SSMET 2014, Session “Composites“
Braunschweig, April 1-3, 2014, 25min

The World-Wide-Failure-Exercises -I and - II for UD-materials

- valuable attempts to validate failure theories on basis of more or less applicable test data sets -

Topics addressed:

- **Introduction, Definitions, State-of-the-Art Strength Failure Conditions (SFC)**
- **Strength Failure Conditions based on Cuntze’s Failure-Mode-Concept**
- **Survey on the World-Wide-Failure-Exercises (WWFEs)**
- **Discussion of Quality of provided Test data Sets**
- **Validation Examples WWFE-I (2D) and WWFE-II (3D)**
- **Practical Relevance with ‘Lessons Learnt’**

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State-of- the-Art in *Static Strength Analysis of UD laminas* *best* represented by the results of the *World-Wide-Failure-Exercises*

Organizer : *QinetiQ , UK* *(Hinton, Kaddour, Soden, Smith, Shuguang Li)*

Aim: *'Testing Predictive Failure Theories for*

Fiber–Reinforced Polymer Composites to the full !'

(was for the transversely-isotropic UD materials , only)

Procedure of the World-Wide-Failure-Exercises-I, -II (1992-2013)**:**

Part A of a WWFE: ***Blind Predictions on basic strengths, only***

Part B of a WWFE: ***Comparison Theory-Test*** *with (reliable)*
Uni-axial 'Failure Stress Test Data' (= basic strength) and
Multi-axial 'Failure Stress Test Data'

(plain test specimens, no notch)

What is <u>Failure</u>?

If the structural part does not fulfil its functional requirements
(FF, IFF, leakage, deformation limit, delamination size limit, …)

What does Failure Theory in the WWFE definition comprise?

*** UD strength failure conditions (SFCs) to predict interactive FF with IFF**

*** Non-linear modelling of the lamina** (hardening with softening)

*** Implementation of SFCs into a computer code for non-linear analysis**

*** Consideration of 2nd-Tg effect of matrix material for $\boldsymbol{p_{hyd} > 200}$ MPa**

FF := Fiber Failure, IFF := Inter-Fiber Failure (matrix failure)

WWFE-I: ***2D in-plane loading Test Data Packs for 14 Test Cases***

WWFE-II: ***3D loading Test Data Packs for 12 Test Cases***

WWFE-III: ***Application of advanced failure models based on Damage and Fracture Mechanics Models***

Deals with validating and benchmarking failure theories that are capable of predicting damage, such as

- matrix crack initiation and development,
- delamination initiation triggered by transverse cracks, and
- deformation up to final fracture.

Task: For endless fiber-reinforced polymers
courses of test data must be mapped
by the contributors
with their strength failure conditions (criteria).

WWFE-I Objective : 2D-Validation with 2D Failure StressTest Data

TC1-TC3 ***UD lamina : for validation of UD models***

TC4-TC14 ***UD lamina-composed Laminates :***

(quasi-isotropic, angle-ply, cross-ply):

for verification of laminate design by multi-axial failure stress envelopes *and* ***stress-strain curves .***

WWFE-II Objective : 3D–Validation with 3D Failure StressTest Data

involving hydrostatic pressures up to > 10000 bar = 1000 MPa

TC1 ***Epoxide matrix : for validation isotropic model***

TC2-TC7 ***UD lamina : for validation UD model***

TC8-TC12 ***Laminates*** **:** ***for verification of laminate design.***

WWFE Assumptions for UD Modelling and Testing

- **The UD-lamina is macroscopically homogeneous**.

  It can be treated as a homogenized (‘smeared‘ material)
- **The UD-lamina is transversely-isotropic:**

  On planes, parallel to the fiber direction it behaves orthotropic and on planes transverse to fiber direction isotropic (quasi-isotropic plane)
- **Uniform stress state about the critical stress ‘point‘**

Test specimen:

Pore-free material, specimen surfaces polished, well sealed (WWFE-II) , fiber volume is constant, tube specimens show no warping and do not bulge, perfect bonding, no layer waviness, edge effects do not exist, …

Basic Features of the presenter‘s Failure-Mode-Concept

- **Each failure mode represents 1 independent failure mechanism and 1 piece of the complete *failure surface***

- **Each failure mechanism is governed by 1 basic strength**

- **Each failure *mechanism* is represented by 1 failure *condition***

- **Interaction of Failure Modes:**

  Probabilistic-based 'rounding-off' approach (series model)

Information available when generating Strength Failure Conditions (SFCs)

1 If a UD material element can be homogenized to an ideal (= frictionless) crystal,

then, material symmetry demands for the **transversely-isotropic UD-material**

- 5 *elastic 'constants' , 5 strengths, 5 fracture toughnesses , 5 invariants* and
- *2 physical parameters (such as CTE, CME, material friction, etc.)*

(generic numbers 5 and 2)

2 Mohr-Coulomb requires for the real crystal another inherent parameter,

- the *physical parameter 'material friction' : UD* $\mu_{\perp\parallel}, \ \mu_{\perp\perp}$

3 Fracture morphology gives evidence:

Each strength corresponds to a distinct *failure mode*

and to a *fracture type* as Normal Fracture (NF) or Shear Fracture (SF) !

Reasons for Chosing Invariants when generating Failure Conditions

From Beltrami, Mises (HMH), and Mohr / Coulomb (friction) can be concluded:

Below invariant terms - used in a *failure function* ***F*** - can be dedicated to a physical mechanism in the solid = cubic material element:

- volume change : I_1^2 … ***(dilatational energy)*** I_1^2 , I_2^2
- shape change : J_2 (Mises) … ***(distortional energy)*** I_3 , I_4

and - friction : I_1 … ***(friction energy)*** I_2

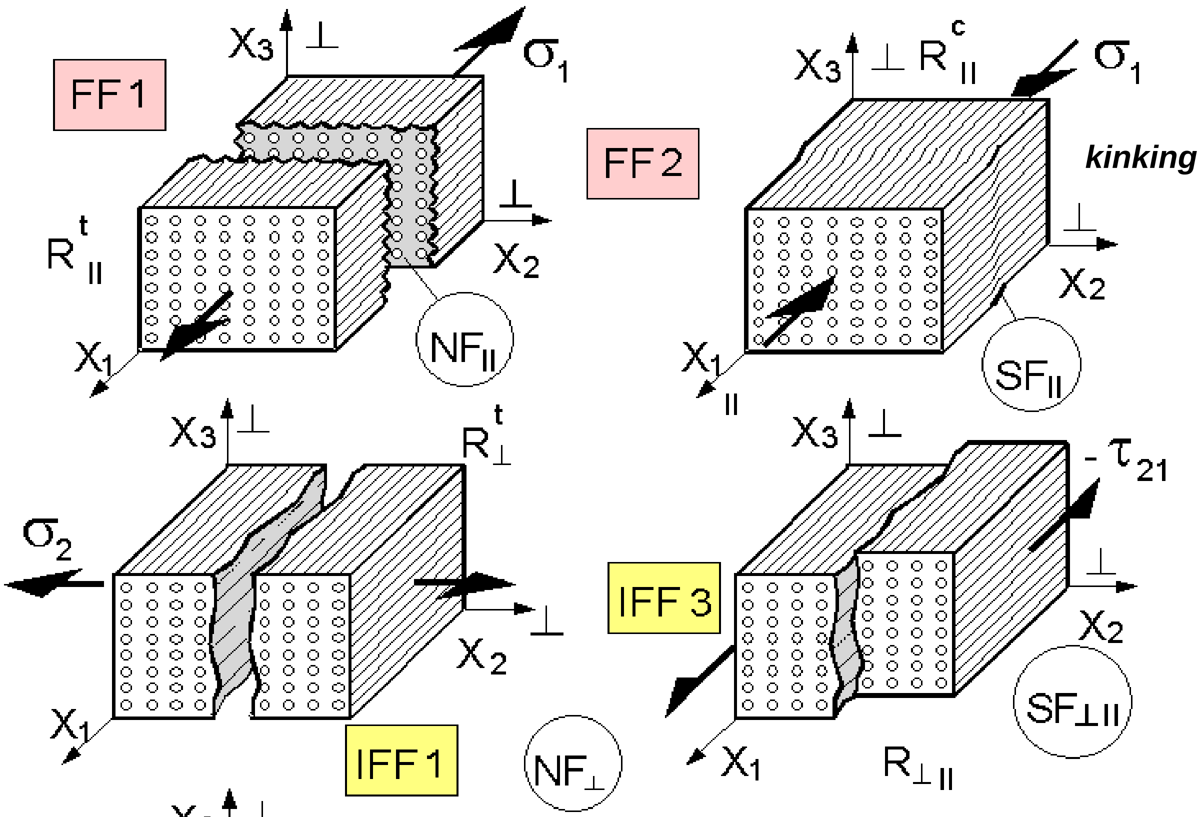
Stress Invariants: ***isotropic materials*** *and* ***: UD materials***

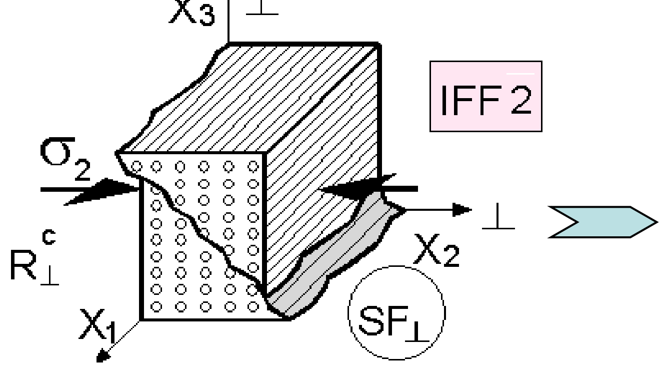
Mohr-Coulomb

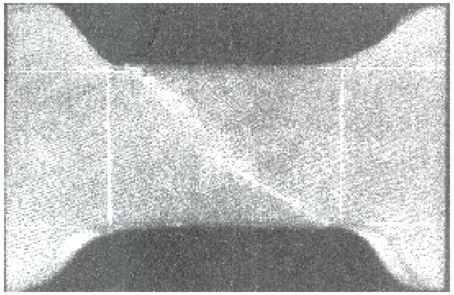
These I_1
are different !

Invariant := Combination of stresses, the value of which does not change when altering the coordinate system.

Observed Strength Failure Modes with Strengths of brittle UD Materials





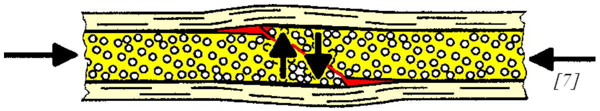


wedge failure type

t = tension
c = compression

▶ 5 Fracture modes exist

= 2 FF (Fibre Failure)
+ 3 IFF (Inter Fibre Failure)



Fracture Types:

NF := Normal Fracture

SF := Shear Fracture

two Types of Strength Failure Conditions (SFCs) used in the WWFE

Tsai-Wu

1 Global ***strength*** **failure condition** : **F ({*σ*}, {*R*}) = 1** (usual formulation)

Set of Modal **strength failure conditions: F ({*σ*}, *R*^{mode}) = 1** (addressed in FMC)

Puck, Cuntze

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of 6 stresses (general)

$$\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^T$$

vector of 5 strengths (UD)

Test data mapping : $R \Rightarrow \bar{R}$ average strength value (here addressed)

Design Verification : R strength design allowable,

- needs an ***Interaction of Failure Modes*:**

  Probabilistic-based 'rounding-off' approach (series model)

  directly delivering the (material) reserve factor in linear analysis

Benefits of the <u>modal</u> strength failure conditions (SFCs):

- No more input required than for the usually applied

  <u>global</u> strength failure conditions

- Have not the short-comings of the global conditions that

  - mathematically combine independent failure domains
  - do not directly use the physically necessary friction

    which means a bottle-neck if too few multi-axial

    failure stress data are available *and*

    if a test point change is required in a distinct mode this will change the shape of the failure surface in independent mode domains

Interaction of Single Strength Failure Modes in the modal FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ ....}$$ ***= 1 = 100% , if failure***

with mode-interaction exponent *2.5 < m < 3 from mapping experience*

as modal material stressing effort (in German Werkstoffanstrengung)

and

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} \,/\, \bar{R}^{\text{mode}}$$

equivalent mode stress

mode associated average strength

later → example

WWFE-II Set of Modal 3D UD Strength Failure Conditions (criteria)

strains from FEA

[Cun04, Cun11]

FF1 $Eff^{\parallel\sigma} = \breve{\sigma}_1 / \bar{R}_{\parallel}^{t} = \sigma_{eq}^{\parallel\sigma} / \bar{R}_{\parallel}^{t}$, $\breve{\sigma}_1 \cong \varepsilon_1^{t} \cdot E_{\parallel}$ *

FF2 $Eff^{\parallel\tau} = -\breve{\sigma}_1 / \bar{R}_{\parallel}^{c} = +\sigma_{eq}^{\parallel\tau} / \bar{R}_{\parallel}^{c}$, $\breve{\sigma}_1 \cong \varepsilon_1^{c} \cdot E_{\parallel}$

2 filament modes

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / 2\bar{R}_{\perp}^{t} = \sigma_{eq}^{\perp\sigma} / \bar{R}_{\perp}^{t}$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / \bar{R}_{\perp}^{c} = +\sigma_{eq}^{\perp\tau} / \bar{R}_{\perp}^{c}$

3 matrix modes

IFF3 $Eff^{\perp\parallel} = \{[\mu_{\perp\parallel} \cdot I_{23-5} + (\sqrt{\mu_{\perp\parallel}{}^2 \cdot I_{23-5}{}^2 +_{\varepsilon} 4 \cdot \bar{R}_{\perp\parallel}{}^2 \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,] / (2 \cdot \bar{R}_{\perp\parallel}{}^3)\}^{0.5} = \sigma_{eq}^{\perp\parallel} / \bar{R}_{\perp\parallel}$

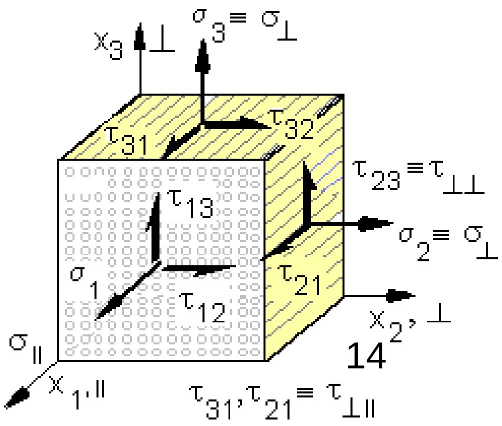
with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

Modes-Interaction :

$$Eff^{m} = (Eff^{\parallel\tau})^m + (Eff^{\parallel\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

with mode-interaction exponent $2.5 < m < 3$ **from mapping test**

Typical friction value data range: $0.05 < \mu_{\perp\parallel} < 0.3$, $0.05 < \mu_{\perp\perp} < 0.2$



Poisson effect * : bi-axial compression strains the filament without any σ_1

t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre

Cuntze‘s Pre-design Input for 3D UD SFCs

Test Data Mapping Design Verification

- **5 strengths :** $\{\overline{R}\} = (\overrightarrow{R}_{\parallel}^{t}, \overline{R}_{\parallel}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp\parallel})^{T}$ $\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^{T}$

  average (typical) values **strength design allowables**

- **2 friction values : for 2D** $\mu_{\perp\parallel}$ **, for 3D** $\mu_{\perp\parallel}, \mu_{\perp\perp}$

  $\mu_{\perp\parallel} = 0.1$ $\mu_{\perp\perp} = 0.1$

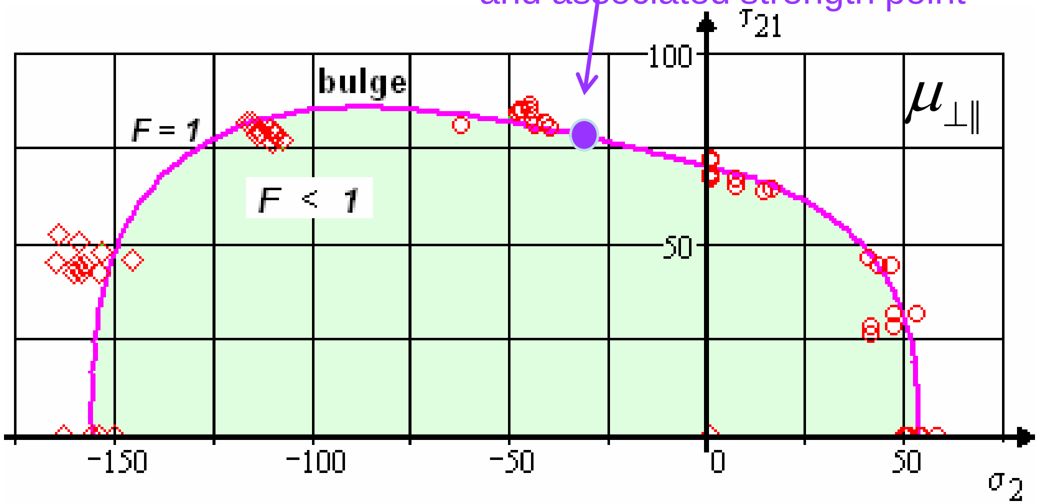
- **1 mode-interaction exponent :** *m = 2.6 .*

values, recommended
for pre-design

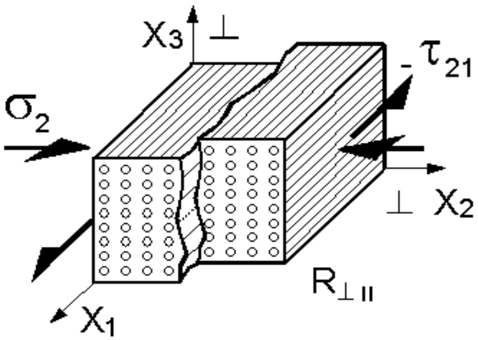
Estimation of Friction Values from bi-axial test points $\mu_{\perp\parallel}$, $\mu_{\perp\perp}$

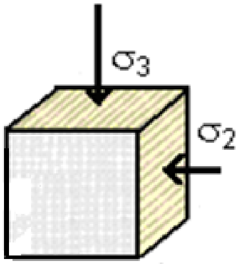
$$|\tau_{21}| = \bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2$$

Estimation:
Straight line through magenta point and associated strength point

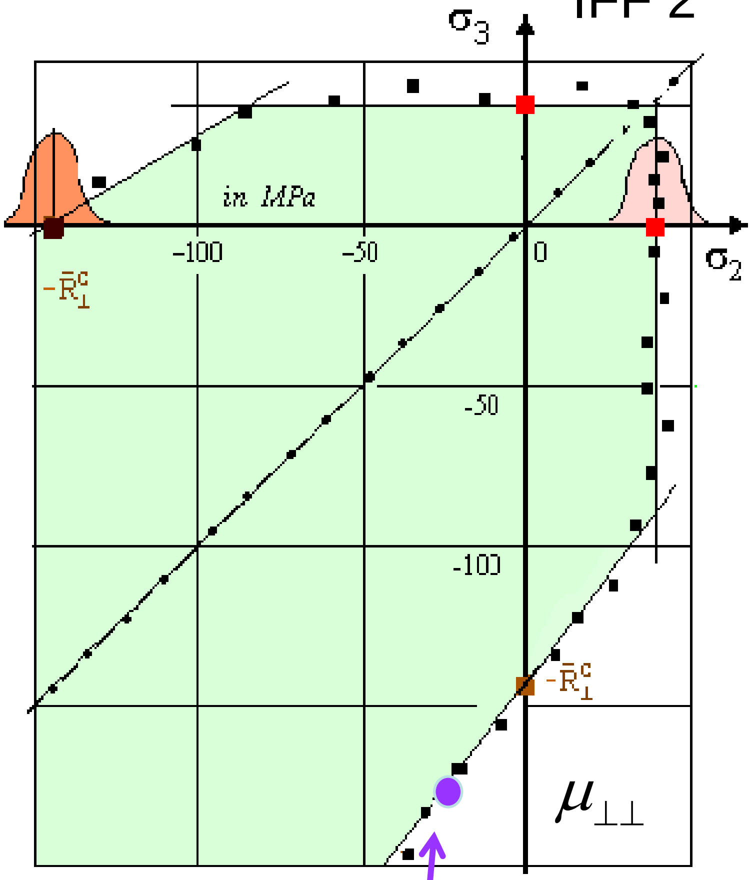


IFF 3





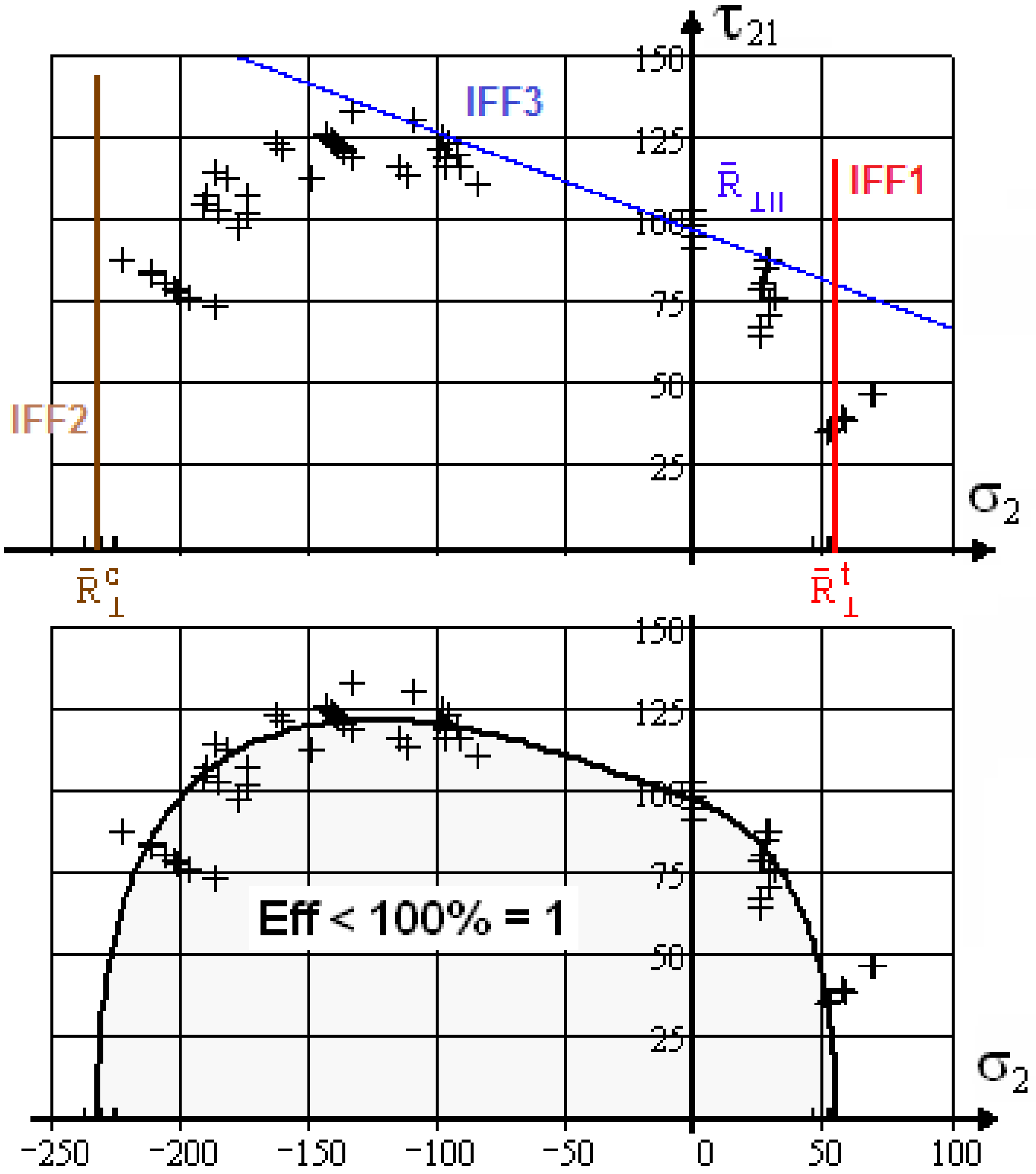
IFF 2



1. Fitting of course of test data (min error square) in ‘pure‘ failure mode domains
2. Estimation with one strength value and one multi-axial failure stress point
3. For $\mu_{\perp\perp}$ in addition : derivation from fracture plane measurements possible.

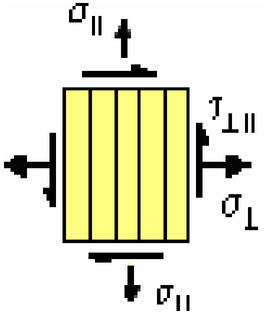
minimum error square

Interaction Visualization of UD Failure Modes $\tau_{21}(\sigma_2)$ $\breve{\sigma}_1 = 0$



Mapping of course of IFF test data in a pure mode domain by the *single Mode Failure Condition.*

3 IFF pure modes = straight lines !.



IFF 1 : $\frac{\sigma_2}{\bar{R}_\perp^t} = 1$

IFF 2 : $\frac{-\sigma_2}{\bar{R}_\perp^c} = 1$

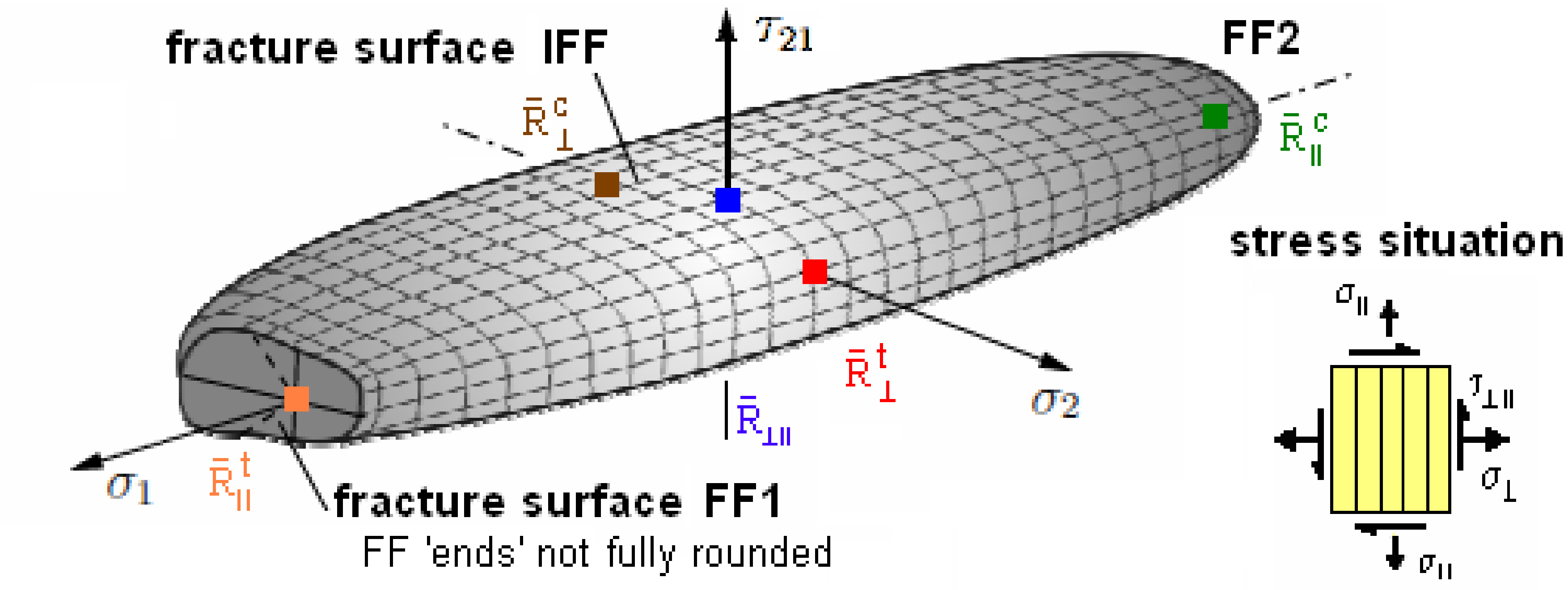
IFF 3* *(2D simplified) : $\frac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 1$

Mapping of course of test data by *Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

$$m = 2.5, \quad \mu_{\perp\parallel} = 0.3$$

Visualization of 2D UD SFCs as Fracture Failure Surface (Body)



$$\{\sigma\}=(\sigma_1,\sigma_2,0,0,0,\ \tau_{21})^T$$

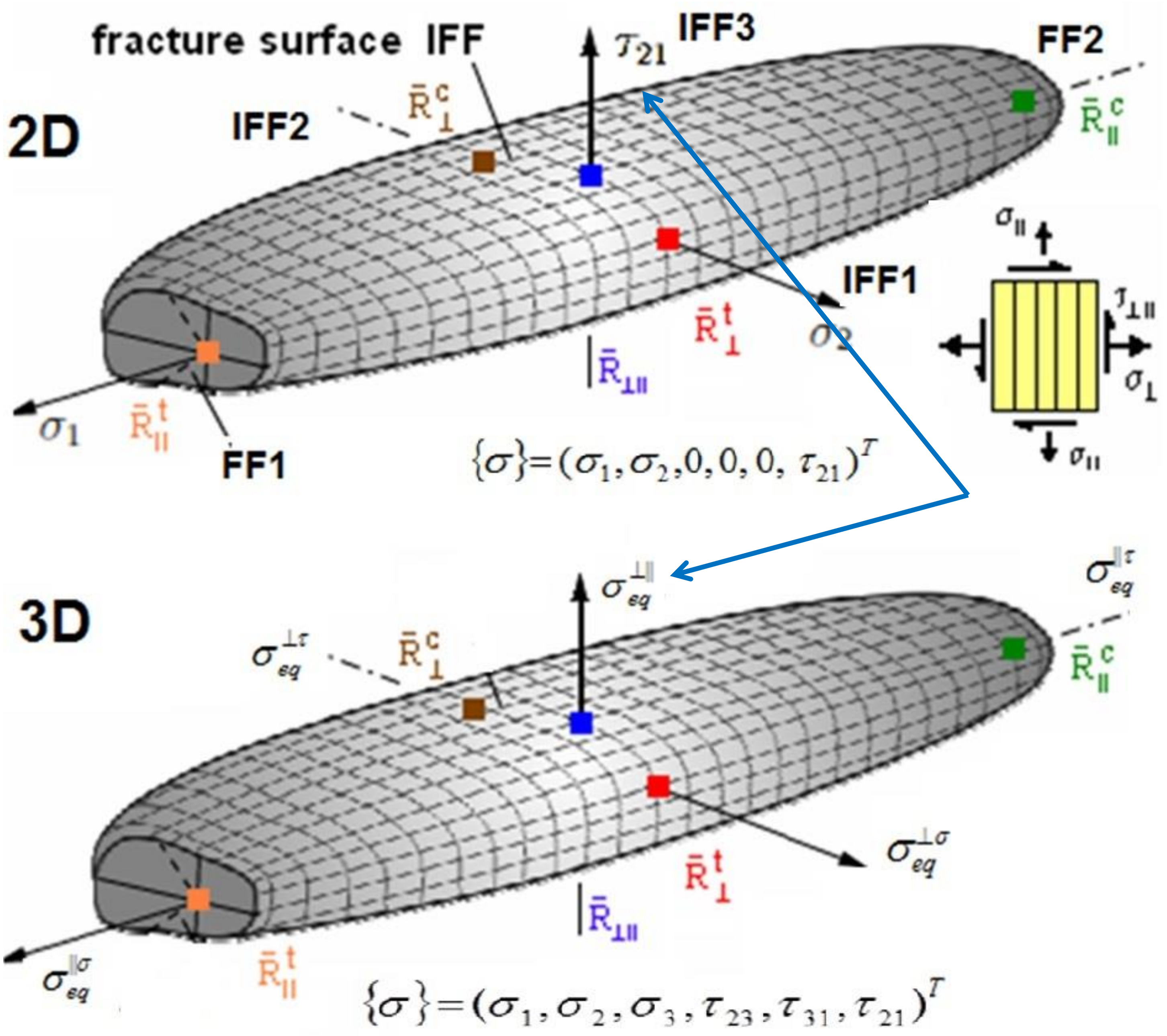
Mode interaction fracture failure surface *of FRP UD lamina*

$$Eff^{\,m} = (Eff^{\,\parallel\tau})^m + (Eff^{\,\parallel\sigma})^m + (Eff^{\,\perp\sigma})^m + (Eff^{\,\perp\tau})^m + (Eff^{\,\perp\parallel})^m = 1$$

(courtesy W. Becker) .

Mapping: Average strengths indicated

2D = 3D Fracture surface by replacing the stress by the equiv. stress

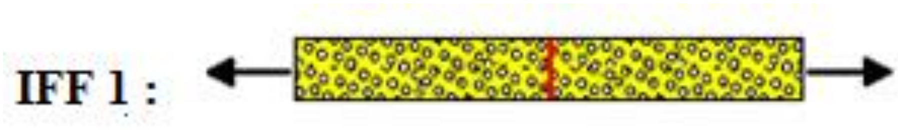


$$\{\sigma\}=(\sigma_1,\sigma_2,0,0,0,\ \tau_{21})^T$$

$$\{\sigma\}=(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{31},\tau_{21})^T$$

Isolated and Embedded Laminas (WWFE-II, TC 3)

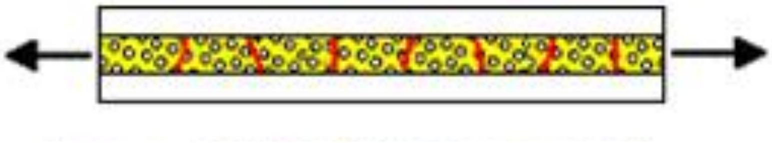
Isolated behaviour:



weakest link problem

$$\{R\} = (R^{t}_{\parallel}, R^{c}_{\parallel}, R^{t}_{\perp}, R^{c}_{\perp}, R_{\perp\parallel})^{T}$$

Embedded behaviour:



redundancy problem

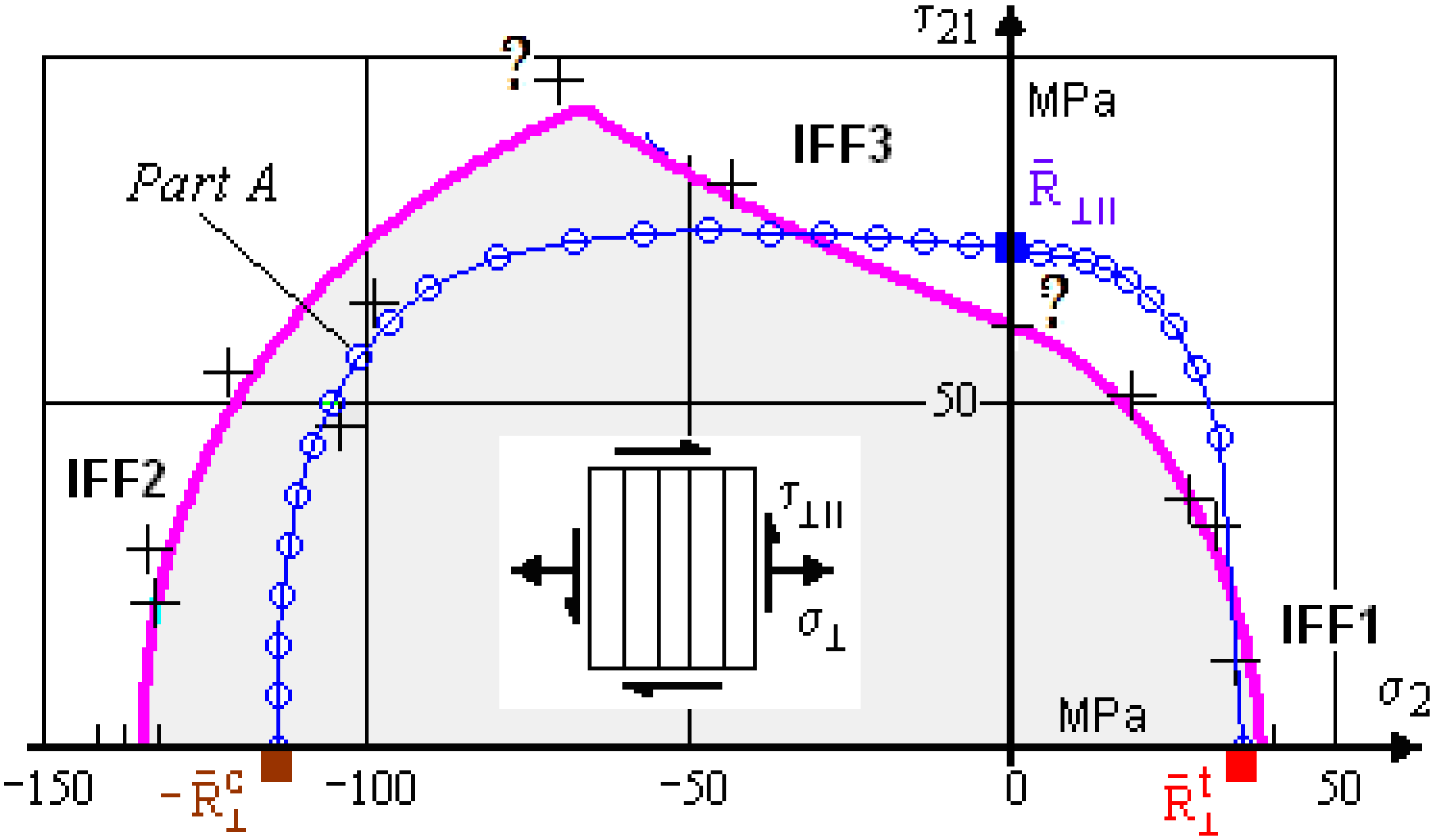
'healing' versus 'notching'

of neighbour lamina surfaces

in-situ

Lesson Learnt: Basic strengths are weakest-link data !

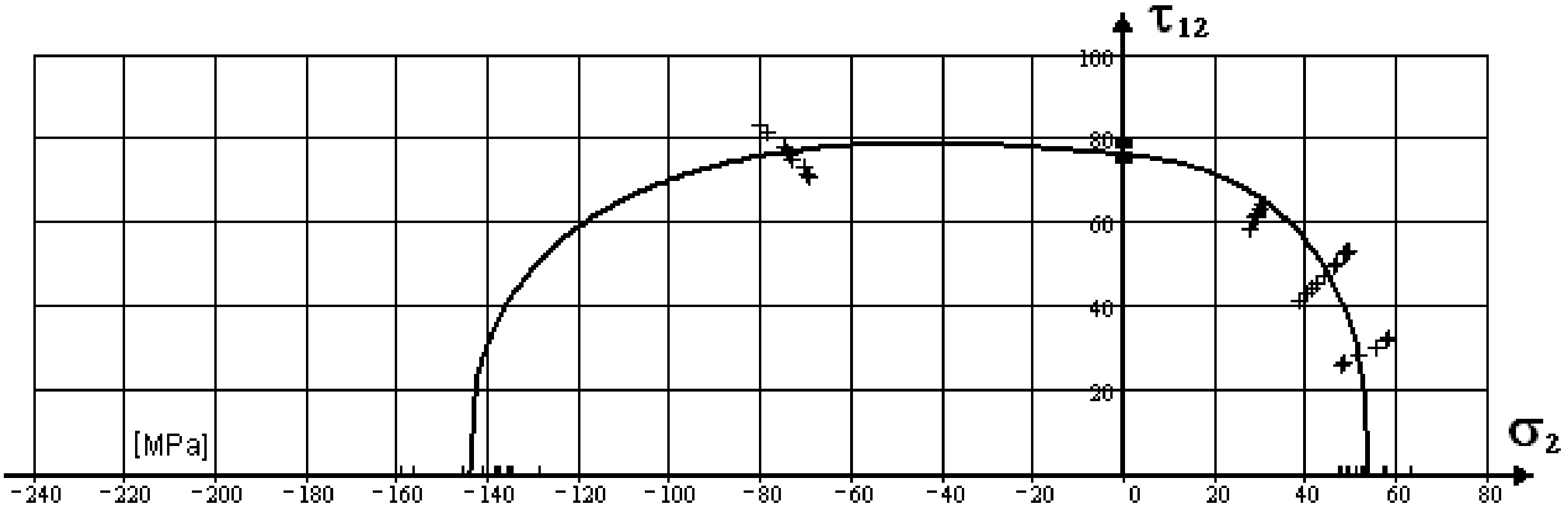
Test Case 1, WWFE-I, ***IFF curve*** $\tau_{21}(\sigma_2)$

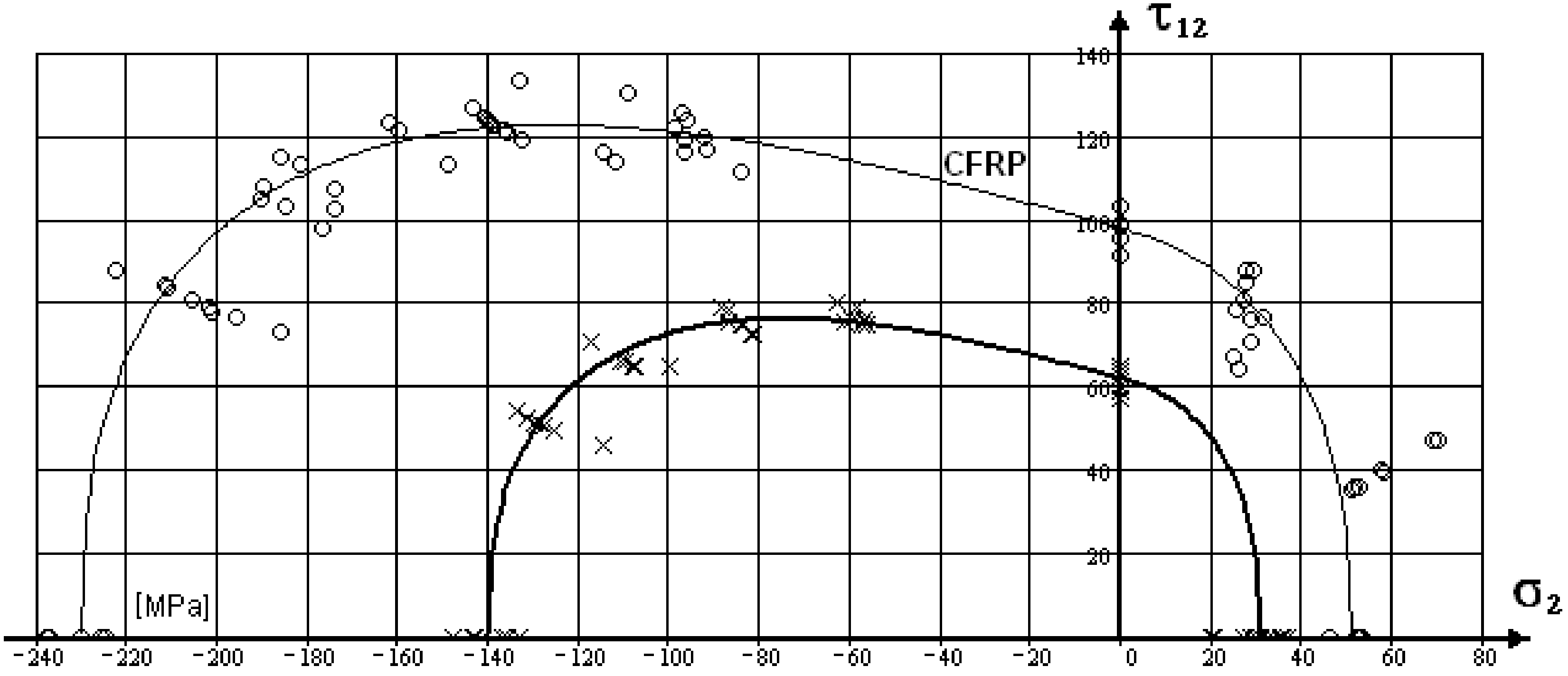


Part A, prediction: 3 Strength data provided, only. No friction value (slope) $\mu_{\perp\parallel}$ given !

Part B, comparison: 3 Strength points altered! 2 doubtful (**?**) single failure stress points

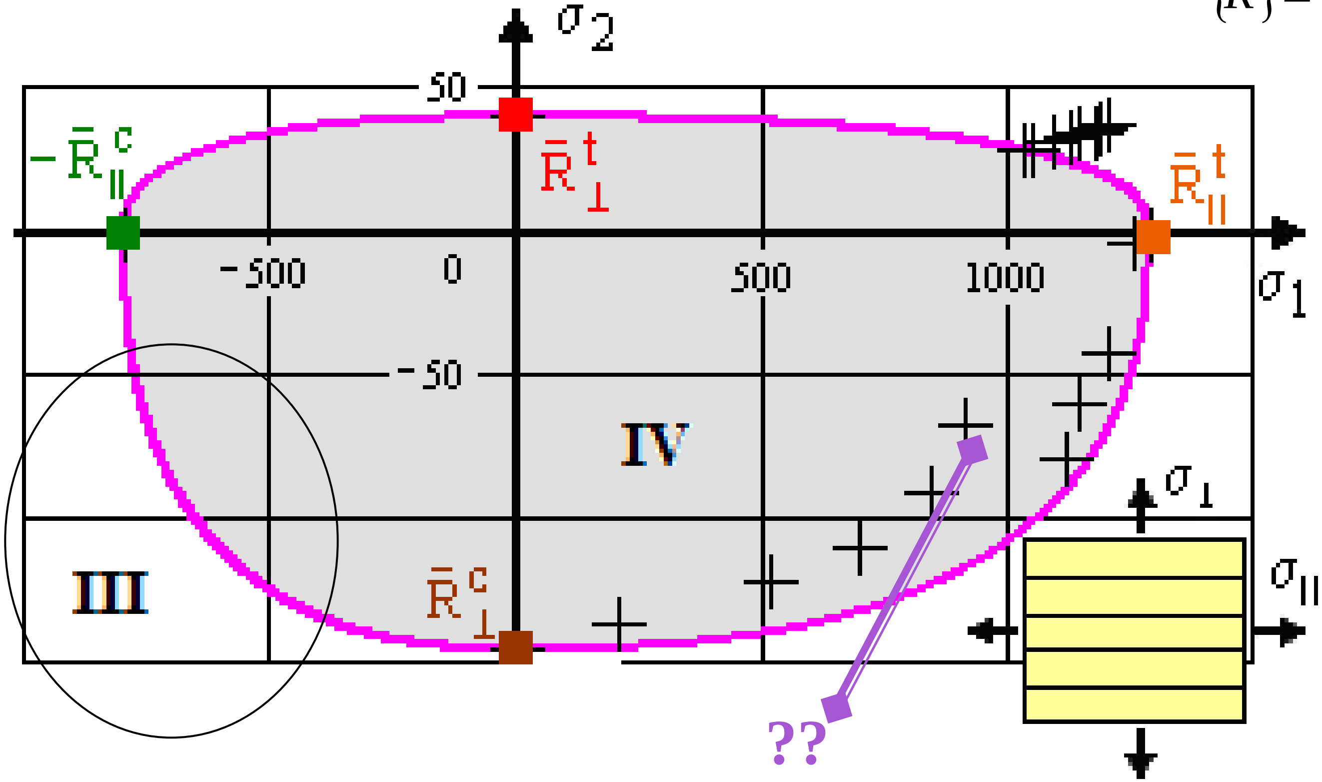
Own test results: 2 GFRP, 1 CFRP Test Series





Test Case 3, WWFE-I $\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$



Hoop wound tube
UD-lamina.
E-glass/MY750epoxy +

$$\sigma_1 = \sigma_{hoop}$$

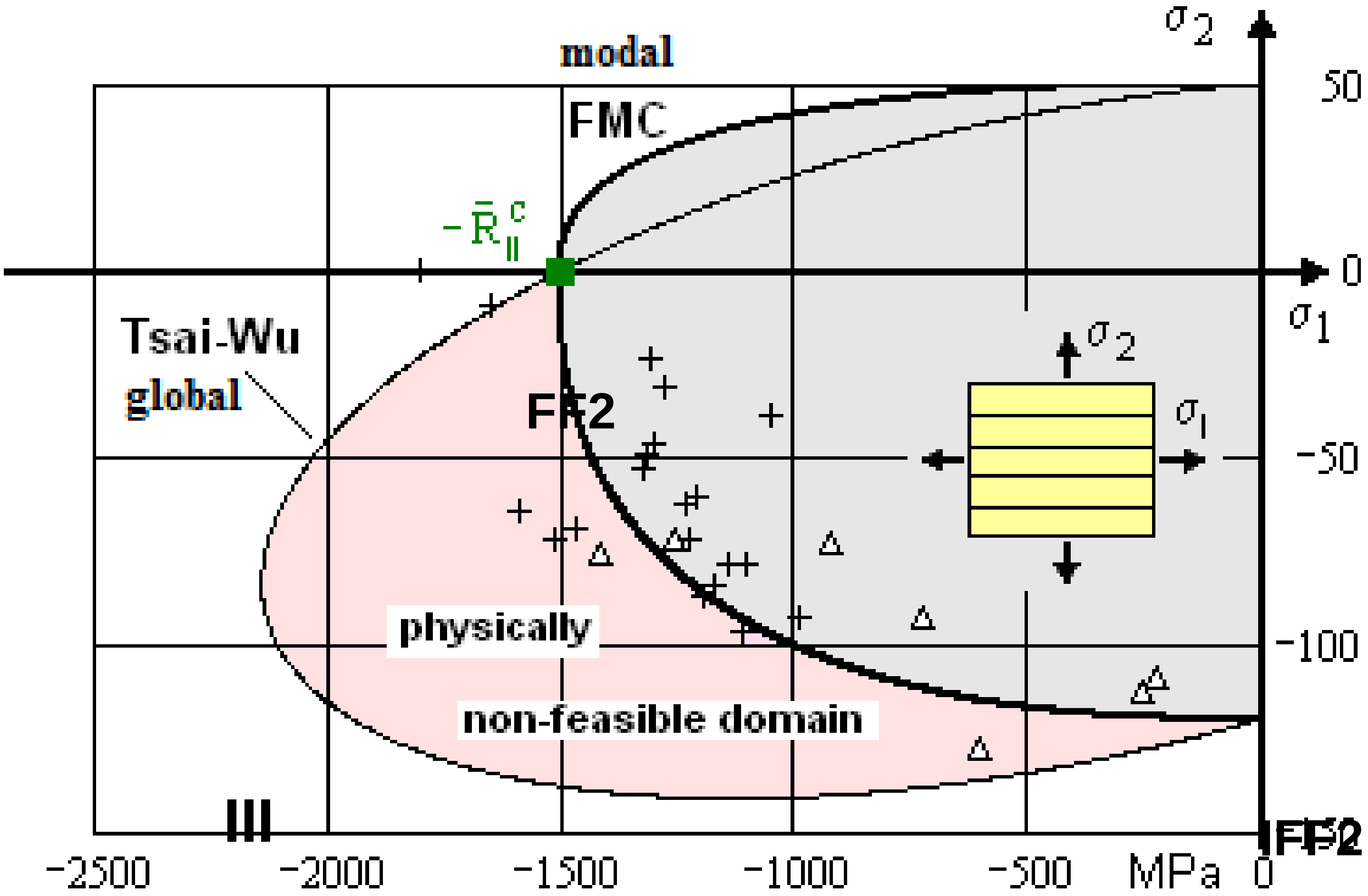
$$\sigma_2 = \sigma_{axial}$$

Part A: Data of strength points were provided, only

Part B: Test data in quadrant IV show discrepancy , testing?

No data for quadrants II, III was provided ! But, ..

Mapping in the ‘Tsai-Wu non-feasible domain‘ (quadrant III)



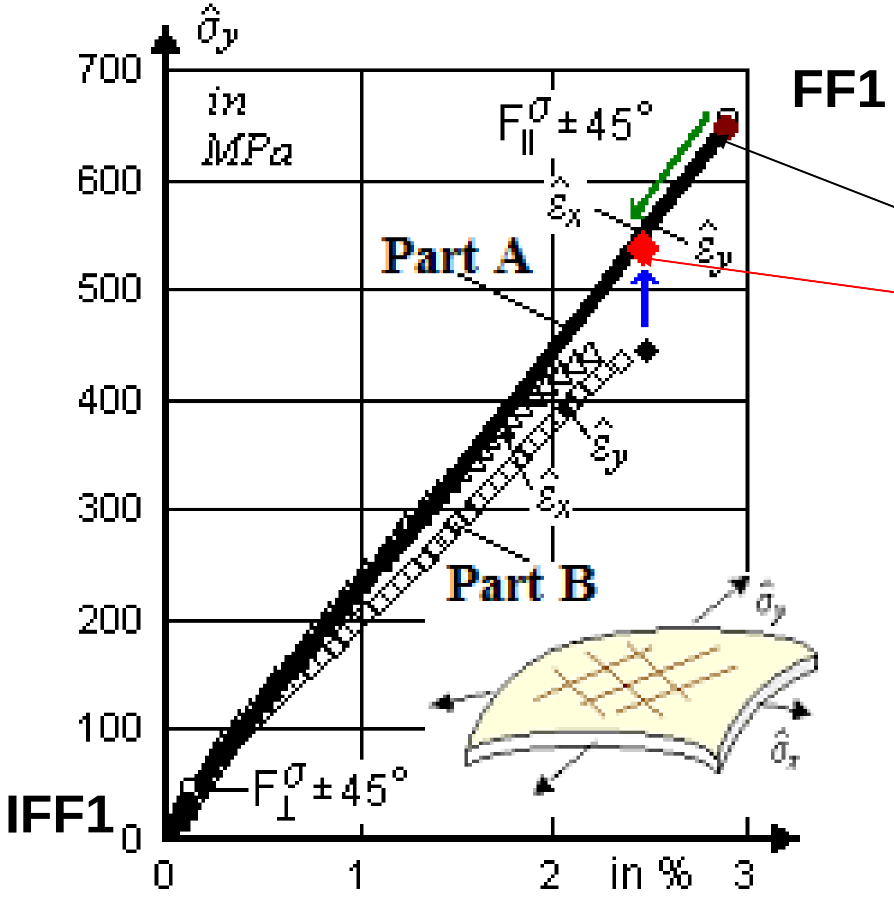
$\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

Data: courtesy IKV Aachen, Knops

Lesson Learnt: The modal FMC maps correctly, the *global* Tsai-Wu formulation predicts a non-feasible domain !

Test Case 13, WWFE-I, Laminate Stress-Strain Curve

$\hat{\sigma}_y : \hat{\sigma}_x = 1 : 1$



$\{\overline{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$

Loading of tube: internal pressure + axial tension.

Laminate: E-glass/MY750. [+45/-45/45/-45]-

Bulging (widening) reported in experiment.

- ● Final blind prediction point.
- ◆ Maximum test value *after* correction and shifting.

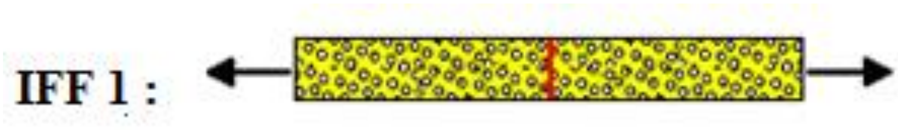
Mapping quality very good after re-evaluation !

Part A: Data of strength points and the fracture strain were provided

Part B: Increased test data information caused **a reduction of *fracture strain* and to *increase the failure stress* after widening of the tube was reported**

Isolated and Embedded Laminas (for II-TC 3 essential)

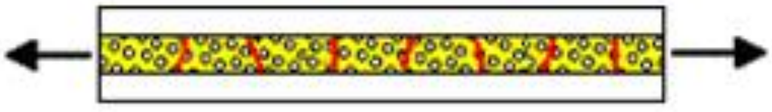
Isolated behaviour:



weakest link problem

$$\{R\} = (R^t_{\parallel}, R^c_{\parallel}, R^t_{\perp}, R^c_{\perp}, R_{\perp\parallel})^T$$

Embedded behaviour:



redundancy problem

'healing' versus 'notching'

of neighbour laminas surfaces

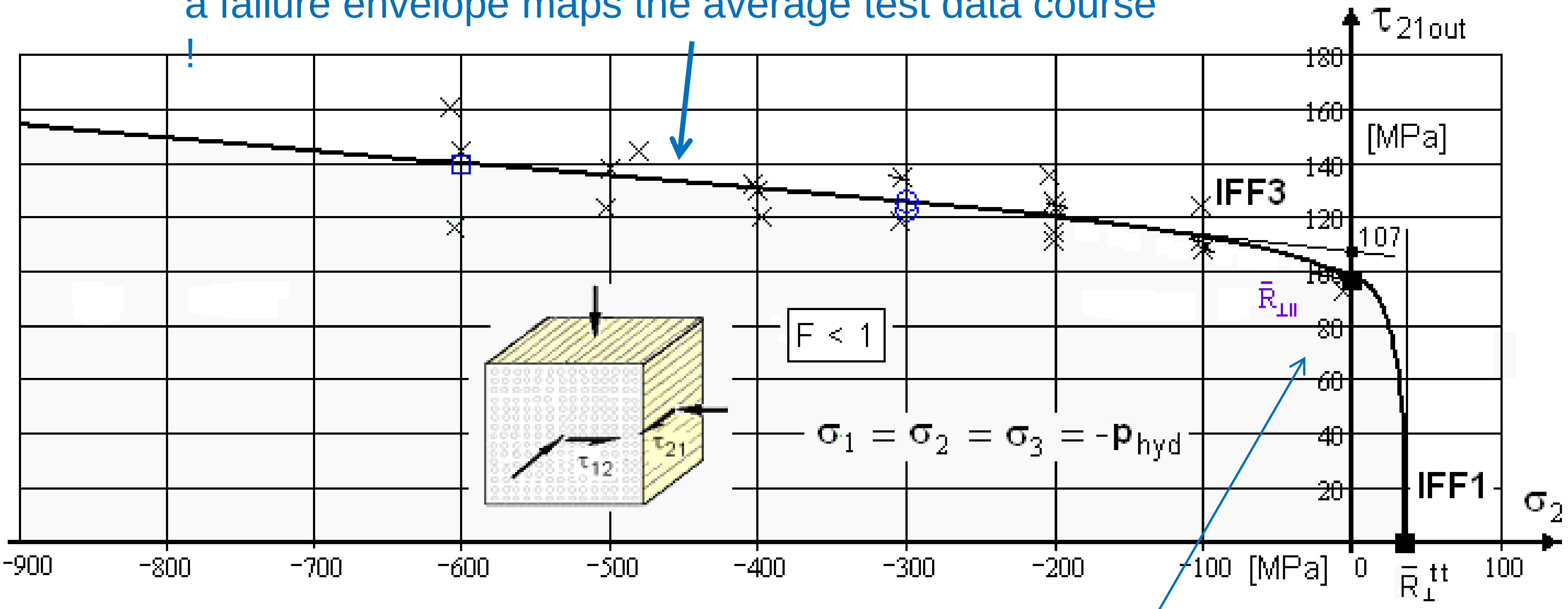
in-situ

Lesson Learnt: Basic strengths are weakest-link data !

Test Case 3, WWFE-II, UD Test Specimen

$\tau_{21}(\sigma_2 = -p_{hyd})$

a failure envelope maps the average test data course !



Good Mapping,

<u>after</u> re-evaluation of provided data and use of an average stress-strain curve instead of the provided ‘upper‘ stress-strain curve

<u>and</u> a novel physical interpretation of test data, discriminating near $\sigma_2 = 0$, ‘isolated‘ and ‘embedded (redundant)‘ ones !

Test Case 5, WWFE-II, UD test specimen

$\sigma_2(\sigma_1 = \sigma_3)$

= hydrostatic pressure with additional loading

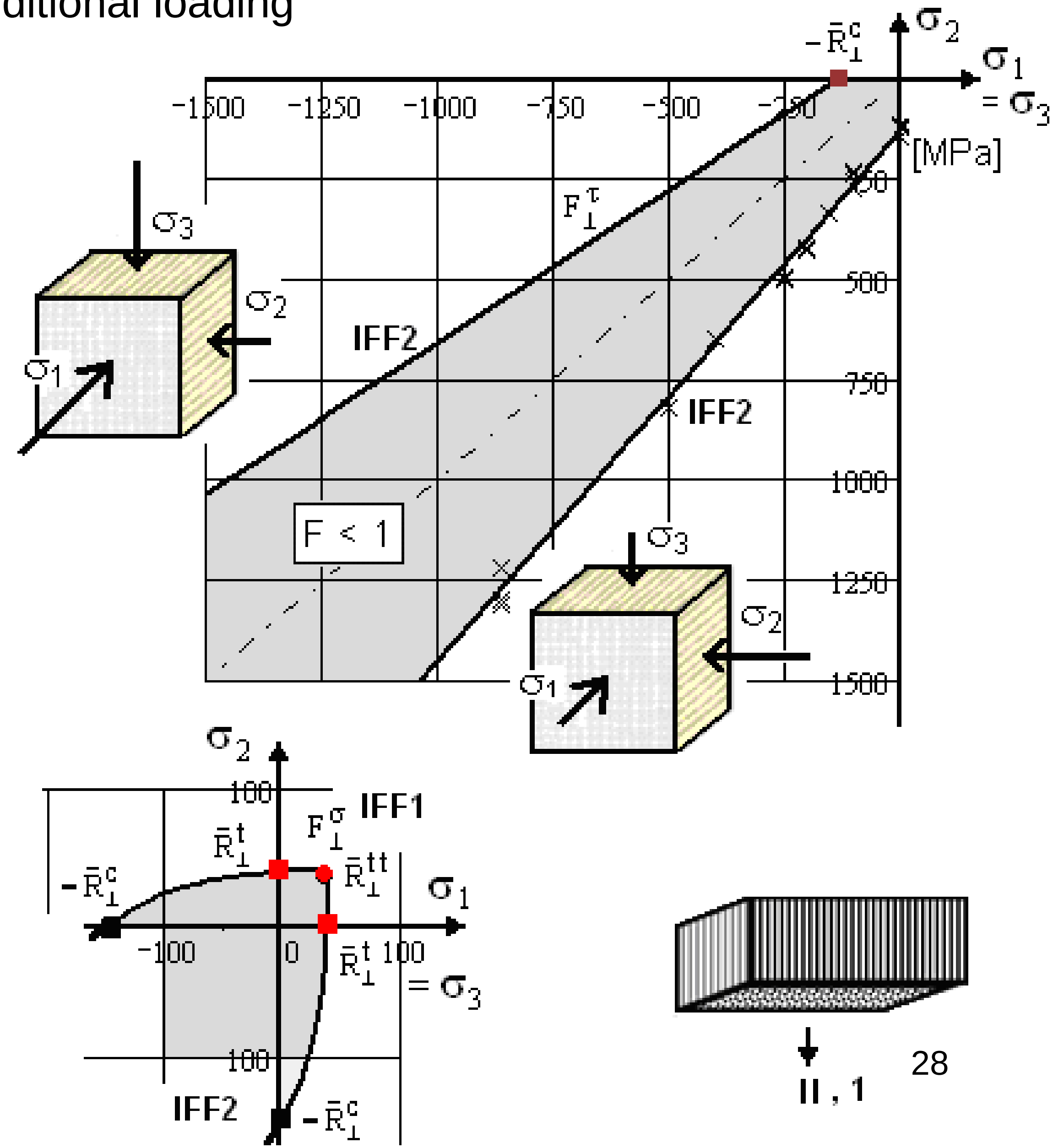
UD E-glass/MY750epoxy.

$\nu_{\perp||} = 0.28$ $b_{\perp\perp} = 1.16$ $m = 2.8$

$\{\bar{R}\} = (1280,\ 800,\ 40,\ 132,\ 73)^T\ MPa$

Good Mapping, <u>after</u> QinetiQ was asked to re-evaluate the lower branch test data ! Then,the upper branch was fitted other test data.

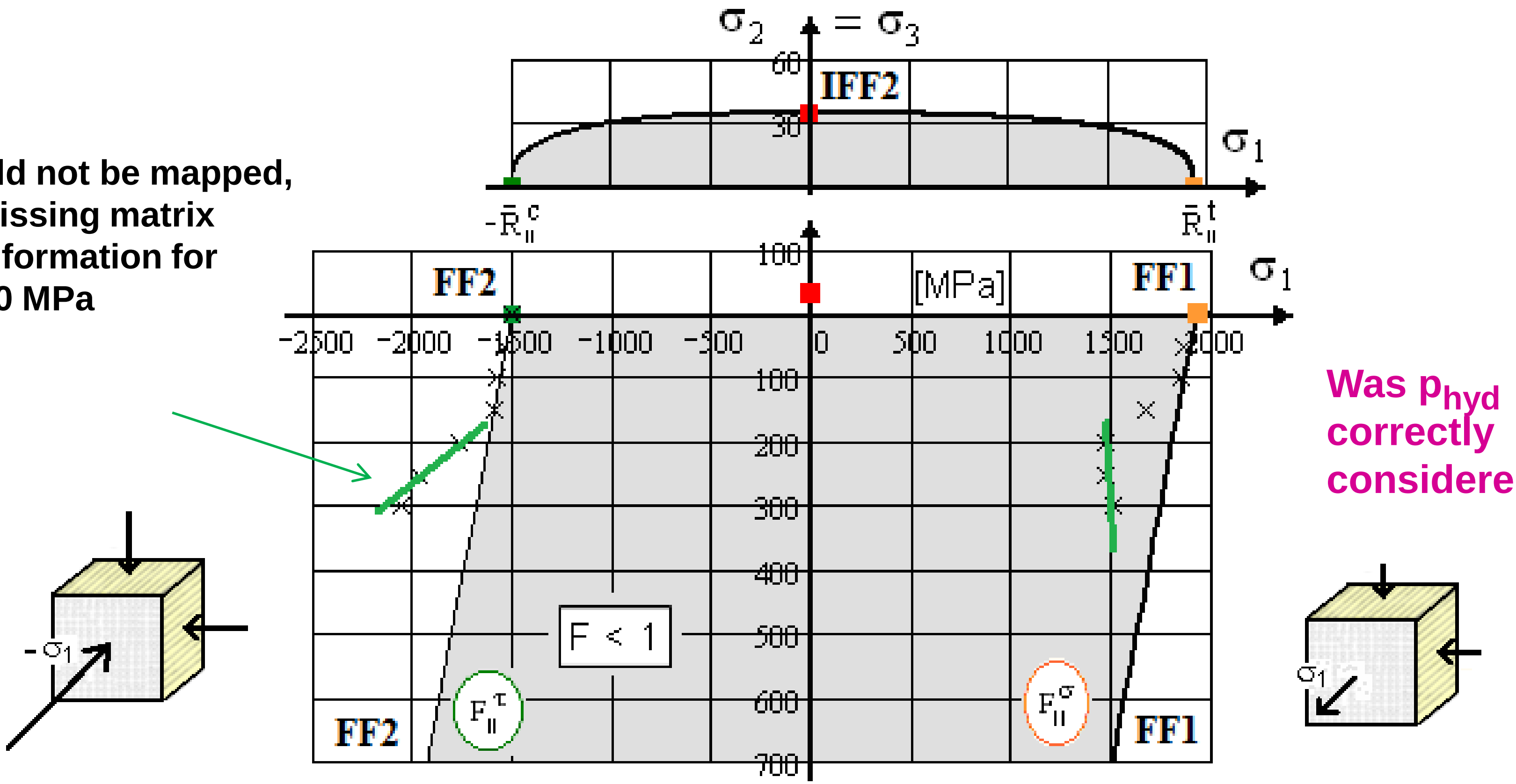
Both branches are reliable and can be used for validation of the model



Test Case 6, WWFE-II, UD test specimen

$\sigma_1(\sigma_2 = \sigma_3)$

kink could not be mapped, due to missing matrix 2ndTg-information for p_{hyd} >200 MPa



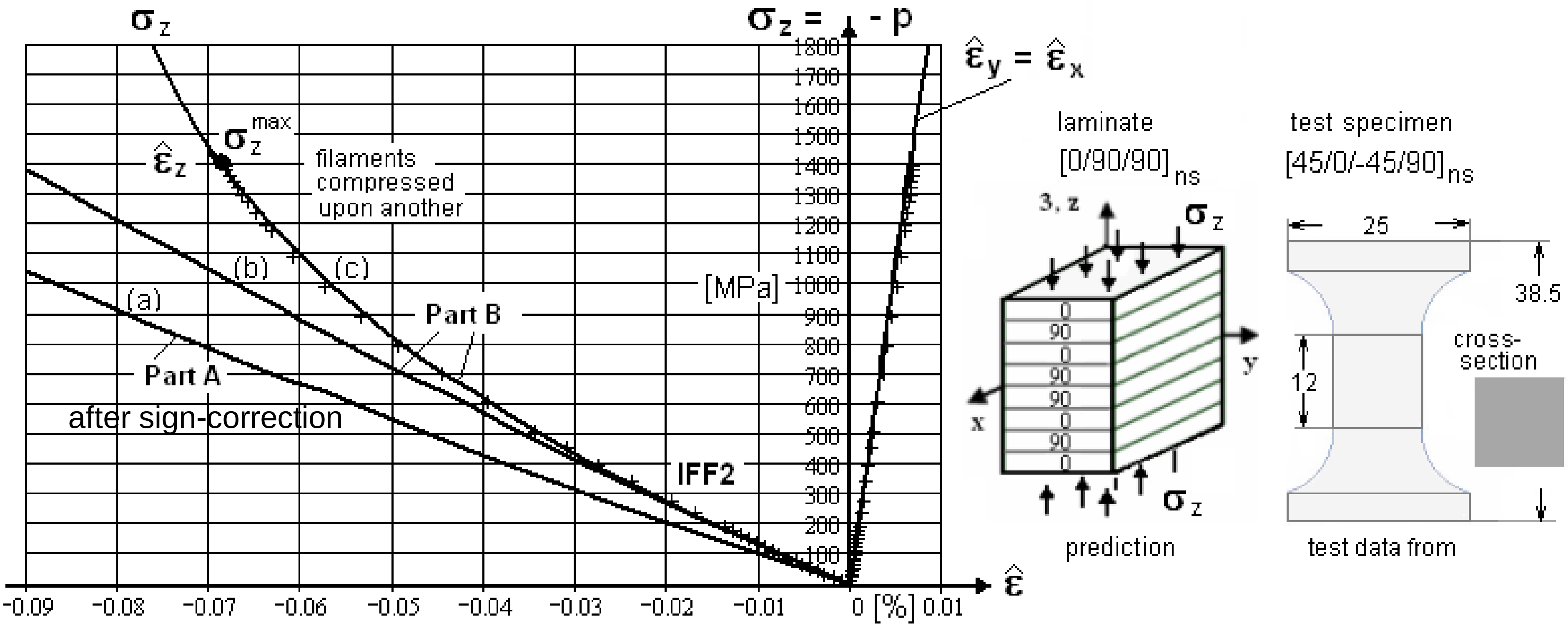
Was p_{hyd} correctly considered ?

Lesson Learnt: (1) No mapping possible, due to missing 2ndTg information!

(2) No explanation for oppositely directed slopes !

Not acceptable for model validation and design verification!

Test Case 12, WWFE-II, Laminate Test Specimen



Good Mapping <u>after</u> novel physical interpretation of test data

„Filaments are finally compressed to another which stiffens!“ by fitting the Part B data-improved **curve (b) to (c) as pressure-dependent increase of the lateral stiffness** → filament perpendicular E

Lesson Learnt: A structural failure cannot be described by a (material) SFC !

Conclusions wrt. Beltrami-based *Failure Mode Concept*

- **The FMC – applied to UD material - is an efficient concept,**
  that improves prediction + simplifies design verification.
  Formulation basis is whether the material element experiences
  a *volume* change, a *shape change* and *friction* .

- **Delivers a combined formulation of *independent modal failure modes*,**
  without the well-known drawbacks of global SFC formulations
  (which *mathematically combine in-dependent failure modes)* .

- **The FMC-based 3D UD Strength Failure Conditions are simple but**
  describe physics of each single failure mechanism pretty well.

Conclusions WWFE

<u>FMC</u>-based UD Static Strength Failure Conditions :

1) 2D stress case: Test data mapping was successful, validation achieved
2) 3D stress case: Was successful if <u>reliable</u> 3D test data were available.
   This was just partly the case.

The never funded single author is proud on this success, against institutes in the world!

<u>QINETICS statement:</u> The reader shall form a view of my mapping accuracy in the WWFE-II - TCs 2, 3, 4, 12 (doubting my physically-based interpretations). Please, form a view.

General Lessons Learnt *from the WWFEs:*

- **Prediction is not possible if physically necessary friction values must be considered (for shear fracture prediction).**

  Global SFCs do not consider them, therefore have shortcomings.
- **Validation of failure conditions requires a <u>uniform stress field in the critical domain</u>. This was not always given for the WWFE test cases.**

Generating reliable 3D test data
is a bigger challenge than generating a theory !

I think
we all must step *on the accelerator*
improving non-sufficient strength criteria for textile business,
like me, in the unpleasant situation, below !

Kaiman mother "Maria" (4 m) protecting her eggs

Literature

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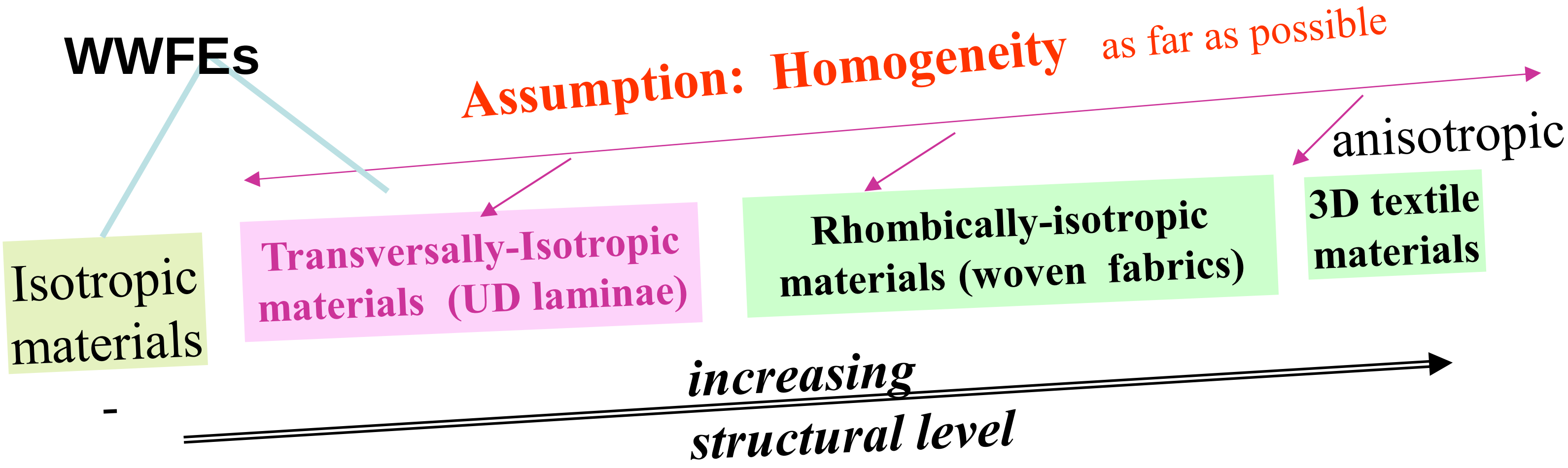
[VDI2014] VDI 2014: German Guideline, Sheet 3 *"Development of Fiber-Reinforced Plastic Components, Analysis".* Beuth Verlag, 2006. (in German and English).

Material Homogenizing (smearing) and Modelling



WWFEs

Assumption: Homogeneity as far as possible

anisotropic

Isotropic materials

Transversally-Isotropic materials (UD laminae)

Rhombically-isotropic materials (woven fabrics)

3D textile materials

-

increasing structural level

For the assessment of the stresses
in the critical ‘points‘ of the homogenized material
validated Strength Failure Conditions are to be provided !

Driving Motivation of the author for his contribution

1. **As an engineer from industry - to figure out an**

   Engineering Approach where all Model Parameters can be Measured and

2. **As learned from an insistent experience, that basically**

   or grey cast iron
   a tri-axially compressed brittle *porous* concrete can be described like
   a tri-axially tensioned ductile *porous* metal (in final ‘*Gurson’ domain*) !

   which means that a linking information can be helpfully used, *because*

   mechanical behaviour of very different structural materials can

   - ***possess similar material behaviour***
   - ***belong to the same class of material symmetry .***

Reasons to perform the WWFE-II for tri-axial stress states

with hydrostatic pressures far beyond usual structural applications:

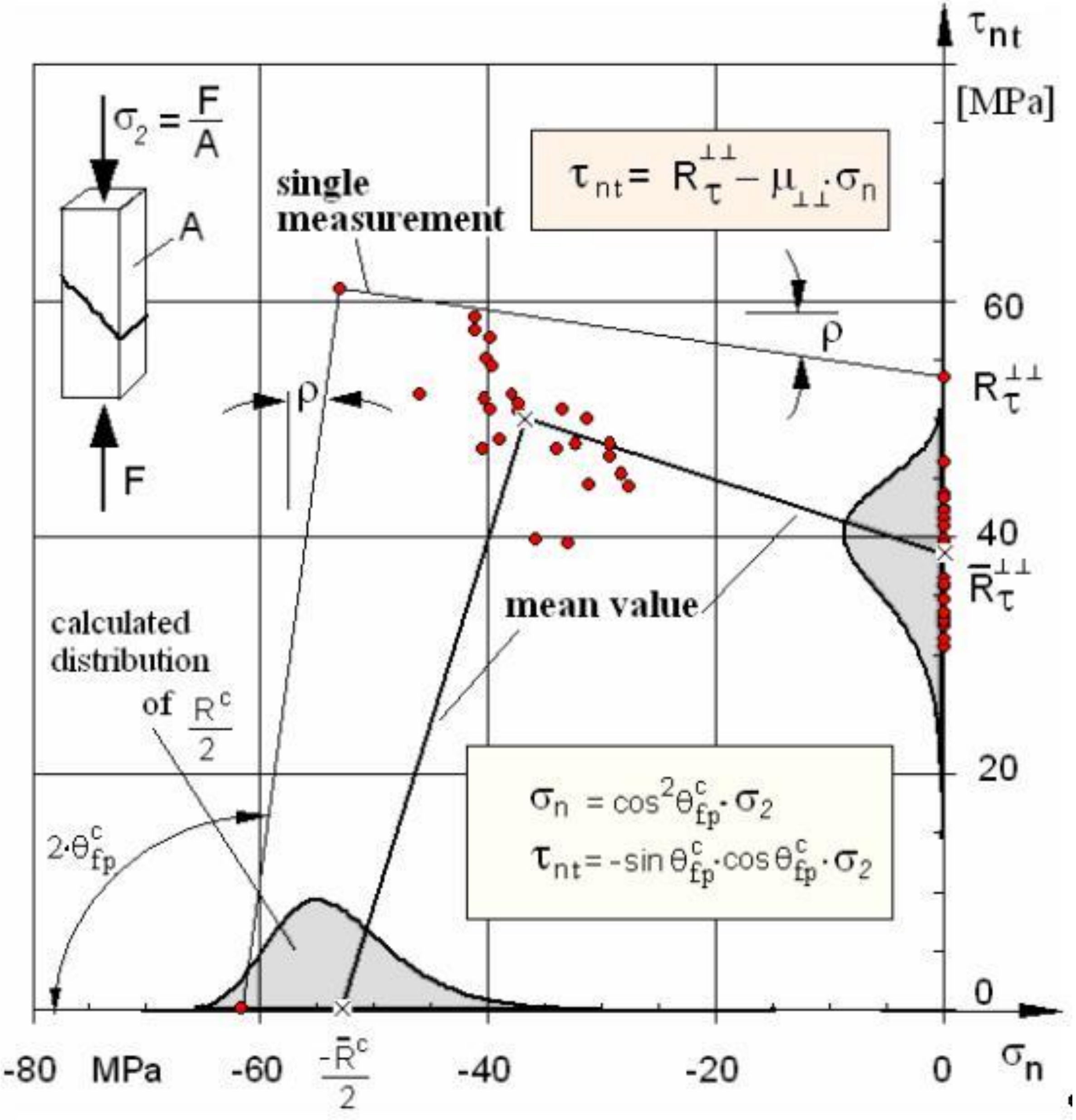
Tri-axial failure states are often encountered :

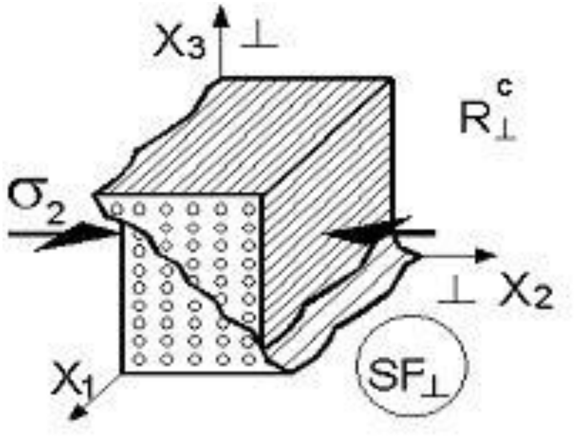
** in submarines, rotor blade roots, bolted and screwed joints,*
** bearings such as sealed polymer bearing cartridges pressurized up*
to 600 MPa = 6000 bar,
** in cases of impact and ballistics, and other applications like*
high pressure vessels,
anchor points of tension cables in civil engineering,
** load carrying UD hangers of helicopter blades,*
load introduction points,
** CFRP tubes for deepwater umbilicals, underwater blast.*

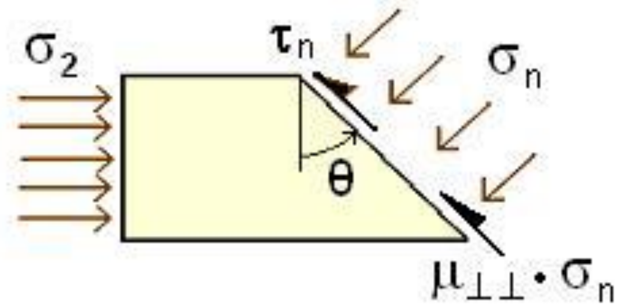
In consequence,

there is a strong need to validate strength failure conditions in the multi-axial, very high compression domain, too.

Estimation of $\mu_{\perp\perp}$ from Measurement of Friction Angle







or $\tan\rho = \mu_{\perp\perp}$

$$\mu_{\perp\perp} = -\cos\left(\frac{2 \cdot \Theta_{fp}^{c}{}^{\circ}}{180^{\circ}} \cdot \pi\right)$$

Design Verification :

Reserve Factor is load-defined : *RF = Failure Load / applied Design Load*

Material Stressing Effort : *Eff = 100% if RF = 1* (Anstrengung)

Material Reserve Factor : f_{Res} = *Strength / Applied Stress*

If linear situation: $f_{Res} = RF = 1 / Eff$

Demonstration of MoS > 0 or RF = MoS + 1 > 1