Trace-based Design, Testing and Manufacturing of Composites

Stephen W. Tsai
Stanford University
June 9, 2014

Outline

- Trace-based theory
- Supporting data
- Prelminary design
- Omni strain allowable

Stiffness and Compliance Matrices

$$[Q] = \begin{bmatrix} \frac{E_x}{1-\nu_x\nu_y} & \frac{\nu_y E_x}{1-\nu_x\nu_y} & 0 \\ \frac{\nu_x E_y}{1-\nu_x\nu_y} & \frac{E_y}{1-\nu_x\nu_y} & 0 \\ 0 & 0 & E_s \end{bmatrix} \qquad [S] = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_y}{E_y} & 0 \\ -\frac{\nu_x}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & 1/E_s \end{bmatrix}$$

Reciprocal relation: $\nu_x E_y = \nu_y E_x$

Laminate in-plane stiffness in terms of ply stiffness [Q]:

$$[\mathbf{A}^*] = \frac{1}{h}[A] = \frac{1}{h}\sum_{i=1}^{m}[Q']^{(i)}h^{(i)} = \sum_{i=1}^{m}[Q']^{(i)}\frac{h^{(i)}}{h} = \sum_{i=1}^{m}[\mathbf{Q}']^{(\mathbf{i})}\mathbf{v}^{(\mathbf{i})}$$

where $v^{(i)}$ = fraction of the i-th ply group

Ply & Laminate Stiffness Matrix & Trace

Laminate

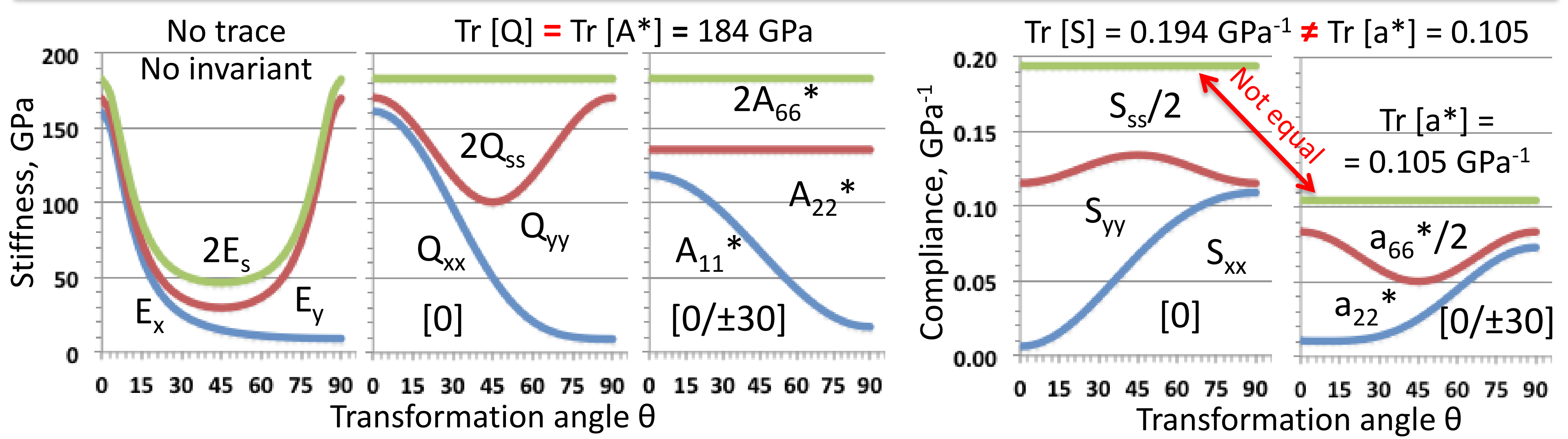
$$\begin{vmatrix} E_1^\circ & \nu_{12}^\circ & 0 \\ \nu_{21}^\circ & E_2^\circ & 0 \\ 0 & 0 & E_6^\circ \end{vmatrix} \Leftrightarrow \begin{bmatrix} A_{11}^* & A_{12}^* & 0 \\ A_{21}^* & A_{22}^* & 0 \\ 0 & 0 & A_{66}^* \end{bmatrix} \Leftrightarrow \begin{bmatrix} a_{11}^* & a_{12}^* & 0 \\ a_{21}^* & a_{22}^* & 0 \\ 0 & 0 & a_{66}^* \end{bmatrix} \Leftrightarrow \begin{vmatrix} E_1^\circ & \nu_{12}^\circ & 0 \\ \nu_{21}^\circ & E_2^\circ & 0 \\ 0 & 0 & E_6^\circ \end{vmatrix}$$

Eng'g constants array | In-plane stiffness matrix | Compliance matrix | Eng'g constants array

Ply data

$$\begin{vmatrix} E_x & \nu_y & 0 \\ \nu_x & E_y & 0 \\ 0 & 0 & E_s \end{vmatrix} \Leftrightarrow \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{yx} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} \Leftrightarrow \begin{bmatrix} S_{xx} & S_{xy} & 0 \\ S_{yx} & S_{yy} & 0 \\ 0 & 0 & S_{ss} \end{bmatrix} \Leftrightarrow \begin{vmatrix} E_x & \nu_y & 0 \\ \nu_x & E_y & 0 \\ 0 & 0 & E_s \end{vmatrix}$$

Eng'g constants array | Ply stiffness matrix | Ply compliance matrix | Eng'g constants array



Input Data: Ply Stiffness and Strength

Ply name	Ex, GPa	Ey, GPa	nu/x	Es, GPa
X, MPa	X', MPa	Y, MPa	Y', MPa	S, MPa

T700 C-Ply 55[S	121	8	0.3	4.7
2530	1669	66	220	93
T700 C-Ply 64[S	141	9.3	0.3	5.8
2944	1983	66	220	93
IM7/977[SI]	191	9.94	0.35	7.79
3250	1600	62	98	75
T800/Cyt[SI]	162	9	0.4	5
3768	1656	56	150	98
IM7/8552[SI]	171	9.08	0.32	5.29
2326	1200	62	200	81.5

IM7/MTM [SI]	175	8.2	0.33	5.5
2500	1700	69	169	43
AS4/H3501[SI]	138	8.96	0.3	7.1
1447	1447	52	206	93
IM6/ep[SI]	203	11.2	0.32	8.4
3500	1540	56	150	98
T3/F93[SI]	148	9.65	0.3	4.55
1314	1220	43	168	48
T3/N52[SI]	181	10.3	0.28	7.17
1500	1500	40	246	68

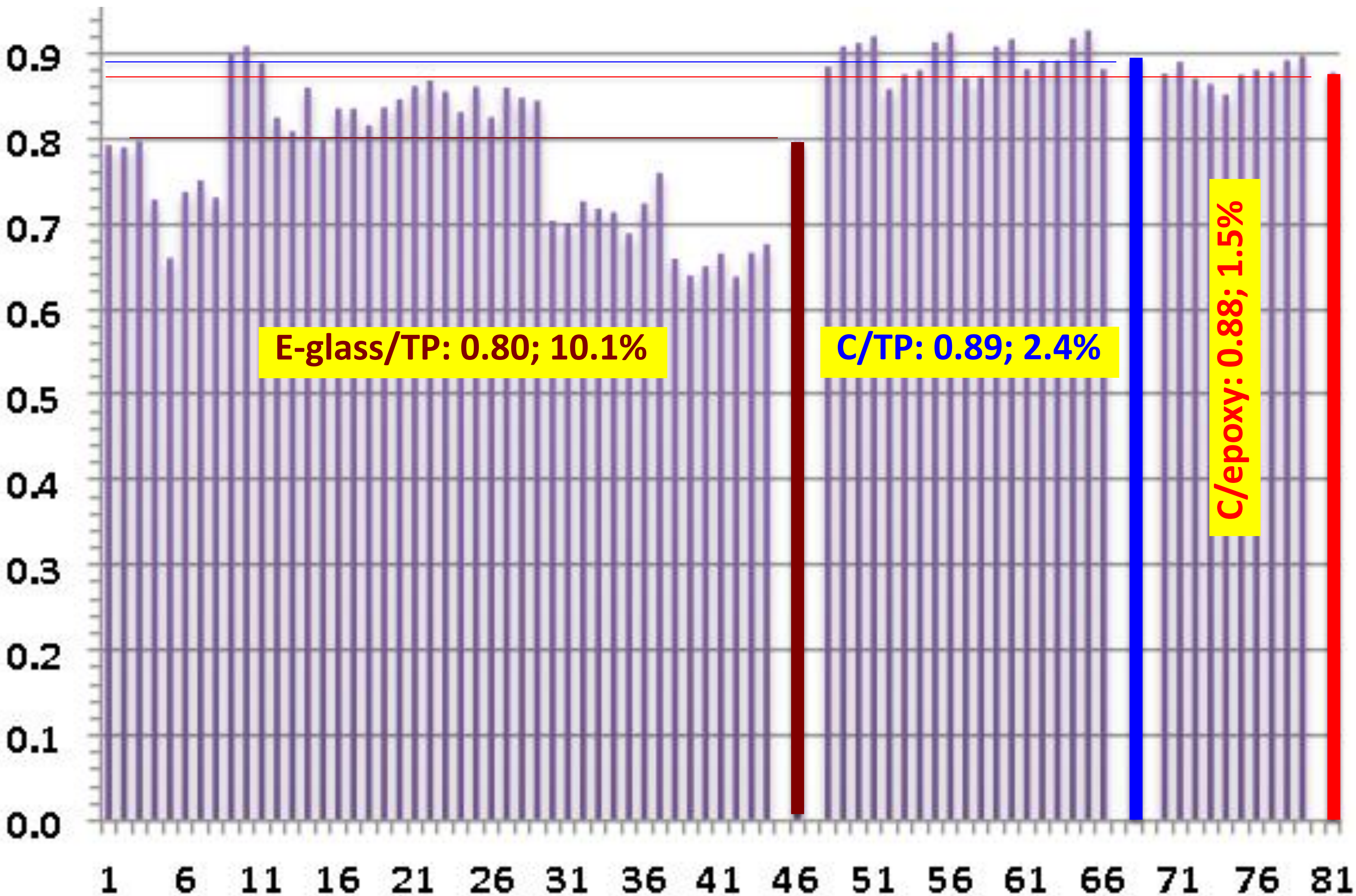
Master Ply Stiffness: Trace Normalized

Carbon/epoxy ply stiffness in trace normalized factors

Material [0]	Qxx*	Qyy*	Qss*,Es	Tr, GPa	Trace*	Ex*	Ey*	nu/x*	Ey/Ex
IM7/977-3	0.88	0.046	0.036	218	1.00	0.88	0.046	0.35	0.052
T800/Cytec	0.90	0.050	0.027	183	1.00	0.89	0.049	0.40	0.056
T7 C-Ply 55	0.88	0.057	0.034	139	1.00	0.87	0.058	0.30	0.066
T7 C-Ply 64	0.87	0.057	0.036	163	1.00	0.86	0.057	0.30	0.066
AS4/3501	0.86	0.056	0.044	162	1.00	0.85	0.055	0.30	0.065
IM6/epoxy	0.88	0.049	0.036	232	1.00	0.88	0.048	0.32	0.055
AS4/F937	0.89	0.058	0.027	168	1.00	0.88	0.057	0.30	0.065
T300/N5208	0.88	0.050	0.035	206	1.00	0.88	0.050	0.28	0.057
IM7/8552	0.90	0.048	0.028	192	1.00	0.89	0.047	0.31	0.053
IM&/MTM45	0.90	0.042	0.028	195	1.00	0.90	0.042	0.33	0.047
Master ply	0.883	0.050	0.034	187	1.00	0.877	0.050	0.305	0.0609
Std dev	0.013	0.005	0.005	28.468	0.001	0.014	0.006	0.034	0.5%
CV	1.5%	10.9%	15.8%		0.1%	1.5%	11.1%	11.3%	9.0%
Master GPa	165	9.35	6.42		187	164	9.30	0.31	11.39

$Q_{xx} = Q_{xx}{}^{*} \times Tr = 0.883 \times 187 = 165$ GPa

Median and Coeff Var of E_x/Trace [Q]



[0] ply data

Material [0]	Qxx*	Qyy*	Qss*	Tr, GPa	Trace*	Ex*	Ey*	nu/x*
IM7/977-3	0.88	0.046	0.036	218	1.00	0.88	0.046	0.35
T800/Cytec	0.90	0.050	0.027	183	1.00	0.89	0.049	0.40
T700 C-Ply	0.88	0.058	0.034	139	1.00	0.87	0.058	0.30
AS4/3501	0.86	0.056	0.044	162	1.00	0.85	0.055	0.30
IM6/epoxy	0.88	0.049	0.036	232	1.00	0.88	0.048	0.32
AS4/F937	0.89	0.058	0.027	168	1.00	0.88	0.057	0.30
T300/N5208	0.88	0.050	0.035	206	1.00	0.88	0.050	0.28
Master ply	0.883	0.0502	0.0348	183	1.000	0.876	0.0500	0.300
Coeff var	1.1%	0.44%	0.53%			1.2%	0.5%	4.1%

Laminates have lower cv than plies

* normalized by Trace

[0/±30]

[0/±30]: 1:0	A11*	A22*	A66*	Tr, GPa	Trace*	E1*	E2*	nu/21*
IM7/977-3	0.65	0.091	0.13	218	1.00	0.52	0.072	1.2
T800/Cytec	0.66	0.091	0.13	183	1.00	0.50	0.069	1.3
T700 C-Ply	0.64	0.099	0.13	139	1.00	0.52	0.079	1.1
AS4/3501	0.64	0.101	0.13	162	1.00	0.53	0.084	1.0
IM6/epoxy	0.65	0.093	0.13	232	1.00	0.52	0.074	1.2
AS4/F937	0.65	0.096	0.13	168	1.00	0.50	0.074	1.2
T300/N5208	0.65	0.093	0.13	206	1.00	0.52	0.075	1.2
Master ply	0.647	0.0930	0.130	183	1.000	0.515	0.0745	1.18
Coeff var	0.57%	0.36%	0.16%			1.0%	0.5%	8.4%

$[0_2/\pm30_2/\pm60/90]$

[0/±30]; 2:1	A11*	A22*	A66*	Tr, GPa	Trace*	E1*	E2*	nu/21*
IM7/977-3	0.46	0.28	0.13	218	1.00	0.42	0.25	0.40
T800/Cytec	0.47	0.28	0.13	183	1.00	0.42	0.25	0.43
T700 C-Ply	0.46	0.28	0.13	139	1.00	0.42	0.25	0.40
AS4/3501	0.46	0.28	0.13	162	1.00	0.42	0.26	0.37
IM6/epoxy	0.46	0.28	0.13	232	1.00	0.42	0.25	0.39
AS4/F937	0.46	0.28	0.13	168	1.00	0.41	0.25	0.42
T300/N5208	0.46	0.28	0.13	206	1.00	0.42	0.25	0.39
Master ply	0.463	0.278	0.130	183	1.000	0.418	0.252	0.398
Coeff var	0.26%	0.12%	0.19%			0.18%	0.20%	1.75%

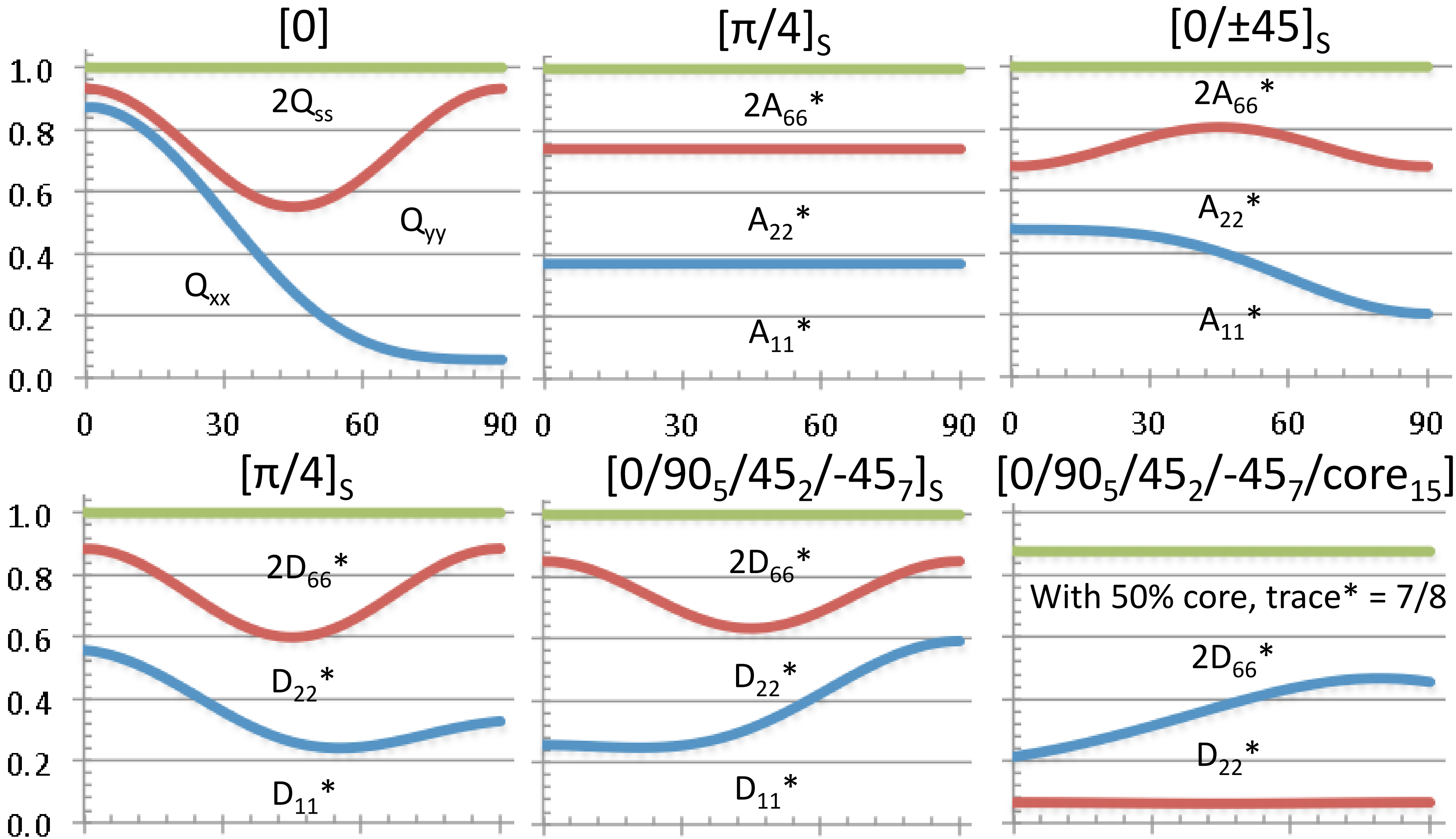
* normalized by Trace

$[0_2/\pm45_3/90]$

[0/±45]; 2:1	A11*	A22*	A66*	Tr, GPa	Trace*	E1*	E2*	nu/21*
IM7/977-3	0.38	0.29	0.16	218	1.00	0.32	0.24	0.48
T800/Cytec	0.39	0.29	0.16	183	1.00	0.31	0.23	0.52
T700 C-Ply	0.38	0.29	0.16	139	1.00	0.31	0.24	0.49
AS4/3501	0.38	0.29	0.16	162	1.00	0.32	0.25	0.46
IM6/epoxy	0.39	0.29	0.16	232	1.00	0.32	0.24	0.48
AS4/F937	0.38	0.29	0.16	168	1.00	0.31	0.23	0.52
T300/N5208	0.39	0.29	0.16	206	1.00	0.32	0.24	0.48
Master ply	0.385	0.293	0.161	183	1.000	0.316	0.240	0.485
Coeff var	0.08%	0.10%	0.09%			0.5%	0.5%	2.2%

Transformed Components of Trace

For all laminates: Tr [Q] = Tr [A*] = Tr [D*]



Normalized Master Laminate Factors

Need only one test: E_x /0.876 = Tr [A°] >>> factors for $E_1°$, $E_2°$, ν_x, $E_6°$

Zero test: If you believe in rule of mixtures that $E_x = v_f E_f$

Or another single test of [π/4]: $E_1°$/0.337 = Tr [A°], …

Master Laminate	E1°/Tr	E2°/Tr	nu/x	E6°/Tr	Trace*
[0]	0.876	0.050	0.300	0.0343	1.000
[0/90]	0.468	0.468	0.036	0.031	0.999
[0/45/90/-45] = [π/4]	0.337	0.337	0.298	0.130	1.000
[0/±30]	0.515	0.0745	1.180	0.130	0.998
[(0/±30)2/±60/90]	0.418	0.252	0.398	0.130	1.000
[0/±30/±60/90] = [π/6]	0.338	0.338	0.297	0.130	1.002
[0/±45]	0.377	0.158	0.709	0.161	1.000
[(0)2/(±45)3/90]	0.316	0.240	0.485	0.161	0.999
[0/(±45)2/90]	0.280	0.280	0.419	0.161	1.001

Examples: For [0/±45], $E_1°$ = 0.377 Tr; $E_6°$ = 0.161 Tr (shear test can be avoided)

For C-Ply 55, Tr = 139 GPa, $E_1°$ = 0.377 x 139 = 52.4 GPa; $E_6°$ = 0.161 x 139 = 22.4 GPa

For T800/Cytec, Tr = 183 GPa, , $E_1°$ = 0.377 x 183 = 69.0; $E_6°$ = 0.161 x 183 = 29.4 GPa

How Many Specimens: 1 or 0

$E_f \overset{v_f}{>>>>>>>>} E_x \overset{0.88}{>>>>>>>>}$ Trace [Q] >>>>> Laminate stiffness:

	E1*	E2*	nu/21*	E6*
[0]	0.876	0.052	0.300	0.0348
	1.2%	0.5%	4.1%	0.52%
[π/4]	0.337	0.337	0.298	0.130
	0.13%	0.13%	1.2%	0.17%
[0/±30]	0.515	0.0745	1.18	0.130
	1.0%	0.5%	8.4%	0.16%
[0/±45]; 4::	0.315	0.240	0.485	0.161
	0.5%	0.5%	2.1%	0.09%

Trace-normalized Stiffness of CFRP

[π/4]

[π/4]	A11*=A22*	A66*	TRACE*	E1*	nu/x*
IM7/977-3	0.370	0.130	1.000	0.337	0.297
T800/Cytec	0.373	0.127	1.000	0.335	0.320
T700 C-Ply 55	0.371	0.129	1.000	0.337	0.305
T700 C-Ply 64	0.370	0.130	1.000	0.337	0.300
AS4/3501	0.368	0.132	1.000	0.338	0.282
IM6/epoxy	0.370	0.130	1.000	0.337	0.295
AS4/F937	0.373	0.127	1.000	0.335	0.314
T300/N5208	0.370	0.130	1.000	0.337	0.294
IM7/8552	0.372	0.128	1.000	0.336	0.312
IM7/MTM45	0.371	0.129	1.000	0.336	0.307
MEDIAN	0.371	0.129	1.000	0.337	0.303
Std dev*	0.15%	0.15%	0.15%	0.10%	1.1%

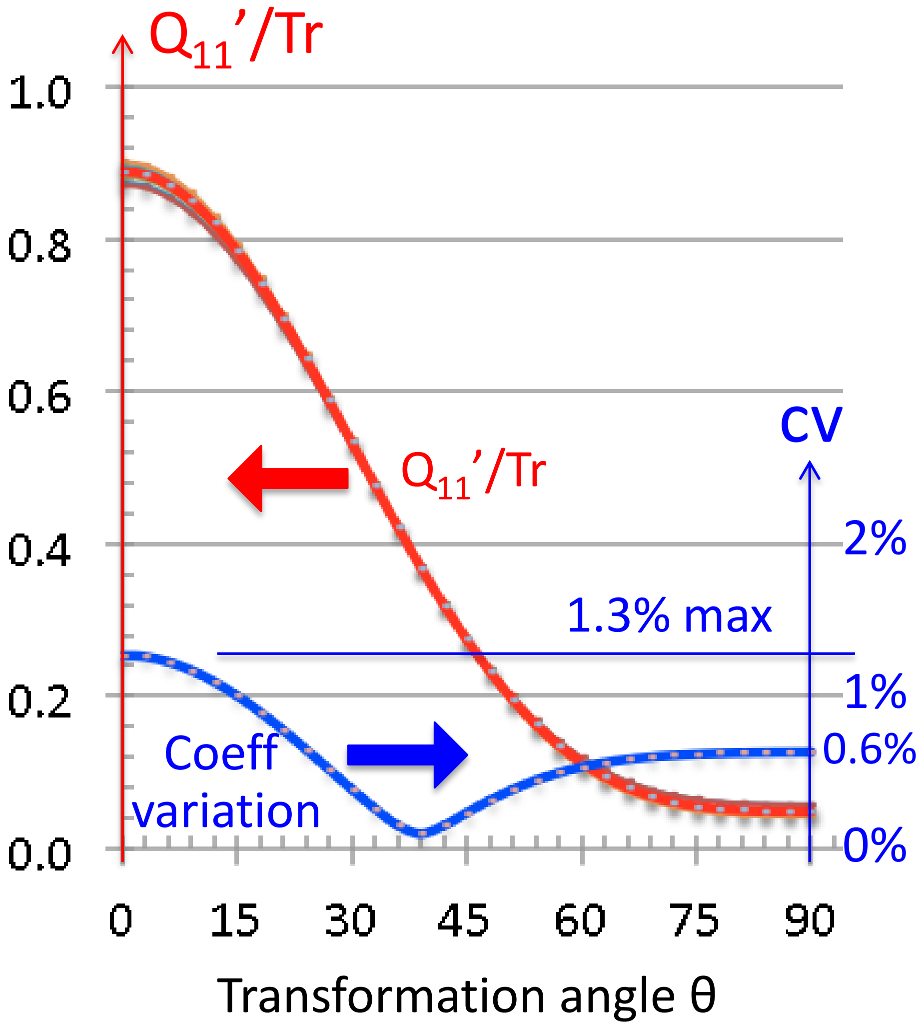
Example: 2(0.371 + 0.129) = 1.000 = Trace*

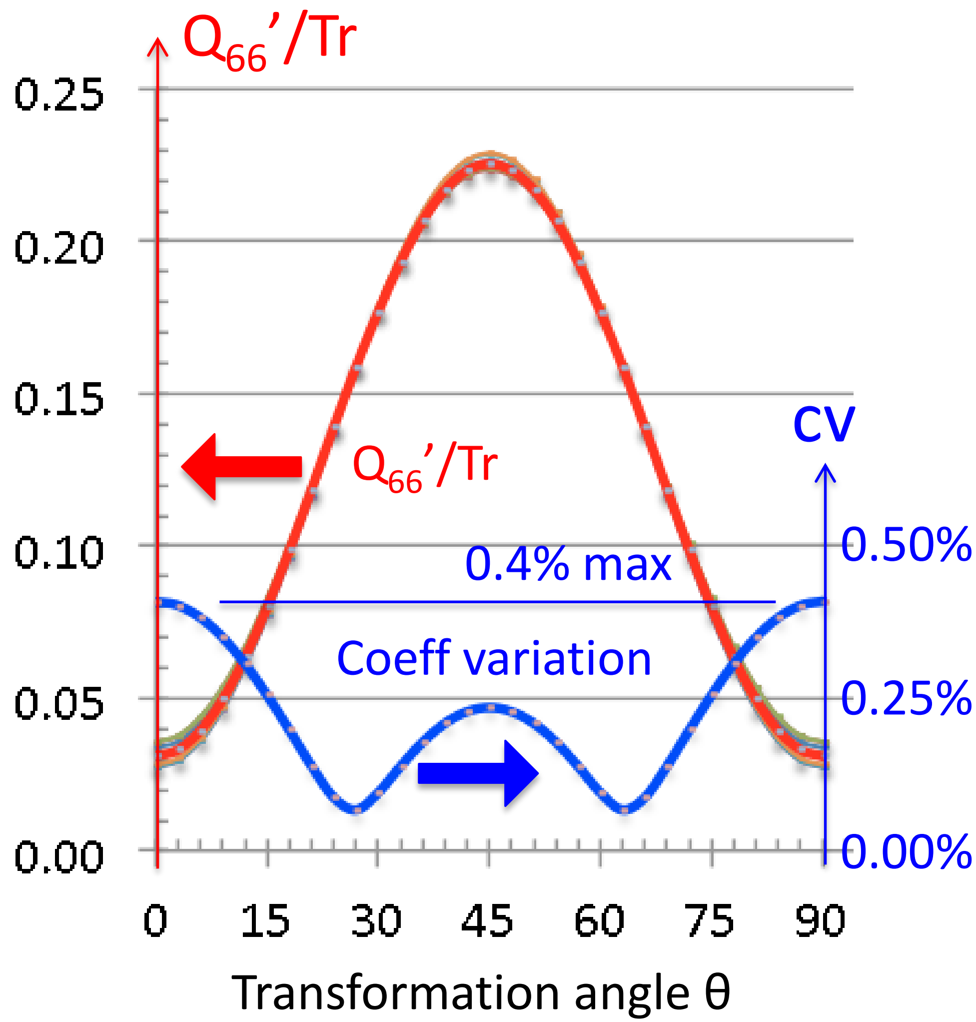
Q_{11}/Tr for CFRP

Theta	C-Ply 55	C-Ply 64	IM7/977	T8/Cytec	IM7/8552	IM7/MTM	Master ply Median	Std dev
0	0.87	0.87	0.88	0.90	0.90	0.90	0.889	1.3%
15	0.77	0.77	0.78	0.79	0.79	0.79	0.784	1.0%
30	0.53	0.53	0.53	0.53	0.53	0.54	0.533	0.4%
45	0.28	0.28	0.28	0.27	0.27	0.27	0.275	0.2%
60	0.12	0.12	0.11	0.11	0.11	0.11	0.113	0.5%
75	0.06	0.07	0.05	0.06	0.05	0.05	0.056	0.6%
90	0.06	0.06	0.05	0.05	0.05	0.04	0.049	0.6%

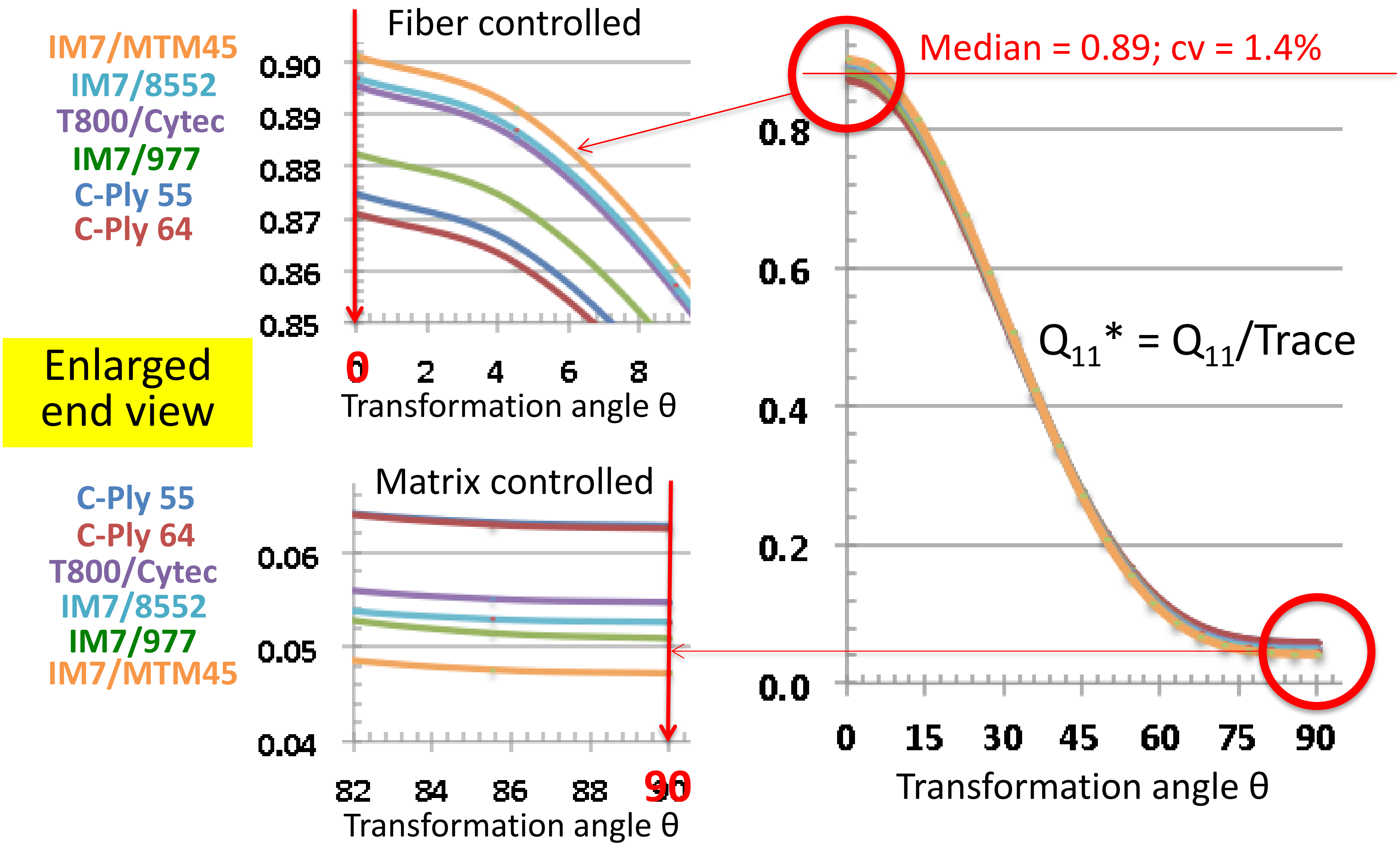
Transformed: Q_{11}'/Tr, Q_{66}'/Tr, & CV

Based on 6 typical CFRP





Dispersion of Q_{11}* at 0° and 90°



Trace in 3D of T300/N5208

Trace [C] = $C_{11} + C_{22} + C_{33} + 2C_{23} + 2C_{13} + 2C_{12}$ = 247 GPa

2D trace = 206 GPa; Ratio = 247/206 = 1.2

[C] at 0 deg	1	2	3	4	5	6
1	184.6	5.88	5.88	0	0	0
2		13.94	7.06	0	0	0
3			13.94	0	0	0
4				3.44	0	0
5					7.17	0
6						7.17

$$[Q] = \begin{bmatrix} 181.8 & 2.90 & 0 \\ & 10.35 & 0 \\ & & 7.17 \end{bmatrix}$$

[C] at 45 deg	1	2	3	4	5	6
1	59.75	45.41	6.47	0	0	42.67
2		59.75	6.47	0	0	42.67
3			13.94	0	0	-0.59
4				5.31	1.87	0
5					5.31	0
6						46.70

3D vs 2D Trace: Constant ratio of 1.2

CFRP	2D Tr, GPa	E1* 2D	3D Tr, GPa	E1* 3D	3D/2D Trace	2D/3D E1*	Ratio Tr/E1
IM7/977-3	218	0.88	261	0.75	1.20	1.17	1.03
T800/Cytec	183	0.89	219	0.77	1.20	1.16	1.04
T700 C-Ply 55	139	0.87	170	0.73	1.22	1.19	1.03
T700 C-Ply 64	163	0.87	199	0.73	1.22	1.19	1.02
AS4/3501	162	0.85	200	0.71	1.23	1.21	1.02
IM6/epoxy	232	0.87	279	0.74	1.20	1.17	1.02
T300/N5208	206	0.88	248	0.74	1.20	1.18	1.02
IM7/8552	192	0.89	227	0.77	1.18	1.16	1.02
IM7/MTM45	195	0.90	228	0.78	1.17	1.15	1.02
T700/2510	144	0.88	175	0.74	1.22	1.19	1.03
T650/epoxy	160	0.87	197	0.73	1.23	1.19	1.03
IM7/8552 N	180	0.88	215	0.76	1.20	1.17	1.02
T4708/MR60H	158	0.90	187	0.78	1.18	1.15	1.03
Cytec/MTM45	143	0.89	171	0.76	1.20	1.17	1.02
Median	171	0.877	208	0.748	1.20	1.17	1.02
Coeff Var	28	1.3%	32	2.1%	1.8%	1.7%	0.4%

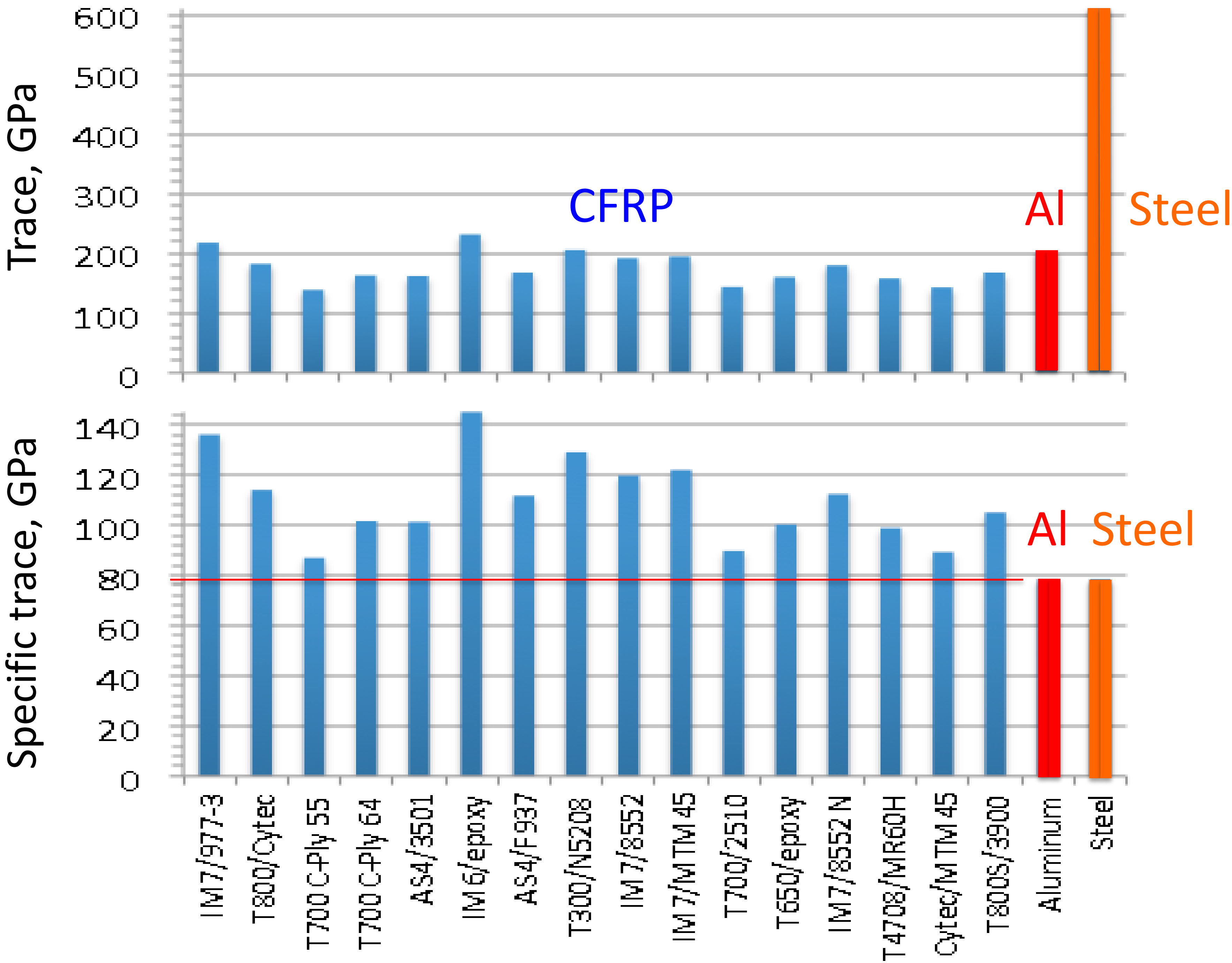
Less fiber, and more matrix contribution to trace: 0.877 for 2D versus 0.748 for 3D

Outline

- Trace-based theory
- Supporting data
- Prelminary design
- Omni strain allowable

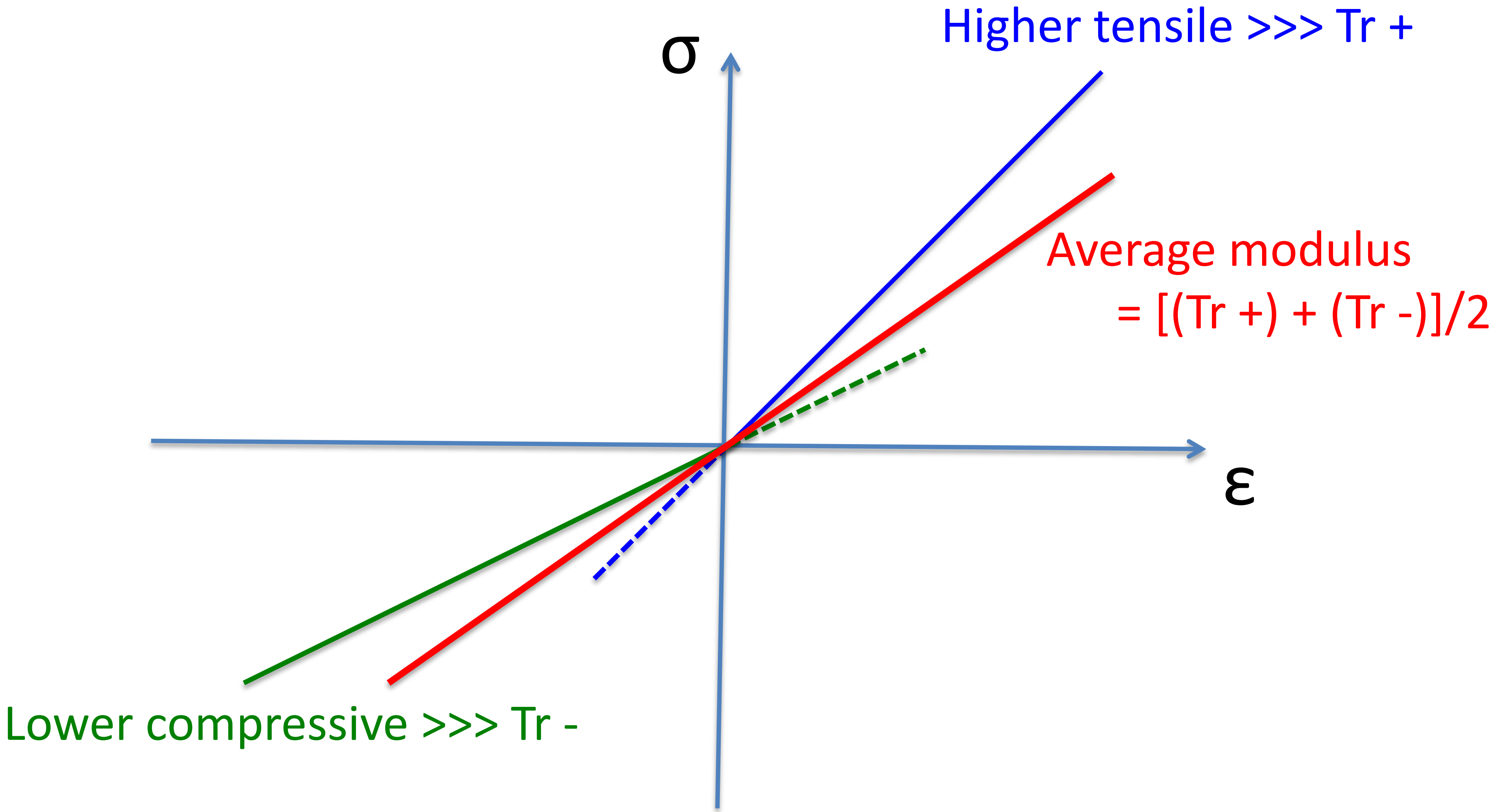
Trace-based [π/4] Stiffness of CFRP

[π/4]	A11*=A22*	A66*	Tr, GPa	E1*	nu/x*	Tr/SG	Tr/Al	Al/Tr
IM7/977-3	0.370	0.130	218	0.337	0.297	136	1.73	0.58
T800/Cytec	0.373	0.127	183	0.335	0.320	114	1.45	0.69
T700 C-Ply 55	0.371	0.129	139	0.337	0.305	87	1.11	0.90
T700 C-Ply 64	0.370	0.130	163	0.337	0.300	102	1.29	0.77
AS4/3501	0.368	0.132	162	0.338	0.282	101	1.29	0.78
IM6/epoxy	0.370	0.130	232	0.337	0.295	145	1.85	0.54
AS4/F937	0.373	0.127	168	0.335	0.314	112	1.42	0.70
T300/N5208	0.370	0.130	206	0.337	0.294	129	1.64	0.61
IM7/8552	0.372	0.128	192	0.336	0.312	120	1.52	0.66
IM7/MTM45	0.371	0.129	195	0.336	0.307	122	1.55	0.64
T700/2510	0.372	0.128	144	0.336	0.314	90	1.14	0.88
T650/epoxy	0.371	0.129	160	0.337	0.307	100	1.28	0.78
IM7/8552 N	0.371	0.129	180	0.336	0.306	112	1.43	0.70
T4708/MR60H	0.373	0.127	158	0.335	0.320	99	1.26	0.79
Cytec/MTM45	0.373	0.127	143	0.335	0.318	89	1.14	0.88
T800S/3900	0.373	0.127	168	0.335	0.320	105	1.34	0.75
Aluminum	0.370	0.130	204	0.337	0.300	79	1.00	1.00
Steel	0.370	0.130	613	0.337	0.300	79	1.00	1.00
MEDIAN	0.371	0.129		0.337	0.306			
Std dev*	0.14%	0.14%		0.1%	1.1%			

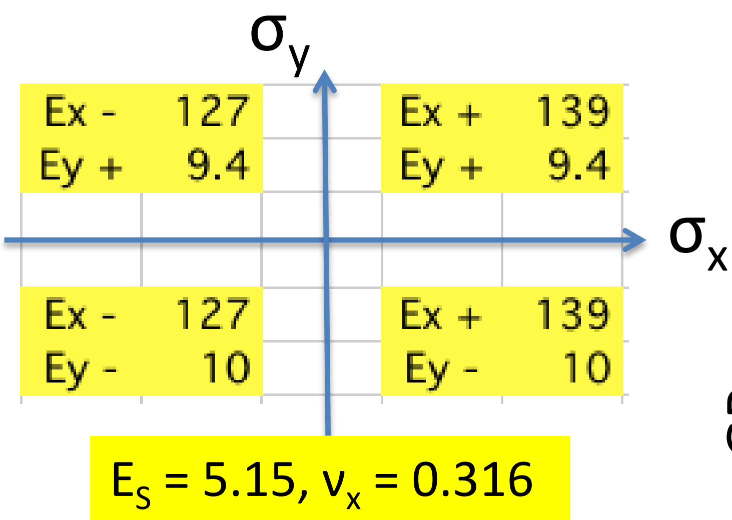


Trace, GPa
600
500
400
300
200
100
0
CFRP
Al
Steel
Specific trace, GPa
140
120
100
80
60
40
20
0
Al
Steel
IM7/977-3
T800/Cytec
T700 C-Ply 55
T700 C-Ply 64
AS4/3501
IM6/epoxy
AS4/F937
T300/N5208
IM7/8552
IM7/MTM45
T700/2510
T650/epoxy
IM7/8552 N
T4708/MR60H
Cytec/MTM45
T800S/3900
Aluminum
Steel

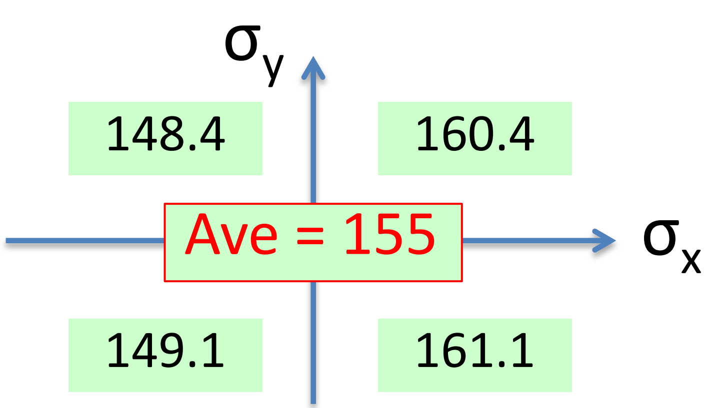
Average Trace for Tr + ≠ Tr -

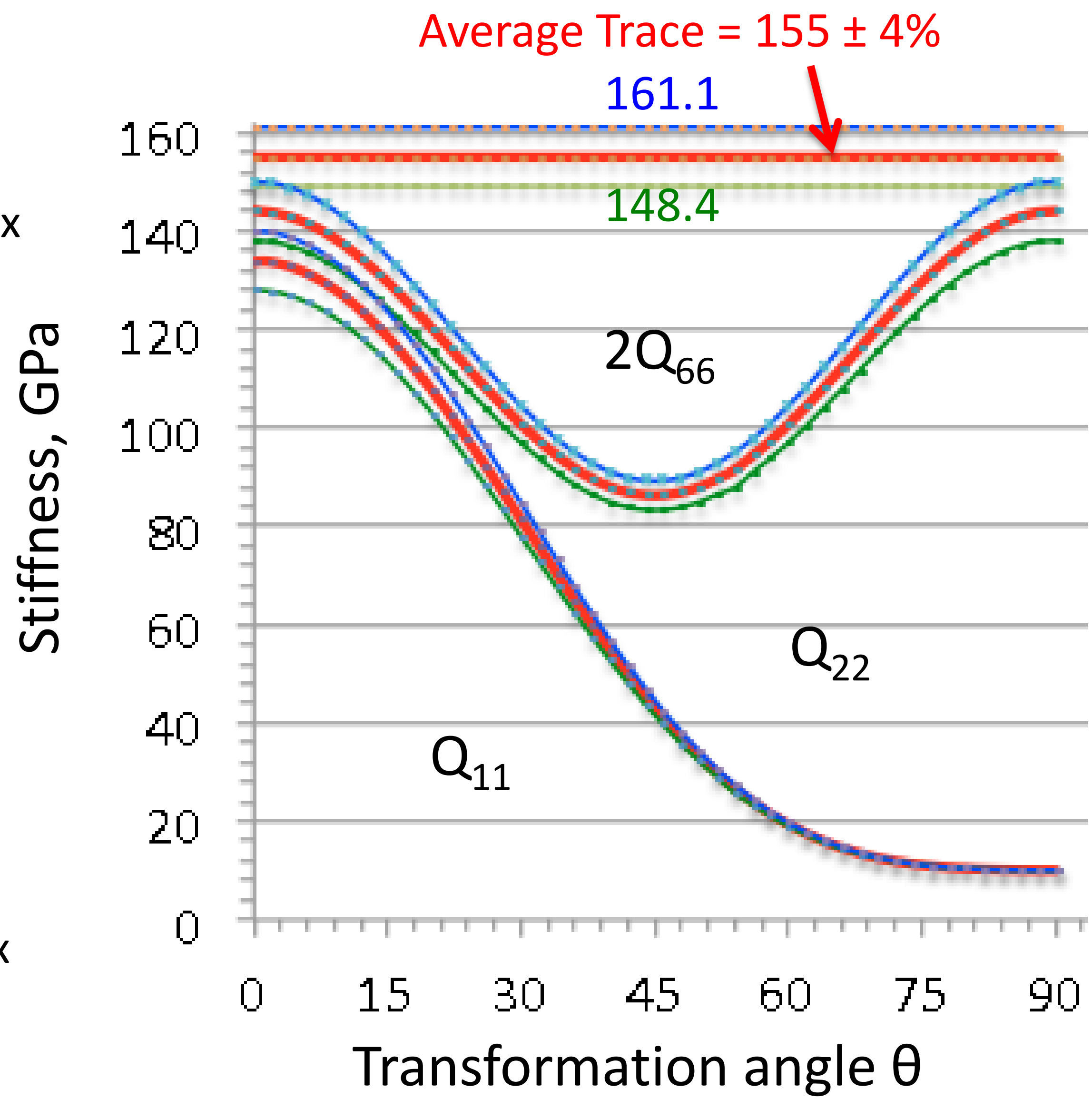


Average Value of Traces for T650



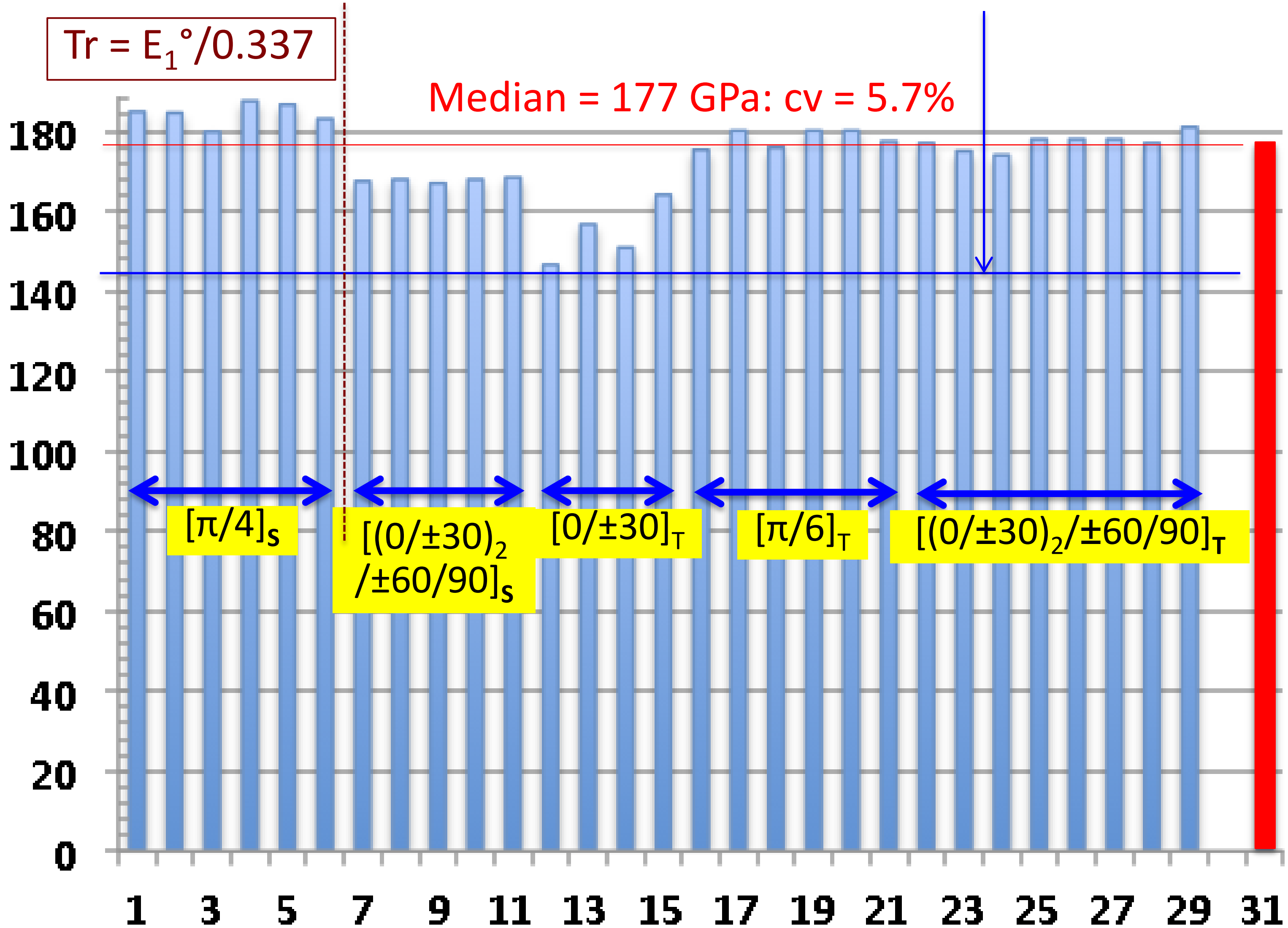
Four possible stiffness combinations leading to four possible traces





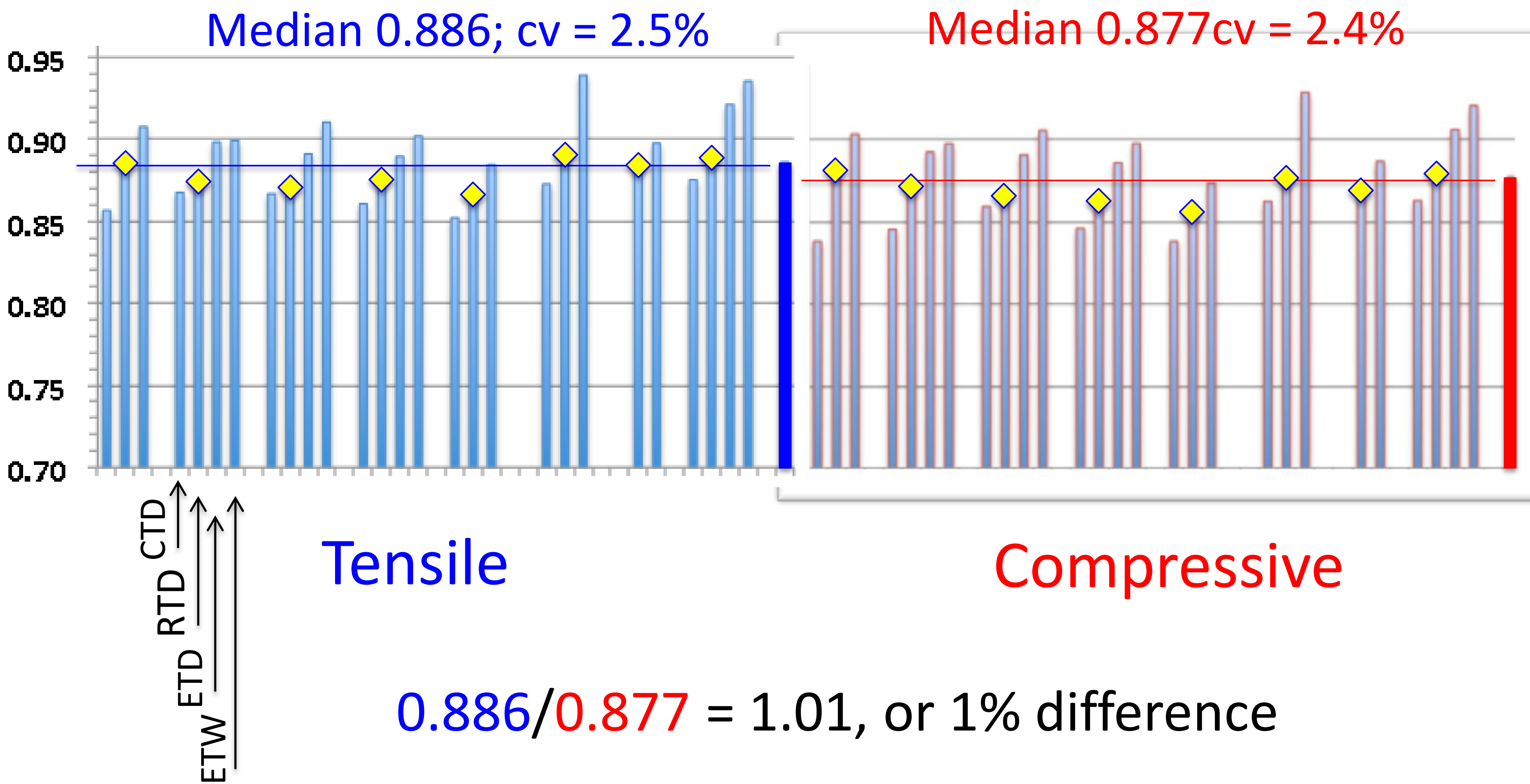
Measurement of Trace from $E_1°$

Material: T800/AR250 = 177 (T700/AR250 = 144)



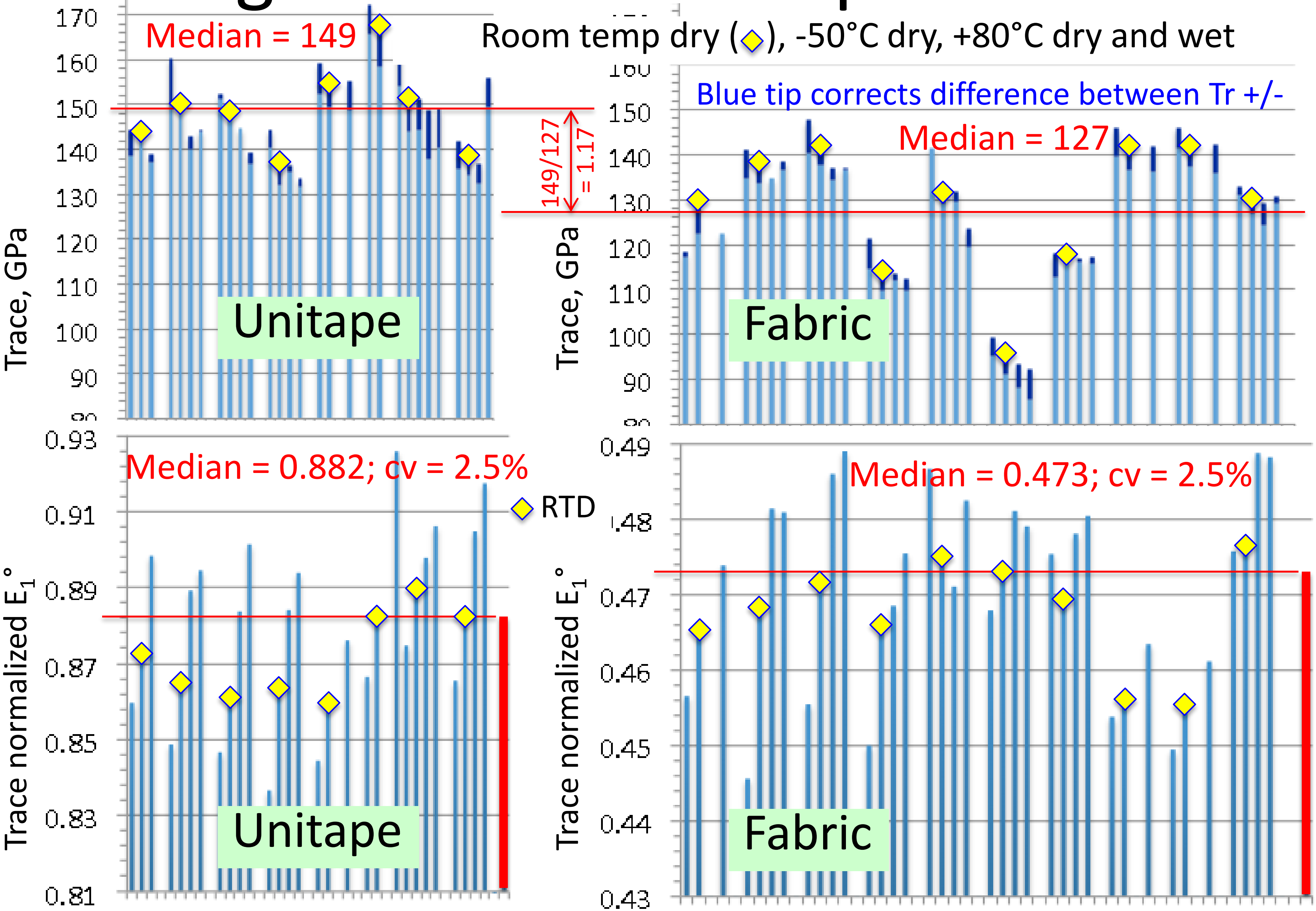
Tensile and Compressive $E_1°$/Trace

Room temp dry (with ◇), -50°C dry, +80°C dry and wet



0.886/0.877 = 1.01, or 1% difference

Average Trace for Unitape and Fabric



Trace-based $E_1°$ vs Unitape Data

Theory: $E_1° = 160 \times 0.207 = 33.1$ GPa

Laminate	Trace	[π/4]	Other laminates . . .						
Percent [0]	100	25	10	50	9	55	8	50	
Percent [±45]	0	50	80	40	73	36	84	43	
Percent [90]	0	25	10	10	18	9	8	7	
Master ply factors	GPa	0.337	0.223	0.518	0.225	0.551	0.207	0.516	
T650/epoxy	160	53.9	35.7	82.9	36.0	88.2	33.1	82.6	
IM7/8552 N	180	60.7	40.1	93.2	40.5	99.2	37.3	92.9	
T4708/MR60H	158	53.2	35.2	81.8	35.6	87.1	32.7	81.5	
Cytec/MTM45	143	48.2	31.9	74.1	32.2	78.8	29.6	73.8	
T650/epoxy	Data	51.3		78.3			29.8	78.3	
IM7/8552 N	Data	57.6	36.0	90.7					
T4708/MR60H	Data	51.3			31.2	86.5			
Cytec/MTM45	Data	48.3	30.0						Median
T650/epoxy	160	0.95		0.94			0.90	0.95	0.95
IM7/8552 N	180	0.95	0.90	0.97					0.95
T4708/MR60H	158	0.96			0.88	0.99			0.96
Cytec/MTM45	143	1.00	0.94						0.97
								Median	0.957

Laminate

Theory GPa

Data, GPa

Ratio

Ratio: 30.0/31.9 = 0.94

Trace-based $E_1°$ vs Fabric Data

Theory: $E_1° = 119 \times 0.432 = 51.4$ GPa

Fabric	Trace	[π/4]	Hi shear	Lo shear	
Percent [0]	100	50	20	80	
Percent [45]	0	50	80	20	
Master ply factors	GPa	0.337	0.223	0.432	
Park T700/E765	119	40.1	26.5	51.4	
Cytec T650 Plain	148	49.9	33.0	63.9	
Cytec T650 8HS	147	49.5	32.8	63.5	
AS4/MTM45	133	44.8	29.7	57.5	
Park T700/E765	Data	41.1	26.8	50.6	
Cytec T650 Plain	Data	48.8	31.9	61.0	
Cytec T650 8HS	Data	49.5	32.5	61.6	
AS4/MTM45	Data	44.1	27.8	55.3	Median
Park T700/E765	119	1.02	1.01	0.98	1.01
Cytec T650 Plain	148	0.98	0.97	0.95	0.97
Cytec T650 8HS	147	1.00	0.99	0.97	0.99
AS4/MTM45	133	0.98	0.94	0.96	0.96

Theory, GPa

Data, GPa

Ratio

Ratio: 44.1/44.8 = 0.98

Higher lamination efficiency

Trace-based Lamination Efficiency

Laminate	[π/4]	Other laminates . . .						
Percent [0]	25	10	50	9	55	8	50	
Percent [±45]	50	80	40	73	36	84	43	
Percent [90]	25	10	10	18	9	8	7	Average
T650/epoxy	0.95		0.94				0.95	0.95
IM7/8552 N	0.97	0.91	0.98					0.95
T4708/MR60H	0.96			0.88	0.99			0.94
Cytec/MTM45	1.00	0.94				1.00		0.98
							Median	0.949

Example: For T650/epoxy unitape Trace [Q] = $Q_{xx} + Q_{yy} + 2Q_{ss}$ = 160 GPa = Tr [A°]

For [π/4], $E_1°$ = 0.337 Tr [A] = 0.337 x 160 = 53.9 GPa

NIAR data = 51.3 GPa; Lamination efficiency: 51.3/53.9 = 0.95

When laminate is perfect, the factor is 0.337. In real laminates, defects reduced laminate efficiency to less than 100 percent. Reduction for CFRP is about 5 percent

Trace-based Manufacturing Efficiency

A Measure of Quality of Lamination

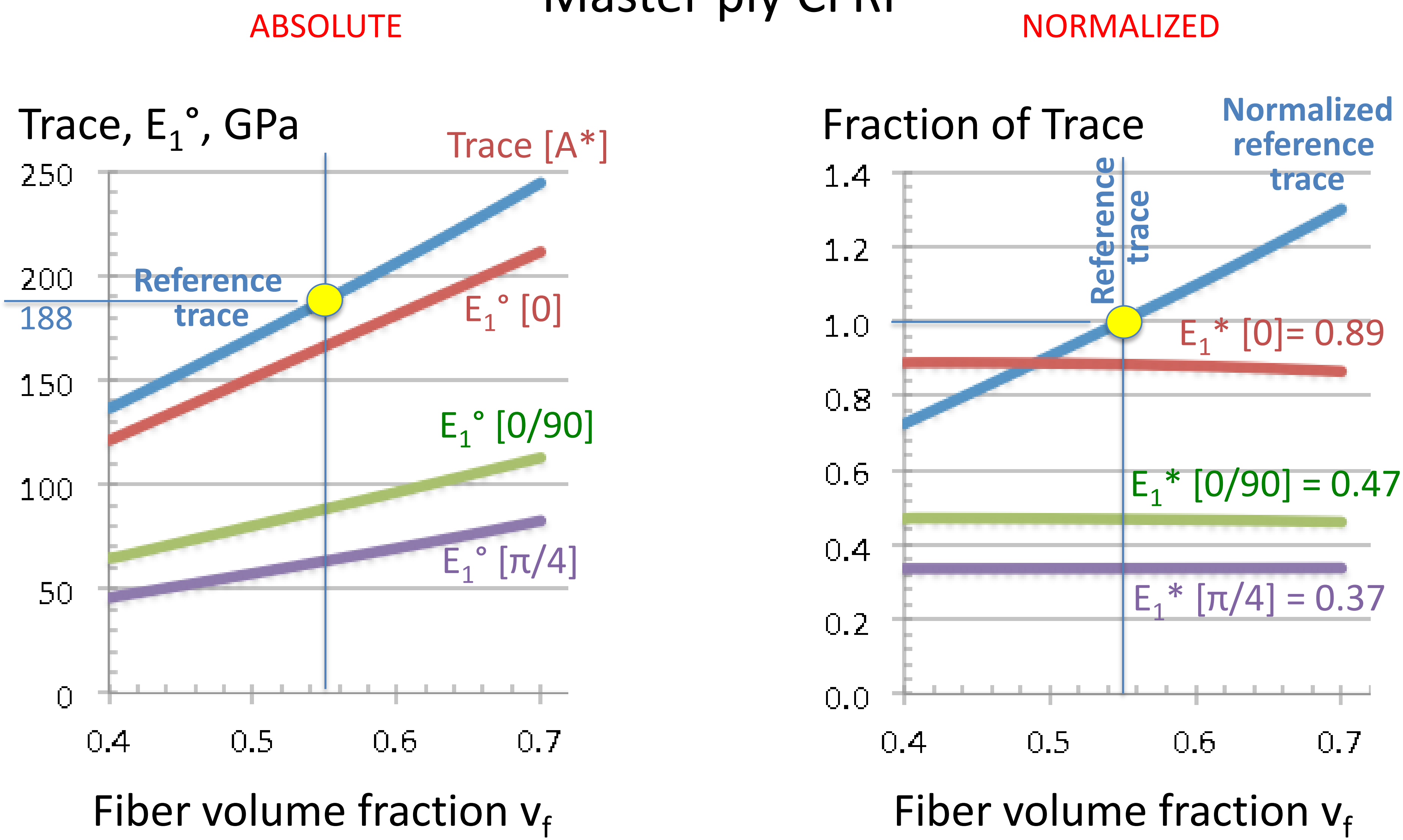
$E_1 = 187 \times 0.337 = 63.0$ GPa

Laminate	Trace [π/4]		Other laminates . . .						
Percent [0]	100	25	10	50	9	55	8	50	
Percent [±45]	0	50	80	40	73	36	84	43	
Percent [90]	0	25	10	10	18	9	8	7	
Master ply factors	187	0.337	0.223	0.518	0.225	0.551	0.207	0.516	
E1 perfect laminate, GPa		63.0	41.7	96.9	42.1	103.0	38.7	96.5	
T650/epoxy	Data	51.3		78.3			29.8	78.3	
IM7/8552 N	Data	57.6	36.0	90.7					
T4708/MR60H	Data	51.3			31.2	86.5			
Cytec/MTM45	Data	48.3	30.0						Median
T650/epoxy	160	0.95		0.94			0.90	0.95	0.95
IM7/8552 N	180	0.95	0.90	0.97					0.95
T4708/MR60H	158	0.96			0.88	0.99			0.96
Cytec/MTM45	143	1.00	0.94						0.97
								Median	0.957

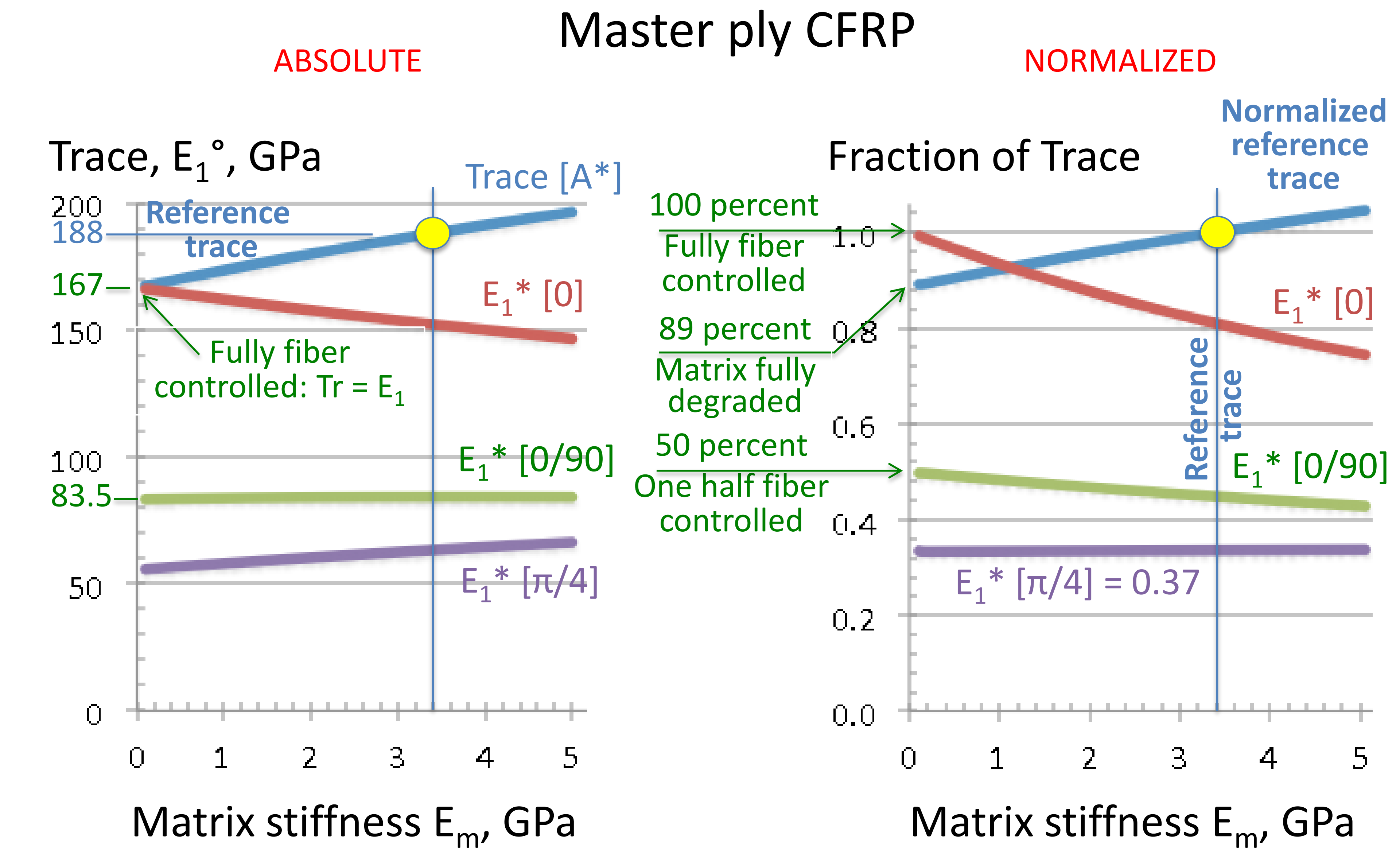
Example: [π/4] T650/epoxy Laminate efficiency = 51.3/63.0*187/160 = 0.95

Laminate Stiffness vs Fiber Volume

Master ply CFRP

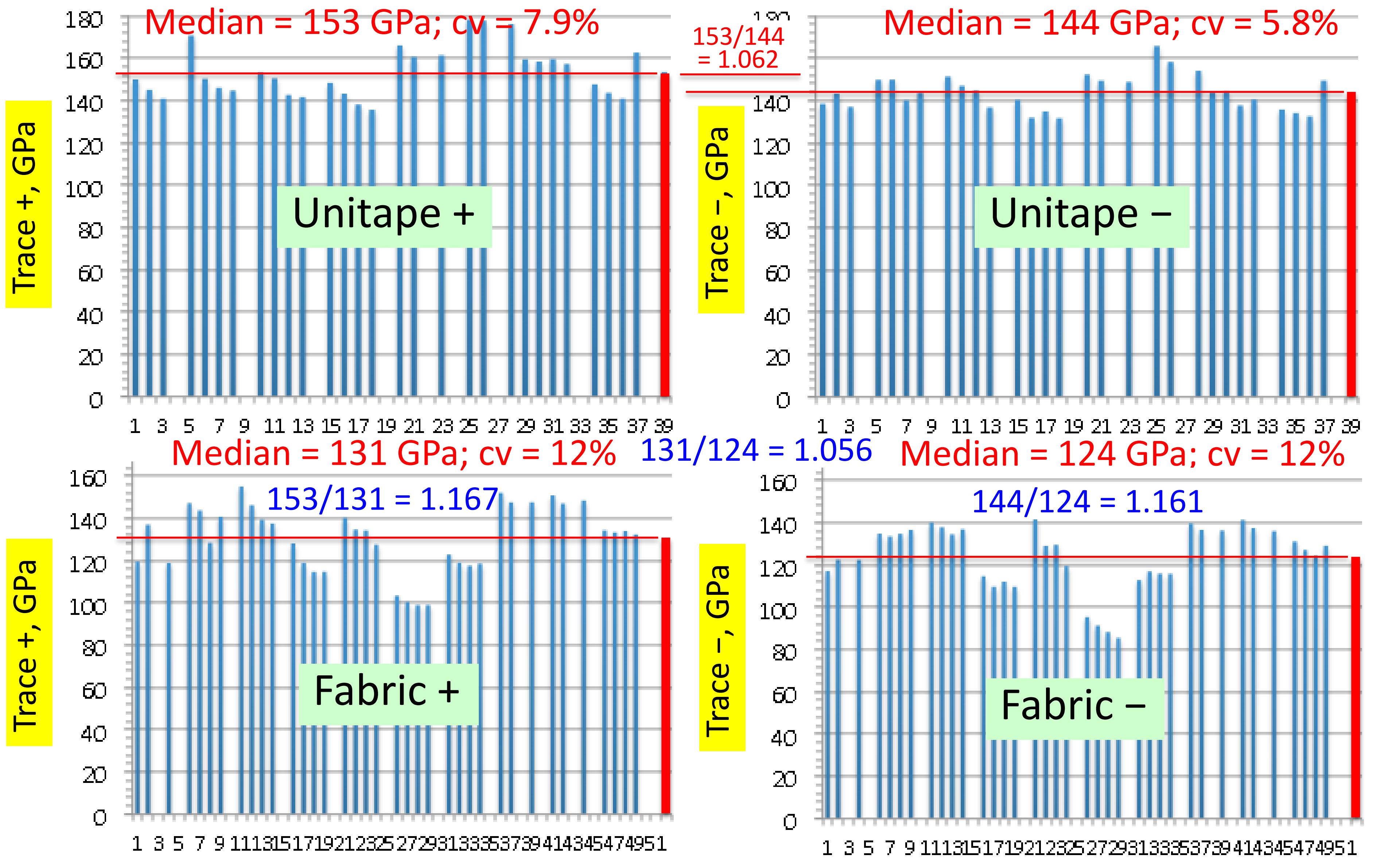


Laminate Stiffness vs Matrix Stiffness



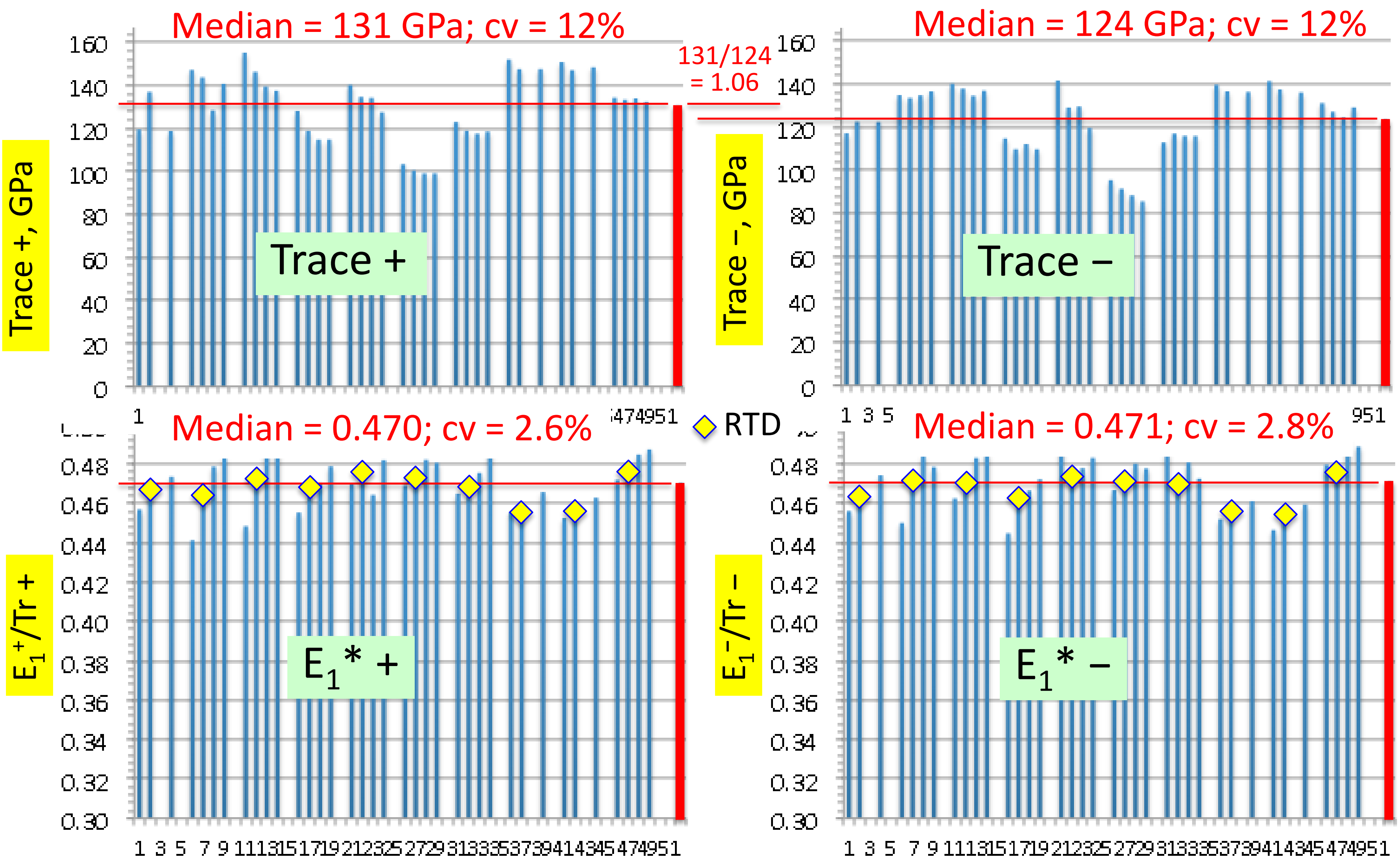
Trace +/-: Unitape vs Fabric = +16%

Trace of CFRP: Room temp dry, -50°C dry, +80°C dry and wet



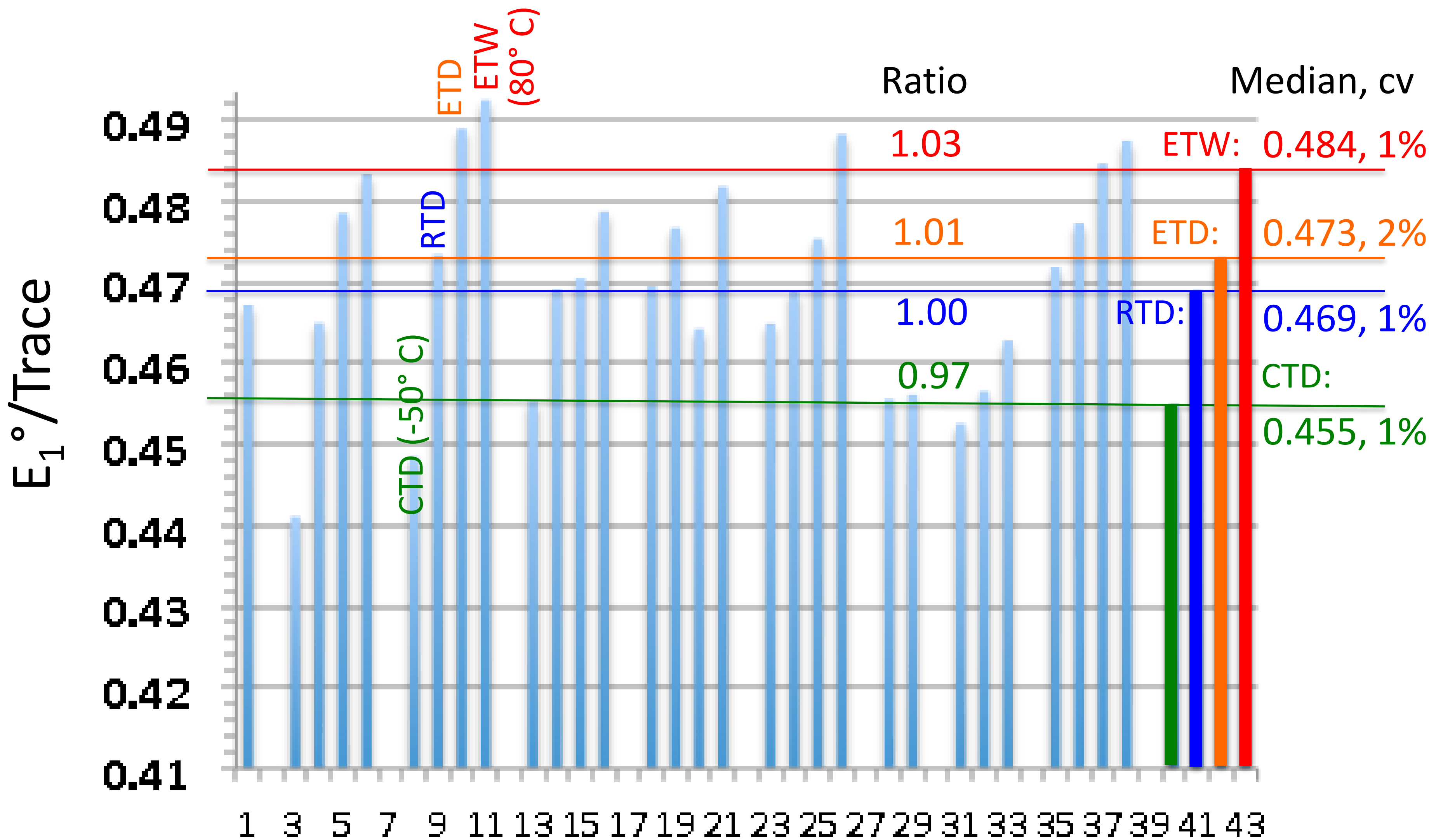
CFRP Fabric: Absolute Trace ±; $E_1^{\pm}/Tr$

Trace of 11 CFRP: Room temp dry (◇), -50°C dry, +80°C dry and wet



Normalized Fabric Stiffness: $E_1°$/Trace

Room temp dry, low temp (-50°C) dry, high temp (+80°C) dry and wet



Plane Elasticity & Bending Equations

Plane elasticity: $$a_{22}^* \frac{\partial^4 F}{\partial x^4} + (2a_{12}^* + a_{66}^*) \frac{\partial^4 F}{\partial x^2 \partial y^2} + a_{11}^* \frac{\partial^4 F}{\partial y^4} = 0.$$

Plate bending: $$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} = 0$$

Homogeneity: [D*] = [A*]; [B] = 0

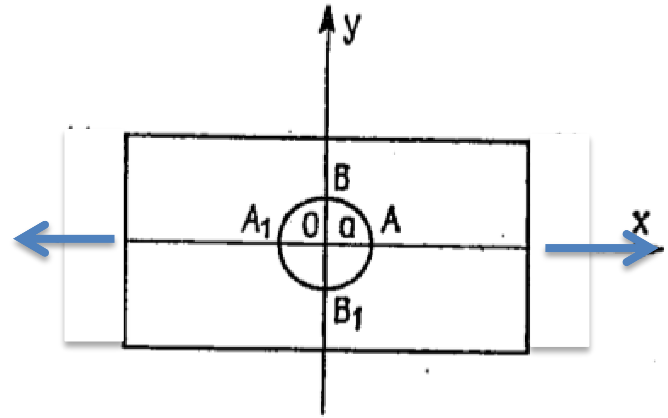
[0/±30]; 2:0	$\frac{2a_{12} + a_{66}}{a_{22}}$	$\frac{a_{11}}{a_{22}}$	$\frac{2(A_{12} + 2A_{66})}{A_{11}}$	$\frac{A_{22}}{A_{11}}$	Trace, GPa
IM7/977-3	2.23	0.58	1.20	0.58	218
T800/Cytec	2.23	0.58	1.20	0.58	183
T7 C-Ply 55	2.23	0.58	1.20	0.58	139
T7 C-Ply 64	2.23	0.58	1.20	0.58	163
AS4/3501	2.18	0.59	1.23	0.59	162
IM6/epoxy	2.23	0.58	1.20	0.58	232
AS4/F937	2.33	0.59	1.19	0.59	168
T300/N5208	2.24	0.58	1.20	0.58	206
Master ply	**2.23**	**0.58**	**1.20**	**0.58**	**175**
Std dev	0.044	0.004	0.013	0.004	
Coeff var	2.0%	0.7%	1.1%	0.7%	

Lekhnitskii's Elasticity Solutions

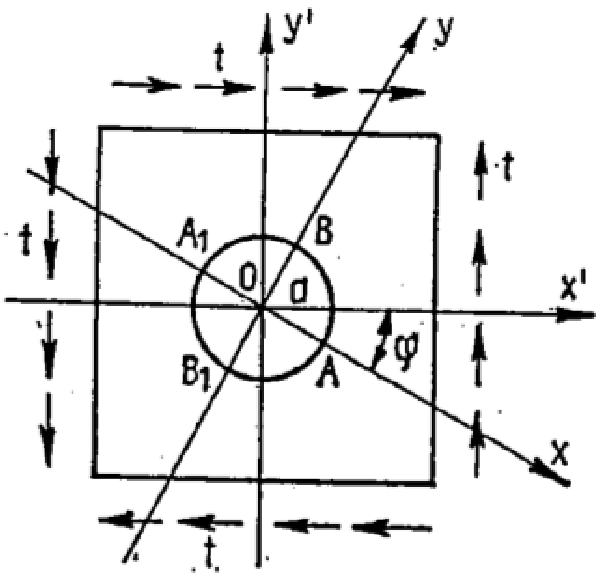
$$k = -\mu_1\mu_2 = \sqrt{\frac{E_1}{E_2}}$$

$$n = -i(\mu_1 + \mu_2) = \sqrt{2\left(\frac{E_1}{E_2} - \nu_1\right) + \frac{E_1}{G}}$$

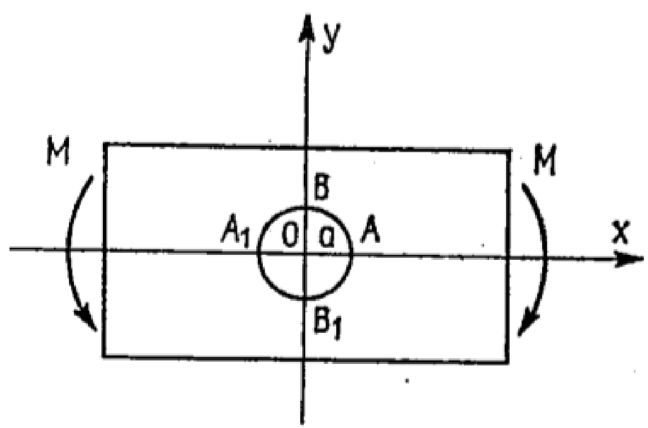
Key parameters: *k, n*



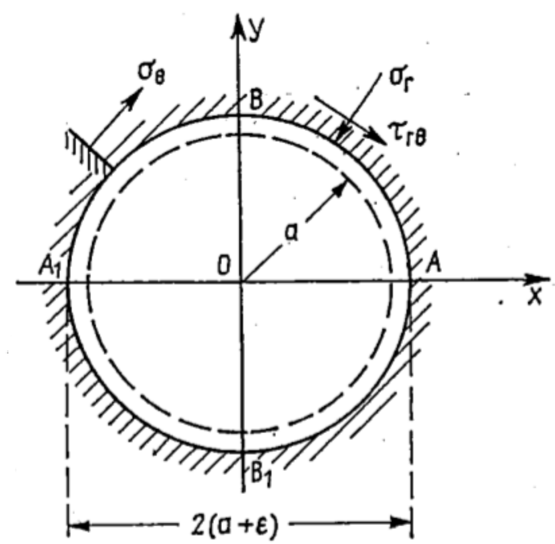
Open hole tension



Shear



Bending



Interference

Same Solutions for 8 CFRP's for [0/±30]

[0/±30]; 2:0	OHT	Press at 0	Shear	Bending	Interference	
Parameters	n+1	(n-1)/k	(1+k+n)n	2k+n	k(1+n)-nu	Tr, GPa
IM7/977-3	2.53	0.20	7.99	6.91	5.60	218
T800/Cytec	2.44	0.16	7.41	6.83	5.28	183
T7 C-Ply 55	2.61	0.24	8.37	6.75	5.61	139
T7 C-Ply 64	2.61	0.24	8.37	6.75	5.61	163
AS4/3501	2.68	0.27	8.71	6.70	5.73	162
IM6/epoxy	2.53	0.20	7.94	6.83	5.51	232
AS4/F937	2.51	0.20	7.74	6.71	5.34	168
T700/5208	2.53	0.20	7.93	6.80	5.47	206
Median	2.53	0.20	7.97	6.77	5.56	
Std dev	0.074	0.033	0.408	0.070	0.152	
Coeff var	2.9%	16.4%	5.1%	1.0%	2.7%	

Median values can be used for most cases with error less than experimental

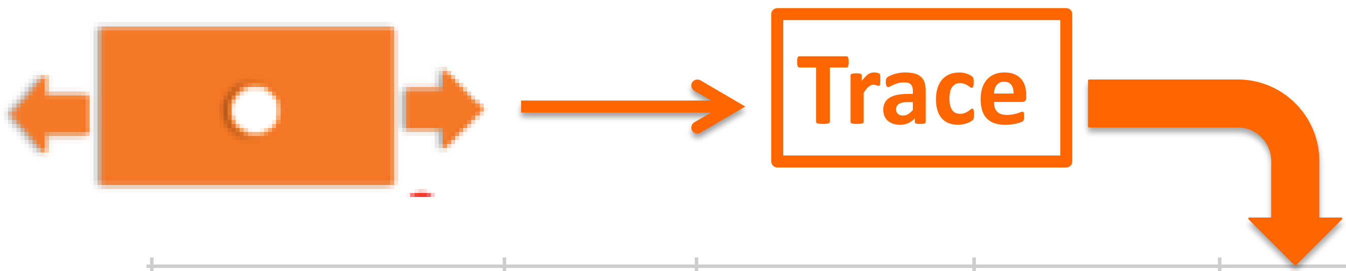
Exact solutions from Lekhnitskii's *Anisotropic Plates*

Solutions for Different Laminates

Laminate; cv	OHT	Press at 0	Shear	Bending	Interferenc	Med cv
[0/±30]: 2:0	2.53	0.20	7.97	6.77	5.56	
Coeff var	2.9%	16.4%	5.1%	1.0%	2.7%	2.9%
[0/±30]; 4:2	2.99	0.77	8.54	4.57	3.45	
Coeff var	0.0%	0.4%	0.1%	0.2%	0.4%	0.2%
[0/±30]; 2:2	3.00	1.00	8.00	4.00	2.70	
Coeff var	0.0%	0.1%	0.1%	0.0%	0.4%	0.1%
[0/±45]; 4:2	2.65	0.57	6.26	3.94	2.55	
Coeff var	0.8%	3.4%	1.8%	0.5%	1.6%	1.6%
[π/4]	3.00	1.00	8.00	4.00	2.70	
Coeff var	0.0%	0.0%	0.0%	0.0%	0.4%	0.0%

Median values can be used for different laminates with error less than experimental

One Test for Trace = Multiple Solutions



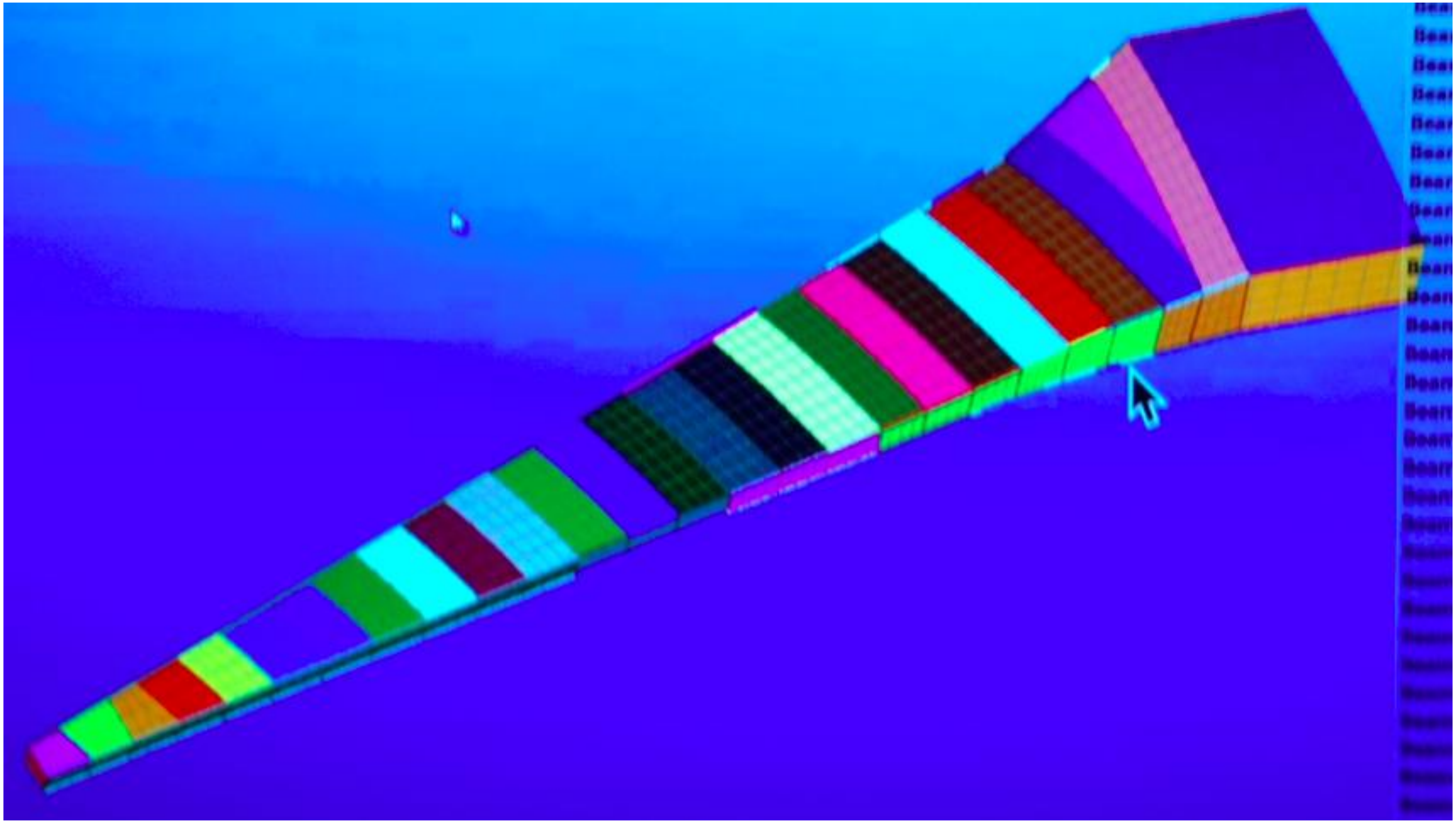
Lam; Ratio	OHT	Press at 0	Shear	Bending	Interference
[0/±30]: 2:0	✔	✔	✔	✔	✔
[0/±30]: 4:2	✔	✔	✔	✔	✔
[0/±30]; 2:2	✔	✔	✔	✔	✔
[0/±45]; 2:0	✔	✔	✔	✔	✔
[0/±45]; 4:2	✔	✔	✔	✔	✔
[0/±45]; 2:2	✔	✔	✔	✔	✔
[0/±phi]; 2:0	✔	✔	✔	✔	✔
[0/±phi]; 4:2	✔	✔	✔	✔	✔
[0/±phi]; 2:2	✔	✔	✔	✔	✔

Outline

- Trace-based theory
- Supporting data
- Prelminary design
- Omni strain allowable

Hypersizer Methodology 1

Each sub-component/element independently sized based on local loading conditions



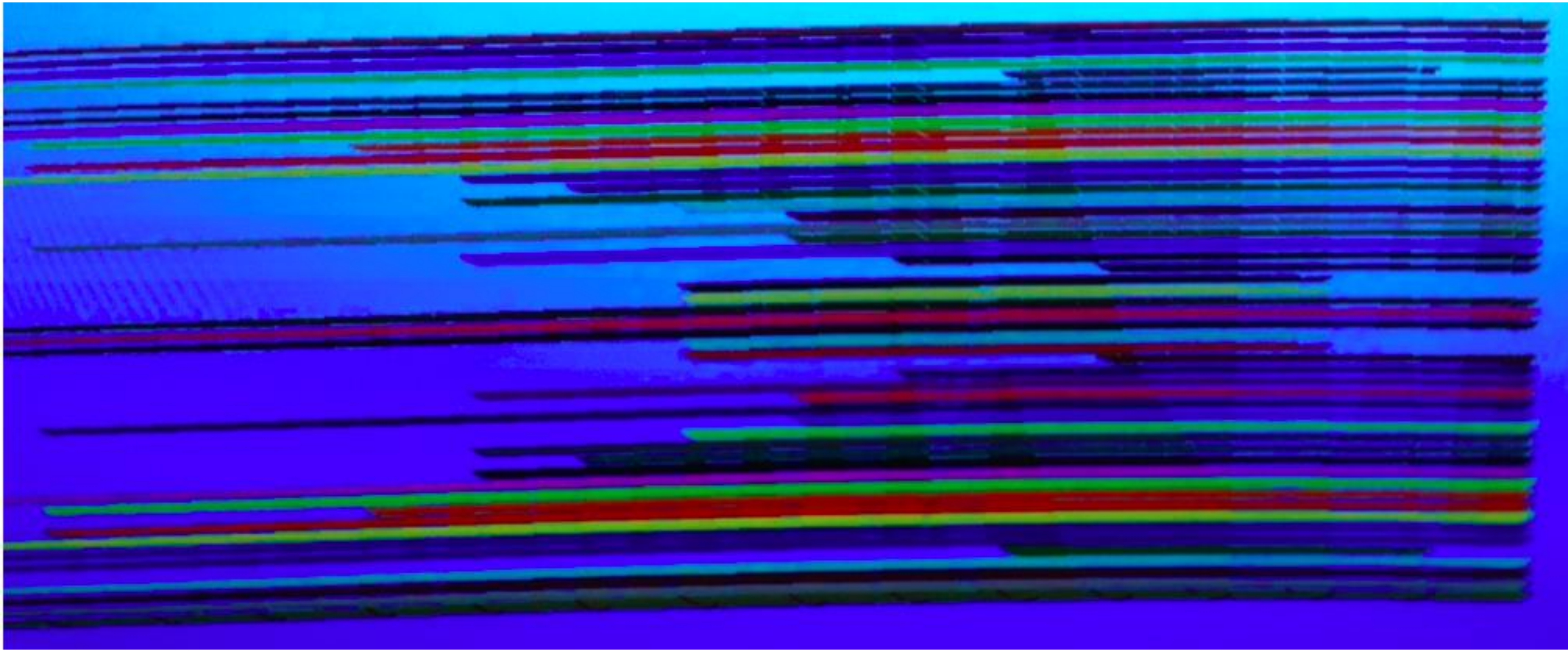
Hypersizer Methodology 2

Ply composition of each element in terms of
ply count of 0, ±45 nd 90

Global Ply	t (in)	Material	θ	FA	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123
51	0.0055	Tape: AS4/3502 Tape DT	45	✓	45	47	51	51	49	47	45	41	35	33	27	25	25	25	19	13	11	11
[illegible]	[illegible]	[illegible]	[illegible]	✓	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
49	0.0055	Tape: AS4/3502 Tape DT	-45	✓	43	45	49	49	47	45	43	39	33	31	25	23	23	23	17	11	9	9
[illegible]	[illegible]	[illegible]	[illegible]	✓	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
46	0.0055	Tape: AS4/3502 Tape DT	0		41	42	46	46	44	43	41	37	31	29	23	21	21	21	15	9		
45	0.0055	Tape: AS4/3502 Tape DT	0		40	41	45	45	43	42	40	36	30	28	22	20	20	20	14			
44	0.0055	Tape: AS4/3502 Tape DT	45		39	40	44	44	42	41	39	35	29	27	21	19	19	19	13			
43	0.0055	Tape: AS4/3502 Tape DT	0		38	39	43	43	41	40	38	34	28	26	20	18	18	18				
41	0.0055	Tape: AS4/3502 Tape DT	45		36	37	41	41	39	38	36	32	26	24	18	17	17	17				
40	0.0055	Tape: AS4/3502 Tape DT	-45		35	36	40	40	38	37	35	31	25	23	17	16	16	16	12			
34	0.0055	Tape: AS4/3502 Tape DT	-45		29	30	34	34	32	31	29	26	21	20	16	15	15	15				
27	0.0055	Tape: AS4/3502 Tape DT	0	✓	24	25	27	27	26	25	24	22	19	18	15	14	14	14	11	[illegible]	7	7
[illegible]	[illegible]	[illegible]	[illegible]	✓	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25	0.0055	Tape: AS4/3502 Tape DT	0	✓	22	23	25	25	24	23	22	20	17	16	13	12	12	12	9	6	5	5
18	0.0055	Tape: AS4/3502 Tape DT	-45		17	18	18	18	18	17	17	16	15	14	12	11	11	11				
12	0.0055	Tape: AS4/3502 Tape DT	-45		11	12	12	12	12	11	11	11	11	11	11	10	10	10	8			
11	0.0055	Tape: AS4/3502 Tape DT	45		10	11	11	11	11	10	10	10	10	10	10	9	9	9				
9	0.0055	Tape: AS4/3502 Tape DT	0		8	9	9	9	9	8	8	8	8	8	8	8	8	8				
[illegible]	0.0055	Tape: AS4/3502 Tape DT	45		7	8	8	8	8	7	7	7	7	7	7	7	7	7	7			
7	0.0055	Tape: AS4/3502 Tape DT	0		6	7	7	7	7	6	6	6	6	6	6	6	6	6	6			
6	0.0055	Tape: AS4/3502 Tape DT	0		5	6	6	6	6	5	5	5	5	5	5	5	5	5	5	5		

Hypersizer Methodology 3

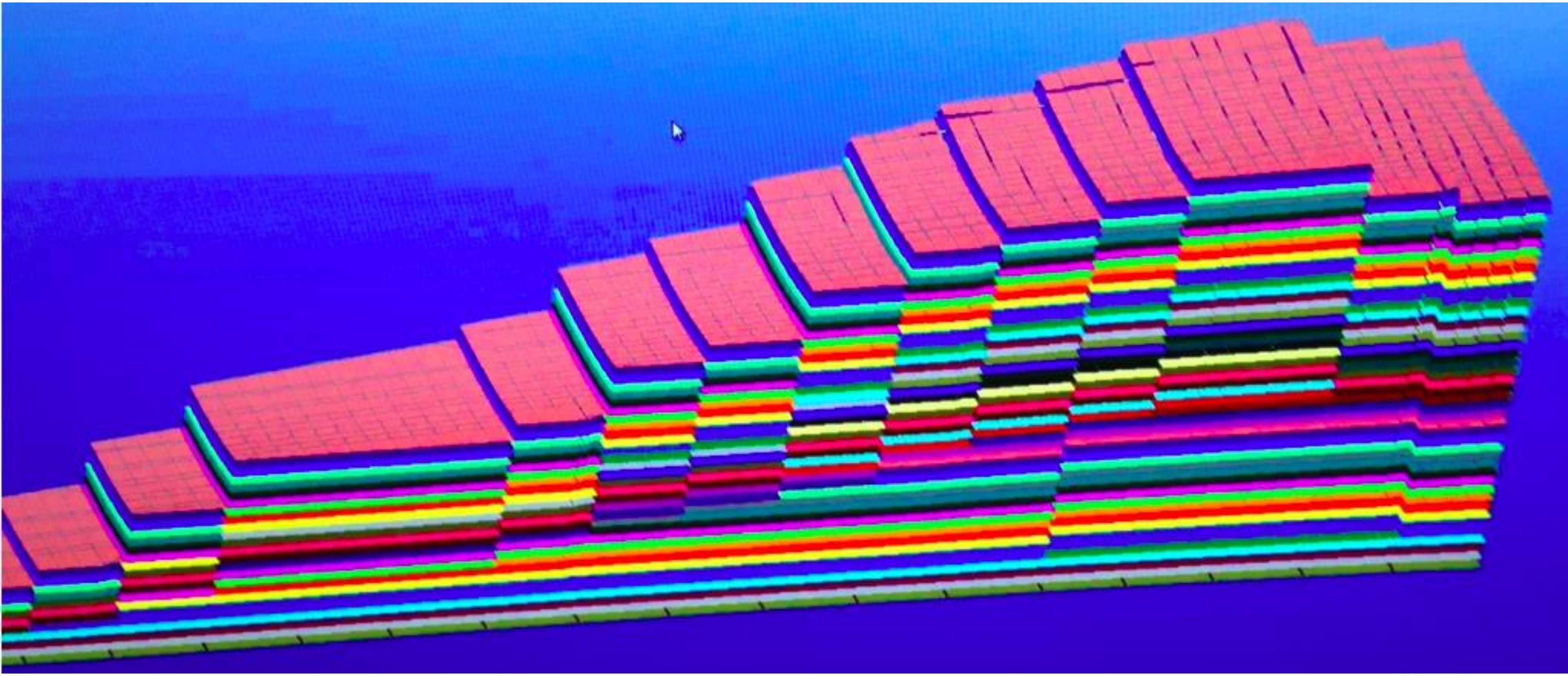
Ply layup sequence to make it compatible between elements showing ply drop of 0, ±45 nd 90



Note mid-plane symmetry, and complex ply drop

Hypersizer Methodology 4

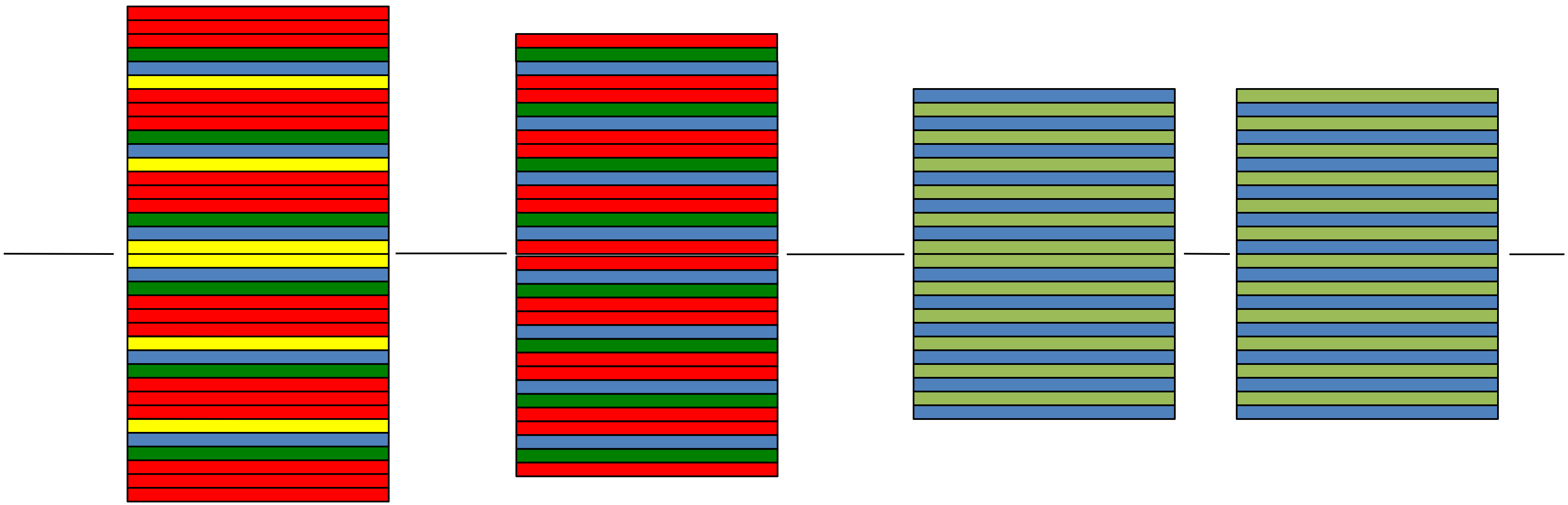
Ply layup sequence or stacking showing ply drop of 0, ±45 and 90



A manufacturing nightmare

Laminate Design: Homogenization

$[0_3/\pm 45/90]_{3S}$ $[0_2/\pm\Pi]_{4S}$ $[\pm\Pi]_{6S}$ $[\pm\Pi]_{12T}$



n = 36 n = 32 n = 24 n = 24

Symmetric **Symmetric** **Symmetric** **Asymmetric**

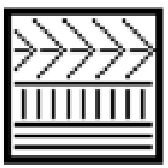
Degree of homogenization

Fewer angles, thinner plies

Homogenization Opens a New World

$[0/\pm45/90]_{rT}$, T800/52

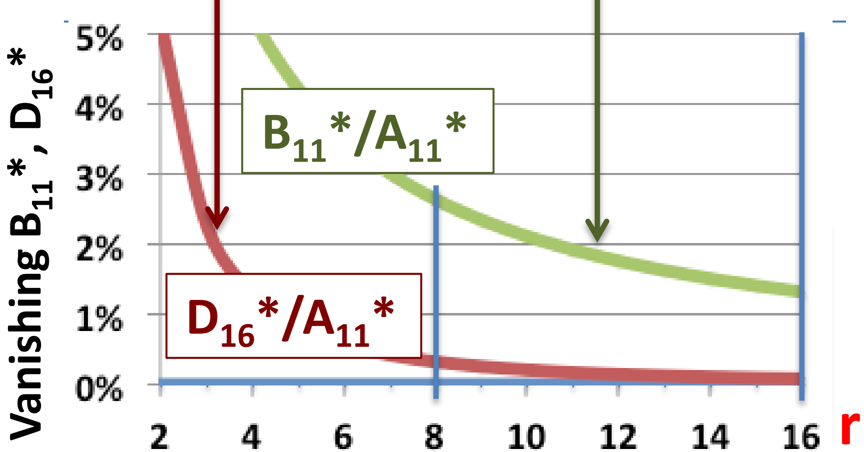
r = repeat = 2, T = total

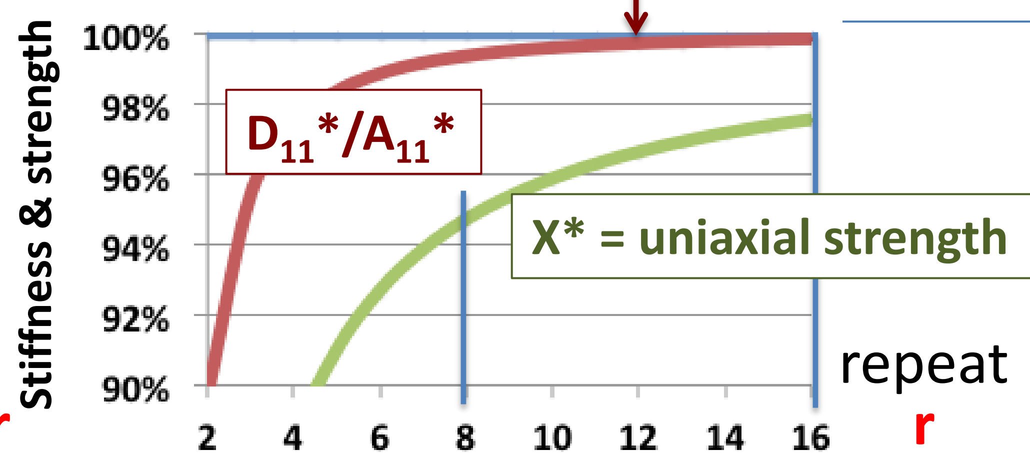


$$\begin{bmatrix} A^* & B^* \\ 3B^* & D^* \end{bmatrix} = \begin{bmatrix} 76.4 & 22.6 & 0 & -\frac{20.6}{r} & \frac{9.8}{r} & -\frac{5.4}{r} \\ 22.6 & 76.4 & 0 & \frac{9.8}{r} & \frac{0.9}{r} & -\frac{5.4}{r} \\ 0 & 0 & 26.9 & -\frac{5.4}{r} & -\frac{5.4}{r} & \frac{9.8}{r} \\ -\frac{61.7}{r} & \frac{29.6}{r} & -\frac{16.1}{r} & 109.(r) & 22.6 & -\frac{16.1}{r^2} \\ \frac{29.6}{r} & \frac{2.6}{r} & -\frac{16.1}{r} & 22.6 & 44.(r) & -\frac{16.1}{r^2} \\ -\frac{16.1}{r} & -\frac{16.1}{r} & \frac{29.6}{r} & -\frac{16.1}{r^2} & -\frac{16.1}{r^2} & 26.9 \end{bmatrix}$$

To be homogeneous:

[A*] = [D*]; [B*] = 0





Asymmetric Panels Do Not Warp

Nonstop stacking, no mid-plane symmetry

600 x 900 x 2 mm thick panel

$[0/25]_{16T}$

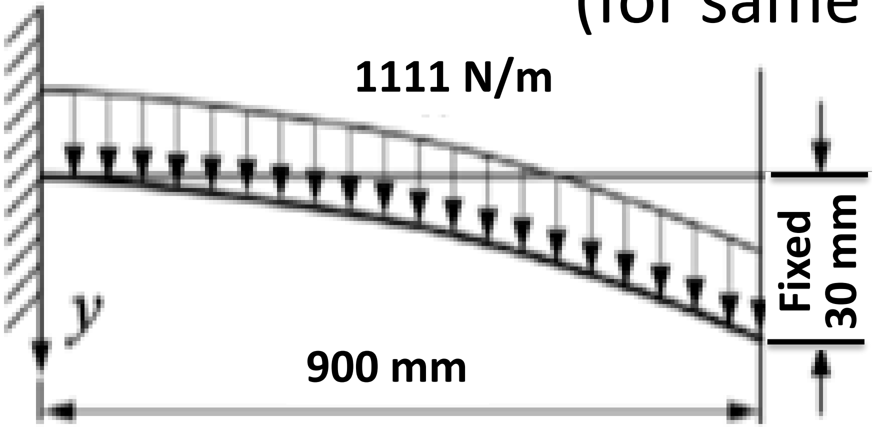
Bob Skillen
VX Aerospace

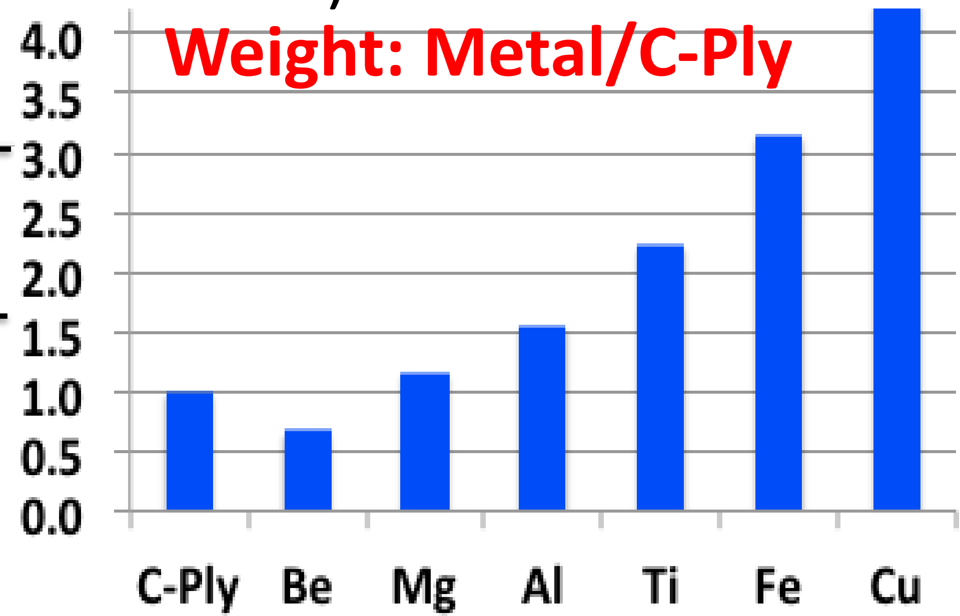
No warpage

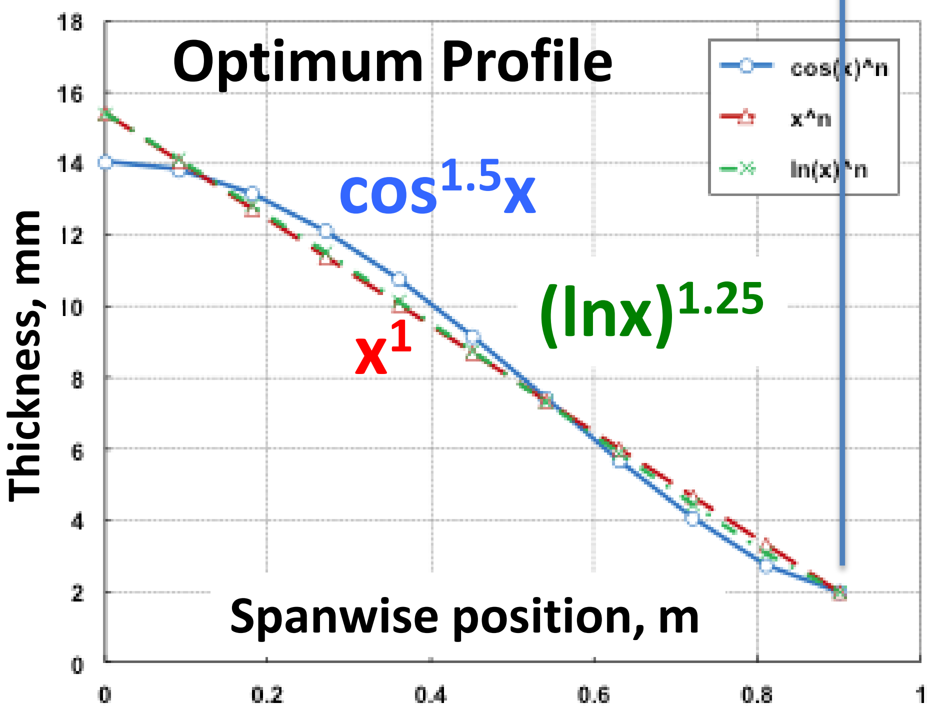
Optimized [π/4] C-Ply vs Metals

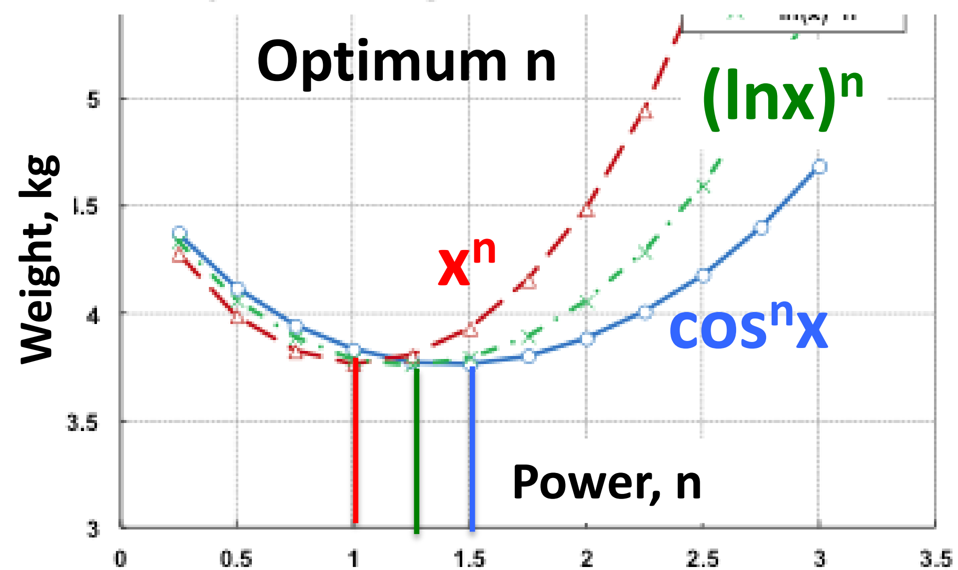
(for same deflection)

Sangwook Sihn



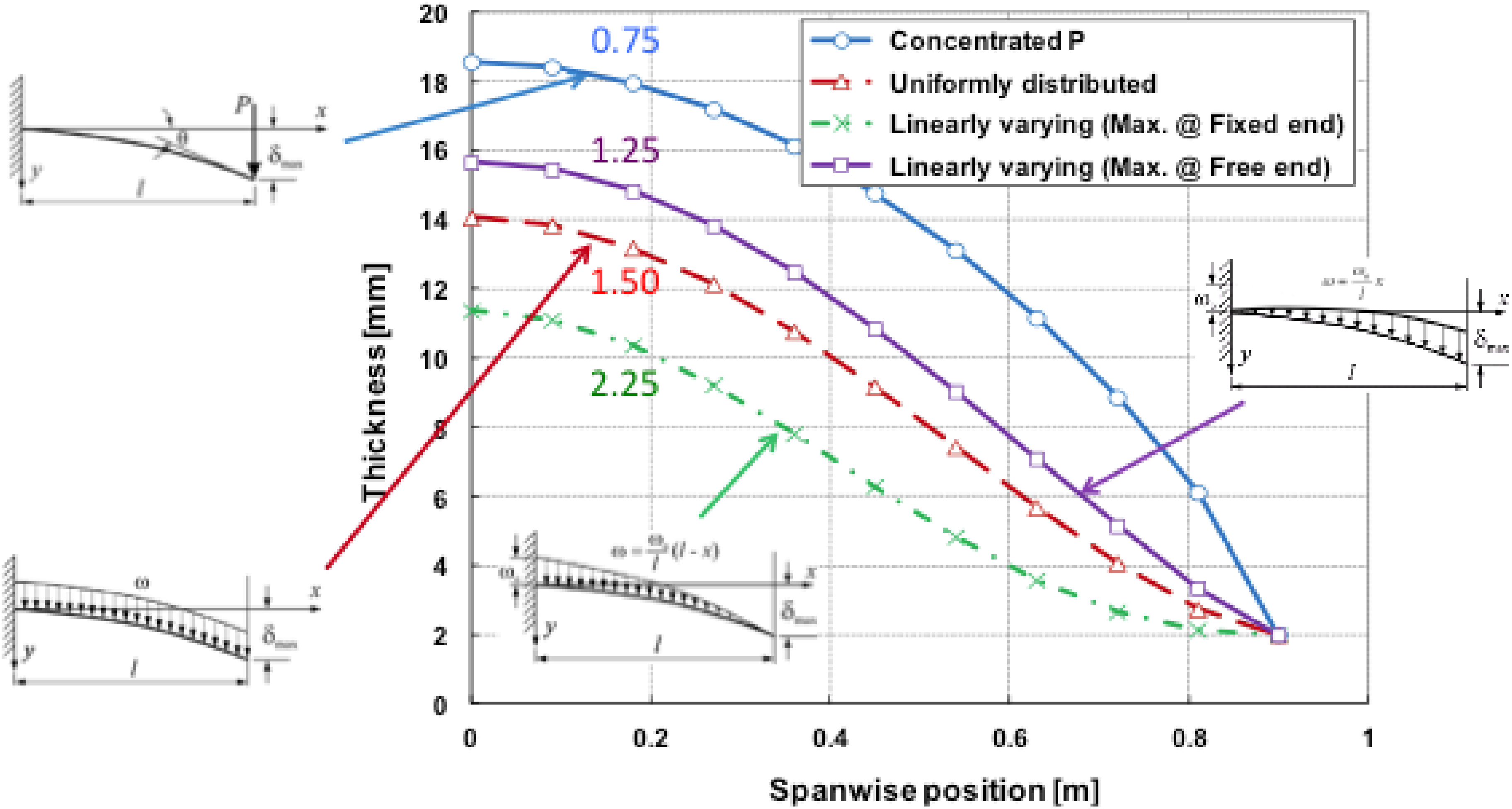




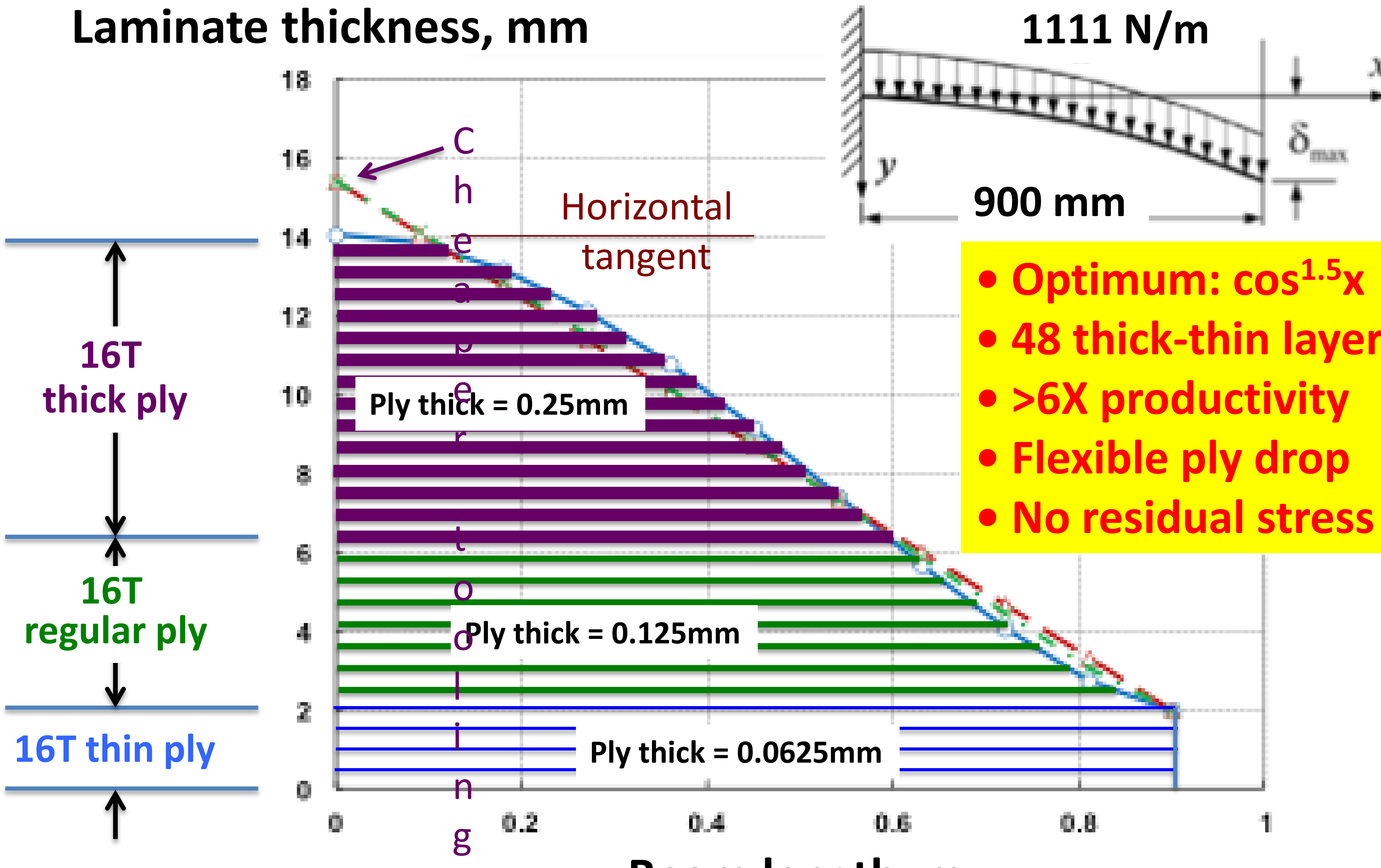


Optimum Profiles for Cantilever Beams

Sangwook Sihn



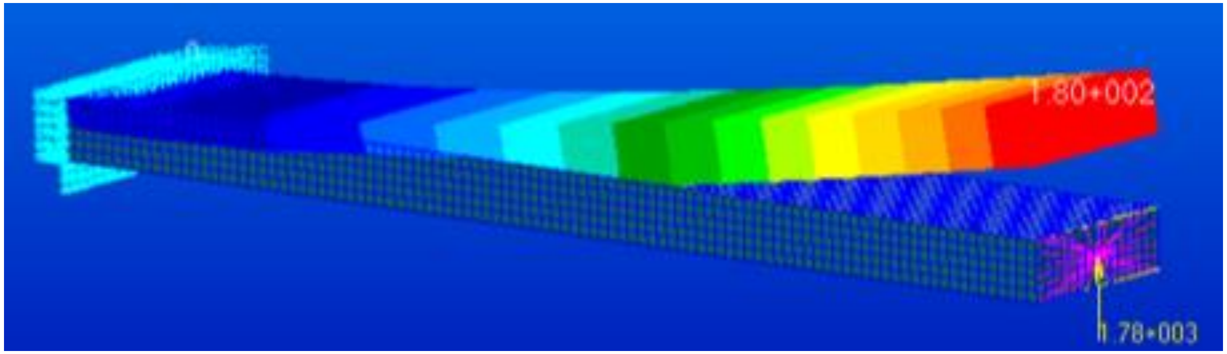
Optimum 48-layer Thick-thin C-Ply Layup



- Optimum: $\cos^{1.5}x$
- 48 thick-thin layers
- >6X productivity
- Flexible ply drop
- No residual stress

Scaling by Trace for Material/Laminate

Giulio Romeo



Scale materials: same $[0/\pm45/90]_{8S}$

Material	η	η_{scaled}	Err. %
IM6/Epoxy	180,27	180,27	0,00
AS4/3501	257,60	258,36	-0,30
C-Ply	301,42	300,79	0,21
T300/N5208	202,72	202,72	0,00

Scale laminates: same T300/N5208

Material	θ	η	η_{scaled}	Err. %
$[0/\pm\theta/90]_{8S}$	15	120,77	120,58	0,16
	30	147,98	150,68	1,79
	60	249,34	259,48	3,91
	75	270,35	286,56	5,66
$[0/\pm\theta/90]_{16T}$	15	121,059	121,061	0,00
	30	148,84	151,284	1,62
	60	250,63	260,519	3,80
	75	271,87	287,706	5,50

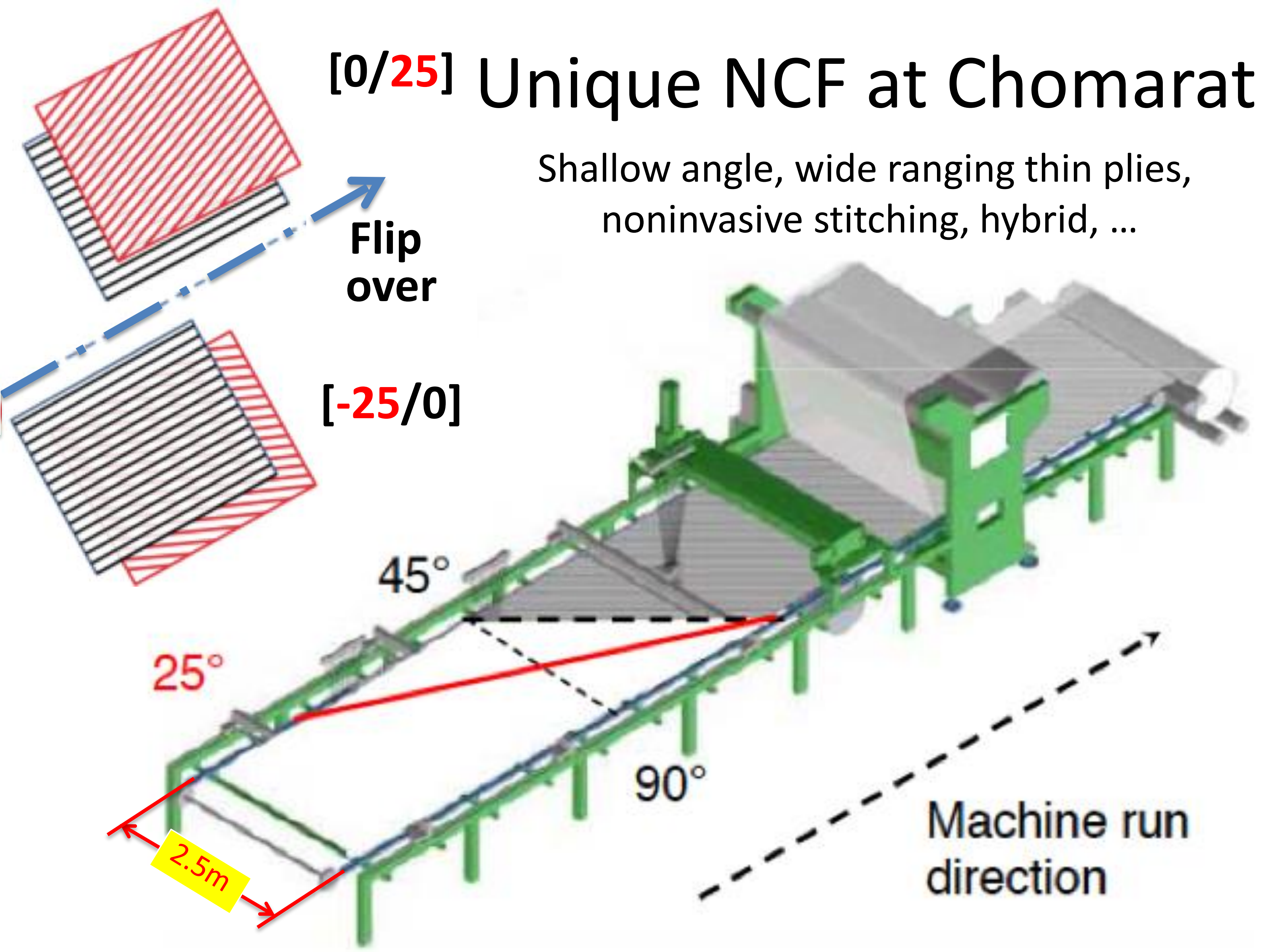
Scaling by Trace for Panel Buckling

Giulio Romeo

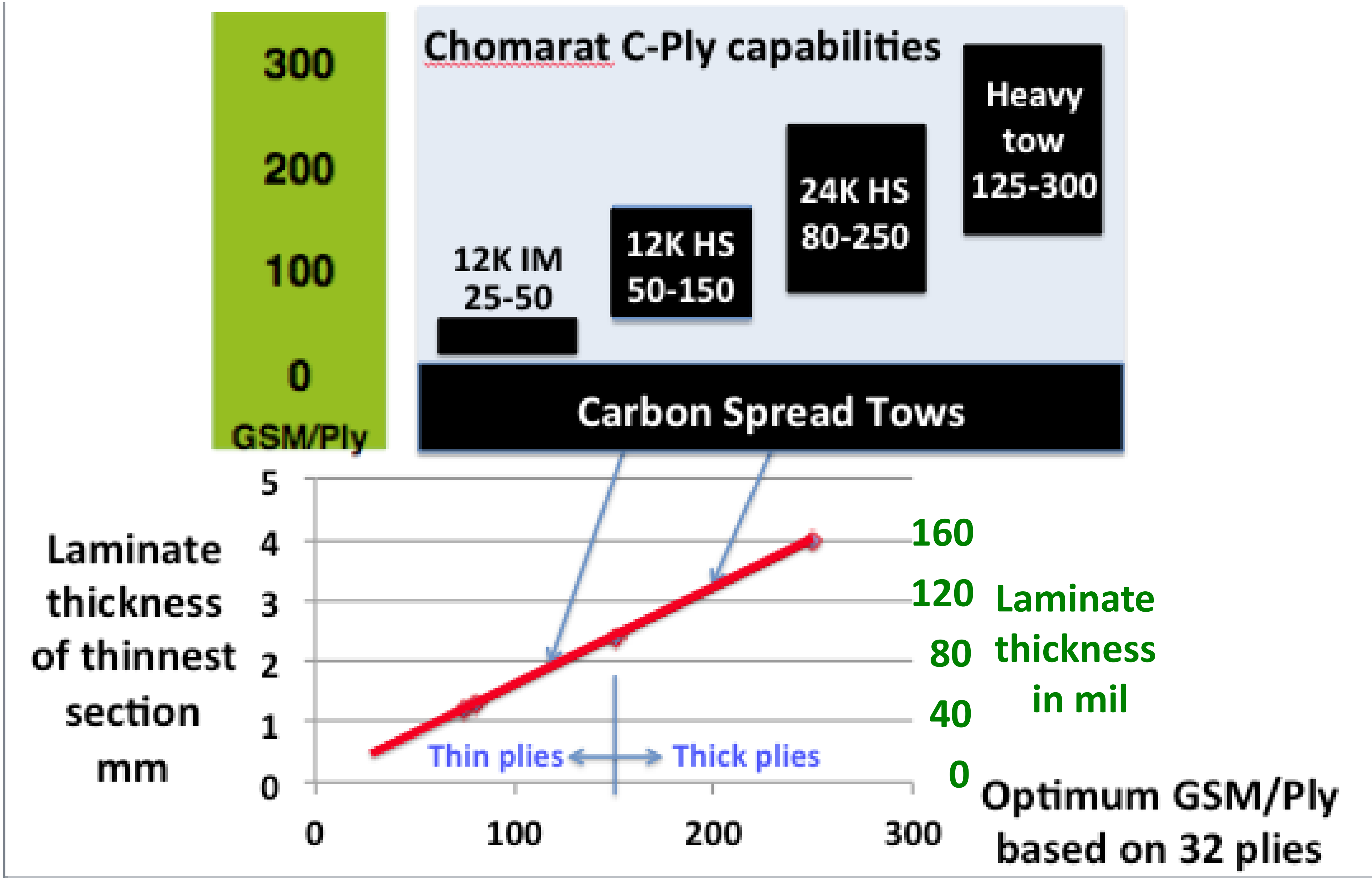
Material	Load case	N_x	$N_{x\ scaled}$	Err, %	N_y	$N_{y\ scaled}$	Err, %	N_{xy}	$N_{xy\ scaled}$	Err, %
IM6/Epoxy	1	-307,3	-302,3	1,66	-	-	-	-	-	-
	2	-173,4	-169,3	2,44	-69,30	-67,69	2,38	-	-	-
	3	-166,0	-162,8	1,98	-66,40	-65,11	1,98	83	82,4	0,70
	4	-150,0	-145,8	2,90	-60,00	-58,31	2,90	-75	-73,1	2,53
AS4/3501	1	-215,4	-210,9	2,13	-	-	-	-	-	-
	2	-122,0	-118,1	3,30	-48,80	-47,23	3,33	-	-	-
	3	-116,5	-113,6	2,58	-46,60	-45,43	2,58	58,3	57,5	1,38
	4	-106,0	-101,7	4,21	-42,40	-40,69	4,21	-53	-51,0	3,85
C-Ply	1	-186,0	-189,7	1,97	-	-	-	-	-	-
	2	-105,0	-107,1	1,93	-42,00	-40,57	3,53	-	-	-
	3	-100,0	-102,5	2,44	-40,00	-39,02	2,50	50	49,4	1,22
	4	-91,0	-92,62	1,75	-36,5	-34,95	4,44	-45,6	-43,8	4,02
T300/N5208	1	-273,4	-268,8	1,71	-	-	-	-	-	-
	2	-154,3	-150,5	2,51	-61,70	-60,19	2,51	-	-	-
	3	-147,6	-144,7	1,97	-59	-57,9	1,90	74	76,0	2,68
	4	-133,7	-129,6	3,14	-53,5	-51,853	3,18	-67	-68,7	2,49

Unique NCF at Chomarat

Shallow angle, wide ranging thin plies, noninvasive stitching, hybrid, …



Wide-range GSM to Meet Requirement

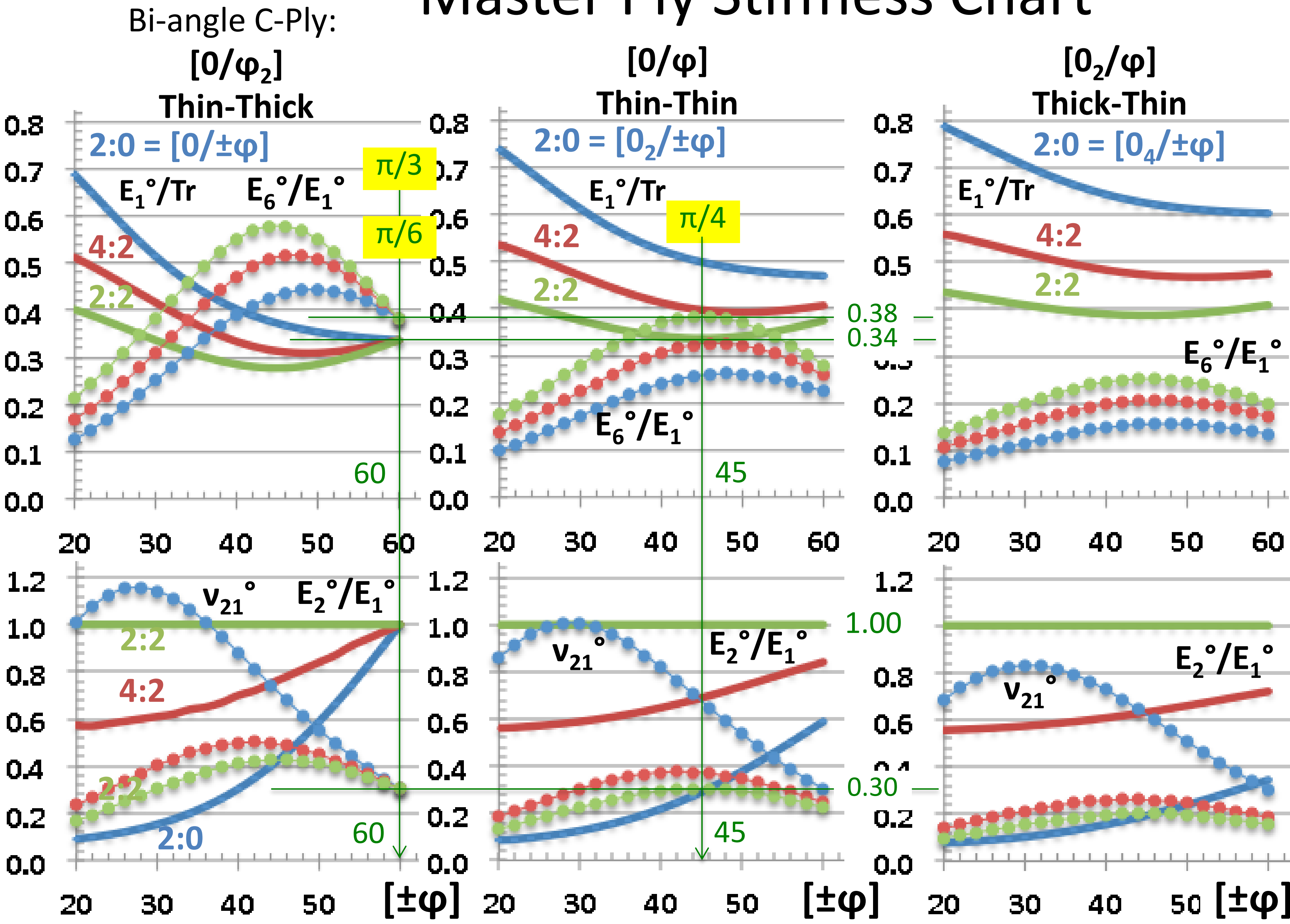


Lowest Cost Layup of Thick-thin C-Ply

Starting C-Ply	$[0/\varphi_2]$ - Thin-Thick (33/67/0) – 150 gsm	$[0/\varphi]$ - Thin-Thin (50/50/0) – 100 gsm	$[0_2/\varphi]$ - Thick-Thin (67/33/0) – 150 gsm
1-axis 2:0 **ATL: 6X**	$[0/\pm\varphi]_2$ = $[\pi/3]_2$ for $\varphi = 60$ (33/67/0)	$[0_2/\pm\varphi]$ (50/50/0)	$[0_4/\pm\varphi]$ (67/33/0)
2-axis 4:2 **ATL: 4X**	$[(0/\pm\varphi)_2/(\pm\psi/90)]_2$ $\Psi = 90 - \varphi$ (22/67/11)	$[(0_2/\pm\varphi)_2/\pm\psi_2/90_2]$ $\Psi = 90 - \varphi$ (33/50/17)	$[(0_4/\pm\varphi)_2/\pm\psi/90_4]$ $\Psi = 90 - \varphi$ (44/33/22)
2-axis 2:2 **ATL: 2X**	$[0/\pm\varphi/\pm\psi/90]_2$ = $[\pi/6]_2$ for $\varphi = 30$ (17/66/17)	$[0_2/\pm\varphi/\pm\psi/90_2]$ = $[\pi/4]_2$ for $\varphi = 45$ (25/50/25)	$[0_4/\pm\varphi/\pm\psi/90_4]$ $\Psi = 90 - \varphi$ (33/33/33)

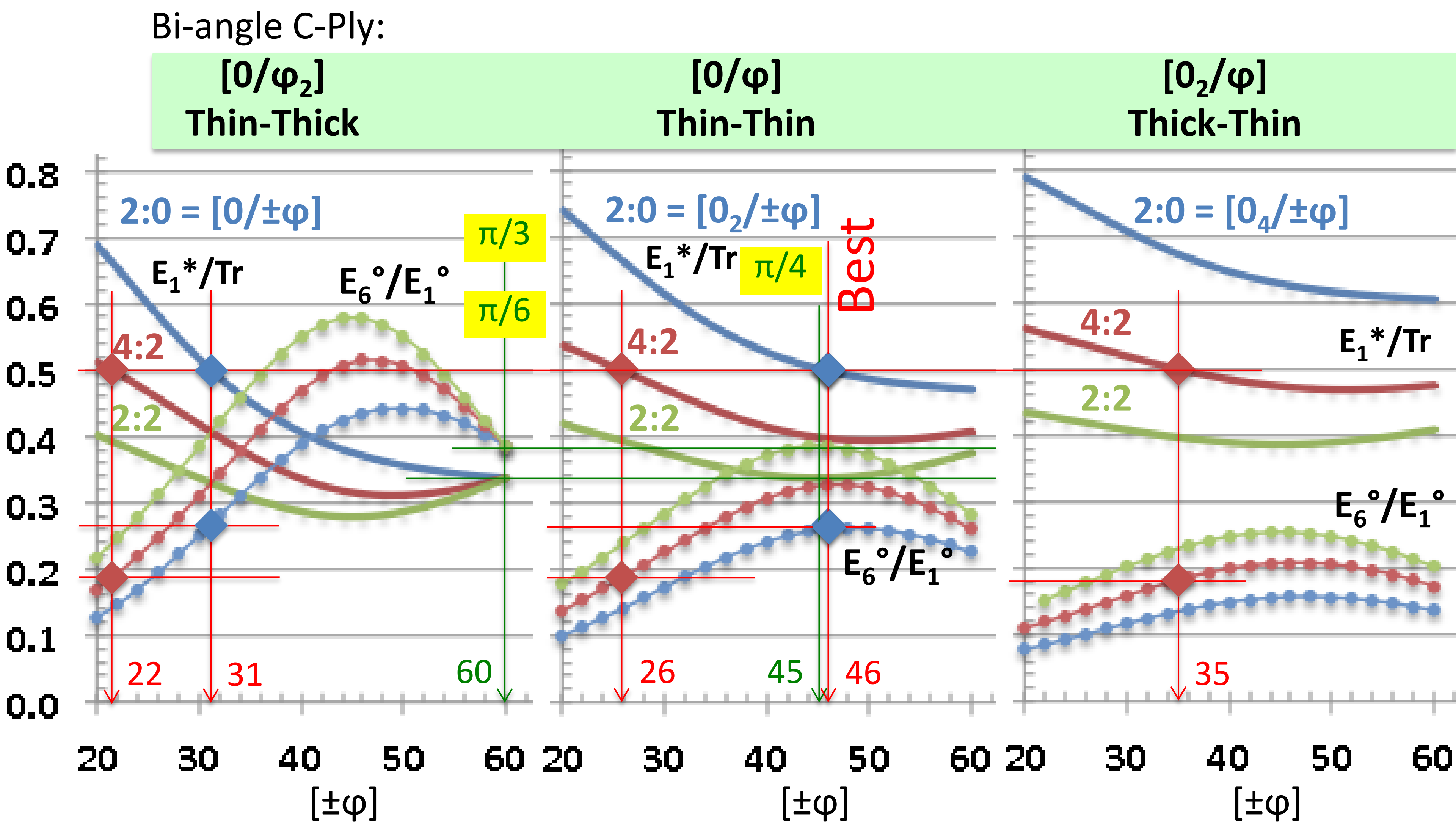
4-axis ATL as unity base-line

Master Ply Stiffness Chart



A Master Ply Design Chart: Best E_1

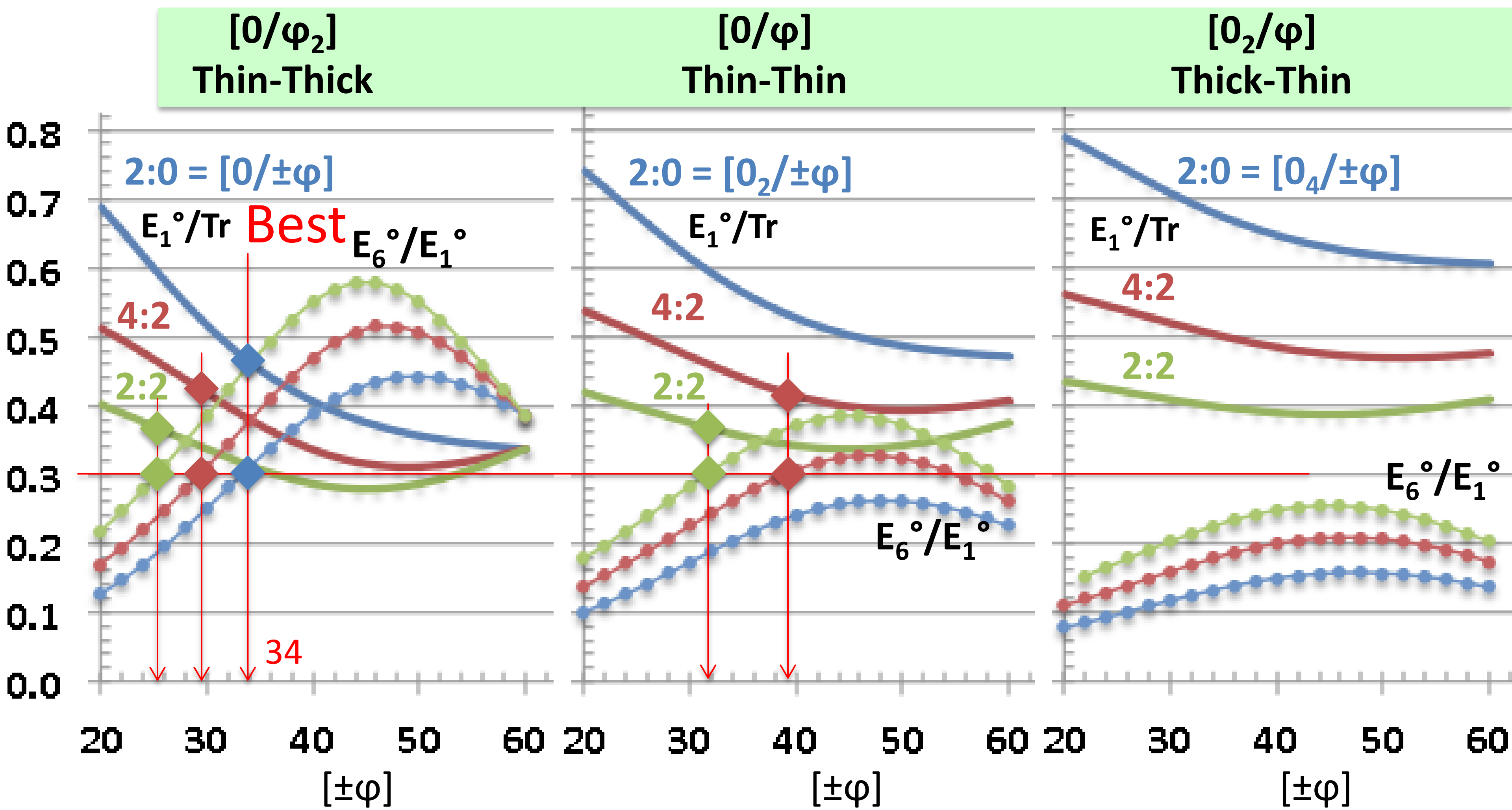
Smooth lines = trace normalized = E_1^*, GPa ; Dots = $E_6°/E_1°$



A Master Ply Design Chart: Best G/E

Smooth lines = normalized = E_1^*, GPa ; Dots = $E_6°/E_1°$

Bi-angle C-Ply:

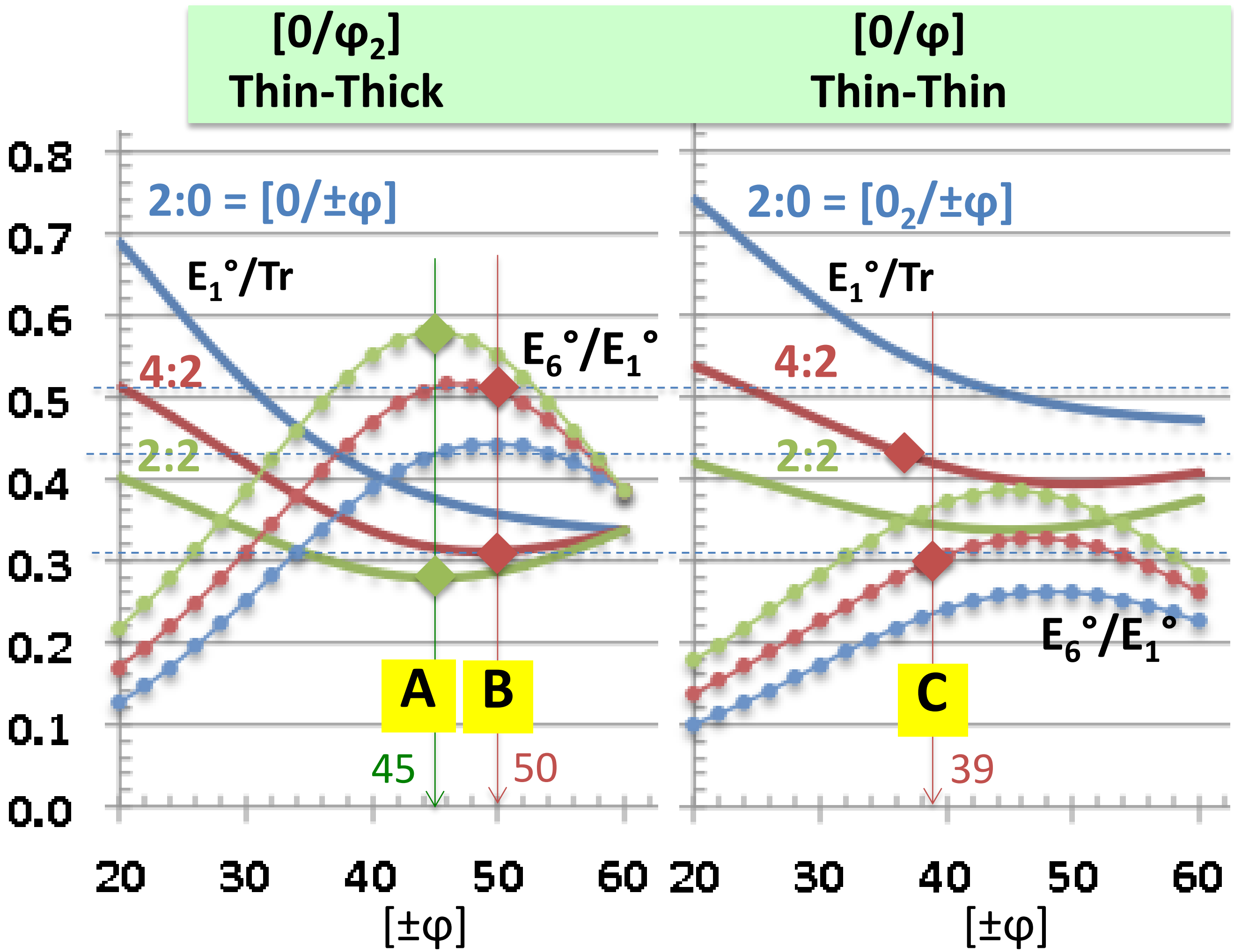


Master Chart for C-Ply to Replace

A: $[0/\pm45_4/90]$; B: $[0_2/\pm45_3/90]$; C: $[0_3/\pm45_2/90]$

Smooth lines = normalized = E_1^*, GPa ; Dots = $E_6°/E_1°$

Bi-angle C-Ply:



Low cost replacement

Lam A: C-Ply $[0/45_2)$

Tape: $[0/\pm45]$; 2:2

$[(0/\pm45)/(\pm45/90)]$

$E_1° = 26.3$ (30.8, 85%)

$E_6° = 25.3$ (25.9, 98%)

Lam B: C-Ply $[0/50_2]$

Tape: $[0/\pm50]$; 4:2

$[(0/\pm50)_2/(\pm40/90)]$

$E_1° = 43.0$ (43.6, 99%)

$E_6° = 21.8$ (22.4, 97%)

Lam C: C-Ply [0/39]

Tape: $[0_2/\pm39]$; 4:2

$[(0_2/\pm39)_2/(\pm51/90_2)]$

$E_1° = 58.0$ (59.3, 98%)

$E_6° = 17.4$ (18.0, 97%)

A Master Chart for C-Ply to Replace

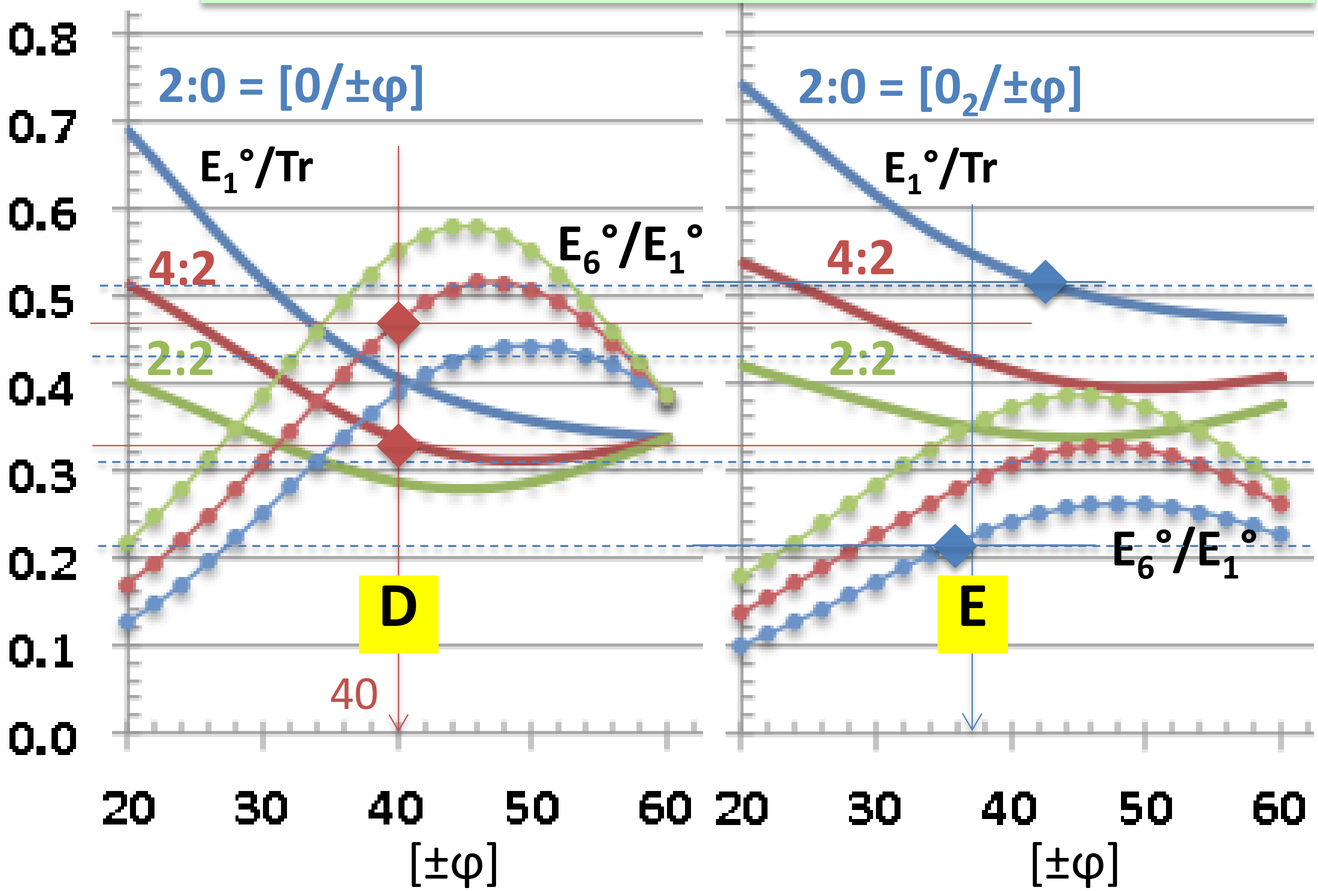
D: $[0_{1.5}/\pm45_2/90]$; E: $[0_5/\pm45_2/90]$

Smooth lines = normalized = E_1^*, GPa ; Dots = $E_6°/E_1°$

Low cost replacement

Bi-angle C-Ply:

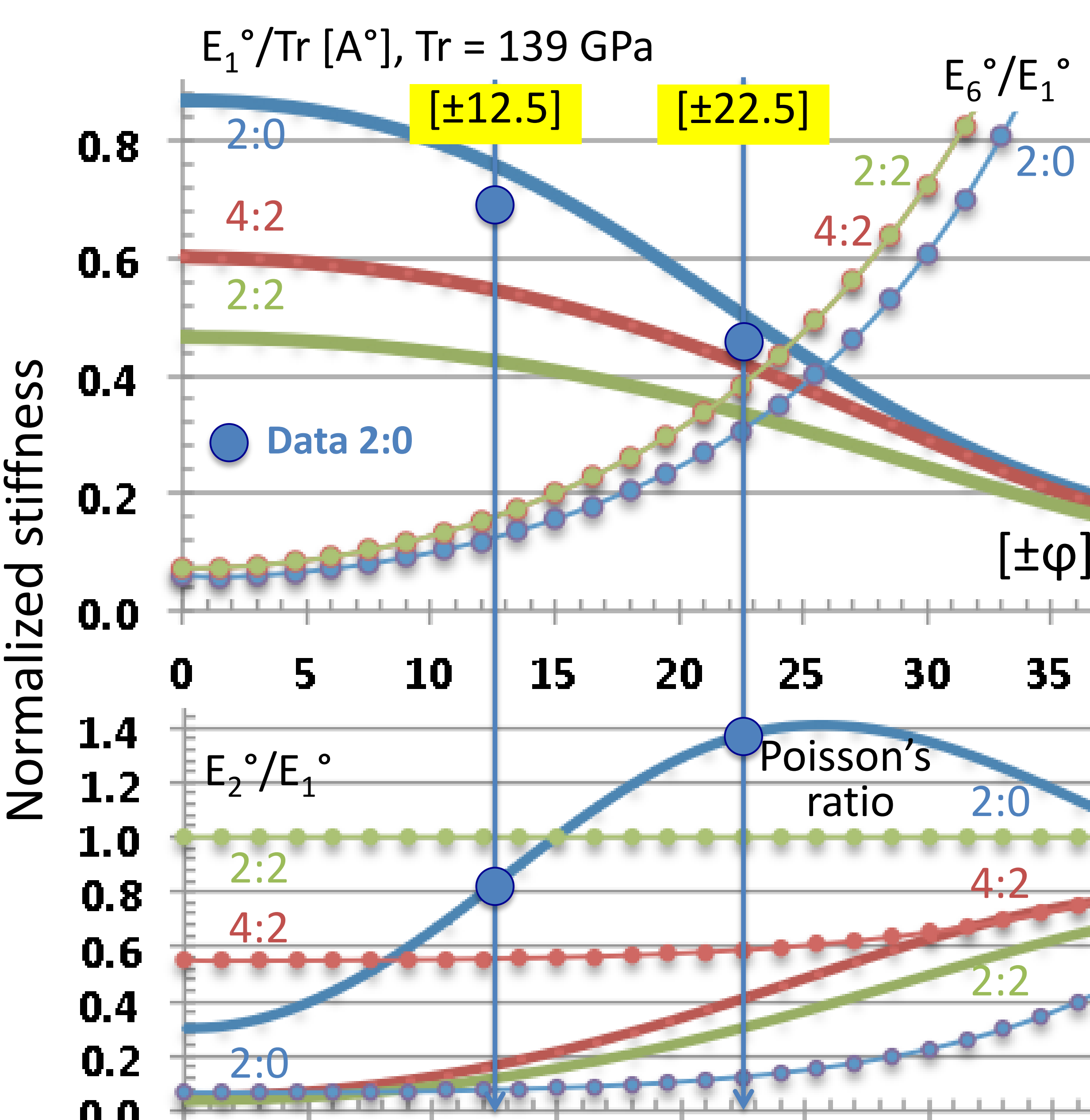
$[0/\varphi_2]$ Thin-Thick	$[0/\varphi]$ Thin-Thin

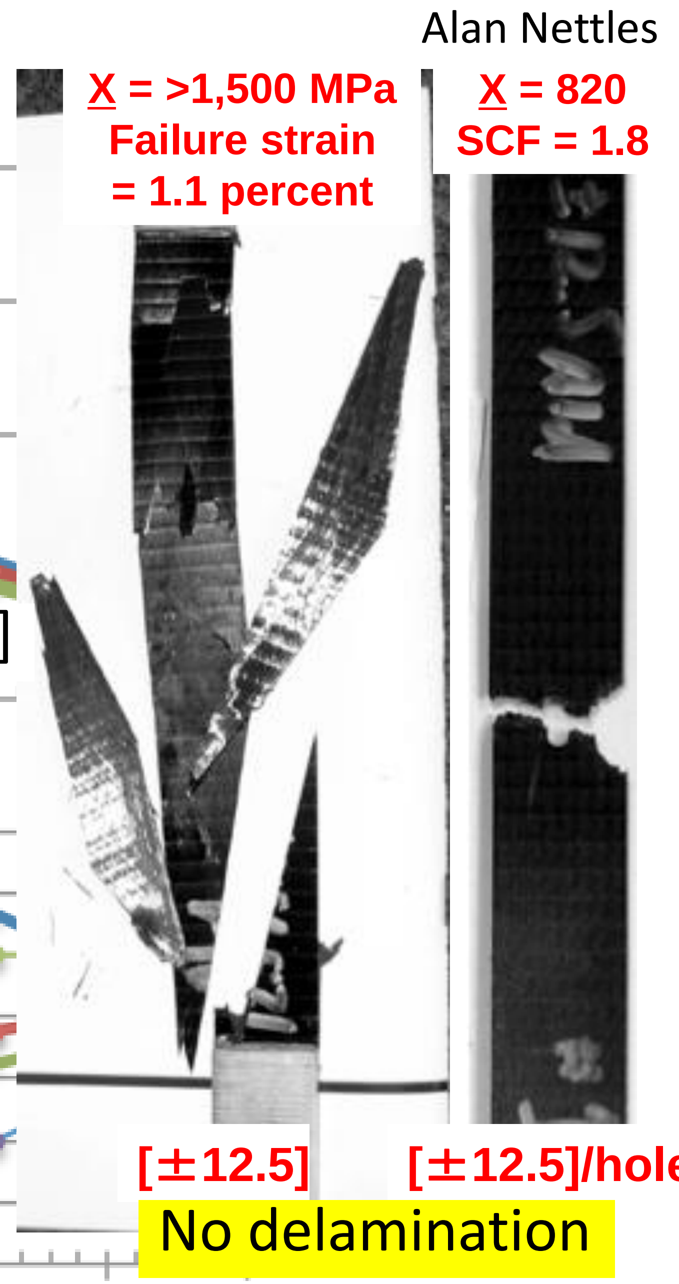


Lam D: C-Ply $[0/40_2]$
Tape: $[0/\pm40]$; 4:2
$[(0/\pm40)_2/(\pm50/90)]$
$E_1°$ = 45.0 (46.5, 103%)
$E_6°$ = 21.0 (21.9, 104%)

Lam E: C-Ply [0/37]
Tape: $[0_2/\pm37]$; 2:0
$[(0_2/\pm37)]$
$E_1°$ = 75.7 (71.6, 106%)
$E_6°$ = 17.0 (15.3, 111%)

Ultra-shallow-angle C-Ply: 5 ≤ φ ≤ 20

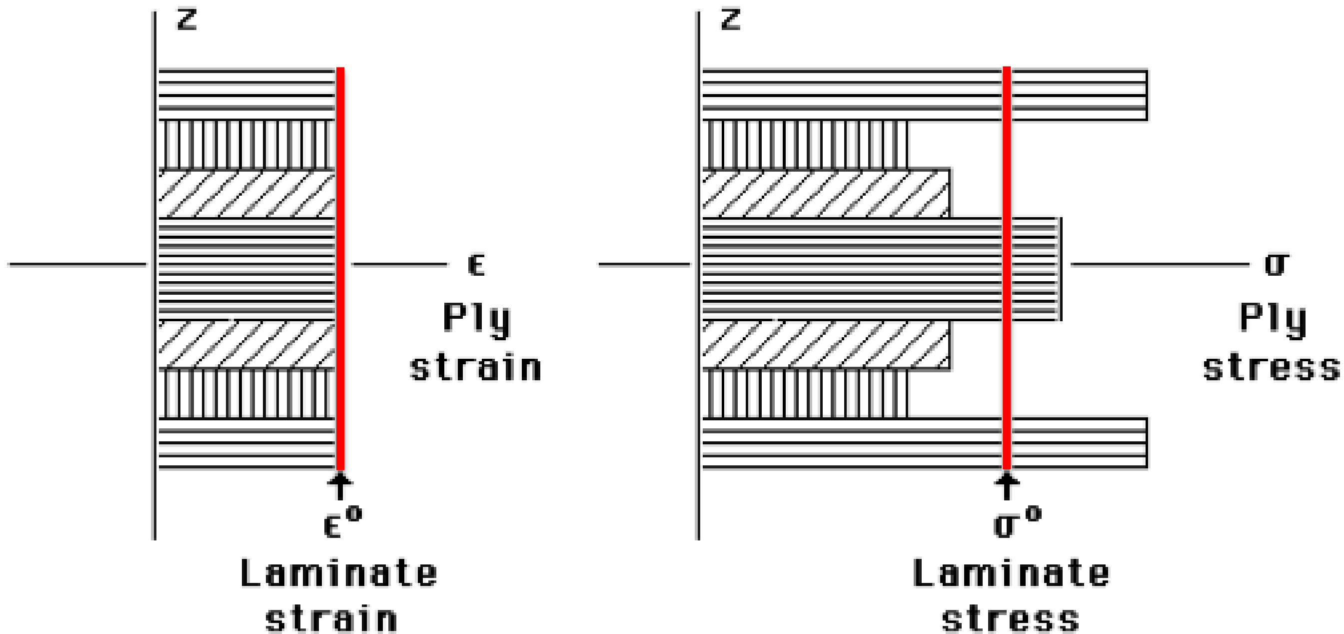




Outline

- Trace-based theory
- Supporting data
- Prelminary design
- Omni strain allowable

Ply Strain and Stress of a Laminate



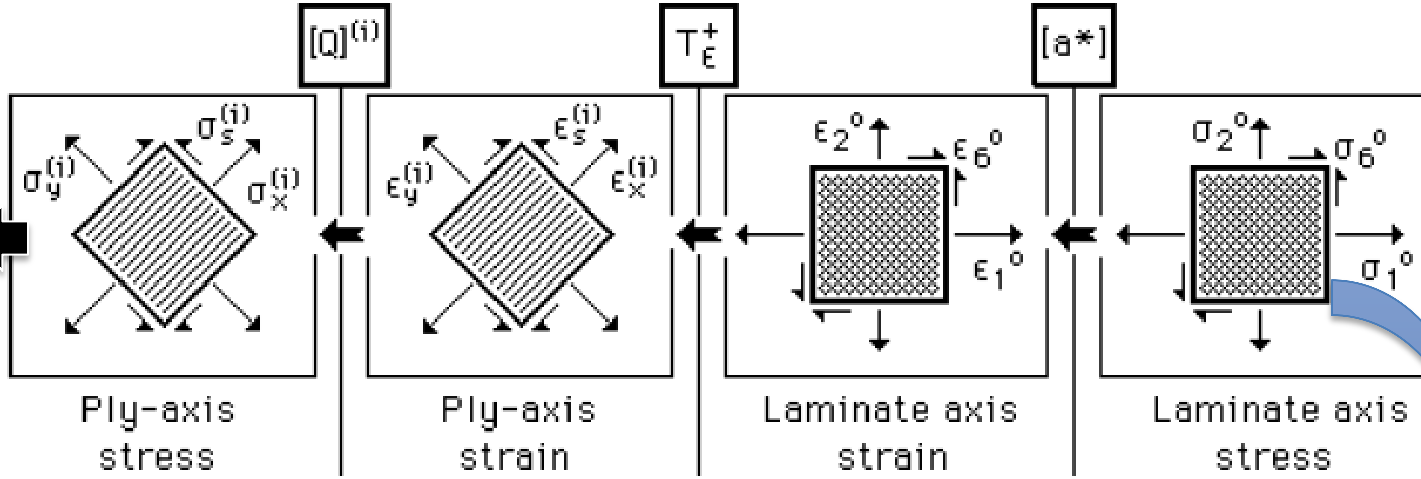
Since ply and laminate strains are equal, strain-based failure criteria are functions of ply angles only, independent of ply composition of the laminate. So a strain-based criterion is the same for all laminates

Ply stress various from ply to ply depending on the ply angles. The stiffer ply will have higher ply stress. Unlike strain-based failure, stress-based failure tensors [F] and {F} are functions of not only each ply angle but also ply composition of the laminate. Thus each laminate has its own failure envelope.

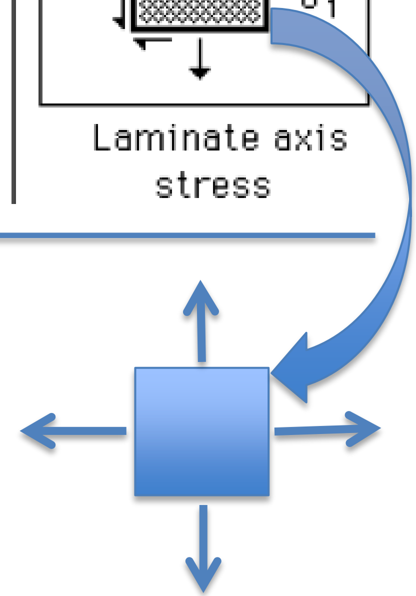
Ply-by-Ply vs Homogenized Plate

Ply-by-ply $R^{(i)}$ of a laminated anisotropic or orthotropic plate

$R^{FPF} \Leftarrow R^{(i)} \Leftarrow$



R = strength ratio
= safety factor



Homogeneous anisotropic plate: one <u>R</u>

$\underline{R}^{omni}$

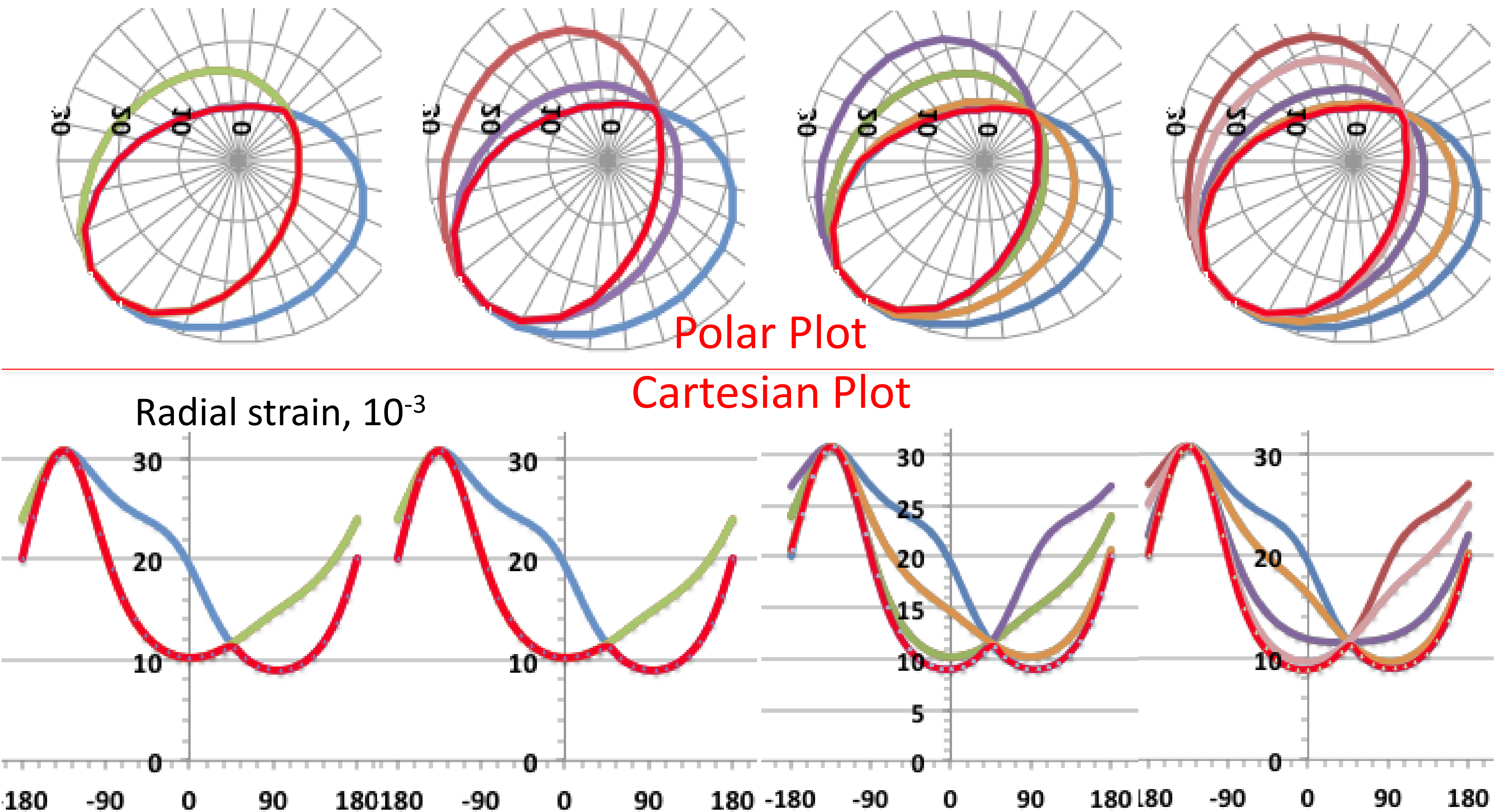
$E_1° = 1/a_{11}*$, $E_2° = 1/a_{22}*$, . . . $nu_{61}° = a_{61}*/a_{11}*$

Anisotropic Tsai-Wu criterion: $\underline{F}_{11}$, . . . $\underline{F}_{16}$; $\underline{F}_1$, $\underline{F}_2$, $\underline{F}_6$

Back to the basics: many closed-form and FEM solutions easily applied; speed increases by n (number of plies) in model formation and stress recovery

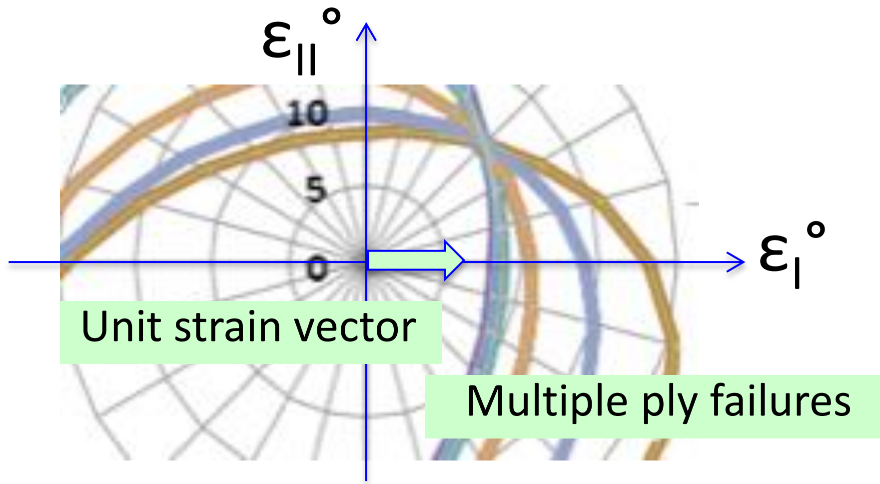
Successive Increase in Ply Angles

C-Ply 55 [π/3]; 8.94 C-Ply 55 [π/4]; 8.85 C-Ply 55 [π/6]; 8.94 C-Ply 55 [π/8]; 8.85



Transformation angle of strain envelope radii θ

Omni Strain FPF Envelopes: C-Ply 64



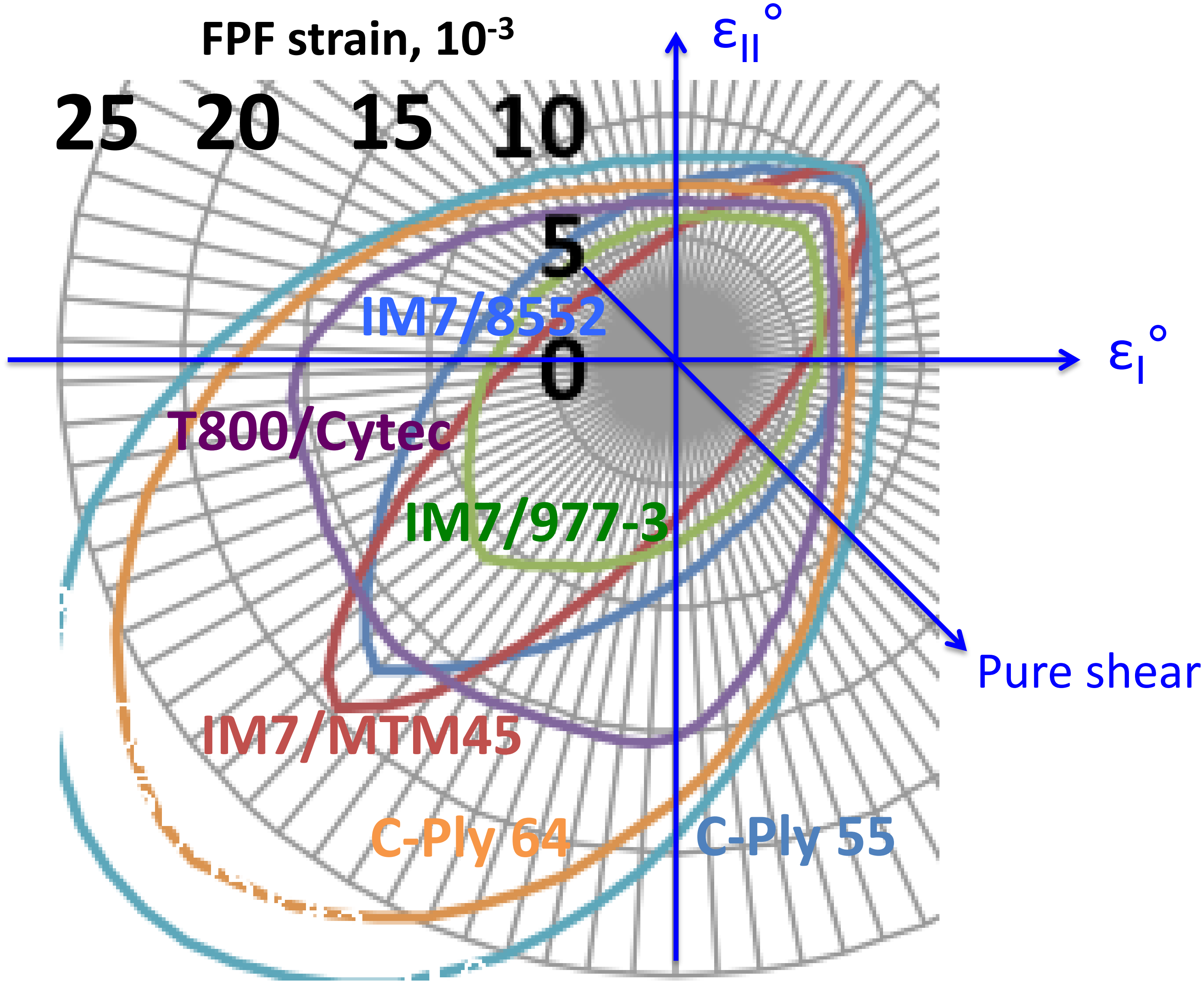
Ply angle: 0 to 2π @15° delta ↓ ↓ ↓

Polar angle of radial strain vector: 0 to 2π @15° increments➔➔➔

+/- Teta	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
0	18.36	14.37	11.18	9.12	7.91	7.29	7.10	7.30	7.95	9.22	11.45	15.23	20.83	26.46	28.98	28.38	26.60	24.90	23.73	23.13	23.01	23.14	22.97	21.58	18.365
15	15.76	13.17	10.82	9.12	8.05	7.47	7.29	7.49	8.12	9.31	11.36	14.72	19.68	25.17	28.43	28.38	26.49	24.29	22.46	21.15	20.28	19.66	18.99	17.80	15.764
30	11.88	10.97	10.00	9.12	8.45	8.06	7.96	8.18	8.77	9.84	11.59	14.32	18.29	23.21	27.33	28.38	26.34	23.10	20.04	17.61	15.81	14.50	13.52	12.70	11.876
45	9.345	9.18	9.13	9.12	9.13	9.18	9.35	9.70	10.32	11.34	12.89	15.17	18.38	22.49	26.55	28.38	26.55	22.49	18.38	15.17	12.89	11.34	10.32	9.70	9.3451
60	7.963	8.06	8.45	9.12	10.00	10.97	11.88	12.70	13.52	14.50	15.81	17.61	20.04	23.10	26.34	28.38	27.33	23.21	18.29	14.32	11.59	9.84	8.77	8.18	7.9628
75	7.294	7.47	8.05	9.12	10.82	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24.29	26.49	28.38	28.43	25.17	19.68	14.72	11.36	9.31	8.12	7.49	7.2942
90	7.097	7.29	7.91	9.12	11.18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24.90	26.60	28.38	28.98	26.46	20.83	15.23	11.45	9.22	7.95	7.30	7.0966
105	7.294	7.47	8.05	9.12	10.82	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24.29	26.49	28.38	28.43	25.17	19.68	14.72	11.36	9.31	8.12	7.49	7.2942
120	7.963	8.06	8.45	9.12	10.00	10.97	11.88	12.70	13.52	14.50	15.81	17.61	20.04	23.10	26.34	28.38	27.33	23.21	18.29	14.32	11.59	9.84	8.77	8.18	7.9628
135	9.345	9.18	9.13	9.12	9.13	9.18	9.35	9.70	10.32	11.34	12.89	15.17	18.38	22.49	26.55	28.38	26.55	22.49	18.38	15.17	12.89	11.34	10.32	9.70	9.3451
150	11.88	10.97	10.00	9.12	8.45	8.06	7.96	8.18	8.77	9.84	11.59	14.32	18.29	23.21	27.33	28.38	26.34	23.10	20.04	17.61	15.81	14.50	13.52	12.70	11.876
165	15.76	13.17	10.82	9.12	8.05	7.47	7.29	7.49	8.12	9.31	11.36	14.72	19.68	25.17	28.43	28.38	26.49	24.29	22.46	21.15	20.28	19.66	18.99	17.80	15.764
180	18.36	14.37	11.18	9.12	7.91	7.29	7.10	7.30	7.95	9.22	11.45	15.23	20.83	26.46	28.98	28.38	26.60	24.90	23.73	23.13	23.01	23.14	22.97	21.58	18.365
195	15.76	13.17	10.82	9.12	8.05	7.47	7.29	7.49	8.12	9.31	11.36	14.72	19.68	25.17	28.43	28.38	26.49	24.29	22.46	21.15	20.28	19.66	18.99	17.80	15.764
210	11.88	10.97	10.00	9.12	8.45	8.06	7.96	8.18	8.77	9.84	11.59	14.32	18.29	23.21	27.33	28.38	26.34	23.10	20.04	17.61	15.81	14.50	13.52	12.70	11.876
225	9.345	9.18	9.13	9.12	9.13	9.18	9.35	9.70	10.32	11.34	12.89	15.17	18.38	22.49	26.55	28.38	26.55	22.49	18.38	15.17	12.89	11.34	10.32	9.70	9.3451
240	7.963	8.06	8.45	9.12	10.00	10.97	11.88	12.70	13.52	14.50	15.81	17.61	20.04	23.10	26.34	28.38	27.33	23.21	18.29	14.32	11.59	9.84	8.77	8.18	7.9628
255	7.294	7.47	8.05	9.12	10.82	13.17	15.76	17.80	18.99	19.66	20.28	21.15	22.46	24.29	26.49	28.38	28.43	25.17	19.68	14.72	11.36	9.31	8.12	7.49	7.2942
270	7.097	7.29	7.91	9.12	11.18	14.37	18.37	21.58	22.97	23.14	23.01	23.13	23.73	24.90	26.60	28.38	28.98	26.46	20.83	15.23	11.45	9.22	7.95	7.30	7.0966
285	7.294	7.47	8.05	9.12	10.82	13.17	15.76	17.80	18.99	19.66	20.28	21.15	22.46	24.29	26.49	28.38	28.43	25.17	19.68	14.72	11.36	9.31	8.12	7.49	7.2942
300	7.963	8.06	8.45	9.12	10.00	10.97	11.88	12.70	13.52	14.50	15.81	17.61	20.04	23.10	26.34	28.38	27.33	23.21	18.29	14.32	11.59	9.84	8.77	8.18	7.9628
315	9.345	9.18	9.13	9.12	9.13	9.18	9.35	9.70	10.32	11.34	12.89	15.17	18.38	22.49	26.55	28.38	26.55	22.49	18.38	15.17	12.89	11.34	10.32	9.70	9.3451
330	11.88	10.97	10.00	9.12	8.45	8.06	7.96	8.18	8.77	9.84	11.59	14.32	18.29	23.21	27.33	28.38	26.34	23.10	20.04	17.61	15.81	14.50	13.52	12.70	11.876
345	15.76	13.17	10.82	9.12	8.05	7.47	7.29	7.49	8.12	9.31	11.36	14.72	19.68	25.17	28.43	28.38	26.49	24.29	22.46	21.15	20.28	19.66	18.99	17.80	15.764
360	18.36	14.37	11.18	9.12	7.91	7.29	7.10	7.30	7.95	9.22	11.45	15.23	20.83	26.46	28.98	28.38	26.60	24.90	23.73	23.13	23.01	23.14	22.97	21.58	18.365

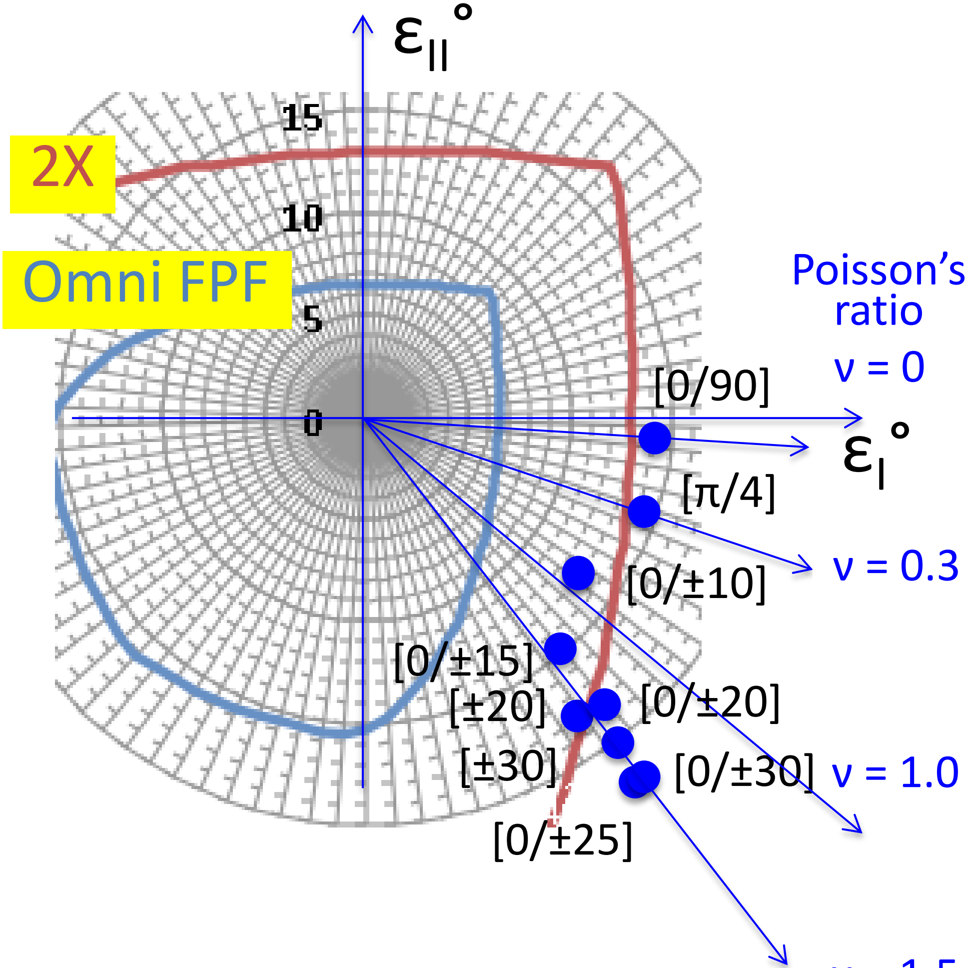
Controlling ply angles

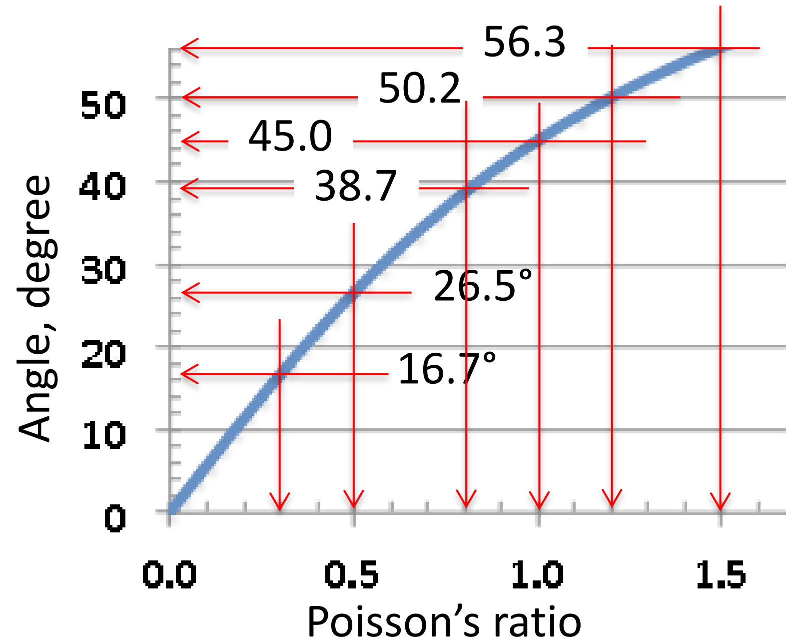
CFRP Omni Envelopes in Polar Plot



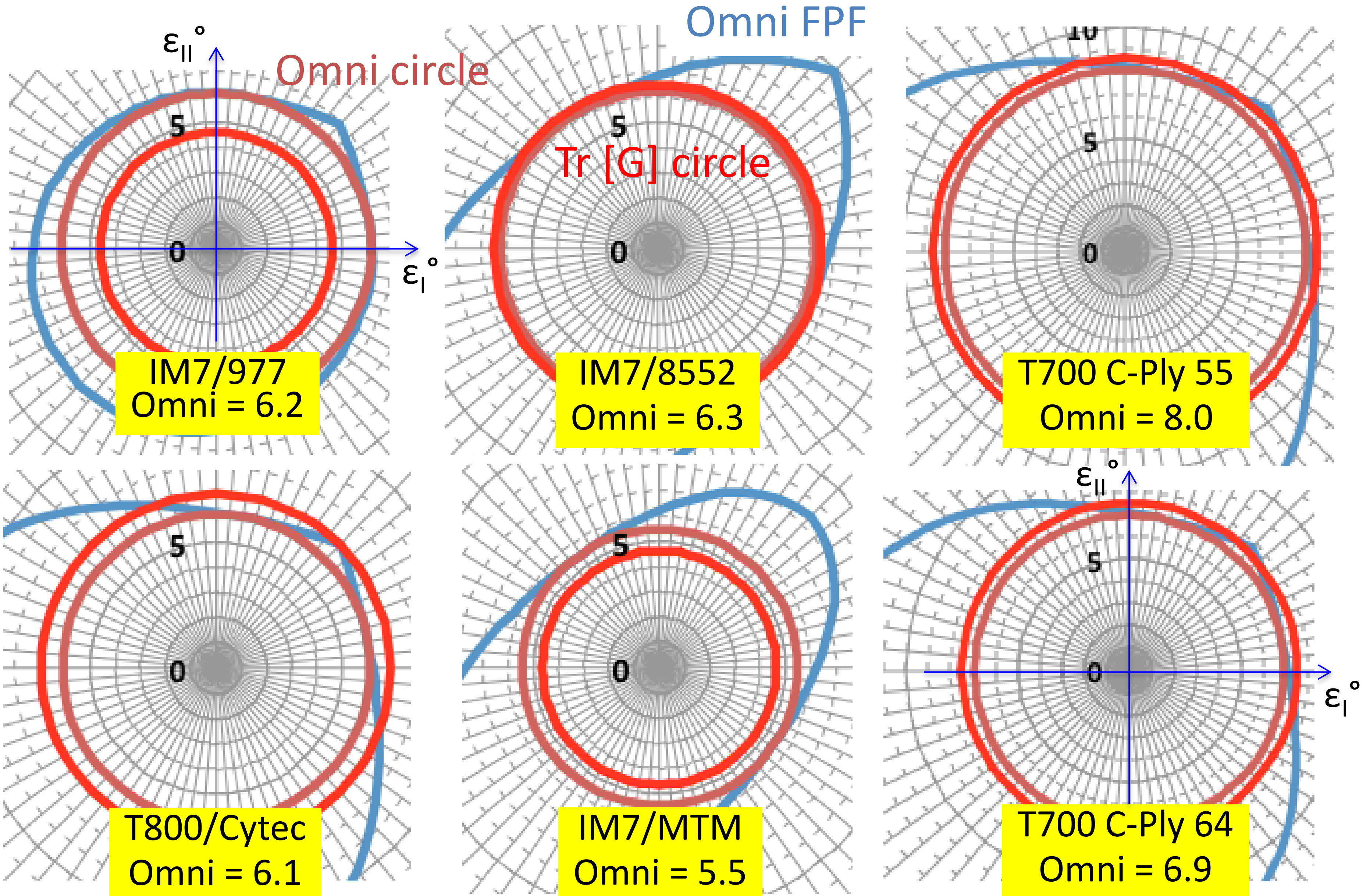
Omni Strain Envelope for T800/Cytec

All uniaxial tensile data can be placed on this principal strain plane

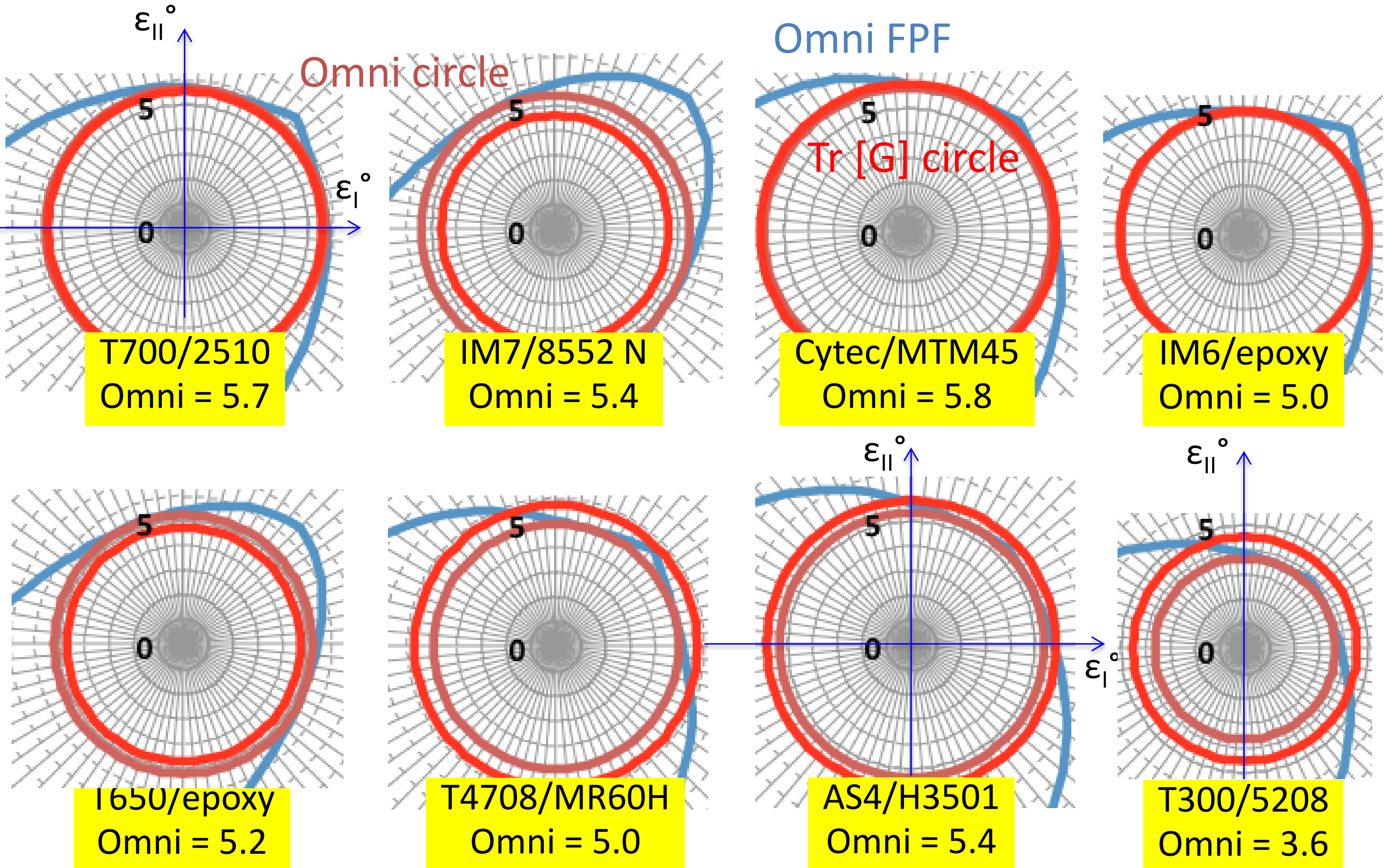




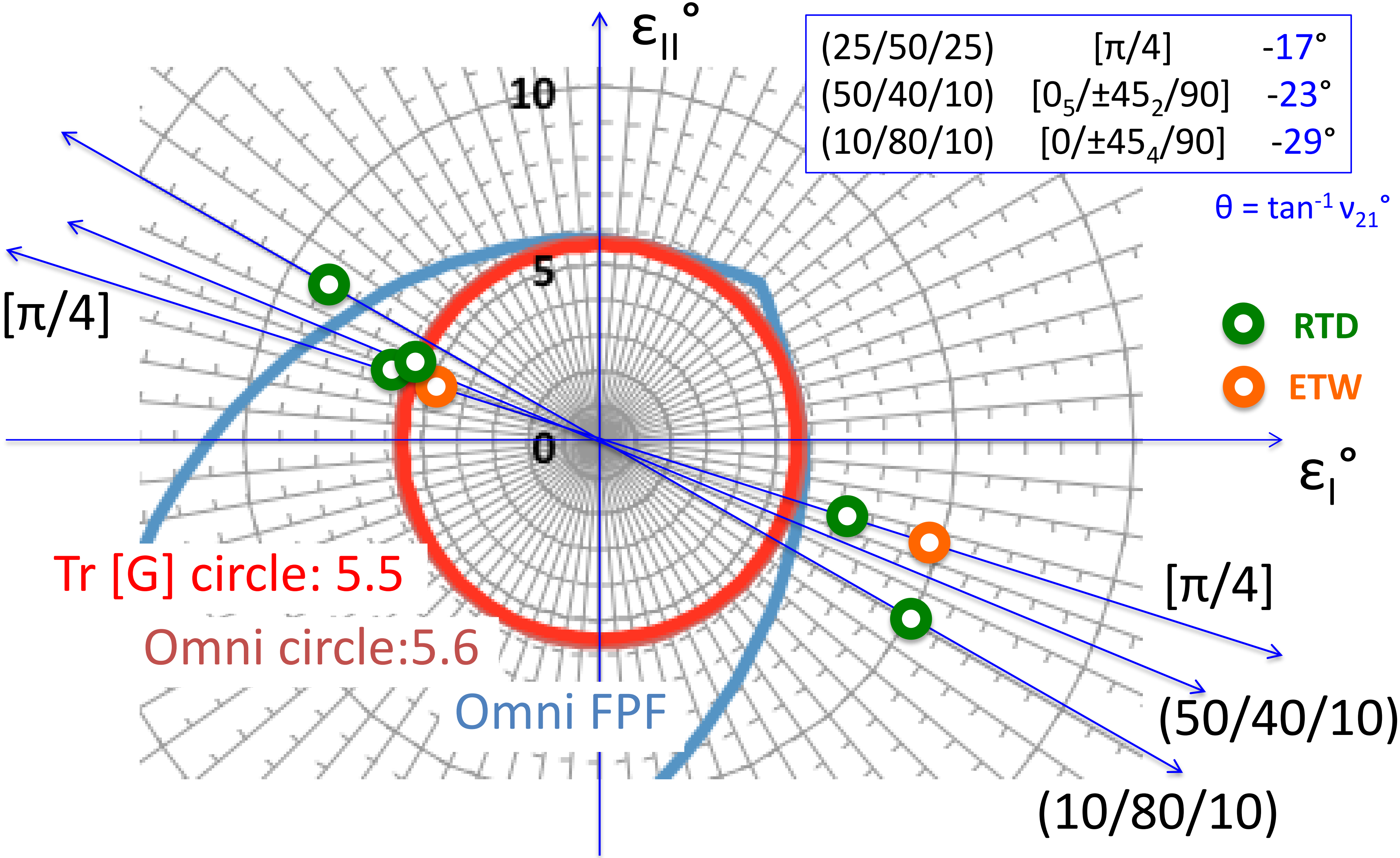
Omni Strain FPF and Circle, and Tr [G]



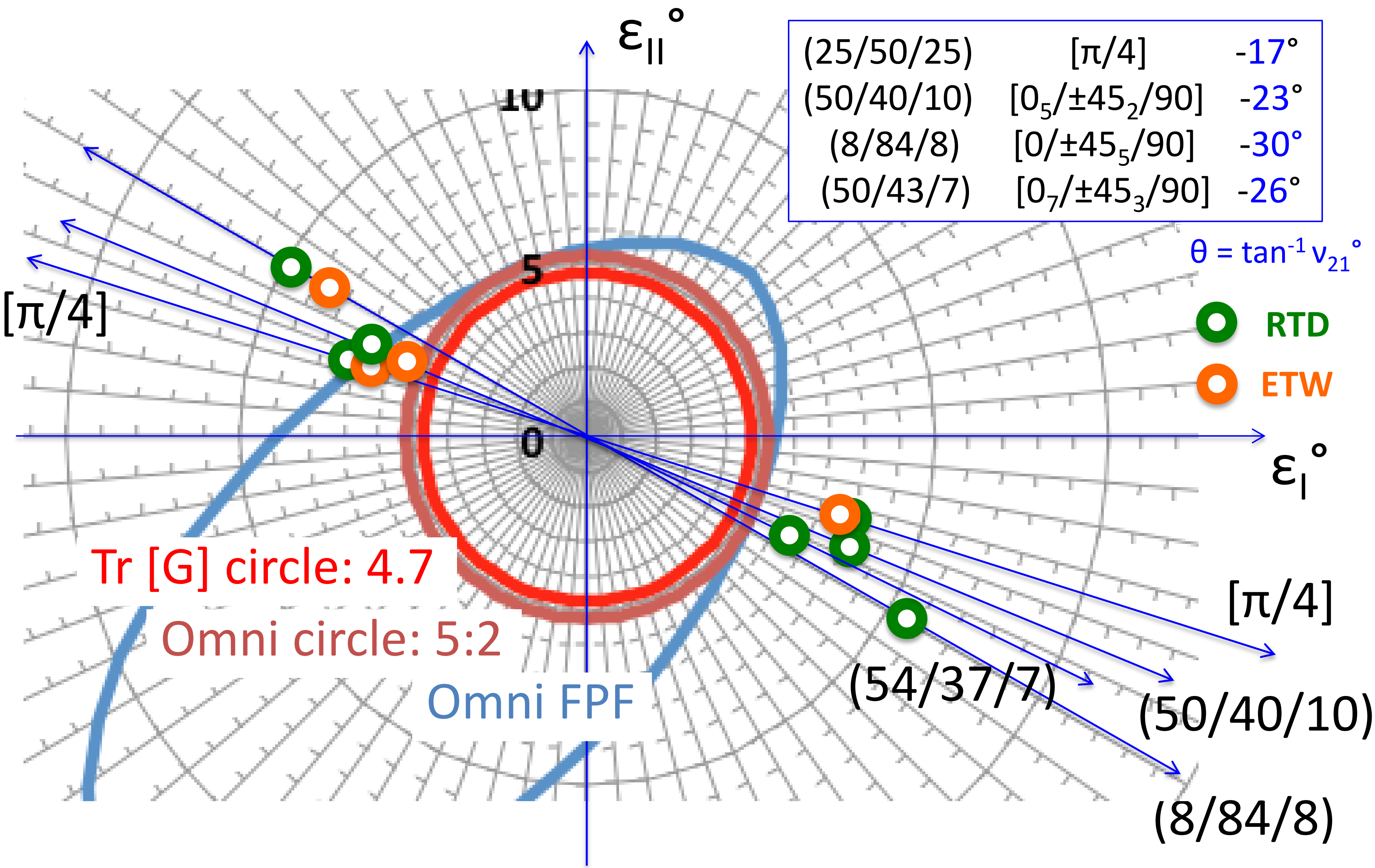
Omni Strain FPF and Circle, and Tr [G]



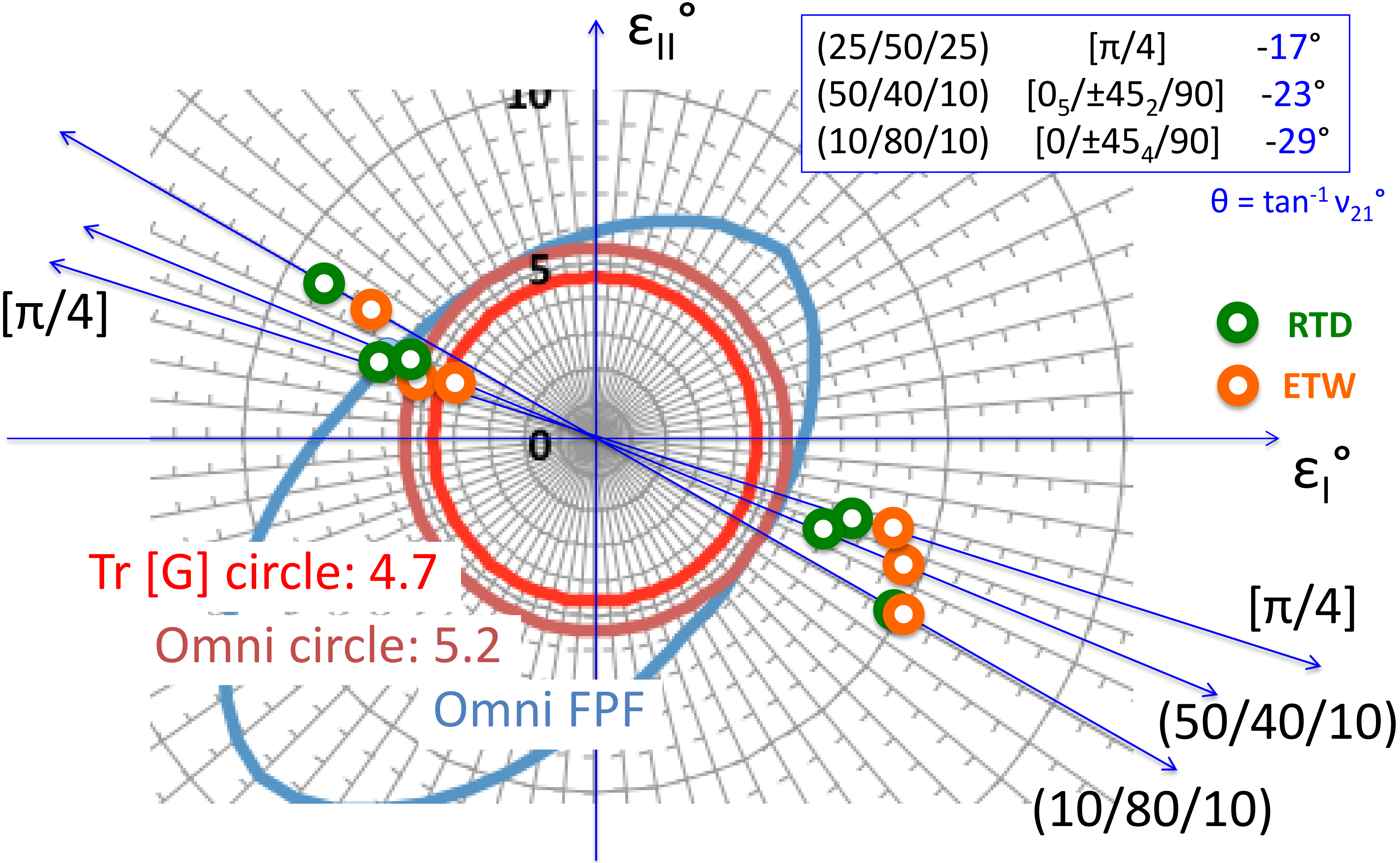
T700/2510: Failure Strain of OHT/OHC



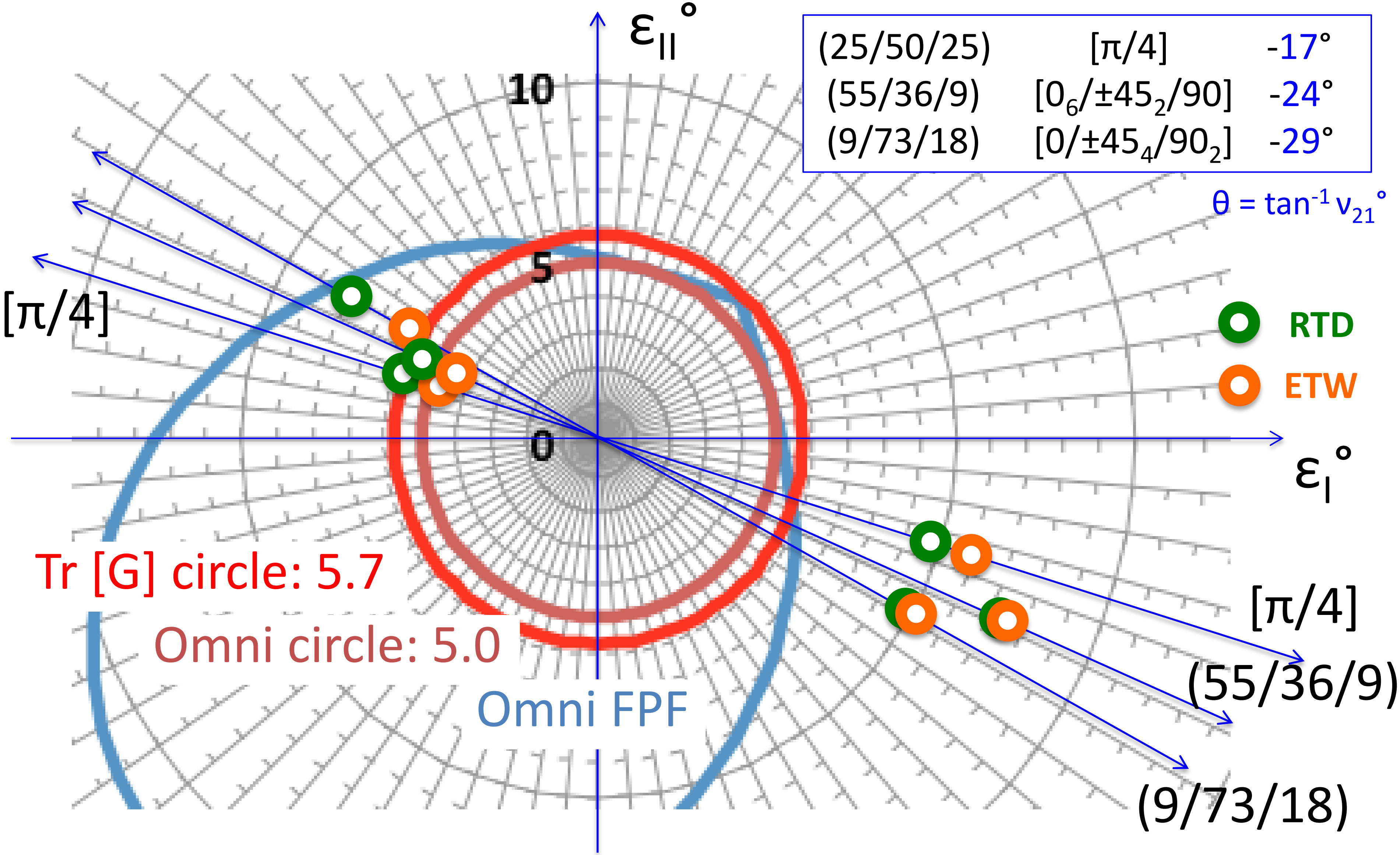
T650/epoxy: Failure Strain of OHT/OHC



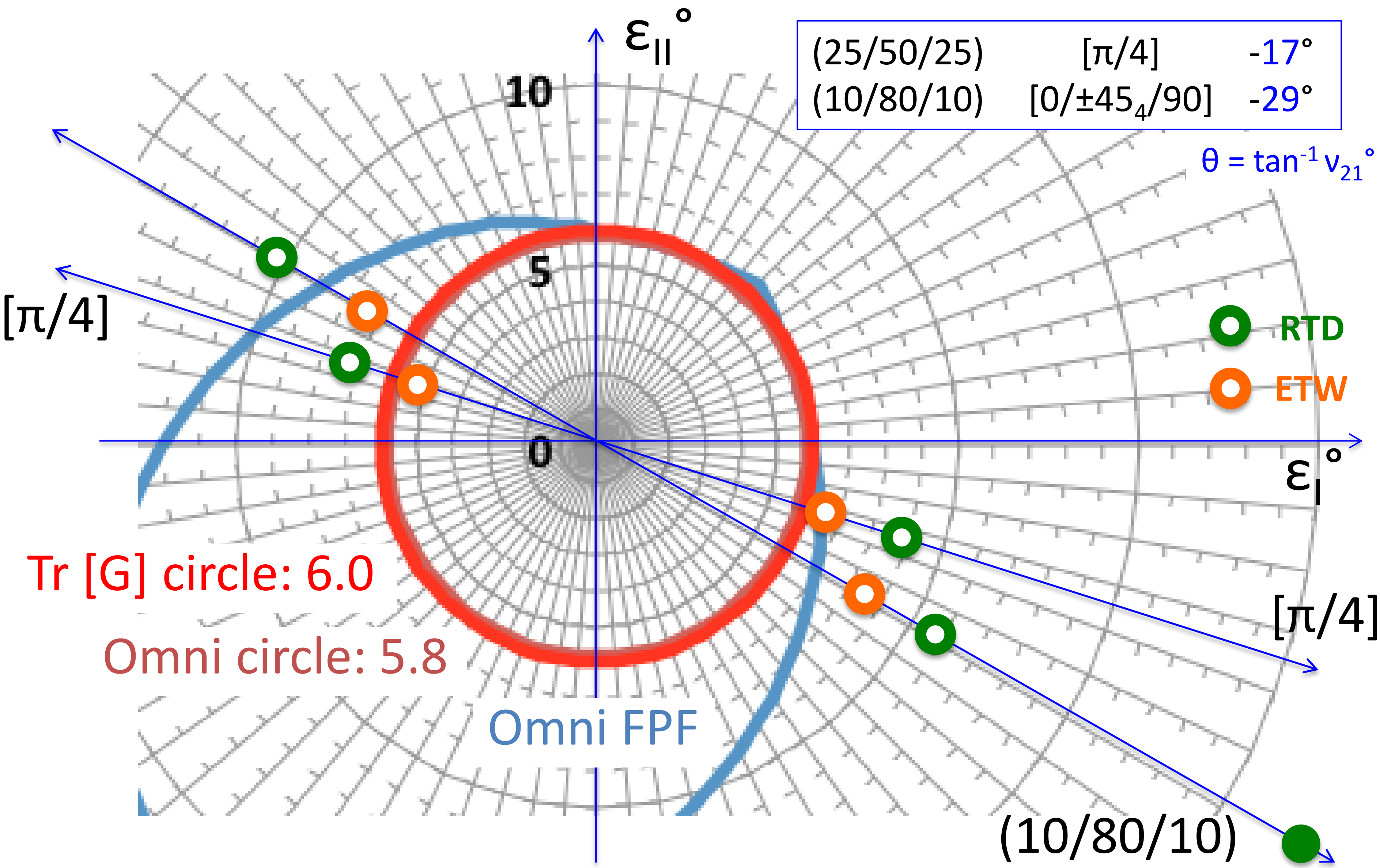
IM7/8552: Failure Strain of OHT/OHC



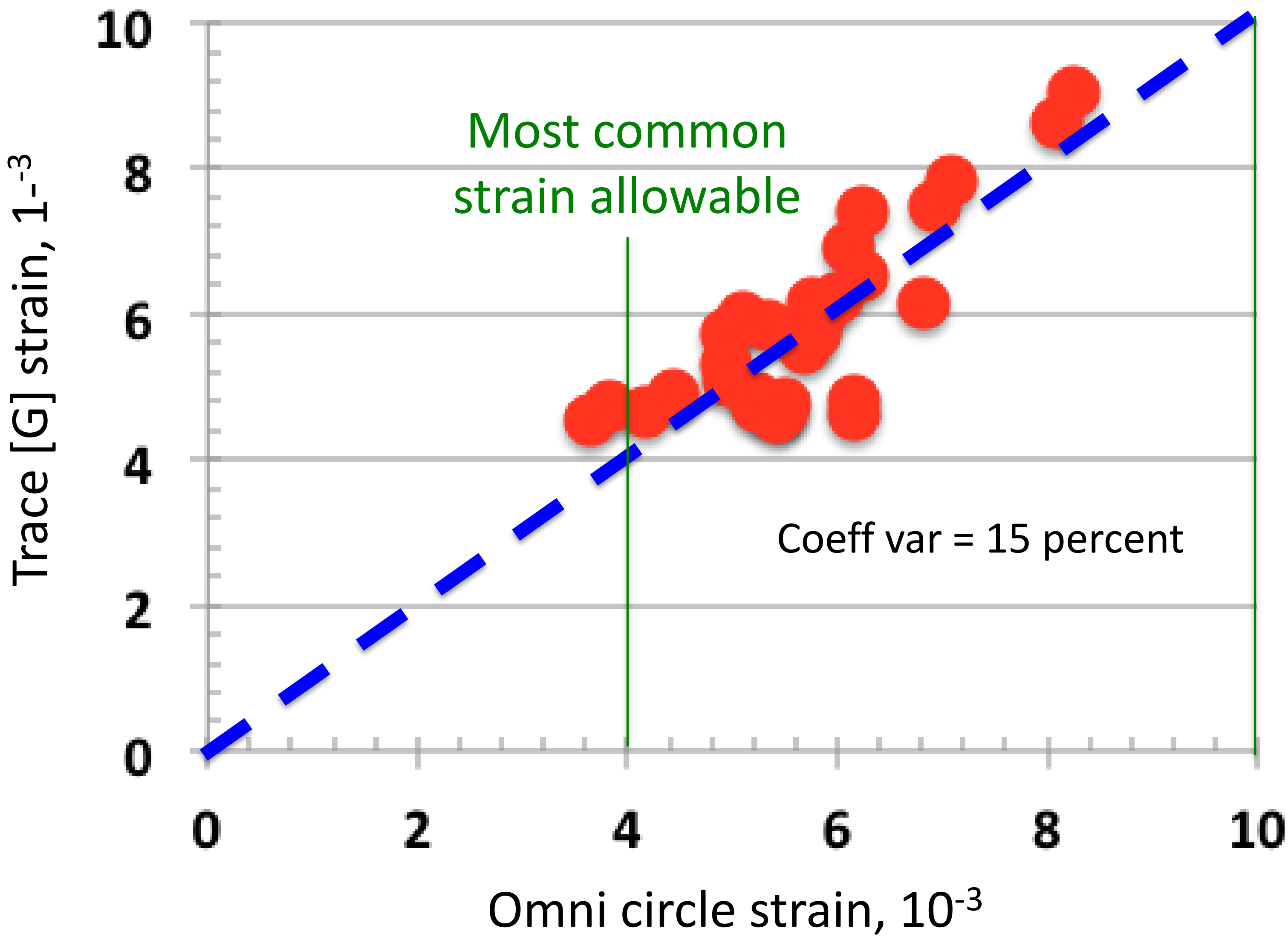
T4708/MR60H: Failure Strain OHT/OHC

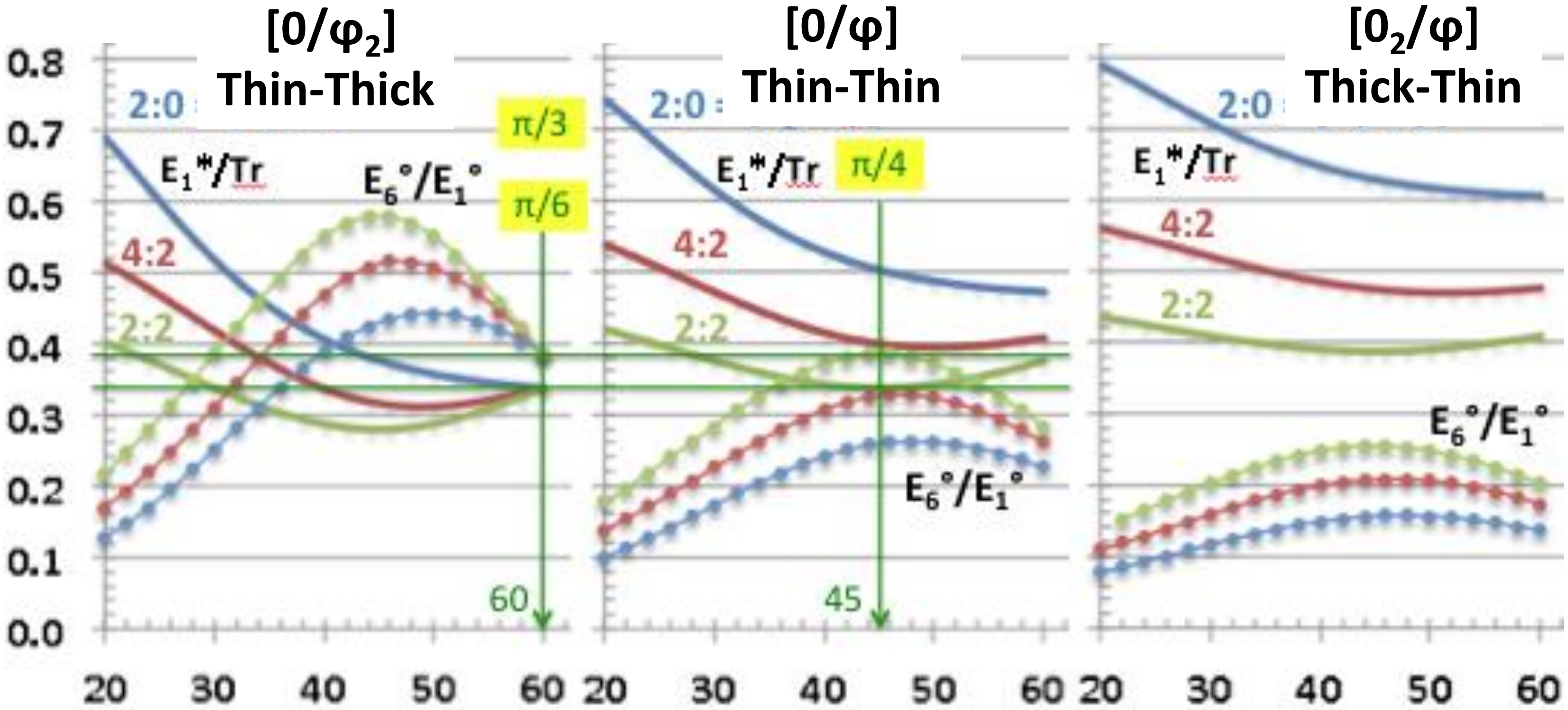


Cytec/MTM: Failure Strain OHT/OHC

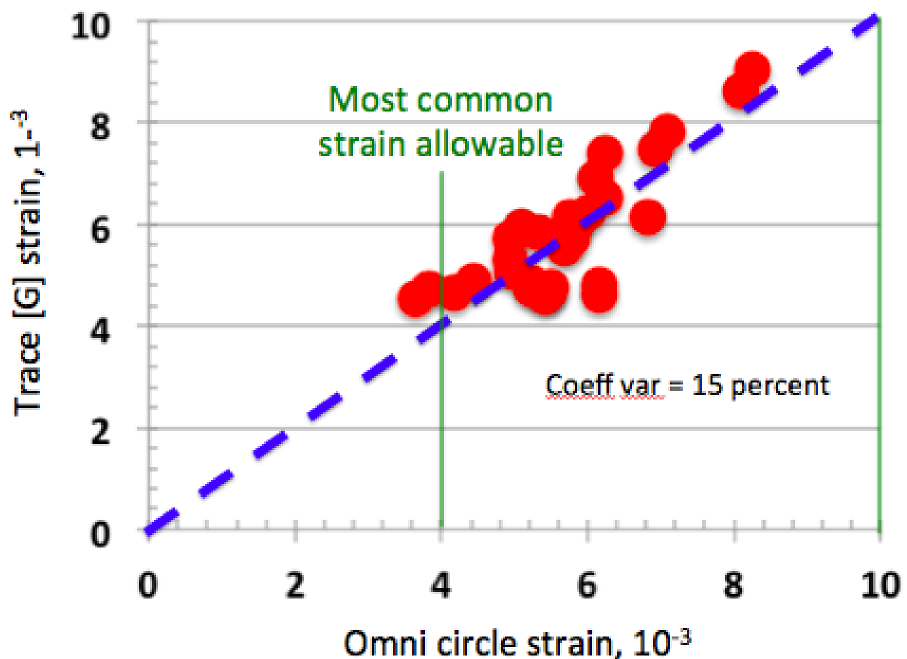


Omni Circle vs Tr [G] Circle

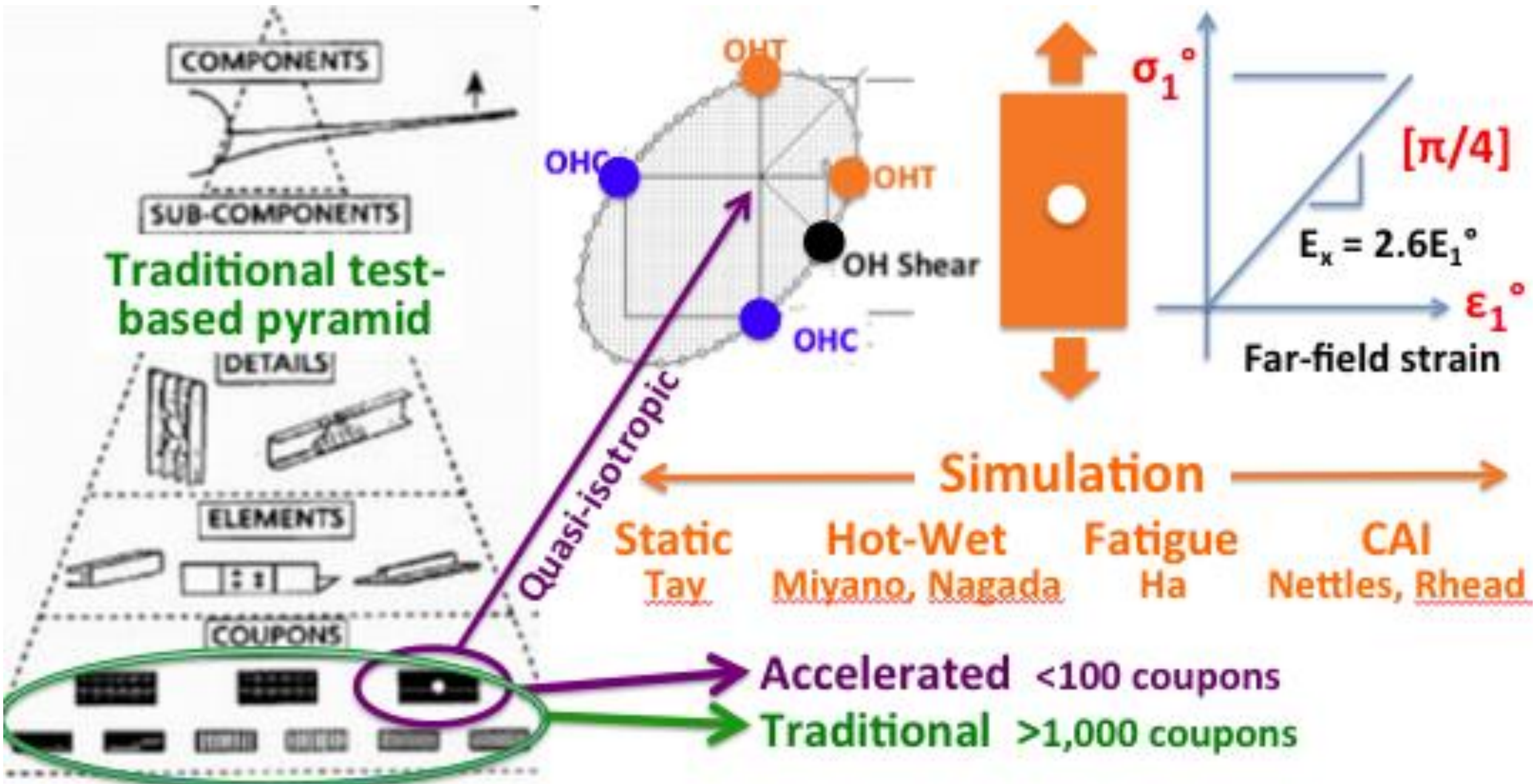




Need only [π/4] coupons
to define all CFRP laminates:
1) Trace [Q] = $E_1°/0.337$
2) Omni circle for strain
allowable from OHC to
include defect and damage



Accelerated Allowable Generation



Master Ply and its Laminates

- Plane stress stiffness [Q] is better represented by its invariant trace: $Q_{xx} + Q_{yy} + 2Q_{ss}$ - - a linear scaling factor
- When normalize by trace [Q*] plies and laminates are insensitive among many composite plies justifying a master ply
- The same invariance holds from ply to in-plane, and to flexure – to scale design is made easy
- Power of bi- and tri-angle tapes can save cost through 1- or 2-axis; increase CAI through 6-angle laminates
- Certification of asymmetric layup and homogenization of composite laminates can be accelerated with fewer coupons, and more simulation guided by invariants
- Recommend laminates with holes as test coupons

Opportunities in Composites Design

- Trust fundamental theories, like invariants for master ply, a one parameter for design
- Multi-angle tape layup can achieve >2X in speed and 6-angle laminates while limited to 1- or 2-axis layup, no more 4-axis
- Thin plies can increase toughness and homogenization - amenable to optimization, and ply angle used as a continuous variable
- Simulation will guide tests for hot-wet, fatigue, CAI, damage tolerance, and micromechanics
- Design allowable and certification can be simplified by testing laminates with open hole replacing smooth coupons of plies and laminates

ONSITE/ONLINE COMPOSITES DURABILITY WORKSHOP-19

Department of Aeronautics & Astronautics, Stanford University, July 27-29, 2014

Βρεακτηρουγη αδϖανχεσ τηατ χαν σιμπλιφψ δεσιγν αλλοωαβλε γενερατιον ανδ χερτιφιχατιον ωιλλ βε προποσεδ βψ ρεπρεσεντατιϖεσ οφ Στανφορδ, Ωιχηιτα Στατε ανδ οτηερ υνιϖερσιτιεσ, ενδ υσερσ, ανδ ΦΑΑ. Τηε πρινχιπλεσ οφ ινϖαριαντσ, πλυσ τιμε–τεμπερατυρε συπερποσιτιον ανδ μυλτι–σχαλε σιμυλατιονσ χαν χολλαπσε νεαρλψ αλλ χαρβον/πολψμερ χομποσιτεσ το ονε μαστερ πλψ ανδ ωιτη σχαλινγ ονε σινγλε τεστ ωιλλ συφφιχε φορ εαχη ματεριαλ σψστεμ ιν ταπεσ, φαβριχσ ορ βραιδσ. Ηελπ ιν υσινγ τοολσ ανδ τεμπλατεσ φορ ονε μαστερ πλψ ωιλλ βε αϖαιλαβλε δυρινγ ανδ αφτερ ωορκσηοπ. Ρεχεντ αδϖανχεσ ιν Ξ–Πλψ ον μανυφαχτυρινγ ανδ πιονεερινγ ρεσεαρχη τοπιχσ ωιλλ αλσο βε πρεσεντεδ. Ρεγιστρατιον φεε φορ ον–σιτε: ΥΣ∃800; φορ ον–λινε: ΥΣ∃400.

Ρεγιστρατιον φεε ινχλυδεσ αλλ πρεσεντατιονσ, λιϖε ανδ ρεχορδεδ, αλλ τοολσ ανδ τεμπλατεσ, ανδ α χοπψ οφ αλλ–νεω *Χomposite Materials Mechanics and Testing – An Invariant-based Approach* βψ Στεπηεν Τσαι ανδ Δανιελ Μελο (το βε μαιλεδ ιν θυαρτερ 4 τηισ ψεαρ). Ον–σιτε ρεγιστρατιον φεε ινχλυδεσ Συνδαψ ρεχεπτιον, χοφφεε βρεακσ, λυνχηεσ ανδ διννερσ, ασ ωελλ ασ βυσ τρανσπορτατιον. Σπουσε προγραμ αϖαιλαβλε φορ εξτρα φεε οφ ∃100.

Φορ ινφο/ρεγιστρατιον

web.stanford.edu/group/composites/cdw19