigma_2^c(\varepsilon_2)$ and $\tau_{21}(\gamma_{21})$ are normally observed and have to be applied.

1.3.2 Reason for non-linearity

The author believes that the non-linear behaviour of *laminates* composed of *brittle* laminae, similar to those used in the 'failure exercise', originates from damage development around inherent defects or flaws in the constituent *matrix* (a ductile matrix tensile specimen would show necking and so-called crazing, which appears in case of glass fibre composites as whitening in a tensile test) and at the *interface* fibre-matrix. These defects grow to micro-cracks and later to cracks under increased stressing.

1.3.3 Procedure of progressive failure analysis of laminates

The procedure used in the present paper is based on 'ply-by-ply' analysis. For carrying out the non-linear stress analysis the required non-linear stress-strain curves should describe both material hardening and material softening behaviours illustrated in ***Figure 3***. Load-controlled *hardening* describes the behaviour of the lamina up to the point of maximum stress (= strength, eg $\overline{R}_{\perp\parallel}$) and this point corresponds to initial failure of IFF modes. Strain-controlled softening behaviour describes the lamina response beyond that point and this is associated with *progressive failure*. However, according to knowledge about isotropic material behaviour it is quite probable that some minor damage (eg the matrix yields) still takes place during the hardening response.

It is worth mentioning here that the behaviour of the lamina in the laminate beyond initial failure along the softening curve, is referred to as *effective* stress-strain curve (which is still unknown, but has to be determined), and that the embedded lamina's behaviour (curve c in $\sigma_2^t(\varepsilon_2)$) can take various softening forms as shown in ***Figure 4***.

Note: After the onset of IFF only 'average stresses' can be calculated for the micro-cracked lamina and are inserted into the failure conditions. Average stresses are defined as stresses smeared over some length (includes a number of micro-cracks). of the cracked lamina.

The present method requires data for the pure failure mode domains, only! These data is the set of strength values which are always mandatory in the design and a few curve parameters. Data in the *Mixed Failure Domains* (*MiFD)* where the stress state influences various modes is not requested.

Secant modulus approach is employed the non-linear analysis.

A triggering approach is used to describe the effects of the stress state in the MiFD on the secant modulus of the mutually affecting modes in the MiFD. This approach increases the *equivalent stress (*which considers *all* influencing stresses) of each *affected mode* in the case of *hardening* (the secant modulus becomes a little smaller) and decreases the equivalent stress in the case of *softening* (the secant modulus becomes smaller, too).

In the re-worked MATHCAD-based code 'CLT FRP Non-linear', generated for the Failure Exercise, the self-correcting secant modulus method, described in Part A, was applied to describe the successive degradation (the softening, with its *effective* stress-strain curve). Each mode *equivalent stress* is mapped onto the measured associated *uni-axial* stress-strain curve, according to the well-known isotropic procedure.

Note: Non-linear behaviour of well-designed laminates (fibres are placed at least in three directions according to primary and probable secondary load directions) is most often physically (*lamina* behaviour) caused but rarely geometrically (*laminate* behaviour, see unfavourable loaded $[+55/-55]_s$).

2 PART A THEORY : COMPARISON WITH EXPERIMENT

2.1 Brief Review of Theoretical Assumptions and Remarks to the Analysis

2.1.1 Failure conditions applied in non-linear analysis

The following *failure conditions*, derived from the complete FMC-based invariant formulations, were employed in the non-linear analysis carried out in Part A[14]

$$FF1{,}2: \quad \frac{\varepsilon_1 \cdot E_{\parallel}^{t}}{Eff^{\parallel\sigma} \cdot R_{\parallel}^{t}} = 1; \qquad \frac{-\sigma_1}{Eff^{\parallel\tau} \cdot R_{\parallel}^{c}} = 1 \qquad \ldots\underline{\text{Part A set}}$$

$$IFF1{,}2: \quad \frac{\sigma_2}{Eff^{\perp\sigma} \cdot R_{\perp}^{t}} = 1; \qquad \frac{\tau_{21}^{\ 3} + b_{\perp\parallel}\, 2\sigma_2 \tau_{21}^{\ 2}}{(Eff^{\perp\parallel} \cdot R_{\perp\parallel})^3} = 1 \tag{1}$$

$$IFF3: \quad \frac{(b_{\perp}^{\tau} - 1)(\sigma_2 + \sigma_3)}{Eff^{\perp\tau} \cdot R_{\perp}^{c}} + \frac{b_{\perp}^{\tau}(\sigma_2 - \sigma_3)^2 + b_{\perp\parallel}^{\tau} \cdot \tau_{21}^{2}}{(Eff^{\perp\tau} \cdot R_{\perp}^{c})^2} = 1$$

wherein the 3 curve parameters are given as

$$b_{\perp\parallel} = \frac{1 - \left(\tau_{21}^{\perp\parallel} / \overline{R}_{\perp\parallel}\right)^2}{2\sigma_2^{c} \cdot \tau_{21}^{\perp\parallel 2} / \overline{R}_{\perp\parallel}^{3}}, \qquad b_{\perp}^{\tau} = \frac{1 + (\sigma_2^{c\tau} + \sigma_3^{c\tau}) / \overline{R}_{\perp}^{c}}{\sqrt{(\sigma_2^{c\tau} + \sigma_3^{c\tau}) / \overline{R}_{\perp}^{c} + (\sigma_2^{c\tau} - \sigma_3^{c\tau})^2 / \overline{R}_{\perp}^{c2}}}, \tag{2}$$

and $b_{\perp\parallel}^{\tau} = 1 - (b_{\perp}^{\tau} - 1)\sigma_{2\perp\parallel}^{c\tau} / \overline{R}_{\perp\parallel} - b_{\perp}^{\tau}(\sigma_{2\perp\parallel}^{c\tau} / \overline{R}_{\perp\parallel})^2$.

The last parameter will be skipped later in the context of the FMC because, it more has a sometimes helpful mapping character than a physical character. However: How good is the test data? Worthwhile to be mapped better?

The IFF1 and IFF2 modes are generally harmless failures whereas the IFF3 mode could cause catastrophic failure. This so-called wedge failure may occur in a laminate specimen if an internal lamina is laterally compressed. The event of a wedge failure is equal to the onset of delamination damage. In case of a plane laminate specimen, despite the anti-buckling device applied when testing in the compression regime, the wedge may slide and then cause a compressive reaction $\sigma_3^{\,c}$, normal to the lamina's plane, onto the adjacent laminae. Such a 'wedging-off effect' acting at an outer lamina will induce there delamination or might increase the size of an initial delamination.

In order to compute the stress effort $Eff^{(mode)}$, the relevant stresses $(\sigma_1, \sigma_2, 0, 0, 0, \tau_{21})$ are inserted into eqns(1). This will either lead to no failure, if $Eff^{(mode)} < 1$, or to failure if $Eff^{(mode)}$ exceeds *1* which means degradation and stress redistribution occurs.

<u>*Note*</u>: the value of *Eff* then remains constant *1* in the softening region. Both, the damage-dependent equivalent stress and the damage-dependent strength *R* become smaller according to the effective stress-strain curve.

The equivalent stress is defined as $Eff \cdot R = \sigma_{eq}(\{\sigma\})$. (3)

2.1.2 Non-linear analysis in mode interaction zones

In the case of a non-linear analysis, the material effort *Eff* has to be employed instead of the reserve factor f_{Res}. The general relationships between equivalent stresses

$$\left\{\sigma_{equiv.}^{(\mathrm{mod}\,es)}\right\} = \left(\sigma_{eq}^{\|\sigma}, \sigma_{eq}^{\|\tau}, \sigma_{eq}^{\perp\sigma}; \sigma_{eq}^{\perp\tau}; \sigma_{eq}^{\perp\|}\right)^T \tag{4}$$

with

$$\boxed{\begin{aligned} &\sigma_{eq}^{\|\sigma} = \varepsilon_1 \cdot E_{\|}^t, \quad \sigma_{eq}^{\|\tau} = -\sigma_1, \quad \sigma_{eq}^{\perp\sigma} = \frac{I_2 + \sqrt{I_4}}{2}, \\ &\sigma_{eq}^{\perp\|} = (I_3^{3/2} + b_{\perp\|}(I_2 I_3 - I_5))^{1/3}, \qquad \text{.......}\underline{\textit{Part A set}} \\ &\sigma_{eq}^{\perp\tau} = \frac{2 \cdot (b_{\perp}^{\tau} I_4 + b_{\perp\|}^{\tau} I_3)}{(-b_{\perp}^{\tau} - 1) I_2 + \sqrt{(b_{\perp}^{\tau} - 1)^2 I_2^2 + 4 b_{\perp}^{\tau} I_4 + 4 b_{\perp\|}^{\tau} I_3)}} \end{aligned}}, \tag{5}$$

and the efforts read

$$\left\{Eff^{(\mathrm{mod}\,es)}\right\} = \left(\frac{\sigma_{eq}^{\|\sigma}}{\overline{R}_{\|}^t}, \frac{\sigma_{eq}^{\|\tau}}{\overline{R}_{\|}^c}, \frac{\sigma_{eq}^{\perp\sigma}}{\overline{R}_{\perp}^t}, \frac{\sigma_{eq}^{\perp\tau}}{\overline{R}_{\perp}^c}, \frac{\sigma_{eq}^{\perp\|}}{\overline{R}_{\perp\|}}\right)^T . \tag{6}$$

The actual stress effort of a given mode, $Eff^{(\mathrm{mod}\,e)}$, is the ratio of the equivalent stress to the associated mode strength. The procedure of determining the resultant stress effort $Eff^{(res)}$ in each lamina of the laminate is similar to that of $f_{\mathrm{Res}}^{(res)}$ shown in eqn(23a).

2.1.3 Residual stresses

Residual stresses are taken into account by adding their values to the load stresses due to

$$\{\sigma\} = \{\sigma\}_{(L)} + \{\sigma\}_{(R)} \ . \tag{7}$$

The residual stresses in the laminae of the laminate decay with decreasing stiffness caused by the matrix degradation, which accompanies increasing non-linearity. In other words: In parallel to the decay of the stiffness the non-linear analysis releases matrix-dominated stresses. This applies for mechanical as well as for thermal load stresses.

2.1.4 Consideration of 3D-states of stress

In order to take the pressure effects into consideration when testing tubular specimens, the value of the pressure is taken as $\sigma_3 = -p$, and this is inserted into eqn(1). This is valid for both internal and external pressure loadings, and hence, the state of stresses is described as $(\sigma_1, \sigma_2, \sigma_3 = -p, 0, 0, \tau_{21})$

<u>*Note*</u>: For IFF1 and IFF2 modes, the pressure $\sigma_3 = -p_{ext}$ has no effect.

2.2 Comparison of Part A Theoretical Predictions with Experimental Results

2.2.1 Stress- strain curves of the UD-lamina

In the *Figures 5* and ***6*** the course of the mapped test data (solid lines) for the GFRP E/MY750 is displayed as well as the softening curve which is to be assumed for the effective stress-strain curve of the embedded UD-lamina (dotted curve). The dotted part or effective - curve $\sigma_2^c(\varepsilon_2)$ in *Fig.6* is valid if catastrophic failure of the 'delamination initiating' wedge is prevented by the laminate. All the oblique micro-cracks generated are still closed under the compression but deliver some compliance caused by the movement of UD material in thickness direction.

As there are for embedded laminae no test results available, which would verify the dotted curve, unfortunately, one cannot improve this input basis of the analysis.

Note: If the data given in Part B[8] outline more information than was given in Part A, then, the Part A graph is obsolete and its parameter set has to be reworked. For general clarification therefore, the captures of the figures will obtain the associated data set. Parameters, not relevant for the actual graph, are put into brackets.

2.2.2 Biaxial failure envelopes of the UD-lamina (90°tubes)

In the following bi-axial UD failure envelopes the un-known (not provided[8]) *residual stresses* could not taken into account.. Thus, just the so-called *load stresses* from the mechanical load test are considered. For the non-linear analysis the Ramberg/Osgood exponent and the assumed softening parameters are added to each capture.

Several cross-sections of the five-dimensional IFF-body are displayed in the *Figures 7* through ***10***. Where experimental data for Part B were not provided by the organizers the associated Part A figure was included in order to obtain a full survey :

2.2.2.1 Fig. 7, Part B data provided[8], GFRP E/LY556, (τ_{21},σ_2) and (τ_{21},σ_3)

The curve represents the IFF-responsible stresses in the plane of the lamina. Curve *(τ_{31},σ_2)* is obviously different to that of *(τ_{21},σ_2)* which indicates that τ_{31} does not have the same action plane as σ_2^t (known from the Puck[1]-Hashin model). The test data scatter pretty much in the compression domain The highest τ_{21} test point even shows a turn-off tendency in the MiFD.

Mapping of the course of test data is not satisfying. A more 'accurate' mapping was refused because it seems to be fiddling. Careful test series, formerly achieved and in recent German

R&D projects[9], always resulted in a shallower curve, similar to that depicted in the predicted curve (τ_{21},σ_2) of Part A[14]. Such shallower courses of test data can be easily FMC mapped. See test data from Ref[9] for GFRP and CFRP.

A prediction, without having got information in Part A to estimate $b_{\perp\parallel}$, makes not much sense. Therefore, *Fig.7* was reworked after determination of a new value, $b_{\perp\parallel} = 0.3$. Although in this figure the parameter $b_{\perp\parallel}^{\tau} \geq 0$ has a mapping improving influence in the compression domain it is not applied due to two reasons: No fiddling and be as simple as possible in the formulations for the lamina. Due to this, still here is set $b_{\perp\parallel}^{\tau} = 0$, as it is generally done later. Mind: In the next three (UD) graphs this parameter has no effect.

2.2.2.2 Part B data provided[8], CFRP T300/914C, (τ_{21},σ_1)

The test results (τ_{21},σ_1) show a too large scatter. Hence, no attempt is made to compare these experimental data with the predicted curve. Convincing test data are desired.

2.2.2.3 Fig. 8, Part A figure, $(\tau_{21}, v_f\, \sigma_{1f})$

The loading of the 90° tube is torsion with internal pressure plus axial loading. In order to also visualize in Part B the $(\tau_{21}, v_f\sigma_{1f})$-dependability this Part A figure is presented here again.

The graph highlights the (τ_{21},σ_1)-interaction. The notation $v_f\sigma_{1f}$ shall indicate the limited applicability of the homogenized lamina stress σ_1^t, because σ_1^t is not the fracture controlling stress. This is the fibre stress σ_{1f}. To generally maintain the lamina stress level in the graph the fibre stress is multiplied by the fibre volume fraction (approach: $\sigma_{1f}\cdot v_f = \varepsilon_1 \cdot E_\parallel^t$).

<u>*Note*</u>: It is to be learned, fracture may be not always well described by *homogenized* or, in other words, by *average* or smeared stresses!

2.2.2.4 Fig. 9, Part B data provided[8], GFRP E/MY750, (σ_2,σ_1)

The test data provided were not sufficient for the validation of the theory. The sudden shift of the crosses outline problems with the test performance. Some domains are fully missing, the existence of Tsai-Wu's hunch in the third quadrant cannot be discussed.

Mapping was a little adopted to the provided Part B data which led to a slightly improved mapping situation. The effect of the actual winding angle of 85° instead of 90° was not considered due to the fact it will not help to overcome the mapping discrepancies.

One generally missing link is the knowledge about the variation of the ultimate strengths from the strength test series which means, a set of crosses should be given at each strength point, too.

In general, the correlation between predicted and measured data is reasonable. Test data situation is to be improved.

2.2.2.5 Fig. 10, Part A figure, (σ_2=σ_3, σ_1)

*Fig **10*** shows the failure envelope of a lamina with the presence of a through the thickness compressive stress. Some peculiarities of a 2D lateral stressing are depicted.

It outlines that, in the domain $\sigma_2^c = \sigma_3^c > -10R_\perp^c$ and due to Poisson's ratio effects, failure is not caused by the IFF mode $F_\perp^\tau$, but by $F_\parallel^\sigma$. Unfortunately, experimental data[17, 9] are lacking in that domain. It is worth mentioning that bi-axial lateral pressure together with a fibre parallel compression stress is not an un-usual stress situation, as for instance in submarine hulls[15].

Note: Also for *dense* two-phase UD materials is valid "Hydrostatic pressure does not practically lead to fracture".

2.2.3 Initial and final biaxial failure envelopes of the laminates (tube specimens)

For the determination of the failure envelopes the MATHCAD-based code 'CLT-FRP non-linear' was employed, and an *assumed* softening behaviour applied. The symbols used to indicate the *mode of failure* are the symbols which characterize the *failure functions* $F_\parallel^\tau$ etc. The value of the angle marks the associated lamina. Loading of the tubes is achieved by the application of combined axial forces or/and internal or external pressure .

For completeness, the next four graphs are directly taken from Part A, too. According to the task they are just briefly assessed or commented here. In sub-section 5.2 the reworked graphs are presented. In the reworked graphs the given temperature drops are considered and the relevant test data is included.

Information given by the data providers: "The hoop and axial stresses were computed in the test evaluation from the pressure *p* and the axial load *F* by applying $\hat{\sigma}_{hoop} = p(r_{\text{int}} + \Delta r)/t_m$, $\hat{\sigma}_{ax} = pr_{\text{int}}/2t + F/(2\pi \cdot r_{\text{int}} t_m)$ with r_{int} as internal radius and t_m as mean thickness. Correction was made for the axial tensile stress due to the diameter at the centre of the gauge length is becoming greater than the diameter at the ends of the tube".

2.2.3.1 Fig. 11, Part B data provided[8], GFRP E/MY750, filament wound, $[+55/-55]_S$

*Figure **11*** incorporates the initial and the final failure envelope of this GFRP-laminate. Generally, probable remaining corners in the graph become smoothed due to the effect of high interaction of the failure *mode*s of all laminae in the failure system laminate. Eg, in the domain A-B both $F_{\|}^{\sigma}$ of the two adjacent laminae are 'acting together' and, therefore, are increasing the failure probability[13, 11] of the laminate to a higher joint failure probability (jfp). This causes a local shrinking of the multi-dimensional failure body.

In the *positive* quadrant the experimental data (intentionally not included here, see *Figure **11a***, *section 5*) turn away from the predictions. At fracture, the hoop (y) direction is essentially stronger than the axial one. An explanation for this effect will be investigated in sub-section 5.2.

Main difference to the test data is the theoretically higher load-carrying capacity in the biaxial compression domain (3rd quadrant). However it is known, in case of a pressure loaded *tension/compression-torsion tube specimen* and in case of *high pressure vessels* (1000 bar, ARIANE 5 launcher) loaded by external pressure p_{ex} the multi-axial strength is increased ($\sigma_3 = -p_{ex}$ is acting in a favourable manner). Therefore, as the comparison is not satisfactory, the pressure effect on the tube's external surface will be regarded in sub-section 5.2 and compared to the situation on the internal surface.

2.2.3.2 Fig. 12, Part B data provided[8], CFRP AS/3501-6, [0/+45/-45/90], hand lay-up of pre-pregs

*Figure **12*** displays a symmetrical failure envelope for a CFRP *laminate* subjected to a $(\hat{\sigma}_x, \hat{\sigma}_y)$ state of stress. The correlation of experimental and analytical data (see *Figure **12a***) is very bad in the corners of the first and third quadrant. The scatter on the negative $\hat{\sigma}_x$-axis is extreme.

Again, the sharp corners should be rounded-off in a refined procedure by taking into account the joint failure probability of the various failure modes of all laminae of the *laminate* as approximately was performed in the past[13]. In the negative quadrant buckling obviously lets not come $F_{\perp}^{\tau}$ to act. For further ideas, see section 5.2. The test results only partly allow for a verification of the theory. Further tests are to be performed.

2.2.3.3 Fig. 13, Part B data prov.[8], GFRP E/LY556, [90/+30/-30]$_S$, filament winding ($v_f = 62\%$)

*Figure **13*** concerns a laminate subjected to a $(\hat{\tau}_{xy}, \hat{\sigma}_x)$ state of stress. The deficiencies are essentially located in the negative quadrant. Test results (see Figure ***13a***) are not mapped sufficiently well.

The graph will be reworked in sub-section 5.2. Here again, sharp corners would be rounded by regarding the joint failure probability.

2.2.3.4 Fig. 14, Part B data provided[8], GFRP E/LY556, [90/+30/-30]$_S$

*Figure **14*** concerns a laminate subjected to a $(\hat{\sigma}_x, \hat{\sigma}_y)$ state of stress. Highest deficiencies are depicted in the third quadrant. The test results (see *Figure **14a***) in the 'extreme' parts of the first and third quadrant are not interpretable.

Because the predicted failure in the third quadrant is $F_{\parallel}^{\tau}$ as well as the prediction is so far from test it is assumed, instability of the compressed tube might have taken place. Further reasons and improvements are discussed in sub-section 5.2.

2.2.4 Stress-strain curves of the laminates

The following seven stress-strain curves (*Figures **15*** through ***21***) consider the eqns(1) and the data from the *Tables 3* and *6* in Part A[3]. The loading is monotonic, a temperature drop from curing (causes an off-set) is regarded, however, the graphs are shifted to the origin. Some graphs in these figures are *fibre*-dominated. There, the fibre mesh controls the well designed laminate's deformation. In contrast, other graphs are more or less *matrix*-dominated (for a survey, see *Table **2****)*. Whether there might have been generated a bulge (barrel) is not consequently recorded. A bulge is generated if there a widening of the centre tube (where the gage is located on) is recognized which is different to that at the constrained ends. Then, the gauge length is too short because the ends are still impacting the tube's centre part.

All figures are taken from Part A. Where new test was given the graphs are re-worked in sub-section 5.3 (*Figures **15a*** etc.). There, the prescribed[8] temperature drop ΔT will be considered and the relevant test data is incorporated in the graphs.

Fig. 15 and *16* outline the deformation behaviour of a pressure vessel, which is usually designed for one special load case 'internal pressure' that means for ($\hat{\sigma}_y : \hat{\sigma}_x$) = *(2:1)*. Load combinations outside of this ratio -such as *(1:0)*- will lead to too very large shear strains and thereby, to a 'limit of usage' (lou). This shear strain *design limit* or limit of usage (lou) for the embedded lamina was preliminarily assumed to be $\max\gamma = 4\ \%$ shear strain. This value corresponds to the fracture shear strain of the *isolated* lamina.

2.2.4.1 Fig. 15, Part B data provided[8], GFRP E/MY75, [55/-55], (1:0), end-reinforced tube

The predicted final stress-strain points have not been reached in test. Predicted failure was shear fracture by $F_{\perp\parallel}$, an initial failure, followed by limit of usage (lou). As the lou was assumed to be 4%, which corresponds for this stack to ε =2%, the computation stopped induced by numerical instability at too low a level. One idea to improve the prediction is to more correctly respect in computation the fibre angle which increases with increasing widening. Geometrical non-linearity due to an unsound loading condition for this design is to be considered.

2.2.4.2 Fig. 16, Part B data provided[8], GFRP E/MY750, [55/-55], (2/1), end-reinforced tube

The measured fracture load in ***Figure 16*** is about 25% lower than the predicted one. ***Figure 16a*** in sub-section 5.2 is referred to. Even in this case of a well-designed laminate, *non-linear* behaviour exists above initial failure.

Reason for this seems to be: As cylindrical widening cannot explain the difference of the curves, the effect must have come from bulging (barrelling). The $\hat{\sigma}_{ax}$ curve in comparison to the hoop curve indicates to be affected by the end constraints. The boundaries are not so far away from the locations of the strain gauges that bending has been avoided. Fracture by bulging-based bending seems to be the main reason for the difference. The plane CLT analysis cannot treat a probably acting 'bulge effect' of the cylinder that might have taken place.

2.2.4.3 Fig. 17, Part B data provided[8], CFRP AS4/35016, [0/+45/-45/90]s, (2/1)

Figure 17 shows a fibre-dominated behaviour, indicated by the almost flat curve up to fracture. The fracture load was predicted by about 10% too high, see ***Figure 17a***.

The effect of a joint failure probability is acting. It would reduce the fracture load by several percent. For further discussion see sub-section 5.2.

2.2.4.4 Fig. 18, Part B data provided[8], GFRP E/MY750,[45/-45]s, (1/-1)

Figure 18 indicates a numerical instability at reaching initial failure $F_{\perp\parallel}$ of this maximally sheared laminate after initially being too stiff in comparison to the test. And beyond this, a relatively poor mapping of the test data is recognized beyond 60% of the fracture load, ***Figure 18a***.

In sub-section 5.3 some reasoning is provided including the effect of the large deformations occurring.

2.2.4.5 Fig. 19, Part B data provided[8], GFRP E/MY750 [0/90]s, (0/1)

In ***Figure 19*** the predicted and the measured curve are lying almost on another. Unfortunately, for $\hat{\varepsilon}_y$ the stiffness loss by the IFF degradation was not 'subtracted' but added'. That means it initially became stiffer. Further, there is an overestimation of the fracture load by about 10%.

The difference in the fracture load may be partly explained by the fact: A double inner hoop layer [0/90/90/0] is fracture-mechanically more dangerous for the laminate[20,22] than a half-thick centre layer [0/90/0]. Main aspect is that the stress intensity $K_I = \sigma\sqrt{\pi a}$ of the $F_{\perp}^{\sigma}$-induced micro-cracks is proportional to the crack length a, corresponding to the thickness of the 90° layer. Further, fewer micro-cracks (lower crack density) are generated in the [0/90/90/0] laminate but become larger and weaken the fibres locally more due to the more critical 3D state of stress around the micro-crack tip than within a [0//90/0] laminate. With respect to strength a laminate [0/90/0/90/0/90/0] is better than $[0/90_2/0_2/90/0]$.

2.2.4.6 Fig. 20, Part B data provided[8], GFRP E/MY750, [45/-45]s, (1/1)

Obvious in ***Figure 20*** is, a too high fracture load was predicted. Also, the mapping of the non-linear course is not satisfactorily.

It is expected that bulging together with the relatively large deformation will cause this difference. In sub-section 5.3 some interpretations will be presented.

2.2.4.7 Fig. 21, Part B data provided[8], CFRPAS4/3501-6, [0/+45/-45/90]s, (1/0)

Figure 21 shows fibre-dominated behaviour with an almost flat curve up to fracture with a fracture load predicted less than 10% too high, ***Figure 21a***. The predicted stiffness was lower than the measured done. The result is satisfying.

Again, for $\hat{\varepsilon}_y$ the stiffness loss by the initial IFF degradation was not 'subtracted' but 'added'. But, this does not make the curves in total steeper.

2.3 General Comments on Correlation between Theoretical and Experimental Results

2.3.1 Design of Laminates and Tubes

The use of CLT-based analysis, as applied to flat plates, does not take into account some effects associated with the use of tubes as test pieces. In case of anti-symmetrical laminates an un-constrained laminate plate will twist under the action of in-plane normal stresses whereas a tube will not twist but experience in-plane shear straining. ***Table 2*** briefly summarizes the main aspects and describes different classes of laminates and their effects on deformation and fracture behaviours.

It was reported in Ref[8] that “The tubes were designed by utilising linear elastic thin shell theory. Non-linear analysis of the 55° and the 45° GFRP tubes have indicated that the gauge length was too short for the 45° tubes”. Hence, the so-called boundary constraints will influence the strains at the centre of the tube.

Table 3 outlines the different meanings of the theoretical stress and strain data provided by the author and the experimental ones given in the Failure Exercise. From this may be concluded: For the verification of theory a more accurate, finite element code-based large strain / large displacement analysis is required if the real stress state shall be assessed.

It is worth mentioning that the organisers[8], provided a description of the geometries and loading configuration for the tube specimen and asked those participants who possess numerical methods to provide a solution for the state of stresses. The author unfortunately has not the capacity to perform this work.

2.3.2 Constraint effect on an embedded lamina (in-situ behaviour)

If applying test data from tensile coupons of isolated laminae to an embedded lamina in a laminate, one has to consider that tensile coupon tests deliver test results of *weakest link type* (series model). An embedded or even an only one-sided constraint lamina, however, belongs to the class of **redundant type behaviour**, to a failure system of the 'parallel spring model' type. Due to being strain-controlled, the material flaws in a *thin* lamina cannot grow freely up to micro-crack size in thickness direction (called **thin layer effect**), because the neighbouring laminae will act as micro-crack-stoppers[20,22]. In addition to the reasons given in 2.2.4.5 regarding fracture mechanics, a thinner lamina has a lower energy release at increasing of flaws in the 90° layer to micro-cracks then a thicker one. As still mentioned, the actual absolute thickness of a lamina in a laminate is a driving parameter for the initiation or onset of micro-cracks.

2.3.3 Application of an effective stress-strain curve

Cuntze sees the peak value of a so-called *effective* stress-strain curve slightly higher (in-situ effect of the *embedded lamina*) than the strength point $\overline{R}$ of the *isolated* specimen due to the change from the 'weakest link behaviour' to the real redundant behaviour (see *Figures 3* and *4*) of a laminate. For the sake of simplicity this 'peak value' is lowered down to $\overline{R}$ in the analytical description of softening. Due to mapping reasons as calibration point for the softening curve 0.99 $\overline{R}_{\perp\parallel}$ was taken instead of $\overline{R}_{\perp\parallel}$ (see also sub-section 3.2.6).

In the execution of the non-linear analysis the application of an *effective* stress-strain curve is necessary which considers the behaviour of the lamina in the laminate by regarding the stack, its position, and the thickness. In order to provide the non-linear analysis with the input needed, normalized stress-strain curves have been constructed (see Part A) with a hardening part measured and a softening part assumed due to lack of data.

2.3.4 Application of mean properties

In non-linear analysis *mean* values have to be regarded in order to perform a stress and deformation analysis that corresponds to a mean or average *structural behaviour*. This is the best approximation of the structure's physical behaviour. Therefore, the execution of a non-linear *stress* analysis of the structure shall utilize a *mean(σ,ε)-curve*, whereby the associated secant moduli will be *mean* values, too, as performed in the work at hand. The application of a *minimum(σ,ε)-curve*, which is sometimes required, will lead to *lower* stresses.

For simply deriving *clear* data for the secant moduli two regimes have to be distinguished and mapped by 'fitting functions': One below (hardening regime) and one beyond $\varepsilon(\overline{R}_m)$.

<u>*Note:*</u> The strength data provided are assumed to be mean values!

3 NEW IDEAS AND REFINEMENTS TO PART A-THEORY

3.1 FMC-based Set of Lamina Failure Conditions

When describing the various types of failures of a transversally-isotropic lamina, in principle, failure conditions have to be provided -according to Cuntze's FMC- for (a) yielding which represents one physical mechanism, and (b) fracture which is described by five physical mechanisms. For the designer the yield condition is of minor importance.

3.1.1 Yield Condition of the UD Lamina

In conventional FRP materials the matrix is much more ductile (pronounced by a 6% fracture strain, being a very common target) than the fibre. Therefore, for the shear stress driven initial yielding the matrix is responsible, only. The yield strength of the constituent matrix plays a role at relatively low strains of the lamina (model: smeared material). This effect is obvious in the stress-strain curves (τ_{21}, γ_{21}) and (σ_2, ε_2), only.

A practical approach to establish an *initial yield condition* is to assume the existence of a 3D *yield* failure condition in terms of *macro*-mechanical quantities such as the lamina stresses. It is furthermore assumed a perfect bond exists between fibre and matrix. Such a yield failure condition shall be developed now:

Beltrami[2], Schleicher et al. assume at initiation of yield that the strain energy (denoted by W) in a cubic element of a material will consist of two portions:

$$W = \int \{\sigma\}\{\varepsilon\}\, d\{\varepsilon\} = W_{Vol} + W_{shape} \quad \text{with} \quad \{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{13}, \tau_{12})^T . \quad (8)$$

Including Hooke's law in the case of a ***transversally-isotropic*** (UD) body the expression will take the form (s_{ik} = compliance coefficients)[4]

$$W_{UD} = [s_{11}\,\sigma_1^2 + s_{22}\,\sigma_2^2 + s_{33}\,\sigma_3^2 + s_{44}\,\tau_{23}^2 + $$

$$+ s_{55}(\tau_{12}^2 + \tau_{13}^2)]/2 + s_{12}\,(\sigma_1\,\sigma_2 + \sigma_1\,\sigma_3) + s_{23}\,\sigma_2\,\sigma_3$$

$$= \underbrace{\frac{I_1^2}{2E_\parallel}}_{\text{volume}} + \underbrace{\frac{I_2^2(1-\nu_{\perp\perp})}{4E_\perp}}_{\text{volume}} - \underbrace{\frac{\nu_{\perp\parallel} I_1 I_2}{E_\parallel}}_{\text{volume}} + \underbrace{\frac{I_3}{2G_{\parallel\perp}}}_{\text{shape}} + \underbrace{\frac{I_4(1+\nu_{\perp\perp})}{4E_\perp}}_{\text{shape}} \quad . \quad (9)$$

with the invariants $I_1 = \sigma_1,\ I_2 = \sigma_2 + \sigma_3\ ;\ I_3 = \tau_{31}^2 + \tau_{21}^2\ ;\ I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2\ ;$

$$I_5 = (\sigma_2 - \sigma_3)\,(\tau_{31}^2 - \tau_{21}^2) - 4\tau_{23}\,\tau_{31}\,\tau_{21} \quad (10)$$

and the associated volume or shape change of the UD material element indicated above.

The procedure how to apply eqn(10) may be learned from the well-known isotropic results. There, including Hooke's law for ***isotropic*** material, it follows

$$W = \left[\underbrace{\frac{1-2\nu}{3} I_1^{iso\,2}}_{\text{volume}} + \underbrace{\frac{2+2\nu}{3} 3J_2^{iso}}_{\text{shape}} \right] / 2E , \quad (11)$$

with $I_1^{iso} = \underline{f(\sigma)} = \sigma_I + \sigma_{II} + \sigma_{III},$, *and* $\sigma_{eq}^{Mises} = \sqrt{3J_2^{iso}}$,

$$6J_2^{iso} = \underline{f(\tau)} = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2,$$

E:=elasticity modulus, ν:=Poisson's ratio. The first term in eqn(11) describes the volume change of the cubic material element (dilatation) and the second the change of its shape (distortion). One knows, both portions in the bracket above are used to formulate a failure function for subsequent failure surfaces

$$F = c_1 \underbrace{\frac{(1-2\nu) I_1^{iso\,2}}{3\bar{R}^2}}_{\text{volume}} + \underbrace{\frac{(2+2\nu) 3J_2^{iso}}{3\bar{R}^2}}_{\text{shape}} = c_2^2 = \frac{\sigma_{eq}^2}{\bar{R}^2} \quad , \quad (12)$$

wherein c_1 is a shape determining curve parameter, c_2 is the size governing parameter of the failure body, and $\overline{R}$ is a practically chosen strength value. For $\overline{R}$ most often $\overline{R}_m^t$ is employed in the provision of the fracture conditions; whereas in the case of yield conditions $\overline{R}_{p0.2}$ is applied.. When reaching the full-plastic domain a clear value for the varying Poisson's ratio ν is given : at yield limit $\sigma = \overline{R}_{p0.2}^t \Rightarrow \nu = 0.5$.

In order to formulate a relatively simple *yield* failure condition one chooses as basic invariant that term in eqn(11) which respects that the cubic material element experiences in the considered mode a shape change. This means in the isotropic case, J_2 is the choice, thus leading to the Mises model $3J_2^{iso} = \sigma_{eq}^2$. If σ_{eq} reaches $\overline{R}_{p0.2}$, then, the size parameter c_2 equals 1.

The result is a single yield failure surface which represents initial yielding under an arbitrary combination of the 3 principal stresses or of the 6 structural stresses.

Similarly, for the **transversally-isotropic** material just terms describing the shape change of the UD material cube can contribute to a failure function. Based on this, the approach reads

$$\frac{I_3}{(\overline{R}_{\perp\parallel p0.2})^2} + \frac{I_4}{(\overline{R}_{\perp p0.2}^c)^2} = c^2 \quad . \tag{13}$$

with the size parameter c and the two yield strengths dedicated to the two non.linear stress-strain curves $\sigma_2^c(\varepsilon_2)$ and $\tau_{21}(\gamma_{21})$. This single (global) *an-isotropic yield* surface, depicted in the ***Figures 23a*** (plane (τ_{21},σ_2) *and* ***b*** (plane (σ_2,σ_3)) is confined by the five partial fracture surfaces.

The subsequent yield surface is indicated by a vector *normal* to the actual global yield surface determined by its associated *flow rule* (normality criterion) which is not expressed here.

This not anymore the case for the partial *fracture* surfaces. The normality criterion is replaced by the 'idea' of proportional stressing, which means, the failure surface increases in the direction of the actual stressing which is seldom the normal direction. Of course, the loading of the laminate may be a proportional one. However, the stressing in the laminae of the laminate will usually not be proportional due to non-linearity.

From testing known is the fact: Fracture stresses of hoop wound tubes under combined (τ_{21},σ_2) stressing depend on the load path performed in test (ses Ref[16], too). According to the path, this is obvious from the bi-axially UD failure curves, different failure modes may be passed on different ways to the 'combined' fracture stress point allowing for different degradation on these paths. The onset of yielding, eqn(13), gives a hint at which stress combination this will become essential. Furthermore, the yield zone displays at which state of stress combination creep might have an impact on test results, and, when a high load rate has to be applied if creep has to be suppressed.

3.1.2 Fracture Conditions of a UD Lamina

Failure conditions should exhibit -besides a sound physical basis- the numerical advantages: mathematical homogeneity in the stress terms, stress terms of the lowest degree, simplicity, scalar formulations (stress potential, and therefore demand for the application of invariants), and eventually, numerical robustness and rapid computation.
Applying the FMC the choice of the stress invariants is based on whether there are volume and/or shape changes of the material element, whereas the choice of linear or other terms is determined by curve fitting considerations in respect of the advantages given above.

It was learned in the past that failure conditions for a UD lamina have to consider besides the aspects above, in any case *all* physical effects (eg embedding effects). However then, the *simplest* formulation is desired. From the Part A set of fracture conditions one equation, $F_{\perp}^{\tau}$ $=1$, is still not simplified enough without loosing much mapping quality. In this sense the condition is reformulated to become a homogeneous function

$$F_{\perp}^{\tau} = (b_{\perp}^{\tau} - 1)\frac{I_2}{\overline{R}_{\perp}^{c}} + b_{\perp}^{\tau}\frac{\sqrt{I_4}}{\overline{R}_{\perp}^{c}} = 1 \quad , \qquad (14)$$

and it was further set in it $b_{\perp\|}^{\tau} = 0$. A form was chosen, in which all stress terms shall be of the same power (grade) which consequently leads to a replacement of I_4 by $\sqrt{I_4}$. It is be noted that in eqn(14) the value of $b_{\perp}^{\tau}$ is different to that of the former one in the Part A set, of course, because $F_{\perp}^{\tau}$ is a new function. Now, the *reserve factor* is simpler (linearly) to compute due to

$$f_{\mathrm{Res}}^{\perp\tau} = \overline{R}_{\perp}^{c} / \sigma_{eq}^{\perp\tau} \;\; = \overline{R}_{\perp}^{c} / [(b_{\perp}^{\tau} - 1) I_2 + b_{\perp}^{\tau}\sqrt{I_4}\,] \quad , \qquad (15a)$$

as well as the *stress effort*

$$Eff_{\mathrm{Res}}^{\perp\tau} = \sigma_{eq}^{\perp\tau} / \overline{R}_{\perp}^{c} \quad . \qquad (15b)$$

in non-linear analysis. The visualisation of eqn(14) is presented in *Figure **A1*** depicting a linearly running fracture curve .

3.1.3 Listing of Fracture Conditions achieved

In engineering application due to property scatter the simplest strength criteria which still describe the physical effects should be applied. This always reduces the number of curve parameters to be determined and, besides this, the numerical effort. Based on the simplifications outlined, the following set of *failure conditions*, $F(\{\sigma\}) = 1$,

$$\boxed{\begin{aligned} &FF1: \quad F_{\parallel}^{\sigma} = \frac{I_1}{\overline{R}_{\parallel}^{t}} = 1^{*}, \qquad \underline{\textit{Part B , set}} \\ &FF2: \quad F_{\parallel}^{\tau} = \frac{-I_1}{\overline{R}_{\parallel}^{c}} = 1, \\ &IFF1: \quad F_{\perp}^{\sigma} = \frac{I_2 + \sqrt{I_4}}{2\overline{R}_{\perp}^{t}} = 1 \ , \\ &IFF2: \quad F_{\perp\parallel} = \frac{I_3^{3/2}}{\overline{R}_{\perp\parallel}^{\ 3}} + b_{\perp\parallel} \frac{I_2 I_3 - I_5}{\overline{R}_{\perp\parallel}^{\ 3}} = 1, \\ &IFF3: \quad F_{\perp}^{\tau} = (b_{\perp}^{\tau} - 1)\frac{I_2}{\overline{R}_{\perp}^{c}} + \frac{b_{\perp}^{\tau}\sqrt{I_4}}{\overline{R}_{\perp}^{c}} = 1 \ , \end{aligned}} \tag{16}$$

have been derived with two free *curve parameters* $(b_{\perp\parallel},\ b_{\perp}^{\tau})$ to be determined from multi-axial test data or estimated by experience. $\overline{R}$ marks mean strength value. <u>*Note*</u>*: w.r.t $F_{\parallel}^{\sigma}$ $I_1 = \sigma_1 \rightarrow v_f \cdot \sigma_{1f} = v_f \cdot \varepsilon_1 \cdot E_{1f} = \varepsilon_1 \cdot E_{\parallel}$ with σ_{1f} = tensile stress fibre and v_f := fibre volume fraction. The very small load-carrying capacity of the matrix is neglected in relation to the fibre's portion.

The two curve parameters have to be determined from a test point (several measurements in this calibration point □) or correctly, by curve fitting of a course of test data in the associated <u>pure</u> domain applying a regression method. The calibration points (see *Fig, A1* and *7)* deliver, after inserting them into the IFF conditions for $F_{\perp\parallel}$ and $F_{\perp}^{\tau}$ (Part A set, eqn(16)), and after a further resolution, the equations

$$\boxed{\begin{aligned} &b_{\perp\parallel} = \frac{1 - \left(\tau_{21}^{\perp\parallel} / \overline{R}_{\perp\parallel}\right)^2}{2\sigma_2^c \cdot \tau_{21}^{\perp\parallel 2} / \overline{R}_{\perp\parallel}^3} \qquad \text{from } \left(\sigma_2^{\ c},\ \tau_{21}^{\perp\parallel}\right), \ \underline{\textit{Part B set}} && (17a) \\ &b_{\perp}^{\tau} = \frac{1 + (\sigma_2^{c\tau} + \sigma_3^{c\tau}) / \overline{R}_{\perp}^{c}}{(\sigma_2^{c\tau} + \sigma_3^{c\tau}) / \overline{R}_{\perp}^{c} + \sqrt{(\sigma_2^{c\tau} - \sigma_3^{c\tau})^2} / \overline{R}_{\perp}^{c}} \quad . && (17b) \end{aligned}}$$

The value of the curve parameter $b_{\perp}^{\tau}$ differs to that of eqn(2). Both parameters $b_{\perp\parallel}$ and $b_{\perp}^{\tau}$ depend on the material behaviour and on the IFF formulation applied. Bounds on the safe side for GFRP, CFRP and AFRP were assumed to be $0.05 < b_{\perp\parallel} < 0.15$, $1.0 < b_{\perp}^{\tau} < 1.1$ (instead of 1.5 for the old Part A quadratic equation). A value $b_{\perp\parallel} = 0$ means 'no bulge effect' and $b_{\perp}^{\tau} = 1$ means 'no friction' in the ⊥⊥-plane. As calibration points for $b_{\perp}^{\tau}$ are still missing : Assuming $b_{\perp}^{\tau} = 1$ during pre-dimensioning. will keep the engineer in the compression domain on the safe side. It furthermore will simplify the failure function. To assume in pre-dimensioning for the computation $b_{\perp\|} = 0$ is a good approach.

With respect to the 3D character of the IFF conditions above IFF1 and IFF3 may also serve ($F_{\perp}^{\tau}$: *wedge failure*, $F_{\perp}^{\sigma}$: *transversal tensile failure*) as criteria for the *onset of delamination* generated by the inter-laminar stresses $(\sigma_3, \tau_{32}, \tau_{31})$. Hydrostatic compressive and tensile stressing is automatically considered when respecting the full 3D stress state. The wedge failure is also responsible for a delamination-caused *local buckling* of the laminate.

3.2 Refinement of Non-linear Analysis Procedure utilized

As the non-linear analysis is of major challenge in the failure exercise the main topics will be repeated here. First topic is the mapping.

3.2.1 Mapping

- ***Hardening-*** The degree of non-linearity essentially depends on the nonlinearly behaving matrix material which affects $E_{\perp}^{c}$ and $G_{\|\perp}$. For the secant moduli to be applied in the non-linear stress analysis the following values are determined by the Ramberg/Osgood equation

$$\varepsilon = \sigma / E_{(o)} + 0.002(\sigma / R_{p0.2})^{n} \qquad (19)$$

which maps the course of non-linear stress-strain data very well ($E_{(o)}$ is the initial tangent modulus = Young's modulus) with the Ramberg/Osgood exponent

$$n = \ell n\left(\varepsilon_{pl}(R_m)\right) / \ell n\left(R_m / R_{p0.2}\right) \qquad (20)$$

estimated from the strength point $\left(R_m, \varepsilon_{pl}(R_m)\right)$ in *Fig.**3***, eg. Then, data for the secant moduli of $E_{\perp}, G_{\|\perp}$ may be provided from above Ramberg/Osgood mapping due to

$$E_{(sec)} = E_{(o)} / (1 + 0.002 \cdot E_{(o)} / R_{p0.2} \cdot (\sigma / R_{p0.2})^{n-1}) \qquad (21)$$

(The isotropic notations were taken here for the sake of simplicity).

- ***Softening-*** Beyond *Initial Failure* (IFF) an appropriate progressive failure analysis method has to be employed, or in other words, a *Successive Degradation Model* for the description of *Post Initial* failure. This can be performed best by using a failure mode condition that indicates failure type and damage danger of the material (*material* level) predicted by the size of the stress effort. *Final Failure* (*structural* level) occurs after the laminate, and thereby the structure, has experienced a stiffness reduction and has degraded to a level where it is no longer capable of carrying additional load.

Fig. 3 depicts hardening together with softening. The curves for an *isolated,* eg tensile coupon specimen, are (a) in the usual load controlled test, (b) in a strain controlled test. The curve (b) is assumed here due to the lack of experimental data.

Modelling of *Post Initial Failure* behaviour of a laminate requires that assumptions have to be made regarding the decaying elastic properties of the actually degrading embedded lamina (curve (c) in *Fig. 4*). $E_\perp^c$ and $G_{\parallel\perp}$ are decreasing gradually rather than being suddenly annihilated. A rapid collapse (often named 'ply discount method') of $E_\perp^t$ is unrealistic and further, probably leads to convergence problems in numerics. A simple exponential function was used to map this softening in order to later derive the secant moduli. It generally reads in simplified isotropic notation (the suffix s denotes softening)

$$\sigma_s = R_m /(1+\exp[(a_s + \varepsilon)/b_s]) \tag{22a}$$

with two curve parameters a_s, b_s usually estimated from the data of two calibration points, eg

$$(R_m,\ \varepsilon\ (R_m)) \quad \text{and} \quad (R_m \cdot 0.1,\ \varepsilon\ (R_m \cdot 0.1)) \qquad \text{or another, as applicable} \tag{22b}$$

Equation(22) practically models the softening part of the stress-strain curve of a lamina which is embedded in a laminate, and thus, it includes the effect of the altering microcrack density up to the critical damage state (CDS). Curve (c) is therefore termed an *effective* curve.

*Annex **3*** visualizes the hardening function (19) and the softening function (22).

3.2.2 Interaction of Failure Modes

Mechanical and probabilistic interactions can not be clearly distinguished and therefore, the author models the mode interactions by a simple probabilistic series system model. This so-called 'logical model of the failure system' acts as a rounding-off' procedure linked to the determination of the desired values for $f_{Res}^{(res)}$ or $Eff^{(res)}$.

The **(resultant) Reserve Factor** (super-script res) takes account of the interactions of all modes. It may be estimated by the rounding-off equation, describing the series spring model,

$$(1/f_{Res}^{(res)})^{\dot m} = f(f_{Res}^{(modes)}) \qquad \text{if linear state of stress}$$

$$= (1/f_{Res}^{\perp\sigma})^{\dot m} + (1/f_{Res}^{\perp\parallel})^{\dot m} + (1/f_{Res}^{\perp\tau})^{\dot m} + (1/f_{Res}^{\parallel\sigma})^{\dot m} + (1/f_{Res}^{\parallel\tau})^{\dot m} \tag{23a}$$

and

$$Eff^{(res)^{\dot m}} = \sum_{1}^{5} Eff^{(modes)} \qquad \text{if non-linear}$$

$$= \left(\sigma_{eq}^{\parallel\sigma}/\bar{R}_\parallel^t\right)^{\dot m} + \left(\sigma_{eq}^{\parallel\tau}/\bar{R}_\parallel^c\right)^{\dot m} + \left(\sigma_{eq}^{\perp\sigma}/\bar{R}_\perp^t\right)^{\dot m} + \left(\sigma_{eq}^{\perp\tau}/R_\perp^c\right)^{\dot m} + \left(\sigma_{eq}^{\perp\parallel}/\bar{R}_{\perp\parallel}\right)^{\dot m} \tag{23b}$$

as the **resultant Effort** (interaction of failure modes), with $\dot m$ as the mode interaction coefficient (rounding-off exponent). As a simplifying *assumption* $\dot m$ is taken the same for each interaction zone! The value of $\dot m$ has to be set by fitting experience and by respecting the fact -still given above- that in the interaction zones micro-mechanical and probabilistic effects will commonly occur and cannot be discriminated. Some skill has to be put into the choice of the value of $\dot{m}$, but $\dot{m} = 3.1$ is a good approach.

If eg inserting a unidirectional fracture stress (this is the strength value) into the equation above, then a point on a 2D- or 3D-failure curve or failure surface, described by $f_{Res}^{(res)} = 1$, is achieved, the strength point.

3.2.3 Determination of Mode Efforts (interaction of the lamina stresses)

In the following set of formulae the so-called *equivalent stress* as well as the *effort* of each mode is provided. An equivalent stress includes all load stresses and residual stresses which are acting together in a mode equation.

The so-called *Mode Efforts* explicitly read according to the general equation $Eff_{Res}^{(mode)} = \sigma_{eq}^{(mode)} / \overline{R}^{mode}$,

$$Eff^{\|\sigma} \mathrel{\hat{=}} \varepsilon_1 \cdot E_{\|}^{t} / \overline{R}_{\|}^{t} = \sigma_{eq}^{\|\sigma} / \overline{R}_{\|}^{t} \;, \qquad \text{Part B set} \tag{24a}$$

$$Eff^{\|\tau} = -\sigma_1 / \overline{R}_{\|}^{c} = -\sigma_{eq}^{\|\tau} / \overline{R}_{\|}^{c} \;; \tag{24b}$$

$$Eff^{\perp\sigma} = [I_2 + \sqrt{I_4}\,] \;/ 2\overline{R}_{\perp}^{t} = \sigma_{eq}^{\perp\sigma} \;/ \overline{R}_{\perp}^{t} \;, \tag{24c}$$

$$Eff^{\perp\|} = [I_3^{3/2} + b_{\perp\|}(I_2 I_3 - I_5)]^{1/3} / \overline{R}_{\perp\|} = \sigma_{eq}^{\perp\|} / \overline{R}_{\perp\|} \;. \tag{24d}$$

$$Eff^{\perp\tau} = [(b_{\perp}^{\tau} - 1)I_2 + b_{\perp}^{\tau}\sqrt{I_4}\,] \;/ \overline{R}_{\perp}^{c} = \sigma_{eq}^{\perp\tau} / \overline{R}_{\perp}^{c} \;, \tag{24e}$$

<u>Note</u>: If an $Eff_{Res}^{(mode)}$ becomes negative, caused by the numerically advantageous automatic insertion of the FEM stress output $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{13}, \tau_{12})^T$ into *all* 5 failure conditions, a value of 0 shall replace the negative value. A negative value eg results if a positive σ_1 is inserted into eqn(24b). The situation of an imaginary $Eff^{(mode)}$, which is only possible for $Eff^{\perp//}$, is bypassed by a query (see <u>*Annex **2***</u>).

3.2.4 Degradation 'Triggering'

In the laminae of a laminate multi-axial states of stress are acting which in the interaction domains have an impact on more than one of the five failure modes. Adjacent failure modes are commonly affected. One has to pay attention to a proper interaction of the interacting modes in the stress and strain analysis in the following manner: In order to take into account the interaction of the failure modes the secant moduli $E_{2(sec)}$ and $G_{21(sec)}$ are taken from the $\sigma_2(\varepsilon_2)$-curve or the $\tau_{21}(\gamma_{21})$-curve not just at the stresses σ_2 or τ_{21} resulting from the stress and strain analysis for the actual level load. Their values are taken at a little higher stress in the ' hardening branch' with increasing stress and at a little lower stress in the 'softening branch' with decreasing stress. This 'stress correction' is controlled by the so-called ' triggering approach'. Therefore, a corresponding degradation (displayed by a stiffness reduction) has to be considered.:

- for *increasing* stress *(Hardening)* $\Delta\sigma > 0$

$$corr\,\sigma_{eq}^{>(\mathrm{mod}\,e)} = \sigma_{eq}^{(\mathrm{mod}\,e)} \cdot TrF \quad \text{being a modulus decrease,} \tag{25a}$$

- for *decreasing* stress *(Softening)* $\Delta\sigma < 0$

$$corr\,\sigma_{eq}^{<(\mathrm{mod}\,e)} = \sigma_{eq}^{(\mathrm{mod}\,e)} \,/\, TrF \quad \text{being a modulus decrease .} \tag{25b}$$

The controlling parameter is the ratio of the resultant stress effort $\mathrm{Eff}^{(\mathrm{res})}$ to the maximum mode effort $\max\mathrm{Eff}^{(\mathrm{mode})}$. In Part B a revised ($\dot{m}$ 'th root) trigger-factor is utilized

$$\mathrm{TrF} = \sqrt[\dot{m}]{Eff^{\,(res)} / \max Eff^{\,(\mathrm{mode})}} \ , \tag{26}$$

which damps the formerly too sudden stiffness decrease. In the equations above the stress effort of the maximum stressed mode governs the 'triggering', although TrF is dedicated to all IFF modes affected in order to really regard the combined degradation effect. By this triggering approach slightly lower secant moduli $E_{2(\mathrm{sec})}$ and $G_{21(\mathrm{sec})}$ are provided for the next calculation loop lower as those which would result without the correction by the triggering approach. In other words : As adjacent failure modes mutually degrade their elastic stiffnesses, in order to apply correct secant moduli, equivalent stresses

$$\left\{ \sigma_{eq}^{(\mathrm{mod}\,es)} \right\} = \left(\sigma_{eq}^{\parallel\sigma}, \sigma_{eq}^{\parallel\tau}, \sigma_{eq}^{\perp\sigma}; \sigma_{eq}^{\perp\tau}; \sigma_{eq}^{\parallel\perp} \right)^T ,$$

corrected by TrF , have to be inserted into the equations for the secant moduli.

__Figure 22__ visualizes Cuntze's 'triggering approach'. In his theory for the actual load the degradation of $E_{2(\mathrm{sec})}$ and $G_{21(\mathrm{sec})}$ is performed with the same trigger factor TrF. In contrast to Puck's theory, if one of the corrected equivalent mode stresses has reached its strength level, a relatively rapid decrease of the mode's *average* (smeared over the micro-cracks) *equivalent stress* will follow. The triggering approach is already active before the onset of IFF. In section 5 the diminishing effect of triggering with increasing large strains will be considered.

3.2.5 Non-linear Analysis

The solution procedure of the non-linear analysis aims to establish static equilibrium at each load step after material properties have been changed. For each iteration the procedure is repeated until convergence (equilibrium) is reached or total failure. A correction of the fibre angle in accordance with the change of the specimens geometry as consequence of large strain behaviour has been considered.

In the non-linear computations for a small load increment sometimes just one iteration step is practically needed in a secant modulus procedure in order to roughly consider stress-redistribution, that means, load from the weakening matrix (matrix-dominated modes) is transferred to a fibre (fibre mode).

3.2.6 Determination of the Degrading Elasticity Properties of the Lamina

By employing the equivalent stress reached in each failure mode the associated secant modulus of each mode was determined for the hardening and the softening regime.

Considering a consistent stress concept for all $\sigma_{eq}^{(\text{mod}\,es)}$ an *explicit* dependency $E_{\sec}(\sigma_{eq}^{(\text{mod}\,e)})$ has to be provided. For reasons of achieving such an explicit formulation two separate formulae are discriminated which are linked in the strength point. This automatically respects that the chosen non-linear calculation procedure demands for the dependencies of the secant moduli on the corresponding equivalent stress. These dependencies are (see *Fig. 4*)

- Pre-IFF analysis of lamina : $\Delta\sigma > 0$ (*increasing stress, hardening*)

$$E_{\perp(\sec)}^{t} = E_{\perp(o)}^{t}$$

$$E_{\perp(\sec)}^{c} = E_{\perp(o)}^{c} / [1+0.002 \cdot (E_{\perp(o)}^{c} / R_{p0.2}^{\perp c}) \cdot (\sigma_{eq}^{\perp\tau} / R_{p0.2}^{\perp c})^{n_{\perp}^{c}-1}] \qquad (27)$$

$$G_{\|\perp(\sec)} = G_{\|\perp(o)} / [1+0.002 (G_{\|\perp(o)} / R_{p0.2}^{\perp\|}) \cdot (\sigma_{eq}^{\perp\|} / R_{p0.2}^{\perp\|})^{n_{\perp\|}-1}]$$

- Post-IFF analysis of lamina : $\Delta\sigma < 0$ (*decreasing stress, softening*)

$$E_{\perp(\sec)}^{t} = \sigma_{eq}^{\perp\sigma} / \varepsilon(\sigma_{eq}^{\perp\sigma}) = (\sigma_{eq}^{\perp\sigma} / b_{s}^{\perp t}) / \left[\ell n\left(\frac{R_{\perp}^{t} - \sigma_{eq}^{\perp\sigma}}{\sigma_{eq}^{\perp\sigma}}\right) - \frac{a_{s}^{\perp t}}{b_{s}^{\perp t}} \right] \quad \text{etc.} \qquad (28)$$

The branches with increasing stresses ('hardening') of these stress/strain curves are found by the usual experiments with uni-axial σ_2-stress or pure τ_{21}-stress, respectively. The branches with rapidly decreasing stresses (called 'softening') are preliminarily assumed, see *Fig. 4*. For the further modes the same formula is valid, however, the mode parameters are different.

After having reached $Eff^{(res)} = 1$ this value *1* is kept as maximum value in the further degradation procedure which causes a stress redistribution towards the fibres as far as the fibre net allows it. Thereby, also the residual stresses are reduced similar to the situation with metallic materials where increasing non-linearity reduces stiffness, and the residual stresses.

3.2.7 Laminate Failure

The approach may be called a self-correcting secant modulus procedure. The laminate's stiffness matrix is recomputed after each step. Then, the stresses σ_2 and τ_{21} in the laminae of the laminate are computed by using secant moduli from the $\sigma_2(\varepsilon_2)$- and $\tau_{21}(\gamma_{21})$-stress/strain curves and from them the equivalent stresses are determined.

Most engineers assume that FF in at least one lamina of a laminate means final failure of the laminate. Therefore, the biaxial failure envelopes for final failure of laminates predicted by the various authors do not differ that much, as long as the laminates are 'well-designed and have three or more fibre directions. The 'strengths' of these laminates are 'fibre dominated'. Also, the predicted stress/strain curves of such laminates look very similar because the fibres which are much stiffer than the matrix carry the main portion of the loads. Different degradation procedures after the onset of inter-fibre failure (IFF) do therefore not influence the predicted strains very much. This is especially true for CFRP laminates.

4 SOME COMMENTS ON NATURE AND EFFECT OF FAILURES PRE-DICTED

4.1 Remarks on Design to Failure Modes and Modelling

- In composite structures composed of stiff fibres and well-designed by netting theory the fibre net controls the strain behaviour
- The FMC considers the inter-laminar stresses and classifies the failure modes. Therefore, associated degradation models are inherent and make a gradual degradation of the affected property possible
- Above the initial failure level an appropriate progressive failure analysis method has to be employed by taking a *Successive Degradation Model* and by using a failure mode condition that indicates failure type and quantifies damage danger or fracture risk
- Multidirectional laminates are usually still capable of carrying load beyond *initial failure* which usually is determined by IFF
- *Final failure* occurs after the structure has degraded to a level where it is no longer capable of carrying additional load. This is most often caused by FF, however in specific cases by an IFF, too. An inclined wedge-shaped inter-fibre crack caused by $F_{\perp}^{\tau}$ can lead to final failure if it damages the neighbouring layers by its capability to cause delamination.

4.2 Relevance to Experimental Data w.r.t. Evaluation of Measurements

A correct analysis of boundary conditions and stress state of the test specimen is mandatory before evaluating and applying the data. In this sense tubes instead of the flat coupon specimens will help to avoid problems associated with the 'free edge effect'. A wide range of bi-axial stresses can be achieved. Real tri-axial stress states require refined specimens (see VDI report[9], page 107).

The tubular specimens may be subjected to internal and external pressure, to torsion and axial forces. However, also the testing of tubular specimens is not free of problems such as bulging, caused by end constraints, or buckling of the cylinder. Further, tubes may exhibit non-linear changes in geometry during loading. Therefore, a non-linear analysis has to take into account both, large strains *and* large deformations. If these facts are not considered in test evaluation *and* in analysis one has compared apples and oranges.

As indications for IFF are more or less not provided as Failure Exercise information[8] a verification of the predicted values is hardly possible. In the matrix-dominated cases the IFF have a strong influence on the stress-strain behaviour.

In Ref[3] was stated: "Pressures and forces were usually increased continuously until fracture occurred. During the tests, the pressure was increased at a steady stress rate (2MPa/min) and the axial load was increased continuously to maintain a constant predetermined *stress ratio* of the *laminate's mean stresses* $\hat{\sigma}_{hoop} / \hat{\sigma}_{ax}$ within the gauge length until fracture. During loading *bulging* was not regarded (or was not observed?). Fibre volume fraction v_f in the laminates is approximately the same as for the UD laminae".

Whether the stress rate above might have caused some *creeping* in the highly loaded matrix-dominated cases, is not made clear. Did bulging still occur, probably?

Unless otherwise stated the so-called applied stresses in the test (dots) were calculated from measured pressure p and axial load F based upon un-deformed initial geometry applying

$$\hat{\sigma}_{hoop} = p \cdot r_{\text{int}} / t \quad \text{and} \quad \hat{\sigma}_{ax} = p \cdot r_{\text{int}} / 2t + F / (2\pi \cdot r_{\text{int}} \cdot t \quad , \qquad (29)$$

where r_{int} is the internal radius of the un-deformed tube and t the laminate thickness. For $\hat{\sigma}_{ax}$ in case of large displacements corrections were sometimes reported due to the fact that r_{int} at the centre of the gauge length becomes greater than the radius at the ends of the tube.

<u>*Note:*</u> The test stresses given are 'linear' stresses. They cannot be utilized like the analytical ones which are tried to be determined as non-linear, real stresses including bulging as far as possible.

The pressure loading produces a compressive stress $(\sigma_3 = -p)$ on the loaded surface whereas the other surface is free of radial stress. Despite of the fact that with increasing degradation the highest hoop stress in a vessel wanders beyond the IFF from the internal surface to the external surface the most severe stress state will occur at the internal surface because there is no beneficial bi-axial compression, however, some stabilizing by the curvature.

The differences between uni-axially and bi-axially compression-loaded surfaces will have to be analysed. From *Fig. A1* and *Fig.10* can be concluded that uni-axial lateral pressure is more severe than having acting together σ_2^c with σ_3^c. The pressure stabilized surface lamina will not fracture first, in general.

5 APPLICATION OF REFINED THEORY

From the 'refined' theory, which includes failure conditions and some convergence ideas in the non-linear analysis coding, the following impacts can be reported: The effect of the new $F_{\perp}^{\tau}$ is marginal. So, only the coding is of importance. Any difference of the revised graphs to the Part A graphs is essentially contributed to the improvements by a more stabile numerical procedure. In addition, the '$(\sigma_3 = -p)$-effect' was considered in the associated graphs. Lamina stress strain curves have not been modified.

Before the assessment of the following graphs a few statements shall be given regarding (a) can the current theory (= fracture conditions with non-linear coding) predict the final failure stresses and strains observed in tests, (b) what can be done in order to capture the final measured points, (c) should the same $b_{\perp}^{\tau}, b_{\perp\|}$ and $\max \tau_{21}$ values be used in *Figs 11, 11a, 15, 15a, 16, 16a, 18 and 18a*? and, (d) is the max shear strain of 10% used in the analysis based on experimental data? The statements are:

a) The author believes the FMC-based fracture conditions not need to be improved

b) Their capability to predict IFF highly depends on the experimental input. To better predict final failure is more a question of accurate non-linear analysis that eg tackles bulging

c) If the material is the same, and, if the test specimens are identically manufactured as it is guessed in the given test cases, the same curve parameters must be applied

d) The reason to not utilize 4% anymore is based on the fact that 4% belong to a load-controlled behaviour but it is a strain-controlled one. The test results provided, however, enable to assume a strain-controlled value of 10%.

The following sections demonstrate where the refinements have an effect or not.

5.1 Bi-axial Failure Envelopes of the UD-lamina

Fig. ***A1*** depicts the fracture curve $F_{\perp}^{\tau}$. It just outlines for this wedge failure mode (IFF3) the difference of the new linear $F_{\perp}^{\tau}$ to the former quadratic $F_{\perp}^{\tau}$. The curve $\sigma_2(\sigma_3)$ is now fully linear in comparison to the former one, the quadratic behaviour of the former function was pronounced in the first stress quadrant, only. Practically, there is no impact on the mapping capabilities.

5.2 Initial and Final Bi-axial Failure Envelopes of the Laminate

"Unless otherwise stated[8], and in the evaluation of the experimental hoop and axial stresses $\hat{\sigma}_{hoop}, \hat{\sigma}_{ax}$ no allowance was made for bulging or large deformations".

This is to be considered in the correlation of predicted and test data.

5.2.1 Fig. 11a, GFRP E/MY750 [+55/-55]s , tube

*Figure **11a*** incorporates the initial and the final failure envelope of this GFRP-laminate. There are three areas worth discussing: (a) behaviour under biaxial tension, (b) behaviour under biaxial compression and (c) pressure effects.

(a) The predicted curve in the biaxial tension, especially near the horizontal axis, is much lower than the measured values[8] indicate. Here, the know-how from the investigations on the (2:1) stress-strain curves (see ***Figure 15a** and **16a***) is to be included: The test data have to be shifted into the $\hat{\sigma}_x$ direction.

The simulations there further confirm bulging and stress redistribution to the fibres which are then loaded above $\overline{R}_{\parallel}^{t}$! It outlines that after reaching the $F_{\perp\parallel}$ a non-critical (for this stack and loading) $F_{\perp}^{\tau}$ is achieved and then, numerics fails.

(b) The about doubly high test values in the biaxial compression domain could be partly explained by a higher $\overline{R}_{\parallel}^{c}$, effective in the externally pressurized tube. A description of the (-2/-1) test executed incl. boundary constraints would have given much insight to better interpret the *Fig.11a* results. Other ideas to obtain a better fit cannot be given

(c) In order to demonstrate the strengthening effect of the normal pressure p_{ext} on the biaxial strength, a simple example is given which explains the higher experimental results in the *negative quadrant (biaxial compression quadrant)*. For a stress ratio of (–2/-1), the theoretical predicted laminate failure stresses are $\hat{\sigma}_{hoop}$ =*-436MPa*, $\hat{\sigma}_{ax}$ =*-218MPa*), taken at initial failure. The corresponding lamina stresses are collected in the stress vector

$$(\sigma_1,\sigma_2,\sigma_3 = -p_{ext},0,0,\tau_{21})^T = (-524, -123, -p_{ext}, 0, 0, 43)^T.$$

Assuming that the thickness to radius ratio of the tube is *t/r = 0.2* and using 'thin shell' theory, the failure pressure is estimated as $p_{ext} = \hat{\sigma}_{hoop} \cdot (t/r)$ =*-436MPa* $\cdot$*0.2* =*-91.8MPa* = *918bar*.

From this data set and utilizing $\overline{R}_{\perp}^{c} = 144MPa,\ b_{\perp}^{\tau} = 1.09$, an increase of fracture loading can be estimated by a simplifying linear analysis computation. *Without pressure* on the outside of the tube, the reserve factors $f_{Res}^{\perp\tau} = 1.17,\ f_{Res}^{(res)} = 1.0$ *are computed. With pressure*, a remarkable increase is achieved *for* $f_{\mathrm{Res}}^{\perp\tau} \le \dfrac{-\overline{R}_{\perp}^{c}}{[(b_{\perp}^{c}-1)(\sigma_2+\sigma_3)+b_{\perp}^{c}\sqrt{(\sigma_2-\sigma_3)^2}\,]}$ $= 7.0$. *Regarding the other* mode reserve factors $f_{\mathrm{Res}}^{\parallel c} = -\overline{R}_{\parallel}^{c} / \sigma_1 = -800 \ /(-524) = 1.53$, $f_{\mathrm{Res}}^{\perp\parallel} = 2.71$, the resulting reserve factor, computed from $(1/\,f_{\mathrm{Res}}^{(res)})^{\dot m} = (1/\,f_{\mathrm{Res}}^{\perp\parallel})^{\dot m} + (1/\,f_{\mathrm{Res}}^{\perp c})^{\dot m} + (1/\,f_{\mathrm{Res}}^{\parallel c})^{\dot m}$, *becomes* $f_{\mathrm{Res}}^{(res)} = 1.45$, which is much larger than *1* !. This increase outlines reserves.

Above value demonstrates that, taking into account the radial pressure (via 3-D analysis), the predicted failure stresses are much higher than those obtained using 2-D analysis (it is not possible to take pressure effects into account). For the estimated 918 bar, the increased multi-axial strength is depicted in *Fig. 11a* by a filled rhombus sign. It can be seen that the consideration of $\sigma_3 = -p_{ex}$ leads to an increase in the biaxial compression strength.

The inner layer is weaker than the outer layer in case of biaxial compression, however there is some stabilizing effect from the curvature.

5.2.2 Fig. 12a, CFRP AS/3501-.6 [0/+45/-45/90]s, quasi-isotropic, hand lay-up, lined tube

"Hoop and axial stresses[8] were calculated in the test data evaluation using the formula below, where r_m is the mean radius and A is the cross sectional area of the tube. Buckling might have caused the low compression values of the *tube structure* and not *crushing* of the *UD material*". *Formula:* $\hat\sigma_{hoop} = pr_m / t$, $\hat\sigma_{ax} = F / A$.

The failure envelope in *Fig. 12a* is slightly different to that in *Fig 12* due to the improvements with the programme. The coincidence with the experiment partly is very bad. Remarkable for the test data provided is the high scatter. Especially, the large scatter on the $\hat\sigma_x^c$ axis (F_{ax} loading) is astonishing and makes a prediction doubtful.

Buckling in test will be responsible for the large discrepancy in the compression domain ($\hat\sigma_y^c$, p_{ext}, $negF_{ax}$).

According to joint failure probability of the laminae the sharp corners are smoothed artificially, but to be honest, such a 'laminate smoothening process' requires a high effort [13] and is not yet matured to an engineering tool. Shortcomings, not allowing for fast improving this situation, are the normally poor data knowledge about the uncertain design parameters (stochastic model of the strengths, load,..) and the non-linear stress situation to be tackled.

5.2.3 Fig. 13a, GFRP E/lY556 [90/+30/-30]s, torsion with axial load, tube

"The material[8] is not quasi-isotropic because t^{90} is not 1/3 but 17.2 %. Large deformation effect is considered in the graph !".

The differences between Part A prediction and Part B prediction are caused by the code improvements (still not sufficient for high non-linearity).

Large deformation cannot be considered in the 2D *plate* analysis performed, a non-linear *cylinder analysis* does it. The stresses computed by the (plane) CLT are real stresses in contrast to the test stresses.

According to in-house measurements and general strength data experience[9] with this GFRP material the provided compression strengths $\bar{R}_{\parallel}^{c} = 1140\ MPa$ and $\bar{R}_{\perp}^{\tau} = 570\ MPa$ seem to be pretty low. Plausibility check: There is no rationale that for the similar composites E/LY556 and E/MY750 (glass fibres and matrix have the same properties, see Part A) the two strengths above should differ that much. In any case, the transfer of UD properties to the laminate looks questionable, here.

It is obvious that the test data in the compression domain $\hat{\sigma}_x^c$ is higher than the predicted data. Wedge failure $F_{\perp}^{\tau}\,90$ is indicated for $(\hat{\sigma}_x^c, \hat{\tau}_{xy} = 0)$, however, in this specific laminate configuration the outside lying 90° layer does not crucially harm the inner embedded 30° laminae! Also, the 90° layer at the inner surface has not a detrimental impact on the +30° neighbour layer. The compression values, achieved in experiment, can be explained by an in reality higher $\bar{R}_{\parallel}^{c}$ value. By the way, the value for the failure $F_{\parallel}^{\tau}\,90°$ is about the same as for $F_{\perp}^{\tau}\,90°$. The dent at the $\hat{\tau}_{xy}$ axis is caused by some numerical instability and further work is needed to obtain a robust post failure prediction in all domains.

Wedge failure prediction omitted in Part B ?? Sam. I do not understand this remark.

5.2.4 Fig. 14a, GFRP E/LY556 [90/+30/-30]s, pressure with axial load, tube

"The few tests carried out (3rd quadrant) under external pressure and axial compression are reported[8] to be governed by buckling". *Formulae* $\hat{\sigma}_{hoop} = p(r_{\text{int}} + \Delta r)/t$ and $\hat{\sigma}_{ax} = pr_{\text{int}}/2t + F/(2\pi \cdot r_{\text{int}} t)$.

As mentioned in 5.2.3, the compression strengths $\bar{R}_{\parallel}^{c}$ as well as $\bar{R}_{\perp}^{c}$ seem to be too low.

For the discrepancies, there might be some explanations:

(1) On the negative hoop axis ($\hat{\sigma}_y$) the maximum load achieved is lower than that on the negative $\hat{\sigma}_x$-axis, because: the pure axial (x) loading of the cylinder is less buckling-critical than external pressure combined with axial tension load (failure caused by $F_{\perp}^{\tau}\,90°$). Whereby,

the external 90° layer ($\hat{\sigma}_y^c$, p_{ext}, F_{ax}) becomes stabilized by twofold lateral compression ($\sigma_2^c, \sigma_3^c = -p_{ext}$). The internal 90° layer is a little stabilized by the curvature of the shell.

(2) An increase of $\bar{R}_{\parallel}^c$ would help to fit the test data on the $\hat{\sigma}_x$-axis. The author believes coupon buckling led to the low value of $\bar{R}_{\parallel}^c = 570MPa$. Its failure stress decreasing effect is the same in *Fig.14a* ($F_{\parallel}^{\tau}\ 90°$) as in 13a ($F_{\parallel}^{\tau}\ 30°$, $F_{\parallel}^{\tau}\ 90°$).

(3) An increase of $\bar{R}_{\perp}^c$ does the same in the fourth quadrant

(4) A non-linear buckling analysis which considers the real imperfections can probably help to sort out where the difference is coming from, because the failure criticality of all affected modes is dependent on the imperfection geometry.

5.3 Stress-strain Curves of the Laminates

For the 45° tubes, bulging is reported in Ref[8] and the same can be assumed for the 55° tubes. The size of bulging is characterized by the difference axial to hoop deformation (not clear). The strain gages were placed on the surface. They measure the actual real deformations.

Both, axial and hoop direction experience a non-linear 'large deformation' in lengthening and circumferential widening, respectively. Of course, large strains are considered anyway. Decisive for the analysis is: As long as fibres remain intact during matrix degradation (IFF) they are able to carry additional load in the lamina. In a compound such as a laminate 'healing effects' are given, then, when the weakest-link-behaviour can not freely act.

The gap between the non-linear plane CLT prediction and the test data can be closed if respecting: the test data is not consistent w.r.t. to the stresses which are simplified evaluated from measured loads, and the strains monitored are real strains including cylindrical widening and bulging (*Table 3*). Respecting these facts test data may be corrected (see filled rhombus).

5.3.1 Fig. 15a, GFRP E/MY750, [55/-55]s, $(\hat{\sigma}_y : \hat{\sigma}_x)$=(1:0), radial loading, tube

"Axial load was taken over by the tension rod of a sealed piston, lined specimen. Bulging assumed to have take place". *Formula* $\hat{\sigma}_{hoop} = p(r_{\text{int}} + \Delta r)/t$.

Figure 15a depicts a theoretical curve stiffness in hoop direction (y) higher than the evaluated measured one because the analysis deals with the given stresses. Furthermore, the maximum test load was not achieved in the analysis. The first IFF is caused by $F_{\perp\parallel}$ and then in the wedge failure mode $F_{\perp}^{\tau}$. By chance, the predicted final point coincides with the stress at leakage.

First assessments are: 1)A large deformation/large strain correction will make the test curve steeper or -the other way around- the theoretical curve shallower. 2) In this application $F_{\perp}^{\tau}$ is not a '*catastrophic'* failure mode, because the wedge cannot harm any existing load-carrying hoop layer lying above. 3) Leakage can not be predicted by the current theory, as will be detailed later.

In order to support findings a check utilizing the measured strains is performed. From measurement both strains on the external surface are known. They respect large strains (the real strains) and large deformation. The 'test' stresses given do not respect this. Therefore, a trial is made to close the gap between the non-linear plane CLT prediction, the measured strain data, and the given stresses, which -due to this fact- are not consistent w.r.t. the analytical stresses. At first the failure strains are depicted from the test curve. Secondly, a short analysis delivers the change of the fibre direction by a simple estimation using the measured fracture strains $\hat{\varepsilon}_{hoop}^{fracture} = 0.09 \equiv 9\%$, $\hat{\varepsilon}_{ax}^{fracture} = -0.11,$ $\widehat{\alpha} = 55 \cdot \pi / 180 \Rightarrow$ $\widehat{\alpha}_{final} = \arctan(\tan \widehat{\alpha} \cdot (1 + \hat{\varepsilon}_{hoop}^{fracture}) / (1 + \hat{\varepsilon}_{ax}^{fracture}) = 1.o5$ and $\alpha_{final} = 60°$.

Then, the lamina strains and associated secant moduli are estimated from the (assumed) softening curve. Eventually, via CLT computation the lamina stresses are derived and assessed. From these analyses at first a surprising fact seems to have been detected: Failure strain $\varepsilon_{\parallel}^{t}$ and stress $\sigma_{\parallel}^{t}$ determined in the tube are much higher than fracture strain $\bar{\varepsilon}_{\parallel}^{fracture}$ and 'weakest link strength' $\overline{R}_{\parallel}^{t}$ from the coupon test ! This has to be investigated. The stresses $\sigma_{\perp}$ and $\tau_{\perp\parallel}$ are zero. And a further finding, bulging must have occurred.

The numerical analysis of the MATHCAD code used might be made more stable with an improved softening curve for $\tau_{\perp\parallel}$ and $\sigma_{\perp}^{c}$ based on an improved understanding. These softening curve will have a finally steeper decay which better allows for stress redistribution to the fibres.

5.3.2 Fig. 16a, GFRP E/MY750, [55/-55]s, $(\hat{\sigma}_y : \hat{\sigma}_x)$=(2:1), vessel loading, tube

"Bulging is assumed to have taken place". *Formula guessed* $\hat{\sigma}_{hoop} = p(r_{\text{int}} + \Delta r)/t,$ $\hat{\sigma}_{ax} = pr_{\text{int}} / 2t + F / (2\pi \cdot r_{\text{int}} t)$.

Bulging will degrade the axial direction more than the hoop direction and probably change the angle α to a higher value (eg 57°), which results in a 'weakening' of the stress-strain curve. This bulging effect is not simulated here. This would need a FEA. The twofold $F_{\parallel}^{\sigma}$ failure in both 55° layers is subjected to 'joint failure probability' reducing the load carrying capacity a little. Furthermore, in net theory, which should be increasingly valid after IFF has occurred, the ratio of $\varepsilon_x / \varepsilon_y \approx 1$ is not found.

According to the 'short analysis' applied in *Fig.15a* the measured fracture strains are taken, again. From them $\{\hat{\varepsilon}_{outer}^{fracture}\} = (+4.2\%, +2.4\%, 0)^T$, at first the final angle at fracture -via the initial angle $\widehat{\alpha} = 55 \cdot \pi / 180$ - $\widehat{\alpha}_{final} = \arctan(\tan \widehat{\alpha} \cdot (1 + \hat{\varepsilon}_{hoop}^{fracture}) / (1 + \hat{\varepsilon}_{ax}^{fracture}) \Rightarrow$ $\alpha_{final} = 54.6°$, the lamina strains $\{\varepsilon^{fracture}\} = (3.1, 3.6, -1.6)^T$ are computed and further the secant moduli (softening regime). Eventually, the stresses $\{\hat{\sigma}_{outer}^{fracture}\} = [A^{fracture}]\{\hat{\varepsilon}_{outer}^{fracture}\}$ and the lamina stresses $\{\sigma_{outer}^{fracture}\} = [T_\sigma]^{-1}\{\hat{\sigma}_{outer}^{fracture}\}$. The value derived (marked by a filled rhombus in the graph) confirm that the *real* test stresses are higher for a distinct strain level than those given in the test graph. Of course, deformation caused by bulging cannot be respected in analysis.

5.3.3 Fig. 17a, CFRP AS/3501-6, [0/+45/-45/90]s, $(\hat{\sigma}_y : \hat{\sigma}_x)$ =(2:1), tube

Formula: $\hat{\sigma}_{hoop} = pr_m / t, \ \hat{\sigma}_{ax} = F / A$

The initial failure stresses in *Fig 17* are lower than those in *Fig 17a*. The reason for this is the improved coding.

The comparison of this fibre-dominated laminate behaviour is satisfying. For such 'well' designed laminates good correlation will be achieved.

5.3.4 Fig. 18a, GFRP E/MY750 [45/-45]s, $(\hat{\sigma}_y : \hat{\sigma}_x)$ =(1:-1), lined tube

Shear loading generated by internal pressure, over-compressed in axial direction, r_{int}=50mm, t=5.9mm. "Thick cylinder theory was used in test evaluation. Strains were measured at the inside surface of the tube, stresses are given for the inside. Bulging reported". Formula applied for thick cylinder is missing.

In case of thick cylinder theory inside hoop strain and stress are higher than the outside ones as long as increasing degradation (plastic effect) will not have smoothed out the difference. According to the fact that the mutually strain-controlling layers (embedded laminae) are redundant the author increased the max shear strain from 4% in *Fig.11* to 10% in *Fig.11a.*

Following the plane CLT analysis the comparison of this highly matrix-dominated laminate was a little disappointing because the analysis stops before reaching the fracture strain domain. Again, the simple estimation

$$\widehat{\alpha} = 45 \cdot \pi / 180 \ , \ \hat{\varepsilon}_{hoop}^{fracture} = 0.10, \ \hat{\varepsilon}_{ax}^{fracture} = -0.11 \quad \text{and} \quad \alpha_{final} = 51° .$$

helps to get checking values for the interpretation of the test data. Bulging must occur. Also here, the simulation of the test results confirms both:

Regarding large strain/large deformation, the test curve will become stiffer in the upper region. The utilized softening curve does not allow for further stress-redistribution to the fibres which seem, due to redundancy effects, to still act above the 'weakest link value' $\overline{R}_{\parallel}^{t}$!

5.3.5 Fig. 19a, GFRP E/MY750 [0/90]s, $(\hat{\sigma}_y : \hat{\sigma}_x)$ =(1:0), coupon

The predicted initial and intermediate failure stresses are lower than those measured in the tests. This difference can be explained by the improved coding. Due to the error in stiffness addition, the initial failure stresses in Fig 19 are a little different to that in Fig 19a.

The comparison of test and analytical curve shows the theoretical curve is less stiff almost up to fracture than the experimental one. This effect cannot be explained by the author.

5.3.6 Fig. 20a, GFRP E/MY750 [45/-45]s, $(\hat{\sigma}_y : \hat{\sigma}_x)$ =(1:1), int. pressure with axial tension

"No allowance was made for large deformation. The readings from the individual strain gauges varied up to 22%. Reasons for the divergence of the hoop and the axial strain for this symmetric lay-up are not clear. The strength of the tubes is believed to be higher as in the graph outlined. Extensive crack spacing was recorded. Bulging reported" *Formulae:* $\hat{\sigma}_{hoop} = p(r_{\text{int}} + \Delta r)/t$ and $\hat{\sigma}_{ax} = pr_{\text{int}} / 2t + F/(2\pi \cdot r_{\text{int}} t)$.

The curves should lie on another w.r.t. the symmetric geometry and the loading. Different curves for $\hat{\varepsilon}_x$ and $\hat{\varepsilon}_y$ indicate bulging. Bulging brings on top of the circumferential widening higher strains at a distinct load level $\hat{\sigma}$ of the test evaluation. Again, a twofold $F_{\parallel}^{\sigma}$ failure in both 45° layers will reduce the theoretical fracture value a little, and, applying the previous fracture strain estimation the real test stresses are lifted to the value indicated by a filled rhombus.

The test results show a leakage at around 210MPa[8] stress. This structural failure mode cannot be predicted, because leakage is determined by the stochastic IFF-based micro-crack system of the laminate.

5.3.6 Fig. 21a, CFRP AS/3501-6, [0/+45/-45/90]s, $(\hat{\sigma}_y : \hat{\sigma}_x)$ =(1:0), tube

Formula: $\hat{\sigma}_{hoop} = pr_m / t,\ \hat{\sigma}_{ax} = F / A$.

The comparison of this fibre-dominated laminate behaviour is satisfying. Again, the initial failure stresses in *Fig, 21* are different to those in *Fig. 21a* due to the corrected stiffness error.

5.4 Application to Stress-strain Curves of Distinct Laminates

* The *Symmetric* Laminate GFRP E/MY750 [+45/-45/-45/+45], $(\hat{\sigma}_y : \hat{\sigma}_x) = (1:1)$

In comparison (see *Figures 18a, 20a)* to the following anti-symmetric case the inner lamina consists of two equal layers. This has an effect on micro-cracking which eventually has a deteriorating impact on FF as still mentioned in sub-section 2.3.2..

* The *Anti-Symmetric* Laminate [+45/-45/+45/-45], $(\hat{\sigma}_y : \hat{\sigma}_x) = (1:1)$:

Due to the stacking sequence some twisting is to be expected for a flat specimen composed of this stack.

In reality, due to the winding process (fibre directions alter), an anti-symmetric stack is manufactured with often many lay-up repetitions. However, circumferentially closed, wound tube specimen in contrast will experience no twisting under in-plane normal loading, but, any 'internal state' of residual stresses will turn the front sections to another. In case of winding a positive winding angle normally follows a negative one and vice versa, if the winding procedure is not interrupted and the winding direction back-changed (increases winding costs). Furthermore it is to be mentioned, the wound anti-symmetric tubes in contrast to the symmetrically stacked ones have no double central layer. A double layer has a detrimental impact on fracture[20].

5.5 Application of Refined Theory to the 55°-tube Test Specimen

In order to better assess the test results the specimen requires a non-linear analysis allowing for large strain and large displacements. It further demands for a FE code with integrated failure analysis respecting degradation.

For the laminates above no analysis is performed w.r.t. the Part B deadline althougrh such a non-linear FEA of this specimen is desired. There is neither a chance nor time for the implementation of the procedure as a subroutine into MARC, which is usually applied in non-linear analysis at MAN.

6 CONCLUDING COMMENTS

6.1 Some Conclusions, Outlooks

Author's main assessments are from engineering point of view:

- The UD fracture conditions are proven to practically work
- The fracture conditions are robust for use in design because the $F_{\perp\parallel}$ problem was solved
- The developed computer code including the non-linear analysis procedure requires further work to eliminate any convergence problems in high shear strain areas. An implementation into a FE code will create a generally accepted tool, only

- The theory is expected to give the largest discrepancy with test data in the high shear area. However, high shear strains do not occur in well-designed laminates.
- Softening curves have to be provided.

6.1.1 Regarding the FMC-based conditions:

• The establishment of FMC-based Fracture Conditions (F = 1) for *Initial Failure* (corresponding to IFF) of dense, brittle laminae and for final Failure of the laminate has been made. Each fracture condition describes the *interaction of stresses* affecting the same fracture mode and assesses the actual state of stress in a 'material point'

• The *complete* failure surface consists of piecewise smooth regimes (partial failure surfaces). Each regime represents *one failure mode* and is governed by *one basic strength.*

• Sufficient for pre-dimensioning are the five basic strengths R. The remaining two unknown *curve parameters* $b_{\perp\parallel}, b_{\perp}^{\tau}$ can be given approximate but fairly representative values if test data are missing. The interaction coefficient $\dot{m}$, after some fitting experience, can be fixed on the safe side. As suitable *low* value of $\dot{m} = 3.1$ is taken, an odd number for numerical reasons.

• The *interaction* (rounding-off) between *adjacent failure modes* is automatically considered when calculating the stress effort $Eff^{(res)}$ as a function of the mode effort $Eff^{(modes)}$. The *'mode fit'* avoids the shortcomings[7] of the *'global fit'* which maps the course of test data by mathematically linking failure modes which are in reality not mechanically linked. One typical shortcoming is that a reduction of the strength of one mode could increase the multi-axial strength in another (independent) mode or part of the global failure surface

• The FMC enables to correctly turn the design key by respecting the most critical mode (mode of highest effort or lowest reserve factor) and to address the location in the Finite Element idealization of the structure[12]

• Homogenisation of the UD-material comes to its limit if a *constituent* stress governs the failure. This is the case for $F_{\parallel}^{\sigma}$, where the macro-mechanical stress σ_1 has to be replaced by the actual fibre stress σ_{1f}. A fibre stress may be zero not even for zero σ_1. Therefore, σ_{1f} has to be estimated as $\sigma_{1f} = \varepsilon_1 \cdot E_{1f}$.

• For the prediction of the level of final failure of the laminate the initial failure (IFF of laminae) approach is not of that high concern, if wedge failure, caused by $F_{\perp}^{\tau} < 1$ and followed by delamination failure, will not occur or has no detrimental effect on load carrying capacity. Wedge failure is not catastrophic for the [+55/-55]s tube, $(\hat{\sigma}_{hoop} / \hat{\sigma}_{ax} = 1/0)$, but there would be a 'wedging off', if a hoop layer additionally had to be wound onto the tube due to a second load case.

• Some multi-axial test data required for a full verification of the shape of the IFF failure body are still missing

• An-isotropic yielding needs (see *Figures 23a,b*) to be studied in order to get a complete image of the behaviour. The benefit is more academic, but, it helps to understand load path dependencies in static and fatigue loading.

• In cases where the scatter in the test data is not known, one may resort to the use of the simplest set of parameters, given as $b_{\perp}^{\tau} = 1,\; b_{\perp\parallel} = 0$. This will usually give a good approximation of *final* failure prediction of a laminate. The criticality of a mode may be classified according to *Figure 1*.

Due to the scatter of experimental data and due to the 'smearing effect' acting in case of laminae embedded in a multi-directional laminate the following set of failure conditions, derived from Eq(16), is recommended as an ***engineering approach***

$$\boxed{\begin{aligned} &FF1:\; F_{\parallel}^{\sigma} = \frac{I_1}{\overline{R}_{\parallel}^{t}} = 1^*, \qquad {}^*\text{with } I_1 = \varepsilon_1 \cdot E_{\parallel} \\ &FF2:\; F_{\parallel}^{\tau} = \frac{-I_1}{\overline{R}_{\parallel}^{c}} = 1, \\ &\text{IFF1}:\; F_{\perp}^{\sigma} = \frac{I_2 + \sqrt{I_4}}{2\overline{R}_{\perp}^{t}} = 1 \\ &\text{IFF2}:\; F_{\perp\parallel} = \frac{I_3^{3/2}}{\overline{R}_{\perp\parallel}^{\;3}} = 1, \\ &\text{IFF3}:\; F_{\perp}^{\tau} = \frac{\sqrt{I_4}}{\overline{R}_{\perp}^{c}} = 1 \end{aligned}} \quad . \tag{30}$$

As constant mode interaction coefficient $\dot{m} = 3.1$ is recommended for all MiFD.

•In parallel to the decay of the stiffness the non-linear analysis sets matrix dominated stresses free: These include thermal residual stresses (curing stresses), thermal stresses, and mechanic stresses across the fibres. This is valid for the curing stresses of the 1st kind (upper or material level). A reduction of the not-respected curing stresses of the 2nd kind (fibre-matrix level) also takes place, less pronounced). Curing stresses of the 2nd kind determine the UD strength however are not evaluated. This non-consideration implies that within the transfer of UD data to the structure it is to be assumed that the curing stresses of the 2nd kind in specimen and structure are approximately the same.

Curing stresses (1st kind) are respected for the laminates. Their determination requires information on thermal shrinking (CTE and temperature difference) as well as on chemical shrinking of the matrix after the gel state in the solid state (data could not provided [9].

The author recommends further work on dealing with residual thermal stresses, because the understanding of the residual stresses about their validation and possible decay with time is not sufficient.

• The minimum number of valid measurements required to establish the 2 curve parameters $b_{\perp\parallel}$, $b_{\perp}^{\tau}$ in eqn(16) is three. Problematic is the performance of the bi-directional compressive test[17,9] in the quasi-isotropic plane. However, these test results are mandatory for the design of highly compression-loaded submarine hulls. For the usual laminate design an application of $b_{\perp}^{\tau} = 1.3$ is sufficient as design value.

• A prediction of leakage as a mode of failure is not possible unless one has not calibrated the IFF behaviour with a sufficient number of tests outlining a critical micro-crack density state and, as characteristic quantity, a limiting strain. It depends on the grade of tightness which is required in the actual case, Design to Leakage covers a very wide range. Cryogenic fluids like LH2, LOx or gases like Helium anyway demand for liners. In case of fluids such like water a rubber sealing is probably enough. Further efforts to investigate this failure mode has to be put on the actually designed pressure vessel. Other existing models predict leakage?

6.1.2 Regarding progressive failure analysis

• An accurate failure prediction involves the application of (a) a physically-sound non-linear stress analysis to cover large shear strains, and (b) a geometrically non-linear analysis to account for large deformation.

• The lamina is the basic building brick (or basic computational element) for the prediction of laminate behaviour. The load, not the stress, was increased monotonically from zero to fracture.

• According to the FMC theory, and in order to perform a reliable non-linear analysis, a clear identification of the dominant failure mode is given

• For stress *concentration* loci in the laminate such as bolt holes a suitable engineering approach for the strength assessment has to be provided (compare ideas for metals). This area lies outside the current failure exercise.

• In the case of stress *intensity* (delamination occurred) a practicable fracture mechanics tool has to be established to assess instabile delamination growth.

Appropriate test specimens and test evaluation have to be discussed

• Triggering looses its influence when approaching the large strain regime beyond the occurrence of an IFF mode

• In order to implement the *multi-fold non-linearity* approach into a commercial FEA code, and taking advantage of the code's solution architecture wrt non-linear laminate analysis, analogue to the isotropic input with one 'yield condition' + associated *flow rule* now three times, for $F_{\perp}^{\tau}, F_{\perp}^{\sigma}, F_{\perp\parallel}$, a condition + '*flow' rule* have to be provided and implemented. This is more complicated than working with one global failure condition such as the isotropic Mises yield condition

• The high internal redundancy of a multi-layered laminate is better modelled by a probabilistic code. *Probabilistic* tools [10,11] should be applied in order to improve the *deterministic* procedures and also to smooth (due to joint failure probability) the sharp corners of a *laminate's* failure envelope, which now represents the 'sum' of all single lamina failure envelopes, only. A non-deterministic approach[10, 11, 13, 20] should be employed to better understand the subsequent failure behaviour (Chris Chamis, NASA, goes this way).

• Loading path effects on IFF have not been considered. Load path dependency requires for some investigation. Deterministic failure path and probabilistic failure path of a laminate may not coincide with each other due to the possibility of having different scatter ranges of the design parameters[10].

6.2 Areas Requiring Further Work

6.2.1 Definitions and Notations

* The author believes that the ***notation*** applied in the failure exercise is very confusing as it was employed in a mixed way. One is never sure after some weeks of break of a correct interpretation of a figure, eg. In order to avoid this bottle-neck for the application of composites, the German working group on the VDI 2014[5] guideline has spent a considerable amount of efforts in the last decade to establish the best practise of the use of ***self-explaining notations***. These notations are pointed out in Part A and B and are recommended by us for use by the FRP community.

* Attention has to be paid to the fact that the expression ***'IFF mode'*** has different meanings in the papers of Puck[1, 14] and Cuntze. Cuntze uses the expression '*mode*' to address his three different invariant IFF *conditions*, based on the idea that for each of these fracture conditions in their 'pure' regimes either the $\sigma_{\perp}$- , the $\tau_{\perp\perp}$-, or the $\tau_{\perp||}$-stressing is 'dominant'. This use of different meanings for the same notation should also be rectified in order to minimise confusion. f_E is the ***stress exposure factor*** used by Puck [1,14]. It has essentially the same

meaning as Cuntze's resultant ***stress effort*** $Eff^{(res)}$. The value of f_E or $Eff^{(res)}$, respectively, quantifies the ' risk of fracture'. Fracture occurs when this value becomes *1 = 100 %*

* ***Initial failure*** *is* usually indicated by the occurrence of a distinct knee in the stress-strain curves of a laminate, and this is determined by the first IFF mode of failure

* As theoretical ***failure load*** often taken is the maximum load achieved when computation stops due to numerical stability problems in non-linear analysis.

* Non-linear analysis in general results in a ***stress redistribution*** within the thickness of a laminated *structure*. This redistribution lowers, due to micro-cracking, the matrix-dominated stress level including the residual stresses in the laminae of the laminate

**** Average stresses*** *are* the *stresses in a lamina*, smeared over a length that includes some micro-cracks generated after the onset of IFF until final failure of the laminate.

* The definition of the **terms** damage, failure, failure modes, flaws, defect, imperfection of a composite structure, etc.. has to be worked out in order to generate a common understanding in the composite community

* ***Effective mode strength*** is the strength $\overline{R}_{ef}^{(IFF\,\mathrm{mod}\,e)}$ of an embedded lamina. Its value is higher, (*Fig.4*), than that of the mode strength $\overline{R}^{(IFF\,\mathrm{mod}\,e)}$, eg $\overline{R}_{\perp}^{t}$, measured by the *isolated* specimen. The value is not a 'weakest link result' (series failure system) as given in case of the isolated specimen but a 'redundancy result' (parallel failure system) due to the embedding of the lamina into the laminate.

6.2.2 Industrial needs

• Industry seeks to replace the expensive 'Make and Test' design method by verified and benchmarked predictive tools that engineers could use with confidence. The limitations of the predictive tools 'failure conditions' should be clearly indicated

• The 2D/3D-*strength* analysis, using the fracture conditions defined in this paper, is not yet fully validated/verified. Similarly, 3D-*stress* analysis of laminated shells obtained by commercial FEA codes (MARC, NASTRAN etc) is not adequate, it is still too time-consuming and pre-processing does not inform about the design driving modes[12] as well as reserve factors[12] demanded (see Part A, FigA3/1) for the 'Proof of Design'. Here, an improvement is highly appreciated

• There is still a need for generating reliable multi-axial test data (not all section planes of the multi-axial fracture body are verified as one may conclude from the *Figures 8,9,10,A1*).This could be achieved through a coordinated and collaborative research programme between leading research institutions. Other areas include the development of probabilistic models and the encouragement of an improved world-wide standardization where manufacturers, technical associations and authorities are all involved

• Verification of engineering approaches, based on qualified FEA-output level when analysing the test specimen, is necessary

• Regarding the investigations in theory and test carried out in Germany on the *lamina* material level in the last years[9] (still going on) the understanding has improved greatly and seems to be a good basis to tackle *laminates* stacked-up of UD-laminae or fabric laminae[18]. For other 'textile pre-forms' (3D, stitched etc.) *engineering models* have to be developed. The work in this field has been initiated. First steps indicate the transferability to fabrics (rhombically-orthotropic) composites should work[18]

• A practical 'progressive failure analysis procedure' has to be provided to designers

• Industry has to cope with damage and the Proof of Design (justification) of damaged structures or laminates, too. Practical criteria for the assessment of damage size and criticality of delamination are needed. Sufficiently well working NDI methods for damage detection are desired in order to avoid *in-stable* (sudden) delamination fracture. A Design guideline for improving damage tolerance analysis has to be provided. The treatment of fatigue and *stable* damage growth has to be enhanced and may be better enhanced on failure mode basis

• As the area 'in-situ behaviour of the embedded laminae' has not attracted much attention further work is highly recommended

• Initial failure stresses are very important where a standard requires a Proof of Design to IFF. This means: if at Design Limit Load level no IFF is permitted. For the higher Ultimate Proof of Design, in case of a well designed laminate, initial failure prediction has not that much impact.

6.2.3 Comparison of Theory with Experiment

Two categories of laminate configurations have been investigated: 1) laminates made of plies where the fibres are oriented in 3 or 4 directions, and 2) laminates containing plies oriented in 2 directions. The latter were loaded in accordance with and without netting analysis (*Table 2*). Correlation between theoretical prediction and experiment is expected to be the more imperfect the less the strength of a laminate can be predicted accurately by netting analysis. *Table 2* provides comments on effect of type of laminates on the deformation and fracture behaviours.

If parts of the predicted initial or final failure envelope do not match the test results this can be attributed to the accuracy of (a) the theory used, (b) the assumptions made in *interpretation* of test data, and (c) the partly questionable data input (test results and evaluation) provided.

Again it has to be mentioned, the theory presented here is capable of dealing with a three dimensional state of stress. However, in the Failure Exercise, some of the test cases required 2D analysis, only, and others involved large through thickness stresses. The latter stresses are generated by the internal or external pressure applied in testing the tubular specimens. Although, a few examples were given in this paper on how to deal with a 3D state of stress, the full potential of the FMC-based IFF conditions is not yet explored.

The comparisons between theoretical predictions and the experimental data helped to identify certain areas where further theoretical and experimental work is required. More and better experimental data is needed for a final verification of theory. In this context a description of tri-axial UD tests (see *VDI progress reports,* series 5, no.506. Tab.6/1)[9] and the determination of the softening curve are addressed.

Well understood experiments have to verify the assumptions made! Avula, in Ref[6], stated 1987 “Experimental observations and measurements are generally accepted to constitute the backbone of physical sciences and engineering because of the physical insight they offer to the scientist for formulating the theory. Concepts that are developed from observations are used as guides for the design of new experiments, which in turn are used for validation of the theory. Thus, experiments and theory have a hand-in-hand relationship”.

But, one has to keep in mind: Experimental results can be far away from the reality like a bad theoretical model, and, theory ‘only’ creates a model of the reality, experiment is ‘just’ one realisation of the reality.

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Annex 1: **Example for the FMC procedure**

Fig. AI refers to the *(σ_2,σ_3)*-plane as one failure plane of the various ones. In the upper part it visualizes the evaluation of test data and in the bottom part the rounding-off (by the spring model) in the multi-fold (MfFD) and mixed failure domains (MiFD) as well as the shrunk design space (mean strength $\overline{R}$ of mapping is replaced by a *strength design allowable* R) to be used by the designer in the 'dimensioning' and in the 'proof of design'.

Additionally to the FMC-based 'Mode Fit' the 'Global Fit' (eg Tsai/Wu's 'single failure surface' criterion[7] describes a global failure surface) is pointed out. The Global Fit interacts between the UD-stresses *and* the *independent* failure modes in one equation, achieving a description of the global (complete) failure surface. This procedure is simple, however error-prone in some domains, due to its physical shortcomings.