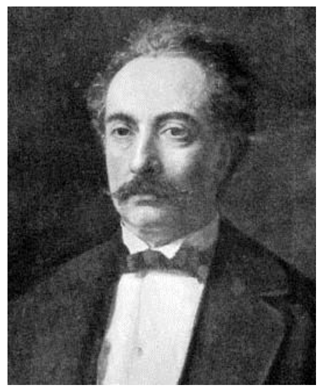
2nd Int. Conf. on **Buckling and Postbuckling Behaviour of Composite Laminated Shell Structures**
Braunschweig, Germany, September 2-5, 2008, (Key-note lecture)
Conference topic met: Failure Criteria

Formulations of Failure Conditions

- Isn't it basically just *Beltrami* and *Mohr-Coulomb*? -

Hencky-
Mises-
Huber

Richard von Mises
1883-1953
Mathematician



Eugenio Beltrami
1835-1900
Mathematician

Otto Mohr
1835-1918
Civil Engineer

Charles de Coulomb
1736-1806
Physician

'Onset of Yielding' **'Onset of Cracking'**

Prof. Dr.-Ing. habil. **Ralf** Georg **Cuntze** VDI (formerly MAN Technologie AG)
D-85229 Markt Indersdorf, Germany/Bavaria, Phone 0049 8136 7754, E-mail: Ralf_Cuntze@t-online.de

Contents of Presentation: (25 min talk)

1 **Introduction to *Design Verification***

2 **Stress States & Invariants**

3 **Observed Strength Failure Modes and Strengths**

4 **Attempt for a Systematization**

5 **Short Derivation of the Failure Mode Concept (FMC)**

6 **Visualizations of some Derived *Failure Conditions***

Conclusions

mandatory input

Motivation for the Work

Existing Links in the Mechanical Behaviour show up: ***Different structural materials***

- ***can possess similar material behaviour or***
- ***can belong to the same class of material symmetry .***

similarity aspect

Welcomed Consequence:

- The same strength failure function F can be used for different materials

- More information is available for pre-dimensioning + modelling

- in case of a newly applied material -

from experimental results of a <u>similarly behaving</u> material.

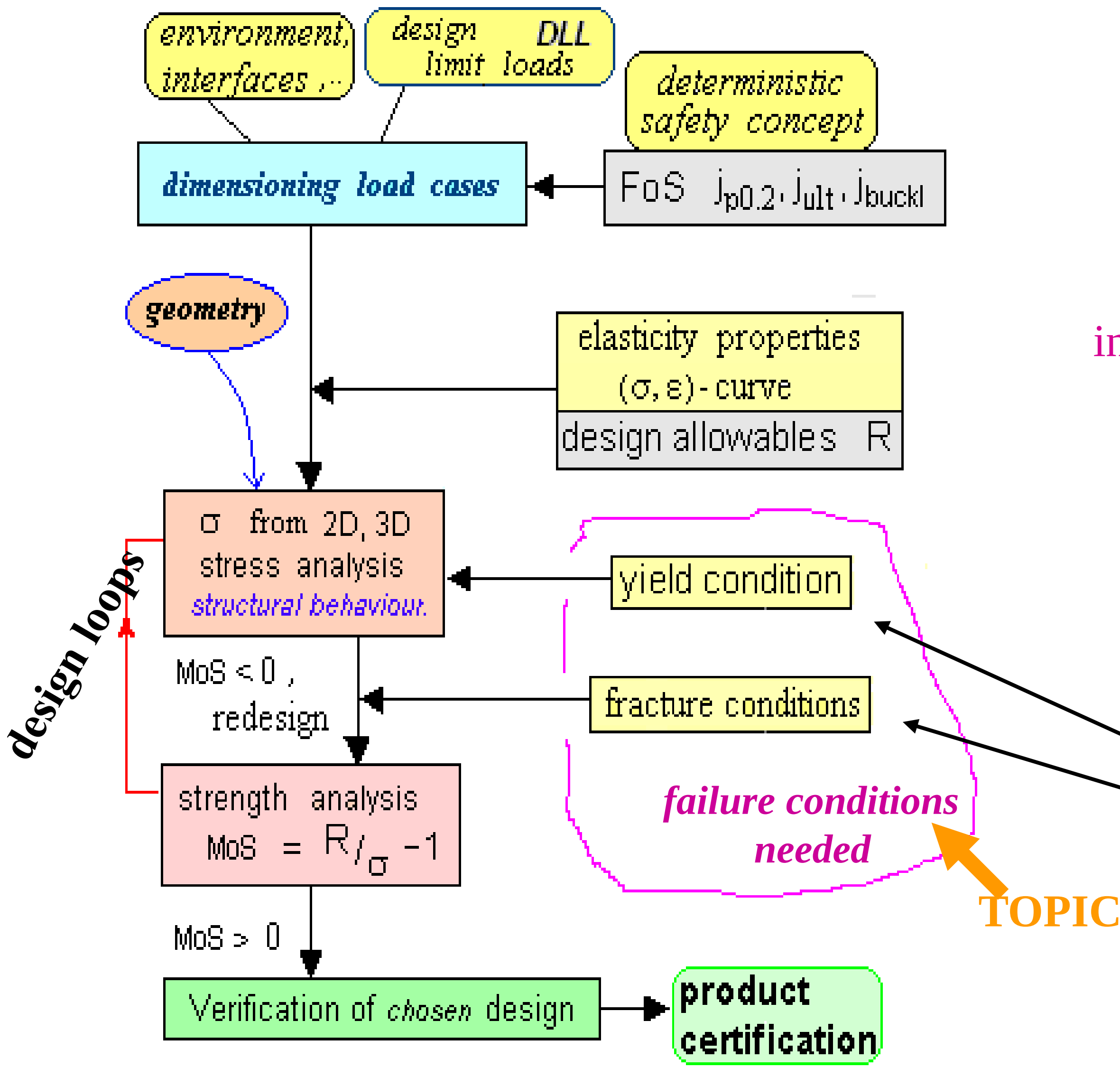
DRIVER: ***Author's experience with structural material applications, range 4 K - 2000 K***

Ariane 1-5 launchers, cryogenic tanks, heat exchanger in solar towers (GAST Almeria), wind energy rotors (GROWIAN), antennas, ATV (JulesVerne), Crew Rescue Vehicle (CMC) for ISS, ….

MESSAGE: **Let's use these benefits!**

1 Introduction to Design Verification

1.1 Static Structural Analysis *Flow Chart* *(isotropic case for simplification)*



FoS := (design) Factor of Safety

MoS := Margin of Safety

R := strength (resistance).

in aerospace usual

Design Verification for:

Design Yield Load (DYL)
$= DLL \cdot j_{p0.2}$ ….. *flight load level*

Design Ultimate Load (DUL)
$= DLL \cdot j_{ult}$ …... *fracture load level*

Design Buckling Load (DBL)
$= DLL \cdot j_{buckl}$ … ***fracture load level***

How can we demonstrate strength of design ?

by the application of strength failure conditions! These are mandatory for the prediction of ***Onset of Yielding*** + ***Onset of Fracture*** for non-cracked materials.

What are Failure Conditions for? *They shall*

• ***assess multi-axial stress states in the critical material point,***

by **utilizing the uniaxial strength values R** and an **equivalent stress σ_{eq}, representing a distinct actual multi-axial stress state.**

for *** dense & porous,**
*** ductile & brittle behaving materials,**
brittle : $R_m^c \geq 3R_m^t$ ductile : $R_{p0.2} \cong R_{c0.2}$

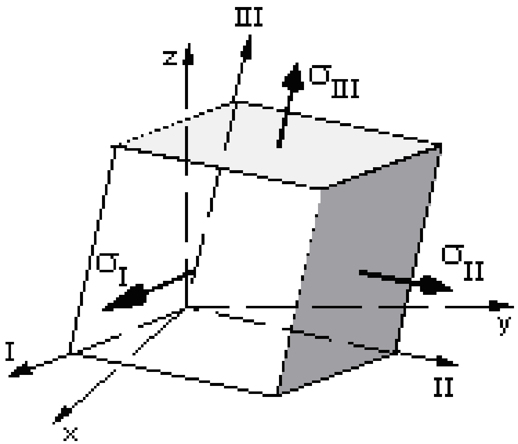
for *** isotropic material**
*** transversally-isotropic material (UD := uni-directional material)**
*** rhombically-anisotropic material (fabrics) + 'higher' textiles etc.**

• ***allow for inserting stresses from the utilized various coordinate systems into stress-formulated failure conditions, -and if possible- invariant-based.***

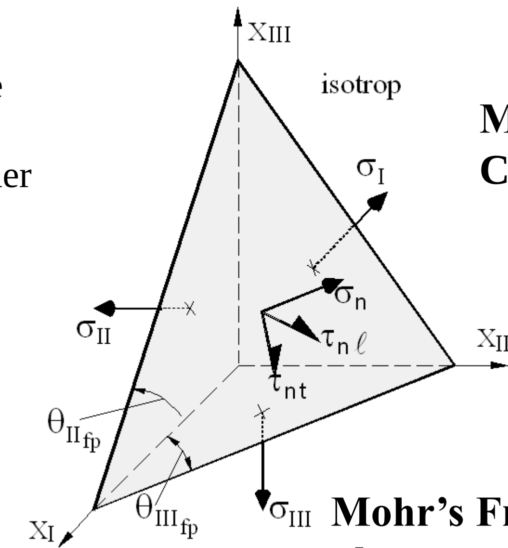
Which kinds of stresses may have to be inserted?

2 Stress States and Invariants

2.1 Isotropic Material (3D stress state), viewing Stress Vectors & Invariants



The stress states in the various COS can be transferred into each other



Mohr's COS

Principal Stresses

Structural Component Stresses

Mohr's Fracture plane Stresses

$$\{\sigma\}_{principal} = (\sigma_I \,,\; \sigma_{II} \,,\; \sigma_{III}\,)^T$$

$$\{\sigma\}_{comp} = (\sigma_x \,,\; \sigma_y \,,\; \sigma_z \,,\; \tau_{yz} \,,\; \tau_{xz} \,,\; \tau_{xy}\,)^T$$

$$\{\sigma\}_{Mohr} = (\sigma_\ell \,,\; \sigma_n \,,\; \sigma_t \,,\; \tau_{nt} \,,\; \tau_{t\ell} \,,\; \tau_{\ell n}\,)^T$$

'isotropic' invariants !

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III}\,)^T = 3\sigma_{oct} \equiv f(\sigma),$$

$$I_1 = (\sigma_x + \sigma_x + \sigma_z\,)^T$$

$$I_1 = (\sigma_\ell + \sigma_n + \sigma_t\,)^T$$

$$6J_2 = (\sigma_I - \sigma_{II}\,)^2 + (\sigma_{II} - \sigma_{III}\,)^2 + (\sigma_{III} - \sigma_I\,)^2$$
$$= 4(\tau_{III}{}^2 + \tau_{II}{}^2 + \tau_I{}^2\,) = 9\tau_{oct}{}^2 \equiv f(\tau)$$

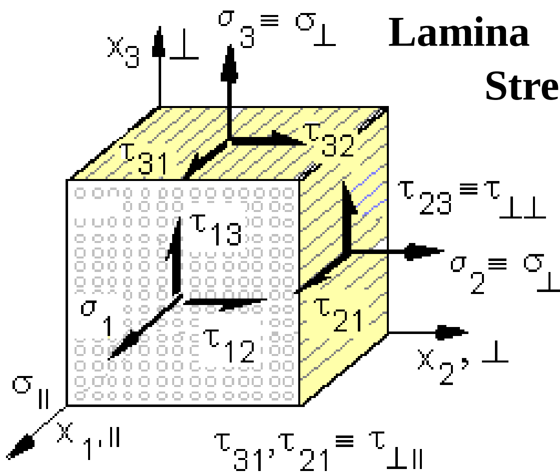
$$6J_2 = (\sigma_y - \sigma_z\,)^2 + (\sigma_z - \sigma_x\,)^2 + (\sigma_x - \sigma_y\,)^2$$
$$+6(\tau_{yz}{}^2 + \tau_{zx}{}^2 + \tau_{xy}{}^2\,) \quad (Mises, HMH\,)$$

$$6J_2 = (\sigma_n - \sigma_t\,)^2 + (\sigma_t - \sigma_\ell\,)^2 + (\sigma_\ell - \sigma_n\,)^2$$
$$+6(\tau_{nt}{}^2 + \tau_{t\ell}{}^2 + \tau_{n\ell}{}^2\,)$$

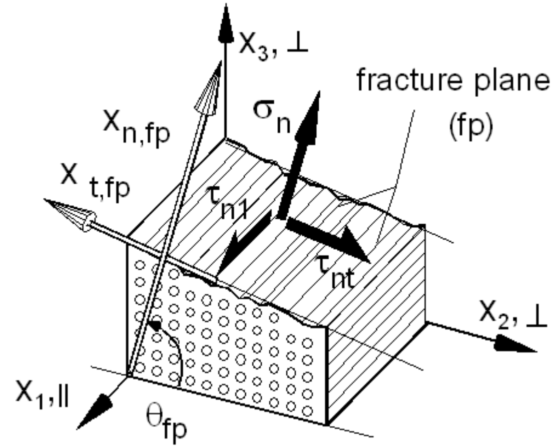
$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III}\,)(2\sigma_{II} - \sigma_I - \sigma_{III}\,)(2\sigma_{III} - \sigma_I - \sigma_{II}\,), \quad I_\sigma = 4J_2 - I_1^2/3, \quad \sigma_{mean} = I_1/3$$

2.2 Transversely-Isotropic Material (◄ Uni-Direct. Fibre-Reinforced Plastics)

Lamina Stresses



Transformation of lamina stresses into the quasi-isotropic plane stresses



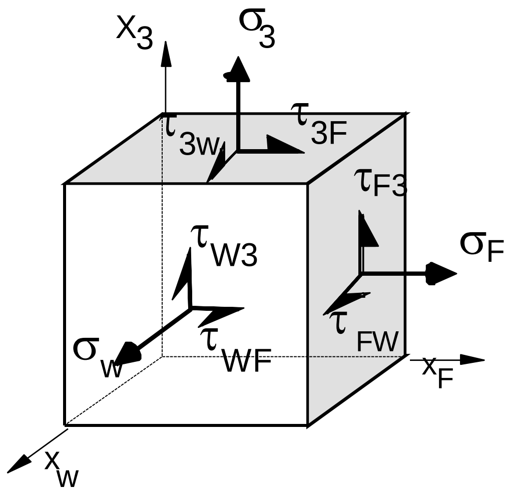
Mohr, Puck, Hashin: Fracture is determined by the (Mohr) stresses in the fracture plane !

$\{\sigma\}^{quasi-isotropic\ plane}_{principal} = (\sigma_1, \sigma_2^p, \sigma_3^p, 0, \tau_{31}^p, \tau_{21}^p)^T$	$\{\sigma\}_{lamina} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$	$\{\sigma\}_{Mohr} = (\sigma_\ell, \sigma_n, \sigma_t, \tau_{nt}, \tau_{t\ell}, \tau_{\ell n})^T$
$I_1 = \sigma_1$, $I_2 = \sigma_2^p + \sigma_3^p$ $I_3 = \tau_{31}^{p\,2} + \tau_{21}^{p\,2}$	$I_1 = \sigma_1$, $I_2 = \sigma_2 + \sigma_3$ $I_3 = \tau_{31}^2 + \tau_{21}^2$ **'UD invariants'!** *[Boehler]*	$I_1 = \sigma_1$, $I_2 = \sigma_n + \sigma_t$ $I_3 = \tau_{t\ell}^2 + \tau_{n\ell}^2$
$I_4 = (\sigma_2^p - \sigma_3^p)^2 + 0$ $I_5 = (\sigma_2^p - \sigma_3^p)(\tau_{31}^{p\,2} - \tau_{21}^{p\,2}) + 0$	$I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}^2$ $I_5 = (\sigma_2 - \sigma_3)(\tau_{31}^2 - \tau_{21}^2) - 4\tau_{23}\tau_{31}\tau_{21}$	$I_4 = (\sigma_n - \sigma_t)^2 + 4\tau_{nt}^2$ $I_5 = (\sigma_n - \sigma_t)(\tau_{t\ell}^2 - \tau_{n\ell}^2) - 4\tau_{nt}\tau_{t\ell}\tau_{n\ell}$

Invariant := Combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system. Good for an optimum formulation of *desired scalar Failure Conditions.*

2.3 Orthotropic Material (rhombically-anisotropic ◄ woven fabric)

Homogenized = smeared
***woven fabrics* material element**



Warp (W), Fill(F).

3D stress state:
Here, just a formulation in fabrics lamina stresses makes sense!

$$\{\sigma\}_{lamina} = (\sigma_W, \sigma_F, \sigma_3, \tau_{3F}, \tau_{3W}, \tau_{FW})^T$$

Fabrics invariants ! *[Boehler]*:

$$I_1 = \sigma_W,\ I_2 = \sigma_F,\ I_3 = \sigma_3,$$
$$I_4 = \tau_{3F},\ I_5 = \tau_{3W},\ I_6 = \tau_{FW}$$

more, -however simple- invariants necessary

NOTE on limits in *Modelling in buckling analysis*: Avoid anisotropic modelling !

(homogenized) Orthotropic Material is the material of the highest structural rank

buckling test experience is available !

3 Observed Strength Failure Modes and Strengths

3.1a Isotropic Material *brittle* , *dense*

if brittle: failure = fracture

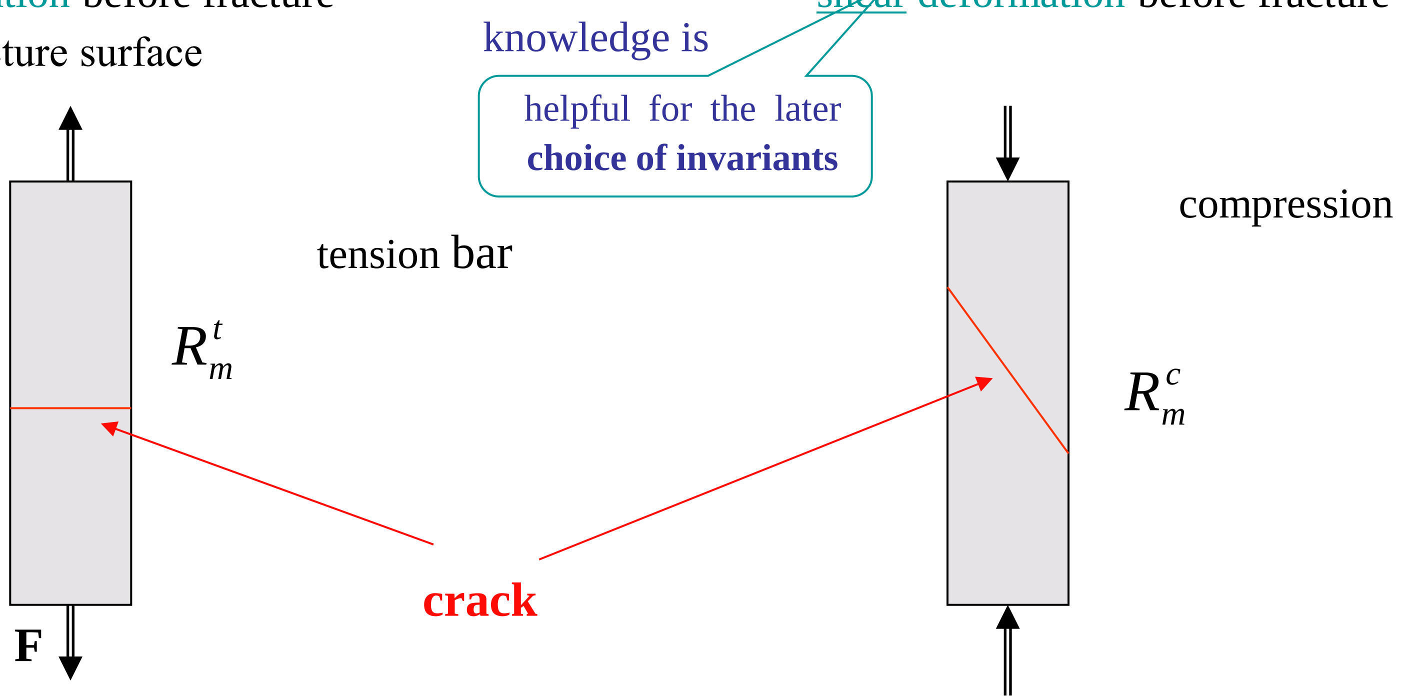
Which failure types (brittle or ductile) are observed ?

Cleavage fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- 'smooth' fracture surface

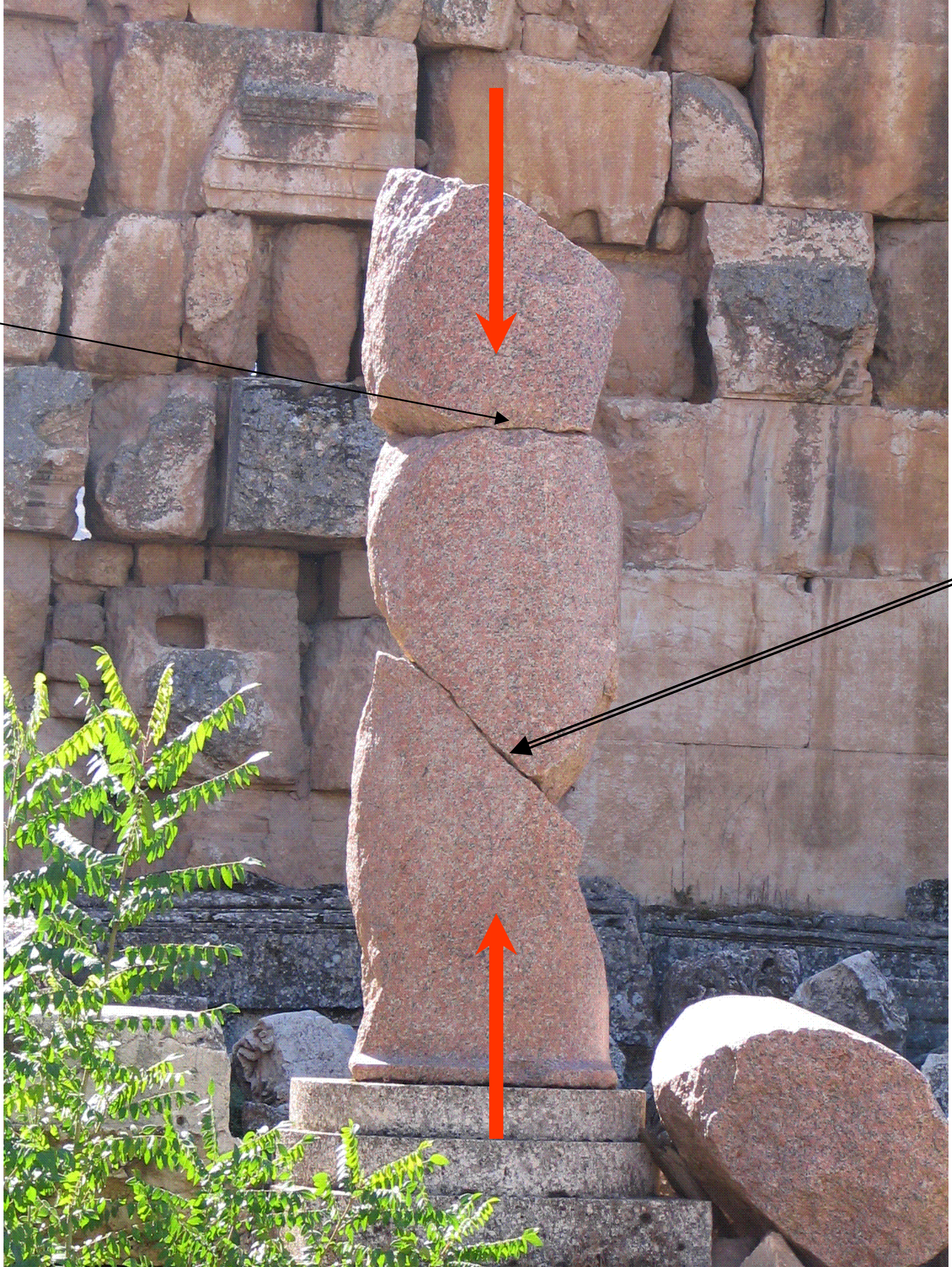
Shear fracture (SF) :

- shear deformation before fracture



conclusion: ▶ **2 strengths** to be measured

just a joint



Example SF : R_m^c

Shear Fracture plane under compression

(**Mohr-Coulomb,** acting *at* **a** ***rock material*** *column,*

at Baalbek, Libanon)

3 Observed Strength Failure Modes and Strengths

3.1b Isotropic Material *brittle, porous*

if brittle: failure = fracture failure

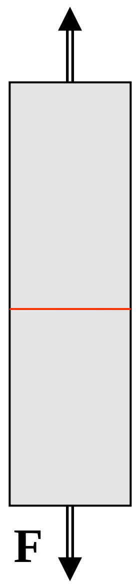
Normal Fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- rough fracture surface

Crushing Fracture (CrF): ⇐ SF

- volumetric deformation before fracture

helpful for the later **choice of invariants**



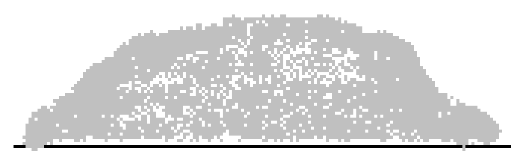
Tension

R_m^t

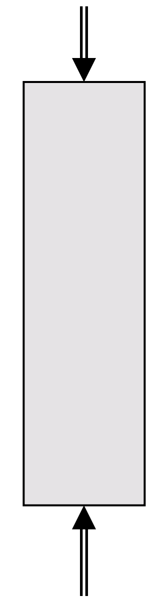
Compression

result of the compression test

= *hill of fragments* (crumbs)



= decomposition of texture



R_m^c

► **2 strengths** to be measured

3 Observed Strength Failure Modes and Strengths

audience familiar ??

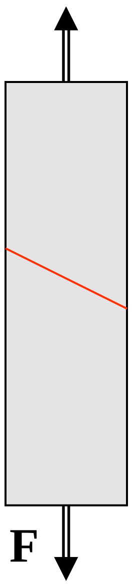
3.1c Isotropic Material *dense, ductile (most of the aerospace nmaterials)*

Shear fracture (SF) :

- *shear deformation* observed before fracture (maximum load)
- later in addition, *volume change* before rupture ('Gurson domain')
- dimples under tension.

Tension

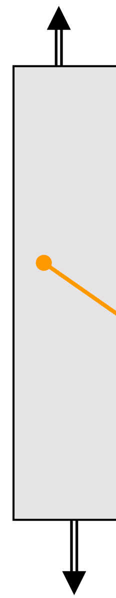
sheet



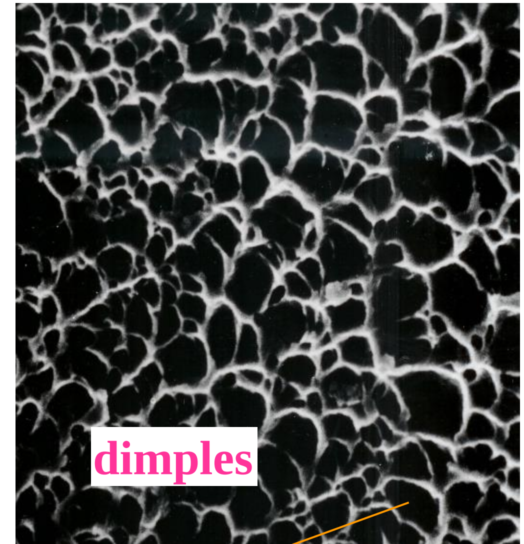
R_m^t

F

round bar



R_m^t

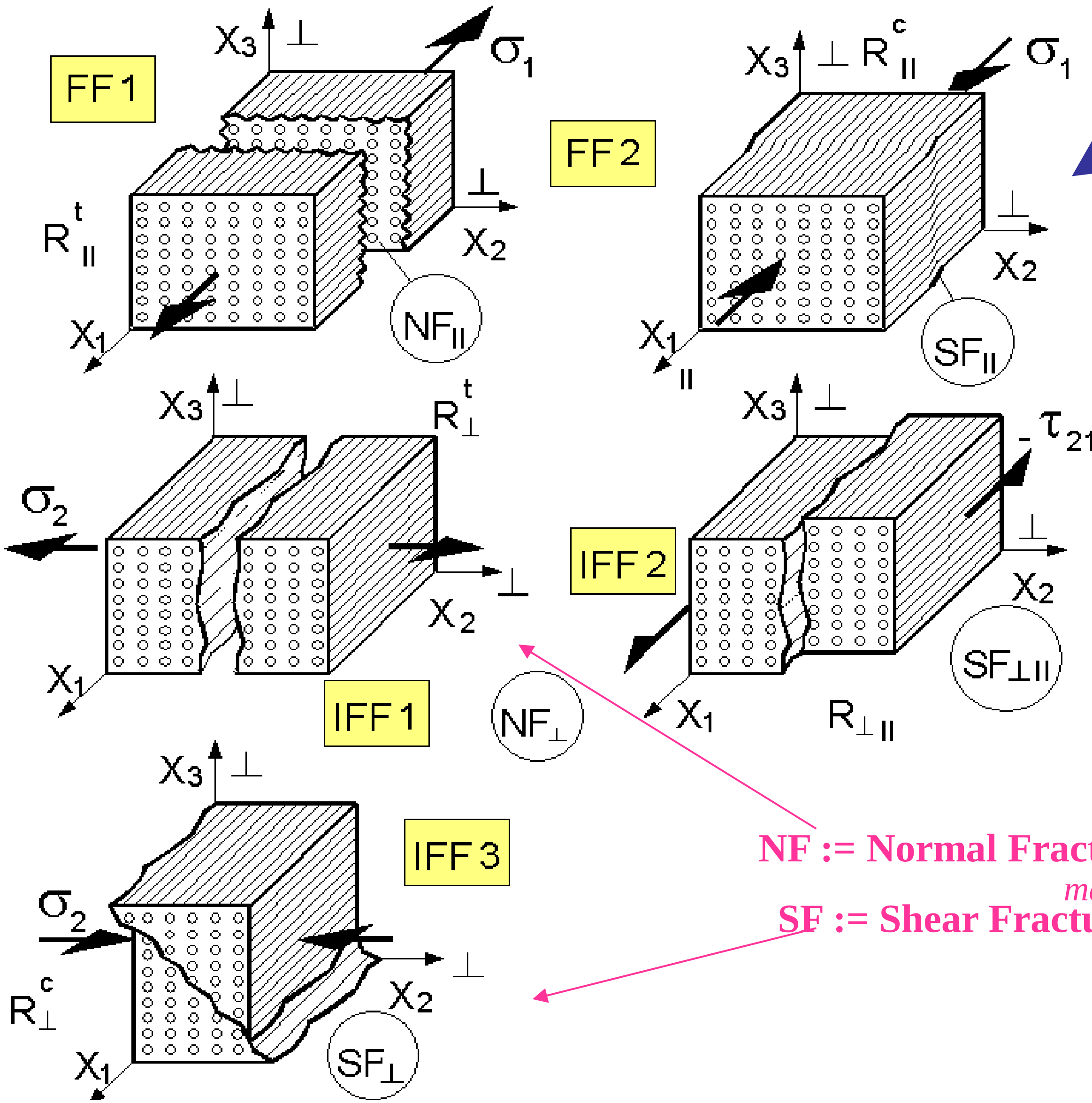


first a **diffuse** and later **local** ***necking*** + **void growth**

▶	**1 strength,** R_m^t to be measured **(=** ***load-controlled*** **value),**
•	R_m^c **is neither existing nor necessary for design ,** $R_{c0.2}$ **is the design driving strength.**

3 Observed Strength Failure Modes and Strengths

3.2a Transversely-Isotropic Material (UD) *brittle. Scheme*



wedge type

Fractography of test specimens reveals:

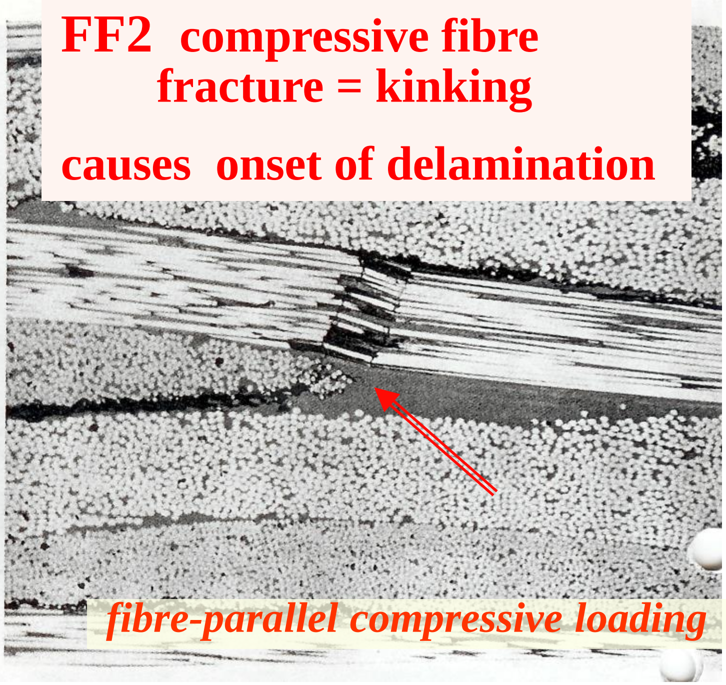
▶ 5 Fracture modes exist in a UD Laminae.
= **2 FF (Fibre Failure)**
+ **3 IFF (Inter Fibre Failure)**

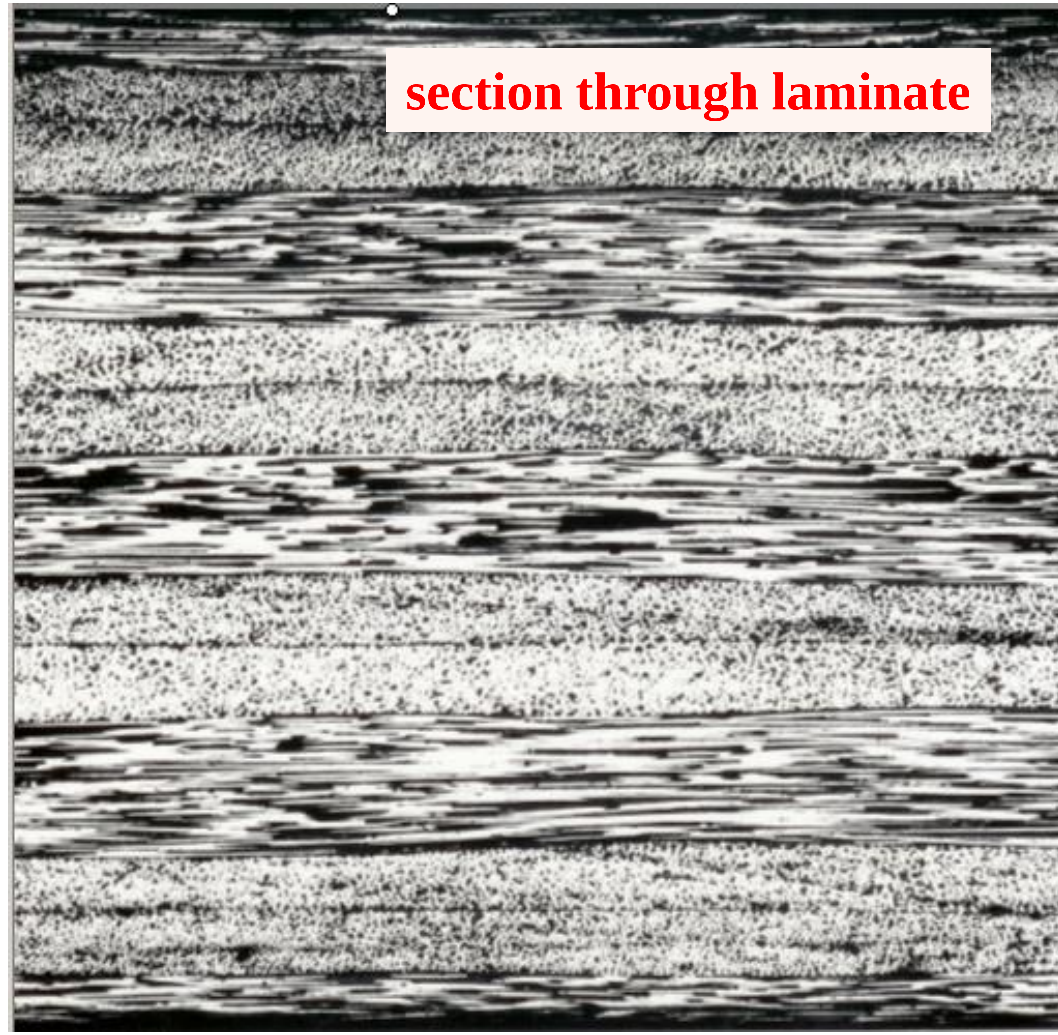
▶ **5 strengths** to be measured

NF := Normal Fracture

macroscopically:

SF := Shear Fracture





Fractography pictures as proofs

3 Observed Strength Failure Modes and Strengths

3.3 Orthotropic Material (woven fabrics)

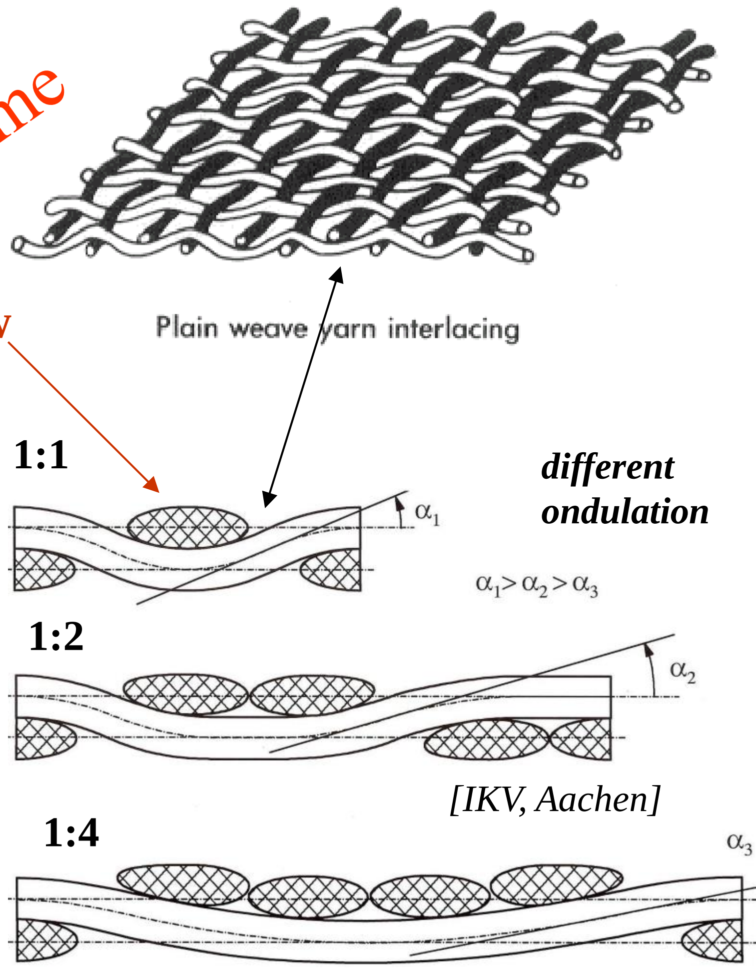
next level above UD

Fibre preforms : from ***roving, tape, weave, braid (2D, 3D), knit, stitch,*** or mixed as in a ***pre-form hybrid***

Fractography exhibits no clear failure modes.

In this material case always multiple cracking is caused under tension, compression, bending, shear !

Due to the presentation time not discussed

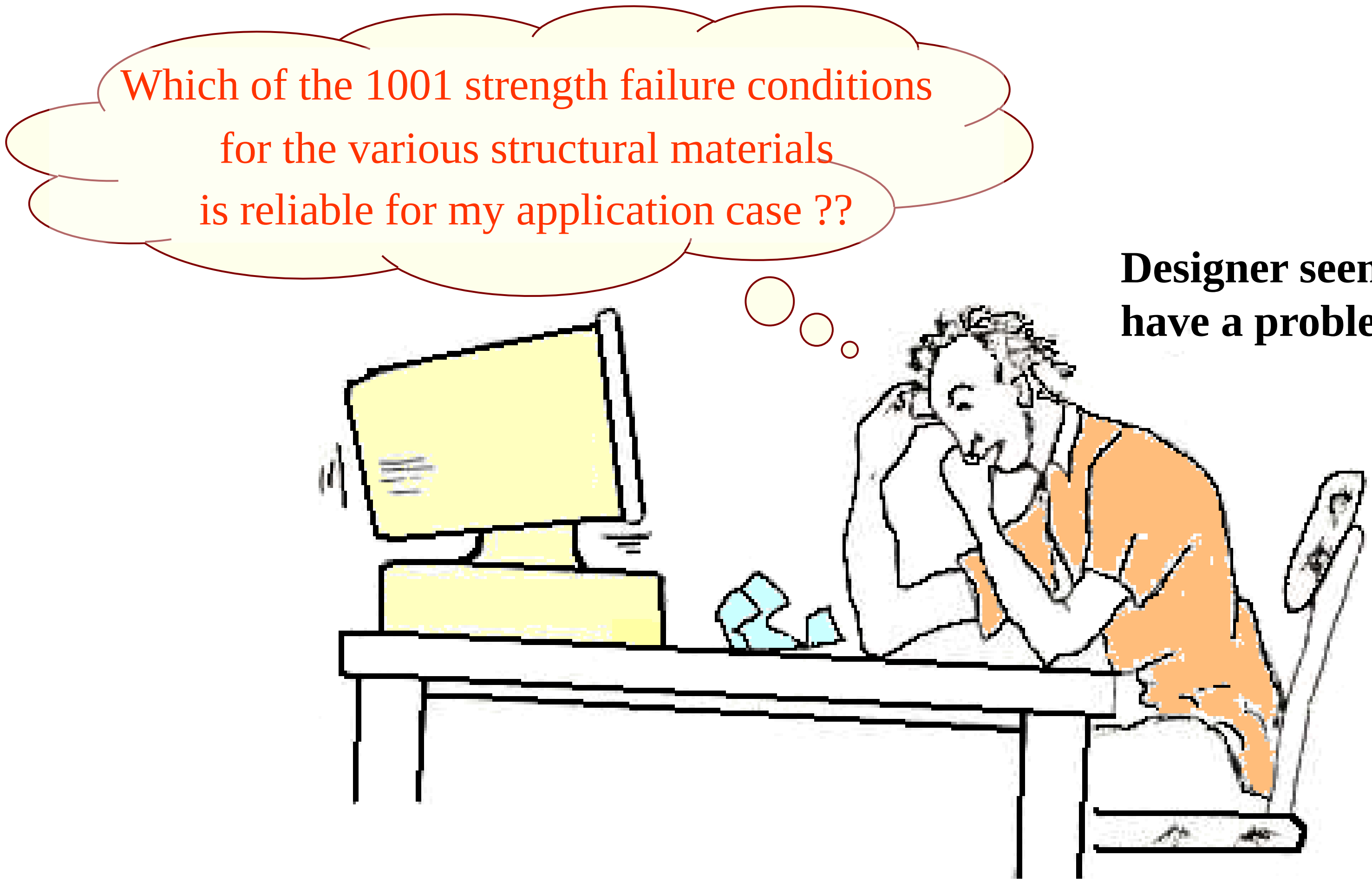


approximately UD-describable

Lessons learned:

- Strengths have to be <u>defined</u> according to material symmetry
- Modelling depends on fabrics type !

► **9 (6** *if F=W* **) strengths** to be measured

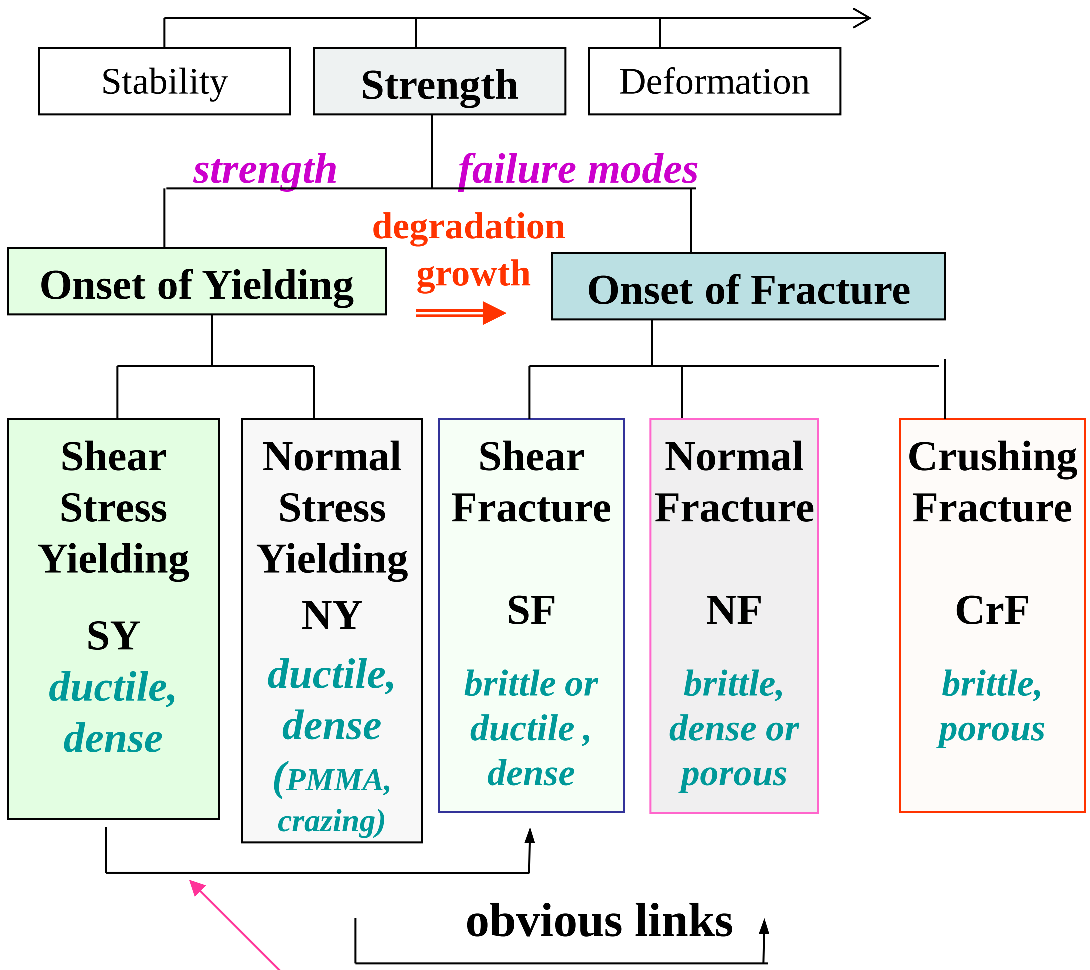


Designer seems to have a problem !

Can one help him by thinking about a systematization based on physical reasoning ?

4 Attempt for a Systematization

4.1a Scheme of Strength Failures for *isotropic materials*



The growing yield body (SY or NY)

is confined by the fracture surface (SF or NF)**!**

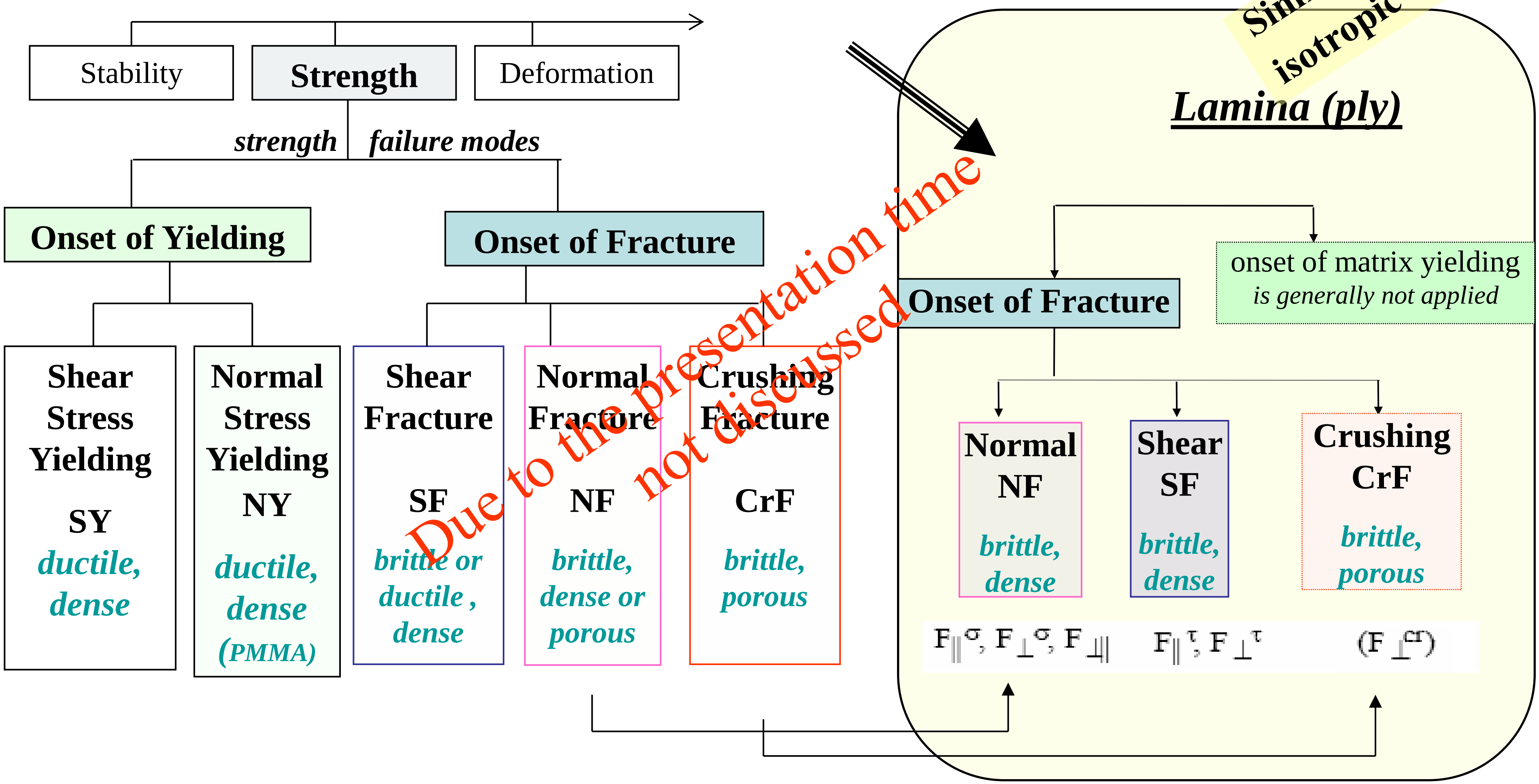
◄ = ***kinds of fracture***

Lesson learned from Mapping Test Data:

The same mathematical form of a failure condition holds - from 'onset of yielding' to 'onset of fracture' - if the physical mechanism remains !

4 Attempt for a Systematization

4.1b Scheme of Strength Failures for *the brittle UD laminae*



+ **delamination failure of *laminate***

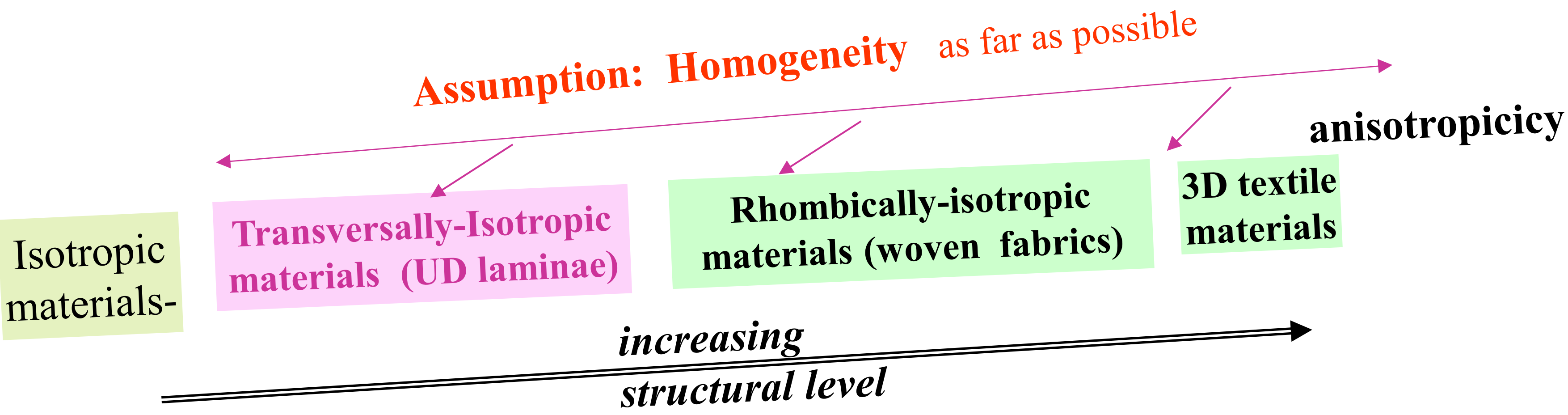
Lessons learned:

** There are coincidences between brittle UD laminae and brittle isotropic materials*

** Increased degradation occurs in the* laminate *beyond onset of the first IFF*

4 Attempt for a Systematization

4.2 Material Homogenizing (smearing) + Modelling, Material Symmetry



Material symmetry shows:

Number of strengths ≡ number of elasticity properties !

Application of material symmetry:

- *Requires that homogeneity is a valid assessment for the <u>task-determined</u> model ,*

but, if applicable

- A *minimum number of properties has to be measured, only* (cost + time benefits) !

It's worthwhile to structure the establishment of strength failure conditions

4.3 Proposed Classification of Homogenized (assumption) Materials

A Classification helps to structure the Modelling Procedure:

Failure Type / Consistency	brittle, semi-brittle Design Ultimate Load	(quasi-) ductile Design Yield Load
dense	fibre re-inforced plastics, mat, woven fabrics, grey cast iron, matrix material, amorphous glass C90-1,.	Glare, ARALL, metal alloys braided textiles
porous	foam, fibre re-inforced ceramics	sponge

← ***design driving*** (Design Yield Load)

failure: fracture (brittle, semi-brittle) — functional or usability limit ((quasi-) ductile)

Conclusion:

Modelling, and Struct. Analysis + Design Verification strongly depend on material behaviour + consistency

5 Short Derivation of the Failure Mode Concept (FMC)

5.1 General on Global Formulation & Mode-wise Formulation

• **A failure condition is the mathematical formulation, F = 1, of the failure surface:**.

1 global failure condition : $F\ (\{\sigma\}, \{R\})\ = 1$ **(usual formulation) ;**

= 'fully interactive conditions' which include several modes

$$\{R\} = (R_1, R_2, \ldots R_i)^T$$

***Several mode* failure conditions** : $F\ (\{\sigma\}, R^{mode})\ = 1$ **(used in Cuntze's FMC).**

mode-associated strength

Lesson learned from application of global failure conditions:

A change, necessary in one failure mode domain, has an impact on other physically not related failure mode domains , however in general, not on the safe side.

F >=< 1 failure criterion.

5 Short Derivation of the Failure Mode Concept (FMC)

5.2 Fundamentals of the FMC (example: UD material)

Remember:

example UD:

- **Each of the observed fracture failure modes was linked to one strength**
- **Symmetry of a material showed :** ***Number of strengths =*** $R_{||}^{t}, R_{||}^{c}, R_{\perp||}, R_{\perp}^{t}, R_{\perp}^{c}$

number of elasticity properties ! $E_{\parallel}, E_{\perp}, G_{\parallel\perp}, \nu_{\perp\parallel}, \nu_{\perp\perp}$

Due to the facts above the

FMC postulates in its '*Phenomenological Engineering Approach*' :

► Number of failure modes = number of strengths, too !

e.g.: isotropic = 2 or above transversely-isotropic (UD) = 5

5.3 Driving idea behind the FMC

A possibility exists to *more generally* formulate failure conditions

- **failure mode-wise** *(shear yielding etc.)* — **Mises, Hashin, Puck etc.**
- **stress invariant-based** *(J_2 etc.)* — **Mises, Tsai, Hashin, Christensen, etc.**

5. Short Derivation of the *Failure Mode Concept (FMC)*

5.4 Detail Aspects

1) 1 failure *condition* represents 1 Failure Mode *(interaction of acting stresses)*.

2) Interaction of adjacent Failure Modes by a *series failure system* model

to map the full course of all test data

$$(Eff)^m = (Eff^{mode1})^m + (Eff^{mode2})^m + \ldots + \ldots\ldots = 1$$

with Stress Effort Eff := portion of load-carrying capacity of the material $\equiv \sigma_{eq}^{mode} / R^{mode}$

and Interaction coefficient m of modes.

NOTE: The presentation shall just provide

with a <u>general view at the material behaviour links</u> and not

with a detailed information on the derived strength failure conditions !

How is above *interaction of modes* performed?

5. Short Derivation of the *Failure Mode Concept (FMC)*

5.5 Interaction of the Strength Failure Modes (example: UD, the 3 IFF)

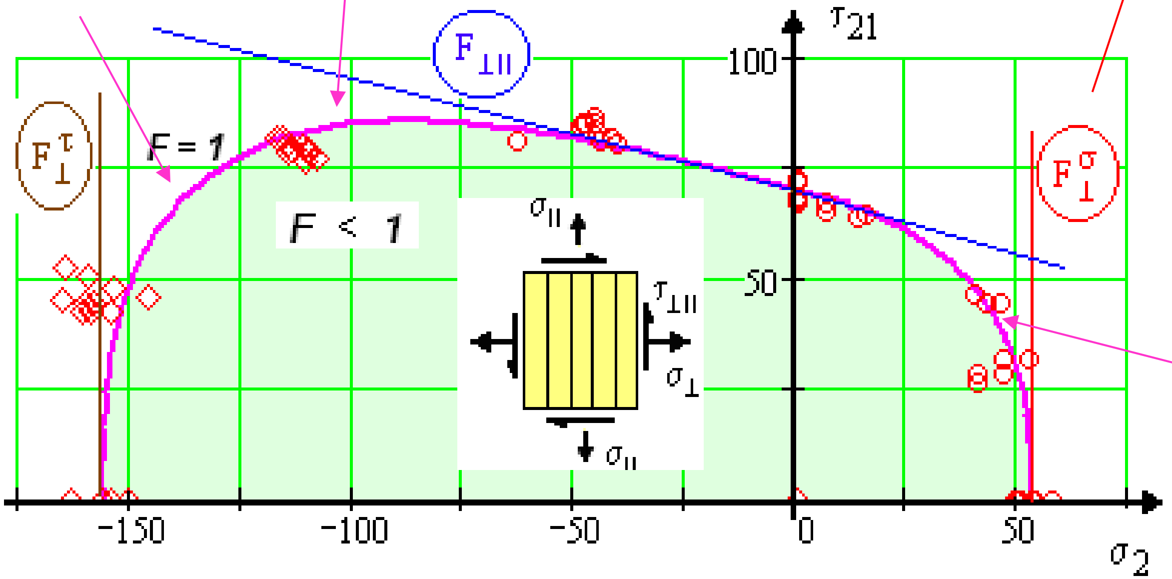
Stress efforts of the 3 pure IFF modes = 3 straight lines :

$$Eff_{\perp}^{\sigma} = \frac{\sigma_2}{\overline{R}_{\perp}^{t}}, \qquad Eff_{\perp\|} = \frac{|\tau_{21}|}{\overline{R}_{\perp\|} - \mu_{\perp\|} \cdot \sigma_2}, \qquad Eff_{\perp}^{\tau} = \frac{-\sigma_2}{\overline{R}_{\perp}^{c}}.$$

All failure modes, 3 IFF + 2 FF, are interacted in one single (*global*) failure equation

magenta curve ; $$Eff^{m} = (Eff_{\|}^{\tau})^m + (Eff_{\|}^{\sigma})^m + (Eff_{\perp}^{\sigma})^m + (Eff_{\perp\|})^m + (Eff_{\perp}^{\tau})^m = 1.$$

by above series failure system model



* *for UD laminae m =2.5 - 3*

* *the same value m is applied for all interaction zones*

IFF curves: (σ_2, τ_{21}) *.Hoop wound GFRP tube: E-glass/LY556/HT976*

5. Short Derivation of the *Failure Mode Concept (FMC)*

5.6 Reasons for Chosing Invariants when generating Failure Conditions

* Beltrami : "At 'Onset of Yielding' the material possesses a distinct ***strain energy*** composed of *dilatational energy* (I_1^2) and *distortional energy* ($J_2 \equiv Mises$) ".

* So, from Beltrami, Mises (HMH), and Mohr / Coulomb (friction) can be concluded:

Each invariant term in the *failure function* ***F*** may be dedicated to one physical mechanism in the solid = cubic material element:

- volume change : I_1^2 … ***(dilatational energy)***
- shape change : J_2 (Mises) … ***(distortional energy)***

and

- friction : I_1 … ***(friction energy)***

Stress Invariants: ***isotropic materials*** *and*

I_1^2 , I_2^2

I_3 , I_4

I_2

: UD materials !

Mohr-Coulomb

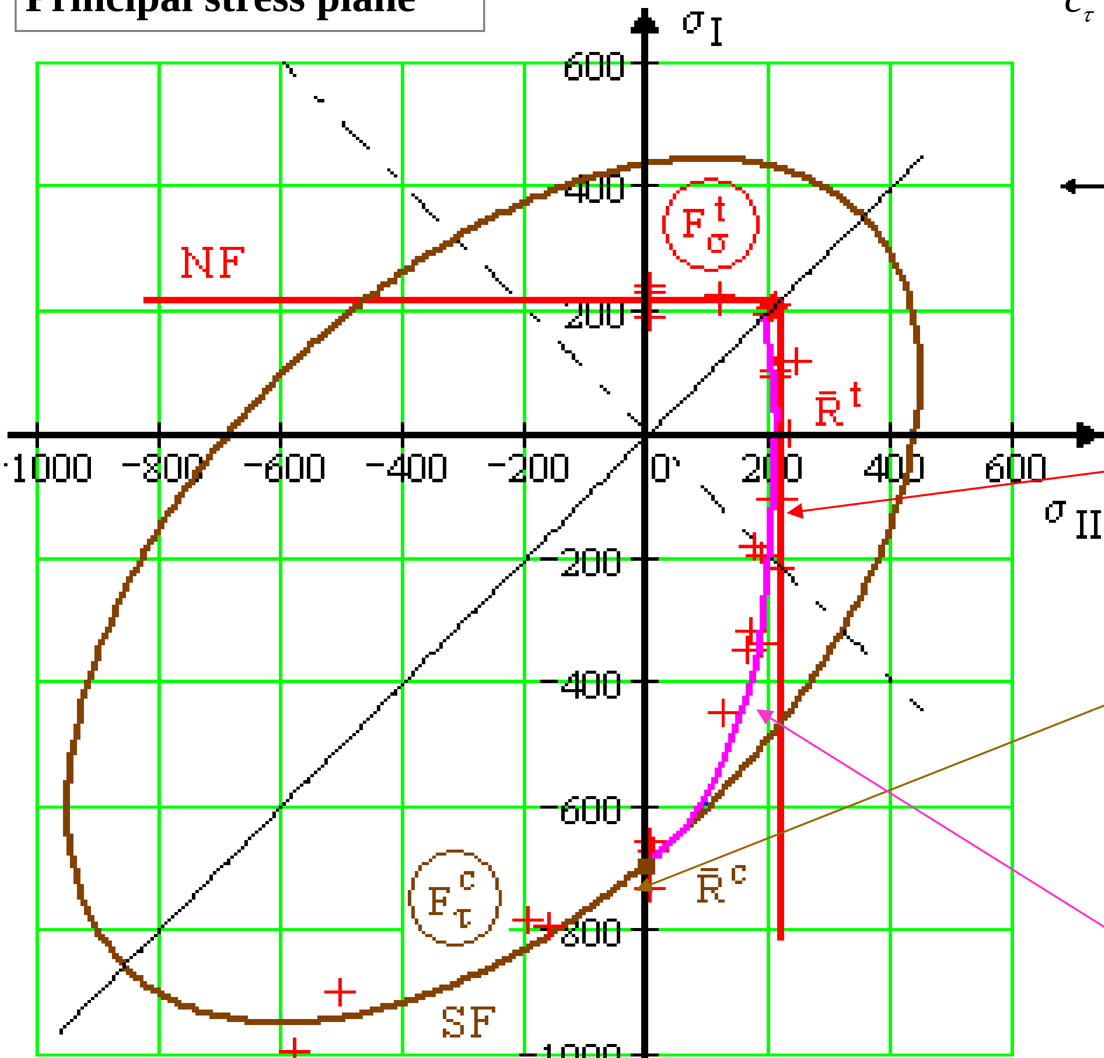
Remember:

These I_1 are different !

6 Visualisation of some Derived Failure Conditions

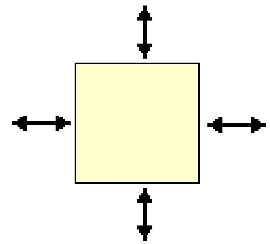
6.1 Grey Cast Iron (brittle, dense, microflaw-rich), *Principal stress plane*

Principal stress plane



$c_\tau^c = a_\tau^c - 1, \quad a_\tau^c = 1.58 \qquad m = 3.1 \qquad \bar{R}^t = 215 MPa;$

$\bar{R}^c = 690 MPa$



$$\{\sigma\}_{principal} = (\sigma_I, \sigma_{II}, 0)^T$$

$$F_\sigma^t = \frac{\sqrt{I_\sigma} + I_1}{2 \cdot \bar{R}^t} = 1$$ deformationless

$$F_\tau^c = a_\tau^c \frac{3J_2}{\bar{R}^{c2}} + c_\tau^c \frac{I_1}{3\bar{R}^c} = 1$$

shear change   friction

= 2 Mode Failure Conditions

Interaction zone

Lessons learned: Basically, *<u>Dense</u> concrete and Glass C 90 will have same failure condition*

6 Visualisation of some Derived Failure Conditions

see Paper for details

6.2a Concrete (isotropic, slightly porous) *Kupfer's data*

Due to presentation time just shown

$$F_\sigma^t = \frac{\sqrt{I_\sigma} + I_1}{2\bar{R}_m^t} = Eff_\sigma^t = 1$$

deformation poor

hyperbola

shape + volume change + friction: Mohr-Coulomb :

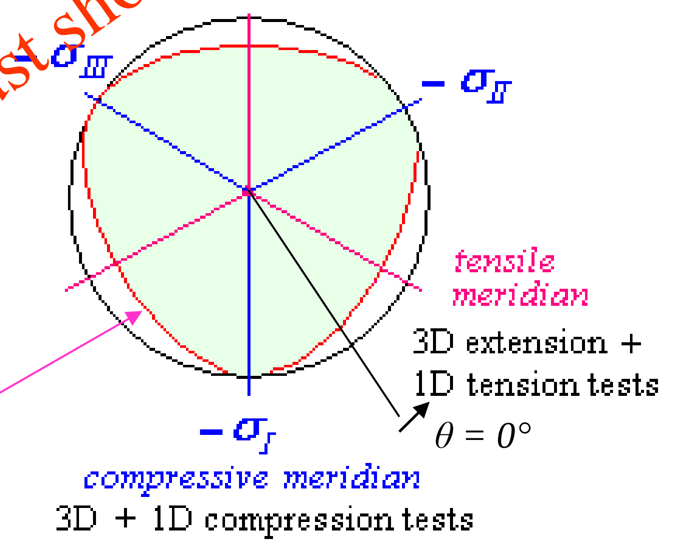
$$F_\tau^c = a_\tau^c \frac{3J_2 \cdot \Theta}{\bar{R}_m^{c\,2}} + b_\tau^c \frac{I_1^2}{\bar{R}_m^{c\,2}} + c_\tau^c \frac{I_1}{\bar{R}_m^c} = 1$$

(closed failure surface)

paraboloide

Octahedral stresses (B-B view)

Isotropic materials possess 120° symmetry :



Lessons learned from test data viewing**:**

- *Course of concrete test data shows a big bandwidth*
- *The reason for the bandwidth is not only the test scatter but the stress-state dependent 'double' failure probability causing non-coaxiality in the octahedral plane. The difference between the so-called tensile (extension) meridian and the compression meridian is to be considered.*

Basically, the differences in the octahedral (deviatoric) plane can be described by :

$$\Theta \Rightarrow \sqrt[3]{1 + d \cdot sin(3\theta)}$$

$$sin(3\theta) = 3\sqrt{3}J_3/(2J_2^{3/2}),$$

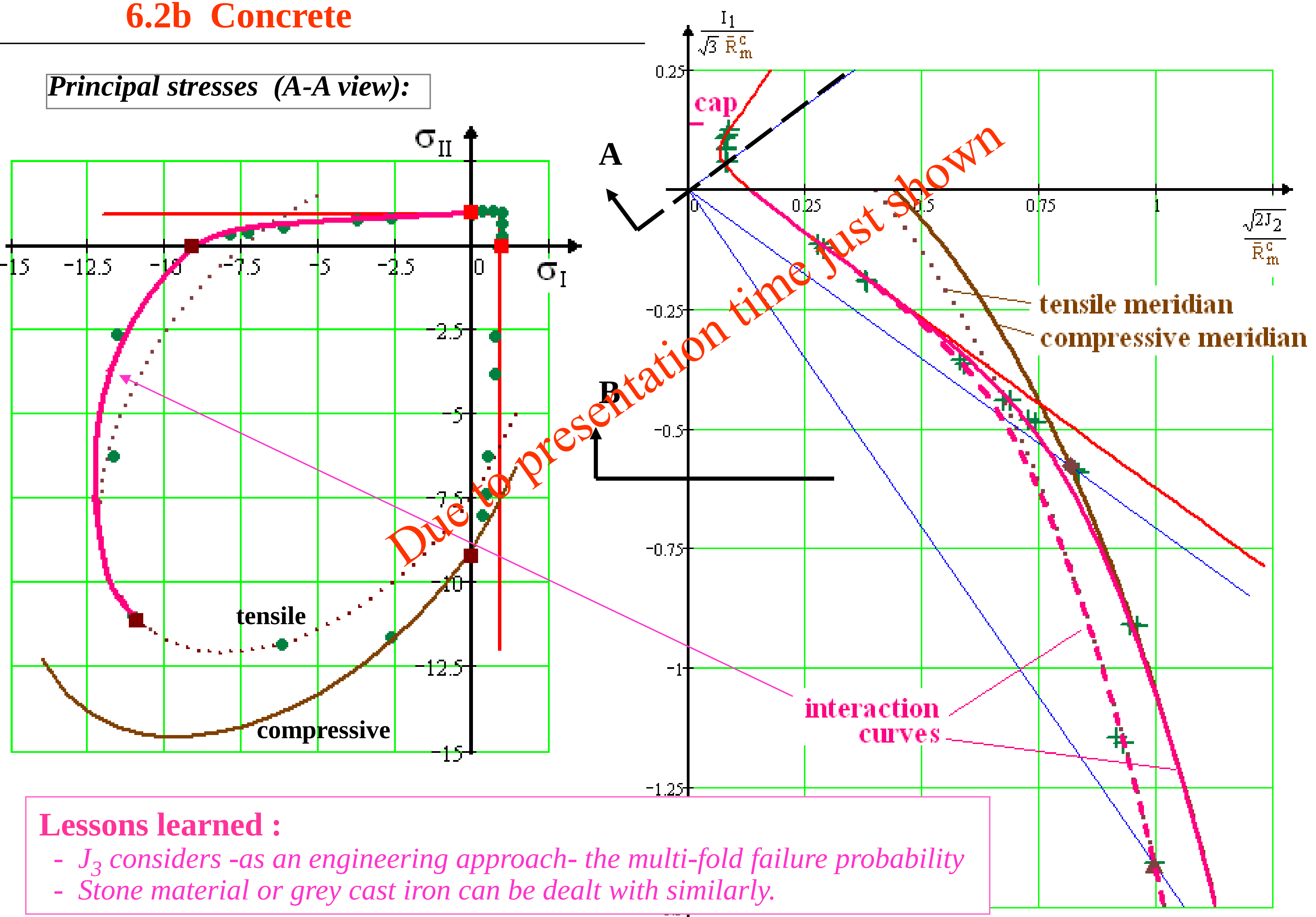
[de Boer, et al] $d \leq 0.5$, *convex*

Remark Cuntze: J_3 practically describes the effect of the doubly acting failure mode, no relation to new special mechanism.

6 Visualisation of some Derived Failure Conditions

see Paper for details

6.2b Concrete



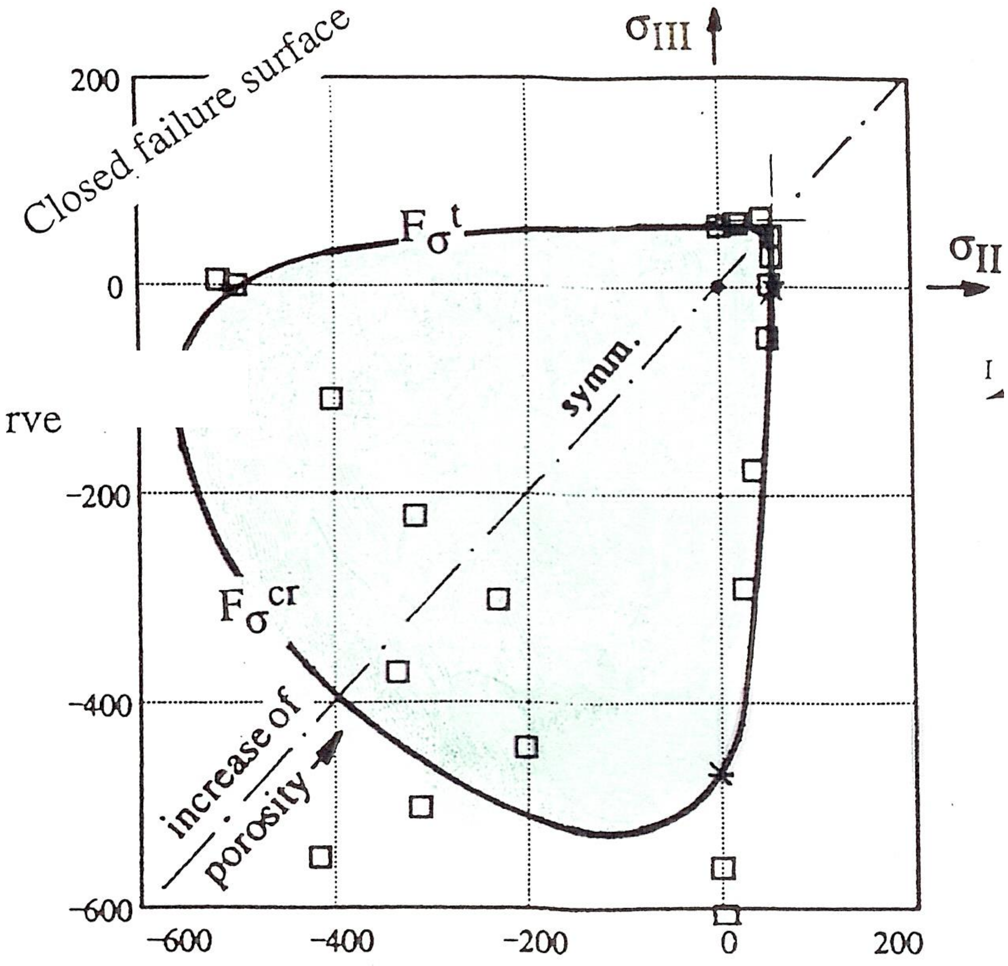
Lessons learned :

- *J_3 considers -as an engineering approach- the multi-fold failure probability*
- *Stone material or grey cast iron can be dealt with similarly.*

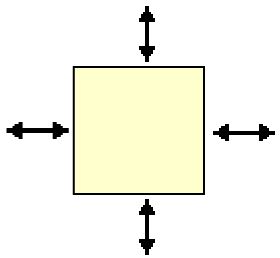
6 Visualisation of some Derived Failure Conditions

6.3 Monolithic Ceramics (brittle, porous isotropic material)

Principal stress plane



$c^{cr} = a^{cr} - 1$ ***[Kowalchuk]***



$$F_\sigma^t = \frac{\sqrt{I_\sigma} + I_1}{2 \cdot \bar{R}^t} = 1$$

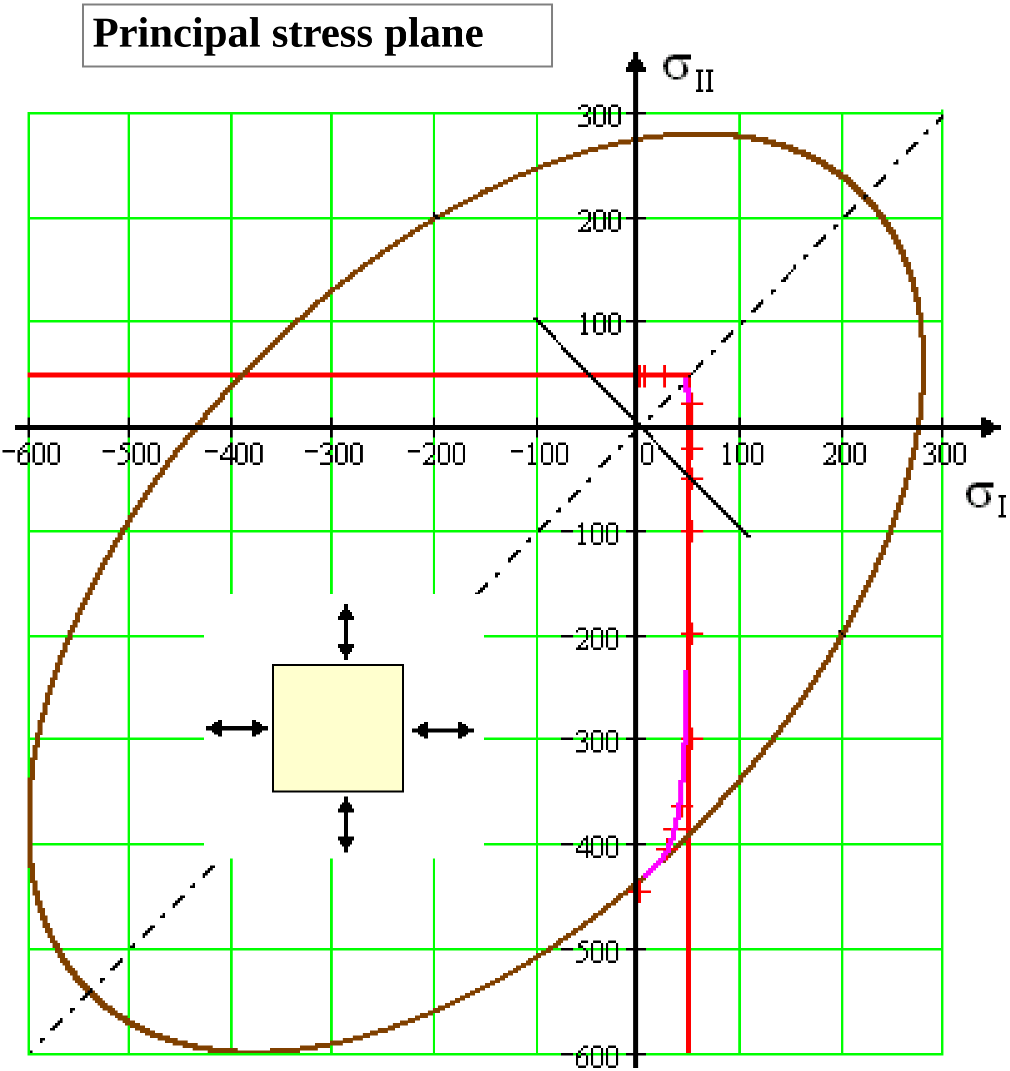
$$F^{cr} = a^{cr} \frac{3J_2}{\bar{R}_m^{c\,2}} + \; + c^{cr} \left(\frac{I_1}{\bar{R}_m^c} \right)^2 = Eff^{cr} = 1$$

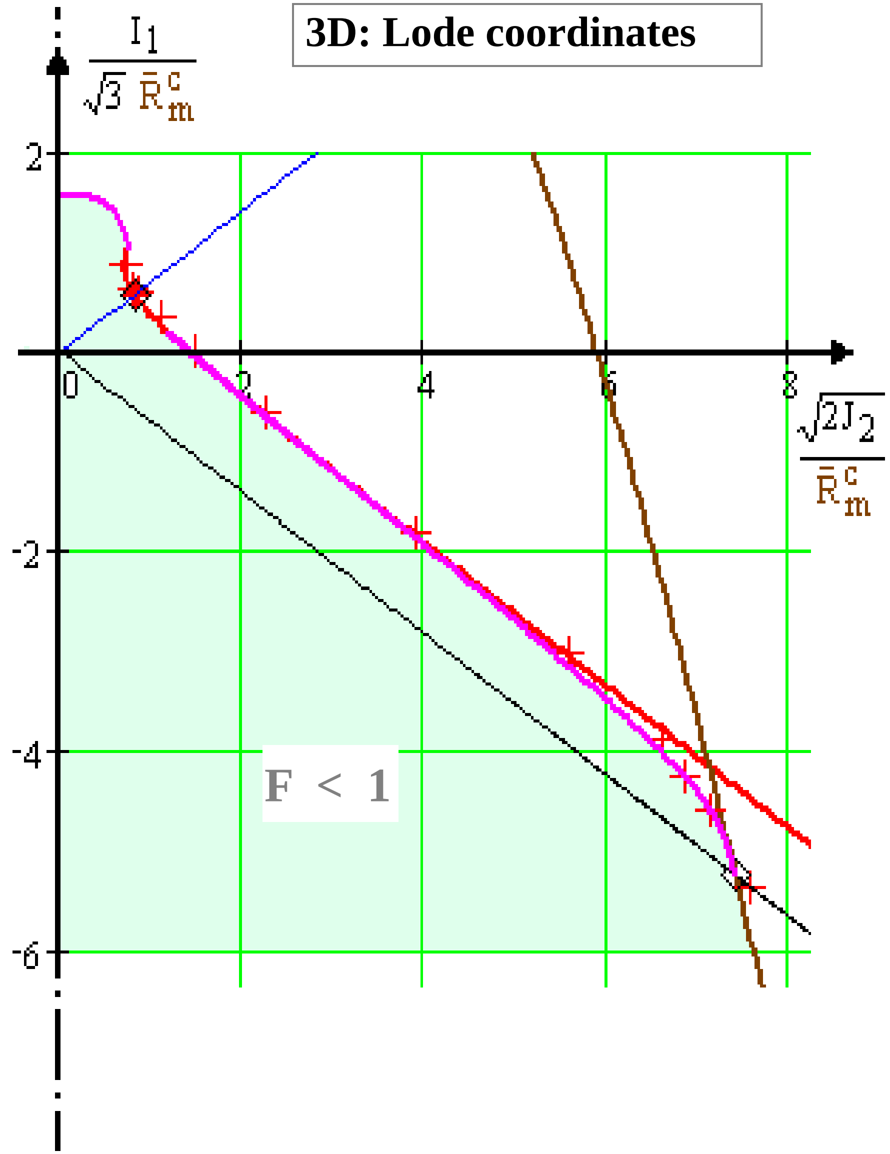
shear change **volume change**

Lessons learned: ***Same failure condition as very porous concrete***

6 Visualisation of some Derived Failure Conditions

6.4 Glass C 90 (brittle, dense isotropic material)



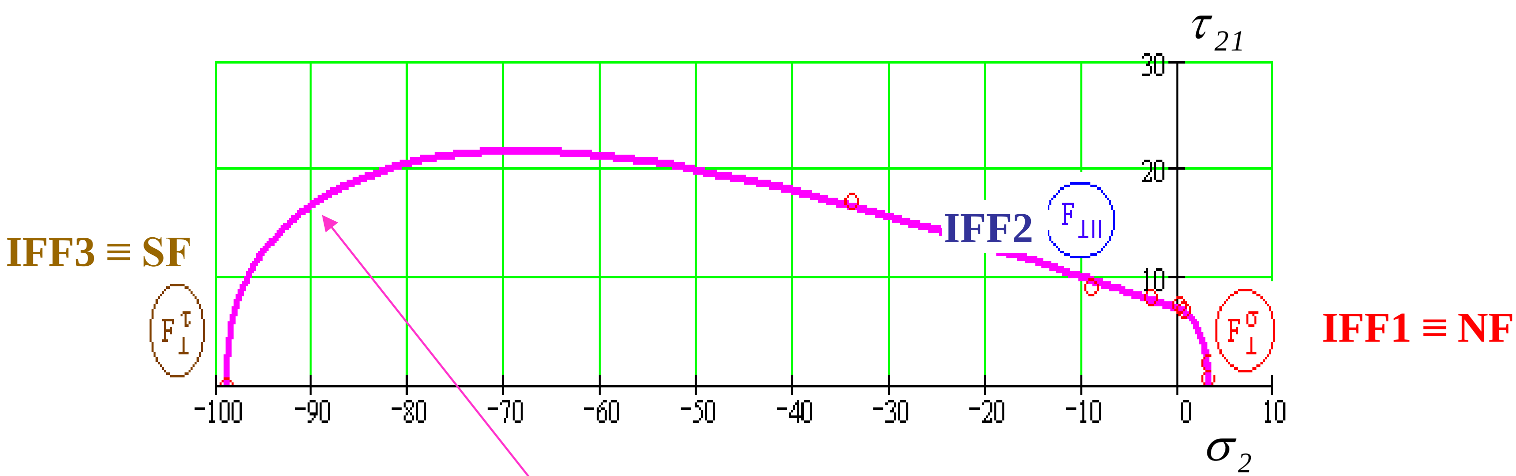


6 Visualisation of some Derived Failure Conditions

6.5 UD Ceramic Fibre-Reinforced Ceramics (C/C) (brittle, porous, tape)

$$\{\bar{R}\} = (\bar{R}_{||}^t,\ \bar{R}_{||}^c,\ \bar{R}_\perp^t,\ \bar{R}_\perp^c,\ \bar{R}_{\perp||}) = (-,\ -,\ 3,\ 99,\ 7)^T,\ m = 2.3,\ \mu_{\perp||} = 0.3$$

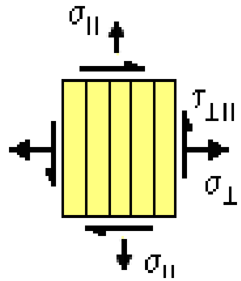
[Diss. B. Thielicke, 1997]



Interaction equation :

$$\left(\frac{\sigma_2}{\bar{R}_\perp^t}\right)^m + \left(\frac{|\tau_{21}|}{\bar{R}_{\perp||} - \mu_{\perp||}\cdot\sigma_2}\right)^m + \left(\frac{-\sigma_2}{\bar{R}_\perp^c}\right)^m = 1$$

$F_\perp^\sigma$ (deformationless), $F_{\perp||}$ (friction (Mohr-Coulomb)), $F_\perp^\tau$ (shear)

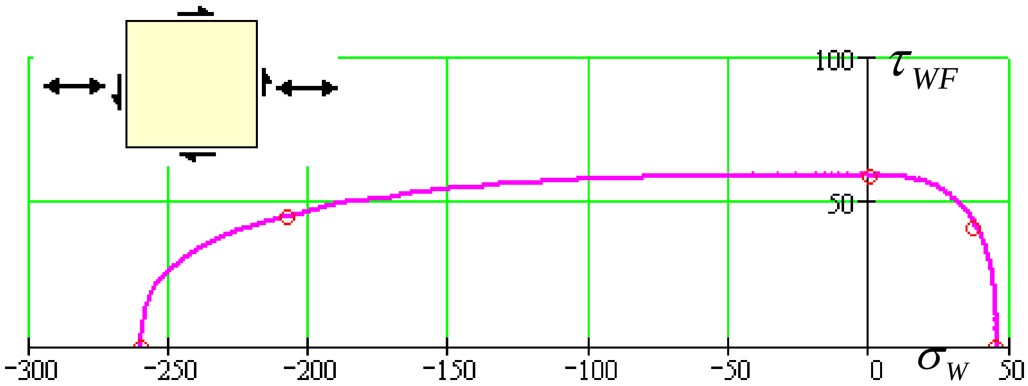


Invariants applied: I3, I2 — I4 , I2

Lesson learned: ***Same failure condition as with UD-FRP***

6 Visualisation of some Derived Failure Conditions

6.6 Fabric Ceramic Fibre-Reinforced Ceramics (CFRC) (brittle, porous)



C/C-SiC, T= 1600°C

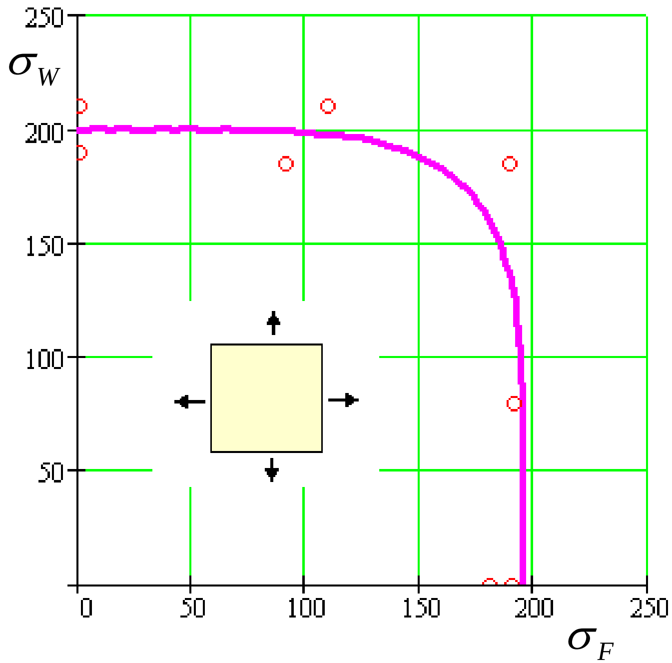
[Geiwitz/Theuer/Ahrendts 1997] ,

*tension/compression-torsion-**tube??***

$$\{\bar{R}\} = (\bar{R}_{\|}^t, \bar{R}_{\|}^c, \bar{R}_{\perp}^t, \bar{R}_{\perp}^c, \bar{R}_{\perp\|}) = (-, -, 45, 260, 59)^T$$

$m = 2.8$

$$\left(\frac{\sigma_W}{\bar{R}_W^t}\right)^m + \left(\frac{-\sigma_W}{\bar{R}_W^c}\right)^m + \left(\frac{\tau_{WF}^2}{\bar{R}_{WF}^2}\right)^m = 1$$



$$\{\bar{R}\} = (\bar{R}_W^t, \bar{R}_W^c, \bar{R}_F^t, \bar{R}_F^c, \bar{R}_{WF}, \bar{R}_3^t, \bar{R}_3^c, \bar{R}_{3F}, \bar{R}_{3W})^T$$

$\{\bar{R}\}$ = *vector of mean strength values*

C/SiC, ambient temperature *[MAN-Technologie, 1996],*

tension/tension tube

$$\{\bar{R}\} = (200, -, 195, -, -,,,,)^T, \; m = 5$$

$$\left(\frac{\sigma_W}{\bar{R}_W^t}\right)^m + \left(\frac{\sigma_F}{\bar{R}_F^t}\right)^m = 1$$

NOTE: For woven fabrics enough test information for a real validation is not yet available!

Conclusions from the Beltrami-based *Failure Mode Concept* applications

- **FMC is an efficient concept, that improves prediction + simplifies design verification**

  is applicable to brittle+ductile, dense+porous, isotropic → orthotropic material
  - if clear failure modes can be identified and
  - if the homogenized material element experiences a *volume* or *shape change* or *friction*

- **Delivers a global formulation of '*individually*' *combined independent failure modes*,**
  without the well-known drawbacks of global failure conditions
  which *mathematically combine in-dependent failure modes* .

- **Failure conditions are simple but describe physics of each failure mechanism pretty well**

- **Material behaviour Links have been outlined:**

  Paradigm*:* Basically, a compressed brittle *porous* concrete can be described like
  a tensioned ductile *porous* metal ('Gurson' domain)

The man years of development of the FMC were never funded !