Gemeinsame Sitzung der 3er-CCeV-AG
‚Engineering, Zerstörungsfreie Werkstoff- u. Bauteilprüfung,
Kleb- und Verbindungstechnik‘. 9.3.2018 TZA , Augsburg
25 min + 5 min

CARBON COMPOSITES

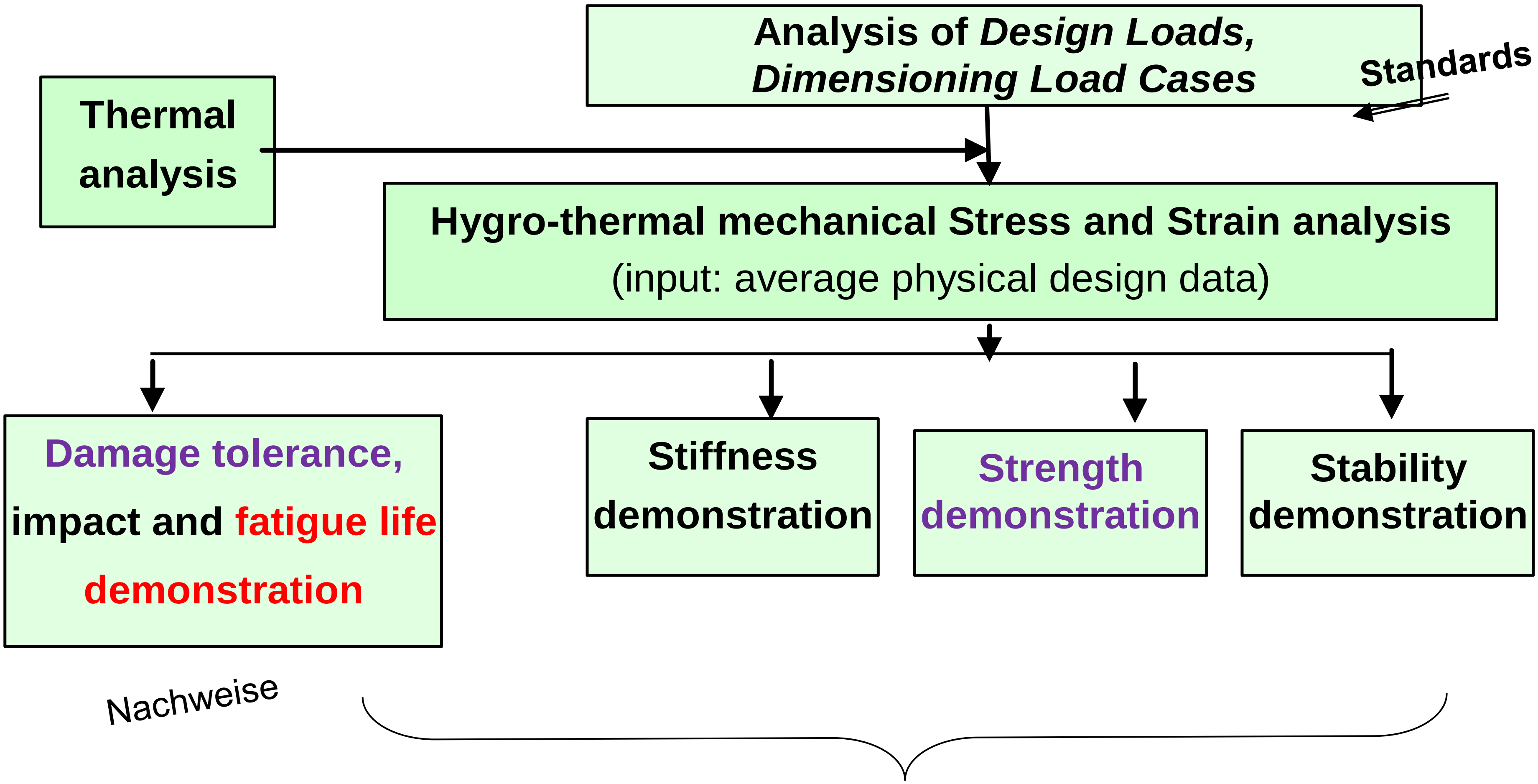
Lifetime Prediction for UD-materials *by using* Failure Mode-linked Modeling of Loading, S-N Curves, Kawai's 'Modified Fatigue Strength Ratio', and Novel Haigh Diagrams

- brittle material behavior -

1 Introduction to Static and Fatigue Design
2 Cuntze's ***Failure Mode Concept-based Strength Criteria***
3 Cuntze's ***Fatigue Life Estimation Concept***
4 Generation and ***Novel Interpretation of*** *UD* ***Haigh Diagrams***
5 Steps of the Fatigue Life Prediction Method Proposed

Prof. Dr.-Ing. habil. Ralf Cuntze VDI, formerly MAN-Technologie AG
linked to Carbon Composite e.V. (CCeV) Augsburg,
Since 1970 in CFRP composite business

Flow Diagram: Structural Design and Design Verification



Analysis of *Design Loads, Dimensioning Load Cases*

Standards

Thermal analysis

Hygro-thermal mechanical Stress and Strain analysis
(input: average physical design data)

Damage tolerance, impact and fatigue life demonstration

Stiffness demonstration

Strength demonstration

Stability demonstration

Nachweise

'Resistances', to be demonstrated
by a *positive Margin of Safety (MoS)*
in order to achieve Structural Integrity *!*

Verification Levels of the Structural Part

focussed here

*≡ **demonstration of strength***

- **Stress**, locally at a critical material ‘point‘: Strength, using strength criteria

  verification by a *<u>basic strength</u>* or a *<u>multi-axial failure stress state</u>*

  Applied stresses are local stresses (continuums mechanics)

  ⟹ *cyclically growth of diffuse and later localized damaging*

- **Stress intensity** (delamination = <u>crack</u>): Damage tolerance, using fracture mechanics tools

  verification by a *fracture toughness (energy –related)*

  Applied stresses are ‘far‘-field stresses.(far from the crack-tip)

  ⟹ *cyclically growth of a detected ‘technical damage‘* (an interlaminar delamination)

Design Verification by theoretical prediction

STATIC :

- Reserve Factor is load-defined :

$$RF = \frac{\text{Predicted Failure Load}}{j \cdot \text{Design Limit Load}} > 1.$$

Material Stressing Effort : if *RF* = 1 then *Eff* = 100% Werkstoffanstrengung (erschöpft)

Material Reserve Factor :

$$f_{\text{Res}} = \frac{\text{Strength Design Allowable}}{\text{Stress at } j \cdot \text{Design Limit Load}} > 1.$$

if linear situation, then : $f_{Res} = RF = 1 / Eff$

Demonstration of MoS > 0 or RF = MoS + 1 > 1

CYCLIC :

$$RF_{life} \approx \frac{\boxed{\text{Predicted Lifetime}}}{j_{life} \cdot \text{Design Limit Lifetime}} > 1.$$

- *$Rf_{life, \text{ Predicted Lifetime}}$*
- Determination of Inspection time
- Determination of Replacement time

j = design factor of safety

Some Definitions needed for Modeling *What is ?*

Material : homogenized (smeared) model of the envisaged complex material which might be a material combination

Failure : structural part does not fulfil its functional requirements such as FF = fiber failure, IFF = inter-fiber-failure (matrix failure), leakage, deformation limit (tube widening, delamination size limit, ..) ⟹ **= a <u>project-defined</u> 'defect'**

Fatigue : process, that degrades material properties

Fatigue Life Stages (1) accumulation of damaging until initiation of a critical damage size (classical fatigue life prediction domain), (2) damage growth until onset of final fracture (domain of damage tolerance concepts), (3) separation (not of interest)

Damaging (not also damage, as used in English literature) **:** process wherein the results, the damaging portions, finally accumulate to a damage size such as a macro-scopic delamination. Accumulation tool usually used is *Palmgren-Miner's Damaging Accumulation Rule* (= model)

Damage : sum of the accumulated damaging or an impact failure, that is judged to be critical. Then, *Damage Tolerance Analysis* is used to predict damage growth under further cyclic loading or static failure under Design Ultimate Load

Haigh Diagram : involves all S-N curves required for fatigue life prediction.

State of the Art: Static Strength Analysis *of UD laminas*

represented best by the results of the *World-Wide-Failure-Exercises*

on ***Static strength criteria for the high-performance UD composite parts***

Organizer : ***QinetiQ , UK*** *(Hinton, Kaddour, Soden, Smith, Shuguang Li)*

Aim: ***'Testing Predictive Failure Theories for***

Fiber–Reinforced Polymer Composites to the full !'

(was for the transversely-isotropic ***UD materials****, only)*

Method of the World-Wide-Failure-Exercises-I, -II (1991-2013)**:**

Part A of a WWFE: ***Blind Predictions on basic strength data***

Part B of a WWFE: ***Comparison Theory-Test*** *with (reliable)*
Uni-axial 'Failure Stress Test Data' (= *basic strength)* and
Multi-axial 'Failure Stress Test Data' (plain test specimens, no notch)

Cuntze's invariant-based strength criteria mapped the provided accurate test data sets best, in WWFE - I (winner) and in WWFE- II !

.. for computation of the damaging portions under cyclic loading applicable

State of the Art : <u>Cyclic</u> Strength Analysis of UD-ply composed Laminates

- **No Lifetime Prediction Method available, applicable to any Laminate**
- **Procedures base – as with metals – on stress amplitudes and mean stress correction**
- **Procedures base on specific laminates and therefore cannot be generally applied**
- **<u>Presently</u>: Engineering Approach**

  <u>Static</u> Design Limit Strain **of** $\varepsilon < 0.3\%$ **,** negligible matrix-microcracking.

  Design experience proved: <u>No</u> fatigue danger given

- **<u>Future</u> : *Design Limit Strain* shall be increased** (EU-project: MAAXIMUS)

  Beyond $\varepsilon \approx 0.5\%$ ***first filament breaks , diffuse matrix-microcracking***

  ***changes to a discrete localized* one.**

Usually, *fiber-dominated laminates* are used in high performance stress applications!

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Note:

* *Failure-Mode-Concept-based Static Strength Criteria* are applicable to isotropic, transversely-isotropic UD and orthotropic woven materials

* as model parameters used are the measurable strengths R_m and friction values μ

* the interaction exponent m is estimated due to mapping experience

* an artificial english denotation for the self-explaining German term Werkstoffanstrengung was created with QinetiQ during the WWFE reading material stressing effort *Eff*

Basic Features of the author‘s Failure-Mode-Concept (FMC), 1995

<u>plus</u> a confirmation that transversely-isotropic UD materials exhibit a *‘5-fold‘* material symmetry characteristic = 5 Strengths, 5 Failure Modes, 5 Es, etc.

- **Each failure mode represents 1 independent failure mechanism**

  and thereby 1 piece of the complete *failure surface*

- **Each failure mechanism is governed by 1 basic strength** (is observed!)

- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !!

  This is of advantage when deriving S-N curves and Haigh diagrams

  with minimum test effort.

⇨ **Consequently, the FMC-approach requires :**

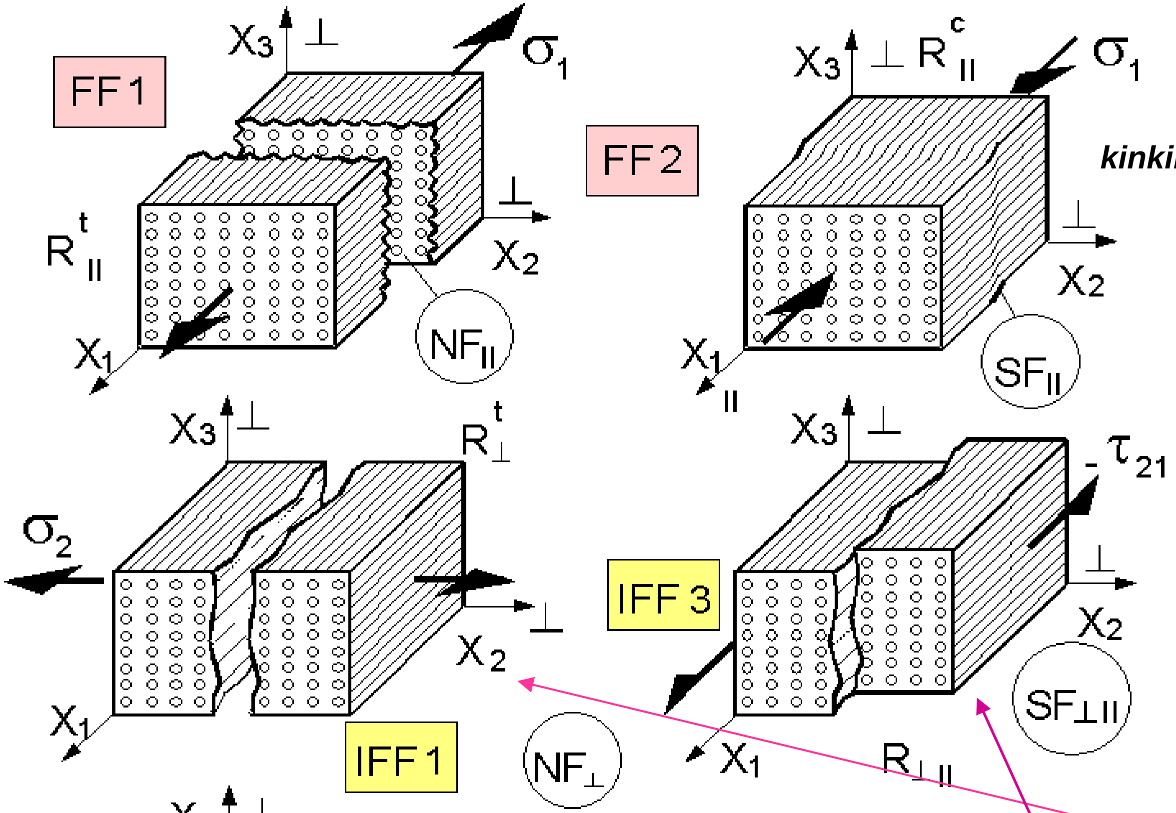
the *interaction of all 5 Modal (fracture) Failure Modes !*

Observed Fracture ‘Planes‘: Transversely-isotropic UD Material

[Cun04]

t = tension

c = compression



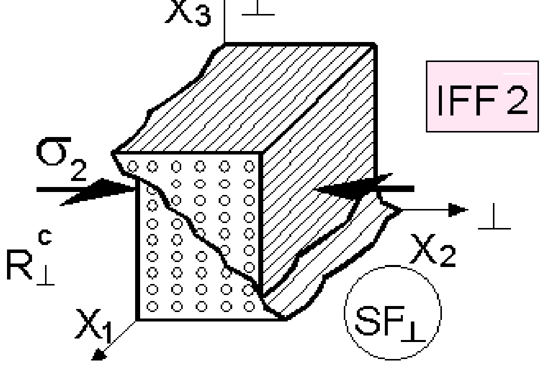
- ▶ **5 Fracture modes exist**

= 2 FF (Fiber Failure)

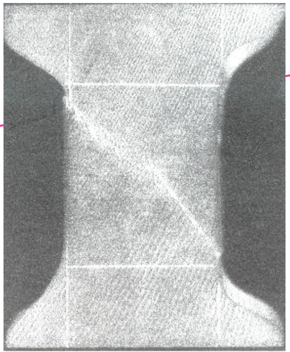
+ 3 IFF (Inter Fiber Failure)

critical in a loaded laminate:
FF1, FF2 + possibly IFF2 !

from the observations follow .. ⇨

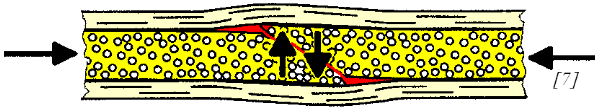


wedge type



NF := Normal Fracture

SF := Shear Fracture



Cuntze's **Set** of **Modal** 3D UD Strength Failure Conditions ('criteria')

Cuntze = Mises amongst the UD criteria

Invariants, replaced by their stress formulations

strains from FEA

[Cun04, Cun11]

FF1 $$Eff^{\parallel\sigma} = \breve{\sigma}_1 / \bar{R}^t_\parallel = \sigma^{\parallel\sigma}_{eq} / \bar{R}^t_\parallel , \qquad \breve{\sigma}_1 \cong \varepsilon^t_1 \cdot E_\parallel \; *$$

FF2 $$Eff^{\parallel\tau} = -\breve{\sigma}_1 / \bar{R}^c_\parallel = +\sigma^{\parallel\tau}_{eq} / \bar{R}^c_\parallel , \qquad \breve{\sigma}_1 \cong \varepsilon^c_1 \cdot E_\parallel$$

2 filament modes

IFF1 $$\boxed{Eff^{\perp\sigma}} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / 2\bar{R}^t_\perp = \boxed{\sigma^{\perp\sigma}_{eq} / \bar{R}^t_\perp}$$

IFF2 $$Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / \bar{R}^c_\perp = +\sigma^{\perp\tau}_{eq} / \bar{R}^c_\perp$$

3 matrix modes

IFF3 $$Eff^{\perp\parallel} = \{[\mu_{\perp\parallel} \cdot I_{23-5} + (\sqrt{\mu_{\perp\parallel}{}^2 \cdot I_{23-5}{}^2 + 4 \cdot \bar{R}_{\perp\parallel}{}^2 \cdot (\tau^2_{31} + \tau^2_{21})^2}\,] / (2 \cdot \bar{R}_{\perp\parallel}{}^3)\}^{0.5} = \sigma^{\perp\parallel}_{eq} / \bar{R}_{\perp\parallel}$$

with $$I_{23-5} = 2\sigma_2 \cdot \tau^2_{21} + 2\sigma_3 \cdot \tau^2_{31} + 4\tau_{23}\tau_{31}\tau_{21}$$

Interaction of modes:

$$Eff^{\,m} = (Eff^{\parallel\tau})^m + (Eff^{\parallel\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

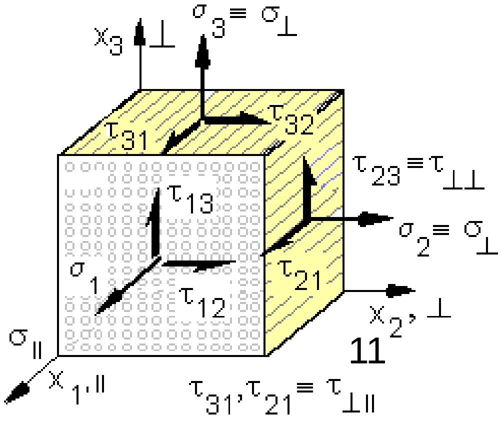
with mode-interaction exponent $2.5 < m < 3$ from mapping tests data

Typical friction value data range: $0.05 < \mu_{\perp\parallel} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$

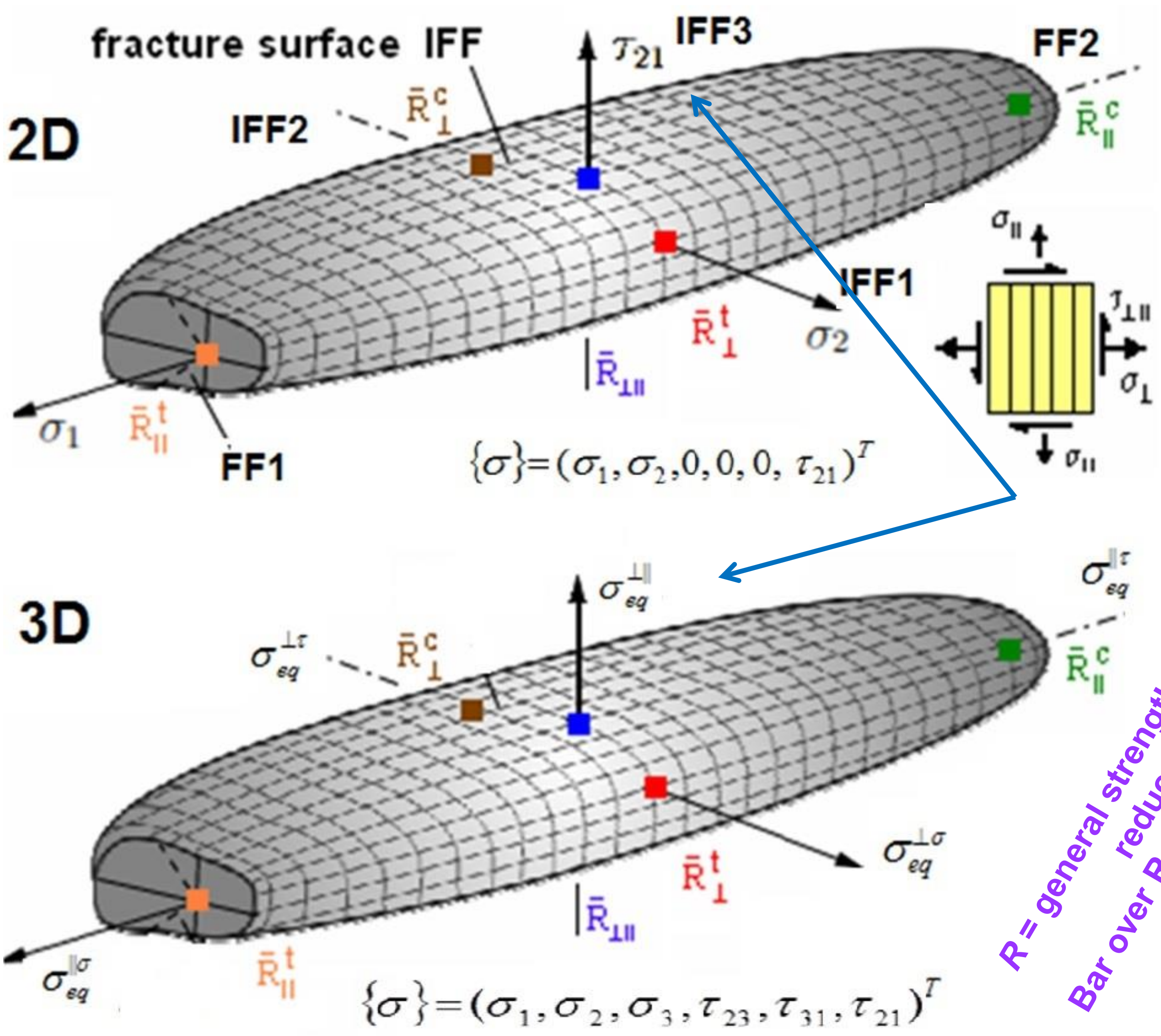
see [Pet16] for measurement

Poisson effect * : bi-axial compression strains the filament without any σ_1

t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre

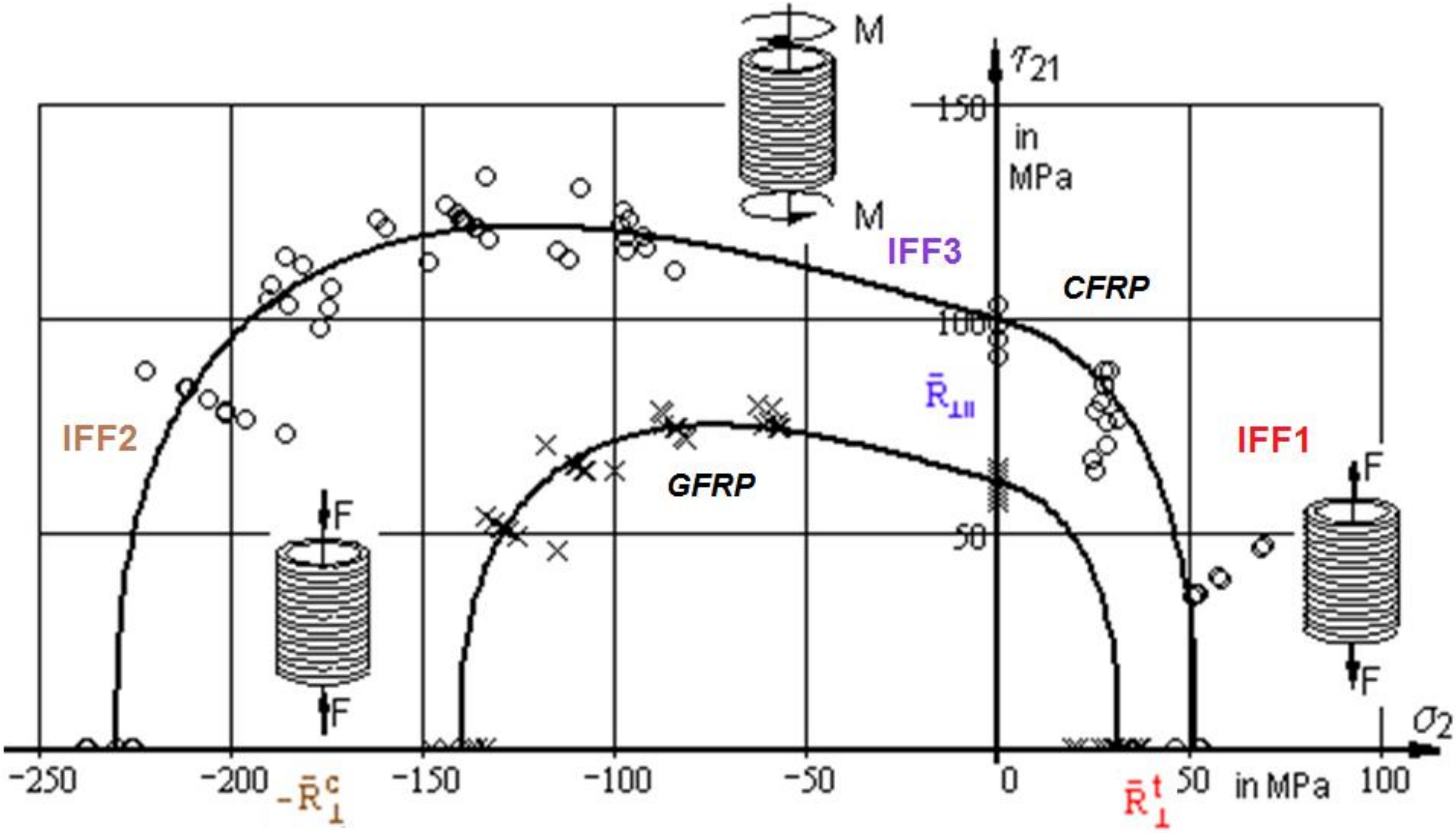


2D ⇨ 3D Fracture surface by replacing the stress by the equival. stress



R = general strength and statistically reduced 'strength design allowable'
Bar over R means average strength, applied when mapping .

IFF 1-2-3 Cross–section of the Fracture Failure Body (surface)



* Above tested were so-called isolated test specimens.

* For the presented fatigue approach embedded laminas are to consider!

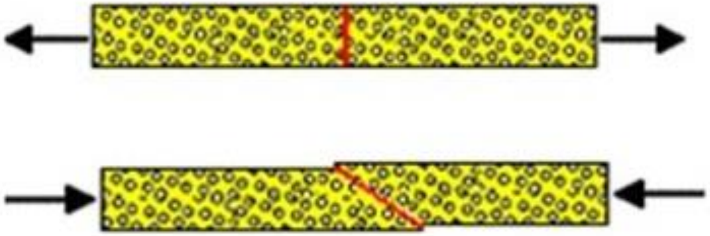
Isolated UD-material (generates hardening curve) and embedded (softening curve)

Isolated‘ lamina test specimens

= weakest link results (series failure system)

IFF 1 :

IFF 2 :



unconstrained lamina

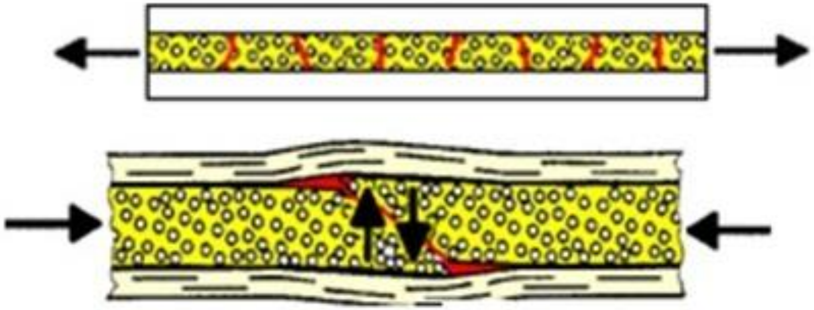
delivers strength property, stress-strain curve

(belongs to hardening)

delivers **basic strength**
as analysis input !

‘Embedded‘ laminas experience in-situ effects

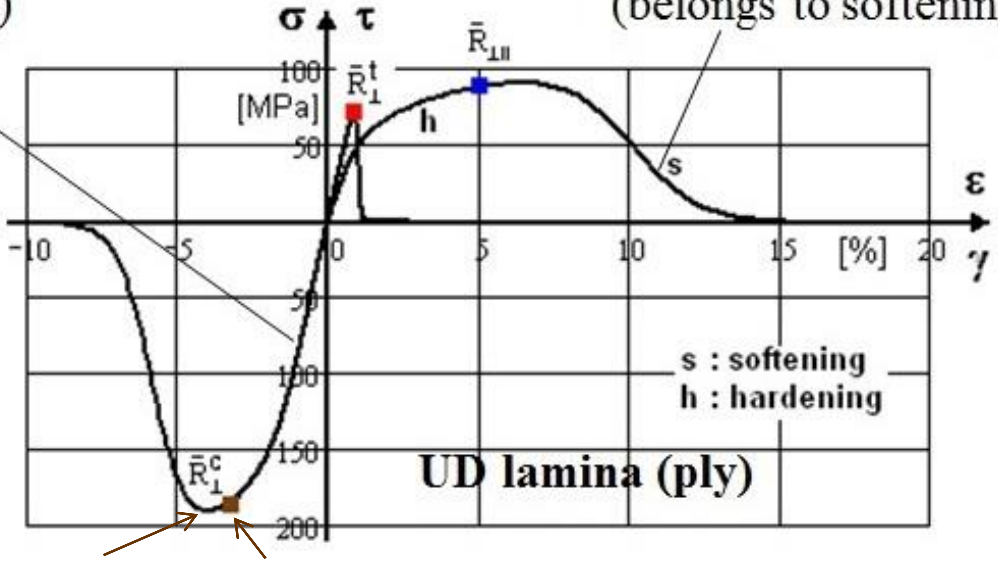
= redundancy result (parallel failure system)



mutually constrained laminas, in laminates

in non-linear laminate analysis

(belongs to softening)



In-situ strength *strength*

Self-explaining, symbolic Notations for Strength Properties

prepared by the author for ESA - Material Handbook

		Fracture Strength Properties									
	loading	**tension**			**compression**			**shear**			
	direction or **plane**	1	2	3	1	2	3	12	23	13	
9	**general orthotropic**	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	**friction properties**
5	**UD**	$R_{\parallel}^{t}$ NF	$R_{\perp}^{t}$ NF	$R_{\perp}^{t}$ NF	$R_{\parallel}^{c}$ SF	$R_{\perp}^{c}$ SF	$R_{\perp}^{c}$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$\mu_{\perp\perp}$, $\mu_{\perp\parallel}$,
6	**fabrics**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	**fabrics general**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	μ_{W3}, μ_{F3}, μ_{WF}
5	**mat**	R_{1M}^t	R_{1M}^t	R_{3M}^t	R_M^c	R_{1M}^c	R_{3M}^c	R_M^τ	R_M^τ	R_M^τ	*(UD, turned direction)*
2	**isotropic matrix**	R_m SF	R_m SF	R_m SF	*deformation-limited*			R_M^τ	R_M^τ	R_M^τ	μ
		R_m NF	R_m NF	R_m NF	R_m^c SF	R_m^c SF	R_m^c SF	R_m^σ NF	R_m^σ NF	R_m^σ NF	μ

NOTE: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m:= 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

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„Fatigue is the black art,

to produce financial black holes“

[J. Draper]

Therefore, in order to reduce very costly cyclic laminate test programs

the German Academic Research Group (BeNa), founded by the author in 2010,

aims at :

A failure mode-based Lifetime Prediction Method,
lamina-oriented on the embedded lamina in order to capture in-situ effects
and using failure mode-based S-N curves.

.

Fatigue Damaging Drivers of Ductile and Brittle behaving Materials

- **Ductile Material Behavior** (example: isotropic metal materials)

  1 damaging mechanism *acts = "slip band shear yielding"*

  drives damaging under cyclic tensile, compressive, shear and torsional stresses:

  Therefore, this single mechanism can be described by one single strength formulation:

  the Mises Yield failure condition!

- **Brittle Behaving Material Behavior :** isotropic Materials

  2 damaging driving mechanisms act = *Normal Fracture failure mode (NF), Shear Fracture failure (SF)*

- **Brittle Behaving UD Material Behavior :** transversely-isotropic UD Materials

  5 damaging driving Fracture failure mechanisms act ≡ 5 Fracture failure modes

Damaging Determination in Brittle behaving cyclically loaded Composites

Assumption:

If the failure mechanism (mode) is cyclically the same as in the static case *then*

- the damaging driving failure parameters are the same and

- the applicability of static stress failure criteria is allowed to quantify the damaging portions !

Measurable Damaging Quantities:

Micorcrackdensity, Residual Strength, Residual Stiffness

Which cyclic Quantities are required for Lifetime Estimation ?

- S-N curves for $R = const = \sigma_{min} / \sigma_{max}$ (= stress ratio)
- Hypothesis for the accumulation of the damaging portions
- Quantification of damaging portions (- increments) *by the*

  application of static fracture strength criteria, if

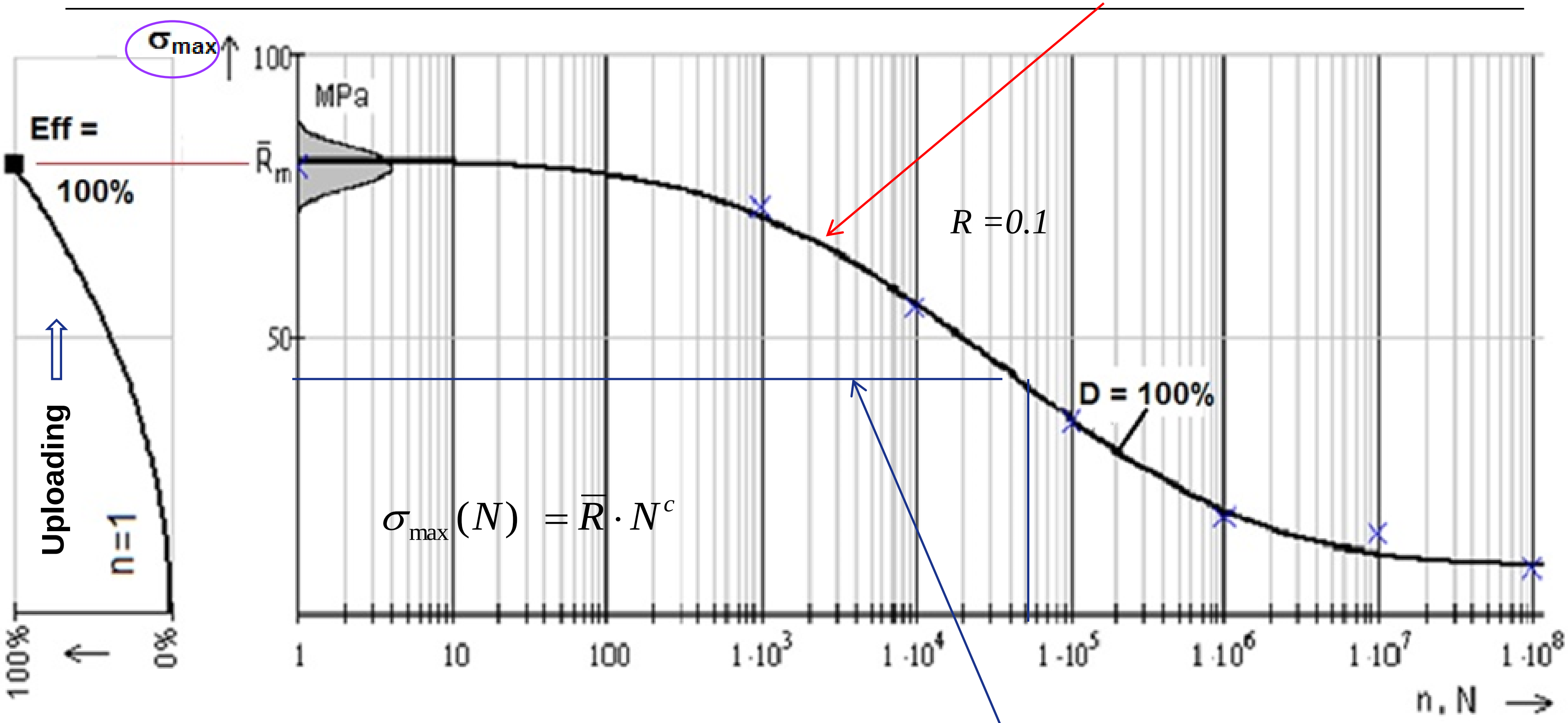
  static strength R_m is replaced by the residual strength $\sigma_{res}(N, R)$.

Thereby, the static material stressing effort *Eff* (Werkstoffanstrengungssumme)

is replaced by the accumulated cyclic damaging *D !*

The same letter is used as for the stress ratio R !

Static and cyclic development of damaging, S-N-curve brittle material



Static

R_m basic strength

Cyclic

residual strength $\sigma(R, N)$

Analogous limits of the material capacities :

- Static : material stressing effort Eff (N=1) = 100 %
- Cyclic : material damaging sum D (N) = 100 %

= *sum of damaging portions*

When designing brittle behaving materials the use of σmax is advantageous compared to amplitude Δσ !

Steps in Fatigue Life Estimation

1 Input

Operational Loading: Load time curves (modeling rain flow, ..)

Time domain: Cycle-by-cycle or block-by-block (less computation effort)

Frequency dom.: Load spectra (loss of Reihenfolge)

Safety Concept: Design safety factor Life $j_{Life} = 3 – 10$, or

an Inspection interval, or an replacement time approach

2 Transfer of operational loadings into stresses using structural analysis

3 Domains of Fatigue Analysis

LCF: high stressing,

HCF: intermediate stressing 10.000 < n < 1.000.000, *rotor tube*

VHCF: low stressing and strains (SPP1466) > 10.000.000 *centrifuges, wind rotor blades*

4 Provision of Haigh Diagrams which involve all necessary S-N curves with Generation of Constant Fatigue Life (CFL) curves

3 Haigh Diagrams are provided : FF1-FF2, IFF1-IFF2, IFF3

.

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In practice, usually , Haigh Diagrams are partly composed by straight lines. Is this reasonable especially for UD materials ?

AIM:

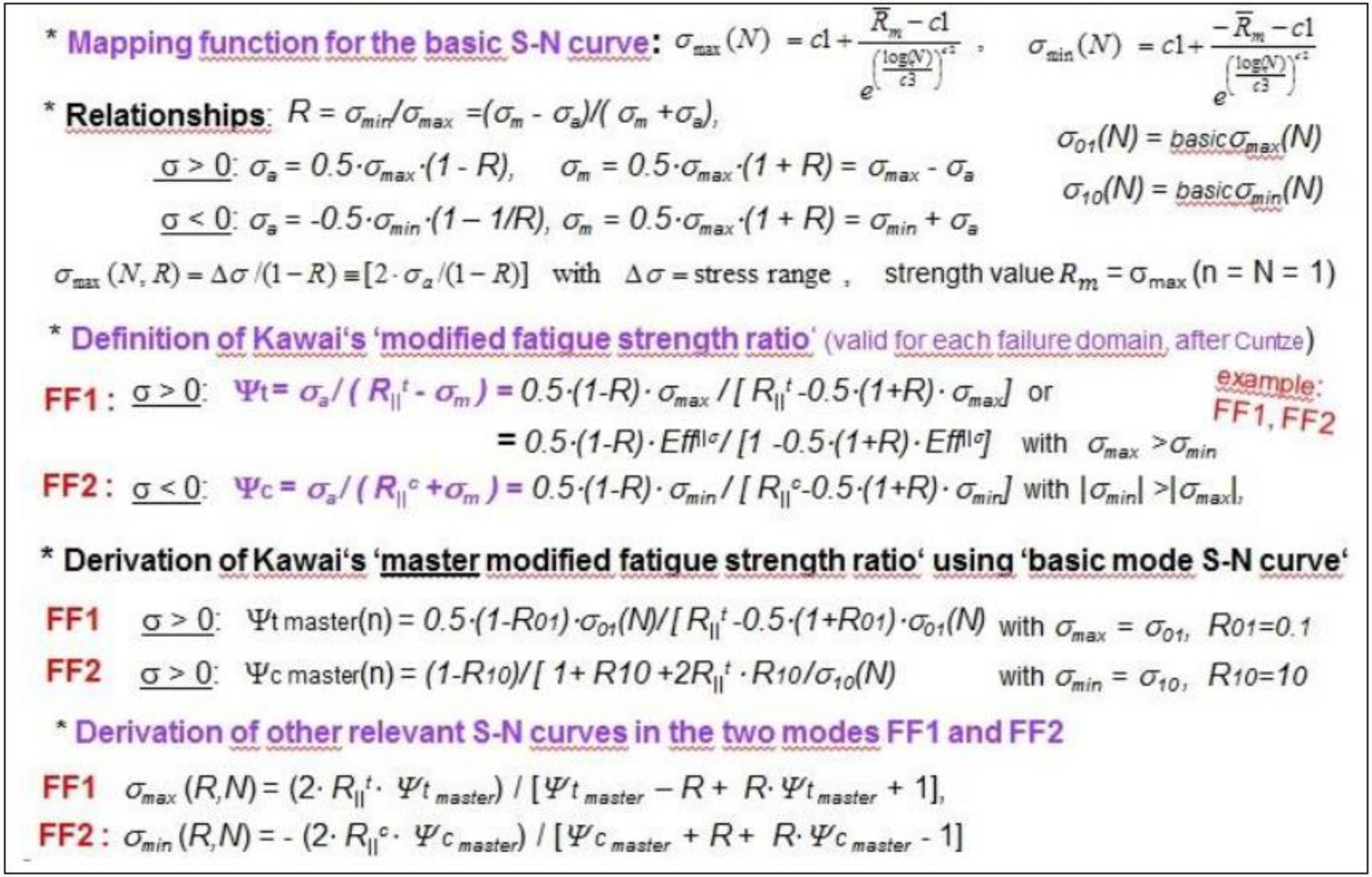
Automatic Establishment of the curved Constant Life Curves (CFL) in Haigh Diagrams *on basis of*

- a measured basic mode-linked S-N curve plus
- a model (Kawai) to predict other necessary S-N curves.

Kawai's Modified Fatigue Strength Ratio Ψ mode-linked applied to predict further necessary S-N curves on basis of one measured mode S-N curve $R = 0.1$ (NF), 10 (SF)

Searched model

Table : Formulas to map the basic S-N curve and Kawai's Master Ψ-model



* **Mapping function for the basic S-N curve:** $\sigma_{max}(N) = c1 + \frac{\overline{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$, $\sigma_{min}(N) = c1 + \frac{-\overline{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$

* **Relationships**: $R = \sigma_{min}/\sigma_{max} = (\sigma_m - \sigma_a)/(\sigma_m + \sigma_a)$,

$\underline{\sigma > 0}$: $\sigma_a = 0.5 \cdot \sigma_{max} \cdot (1 - R)$, $\sigma_m = 0.5 \cdot \sigma_{max} \cdot (1 + R) = \sigma_{max} - \sigma_a$

$\underline{\sigma < 0}$: $\sigma_a = -0.5 \cdot \sigma_{min} \cdot (1 - 1/R)$, $\sigma_m = 0.5 \cdot \sigma_{max} \cdot (1 + R) = \sigma_{min} + \sigma_a$

$\sigma_{01}(N) = basic\,\sigma_{max}(N)$

$\sigma_{10}(N) = basic\,\sigma_{min}(N)$

$\sigma_{max}(N, R) = \Delta\sigma/(1 - R) \equiv [2 \cdot \sigma_a/(1 - R)]$ with $\Delta\sigma$ = stress range , strength value $R_m = \sigma_{max}$ (n = N = 1)

* **Definition of Kawai's 'modified fatigue strength ratio'** (valid for each failure domain, after Cuntze)

FF1 : $\underline{\sigma > 0}$: $\Psi t = \sigma_a / (R_{||}^t - \sigma_m) = 0.5 \cdot (1-R) \cdot \sigma_{max} / [R_{||}^t - 0.5 \cdot (1+R) \cdot \sigma_{max}]$ or

$= 0.5 \cdot (1-R) \cdot Eff^{||\sigma} / [1 - 0.5 \cdot (1+R) \cdot Eff^{||\sigma}]$ with $\sigma_{max} > \sigma_{min}$

FF2 : $\underline{\sigma < 0}$: $\Psi c = \sigma_a / (R_{||}^c + \sigma_m) = 0.5 \cdot (1-R) \cdot \sigma_{min} / [R_{||}^c - 0.5 \cdot (1+R) \cdot \sigma_{min}]$ with $|\sigma_{min}| > |\sigma_{max}|$,

example: FF1, FF2

* **Derivation of Kawai's '<u>master</u> modified fatigue strength ratio' using 'basic mode S-N curve'**

FF1 $\underline{\sigma > 0}$: $\Psi t\,master(n) = 0.5 \cdot (1-R01) \cdot \sigma_{01}(N) / [R_{||}^t - 0.5 \cdot (1+R01) \cdot \sigma_{01}(N)$ with $\sigma_{max} = \sigma_{01}$, $R01 = 0.1$

FF2 $\underline{\sigma > 0}$: $\Psi c\,master(n) = (1-R10)/[1 + R10 + 2R_{||}^t \cdot R10/\sigma_{10}(N)$ with $\sigma_{min} = \sigma_{10}$, $R10 = 10$

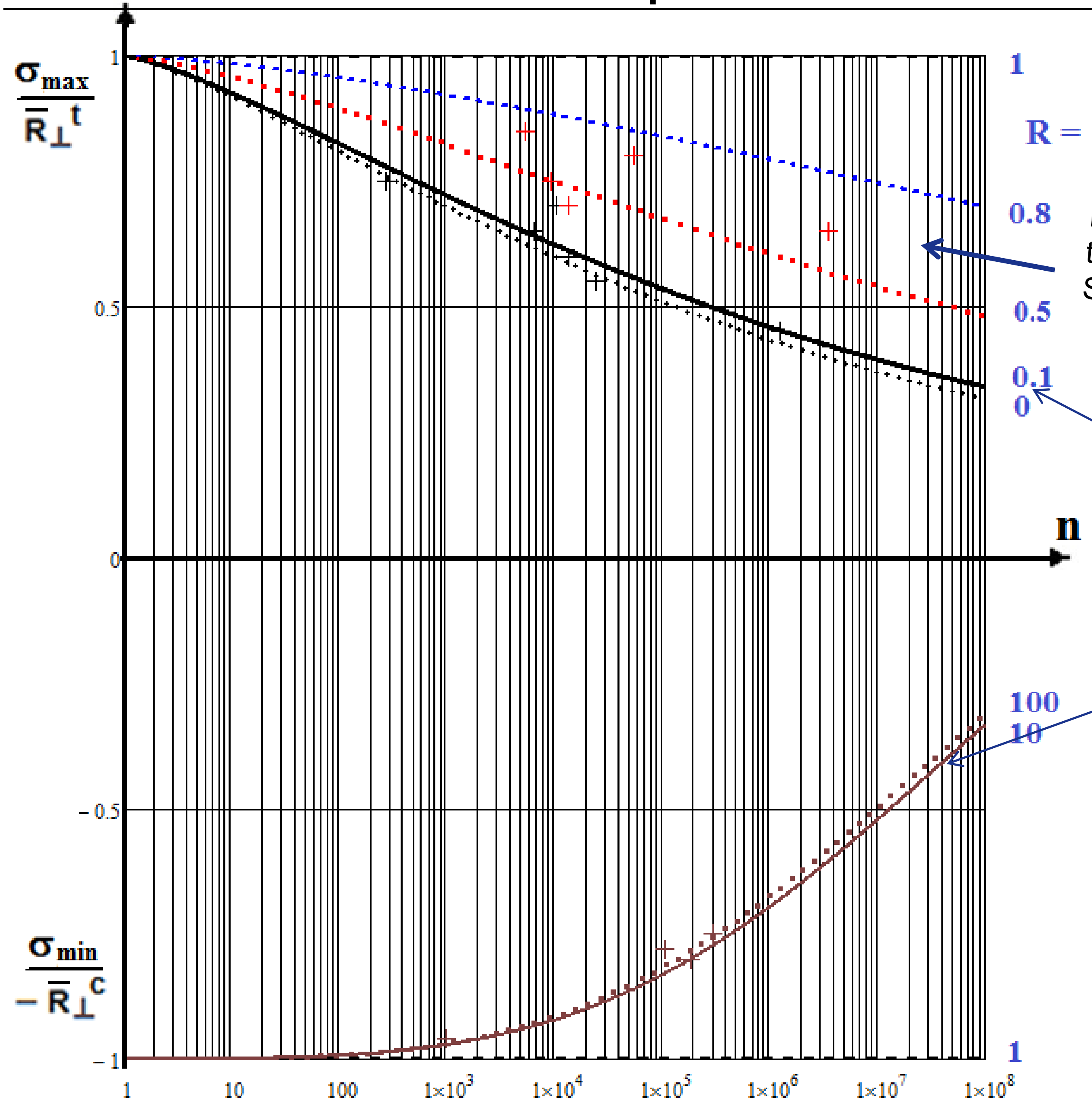
* **Derivation of other relevant S-N curves in the two modes FF1 and FF2**

FF1 $\sigma_{max}(R,N) = (2 \cdot R_{||}^t \cdot \Psi t_{master}) / [\Psi t_{master} - R + R \cdot \Psi t_{master} + 1]$,

FF2 : $\sigma_{min}(R,N) = -(2 \cdot R_{||}^c \cdot \Psi c_{master}) / [\Psi c_{master} + R + R \cdot \Psi c_{master} - 1]$

S-N curve modelled non-linearly

How look Kawai model-predicted ‘Mode S-N curves ?



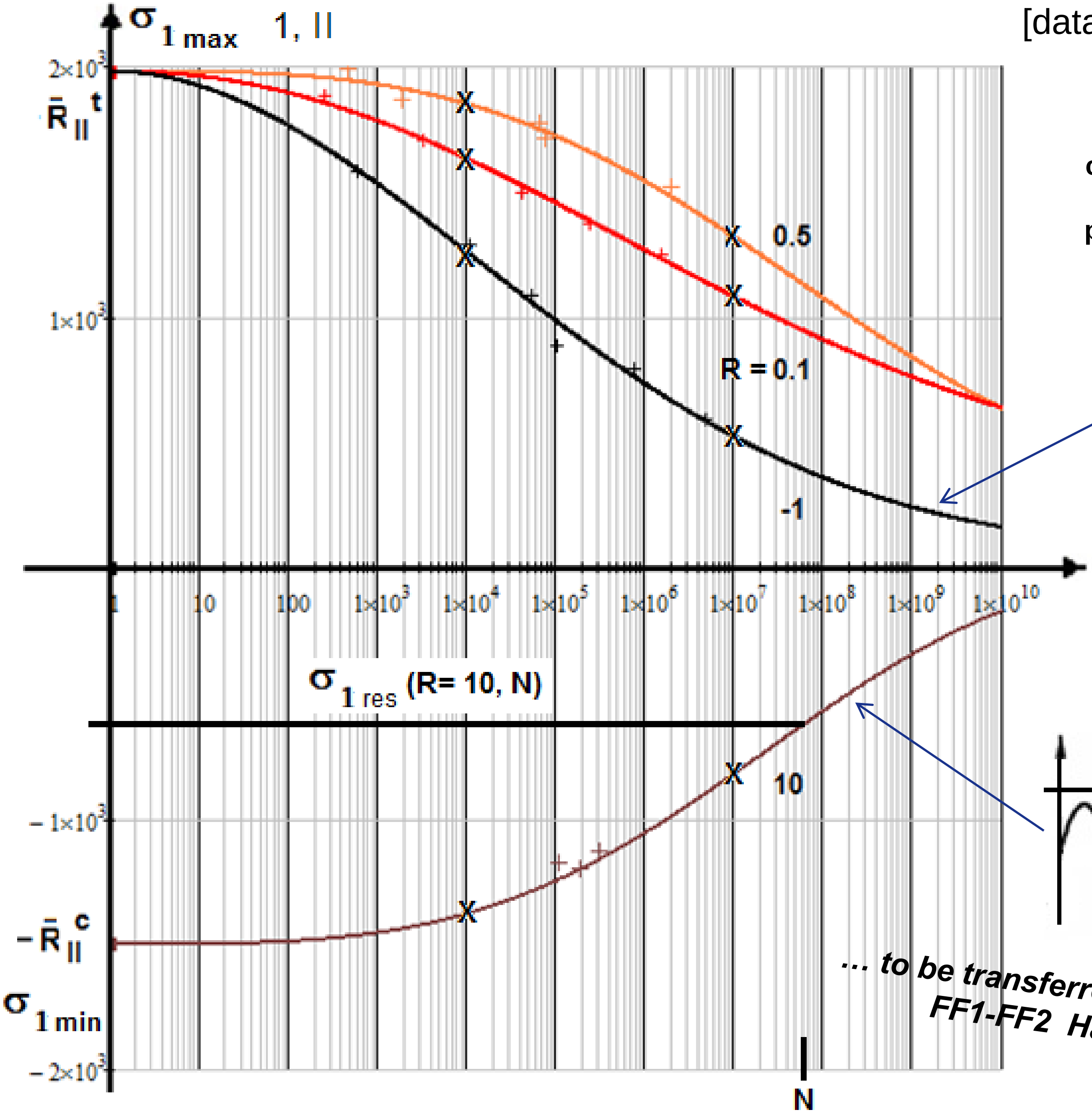
Example *<u>IFF1, IFF2</u>*‘

In the intermediate R-regime, the Kawai model-predicted S-N curves are conservative

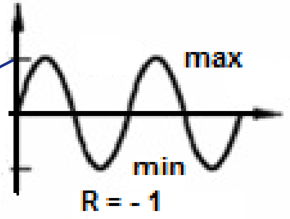
on basis of the two given ‘<u>basic</u> mode S-N curves‘ 0.1, 10 ?

Individually lin-log mapped **FF1-FF2**-linked S-N curves

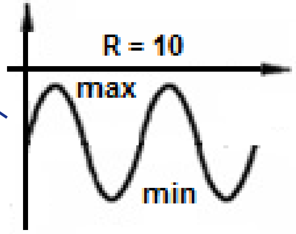
[data, courtesy Kawai-Suda]



From the mapped test data curve analytically determined are anchor points X for the prediction of constant fatigue life (CFL) curves



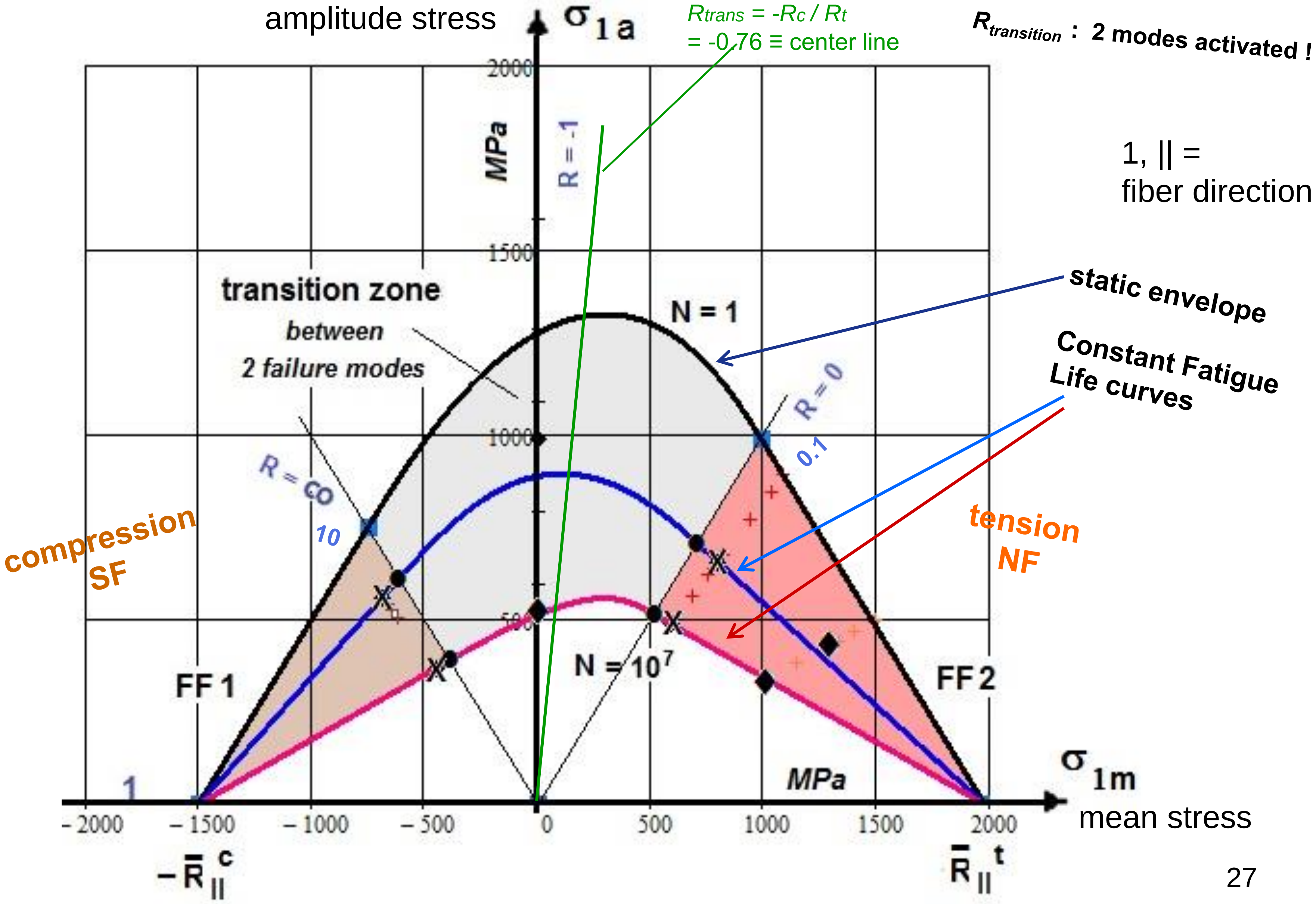
$$\sigma_{\max}(N) = c1 + \frac{\bar{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$$



… to be transferred into the FF1-FF2 Haigh Diagram

Novel Interpretation of the Haigh diagram: example FF1-FF2 UD

displaying the failure mode domains, transition zone test data [Hah14]



$R_{trans} = -R_c / R_t$
$= -0.76$ ≡ center line

$R_{transition}$: 2 modes activated !

1, || =
fiber direction

static envelope

Constant Fatigue Life curves

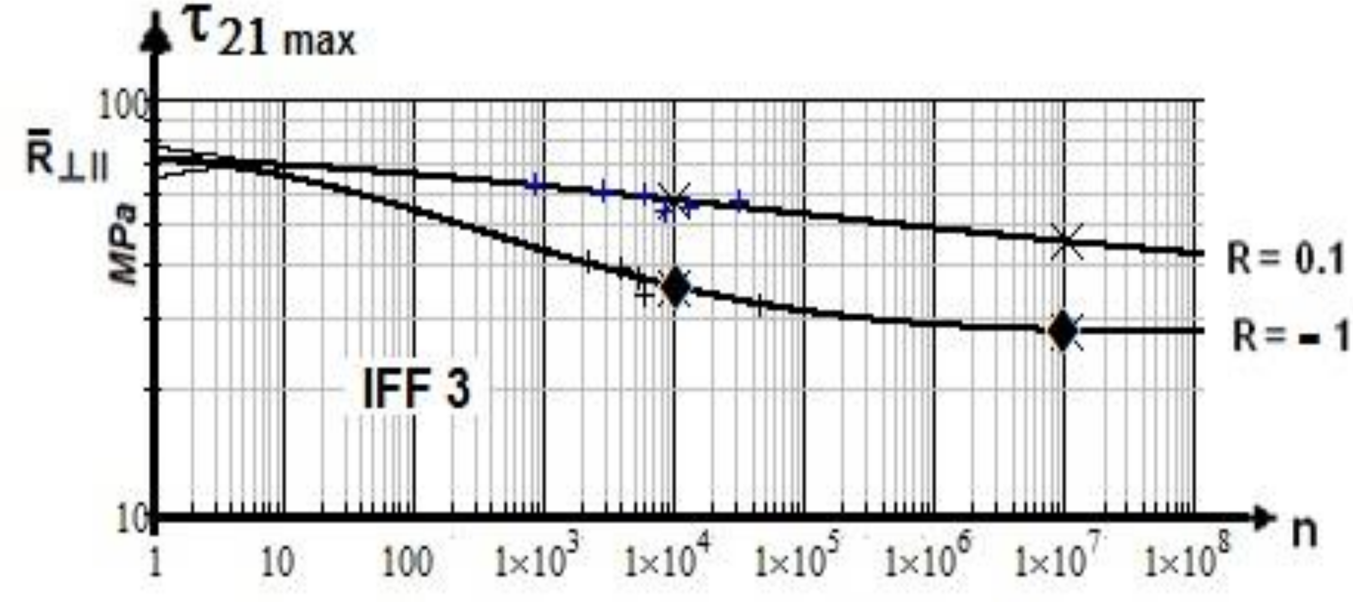
compression
SF

tension
NF

NF = Normal Fracture, SF = Shear Fracture, *N* = fracture cycle number, CFL = Constant Fatigue Life

Individually mapped log-log IFF3-linked S-N curves

[data, courtesy C. Hahne]



21, ⊥|| =
inplane shear

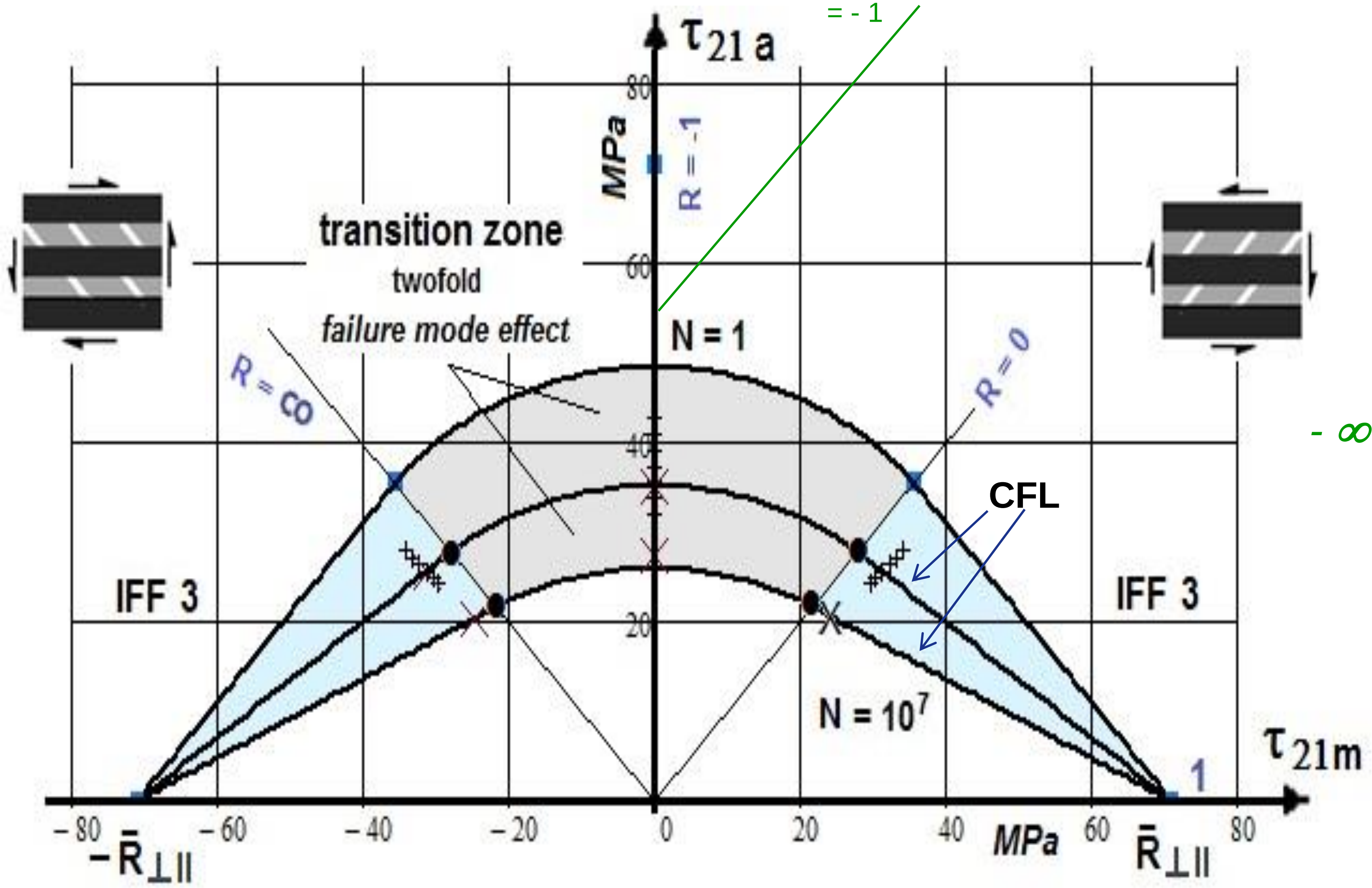
.. to be transferred into the IFF3 Haigh Diagram

IFF3 UD Haigh diagram,

Display of a two-fold mode effect (a:= amplitude, m:= mean, N := number of fracture cycles, R := strength and R := $\sigma_{min}/\sigma_{max}$). Test data CF/EP, courtesy [Hah14]

$R_{trans} = -R_c / R_t = -1$

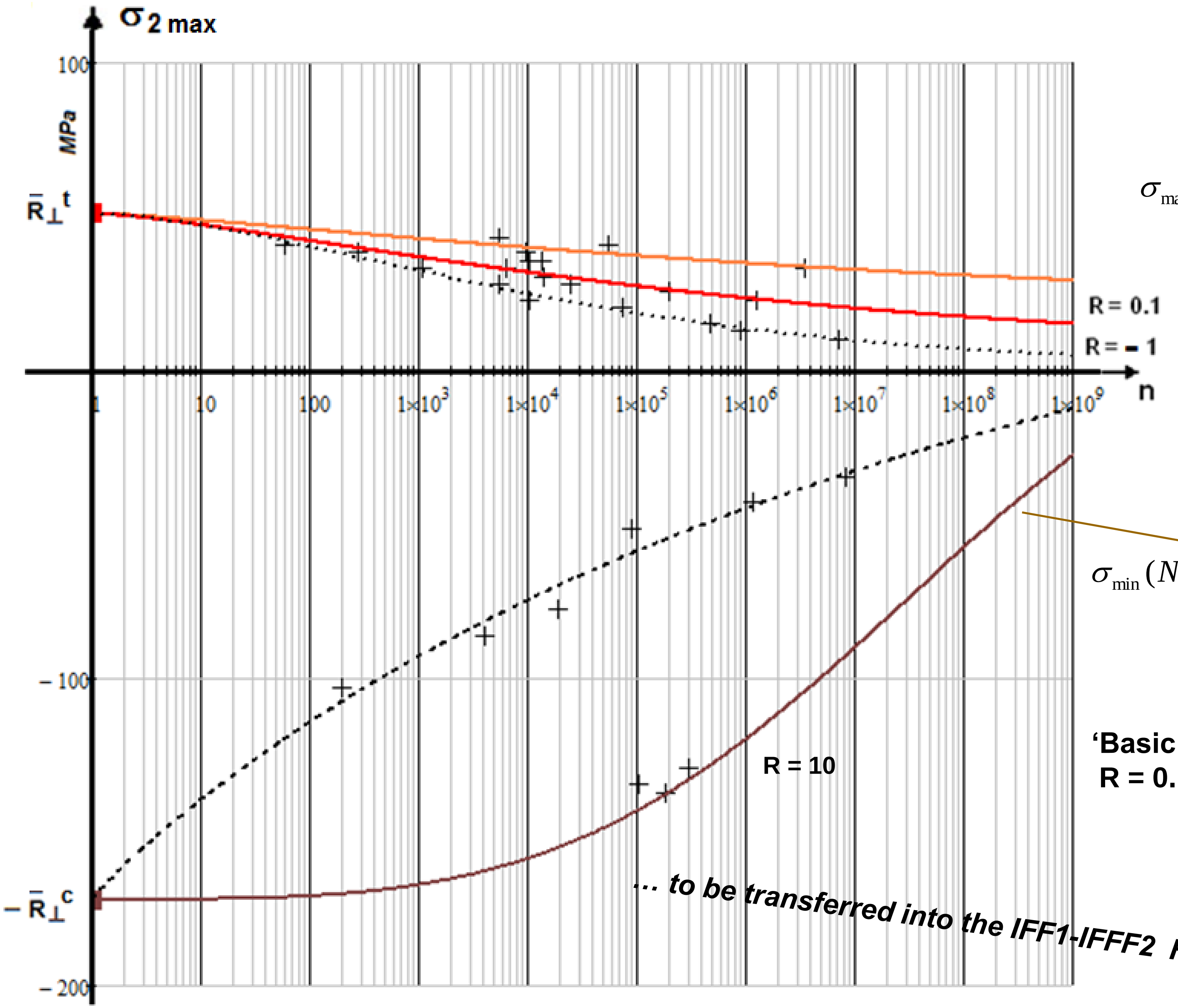
21, ⊥|| = inplane shear



$-\infty < R_{trans} < 0$

CFL

Mapped lin-log IFF1-IFF2-linked S-N curves *[data, courtesy C. Hahne]*



2, ⊥ = across fiber direction

$$\sigma_{\max}(N) = c1 + \frac{\overline{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$$

$$\sigma_{\min}(N) = c1 + \frac{-\overline{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$$

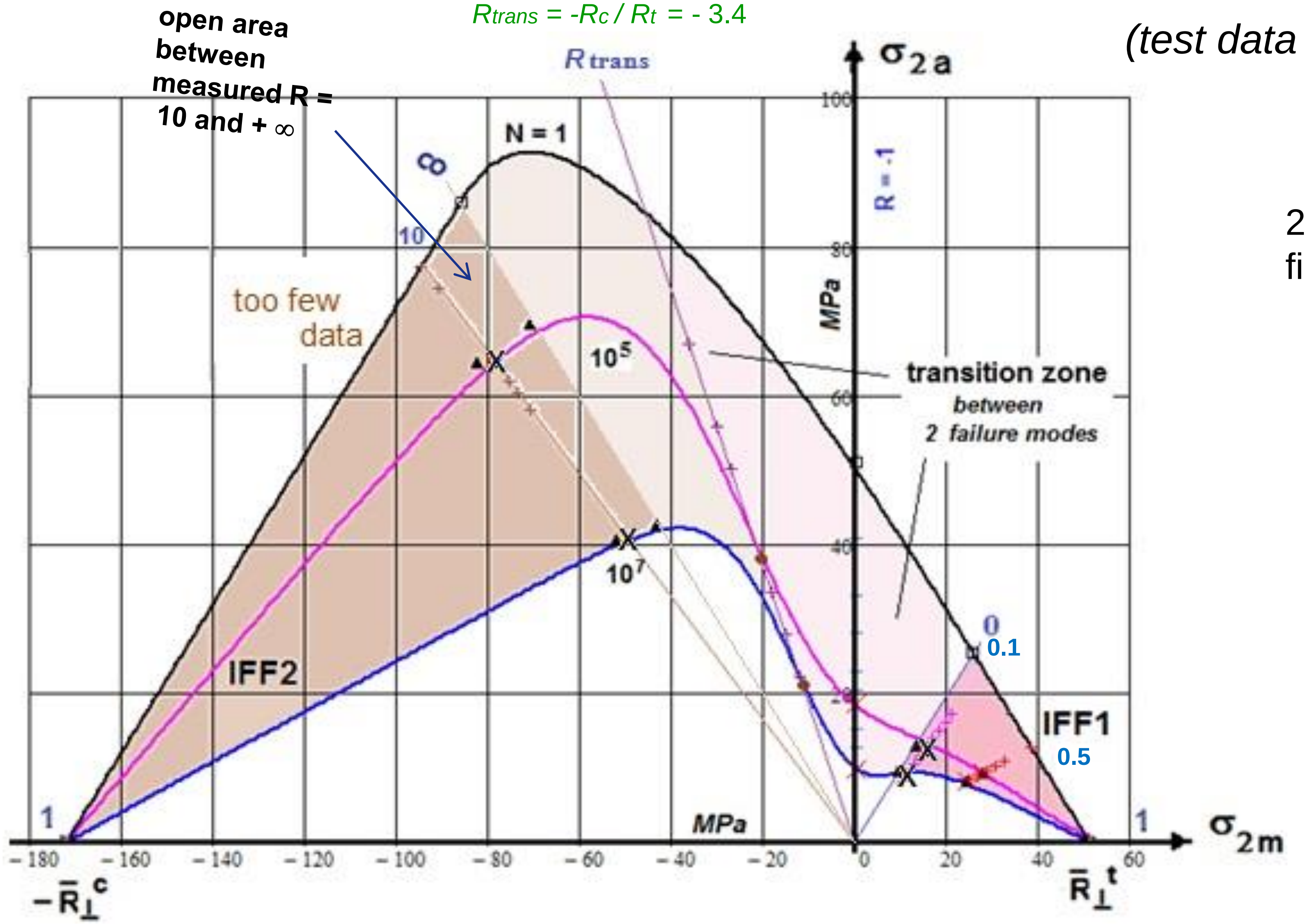
‘Basic mode S-N curves‘ :
R = 0.1 and R = 10

… to be transferred into the IFF1-IFFF2 Haigh Diagram

IFF1-IFF2 UD Haigh diagram

displaying the failure mode domains, transition zone

(test data [C. Hahne])



open area between measured R = 10 and + ∞

$R_{trans} = -R_c / R_t = -3.4$

2, ⊥ = across fiber direction

- ***Curve in the IFF1 domain looks non-linear !***
- ***Check points from Ψ-prediction lie higher than points from S-N test data evaluation***

(The computed S-N curve X-points are anchor (checking) points for the to be predicted CFL curves)

1 Introduction to Static and Fatigue Design
2 Cuntze’s Failure-Mode-Concept-based Strength Criteria
3 Cuntze’s Fatigue Life Estimation Concept
4 Generation and Novel Interpretation of UD Haigh Diagrams
5 Steps of the Full Fatigue Life Prediction Method Proposed

5 Objectives of the Proposed Method

An engineering-like, failure modes-linked lifetime prediction for plain laminates which involves the following topics:

1.) Failure mode-linked *modelling* of the cyclic loading (novel idea)

2.) Measurement of just a minimum number of the failure mode-representative *mode S-N curves* = master R-curve of each mode

3.) Prediction of other necessary stress-ratio ‘*mode S-N curves*’ on basis of the measured mode master curve one (e.g. R=0.5 *from* R=0.1) plus Kawai‘s Model the ‘Modified Fatigue Strength Ratio’

4. Determination of Damaging portions on basis of the static UD strength criteria considering the residual strength $R_{||}(R,N)$)

5.) Failure mode-linked accumulation of Damaging Portions (novel idea) using Palmgren-Miner

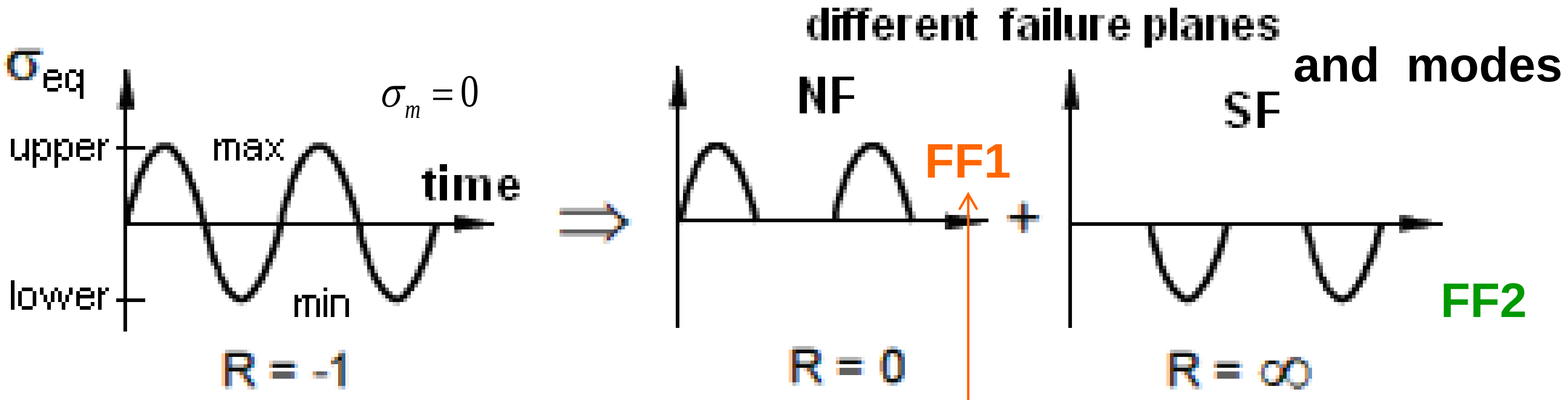
visualized

Novel failure mode-wise modelling of Loading Cycles for high-performance ‘fiber-dominated designed’, UD laminas-composed laminates

---

For simply displaying the approach *it is chosen* : - ***R = -1 loading***

Separation due to the activated inherent different failure modes



NF := Normal Fracture, SF := Shear Fracture

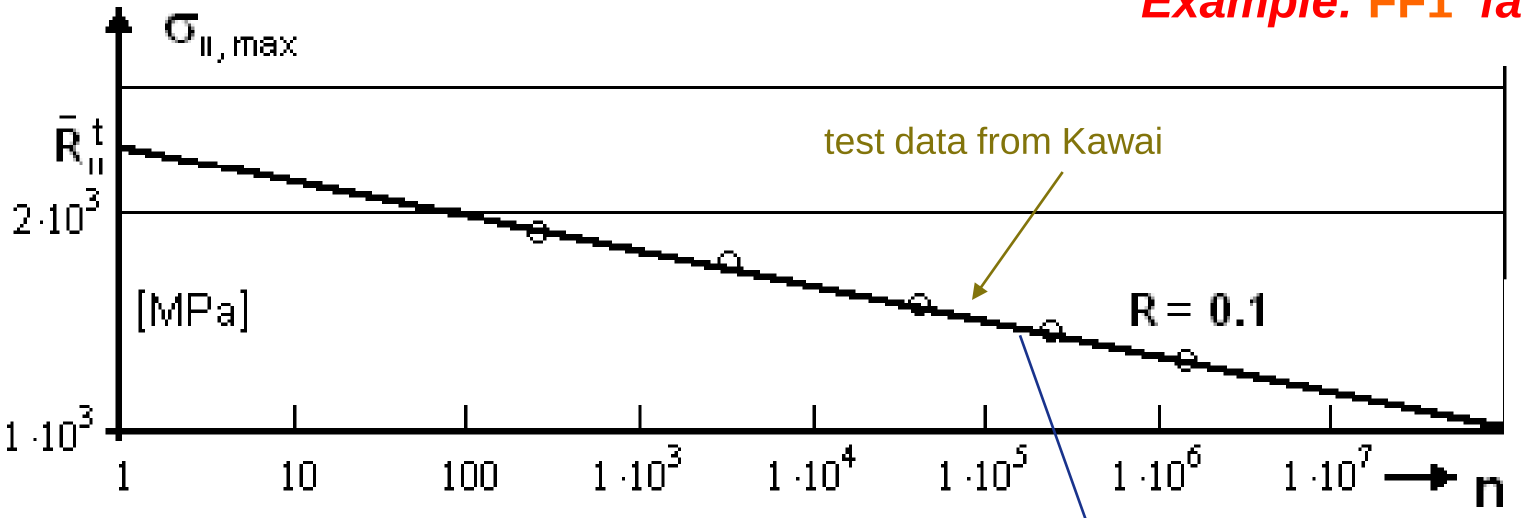
Step 1 : <u>Failure mode-linked</u> apportionment of cyclic loading (<u>novel</u>)

In the hoop-wound, strength capacity delivering layers of the rotor tube the failure mode FF1 is the significant one .

Specific **rain-fall** procedure must be applied

Mapping of S-N data and mode-representative ‘basic‘ *S-N curve*

Example:* FF1 *failure mode



Step 2 **: S-N curve can be mapped, e.g. by a straight * line, in *log-log* graph**

$$\sigma_{\parallel,\ \max}^{Master}(n) \approx \bar{R}_{\parallel}^{t} \cdot n^{c_{Master}}$$

Measured curve used

as mode-representative ***Basic S-N curve*** **for FF1**

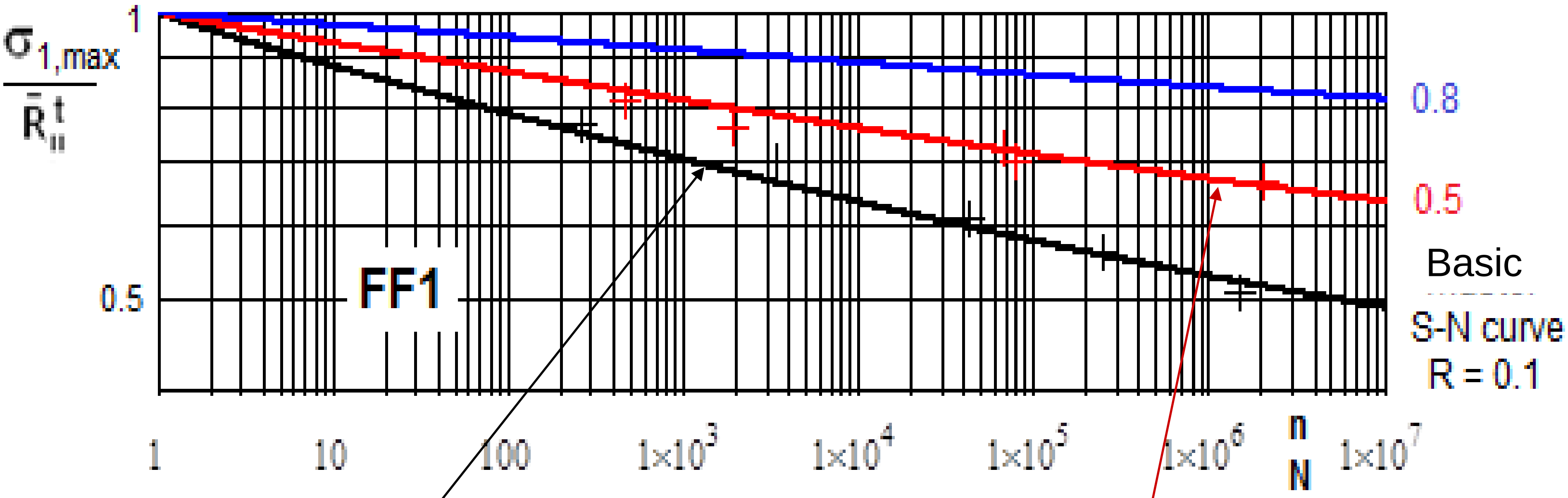
FF1 strength

In the general case of variable loading ⟹ Several S-N-curves are needed !

**** more complicated S-N models are also applied !***

Prediction of needed other FF1 S-N curves from

Basic Mode S-N curve and Kawai model (Ψ Curve)



Given : $\sigma_{\parallel,\ \max}^{Basic}(n) \approx \bar{R}_{\parallel}^{t} \cdot n^{c_{Basic}}$

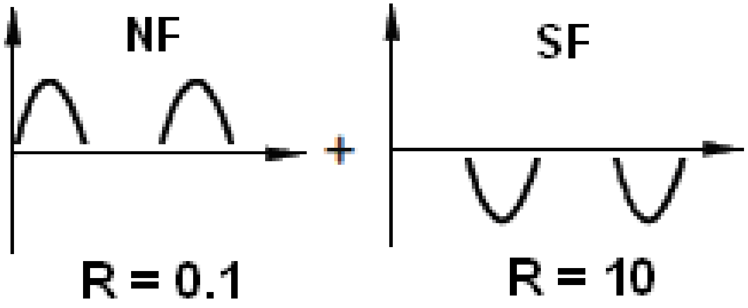
Searched : $\sigma_{\parallel,\max}^{pred} = \bar{R}_{\parallel}^{t} \cdot n^{c_{pred}}$

Slope of R = 0.5 ?

Step 3: Application of Kawai‘s ‘Modified Fatigue Strength Ratio‘.

Application of Miner-‘Rule‘ , for a *simple* loading *example* R = -1

FF1 **FF2**



$$D\ (FF1, FF2) = NF : (n_1 / N_1 + n_2 / N_2 + n_3 / N_3) + SF : (n_4 / N_4)$$

$$\Rightarrow D = D\ (FF1, FF2) + D\ (IFF1, IFF2, IFF3)\ \leq\ D_{feasible}$$

from test experience

Step 4: Determination of Damaging Portions by Static Strength Criteria

Step 5: Mode-wise Accumulation of Damaging Portions (novel)

What was the main Objective of this Investigation ?

For high performance composite parts:
Fatigue pre-dimensioning of
'well-designed', UD laminas-composed laminates
just by single lamina-dedicated mode-representative basic S-N curves,
derived from *sub-laminate* test specimens,
which capture the embedded ply (in-situ) effects,
<u>and</u> on model-predicted (Kawai model), further necessary S-N curves
implemented in automatically constructed CFL curves of the Haigh diagrams.

Everything in the world is terminated by **chance** and **fatigue**.

Heinrich Heine

Thank You !

Literatur

download possible from **carbon~connected.de/Group/CCeV.Fachinformationen/Mitglieder**

[Ban16] Bansemir H.: *Certification Aspects*. Extended Abstract EC16, Augsburg 21-23. September 2016, conference publication

[Cun04] Cuntze R.: *The Predictive Capability of Failure Mode Concept-based Strength Criteria for Multidirectional Laminates*. WWFE-I, Part B, Comp. Science and Technology 64 (2004), 487-516

[Cun05] Cuntze R: *Is a costly Re-design really justified if slightly negative margins are encountered?* Konstruktion, März 2005, 77-82 and April 2005, 93-98 (reliability treatment of the strength problem)

[Cun09] Cuntze R.: *Lifetime Prediction for Structural Components made from Composite Materials – industrial view and one idea*. NAFEMS World Congress 2009, Conference publication.

[Cun12] HSB02000-01 *Essential topics in the determination of a reliable reserve factor*. 20 pages

[Cun13] Cuntze R.: *Comparison between Experimental and Theoretical Results using Cuntze's 'Failure Mode Concept' model for Composites under Triaxial Loadings - Part B of the WWFE-II*. Journal of Composite Materials, Vol.47 (2013), 893-924

[Cun13] Cuntze R.: *Tackling Uncertainties in Design – uncertain design parameters, safety concept, modelling and analysis.* Verbundwerkstoffe, GDM, 18. Symposium, Chemnitz 30.3. – 1.4. 2011

[Cun14] Cuntze R.: *The World-Wide-Failure-Exercises-I and –II for UD-materials – valuable attempts to validate failure theories on basis of more or less applicable test data*. SSMET 2014, Braunschweig, April 1 – 4, 2014, conference handbook

[Cun15] Cuntze, R: *Reliable Strength Design Veriftcation - fundamentals, requirements and some hints*. 3rd. Int. Conf. on Buckling and Postbuckling Behaviour of Composite Laminated Shell Structures, DESICOS 2015, Braunschweig, March 26 ~27, Extended Abstract, Conf. Handbook, 8 pages

[Cun16a] Cuntze R.: *Introduction to the Workshop – from Design Dimensioning via Design Verification to Product Certification*, Extended Abstract, Experience Composites (EC) 16, Symposium Handbook

[Cun16b]: *Classical Laminate Theory (CLT) for laminates composed of unidirectional (UD) laminae, analysis flow chart, and related topics*. Reworked HSB 37103-01, Draft, 2016, 58 pages

[Gai16] Gaier C., Dannbauer H., Maier J. and Pinter G.: *Eine Software-basierte Methode zur Betriebsfestig-keitsanalyse von Strukturbauteilen aus CFK*. CCeV-Austria, AG Engineering, Meeting St. Martin im Innkreis, Sept.8. Magna-Powertrain, Engineering Center Steyr

[Hah15] Hahne C.: *Zur Festigkeitsbewertung von Strukturbauteilen aus Kohlenstofffaser- Kunststoff-Verbunden unter PKW-Betriebslasten*. Shaker Verlag, Dissertation 2015, TU-Darmstadt, Schriftenreihe Konstruktiver Leichtbau mit Faser-Kunststoff-Verbunden, Herausgeber Prof. Dr.-Ing Helmut Schürmann

[HSB] German Aeronautical Technical Handbook *'Handbuch für Strukturberechnung*', issued in English by Industrie-Ausschuss-Struktur-Berechnungsunterlagen. TIB Hannover

[Kad13]Kaddour A. and Hinton M.: *Maturity of 3D failure criteria for fibre-reinforced composites: Comparison between theories and experiments*. Part B of WWFE-II, J. Compos. Mater. 47 (6-7) (2013) 925–966.

[Kaw04] Kawai M.: *A phenomenological model for off-axis fatigue behaviour of uni-directional polymer matrix composites under different stress ratios*. Composites Part A 35 (2004), 955-963

[Koc16] Koch I., Horst P. and Gude M.: *Fatigue of Composites – The state of the art*. EC16, Extended Abstract EC16, Augsburg 21-23. September 2016, conference publication

[Pet15] Petersen E., Cuntze R. and Huehne C.: *Experimental Determination of Material Parameters in Cuntze's Failure-Mode-Concept -based UD Strength Failure Conditions.* Submitted to Composite Science and Technology 134, (2016), 12-25

[Puc02] Puck A. and Schürmann H.: *Failure Analysis of FRP Laminates by Means of Physically based Phenomenological Models.* Composites Science and Technology 62 (2002), 1633-1662

[Rac87] Rackwitz R. and Cuntze R..: *System Reliability Aspects in Composite Structures*. Engin. Optim., Vol. 11, 1987, 69-76

[Roh14] Rohwer K.: *Predicting Fiber Composite Damage and Failure*. Journal of Composite Materials, published online 26 Sept. 2014 (online version of this article can be found at: http://jcm.sagepub.com/content/early/2014/09/26/0021998314553885)

[Sch06] Schürmann H.: *Konstruieren mit Faser-Kunststoff-Verbunden*. Springer-Verlag 2005

[Schu??] Schulte K.:

[Sho06] Shokrieh M.M. and Tahery-Behroz F.: *A unified fatigue model based on energy method.* Composite Structures 75 (2006), 444-450

[VDI2014] VDI 2014: German Guideline, Sheet 3 *Development of Fibre-Reinforced Plastic Components, Analysis*. Beuth Verlag, 2006. (in German and English, author was convenor)

[Wei11] Weinert A. and Gergely P.: *Fatigue Strength Surface: basis for structural analysis under dynamic loads*. CEAS Aeronautical Journal 2011, Vol.2, Issue 1, 243-252

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- **1968, Dr.-Ing.** **Structural Dynamics** (TU Hannover)
- **1968 – 1970, DLR** **FEA-programming**
- **1970 – 2004, MAN-Technologie: Development of Structures**
- **1978, Dr.-Ing. habil.** **Mechanics of Lightweight Structures** (TUM)
- **2004 - 2009** **working on multiple ESA/ESTEC Standards**
- **1972 – 2015** **contributor to the Aerospace Hdbk LTH-HSB, various ESA-Specs, VDI 2014**
- **since 2009** **with Carbon Composites e.V. Head of the**

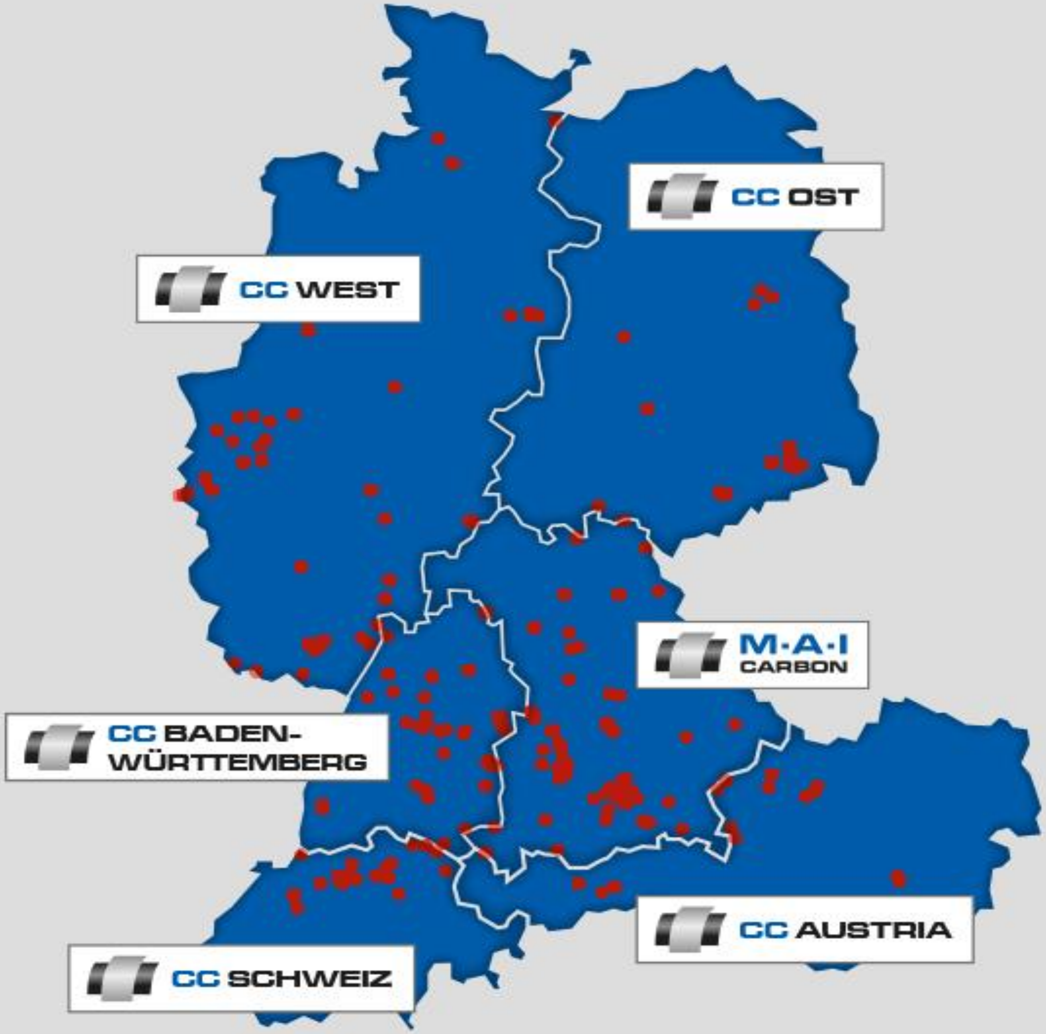
Working Groups

‛Engineering' ***in Mechanical Engineering*** and

‘Design Dimensioning and Design Verification‘ in ***Civil Engineering***

Das Kompetenz-Netzwerk Carbon Composite e.V. (CCeV)

280 Mitglieder



DACH - Netzwerk

CARBON COMPOSITES

Regionalabteilungen

CC BADEN-WÜRTTEMBERG	2014
M·A·I CARBON	2012
CC OST	2012
CC WEST	2016
CC AUSTRIA	2012
CC SCHWEIZ	2012

DACH-weite Fachabteilungen

CERAMIC COMPOSITES	2008
CC BAU	2012

2011

Outlook

* Even in smooth stress regions a strength condition can be only a necessary condition which may be not sufficient for the prediction of ‘onset of fracture’, i.e. for the *in-situ lateral strength in an embedded lamina,* see e.g. [Flaggs-Kural 1982], an energy-based second condition might be applied on top (in the past, this effect was often termed ‘thin-layer effect’).

* In case of discontinuities such as notches with steep stress decays only a *toughness + characteristic length-based energy balance condition* may form a sufficient fracture condition.

* Attempts to link ‘onset of fracture/cracking’ prediction methods for structural components are actually undergone, see e.g. [Leguillon 2002].

ANNEX

Set of Modal 3D Isotropic Strength Failure Conditions, WWFE-I and -II

invariant formulation

$$\text{FF1}: \quad F_{\|}^{\sigma} = \frac{I_1{}^{*}}{\bar{R}_{\|}^{t}} = 1,$$

$$\text{IFF1}: \quad F_{\perp}^{\sigma} = \frac{I_2 + \sqrt{I_4}}{2\bar{R}_{\perp}^{t}} = 1$$

$$\text{FF2}: \quad F_{\|}^{\tau} = \frac{-I_1{}^{*}}{\bar{R}_{\|}^{c}} = 1,$$

$$\text{IFF2}: \quad F_{\perp}^{\tau} = (b_{\perp}^{\tau} - 1)\frac{I_2}{\bar{R}_{\perp}^{c}} + \boxed{\frac{b_{\perp}^{\tau} I_4 + b_{\perp\|}^{\tau} I_3}{\bar{R}_{\perp}^{c2}}} = 1$$

WWFE-I

$$\text{IFF3}: \quad F_{\perp\|} = \boxed{\frac{I_3^{3/2}}{\bar{R}_{\perp\|}{}^{3}}} + b_{\perp\|}\frac{I_2 I_3 - I_5}{\bar{R}_{\perp\|}{}^{3}} = 1,$$

Modifications to by-pass numerical problems when calculating the material stressing efforts of the two IFF modes and to simplify the original equations

WWFE-II
modifications

$$\text{IFF2}: \quad F_{\perp}^{\tau} = (b_{\perp}^{\tau} - 1)\frac{I_2}{\bar{R}_{\perp}^{c}} + \frac{b_{\perp}^{\tau}\sqrt{I_4}}{\bar{R}_{\perp}^{c}} = 1$$

$$b_{\perp\|} = \mu_{\perp\|}, \; b_{\perp\perp} \cong 1/(1-\mu_{\perp\perp})$$

$$\text{IFF3}: \quad F_{\perp\|} = \frac{I_3{}^{2}}{\bar{R}_{\perp\|}{}^{4}} + b_{\perp\|}\frac{I_2 I_3 - I_5}{\bar{R}_{\perp\|}{}^{3}} = 1,$$

No proportional stressing anymore just the mode's driving stress is factored when computing the material stressing efforts 100% = max

$I_1 = \sigma_1$

$I_2 = \sigma_2 + \sigma_3$ (Boehler) $I_4 = (\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2$

$I_3 = \tau_{31}{}^2 + \tau_{21}{}^2$ $I_5 = (\sigma_2 - \sigma_3)(\tau_{31}{}^2 - \tau_{21}{}^2) - 4\tau_{23}\tau_{31}\tau_{21}$

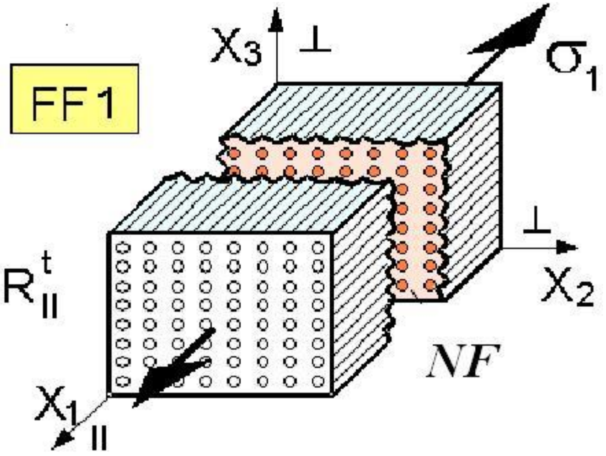
Fibre Failure Mode 1 (FF 1): Consideration of fracture without an acting σ_1

Limit of macro-homogenisation
(micromechanical fibre stress is responsible):

In case of transversal compression (2D or 3D) - due to Poisson's effect- tensile fibre failure (FF1) is possible without σ_1 .

Problem by-passed by taking strains from FEA which considers the full stress-strain behaviour !

$$F_{||}^{\sigma} = \frac{+\sigma_1}{\bar{R}_{||}^{t}} = 1\,,$$



$$I_1 = \sigma_1$$

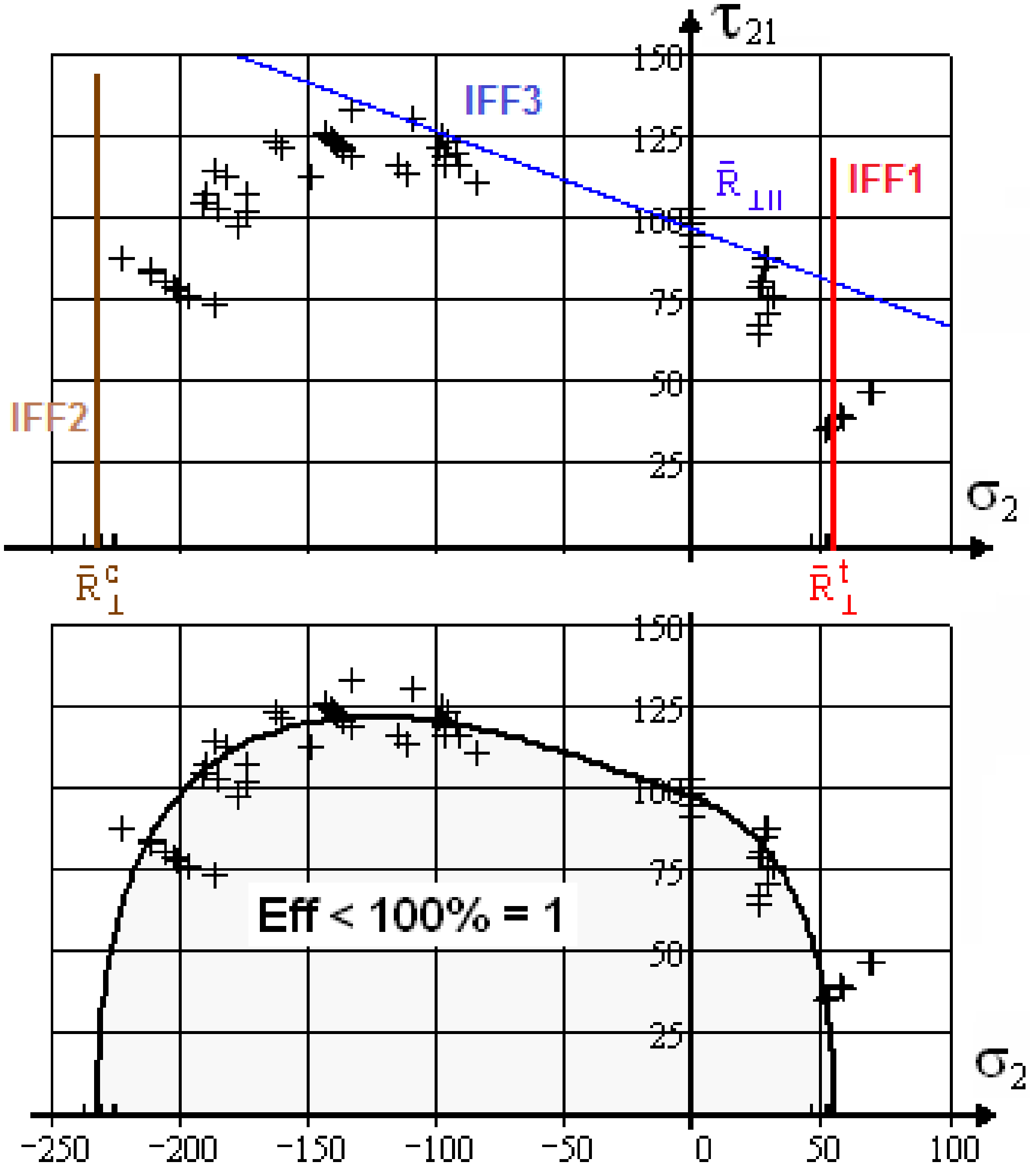
$$\cong \nu_f \cdot \sigma_{1f}^{\;t} = \nu_f \cdot \varepsilon_1^{\;t} \cdot E_{1f} = \varepsilon_1^{\;t} \cdot E_{||}$$

$$F_{||}^{\sigma} = \frac{\varepsilon_1 \cdot E_{||}}{\bar{R}_{||}^{t}}$$

Visualization of Interaction of UD Failure Modes

$\breve{\sigma}_1 = 0$

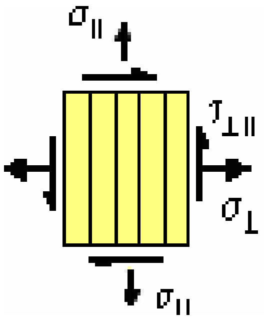
$\tau_{21}(\sigma_2)$ or $\{\sigma\}=(0,\sigma_2,0,0,0,\tau_{21})^T$



Eff < 100% = 1

Mapping of course of IFF test data in a pure mode domain by the *single Mode Failure Condition.*

3 IFF pure modes = straight lines !.

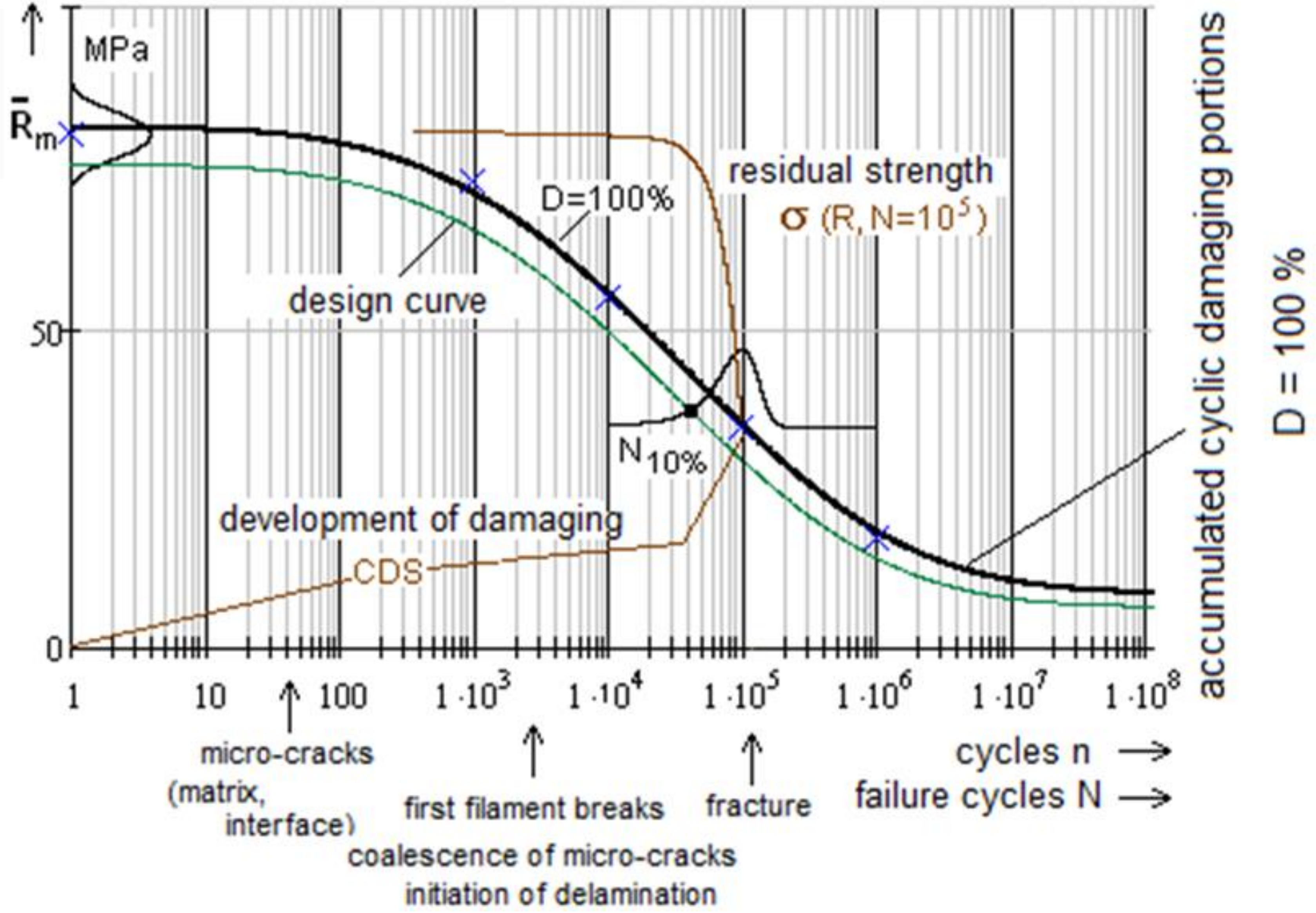


Mapping of course of test data by *Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

Lin-Log S-N Curve: Average Curve (mapping) and Design Curve (verification)

$\sigma_{max} \Rightarrow \sigma_{eq,\,max}$ if more stresses act together within the envisaged failure mode (like Mises)



FF:= fibre failure. IFF:= Inter Fibre Failure, CDS:= characteristic damage state at the end of diffuse damaging

Solution procedure , IFF1-IFF2 Haigh Diagram

Static strength failure $Eff = [(Eff^{NF})^m + (Eff^{SF})^m]^{m^{-1}} = 100\%$

$$\left(\frac{-(\sigma_{2m} - \sigma_{2a}) + |\sigma_{2m} - \sigma_{2a}|}{2 \cdot \overline{R}_{\perp}^{\,c}} \right)^m + \left(\frac{\sigma_{2m} + \sigma_{2a} + |\sigma_{2m} + \sigma_{2a}|}{2 \cdot \overline{R}_{\perp}^{\,t}} \right)^m = 1$$

The used static procedure still works for N = 1 with the interaction formula above delivering the CFL curve for N = 1 cycle, activating both NF + SF .

For higher N the interaction formula is engineeringlike simplified. *It reads*:

$$\sigma_a(\sigma_m) = \left[\left(\frac{\sigma_{aSF}}{1 + e^{\frac{c_{1SF} + \sigma_m}{c_{2SF}}}} \right)^m + \left(\frac{\sigma_{aNF}}{1 + e^{\frac{c_{1NF} + \sigma_m}{c_{2NF}}}} \right)^m \right]^{1/m}$$

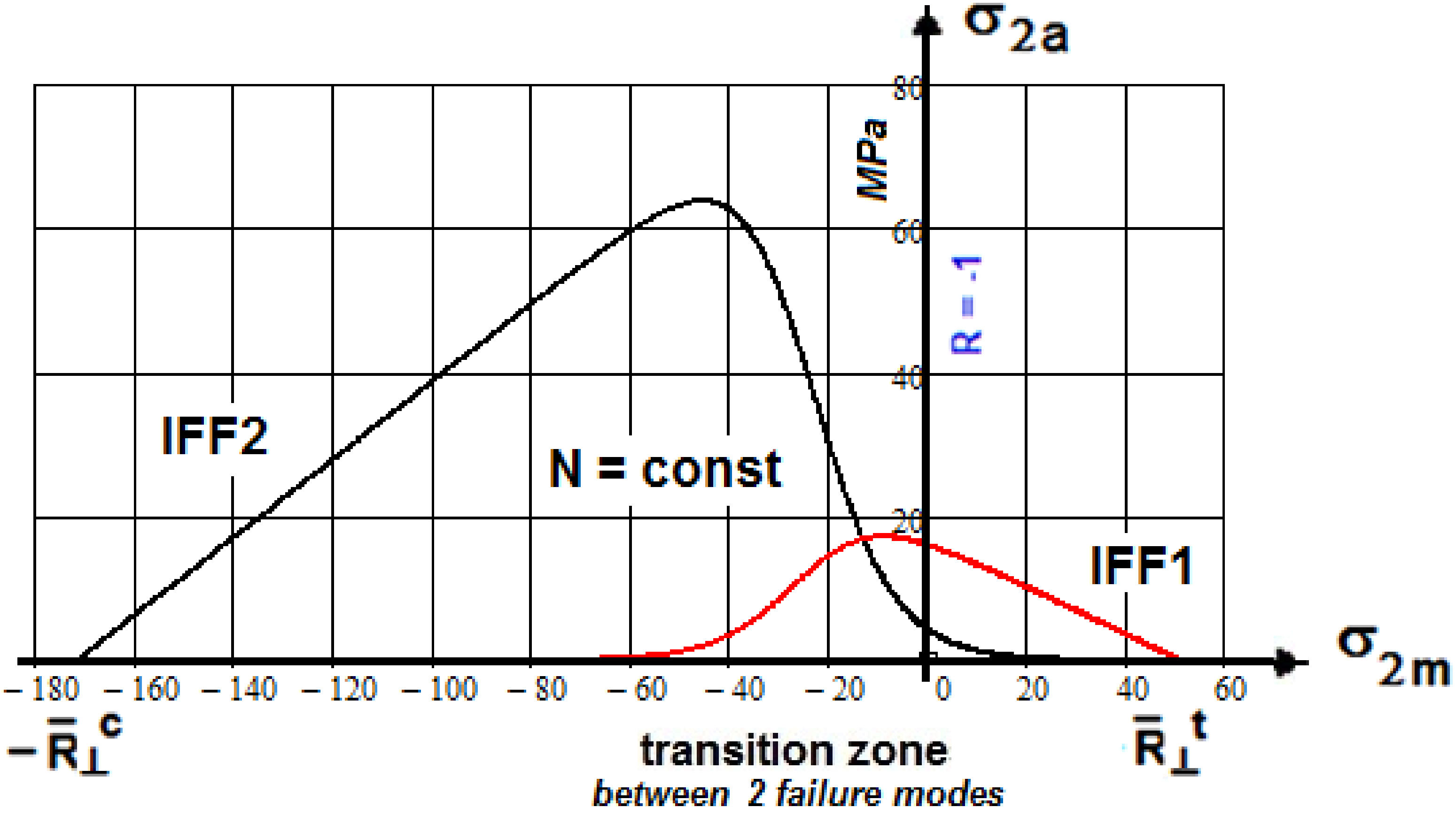
Thereby, an exponential decay function of the SF mode CFL curve for SF from R = ∞ down to zero at the end of the NF CFL curve at R = 0 is applied:

$$f = \left[\frac{1}{1 + e^{\frac{c_1 + \sigma_m}{c_2}}} \right] .$$

How to obtain CFL curves in the Transition Domain ?

No problem for the Haigh diagrams FF and IFF3 due to the strength values being of similar size in each case: The static interaction formula could be used.

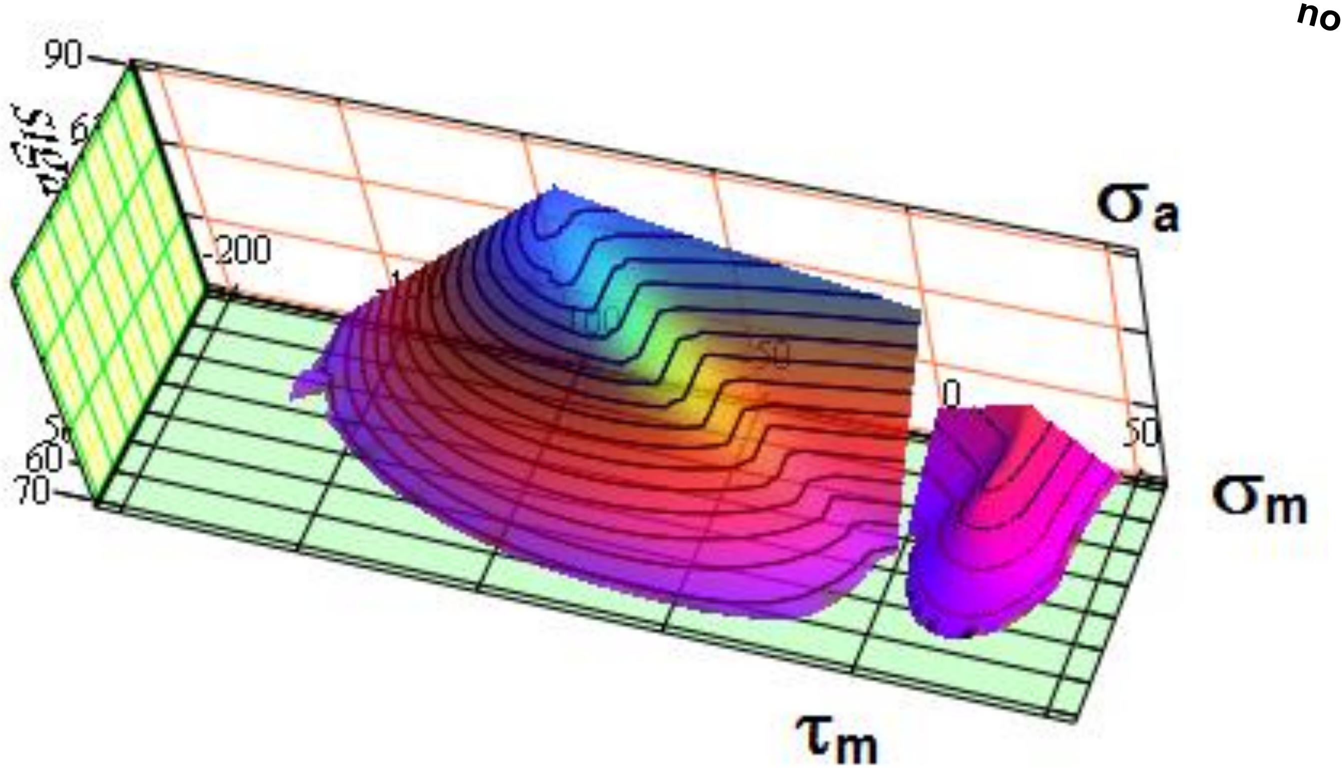
For the IFF1-IFF2 Haigh Diagram a new solution procedure had to be used..



failure domain-linked constant fatigue life (CFL) curves $\sigma_a(\sigma_m, R, N=constant)$.

3D IFF Approach: draft IFF1- IFF2- <u>IFF3</u> = IFF Haigh diagram

gibt es bisher
noch nicht

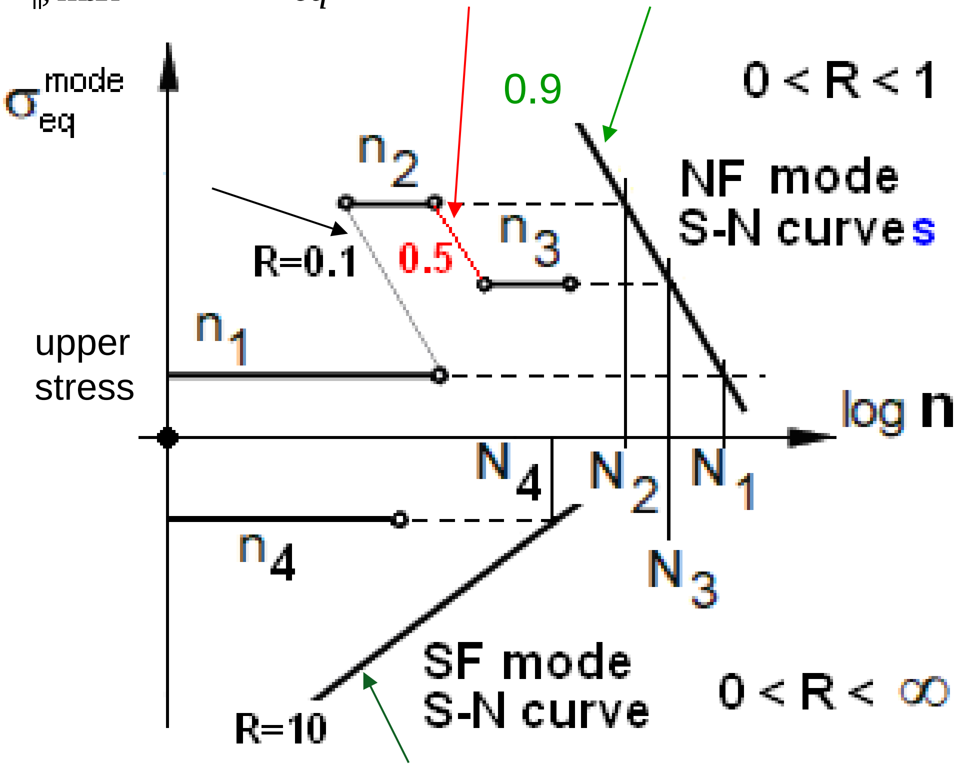


Mathcad 15 – Lösungswegrechnung
Hgeht noch nicht gerschlossen.
Neu-Programmierung derzeit.

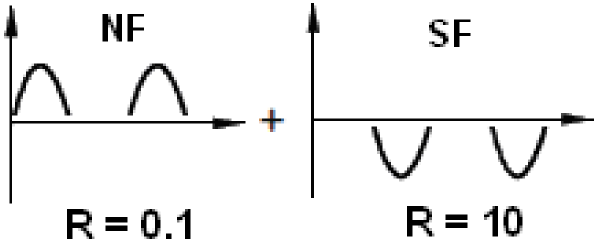
Failure mode-based Lifetime Prediction Method

schematic application (principle: for simple isotropic case as example, 4 blocks)

$$\sigma_{\parallel,\max}^{\mathrm{mode}} \Rightarrow \sigma_{eq}^{\mathrm{mode}}$$



remind



here:

2 master curves

NF: R = 0.1

SF: R = 10

2 predicted curves

NF: R = 0.5, 0.9

Miner application:

$$D = n_1 / N_1 + n_2 / N_2 + n_3 / N_3 + n_4 / N_4$$

Ideas for Experimental Proof

Choice of Test Specimens, Stress Combinations and Loading Types

Demands on test specimens: Consideration of
embedding of ply, ply-thickness effect, fibre volume fraction,
stacking sequence, loadings

1 : Flat coupon material *test specimens* (relatively cheap compared to tubes)

2 : Tension/compression-torsion tube *test specimens* $(\sigma_1, \sigma_2, \tau_{21})$

3 : Sub-laminate *test specimens (with internal* *<u>proof</u>* *ply and outer supporting plies)*

4 : Flat off-axis coupons (shortcomings ‘free edge effect‘ + bi-axial stiffness loss not accurately considered)

To be tested: Combinations of stresses (3D or 2D state of stresses)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T \quad \Rightarrow \quad \sigma_{\parallel}^t,\ \sigma_{\parallel}^c,\ \sigma_{\perp}^t,\ \sigma_{\perp}^c,\ \tau_{\perp\parallel} \quad \text{basic stresses}$$

Model VALIDATION: Loading types applied for the *operational lifetime estimation* are

- *Constant-amplitude loading* : delivers S-N curves (Wöhler curve)
- *Block-loading* : (if appropriate) for a more realistic Fatigue Life estimation
- *Random spectrum loading* : Fatigue Life (Gaßner) curve

The presented Full Lifetime Prediction Approach for UD laminas *for the* often ***fibre-dominated designed UD lamina-composed laminates*** employs

1) Failure mode-linked *load modelling* (novel idea)
2) Measurement of a minimum number of Master S-N curves
3) Prediction of other necessary *mode S-N curves* on basis of the master curve and the use of Kawai's 'Modified Fatigue Strength Ratio'
4) Detrmination of damaging portions basis of the static UD strength criteria . This depends on cycles-linked shrinking of failure surface by FMC strength criteria. In-situ-effect is considered by deformation-controlled testing.
5) Failure mode-linked *damaging accumulation* (novel idea)

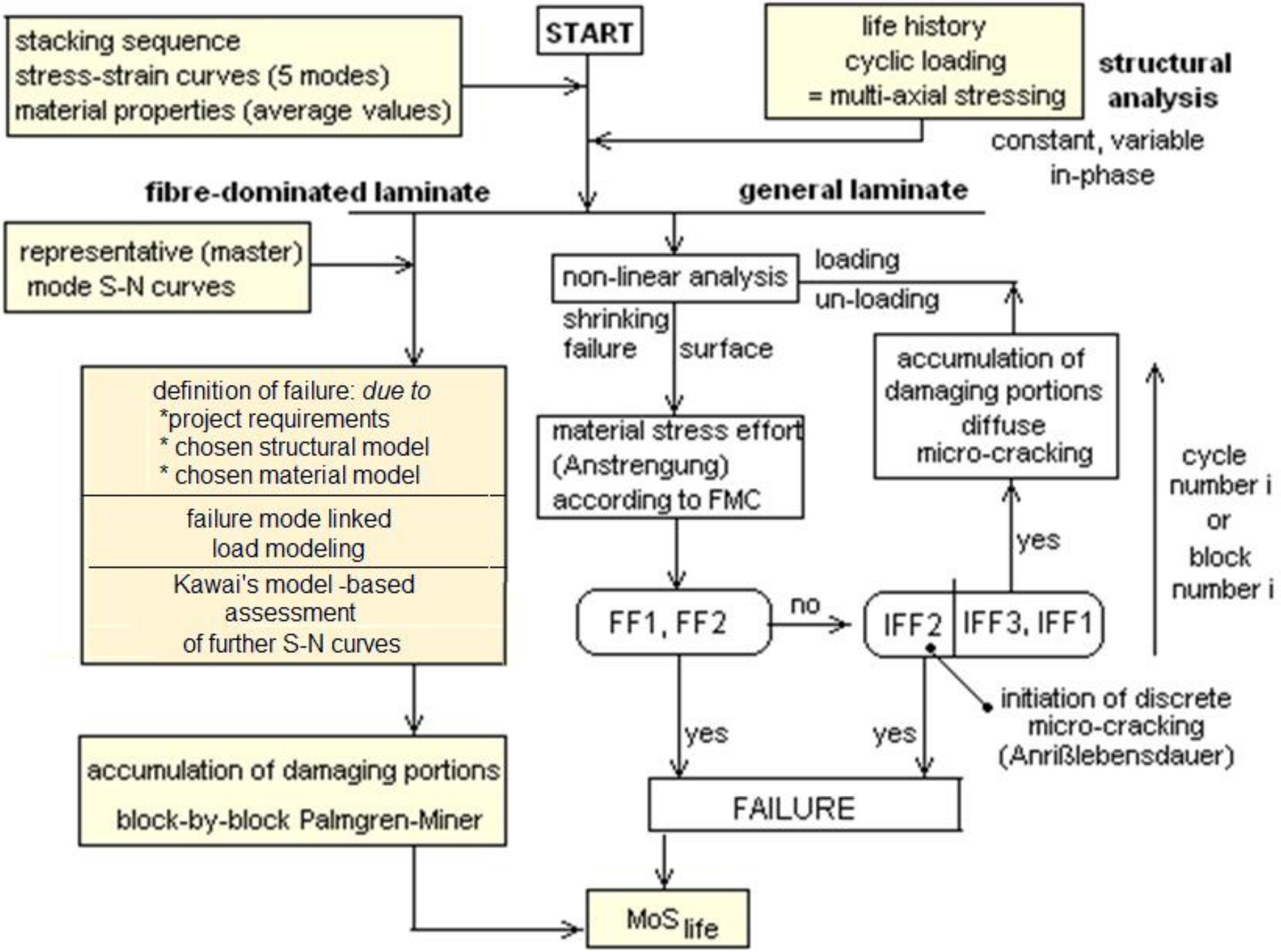
No mean stress correction is performed.

plus the derived 3 Haigh Diagrams.

Cuntze's *5 steps above, including a rigorous failure mode thinking,*
are the decisive BASIS for deriving the depicted novel Haigh Diagrams.

To be done: Deeper investigation of the behavior in the transition domain with the additional damaging caused by mode changes (FF1 to FF2 if $R \cong -1$) including crack-closure effects + investigation of IFF-caused fiber-notching effects.

Failure-Mode-Concept-based UD Lifetime Prediction (HCF)



Haigh Diagrams, brittle-behaving Materials as $\sigma_a(\sigma_m, R)$ and $\sigma(\sigma_m, R)$

Fatigue limit, **endurance limit**, and **fatigue strength** are all expressions used to describe a cyclic property of materials:

Stress Life Fatigue ⇦ approaches ⇨ Strain Life fatigue (***ductile behaviour***) ⇨

Mean stress sensitivity *M of isotropic materials*:

$$M = [\sigma_{endurance}\,(R = -1,\ \sigma_m = 0) - \sigma_{endurance}\,(R = 0,\ \sigma_m = \sigma_a)]\,/\,\sigma_m(R = 0) \quad \text{for} \quad n > 10^6\ cycles$$

Wechselfestigkeit　Schwellfestigkeit

- Brittle behavior: *M* ⇨ 1, max stress (Oberspannung) σ_{max} is responsible for damaging
- Ductile behavior *M* ⇨ 0, amplitude stress σ_a is responsible for damaging (slip)

AIM:

Automatic Establishment of the Curved Constant Life Curves (CFL) in Haigh Diagrams
on basis of
a measured Basic S-N curve plus a model.