Missing Links in the Isotropic Macromechanical Building

Search of the Normal Yielding NY mode, inherent to **Plexiglass**

- Derived on basis of Material Symmetry Facts and Cuntze's Failure Mode Concept FMC –

From Strength Model Validation, SFC $F = 1$, by failure stress mapping **to Strength Design Verification**
For use in mechanical engineering and in civil engineering

CONTENTS

- Searching the Missing Link Normal Yielding in the Macromechanical 'Building'
- Experience with Materials and Hints from Material Symmetry Facts
- Cuntze's Failure-Mode-Concept (FMC), basis to derive Strength Failure Conditions (SFCs)
- Un-usual Behavior of Plexiglass in the Tensile Domain
- Derivation FMC-based Strength Failure Condition (*criterion)* NY for Plexiglass (PMMA)
- NY-SFC, Application to the isotropic Plexiglass Material

Conclusions

Results of a decades-lasting, time-consuming, never funded hobby.

Prof. Dr.-Ing. habil. Ralf Cuntze VDI. 85226 Markt Indersdorf, Ralf_Cuntze@t-online.de, 0049 8136 7754

* Searching Normal Yielding as Partner of Shear Yielding

DESIGN:

In the development of structural components the application of 3D-validated ***strength*** failure conditions SFCs ('criteria'), $F = 1$, is one essential pre-condition for achieving the required fidelity for structural product certification. What is to provide :

- Yield Failure Conditions (ductile behavior) for the non-linear analysis of the material and for the design verification of the Onset-of-Yielding design limit
- Fracture conditions to verify that Onset-of-Fracture does not occur, and this for brittle and ductile behavior. Fracture Failure Conditions confine, when meeting $F_{fracture} = 1$ the load-driven growth of a yield failure surface F_{yield}

Instead of the SFC formulation $F = 1$ with F termed Failure function, equivalently, the more plausible so-called Material Stressing Effort (Werkstoffanstrengung) $Eff = 100\%$ can be used.

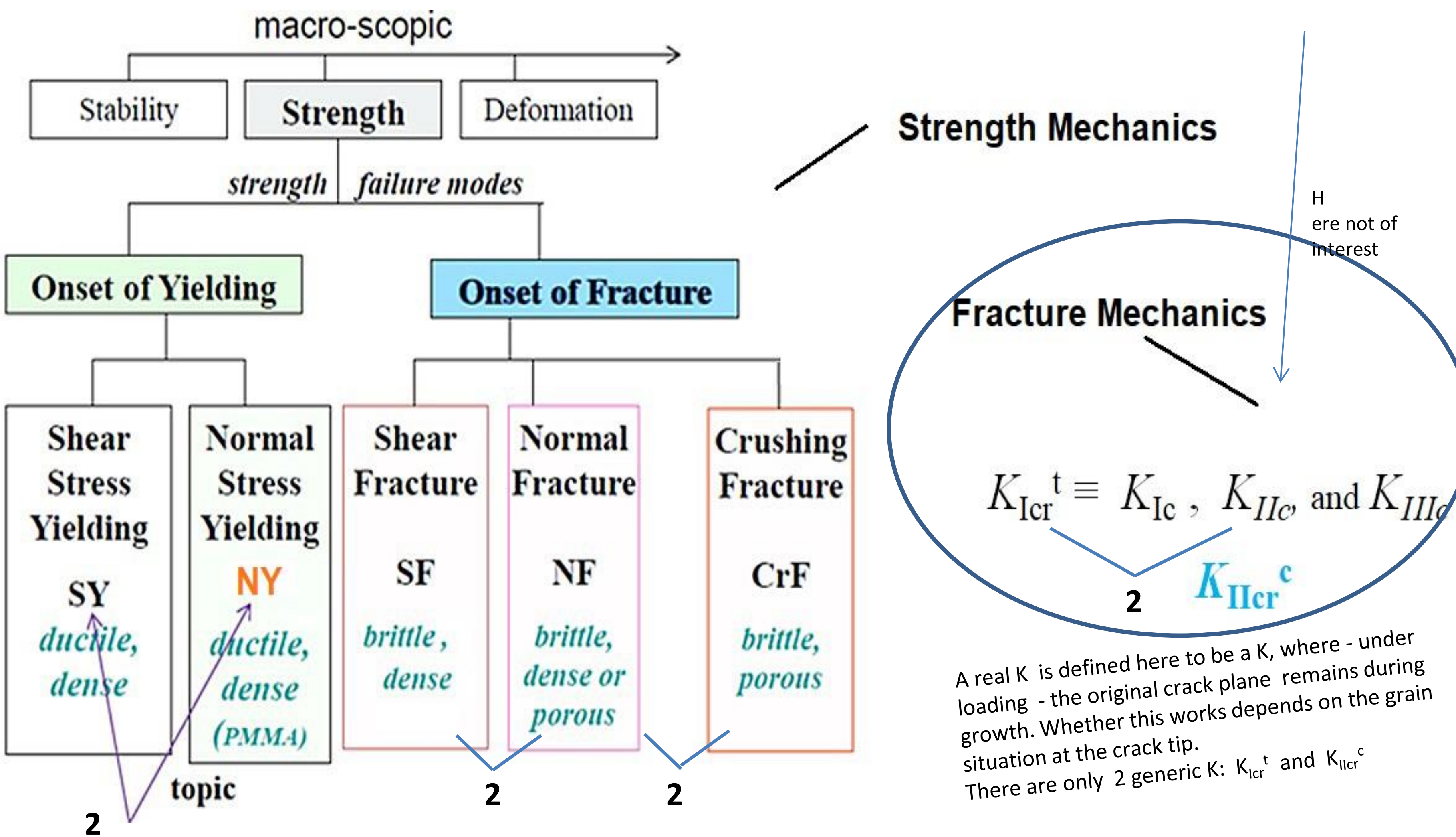
VISION: A 'complete' Macromechanics Building

Since more than two decades the author believes in a macroscopically-phenomenological 'complete classification' system, where all strength failure types are included:

- Dense materials: Normal Fracture NF; Shear stress Yielding SY, followed by Shear Fracture SF
- Porous materials: NF; SF is replaced by Crushing Fracture CrF.

What is missing in a complete system? ⟶ ***Normal Yielding NY***

Assumed system of strength failure modes and the searched missing links *NY*, K_{IIcr}^{c}



NY is known for a long time, but not in structural mechanics.

An explanation for the 'Not known' is that a well describing yield failure condition F^{NY} was missing.

→ Hence a *SFC*, $F = 1$, for NY shall be derived and visualized as Yield Failure Body.

* Experience with Materials and Hints from Material Symmetry Facts

1 If a material element can be homogenized to an ideal crystal (= frictionless), then, material symmetry demands for the Isotropic Material are:

- 2 e*lastic 'constants', 2 strengths, 2 fracture toughness values, 2 strength failure modes* for *yielding* (NY, SY) and for *fracture* (NF, SF), and just *2 'basic' invariants* I_1, J_2 are needed (*This is valid as long as a one-fold acting failure mode is to describe by the distinct SFC and not a multi-fold failure mode)*
- 1 *physical parameter (such as coefficient of thermal expansion CTE, coefficient of moisture expansion CME, material friction, etc.)*

For the transversely-isotropic UD- materials the witnessed respective numbers are 5 and 2. One also needs just 5 invariants to formulate the 5 SFCs)

2. Mohr-Coulomb requires for the real crystal another inherent parameter,
   - the *physical parameter the 'inherent material friction'* μ

3 Fracture morphology finally gives evidence

- Each strength corresponds to a distinct *strength failure mode* and to a distinct *strength failure type*, to (NY, SY) or (NF, SF)

4 Densely packed frictional material experiences dilatation when sheared and 'spherical' grains must move upon another.

A generic number 2 seems to exist for isotropic materials

Material Symmetry-dedicated Derivation of Cuntze's FMC-based SFCs

- Each failure mode represents 1 independent failure mechanism,

  and thereby represents 1 piece of the complete (global) failure surface

- Each failure mechanism is governed by 1 basic strength (this is witnessed)
- Each strength failure mode can be represented by 1 strength failure condition SFC.

  Therefore, equivalent stresses can be computed for each mode. This is of further advantage when deriving S-N curves and Haigh diagrams with minimum test effort

→ Consequently, the FMC-approach requires the interaction of all (isotropic 2) modes!

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + ...} = 1 = 100\% , \; if \quad \text{Onset of Failure}$$

***Eff* = 1 represents the mathematical description of the failure body .**

The interaction of adjacent failure modes is modelled with the 'series failure system". That permits to formulate the total material stressing effort from all activated failure modes = 'accumulation' of *Eff*s or sum of all the failure danger proportions. The value of the interaction exponent *m* depends on the ratio R^c/R^t. For brittle materials with $R^c/R^t > 3$ the value is about $m = 2.6$. A smaller *m* is on the safe side. For slightly brittle materials R^c/R^t is about 5 and more from mapping experience in the transition zone of the two modes.

LL: The use of the entity Eff excellently supports 'understanding the multi-axial strength capacity of materials':
Eff (Werkstoffanstrengung) cannot be > 100%.
Eff must become zero with {σ} is zero. Eff is linked to the German term "Kraftanstrengung".

Hints from Beltrami for the generation of Strength Failure Conditions SFCs

The Hypothesis of Beltrami states:

"At onset-of-yielding, the strain energy density W in a material element consists of two portions; one describing the strain energy due to a change in volume (dilatation, dilation in US) and another strain energy proportion due to a change in shape (distortion)".

These two portions can be related to invariants (now physics-based!) :

* dilatational energy to I_1^2 for a volume change and
* distortional energy to $J_2 \equiv$ ('Mises') for a shear distortion under volume consistency, forming a shape change of the material element.

- If friction is activated under compression then the frictional energy is to consider by applying I_1.

On the following slide the dedication of above invariants is exemplarily applied →

Schematic Example for the use of Invariants $(I_1 < 0)$, slightly porous, brittle

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III}) = f(\sigma) , \quad 6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = f(\tau) \text{ 'Mises' invariant}$$

$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III}) \cdot (2\sigma_{II} - \sigma_I - \sigma_{III}) \cdot (2\sigma_{III} - \sigma_I - \sigma_{II}),$$

$$3 \cdot \sigma_{oct} = \sigma_I + \sigma_{II} + \sigma_{III} = \sigma_t + \sigma_n + \sigma_t \ ; \ 9 \cdot \tau_{oct}^{\ 2} = 6J_2 = 4 \cdot (\tau_{III}^{\ 2} + \tau_I^{\ 2} + \tau_{II}^{\ 2}), \ \tau_{II} = \max \tau (mathem.)$$

$\sigma_I, \sigma_{II}, \sigma_{III}$ are principal stresses, $\sigma_I > \sigma_{II} > \sigma_{III}$ are mathematical stresses (> means more positive)

shape change — friction — volume change

$$F^{SF} = c_1^{SF} \cdot \frac{3J_2 \cdot \Theta^{SF}}{(\bar{R}^c)^2} + c_2^{SF}(\mu) \cdot \frac{I_1}{\bar{R}^c} + c_3^{SF} \cdot (\frac{I_1}{\bar{R}^c})^2 = 1$$

'Mises Cylinder'

Above *general* strength failure condition SFC basically describesShear Fracture. It is normalized by the compressive strength)

Two-fold failure danger can be modelled by using the well known invariant J_3.

The non-circularity function Θ^{SF} includes d^{SF} as non-circularity parameter,

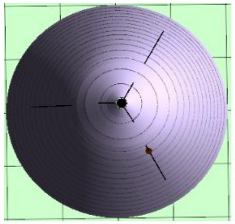
ϑ is Lode angle of the investigated meridian

$$\Theta^{SF}(J_3) = \sqrt[3]{1 + d^{SF} \cdot \sin(3\vartheta)} = \sqrt[3]{1 + d^{SF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}}$$

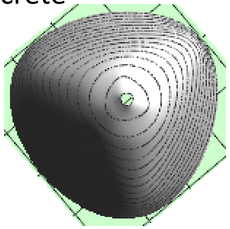
It represents the 120°symmetry of isotropic materials, caused by the equivalency of the 3 principal stresses and directions (see the following fracture bodies).

contour lines of the failure body

Failure body
rotational symmetric (circular): top view



120°-rot. symmetric (non-circular):
Normal Concrete test data



R , f = strength in general. $\bar{R}$ = average strength, used in test data mapping

A Rotational Symmetric Failure Body becomes 120°-symmetric if a failure acts two-fold

In the context above different effects are to discuss:

- Interaction (Mixed) Strength Failure Modes: Different failure modes may be activated by the acting stress state. The interaction of both the activated fracture mode types Normal Fracture NF with Shear Fracture SF under compression increases the danger to fail! → NF with SF and NY with SY
- Multi-fold Failure Modes: The acting stress state with maximally equal orthogonal stresses activates the same mode multi-fold. (Example isotropic material: $\sigma_I = \sigma_{II}$, $\sigma_I = \sigma_{II} = \sigma_{III} \rightarrow \sigma_{hyd}$; 3-fold)

A multi-fold failure mode decreases danger to craze ! $R_{NY}^{tt} > R_{NY}^{t}$ *(weakest-link effect),* $I_1 > 0$ (2 **NY**)

A multi-fold fracture mode decreases danger to fail ! $R^{cc} > R^{c}$ *(redundancy effect),* $I_1 < 0$ (2 **SF**)

Bi-axial compression may activate a critical axial tensile strain, which must be checked.

Physics-based 'isotropic' SFC, usually consider a failure mechanism just one-fold and do not capture the bi-axial effect of $\sigma_I = \sigma_{II}$ or of hydrostatic tensile or compressive failure stress states.

This must be considered by an additional term:

J_3 is used if the same 'failure mode' occurs two times.

→ Then, a 120° rotational-symmetric failure body of isotropic materials can be mapped.

Short Description of the un-usual Behaviour of Plexiglass in the Tensile Domain

Glassy (amorphous, brittle) polymers like polystyrene (PS), polycarbonate (PC) and PolyMethylMethacrylate (PMMA, plexiglass) are often used structural materials.

They experience two different yield failure types, namely crazing and shear stress yielding that is often termed shear-banding, too.

- Crazing may be linked to Normal Yielding (NY) which precedes the crazing-following fracture.
- Crazing occurs with an increase in volume and shear banding does not. Therefore, the dilatational I_1^2 must be employed in the approach for tension $I_1 > 0$.
- Under compression, brittle amorphous polymers usually shear-band (SY) and with it they experience friction. Therefore, I_1 must be employed in the approach for $I_1 < 0$ in order to consider material internal friction in the traditional way.

For obtaining the complete yield failure body its parts NY and SY are to interact, and this is performed like for the fracture failure modes.

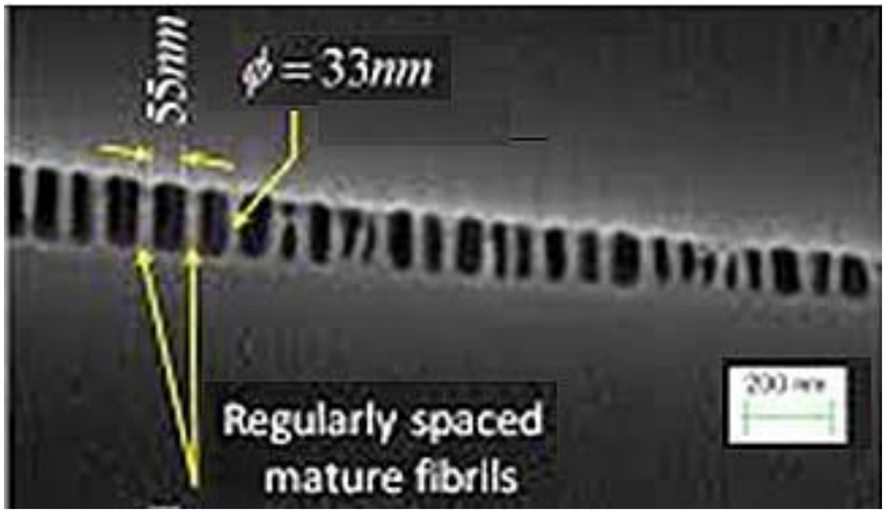
Reminder on HMH-linked 'Mises-cylinder' for 'Onset-of-Shear Stress Yielding SY*: There is no friction acting and therefore yield strengths for compression and tension are the same* $\bar{R}_{0.2}^{\,c} \equiv \bar{R}_{0.2}^{\,t} = R_{p0.2}$, *, in which the superfluous suffix p practically has nothing to do with proportional). HMH means frictionless yielding and therefore it forms a cylinder.*

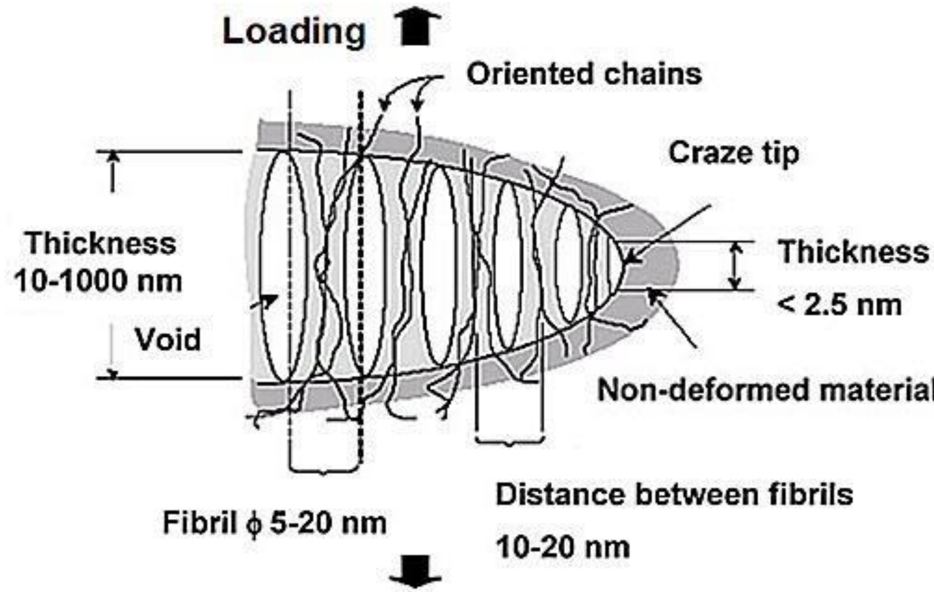
PMMA mechanisms, SEM image of a craze in Polystyrene Image *[created by Y. Arunkumar]*

Crazing involves the formation of fibrils bridging two neighboring layers of the un-deformed polymer.

These elongate and locally fail which leads to a formation or an elongation of an existing micro-crack,

This micro-crack is going to be simulated under Fracture Mode-I loading conditions.





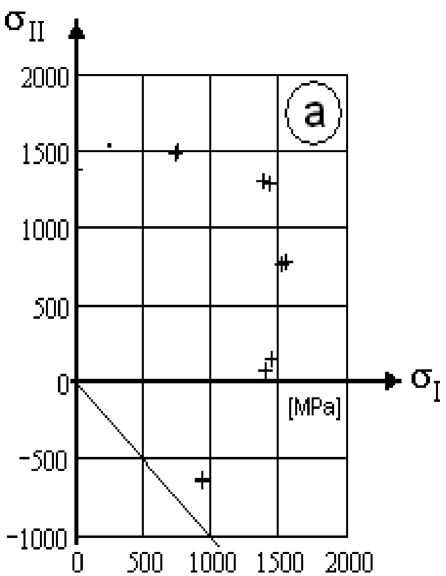
Onset-of-Shear Yielding test data SY for a steel versus Onset-of-Crazing NY

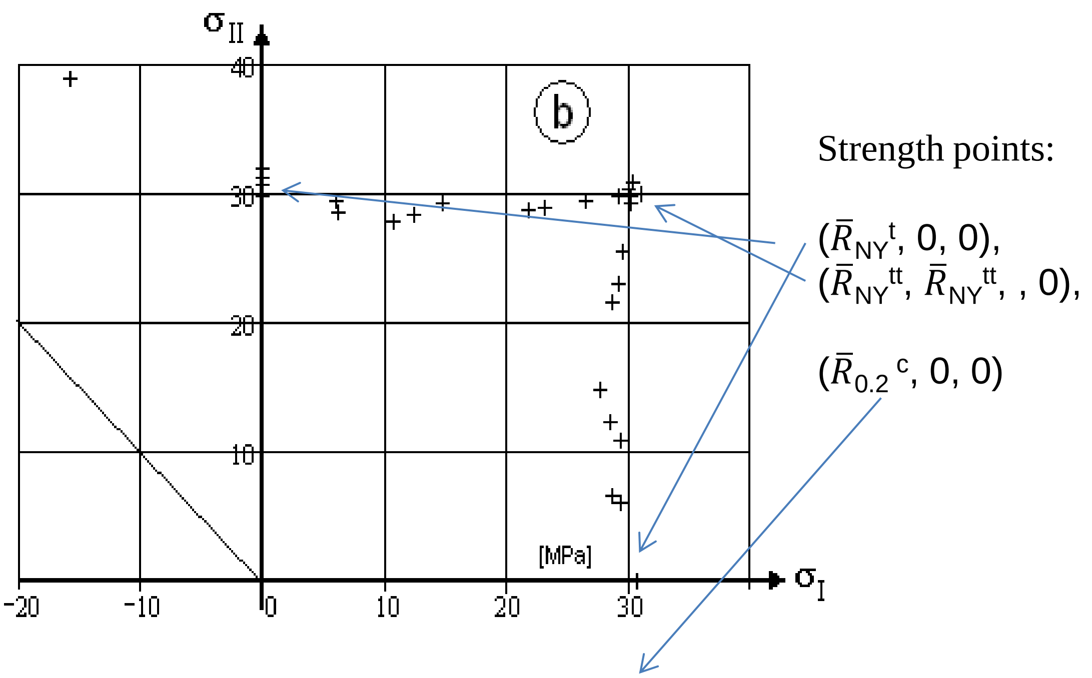
The yield failure type crazing shows a **curiosity** under tensile stress states:

A non-convex shape exists for Onset-of-Crazing $\bar{R}_{NY}^{t}$. NY is followed by the crazing-driven Normal Fracture NF for which - due to the similar shape as reported in literature – the NY-SFC can be used too.

Under compressive stress states the usual shear band yielding SY occurs and later final shear fracture SF occurs. For both, SY and SF, the same SFC can be also applied.

Due to the fact that the Onset-of Crazing and the Onset-of-shear yielding associated stresses ("strengths") are not accurately defined the usual denotations $\bar{R}_{NY}^{t}$ and $\bar{R}_{SY}^{c}$ are used. This has no influence on the logic followed here.



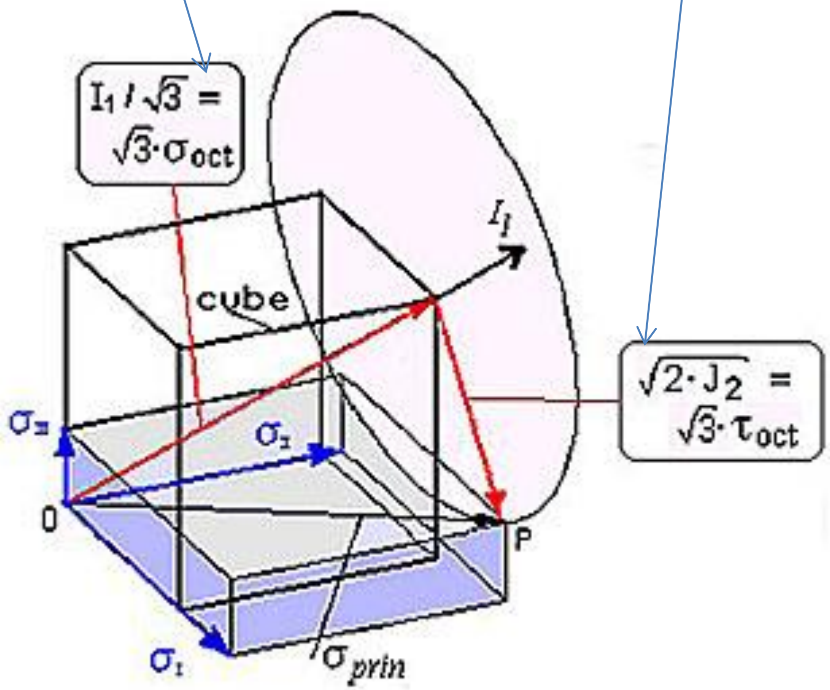


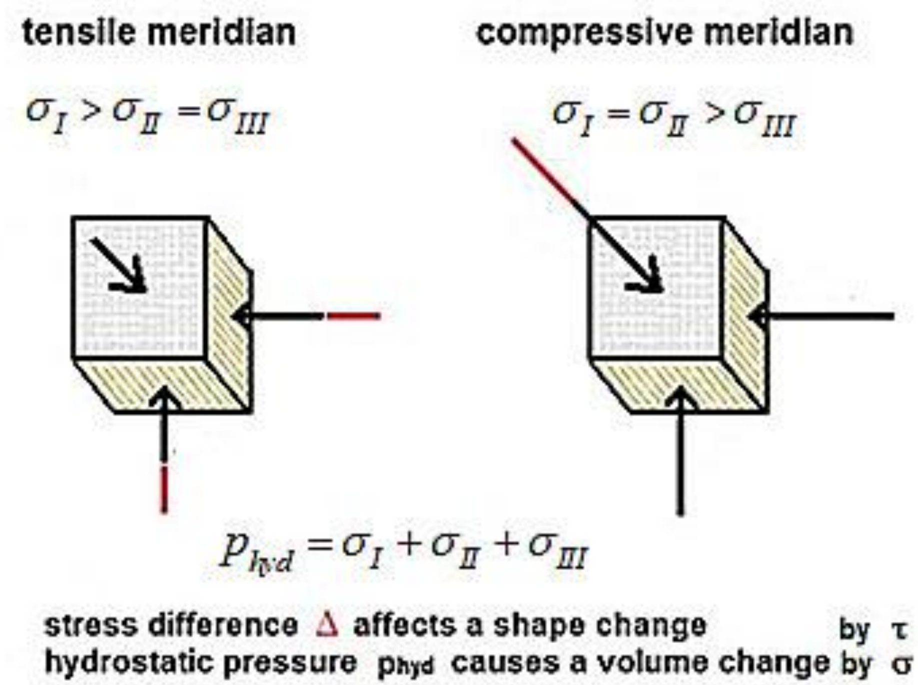
Reminder on HMH-linked 'Mises-cylinder' for 'Onset-of-Shear Stress Yielding SY: *There is no friction acting and therefore yield strengths for compression and tension are the same* $R_{0.2}^{c} = R_{0.2}^{t}$ *(*$\equiv R_{p0.2}$*, in which the superfluous suffix* $_{p}$ *practically has nothing to do with proportional). HMH means frictionless yielding and therefore it forms a cylinder.*

How can a 2D-Test Data Set be Combined with a 3D-Test Data Set ?

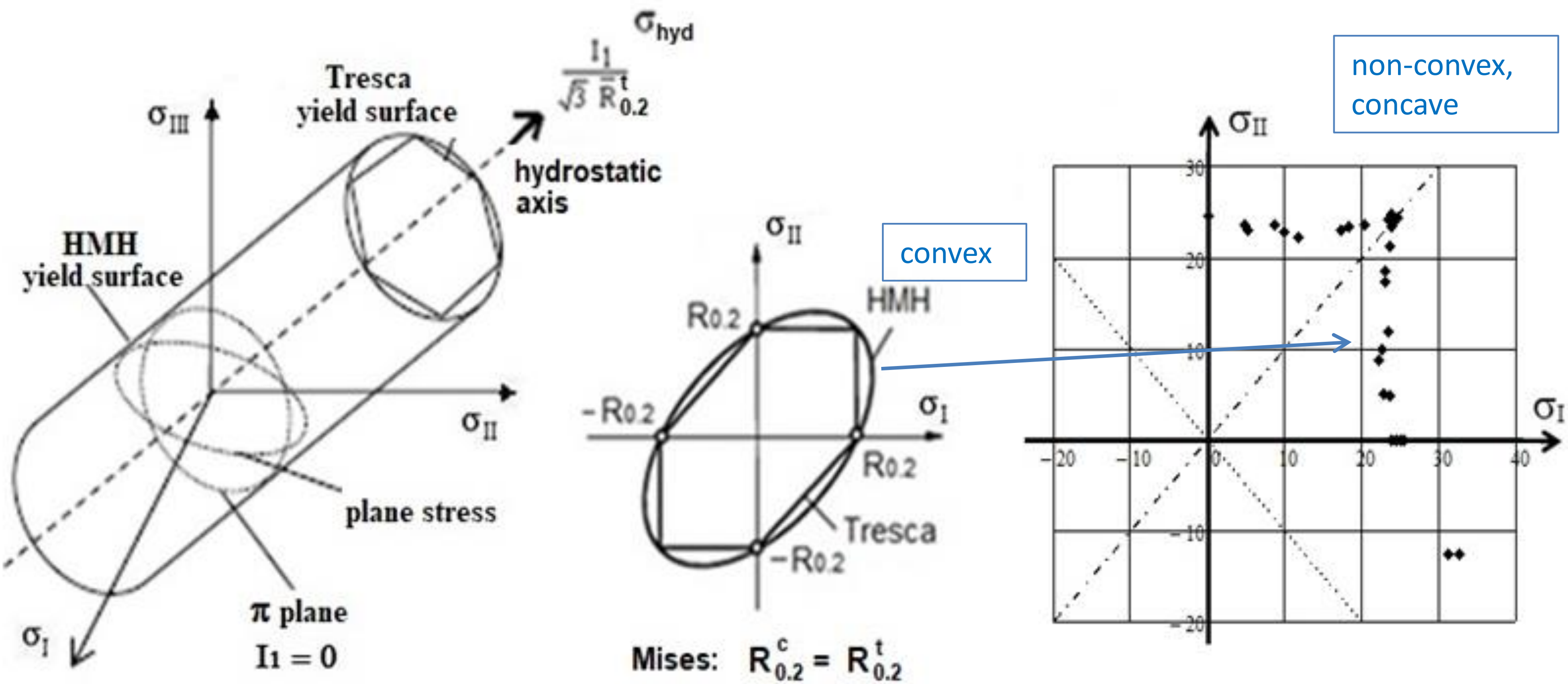
A 2D-data set and a 3D-data set can be put together in a Lode-Haigh-Westergaard diagram. The fracture body is thereby rendered using the Haigh-Westergaard-Lode coordinates with

$I_1/\sqrt{3}$ as y-coordinate and $\sqrt{2 \cdot J_2}$ as x-coordinate.





For Comparison: NY versus 'Mises' and Tresca



Hencky-Mises-Huber HMH yield surface with the Tresca yield surface

(engineering yield strengths are used)

Which Test Series are Typically Run?

Required for the demonstration of a qualified mapping
by the SY-SFC and by the NY-SFC
is the mapping of tensile meridian and compressive meridian
as the essential cross-sections of the yield failure body.
These tests are required and are also usually performed.

$p_{hyd} + F_{axial}^{tension} \quad \leftrightarrow \quad p_{hyd} + F_{axial}^{compression}$

The definitions of the two meridians are given below.
Associated test stress states are formulated in principal stresses and in mathematical stresses:

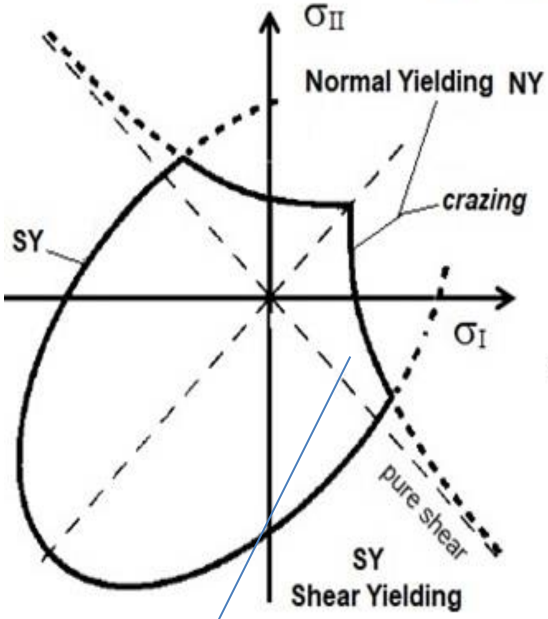
tensile meridian — compressive meridian

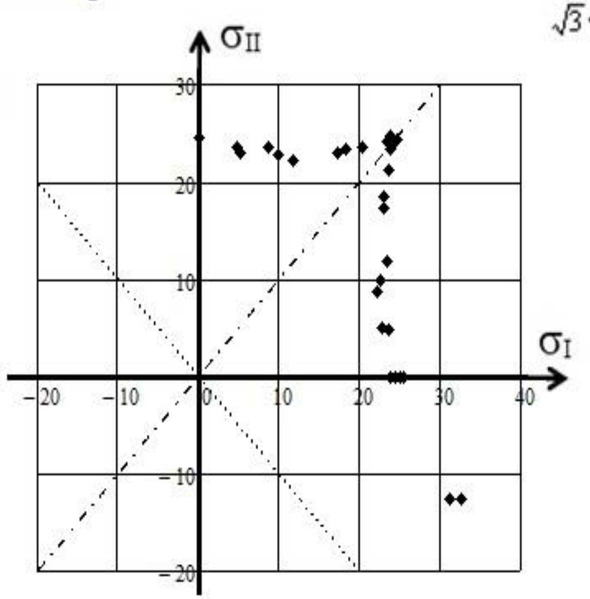
$$(\sigma_{ax}^{t} - p_{hyd}, -p_{hyd}, -p_{hyd}) = (\sigma_I, \sigma_{II}, \sigma_{III}) \rightarrow \sigma_I > \sigma_{II} = \sigma_{III} \quad \Leftrightarrow \quad (-p_{hyd}, -p_{hyd}, \sigma_{ax}^{c} - p_{hyd}) = (\sigma_I, \sigma_{II}, \sigma_{III}) \rightarrow \sigma_I = \sigma_{II} > \sigma_{III}$$

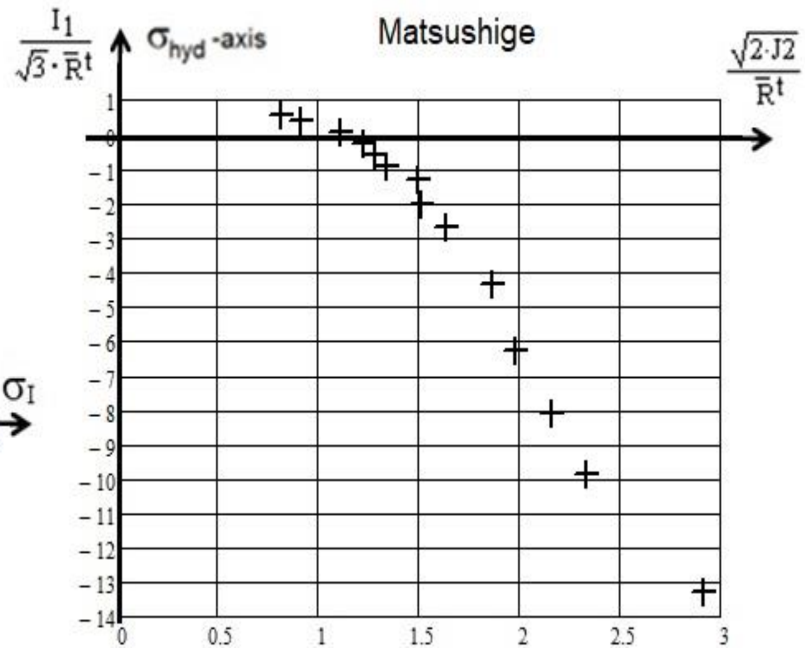
Which NY and SY Test Data Sets were available for PMMA?

Sternstein-Myers [Ste73] and a SY-3D-data set from Matsushige [Mat73]

Sternstein- Ongchin







Sternstein's mapping idea with his
2D test data set in the principal stress plane

Matsushige 3D-PMMA test data set
rendered
in Haigh-Westergaard-Lode coordinates

Only those who search long enough will find

$\rightarrow$ *these test data sets from Sternstein-Myers and Matsushige*
had to be harmonized by the author on basis of literature information !

’Combining’ 2 PMMA Test Data Sets, NY and SY, being of Different Origin

(1) Sternstein-Myers performed **2D** (bi-axial) experiments on craze initiation on the surface of thin-walled cylinders (tubes). The loadings were axial tension plus internal pressure and tension plus torsion. Test temperature was 60°C. Therefore, - following literature - to match with Matsushige’s ambient temperature 23°C data, from consistency reasons the value of $\bar{R}_{NY}^{t}$ is to increase to become comparable with Matsushige.

(2) Matsushige performed **3D** (tri-axial) experiments on sealed (surface crazing is hindered) solid rods at 23° C, under axial tension plus p_{hyd}. The test specimen was pressurized within a chamber. This series along the tensile meridian, characterized by $\sigma_I > \sigma_{II} = \sigma_{III}$, contains the bi-axial point $(-\bar{R}^{cc}_{0.2}, -\bar{R}^{cc}_{0.2}, 0)$.

In comparison to the thin tube the solid rod experiences more bulk crazing than the more dangerous surface crazing. This is essential for test data evaluation.

The two different data sets however clearly outline crazing NY (Sternstein) and shear banding SY (Matsushige).
Therefore they can be used for mapping the course of the 2 yield failure mode test data and for SFC model validation.

However, a harmonization of the two data sets is necessary and follows now: ***in short***

After transferring into MPa, the Matsushige fracture stress values were much higher than the Sternstein ones. Following Sternstein et al the threshold stress value required for crazing (ten minutes hold-time) is about 3900 psi (1000 psi = 6.89 MPa) and for ambient temperature about 5500 psi is guessed, extrapolating his curve approximately. This has the consequence to increase the Sternstein test data by a correction factor of $f \approx 5500/3900$. The choice finally was $f = 1.3$.

FMC-based SFCs for NY and SY

Creator : R. Cuntze

Hyperboloide | Paraboloide

$$F^{NY} = \frac{x^2}{c_2^{NY2}} - \frac{(y-c_1^{NY})^2}{c_3^{NY2}} = 1 \text{ for } I_1 > 0 \quad \Leftrightarrow \quad F^{SY} = c_1^{SY} \cdot \frac{3J_2 \cdot \Theta^{SY}}{\bar{R}_{0.2}^{c\;2}} + c_2^{SF} \cdot \frac{I_1}{\bar{R}^c} = 1 \text{ for } I_1 < 0$$

Considering bi-axial strength (failure mode occurs twice, $\Theta \neq 1$). In Effs now, if possible.

$$Eff^{NY} = \frac{c_3^{NY} \cdot \sqrt{-c_2^{NY2} \cdot y^2 + \Theta^{NY2} \cdot (c_3^{NY2} + c_1^{NY2}) \cdot x^2} + c_2^{NY} \cdot c_1^{NY} \cdot y}{c_2^{NY} \cdot (c_3^{NY2} + c_1^{NY2})} \leftrightarrow Eff^{SY} = \sqrt{c_1^{SY} \cdot \frac{3J_2 \cdot \Theta^{SY}}{\bar{R}_{0.2}^{c\;2}}} = \sigma_{eq}^{SY} / \bar{R}_{0.2}^{c}, \quad Eff^{Mises} = \sqrt{\cdot \frac{3J_2 \cdot 1}{\bar{R}_{0.2}^{c\;2}}}$$

Onset of Crazing = Normal Yielding NY (for fracture similar)

c^{NY}, Θ^{NY} from the two points $(\bar{R}_{NY}^{t}, 0, 0)$ and $(\bar{R}_{NY}^{tt}, \bar{R}_{NY}^{tt}, 0)$ | Θ^{SY} from the point $(-\bar{R}_{0.2}^{cc}, -\bar{R}_{0.2}^{cc}, 0)$

3 curve parameters are to be determined for each mode

Two-fold failure danger can be modelled by using the well known invariant J_3 including d = non-circularity parameter

$$\Theta^{NY} = \sqrt[3]{1 + d^{NY} \cdot \sin(3\vartheta)} = \sqrt[3]{1 + d^{NY} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}} \quad \text{and} \quad \Theta^{SY} = \sqrt[3]{1 + d^{SY} \cdot \sin(3\vartheta)} = \sqrt[3]{1 + d^{SY} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}}$$

Lode angle ϑ, here se tas $\sin(3 \cdot \vartheta)$ with 'neutral'shear meridian angle 0°; tensile meridian angle 30°; compr. m. angle -30°

A yield body is rotational symmetric if $\Theta = 1$

Equation of the yield failure body: $Eff = [(Eff^{NY})^m + (Eff^{SY})^m]^{m^{-1}} = 1 = 100\%$ total effort, interaction

$0 < d^{NY} < 0.5$, $0 < d^{SY} < 0.5$, meridian angles $\vartheta°$: $\bar{R}_{0.2}^{t}$, 30° ; $\bar{R}_{0.2}^{tt}$, -30°; $\bar{R}_{0.2}^{c}$, -30°; $\bar{R}_{0.2}^{cc}$, 30°

with Lode-Haigh-Westergaard coordinates $x = \frac{\sqrt{2 \cdot J_2 \cdot \Theta^{NY}}}{\bar{R}_{NY}^{t}}, \quad y = \frac{I_1}{\sqrt{3} \cdot \bar{R}_{NY}^{t}}$

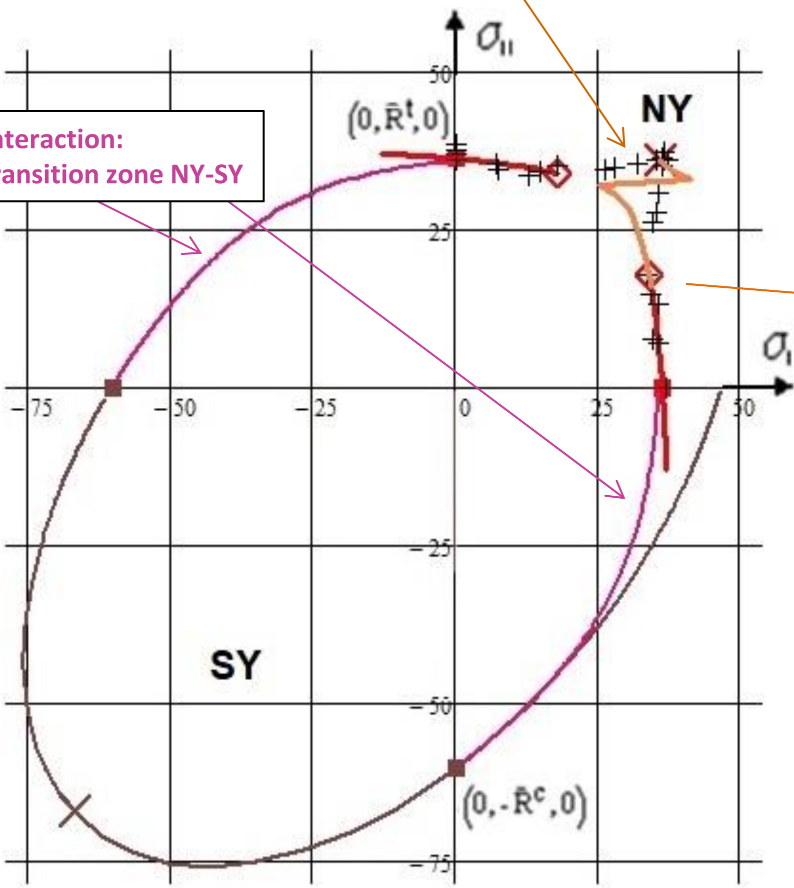
The NY yield failure body is 120°-symmetric in the deviator plane. This is captured by $\Theta(J_3)$, again.

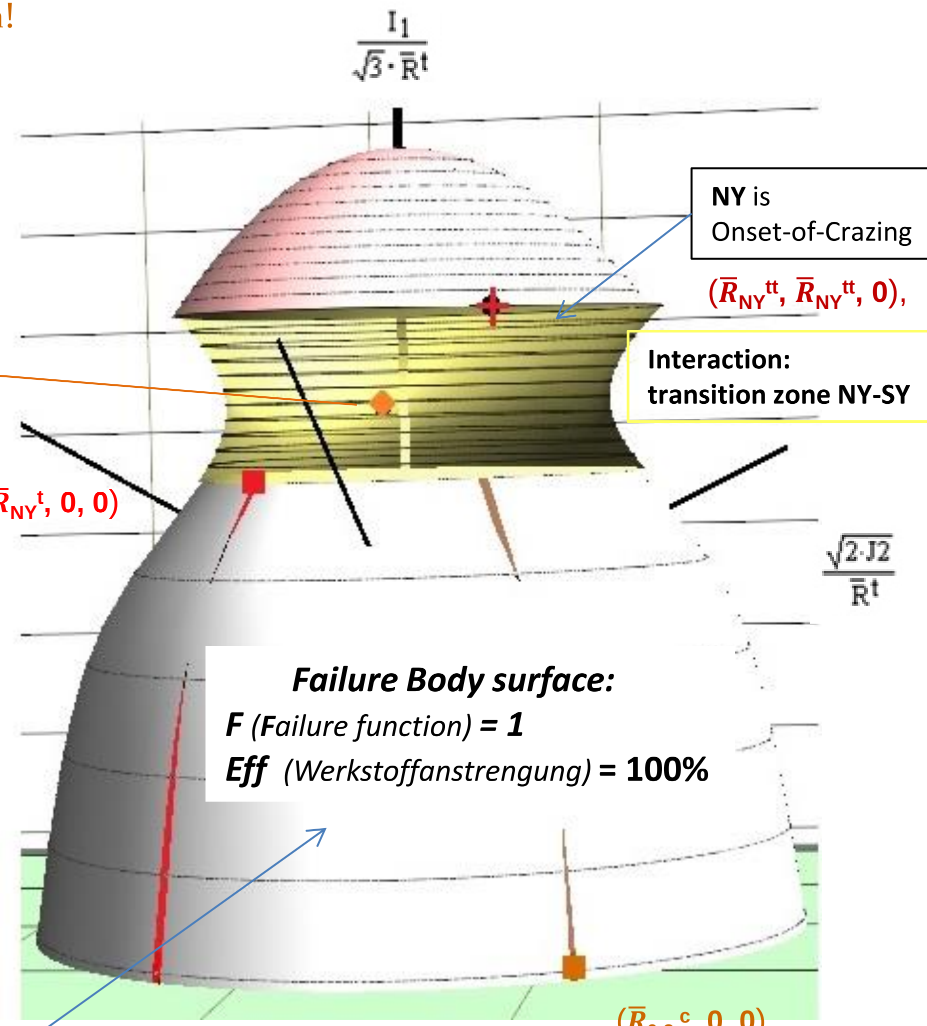
I_1^2 or y^2 stands for the experienced volume change. ***Above formula for NY is new !***

PMMA: Interacted NY-SY Failure Curve in Principal Stress Plane and full Yield Failure Body

The Mathcad 15 solver could not fully capture the non-convex situation!

What about the Drucker stability postulate?





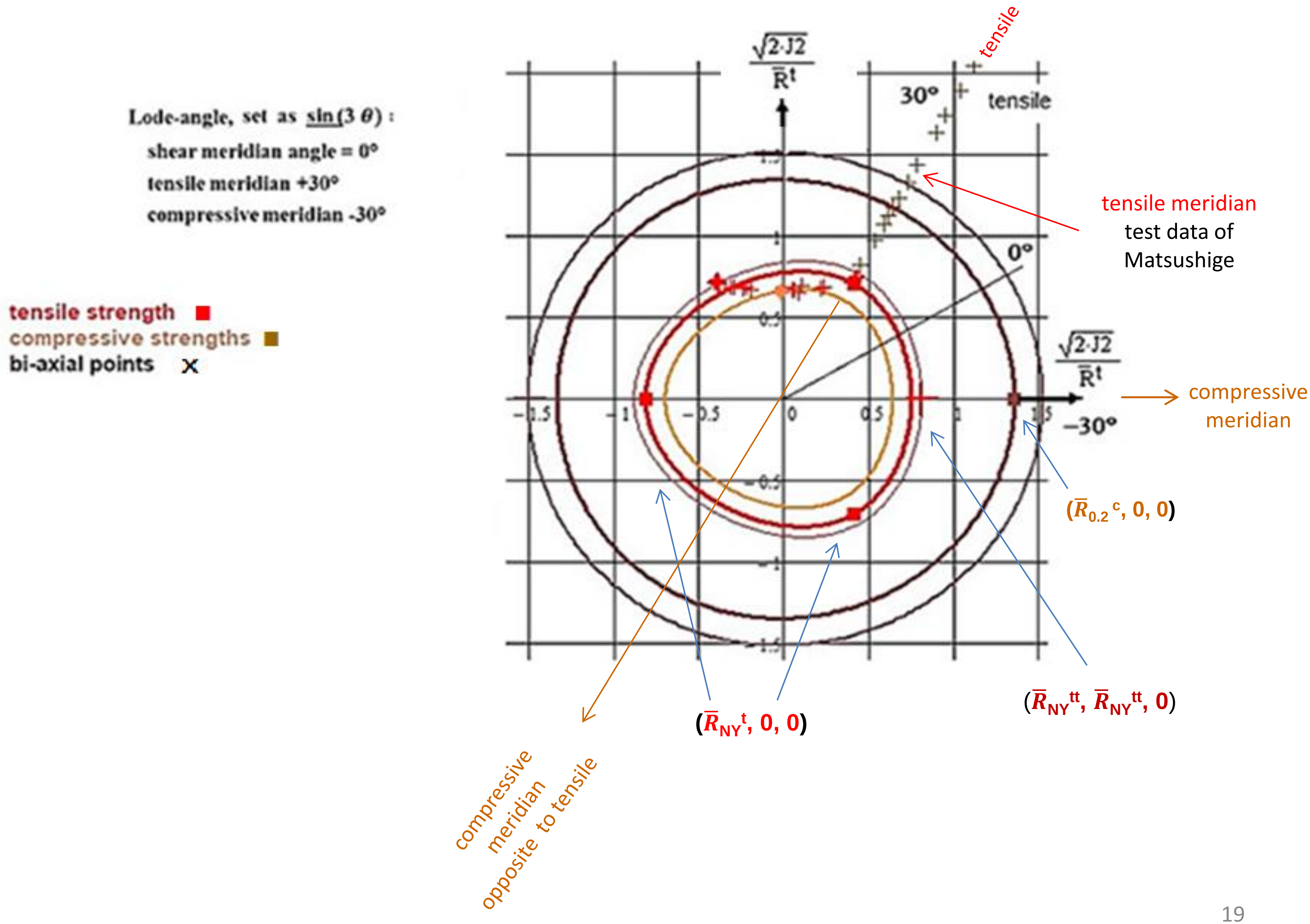
SY is
Onset-of–Shear Yielding

Depiction of the fracture body shape with some representative strength points

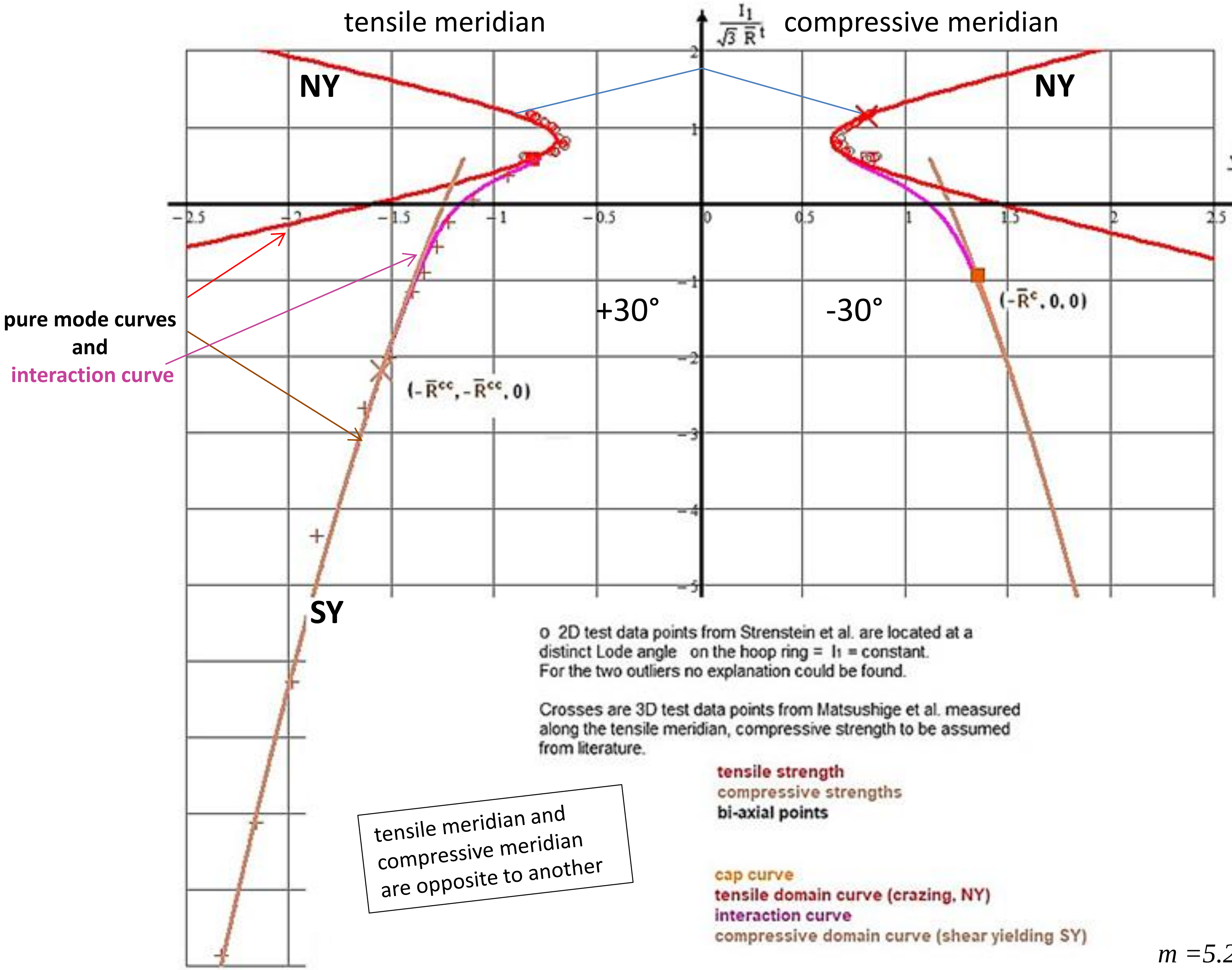
Failure Body surface = site of all failure stress state vectors for Onset-of-yielding or for Onset-of Fracture

I_1 = constant or Hoop Cross-sections of the Yield Failure Body

with 2D Sternstein and 3D Matsushige Test Data, projected onto the hoop cross -section



Main Axial Cross-sections (= meridians) **of the NY-SY Yield Failure Body** (120°-symmetric)



Tensile and compressive meridians of the fracture body (not optimized, the most negative test points are not shown in the figure

'Linear elastic' Strength Design Verification in Critical Stress Hot Spots: Onset-of-Yielding, *PMMA*
Determination of a Reserve Factor *RF* or of a Margin of Safety *MoS=RF - 1*

$$F^{NY} = \frac{x^2}{c_2^{NY2}} - \frac{(y-c_1^{NY})^2}{c_3^{NY2}} = 1 \text{ for } I_1 > 0 \quad \left(x = \frac{\sqrt{2 \cdot J_2 \cdot \Theta^{NY}}}{\bar{R}^t_{NY}},\ y = \frac{I_1}{\sqrt{3} \cdot \bar{R}^t_{NY}} \right) \Leftrightarrow F^{SY} = c_1^{SY} \cdot \frac{3J_2 \cdot \Theta^{SY}}{\bar{R}_{0.2}^{c\ 2}} + 0 = 1 \text{ for } I_1 < 0$$

$$Eff^{NY} = \frac{c_3^{NY} \cdot \sqrt{-c_2^{NY2} \cdot (\frac{I_1}{\sqrt{3}})^2 + \Theta^{NY2} \cdot (c_3^{NY2} + c_1^{NY2}) \cdot 2J_2} + c_2^{NY} \cdot c_1^{NY} \cdot \frac{I_1}{\sqrt{3}}}{c_2^{NY} \cdot (c_3^{NY2} + c_1^{NY2}) \cdot \bar{R}^t_{NY}} = \frac{\sigma_{eq}^{NY}}{\bar{R}^t_{NY}} \Leftrightarrow Eff^{SY} = \sqrt{\frac{3J_2 \cdot 1}{\bar{R}_{0.2}^{c\ 2}}} = \frac{\sigma_{eq}^{SY}}{\bar{R}_{0.2}^{c}} = Eff^{Mises}$$

Onset-of-Crazing = Normal Yielding NY (for fracture the same shape) Onset-of -Shear Yielding ('Mises')

c^{NY}, Θ^{NY} from the two points $(\bar{R}^t_{NY}, 0, 0)$ and $(\bar{R}^{tt}_{NY}, \bar{R}^{tt}_{NY}, 0)$ Θ^{SY} from the point $(-\bar{R}^{cc}_{0.2}, -\bar{R}^{cc}_{0.2}, 0)$

$\bar{R}^t_{NY} = 36.4$ MPa, $\bar{R}^{tt}_{NY} = 36.0$ MPa, ← Design Data → $\bar{R}^c_{0.2}{}^* = 59.6$ MPa , $\bar{R}^{cc}_{0.2} = 67$ MPa

$c_1^{NY} = 0.826$, $c_2^{NY} = 0.664$, $c_3^{NY} = 0.41$, $d_{NY}{}^{**} = -0.26$ $c_1^{SY} = 1.21$, $c_2^{SY} = 0.24$, $d_{SY} = -0.082$

* it is assumed that these values are the conventional yield values with remaining 0.2 % strain

** a negative d marks a 'bi-axial outward dent'

Load-defined Reserve Factor $RF > 1$ or Designer's question: How far can the loading be increased before reaching failure?

Model 'linear-elastic problem', permitted, then load ~ stress and $RF = R / Eff$, $RF = MoS - 1$.

Design Verification demonsatrated by $RF > 1$: using reduced strength values R

Safety Concept: Deterministic, global Design Factor of Safety $j = 1.10$ (= fixed value from a standard !)

Statistically reduced strength values $\bar{R} \to R$: $R^t_{NY} = 30$ MPa, $R^c_{SY} \equiv R^c_{0.2} = 50$ MPa

Loadings $\cdot\ j$: $\{\sigma\} = (j \cdot 0.64 \cdot R^t_{NY}, 0, 0)^T \to \sigma_{eq} = 0.704 \cdot R^t_{NY}$ $\Leftrightarrow$ $\{\sigma\} = (j \cdot 0.7 \cdot R^c_{0.2}, 0, 0)^T \to \sigma_{eq} = 0.77 \cdot R^c_{0.2}$

$$Eff^{NY} = \sigma_{eq}^{NY} / R^t_{NY} = 1.1 \cdot 0.64 \cdot R^t_{NY} / R^t_{NY} = 0.704 = 70,4\ \% \Leftrightarrow Eff^{SY} = 77\ \%$$

Interaction in Mode Transition domain: $Eff = [(Eff^{NY})^m + (Eff^{SY})^m]^{m^{-1}} = 84.6\ \% \Rightarrow RF = 1.4$. $m = 5.1$

Failure Index from *F*, |*F*| versus Material Stressing Effort *Eff* (Werkstoffanstrengung)

***Eff* is based on the traditional Proportional Loading, which means $\sigma_{eq} = R \cdot Eff$ or *Eff* is proportional to σ_{eq}**

Mapping of the 2 Modal Models as Material Model Validation (average values $\bar{R}$ to be used)

$$F^{NY} = \frac{x^2}{c_2^{NY2}} - \frac{(y - c_1^{NY})^2}{c_3^{NY2}} = 1 \text{ for } I_1 > 0 \ \left(x = \frac{\sqrt{2 \cdot J_2 \cdot \Theta^{NY}}}{\bar{R}^t_{NY}},\ y = \frac{I_1}{\sqrt{3} \cdot \bar{R}^t_{NY}} \right) \Leftrightarrow F^{SY}_{Mises} = \frac{3J_2 \cdot 1}{\bar{R}^{c\;2}_{0.2}} = 1 \text{ for } I_1 < 0$$

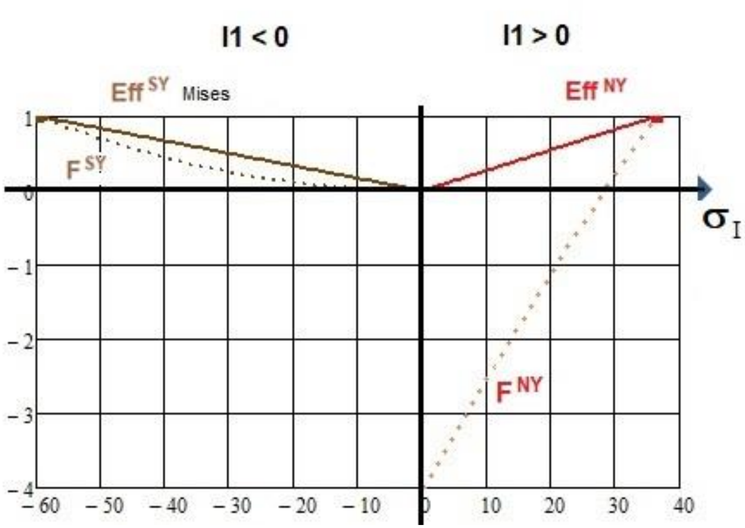
$$Eff^{NY} = \frac{c_3^{NY} \cdot \sqrt{-c_2^{NY2} \cdot (\frac{I_1}{\sqrt{3}})^2 + \Theta^{NY2} \cdot (c_3^{NY2} + c_1^{NY2}) \cdot 2J_2} + c_2^{NY} \cdot c_1^{NY} \cdot \frac{I_1}{\sqrt{3}}}{c_2^{NY} \cdot (c_3^{NY2} + c_1^{NY2}) \cdot \bar{R}^t_{NY}} = \frac{\sigma^{NY}_{eq}}{\bar{R}^t_{NY}} \Leftrightarrow Eff^{SY}_{Mises} = \sqrt{\frac{3J_2}{\bar{R}^{c\;2}_{0.2}}} = \frac{\sigma^{SY}_{eq}}{\bar{R}^c_{0.2}} \leftarrow \frac{3J_2 / (Eff^{SY}_{Mises})^2}{\bar{R}^{c\;2}_{0.2}} = 1$$

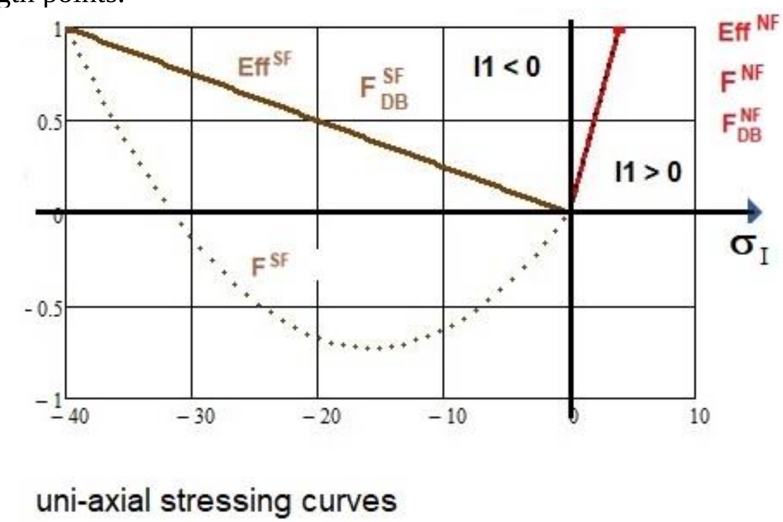
$$F^{NF} = \frac{\sqrt{4J_2 \cdot \Theta^{NF} - I_1^2/3} + I_1}{2 \cdot \bar{R}_t} = 1 \Leftrightarrow F^{SF} = c_1^{SF} \cdot \frac{3J_2 \cdot \Theta^{SF}}{\bar{R}^{c2}} + c_2^{SF} \cdot \frac{I_1}{\bar{R}^c} = 1$$

$$Eff^{NF} = c^{NF} \cdot \frac{\sqrt{4J_2 \cdot \Theta^{NF} - I_1^2/3} + I_1}{2 \cdot \bar{R}^t} = \frac{\sigma^{NF}_{eq}}{\bar{R}^t} \Leftrightarrow Eff^{SF} = \frac{c_2^{SF} \cdot I_1 + \sqrt{(c_2^{SF} \cdot I_1)^2 + 12 \cdot c_1^{SF} \cdot 3J_2 \cdot (\Theta^{SF})}}{2 \cdot \bar{R}^c} = \frac{\sigma^{SF}_{eq}}{\bar{R}^c}$$

For comparison, the Drucker-Prager rotational symmetric Global Model, which combines the 2 modes

$F_{DP} = c_1 \cdot \sqrt{J_2} + c_2 \cdot I_1 = 1$. Curve parameters from the two strength points.





Onset-of-Yielding, PMMA uni-axial stress **Onset-of-Fracture, Normal Concrete**

Eff must be zero, if the stress state is zero. This might be not the case for *F* and | *F* |. Only, if all components of *F* are of the same grade, then *F* ≡ *Eff*. Otherwise one will face problems in Strength Design Verification, as the curves above teach.

Conclusions Demonstrating a ‘Closed’ (and simpler) Macro-mechanical Building

1. It could be shown that this 2nd yield type NY exists in parallel to Shear Stress Yielding SY. This supports the existence of a generic number 2 for isotropic materials
2. The formulations of invariant-based isotropic strength failure conditions (criteria) SFC, $F = 1$, just need 2 invariants I_1, J_2. Due to the fact that a stress state may activate a multi-fold fracture failure type NF or SF the rotational symmetric failure body becomes 120°-symmetric. This is tackled by employing J_3
3. For achieving a reliable 3D-mapping, therefore: The multi-axial failure stress states (R^{tt}, R^{cc}), which generate two-fold failure types and modes, must be known at least. Then, only, the significant inherent 120°-symmetry of brittle isotropic materials can be mapped !

→ Material symmetry seems to have told the author – after 30 years of search - :

In the case of isotropic materials for the associated quantities a generic (basic) number of 2 is inherent. This is valid for yield modes and fracture modes, yield strengths, fracture strengths, fracture mechanics values, invariants to be used when formulating strength criteria, elasticity moduli *and more*.

- A SFC has to map 3D stress states. It can be validated, principally, by 3D test data sets only. If just 2D test data is available, then the 2D-reduced 3D-SFC is applied. This means that the necessary 3D mapping quality is not fully proven. A test series only along a tensile meridian (delivers R^t, R^{cc}) or only along a compressive meridian (delivers R^c, R^{tt}) is not sufficient !
- Considering the non-convex axial failure surface Drucker’s stability postulate is to discuss
- Use *Eff* and not the so-called Failure Index *FI* (value of F or of $|F|$). Only, if the component functions in F are of the same grade, then *Eff* and F correspond
- The author generated failure type-linked or so-called Modal SFCs, which are separate descriptions of each mode. Global SFCs, like Drucker-Prager (on top just rotational symmetric) which was used on the slide before, mathematically combine both the modes. These causes two short-comings: The transition zone between the modes is fixed (accuracy of mapping ?) and if a test data change comes up the full curve must be reworked.

CARBON COMPOSITES

CFK VALLEY™

Fusion zu einem der

weltgrößten Netzwerke **(ca. 400 Mitglieder)**

für den multi-materialen Leichtbau

mit Hochleistungs-Faserverbundwerkstoffen

COMPOSITES UNITED

Composites United e.V.

Kerninformationen

- 50 % KMU-Mitglieder
- Industriegetriebenes Netzwerk
- 5 Cluster und 3 nationale Netzwerke
- Fach-Netzwerk:

  *CU Bau und

  *Ceramic Composites
- Spitzencluster MAI Carbon

Strategie & Ziele

- Entwicklung und Förderung von Hochleistungs-Leichtbau-Verbundwerkstoffen
- Beschleunigung und Förderung von Innovationsprozessen
- Neue Geschäftsmodelle und Technologiezugang