Foam instead of Honeycomb ?

Nowadays, for structural parts of high stiffness, honeycombs are used.

With the new Rohacell Hero a PMI structural foam of an increased tensile fracture strain is available which may replace the expensive honeycombs.

In order to apply this material in structural parts Structural Integrity must be proven.

This requires reliable multi-axial strength test data as well as reliable Strength Failure Conditions SFCs (criteria) for an optimal Design Development process.

Such a foam-describing SFC shall be now validated by test data of similar behaving foam material [courtesy DKI –LBF, Dr. Kolupaev]

PMI = Poly-Methacryl-Imide

CONSTRAINTS in Design Development Process : *Cost and Time Reduction*

Industry looks for robust & reliable analysis procedures in order to replace the expensive ‘Make and Test Method‘ as far as reasonable.

Virtual tests *shall reduce the amount of* physical tests.

In this context:

Structural Design Development

can be only effective and offer fidelity

if

realistic analysis tools and test data input are available

for Design Dimensioning and for Manufacturing as well.

Outline of my talk

The presentation plus further literature may be downloaded from http://www.carbon-composites.eu/leistungsspektrum/fachinformationen/fachinformation-2

CARBON COMPOSITES

Fracture Failure Surface of the Foam *Rohacell 71 G*

derived on basis of the author's Failure-Mode-Concept

1 Introduction
2 Fundamentals when generating SFCs (criteria)
3 Derivation of Cuntze's Failure-Mode-Concept (FMC)
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Results of a time-consuming, never funded „"hobby" *of an engineer, retired from industry*

 Prof. Dr.-Ing. habil. Ralf Georg Cuntze VDI, , linked to Carbon Composite e.V. (CCeV) Augsburg

Situation of the poor Designer:

Is there any Strength Failure Condition ("criterion") I can apply ?

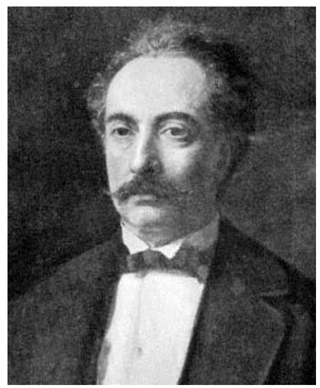
„No. There does not yet exist a validated SFC for isotropic foam material" !

Let's do something to fill the gap!

Some well-known Developers which formulated isotropic 3D Strength Failure Conditions (SFCs)

Hencky-**Mises**-Huber

Richard von Mises
1883-1953
Mathematician



Eugenio Beltrami
1835-1900
Mathematician

Otto Mohr
1835-1918
Civil Engineer

Charles de Coulomb
1736-1806
Physician

'Onset of Yielding'

'Onset of Cracking' **= foam failure**

Hence again, a civil engineer may proceed

What was the Motivation for my non-funded Investigations ?

Existing Links in the Mechanical Strength Behaviour show up:

Different structural materials

- ***can possess similar material behaviour or***
- ***can belong to the same class of material symmetry*** *(see later slide)*

Welcomed Consequence:

- The same strength failure function F can be used for different materials

- More information is available for pre-dimensioning + modelling

in the case of a newly applied material

from experimental results of a similarly behaving material.

Example:
This was a porous concrete, where a multi-axial test data set was available.

Author's experience with structural material applications, range 4 K - 2000 K .

What was the driving idea behind ?

A possibility might exist

for brittle behaving materials

to *more generally* formulate for fracture failure

strength failure conditions (SFCs) :

analogously to **:**

- failure mode-wise *(shear yielding failure, etc.)*

Mises, Hashin, Puck etc.

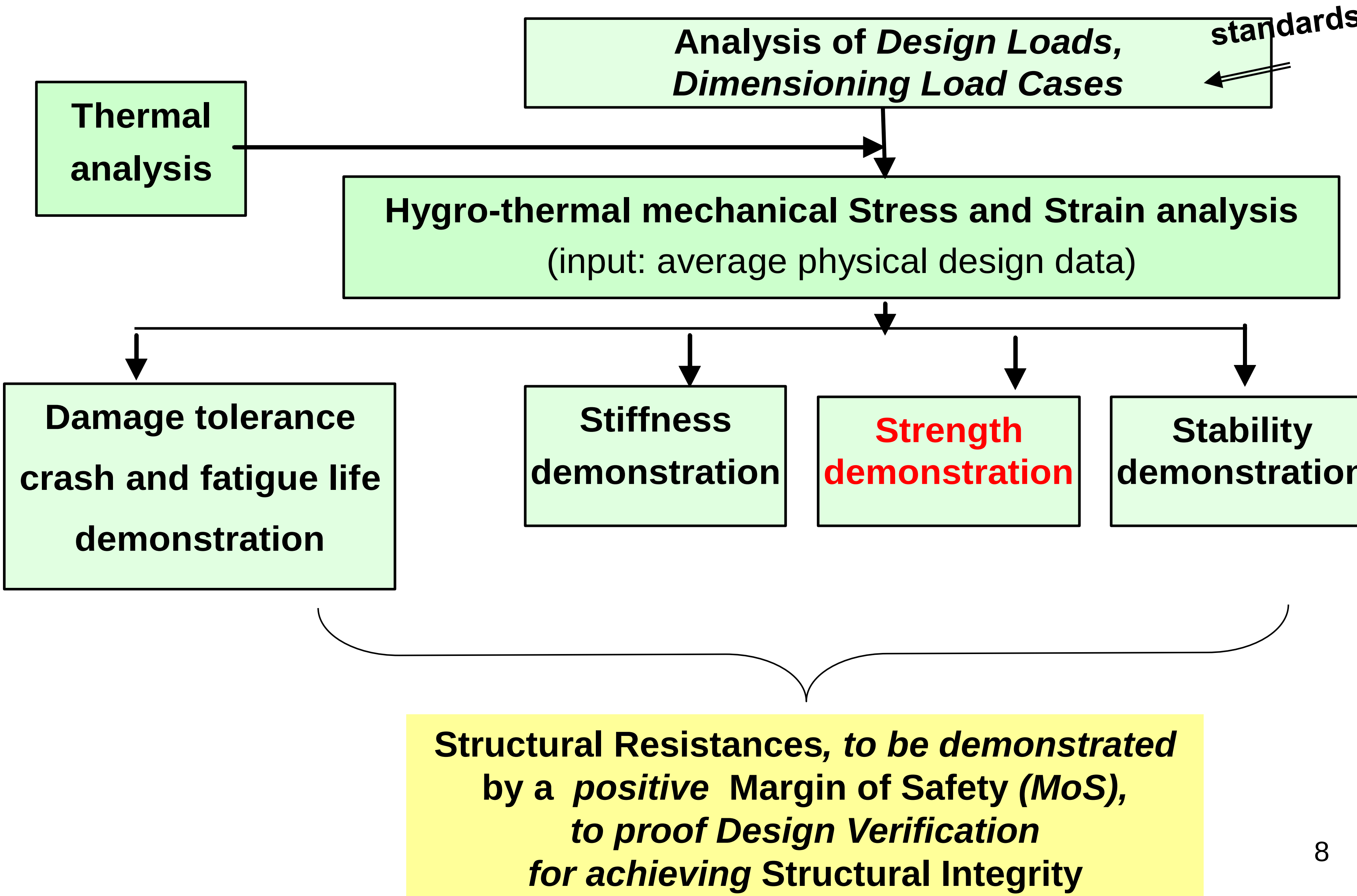
- stress invariant-based *(J_2 etc.)*

Mises, Tsai, Hashin, Christensen, etc.

- obtaining equivalent stresses .

Mises for yielding, Rankine for fracture

Which Design Verifications are mandatory in Structural Design ?



There are two essential tasks the engineer must tackle?

(1) Test Data Mapping and **(2) Design Verification :**

- **Validation of SFCs with Failure Test Data** by

  mapping their course by an average Failure Curve (surface)

- **Finally the Delivery of a reliable Design Verification** by

  calculation of a Margin of Safety or a (load) Reserve Factor

  $$MoS > 0 \quad oder \quad RF = MoS + 1 > 1$$

  on basis of a statistically reduced Failure Curve (surface) .

Design Verification = Achievement of a Reserve against a Limit State

For each distinct Load Case with its single Failure Modes must be computed:

Reserve Factor (is load-defined) **:** ***RF = Failure Load / applied Design Load***

Material Reserve Factor : $f_{Res} = \text{Strength} / \text{Applied Stress}$

if linear situation: $f_{Res} = RF = 1 / Eff$

Material Stressing Effort : $Eff = 100\%$ ***if*** $RF = 1$ **(Anstrengung)**

(Werkstoff-Anstrengung)

Which property value is mandatory as Input in Structural Analysis ?

- The best prediction of the typical behaviour of the structure is performed with typical values = avarage values

- In the design verification – *dependent on the requirements* - the average, the upper or the lower value of the property is used.

Keep in mind:

Be similarly certain/reliable in the design with applied equations, properties, etc. !!

What do the following terms mean ??

Material : homogenized (macro-)model of the envisaged solid

Failure : structural part does not fulfil its functional requirements such as onset of yielding, brittle fracture, FF, IFF, leakage, deformation limit, delamination size limit, frequency bound

= project-fixed **Limit State** of a failure

Failure Theory : tool to predict failure of a structural part

Strength Failure Condition (SFC) : subset of a failure theory

to assess a 'multi-axial failure stress state '

in a critical location of the structural part

= mathematical formulation of the failure surface (body).

Global SFC : describes the full failure surface by one single equation capturing all existing failure modes

Modal SFC : describes parts of the full failure surface by associate equations.

Static Verification Levels

* **<u>Stress</u> at a local material 'point':**
  verification by a *<u>basic strength</u>* or a *<u>multi-axial failure stress state</u>*
  Local stresses are acting and used in the Strength Criteria models

* **<u>Stress concentration</u> at a notch (stress peak at a joint):**
  verification by a *notch strength* *(usually Neuber-like, Nuismer, etc..)*
  'Far'-field stresses are acting, not directly used in the notch strength analysis

* **<u>Stress intensity</u> (at tip of delamination crack):**
  verification by a *fracture toughness (energy –related).*
  Applied stresses are used as 'far'-field stresses.

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Global versus Modal Strength Failure Conditions (SFCs)

1 Global ***strength*** **failure condition** **: F ({σ}, {R}) = 1 (usual formulation)**

Set of Modal **strength failure conditions: F ({σ}, R^{mode}) = 1 (addressed in FMC)**

Example: UD

vector of 6 stresses (general)

$$\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$$

vector of 5 strengths

$$\{R\} = (R_{\parallel}^t, R_{\parallel}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\parallel})^T$$

needs an Interaction of Failure Modes: **performed by a**

probabilistic-based 'rounding-off' approach (series failure system model)

directly delivering the (material) reserve factor in linear analysis

Experience with Failure Prediction:

A Strength Failure Condition (SFC) is a necessary but not a sufficient condition to predict Strength Failure (i.e. thin-layer problem).

Facts of so-called Global SFCs

Global SFCs (one failure surface)

- **Combine all failure modes in one single mathematical equation. This might even capture a**

  - 2-fold acting failure mode (e.g. if $\sigma_I = \sigma_{II}$) or

  - 2-fold acting failure mode under hydrostatic loading ($p_{hyd} = \sigma_I = \sigma_{II} = \sigma_{III}$)

- **Re-calculation of all model parameters in the case of test data change in a distinct domain.**

- **A change in one failure domain deforms the failure surface in all other – physically independent – failure domains. There is a big chance, that a Reserve Factor – to be determined for a stress state in an independent domain - might not be on the conservative side**

- **Often global SFCs just use basic strengths as model parameters. This is physically not permitted because Mohr-Coulomb acts in the case of brittle behaving materials**

Zeigt Unterschied noch nicht gut genug. Lode angle J3

Joint failure probability

Facts of so-called Modal SFCs

Modal (multi-surface) **SFCs:**

- **Describe one single failure mode in one single mathematical formulation** (part of failure surface)**.**
  - determine all model parameters in the respective failure mode
  - capture a twofold acting failure mode (e.g. if $\sigma_I = \sigma_{II}$ (isotropic) or if $\sigma_2 = \sigma_3$ (transversely-isotropic UD material) separately, modal-wise by one additional Ansatz (J_3)
  - capture a threefold acting failure mode under hydrostatic loading alike

- **Re-calculation of the model parameters just in the modal domain if a test data is to be replaced. One Reserve Factor must be freshly determined.**

In short: Main Critics regarding Global SFCs

Globale Festigkeitsbedingungen zwangsverbinden**, **wie** **z. B. bei

Drucker-Prager (isotrop), Tsai-Wu (transversal-isotrop, UD)

die einzelnen Modi in einer Formel,

was generell nachteilig ist und sogar zu Ergebnissen auf der unsicheren

Festigkeits-Seite führen kann,

weil eine Änderung in einem Modusbereich (z. B. Zugbruch), der

durch die Formel insgesamt (global) beschriebenen

Bruchversagensoberfläche, zwangsläufig Änderungen in

<u>unabhängigem</u> anderen Modusbereich nach sich zieht.

Dies ist physikalisch nicht korrekt!

A modal concept

– as found with i.e. Cuntze (general) and Puck (UD) –

builds up the Fracture Failure Surface mode-wise

Material Symmetry Requirements Aspects *(helpful, when generating SFCs)*

1 If a material element can be homogenized to an ideal (= frictionless) crystal,

then, material symmetry demands for the transversely-isotropic UD-material

- 5 *elastic 'constants' , 5 strengths, 5 fracture toughnesses* and

- *2 physical parameters (such as CTE, CME, material friction, etc.)*

(for isotropic materials the respective numbers are 2 and 1)

2 Mohr-Coulomb requires for the real crystal another inherent parameter,

- the *physical parameter 'material friction' : UD* $\mu_{\perp\parallel}$ **, ;**$\mu_{\perp\perp}$ **, *Isotropic*** μ

3 Fracture morphology witnesses:

- Each strength corresponds to a distinct *failure mode*

and to a *fracture type* as Normal Fracture (NF) or Shear Fracture (SF).

Above Facts and Knowledge gave reason

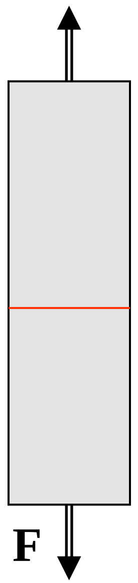
why the FMC strictly employs single *independent* failure modes

by its failure mode–wise concept.

Observed Failure Modes of Brittle behaving porous Isotropic Material

Normal Fracture (NF) (Spaltbruch, Trennbruch) :

- volumetric change before fracture



R_m^t

F

Tension

Failure Mode NF

Crushing Fracture (CrF): SF

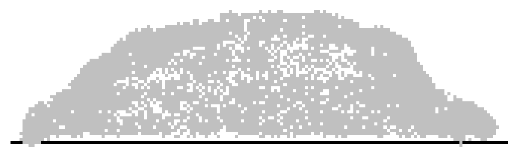
- volumetric change before fracture

helpful knowledge for the ***choice of invariants***

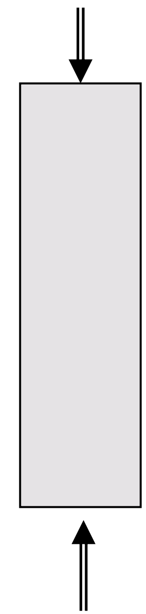
Compression

result of the compression test

= *hill of fragments* *(crumbs)*



= decomposition of texture



R_m^c

Failure Mode CrF

… needs interaction

Observed: ► **Each single Failure Mode is governed by one single strength !!**

if brittle: failure = fracture failure

Interaction of Single Strength Failure Modes in the modal FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots} = 1 = 100\%\text{, if failure}$$

with mode-interaction exponent *m*, *from mapping experience*

***as modal* material stressing effort * (in German Werkstoffanstrengung)**

and

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / \overline{R}^{\text{mode}}$$

equivalent mode stress

mode associated average strength

later → In the example

*** material stressing effort = artificial technical term created together with QinetiQ, UK**

Physically-based Choice of Invariants when generating invariant-based Strength Failure Conditions

* Beltrami : "At 'Onset of Yielding' the material possesses a distinct ***strain energy*** composed of *dilatational energy* (I_1^2) and *distortional energy* ($J_2 \equiv Mises$) ".

* So, from Beltrami, Mises (HMH), and Mohr / Coulomb (friction) can be concluded:

Each invariant term in the *failure function* ***F*** may be dedicated to one physical mechanism in the solid = cubic material element:

- volume change : I_1^2 … ***(dilatational energy)***
- shape change : J_2 (**Mises**) … ***(distortional energy)***

and - friction : I_1 … ***(friction energy)***

relevant for a very brittle behaving foam

Mohr-Coulomb

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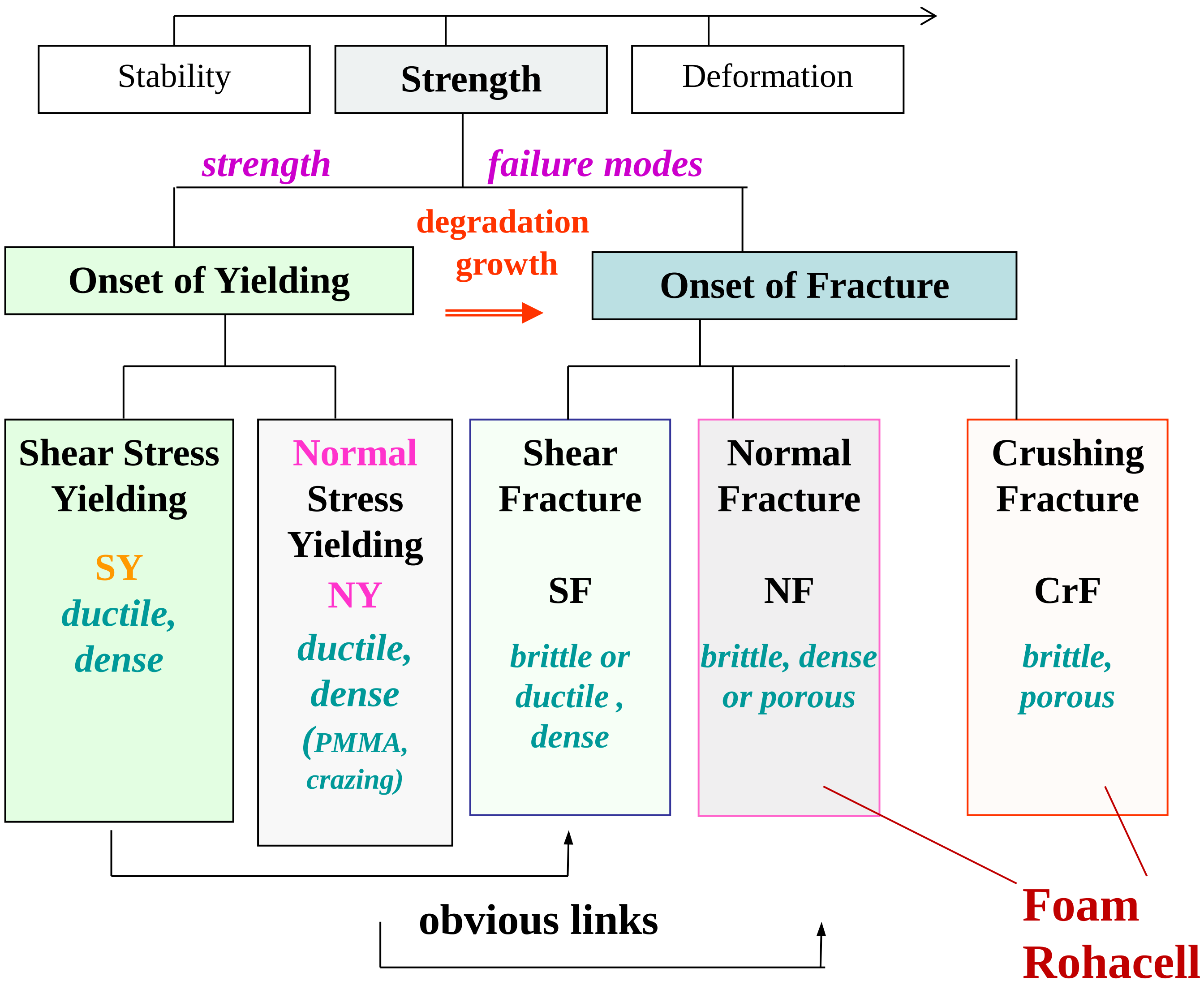
Driver for my research work on Strength Failure Conditions (criteria)

is the achievement of suitable SFCs under some *pre-requisites* :

- ***physically convincing*** **(need minimum test information)**
- ***numerically robust, unique solutions***
- ***simple, as much as possible***
- ***invariant-based*** **(like the Mises yield condition)**
- ***allow to compute an equivalent stress*** **(very helpful for failure mode-based design screw turning)**
- ***rigorous independent treatment of each single failure mode NF, SF, CrF***
- ***using a material behaviour-linked thinking and not a material-linked one***
- ***engineering approach where all model parameters can be measured***
- ***shall allow for a simple determination of the reserve factor RF*.**

Scheme of Strength Failures Types for *isotropic materials*



The growing yield body (SY or NY) **is confined by the fracture surface** (SF or NF)**!**

Material Homogenizing (smearing) **+ Modelling**

Material symmetry shows:

Number of strengths ≡ number of elasticity properties !

Application of material symmetry knowledge:

- *Requires that homogeneity is a valid assessment for the task-determined model* ,

but, if applicable

- A *minimum number of properties has to be measured, only* (cost + time benefits) !

Basic Features of the author's Failure-Mode-Concept (FMC)

- **Each failure mode represents 1 independent failure mechanism and thereby 1 piece of the complete *failure surface***
- **Each failure mechanism is governed by 1 basic strength** (is observed !)
- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !!

- **In consequence, this separation requires :**

  An interaction of the Modal Failure Modes !

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Formulation of Failure-Mode-Concept (FMC)-based Modal SFCs

Use of :

- **Invariants**
- **Hypotheses of**

  Beltrami = dedication of invariants to the deformation of the material element, whether it is a shape change (Mises) or a volume change and
  Mohr-Coulomb = internal friction of a brittle behaving solid material
- **Application of the Reqirements of Material Symmetry** = for isotropic brittle behaving materials the characteristic number of quantities is 2 (2 strengths, 2 strength fracture failure modes, 2 basic invariants)
- advantegeous **equivalent stresses** σ_{eq} and of the physically plausible **material stressing effort** (Werkstoffanstrengung) *Eff*

NOTE:

The characteristic number of quantities for the transversely-isotropic unidirectional material UD is 5

Driver for my research work on Strength Failure Conditions (criteria)

Achievement of practical, physically-based criteria under some *pre-requisites* :

- *physically convincing*
- *simple, as much as possible*
- *invariant-based (like the Mises yield condition)*
- *allow to compute an equivalent stress (very helpful for a distinct failure mode)*
- *rigorous indepent treatment of each single failure mode (2 FF + 3 IFF)*
- *using a material behaviour-linked thinking and not a material-linked one*
- *engineering approach where all model parameters can be measured*.

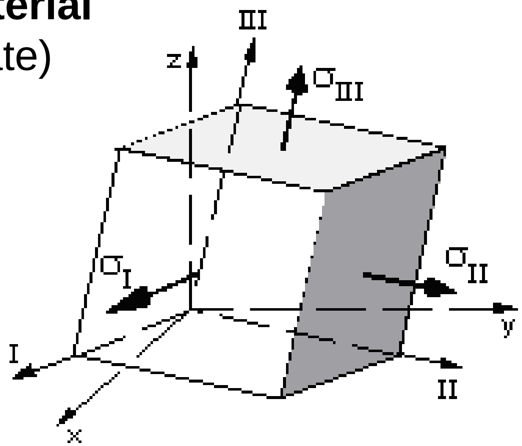
Note on UD strength failure conditions:

Puck's action plane approach involves some basic differences to Cuntzes Failure-mode-concept-based approach:
(1) is not invariant-based, (2) interacts the 3 Inter-Fiber-Failure modes (IFF) by a Mohr-Coulomb-based equation, (3) post-corrects the IFF- influence on FF.

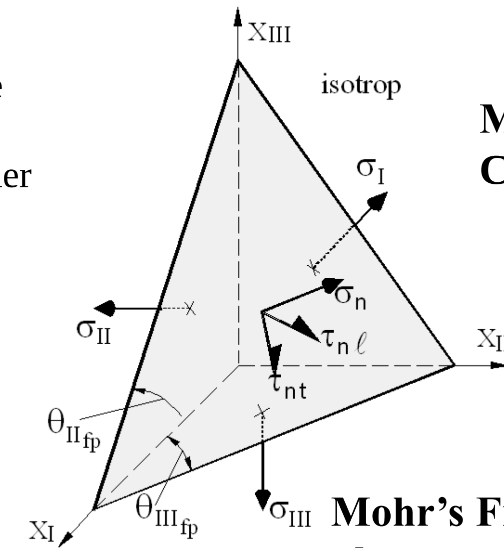
Cuntze provides for each failure mode an equivalent stress, that captures the influence of IFF on FF by his interaction equation, uses less model parameters.

Which are the Stresses & Invariants to be used?

Isotropic Material
(3D stress state)



The stress states in the various COS can be transferred into each other



Mohr's COS

Principal Stresses | **Structural Component Stresses** | **Mohr's Fracture plane Stresses**

$$\{\sigma\}_{principal} = (\sigma_I,\ \sigma_{II},\ \sigma_{III})^T$$

$$\{\sigma\}_{comp} = (\sigma_x,\ \sigma_y,\ \sigma_z,\ \tau_{yz},\ \tau_{xz},\ \tau_{xy})^T$$

$$\{\sigma\}_{Mohr} = (\sigma_\ell,\ \sigma_n,\ \sigma_t,\ \tau_{nt},\ \tau_{t\ell},\ \tau_{\ell n})^T$$

'isotropic' invariants !

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = 3\sigma_{oct} \equiv f(\sigma),$$

$$I_1 = (\sigma_x + \sigma_x + \sigma_z)^T$$

$$I_1 = (\sigma_\ell + \sigma_n + \sigma_t)^T$$

$$6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2$$
$$= 4(\tau_{III}{}^2 + \tau_{II}{}^2 + \tau_I{}^2) = 9\tau_{oct}{}^2 \equiv f(\tau)$$

$$6J_2 = (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2$$
$$+6(\tau_{yz}{}^2 + \tau_{zx}{}^2 + \tau_{xy}{}^2) \quad (Mises, HMH)$$

$$6J_2 = (\sigma_n - \sigma_t)^2 + (\sigma_t - \sigma_\ell)^2 + (\sigma_\ell - \sigma_n)^2$$
$$+6(\tau_{nt}{}^2 + \tau_{t\ell}{}^2 + \tau_{n\ell}{}^2)$$

$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III})(2\sigma_{II} - \sigma_I - \sigma_{III})(2\sigma_{III} - \sigma_I - \sigma_{II}), \quad I_\sigma = 4J_2 - I_1{}^2/3, \quad \sigma_{mean} = I_1/3$$

Invariant := Combination of stresses –powered or not powered- the value of which does not change when altering the coordinate system.

Cuntzes 3D Strength Failure Conditions (criteria) for Foams

Approaches:

$$\frac{\sqrt{4J_2 - I_1^2/3} + I_1}{2 \cdot \overline{R}_t} = 1 \qquad \frac{\sqrt{4J_2 - I_1^2/3} - I_1}{2 \cdot \overline{R}_t} = 1$$

Considering bi-axial strength (failure mode occurs twice**):** **in Effs now**

$$Eff^{NF} = c_{NF} \cdot \frac{\sqrt{4J_2 - I_1^2 \cdot (\Theta_{NF})/3} + I_1}{2 \cdot \overline{R}_t} \qquad Eff^{CrF} = c_{CrF} \cdot \frac{\sqrt{4J_2 - I_1^2 \cdot (\Theta_{CrF})/3} - I_1}{2 \cdot \overline{R}_t}$$

Two-fold failure danger can be excellently modelled by using the often used invariant J3

$$\Theta_{NF} = \sqrt[3]{1 + D_{NF} \cdot \sin(3\theta)} = \sqrt[3]{1 + D_{NF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}} \qquad \Theta_{CrF} = \sqrt[3]{1 + D_{CrF} \cdot \sin(3\theta)} = \sqrt[3]{1 + D_{CrF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}}$$

I_1

Mode interaction: $$Eff^{NF} = [(Eff^{NF})^m + (Eff^{CrF})^m]^{m^{-1}}$$

The failure surface is closed at both the ends: A simple cone serves as closing cap and bottom

$$\frac{I_1}{\sqrt{3} \cdot R_t} = s_{NF} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta_{NF}}}{R_t}\right) + \frac{\max I_1}{\sqrt{3} \cdot R_t} \qquad \frac{I_1}{\sqrt{3} \cdot R_t} = s_{CrF} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta_{CrF}}}{R_t}\right) + \frac{\min I_1}{\sqrt{3} \cdot R_t}$$

The slope parameters ***s*** are determined connecting the respective hydrostatic strength point with the associated point on the shear meridian, *maxI1* must be assessed whereas *minI1* could be measured.

Eff = material stressing effort = Werkstoff-Anstrengung (must be < 1 = 100%)

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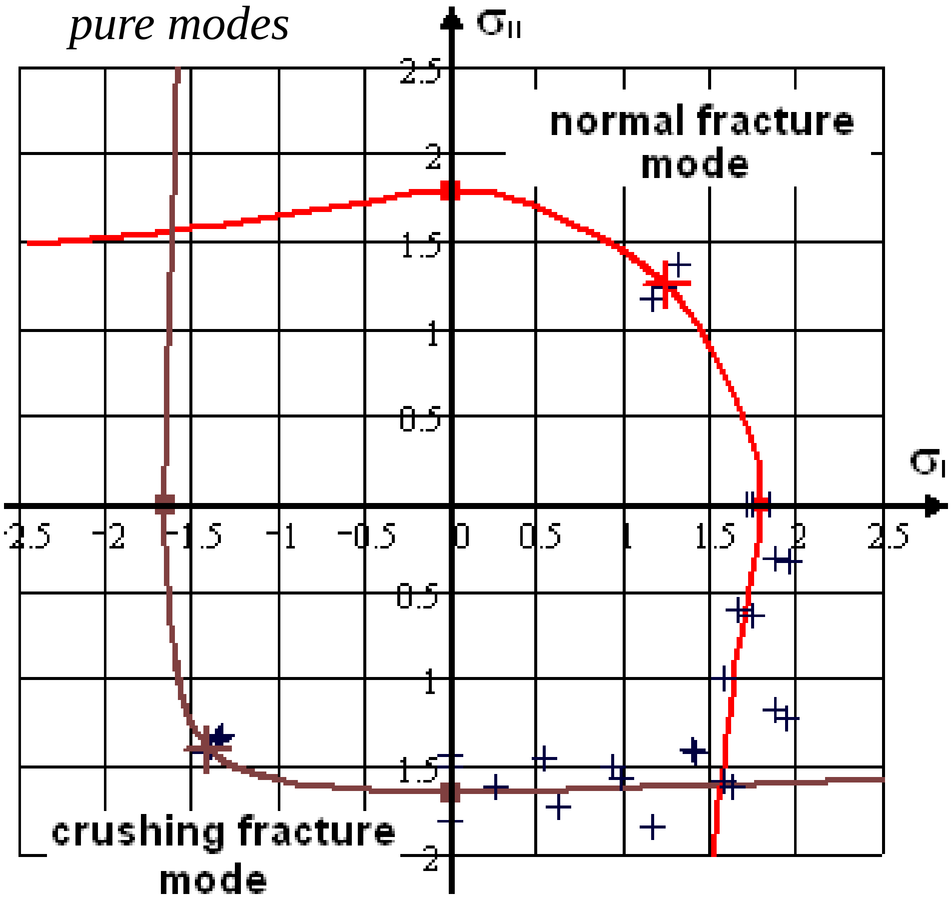
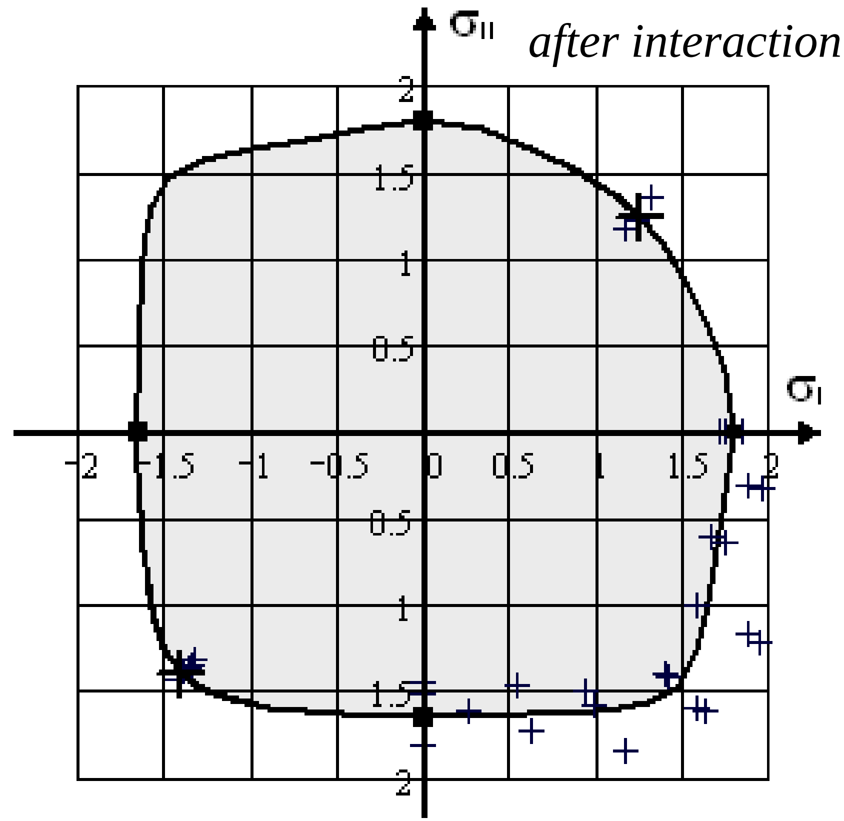
2D - Test Data Set and Mapping in the Principal Stress Plane *(brittle, porous)*

Principal Plane Cross-section of the Fracture Body *(oblique cut)*

pure modes



after interaction

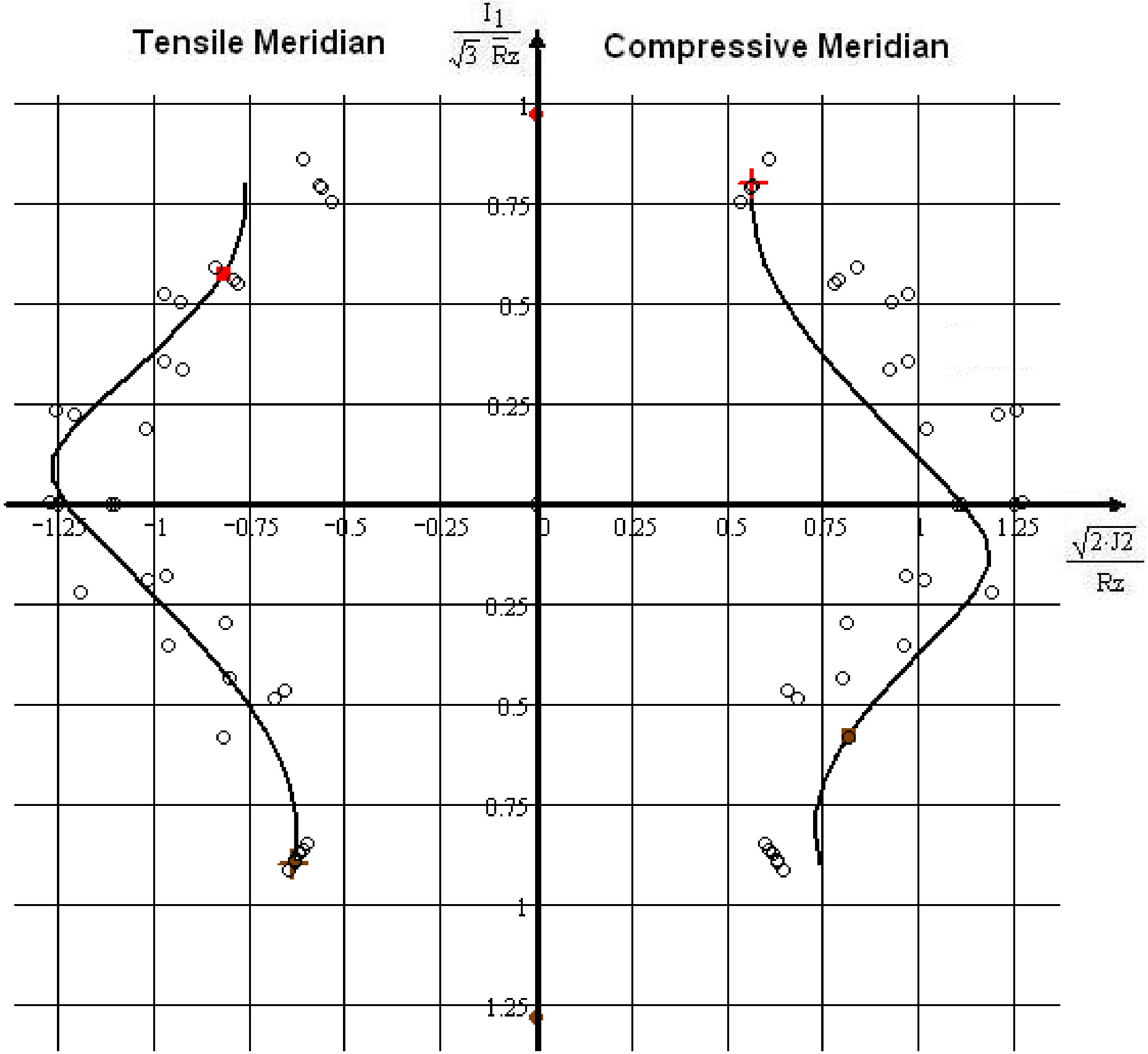


- **Mapping must be performed in the 2D-plane because fracture data set is given there**
- **The 2D-mapping uses the 2D-subsolution of the 3D-strength failure conditions**
- **The 3D-fracture failure surface (body) is based on the 2D-derived model parameters.**

Rohacell 71 IG

Courtesy: LBF-Darmstadt (DKI), Dr. Kolupaev

Generic Lines of Tensile and of Compressive Meridian *(brittle, porous)*



Rohacell 71 IG

Meridional cross-section of the Fracture Body

in **Lode-Haigh-Westergaard coordinates**

bi-axial = +

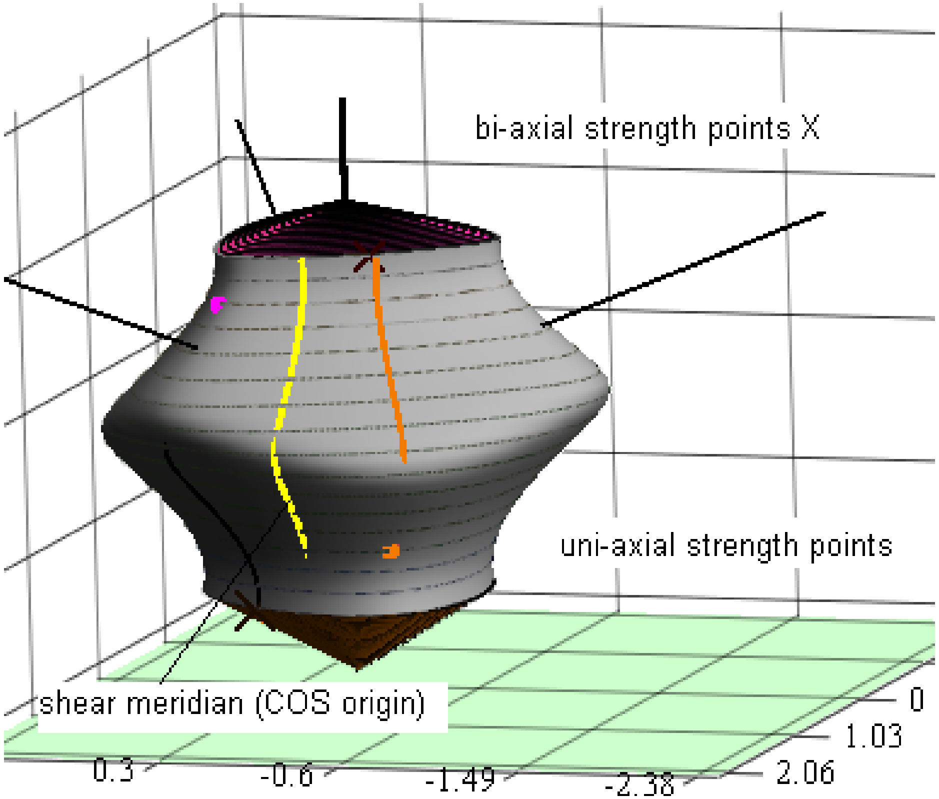
consideration of a ‘twofold mode‘ by above well-known approach

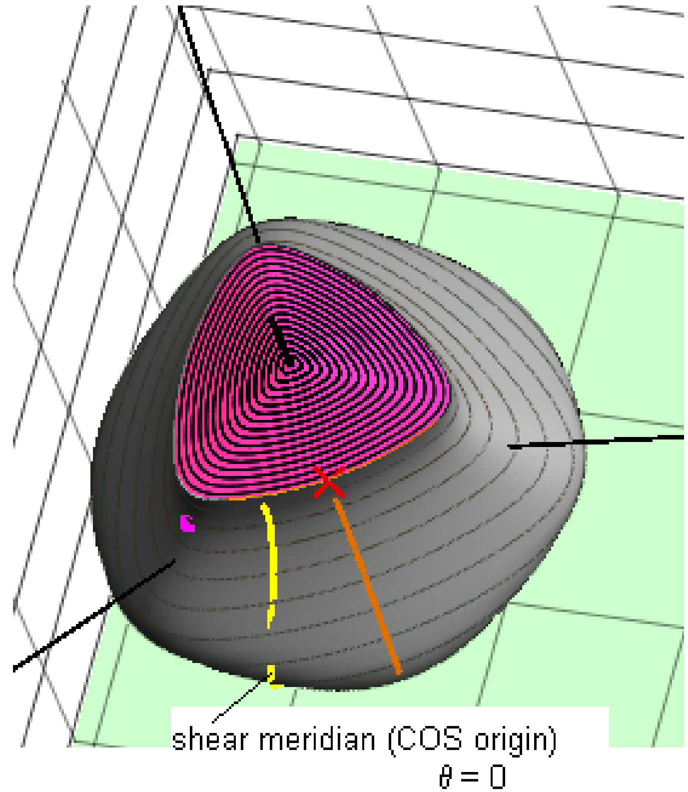
The fracture test data are located at a distinct Lode angle of its associated ring o, 120°-symmetry of the isotropic failure surface (body) .

Cap and bottom are closed by a cone-ansatz, a shape being on the conservative side.

Fracture Failure Surface of *Rohacell 71 IG*

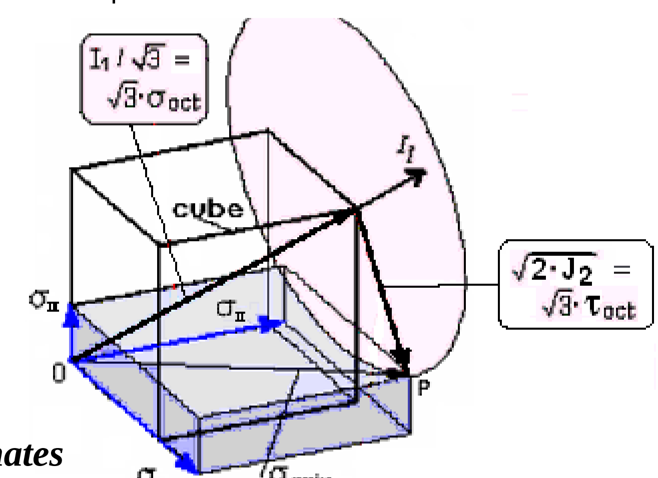
The dent turns !





compressive meridian with tensile meridian form one cross-section shape

The 3D-strength failure condition enables to predict the 120°-symmetric failure body and to judge a 3D- stress state



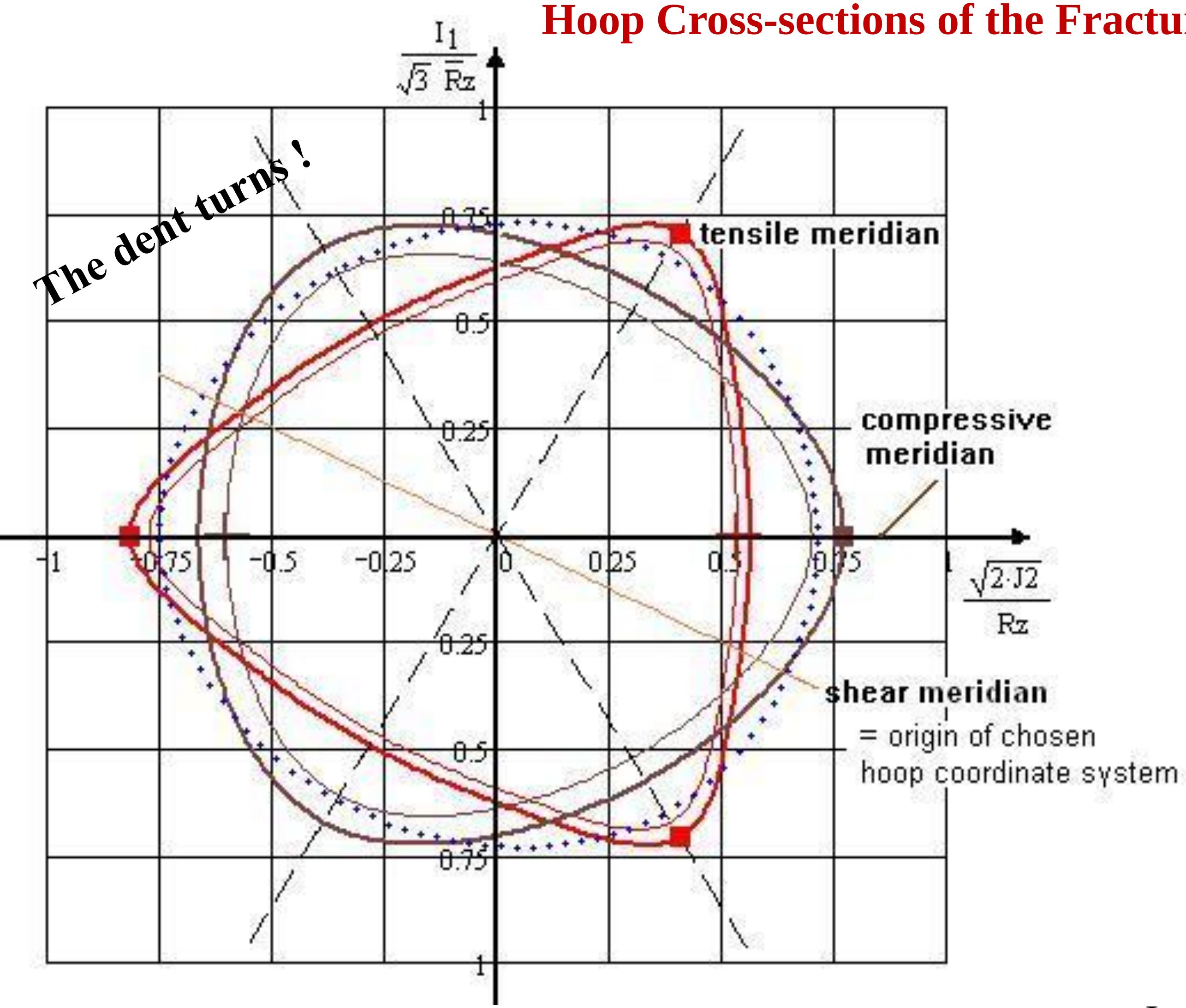
visualization of the Lode-Haigh-Westergaard coordinates

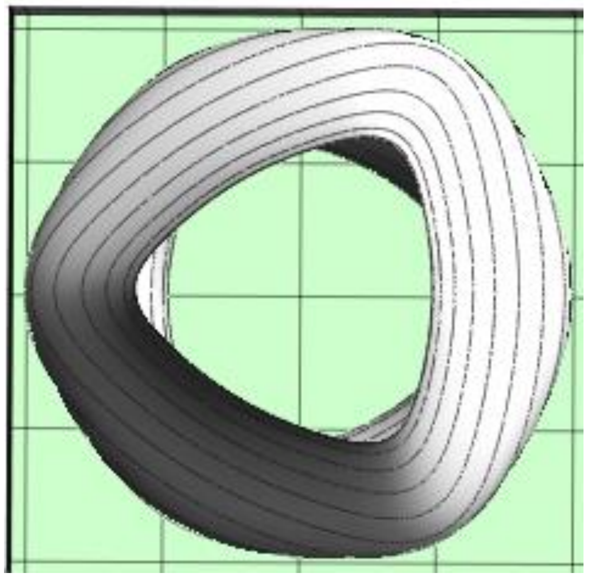
2D Test Data and Mapping in the Orthogonal Stress Plane (brittle, porous)

Hoop Cross-sections of the Fracture Body

Rohacell 71 IG

z = tensile, d = compressive





Caps: No test data, cone chosen

with characteristic uni-axial and bi-axial strength points

+

Lode-angle, here set as <u>sin</u> (3 θ) :

shear meridian angle = 0°

tensile meridian +30°

compressive meridian -30°

+

crushing I1 = 0, interaction domain: Is about a circle.

Determination of the Load-defined Reserve Factor RF

Linear elastic problem for this brittle behaving material

Residual stresses = 0

estimated from given average value

$$\mathbf{RF} = \boldsymbol{f_{Res}} \text{ (material reserve factor)} = \boldsymbol{Eff^{-1}}$$

Stress state:

$\sigma I := 0.9 \quad \sigma II := -0.4 \quad \sigma III := 0.5$

Statistically reduced Strengths:

$Rz := 0.9 \cdot \bar{R}z \quad Rd := 0.85 \cdot \bar{R}d$

Shape parameters: $D\sigma = -0.71 \quad D\tau = 0.21 \quad c1\Theta\sigma = 1.15 \quad c1\Theta\tau = 1.03$

$$I1 := \sigma I + \sigma II + \sigma III \qquad J2 := \frac{\left[(\sigma I - \sigma II)^2 + (\sigma II - \sigma III)^2 + (\sigma III - \sigma I)^2\right]}{6} \qquad J3 := \frac{\left[(2\cdot\sigma I - \sigma II - \sigma III)\cdot(2\cdot\sigma II - \sigma I - \sigma III)\cdot(2\cdot\sigma III - \sigma II - \sigma I)\right]}{27}$$

$I1 = 1 \qquad J2 = 0.44 \qquad J3 = -0.07$

$$Eff\Theta\sigma := c1\Theta\sigma \cdot \frac{\sqrt{4\cdot J2\cdot\sqrt[3]{1 + D\sigma\cdot 1.5\cdot 3^{0.5}\cdot J3\cdot J2^{-1.5}} - \frac{1}{3}\cdot I1^2 + I1}}{2Rz}$$

$$Eff\Theta\tau := c1\Theta\tau \cdot \frac{\sqrt{4\cdot J2\cdot\sqrt[3]{1 + D\tau\cdot\left(1.5\cdot 3^{0.5}\cdot J3\cdot J2^{-1.5}\right)} - \frac{1}{3}\cdot I1^2 - I1}}{2\cdot Rd}$$

$$Eff := \sqrt[9]{Eff\Theta\sigma^{m_{int}} + Eff\Theta\tau^{m_{int}}} \qquad Eff = 0.802 \qquad RF := \frac{1}{Eff} \qquad RF = 1.25$$

The loading may be monotonically increased by the factor RF !

Conclusions

- **The FMC is an efficient concept,**

  that improves prediction + simplifies design verification

  is applicable to brittle and ductile, dense and porous,
  isotropic, transversely-isotropic and orthotropic materials

  if clear failure modes can be identified and the material element homogenized.

  Formulation basis is whether the material element experiences a *volume* change, a *shape change* and *friction* .

  Builds not on the ***material*** but on ***material behaviour !***

- **Delivers a <u>combined formulation</u> of *independent modal failure modes*,**

  without the well-known drawbacks of <u>global</u> SFC formulations

  (which *mathematically combine in-dependent failure modes*) .

- **The FMC-based Failure Conditions are simple but**
  describe physics of each single failure mechanism pretty well.

- **Mapping of the brittle behaving porous foam was successful and with new findings !**

Theory is the Quintessence of all Practical Experience

A. Föppl

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