# Static and Cyclic Strength of Laminates composed of Uni-directional, Endless Fibre reinforced Plies

## - The lighter, the more difficult -

**Ralf Georg Cuntze**

formerly MAN Technologie AG, Augsburg (D); now joined to Carbon Composites e.V., Augsburg
D-85229 Markt Indersdorf, Tel.: 0049 8136 7754, E-mail: Ralf_Cuntze@t-online.de

## Summary:

***Basic features***: The paper presents some background, including views on isotropic materials, in order to find methods to predict the strength of multi-layered laminates composed of UD laminas (plies) and manufactured of fibre-reinforced plastics (FRP).

At first a view of the state of the art of static and cyclic strength is presented. Then, the paper depicts the basics of the author's failure mode concept (FMC) generally applicable to materials that can be homogenized for their analysis. The concept is applied in the following section to static strength conditions of uni-directional (UD) material. Differences between the use of the classical 'global' or interactive conditions and the derived 'modal' conditions are addressed. Of benefit for the design judging engineer is the introduction of the 'material stressing effort' and its associated equivalent stress. On the FMC also builds the proposed lifetime prediction method for the most often 'fibre-dominated' designed laminates of structural walls.

Basic intention of the paper is to show the red line of some new ideas including the use of material symmetry facts, as well as to draw attention to the material behaviour (ductile, brittle, intermediate), to material consistency (dense, porous), and to material element behaviour (volume change, shape change, friction).

***FMC-based Static Strength***: The depicted FMC-based UD strength failure conditions were successful in the World-Wide-Failure-Exercises. In the WWFE-I the courses of multi-axial 2D UD fracture stress test data could be mapped without any difficulties whenever 'clear' data could be provided, [Cun04]. In the running WWFE-II, the provision with reliable 3D UD fracture stress data is a very challenging task. Additionally, a lot of multi-axial fracture stress domains are still not covered by the available test data. Therefore just the remaining 'clear' test data sets have been successfully mapped. As the FMC is based on a very strict thinking in independent failure modes equivalent stresses can be formulated for UD-material, too, which is of benefit for engineering assessment.

The applicability of the conditions is confirmed here by mapping some static test data sets.

***FMC-based Cyclic Strength (lifetime prediction):***

As the determination of a S-N curve is costly, a proposal is made for a so-called master S-N curve which enables the engineer in pre-dimensioning to build lifetime predictions for variable loading on a physically-founded and thereby cheaper basis. Above master S-N curve represents the basic R-ratio S-N curve associated to a failure mode, which may be a normal fracture or a shear fracture mode. Further S-N curves for the same failure mode, required in fatigue analysis, can be predicted from the master curve by utilizing the equivalent strain energy principle.

The prediction method uses fracture failure modes, is lamina-oriented, and applies the well-known stress ratio-dependent S-N curves but failure mode linked, [Cun96, Cun09]. This results in a new failure mode related damage accumulation. With respect to the FMC the cyclic loading has to be 'rainflowed' failure mode-linked. This is a new procedure.

By the proposed method the traditional fatigue life demonstration with its mean stress correction is very much changed. The damaging portions are related to the material stressing efforts that the material experiences under variable loading. As accumulation model for the damaging portions the 'Relative Miner rule' is still recommended.

The method is schematically applied to one set of S-N data, master curve $R = 0.1$.

**Keywords:** Static strength, lifetime prediction, UD laminas, fracture failure mode dependent.

# 1 Introduction

## 1.1 General

Up to now the static analysis of laminates composed of usually brittle behaving fibre-reinforced plastics is not as 'simple' as with isotropic metallic materials. However in the last time, some models have been developed that enable the design engineer to improve the design analyses. Besides Puck's 'Action Plane concept' the author's Failure-Mode-Concept (FMC) also delivers realistic strength failure conditions, which successfully map realistic own multi-axial UD fracture stress data and those of the WWFE. Validation of the conditions for 2D applications is achieved, but not for 3D applications where the basis of reliable test data is not big enough.

The FMC, applied to a UD lamina, maps the UD lamina as a homogenized transversely-isotropic material, i.e. a crystal. Such a crystal possesses as a result of its material symmetry the 'valency' 5. In the case of isotropic material the valency number is 2. This means, that one faces 5 elasticity constants, 5 strengths, 5 (fracture) failure modes and 5 basic invariants.

Corresponding with the number of failure modes 5 associated strength failure conditions have to be set up, which is performed with the full set of 5 basic invariants. This is necessary, because otherwise the different behaviour of $\sigma_2, \tau_{21}$ and $\sigma_2, \tau_{31}$ cannot be described. The result is a set of 5 fibre failure conditions (tension: FF1, compression: FF2) and 3 inter-fibre failure conditions (lateral tension: IFF1, lateral compression: IFF2, and shear: IFF3).

When formulating FMC-based failure conditions it purposefully helps, that a distinct basic invariant is related to the volume change of a material element, another to its shape change and a third to its internal friction which is of course present in the usually not ideal crystal..

In this manner, the performance of a ply-by-ply failure analysis of a laminate is physically-based, ply-oriented and failure mode-linked.

The analysis of cyclically stressed laminates is less developed than the static analysis. Tools to predict the lifetime are still not so far matured that at least 2D stressed laminates can be treated without always carrying out expensive tests.

A reduction of the experimental effort is only possible if a generalization can be made for arbitrary laminates. Very likely, this will be only achieved if the lamina or ply is the building block of a laminate and if the lamina's in-situ behaviour is catched by deformation-controlled cyclic experiments. In this context it is to be considered that a multi-axial stress state will cause a couple of damaging mechanisms which occur at different 'planes' in space and where, for their quantification, the relevant failure condition is required. A newly founded German university working group supposes that the static failure conditions are also capable to describe the degradation of the material properties.

Task is to generate an appropriate damaging accumulation that captures the multi-axiality. Delamination growth models are still available.

For the usually fibre-dominated designed laminates the author proposes a failure mode based engineering model that just needs a minimum of S-N curves. To apply this model the cyclic loading has to be 'rainflowed' failure mode-related. In this context has to be clarified whether an additional interaction of damaging effects has to be considered in the case of mode changes. Open questions with respect to the influences of stress amplitude, mean stress and load sequence on the damaging process have to be discussed.

The paper ins structured according to

1. Introduction
2. Basics of the General Failure Mode Concept (FMC)
3. FMC-based UD Strength Failure Conditions
4. Application by Mapping some Static Test Data Sets
5. FMC-based Lifetime Prediction Method (new ideas)
6. Application to a Cyclic Test Data Set.

Overall intention of the paper is:

- *To show the red line of the basic ideas*
- *To draw attention to :*
  *- material behaviour (ductile, brittle, intermediate),*
  *- material consistency (dense, porous),*
  *- material element behaviour (volume change, shape change, friction).*

### 1.1 The World-Wide-Failure-Exercise (WWFE)

In the WWFE, *failure theories* are tested. Such a failure theory is composed of three parts, at least: 1) The *failure conditions* to assess tri-axial states of stress; 2) Non-linear stress-strain modelling of the embedded UD lamina material (in strain-hardening and strain-softening regime as main degradation domain) as analysis input; and 3) Non-linear coding for obtaining a realistic response of lamina and laminate test specimens considering possible changes of the fibre direction under loading. In the WWFE-II, in addition to WWFE-I, one more point is to be regarded: 4) Modelling of the matrix behaviour in ultra-high pressure domains, [Cun11].

In consequence, the validation of a failure condition is just one part of a failure theory exercise. This has to be considered when judging just the failure conditions of the competing WWFE contributors. Validation of UD failure conditions is performed best with sets of multi-axial UD lamina fracture stress data, and approximately with laminate test data. Laminate test data deliver valuable benchmarks.

Providing the contributors with a reliable test basis was the challenging task for the WWFE organizers from QinetiQ, because proving the capability of the tri-axial FMC theory requires realistic, well evaluated, and well understood experimental data.

Tri-axial failure states are often encountered: in submarines, bolted and screwed joints, bearings such as sealed polymer bearing cartridges pressurized up to 600 MPa, in cases of impact and ballistics, and other applications like composite high pressure vessels, anchor points of tension cables in civil engineering, load carrying UD hangers of helicopter blades, load introduction points, and CFRP tubes for deepwater umbilicals. In consequence, there is a strong need to validate failure conditions in the multi-axial compression domain, too.

### 1.2 State of the Art in Static Strength Analysis of UD Laminas (plies)

The best source for investigating the state of the art is above WWFE which runs since 1991. Its results may be summarized in the following lines

- WWFE-I : Some 2D strength failure conditions can be termed validated. The first places reached the f*ailure mode–based conditions* of Puck and Cuntze, [Cun04]. These ‘modal‘ conditions are not error-prone, e.g. in cases of extrapolation of the application range and of change of the strength data as the basic support points of the (non-)failure envelope than the usual UD ‘global‘ conditions of Tsai etc. or the isotropic global conditions of Drucker-Prager
- WWFE-II : 3D Failure mode–based strength failure conditions could be not yet fully validated due to a lack of sufficient reliable test data in several 3D stress domains, see [Cun10a,b].

In the WWFEs the physically necessary friction values could be not provided $\mu_{\perp\perp}$, $\mu_{\perp\parallel}$, however in general, a strength failure condition cannot be set up by strength data, only! So, there is a big need for the much more challenging 3D testing and its associated research.

The author follows his strict failure mode thinking in the FMC, here applied to UD material, and thereby a specific procedure to achieve strength failure conditions: *Physically-based on failure modes,* and *lamina–oriented. The a*chievement of a concept for their derivation underlies pre-requisites:

- physically convincingly,
- simple, as much as possible, and
- material *behaviour*-linked and not material-linked.

### 1.3 State of the Art in Cyclic Strength Analysis of UD Laminas (plies)

Nowdays, there is no lifetime prediction procedure available, that is applicable to any laminate. The procedures base – as with metals – on stress amplitudes and mean stress correction. They further base on specific laminates and therefore cannot be generally applied.

Until now experience proved: No fatigue danger is given if remaining below the usual s*tatic design limit strain* of about 0.3%, see {[Cun09,10c], however the *Design Limit Strain* shall be increased up to about 0.5%. Then *first filament breaks* will occur –according to the distribution of the fracture strain of filaments (measurements at MAN-NT) - in the usually *fibre-dominated laminates* used in high-stress applications. Caused by this, German researchers investigate this task on a new basis, applying also a p*hysically-based, lamina–oriented procedure*, like in the static case.

## 2 Basics of the General Failure Mode concept (FMC)

### 2.1 Information available when generating Strength Failure Conditions

For the generation of strength failure conditions there is a lot of information available that should be taken as guidance. The basic FMC features are collected in the following list:

1 Pre-condition: Material element can be homogenized to an ideal (frictionless) crystal
Then, material symmetry features may be beneficially used.

2 Material symmetry demands for a distinct crystal a distinct number of material quantities:

2 for an *isotropic* material (5 for a UD material) not all of them must be used

| 2 *Elastic 'constants'* | 2 *Strengths* | 2 *Fracture toughnesses* | 2 *Invariants* |
|---|---|---|---|
| $E, \nu$ | $R^{tensile}, R^{compr}$ | $K_I^{tensile,crit}, K_{II}^{compr,crit}$ | $I_1, J_2$ ('*Mises*') . |

3 Mohr-Coulomb requires for the real crystal another inherent parameter: a friction value

4 Beltrami gave hints to relate invariants to the behaviour of a material element:

Volume change : $I_1^2$ with $I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T \equiv f(\sigma)$,

Shape change: $6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 \equiv f(\tau)$

5 Fracture morphology witnesses

- There are 2 failure modes in case of isotropic materials and 5 in the UD case
- Each strength failure corresponds to a distinct *failure mechanism*
  and to a *failure type* as Normal Fracture (NF) or Shear Fracture (SF)
- Each of the 5 UD *strength failure modes* is governed by one strength, only!

Above knowledge is fully used in the FMC-based development of strength failure conditions and applied to isotropic, transversely-isotropic (UD), and orthotropic materials.
The FMC strictly works with single <u>independent</u> failure modes.

Fig. 1 confirms by its schematics that 5 failure modes can be observed.

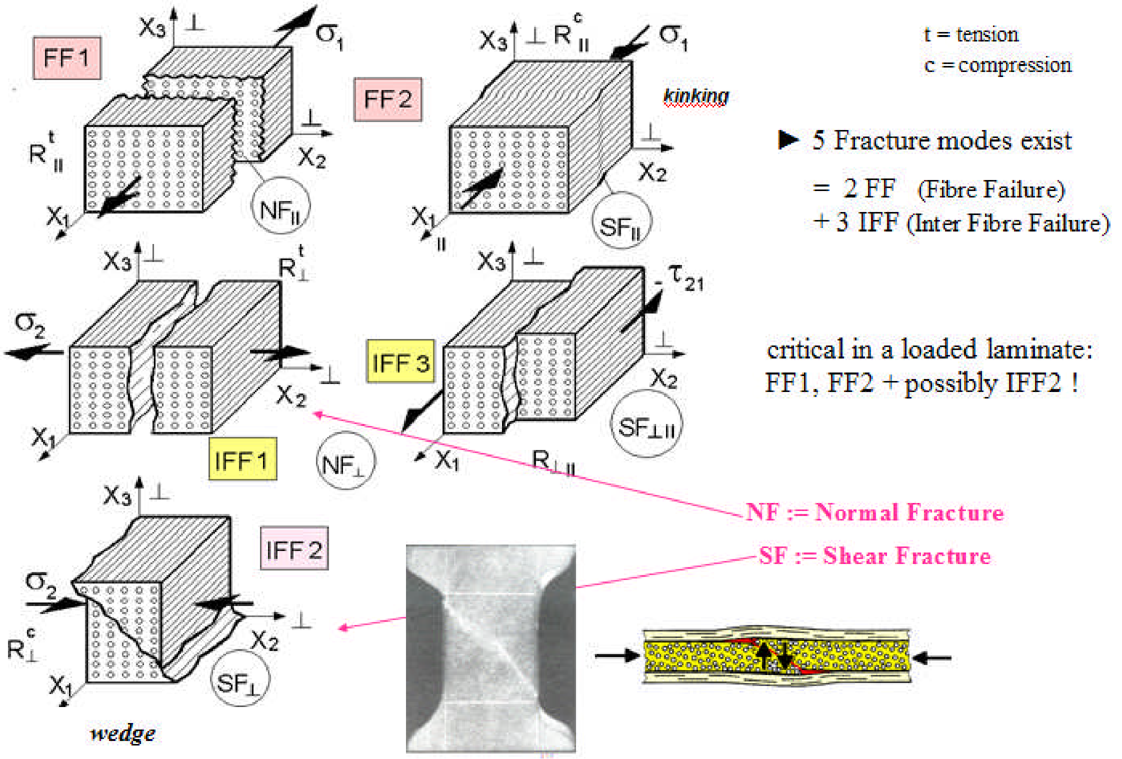


Fig. 1 Fracture 'planes', modes and strengths of UD material

<u>Note</u>: An invariant is a combination of stresses, the value of which does not change when altering the coordinate system. The application of invariants is of benefit when generating failure conditions.

## 2.2 Introduction of 'Material Stressing Effort' *Eff*

Design engineers are familiar with the term reserve factor. According to the Standards this term, however, is load-defined and describes *How much may we upload?* Therefore it may be used for linear analyses, only, when stress is proportional to the load.

In non-linear analysis, including material and geometrical non-linearity, the only accurate quantity is the so-called *material stressing effort* $Eff$ (in Germany, a good old term called Werkstoff-Anstrengung is used. The English lacks for a term). This word has been recently settled with the WWFE organizers as that English term which describes Anstrengung best. Physically, $Eff$ stands for the actual portion of load-carrying capacity of the material.

To be distinguished are two values, a global value and a modal value associated to a distinct mode:

- global *material stressing effort* $Eff$ *,* involves the global responsibility for failure
- modal *material stressing effort* $Eff^{\bmod e}$

$Eff$ corresponds in case of linear analysis with the load reserve factor due to $Eff = 1/f_{\text{Res}}$.

### 2.3 Interaction of Strength Failure Modes in the FMC

Each active failure mode contributes to the global material stressing effort. As model for this interaction of adjacent failure modes a *series failure system* model is employed. It accumulates the interacting failure danger portions as

$$Eff = \sqrt[m]{(Eff^{\text{mode1}})^m + (Eff^{\text{mode2}})^m + \ ....} \tag{1}$$

with the so-called (mode) interaction coefficient $m$ which is in the range of *2.5 < m < 3 for CFRP,* found from mapping many courses of test data.

If just one strength is responsible for a failure mode, then, as with the single mode 'Mises yielding', a *mode equivalent stress* can be formulated. In the case of UD materials it reads

$$Eff^{\bmod e} = \sigma_{eq}^{\bmod e} / \overline{R}^{\bmod e} \tag{2}$$

involving the mode associated average strength.

The next section briefly lists all the FMC-derived failure conditions, see [Cun10a,b].

## 3 FMC-based UD Strength Failure Conditions

### 3.1 Development of the Conditions

Strength Failure Conditions are demanded to be :

- simply formulated, numerically robust,
- physically-based, and therefore need only few information during pre-dimensioning
- shall allow for a simple determination of the design driving mode material stressing effort
- ply-oriented in the case of UD composites.

*One global strength* failure condition : F ( {$\sigma$}, {$R$} ) = 1 (usual formulation); (3a)

*Set of modal* strength failure condi**tio**n : F ( {$\sigma$}, $R^{mode}$) = 1 (addressed in FMC ). (3b)

Test data mapping : $R \Rightarrow \overline{R}$ average strength value (here addressed)

Design Verification : $R$ general and strength design allowable .

with stress vector $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{13}, \tau_{12})^T$ , strength vector $\{R\} = (R_{\parallel}^t, R_{\parallel}^c, R_{\perp}^t, R_{\perp}^c, R_{\perp\parallel})^T$ and $F$ as failure function.

### 3.2 FMC-based 3D Static Failure Conditions for UD material

The FMC enables to formulate an equivalent stress $\sigma_{eq}$ due to the fact that each failure mode is characterized by one strength, only. For each mode, analogously to Mises Yielding can be written

$$Eff^{\bmod e} = \sigma_{eq}^{\bmod e} / R^{\bmod e}. \tag{4}$$

Eq.(2) includes all stresses that are acting together in a given mode.

The vector of the mode equivalent stresses reads (indices $\sigma, \tau$ mark the fracture causing Mohr stress)

$$\{\sigma_{\text{eq}}^{\text{mode}}\} = \left(\sigma_{eq}^{\parallel\sigma},\ \sigma_{eq}^{\parallel\tau},\ \sigma_{eq}^{\perp\sigma},\ \sigma_{eq}^{\perp\tau},\ \sigma_{eq}^{\parallel\perp}\right)^T \ . \tag{5}$$

Replacing the invariants by the associated stress formulations delivers a set of 2 fibre failure (FF)

**FF1** $Eff^{\parallel\sigma} = \sigma_1 / \overline{R}_{\parallel}^{t} = \sigma_{eq}^{\parallel\sigma} / \overline{R}_{\parallel}^{t}$, $\sigma_1 \cong \varepsilon_1^t \cdot E_{\parallel}$ (matrix negligible), * **filament ! modes**

**FF2** $Eff^{\parallel\tau} = -\sigma_1 / \overline{R}_{\parallel}^{c} = +\sigma_{eq}^{\parallel\tau} / \overline{R}_{\parallel}^{c}$, $\sigma_1 \cong \varepsilon_1^c \cdot E_{\parallel}$

**IFF1** $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\ 2}}\,] / 2\overline{R}_{\perp}^{t} = \sigma_{eq}^{\perp\sigma} / \overline{R}_{\perp}^{t}$ **matrix**

**IFF2** $Eff^{\perp\tau} = [(b_{\perp\perp} - 1) \cdot (\sigma_2 + \sigma_3) + b_{\perp\perp}\sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}^{\ 2}}\,] / \overline{R}_{\perp}^{c} = +\sigma_{eq}^{\perp\tau} / \overline{R}_{\perp}^{c}$ **modes**

**IFF3** $Eff^{\perp\parallel} = \{[b_{\perp\parallel} \cdot I_{23-5} + (\sqrt{b_{\perp\parallel}^{\ 2} \cdot I_{23-5}^{\ 2} + 4 \cdot \overline{R}_{\perp\parallel}^{\ 2} \cdot (\tau_{31}^2 + \tau_{21}^2)^2}\,] / (2 \cdot \overline{R}_{\perp\parallel}^{\ 3})\}^{0.5} = \sigma_{eq}^{\perp\parallel} / \overline{R}_{\perp\parallel}$

with $I_{23-5} = 2\sigma_2 \cdot \tau_{21}^2 + 2\sigma_3 \cdot \tau_{31}^2 + 4\tau_{23}\tau_{31}\tau_{21}$

**Modes-Interaction:** ***Eff = 100% = 1***

$$Eff^{m} = (Eff_{\parallel}^{\tau})^m + (Eff_{\parallel}^{\sigma})^m + (Eff_{\perp}^{\sigma})^m + (Eff_{\perp}^{\tau})^m + (Eff_{\perp\parallel})^m = 1$$

**Typical friction value data range:** $b_{\perp\parallel} = \mu_{\perp\parallel}$, $b_{\perp\perp} \cong 1/(1 - \mu_{\perp\perp})$

$0 < \mu_{\perp\parallel} < 0.3,\ 0 < \mu_{\perp\perp} < 0.2,\ 2.5 < m < 3.1$

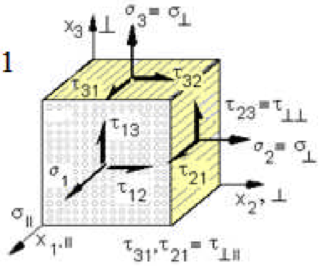


Fig. 2: FMC-based UD strength failure conditions

and 3 inter-fibre-failure (IFF) strength failure conditions that are listed in Fig. 2

Above stresses include the nonlinearly load-dependent load stresses $\{\sigma\}_L$ and the equally nonlinearity dependent residual stresses $\{\sigma\}_R$ from curing. For the measurement of friction values, see [Cun10a].

It has to be paid attention to the limits of homogenization. In fibre direction, failure is not caused by the smeared stress of the fibre 'bundle' but of the filament stress. This can be considered by replacing the smeared stress according to

$$\sigma_1 \cong V_f \cdot \sigma_{1f} = V_f \cdot \varepsilon_1 \cdot E_{\parallel} / V_f = \varepsilon_1 \cdot E_{\parallel} \qquad (6)$$

with $V_f$ as fibre volume fraction and using the filament strain known from the analysis..

An optimal use of the failure conditions above in numerical analysis is described in [Cun10b]. Fig. 3 presents the mapping of the single modes

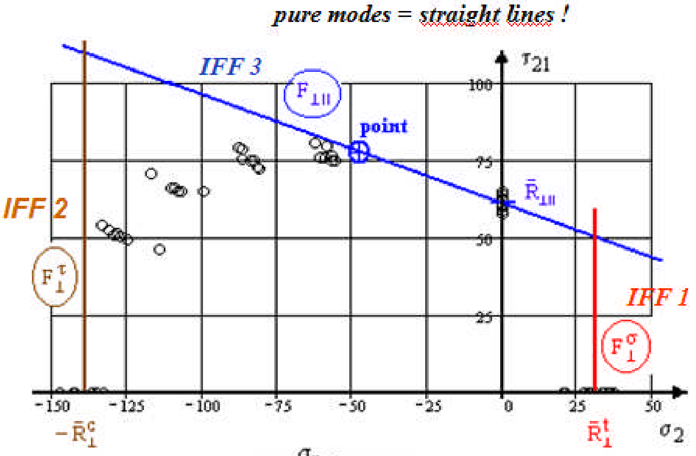
*FF 1* : $\dfrac{\varepsilon_1 \cdot E_{\parallel}}{\overline{R}_{\parallel}^{t}} = 1,$

*FF 2* : $\dfrac{-\varepsilon_1 \cdot E_{\parallel}}{\overline{R}_{\parallel}^{c}} = 1,$

*IFF 1* : $\dfrac{\sigma_2}{\overline{R}_{\perp}^{t}} = 1$

*IFF 2* : $\dfrac{-\sigma_2}{\overline{R}_{\perp}^{c}} = 1$

*IFF 3* : $\dfrac{|\tau_{21}|}{\overline{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 1$



simplified 2D approach: $\tau_{21} = \overline{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2$

$\mu_{\perp\parallel} = \dfrac{\overline{R}_{\perp\parallel} - \tau_{21}^{\oplus}}{\sigma_2^{\ominus}}$ **from friction test or suitable multi-axial fracture stress point**

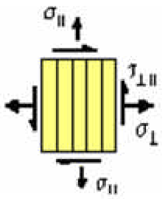


Fig. 3: Modelling of the single, pure failure domains

whereas in Fig. 4 a visualization of the mode-interaction of the developed strength failure conditions is performed for plane stress states $\{\sigma\}=(\sigma_1,\sigma_2,0,0,0,\tau_{21})^T$, and average (typical) stresses are indicated, which are required for mapping. The equation for the global material stressing effort reads

$$\left[\frac{\varepsilon_1^t \cdot E_\parallel}{\bar{R}_\parallel^t}\right]^m + \left[\frac{-\varepsilon_1^c \cdot E_\parallel}{\bar{R}_\parallel^c}\right]^m + \left(\frac{\sigma_2}{\bar{R}_\perp^t}\right)^m + \left(\frac{-\sigma_2}{\bar{R}_\perp^c}\right)^m + \left(\frac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2)}\right)^m = 1. \qquad (7)$$

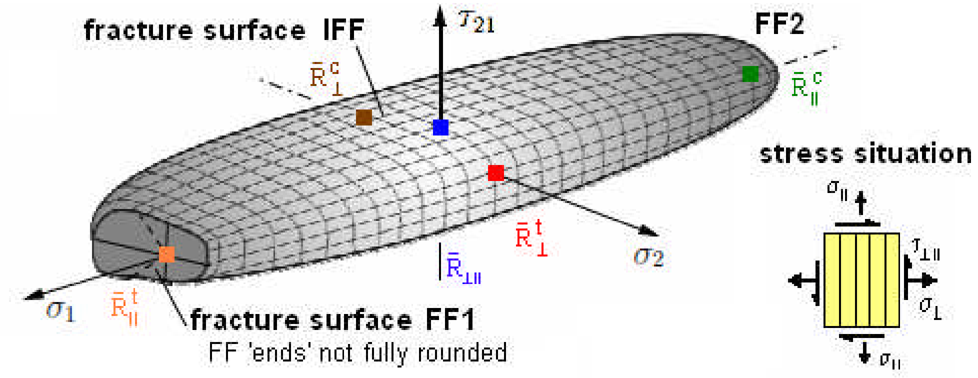


Fig. 4: Modelling of the mode-representative master S-N curve (courtesy W. Becker)

### 3.3 Effects impacting Strength and Strength Values

***Pre-requisites:*** Thereby it is presumed by the author: 1) Pore-free material, no layer waviness, edge effects do not exist; 2) Constant fibre volume content ($V_f = 60\%$); 3) Perfect bonding of the layers.

In this context shall be reminded: Fitting of usually not sufficiently large test data sets means mapping of a random situation. Model parameters are variables. Therefore, a determined parameter is always more or less attributed to such a specific case. Average (typical) data are seldom available.

***'Isolated' and 'embedded 'properties:*** Properties, used as input for the analysis are generally test results from *isolated* UD lamina specimens such as from a tensile coupon or a 90° wound tube. They are load-controlled achieved and are results of *weakest link* type behaviour whereas the in-situ behaviour of a (constraint) one-sided or two-sided UD lamina, *embedded* in a laminate, is deformation-controlled and therefore of *redundant type.* In Fig. 5 for an is*olated* lamina, *weakest link type* test results are clearly witnessed. A single crack is generated at the site of the most unfavourable flaw and this leads to full direct fracture in contrast to the situation on the right sketches.

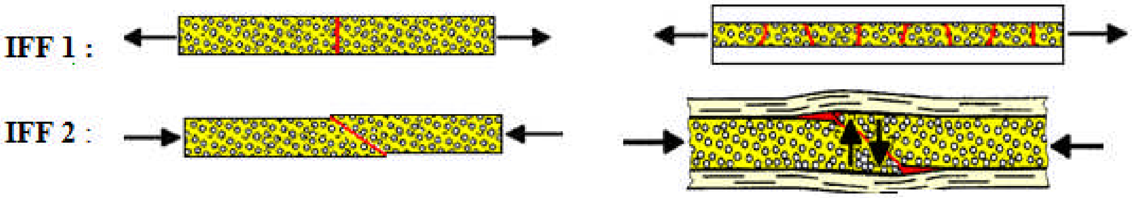


Fig. 5: IFF features in isolated (left) and embedded (right) laminas, [Puck, VDI97]

A non-linear analysis in the strain-softening domain of an embedded lamina has to consider both, the positive effect from obtaining redundancy by the embedding and the adverse effect of notching neighbour layers by localizing of micro-cracks versus the end of diffuse multi-site micro-cracking when approaching the so-called characteristic damage state (CDS). This fact shows up that a good mapping of the course of ‘isolated UD test data’ does not involve the full information necessary for a qualified analysis of laminates which consist of a stack of embedded laminas, [Cun10a].

***Hydrostatic pressure states:*** Generally, mulri-axial compressive stress states have advantages and reduce the material stressing effort or the associated equivalent stress. They exhibit ‘healing’ (smoothing behaviour) effects due to the redundancy effects. The effect is stressed in Fig. 8.

***Residual stresses and thickness effect:*** Residual stresses in a lamina of the laminate are decaying with decreasing stiffness caused by the degradation which accompanies increasing non-linearity. In other words: In parallel to the decay of the stiffness the non-linear analysis releases matrix-dominated stresses. Due to being strain-controlled, the material flaws in a *thin* lamina cannot grow freely up to micro-crack size in the thickness direction (this is sometimes called *thin layer effect*), because the neighbouring laminas act as micro-crack-stoppers. Considering fracture mechanics, the strain energy release rate responsible for the development of damage in the 90° plies from flaws into micro-cracks, increases with increasing ply thickness. Therefore, the actual absolute thickness of a lamina in a laminate is a driving parameter for initiation or onset of micro-cracks.

## 4 Application by Mapping the course of some Static Test Data

Available test data are seldom complete and sometimes show irregularities from testing. For instance Fig. 6 presents a gap in the fourth quadrant.

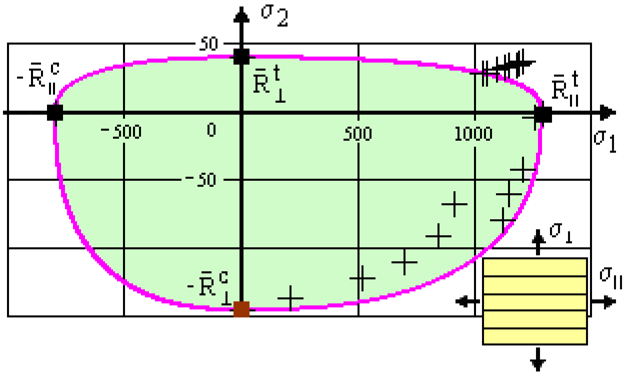


Fig. 6:
*Bi-axial (non-)failure stress envelope*
$(\sigma_2^c, \sigma_1^t)$.
*UD lamina*
*E-lass/MY750epoxy.*

Fig. 7 shows a bi-axial compressive failure curve. As a warning, the Tsai-Wu curve is included in the diagram in order to show the shortcoming of a global failure condition in this quadrant. *Tsai-Wu is not applicable here.*

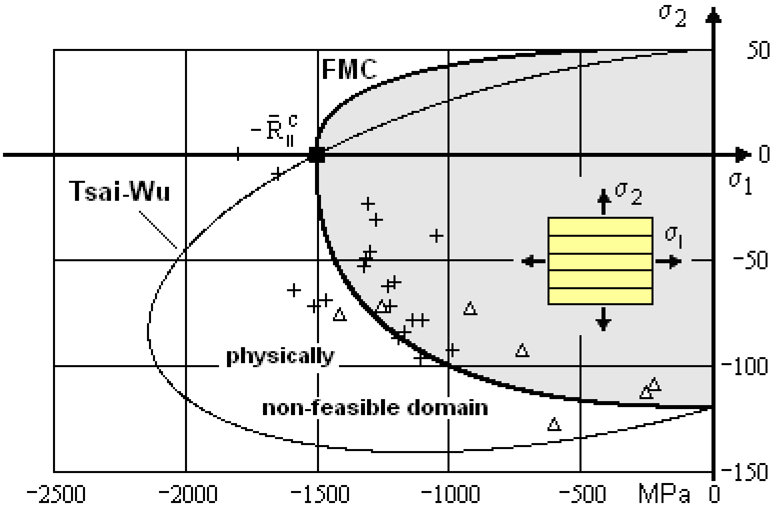


*Fig. 7: Bi-axial failure curve*
$(\sigma_2^c, \sigma_1^c)$
*UD-lamina.*
*T300/LY556/HY917/DY070*
.

Eventually Fig. 8 depicts the failure curve of the fracture shear stress under hydrostatic pressure.

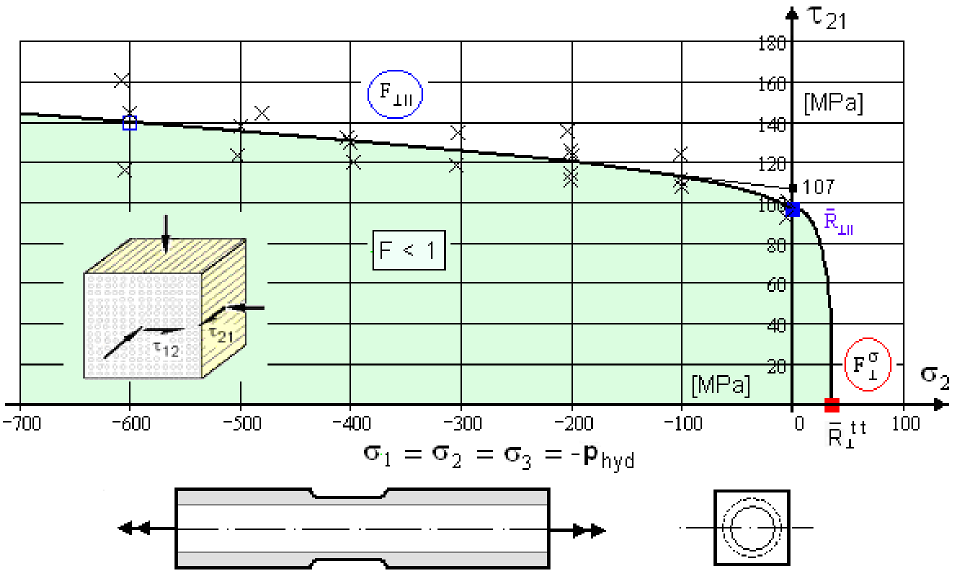


Fig. 8: UD T300 carbon/PR319 epoxy $\tau_{21}(\sigma_1 = \sigma_2 = \sigma_2 = -p_{hyd})$, $m = 2.8$, $\mu_{\perp\parallel} = 0.32$,

$$\{\bar{R}\} = (1378,\ 959,\ 40,\ 125,\ 97)^T,\ \{\varepsilon_{fr}\} = (1.07,\ 0.74,\ 0.43,\ 2.8,\ 8.6)^T\,\%,$$

# 5 FMC-based Lifetime Prediction Method

## 5.1 Modelling of Loading Cycles

The first idea of the proposed method is a, *[Cun96],*

*I: failure mode-linked apportionment of the encountered cyclic loading.*

For the sake of simplicity the principle is shown for isotropic material. As simple loading a constant amplitude loading of a stress ratio $R = -1$ is chosen (Standards-caused the letter $R$ has to be applied as with the strength). Fig. 9 depicts the procedure and an indication of the insertion of the obtained cycle blocks into the Miner 'rule'.

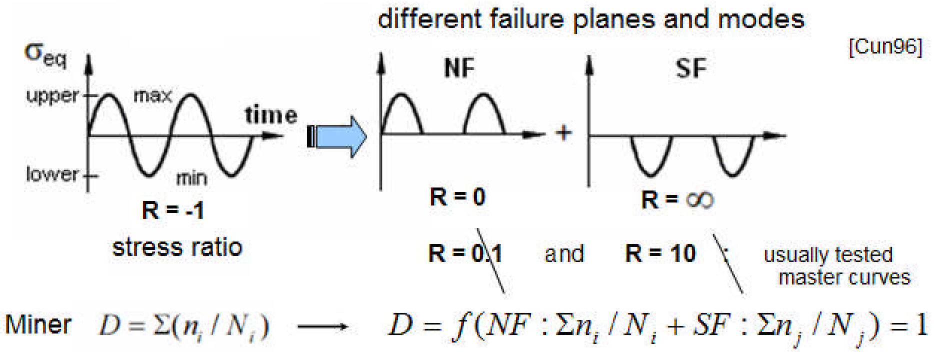


Fig. 9. Failure mode-based modelling of loading cycles

## 5.2 Modelling of the Mode-representative Master S-N curve

The next idea can be formulated as, compare [Sho96],

*II: S-N curve can be mapped by a straight line in the log-log diagram*, Fig. 10.

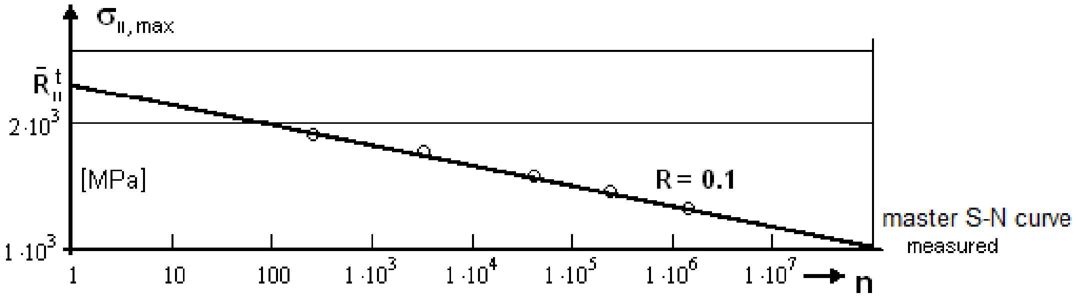


Fig. 10: Modelling of the mode-representative master S-N curve. FF1 test data from [Kaw04]

As mapping function for the S-N curve the simplest type is taken, namely a straight line

$$\sigma^{repr}_{\parallel,\ \max}(n) \approx \bar{R}^t_\parallel \cdot n^{c_{repr}}. \qquad (8)$$

In the case of brittle behaving materials the strength values can be used in the S-N curve as origins at n=1. This depends on the model used. Further applicable curve models were the Weibull-model and the Wearout-model, [VDI 2014].

## 5.3 Mode S-N Curves Prediction on basis of Master Curve

The logic behind the use of the strain energy equivalence is: Fatigue strain energy, required to generate a distinct damage state is equal to the strain energy, which is necessary under monotonic loading to obtain the same damage state. Neglecting heat loss [Kad94], damage is proportional to the supplied energy.

This energy can be formulated as $\Delta W = \frac{1}{2} \cdot (\sigma_{\max} \cdot \varepsilon_{\max} - \sigma_{\min} \cdot \varepsilon_{\min})$.

Ignoring non-linearity and applying Hooke $\sigma = \varepsilon \cdot E$ the formulation takes the shape

$$\Delta W = (\sigma_{\max}{}^2 - \sigma_{\min}{}^2) / 2 \cdot E.$$

Of advantage, however not necessary, for the derivation of the present method, is a normalization of the strain energy [Sho06]

$$\Delta W \cdot \overline{R}^2 \propto \sigma_{\max}{}^2 - \sigma_{\min}{}^2 = \sigma_{\max}{}^2 \cdot (1 - R^2).$$

Brittle materials such as composites the maximum (upper) stress representation is more often used. It can be transferred to the stress range, see Fig. 6, according to

$$\sigma_{\max} = \Delta\sigma / (1-R) = 2 \cdot \sigma_a / (1-R) \quad \text{with} \quad \Delta\sigma = \text{stress range}.$$

Fatigue strain energy, required to generate a distinct damage state is equal to the strain energy, which is necessary under monotonic loading to obtain the same damage state. In other words this may be formulated as

> *III: A distinct strain energy level, associated to a point at the given mode-representative S-N curve (here R = 0.1 as master R-ratio curve) will be reached for R = 0.5 at higher cycles.*

The idea is demonstrated for the simple case of a fibre-dominated laminate under tension, where the change of strain energy between maximum and minimum loading for the FF1 portion reads

$$\Delta W^{\|\sigma} \cdot \overline{R}_{\|}^{t} \propto \Delta\sigma_{eq}^{\|\sigma\,2} \Rightarrow \Delta W^{\|\sigma} \cdot \overline{R}_{\|}^{t\,2} \propto \sigma_{\|,\max}{}^2 \cdot (1 - R^2) = \sigma_{|eq}^{\|\sigma\,2} \cdot (1 - R^2).$$

From experiment is known: maximum stress, tensile strength, stress ratio R, and thereby also the fatigue strain energy.

The strain energy of all mode contributions (5 in UD case) reads $\Delta W_{\text{total}} = \sum_1^5 \Delta W^{\text{modes}}$ .

Prediction of another mode S-N curve from the mode-representative master curve requires the determination of the new slope, here slope of $R = 0.5$, considering

$$\sigma_{\|,\max}^{pred} = \overline{R}_{\|}^{t} \cdot n^{c_{pred}} \quad \Longrightarrow \quad \sigma_{\|,\ \max}^{repr} = \overline{R}_{\|}^{t} \cdot n^{c_{repr}} ..$$

The application of the strain energy equivalence is demonstrated in Fig. 11.

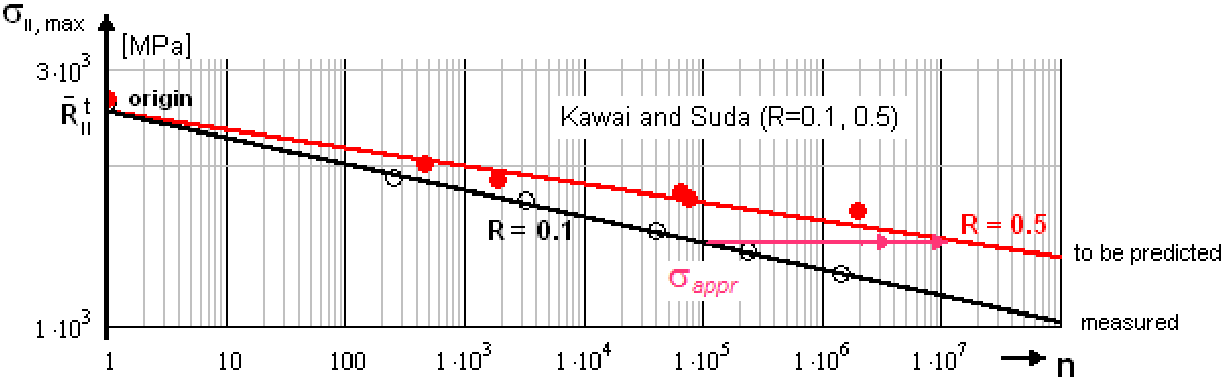


Fig. 11: Prediction of mode S-N curves from a mode-representative master curve

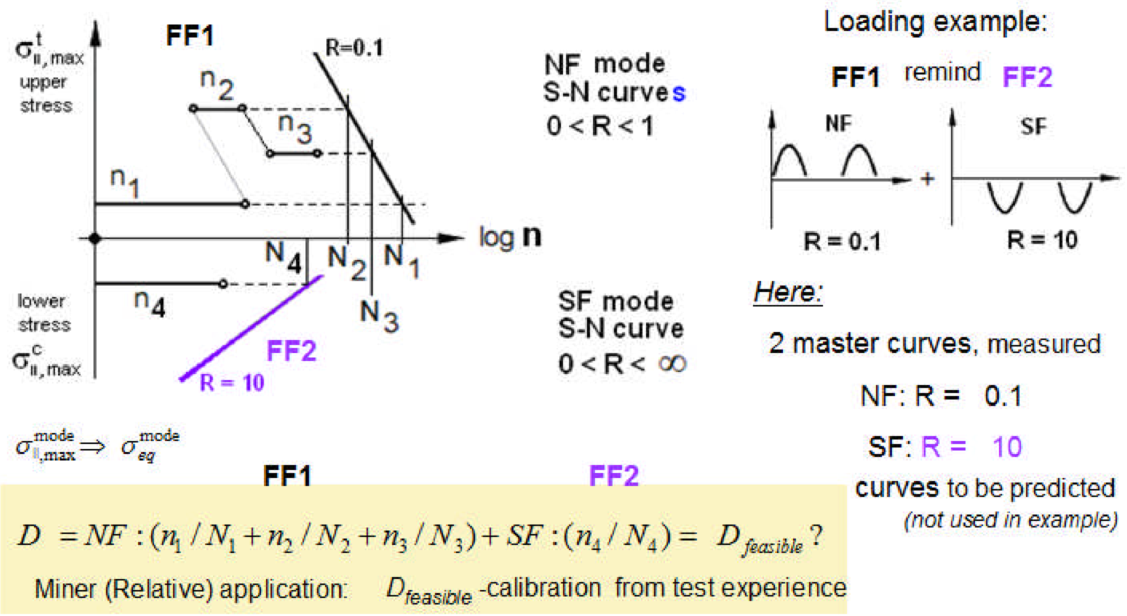


Fig. 12: Schematic application of the method for two modes

## 6 Application of the Method to Miner Rule

Eventually, in Fig. 12 a schematic application is presented. Of course, a maximum allowable damage value $D_{feasible}$ is to be experimentally derived.

What is still open? How essential is the mode interaction damaging and the effect of loading sequence.

## Conclusions & Outlook

***FMC-based Static Strength Failure Conditions:***

1) 2D stress case: Test data mapping was successful. Validation obtained
2) 3D stress case: Looks promising for modal conditions as far as reliable 3D test data were available.
   To be done: Generation of missing 3D strength test data in various stress state domains.

***Idea for a new, failure mode-linked Lifetime Prediction Method:***

*method for the usually fibre-dominated laminates,* which employs

1) Measurement of a minimum number of representative S-N curves
2) Prediction of other necessary stress-ratio *mode S-N curves* on basis of an available representative one, typical for the envisaged mode. Use of *strain energy equivalence*
3) Failure mode-linked damage accumulation (Miner)
4) Damage accumulation depends on cycles-linked shrinking of failure surface.
   In-situ-effect consideration by deformation-controlled testing.
   To be done: Investigation of the new idea and of probable additional damage caused by mode changes (FF, IFF, mixed).

Note: ***Generating reliable 3D test data is a bigger challenge than generating a theory.***

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