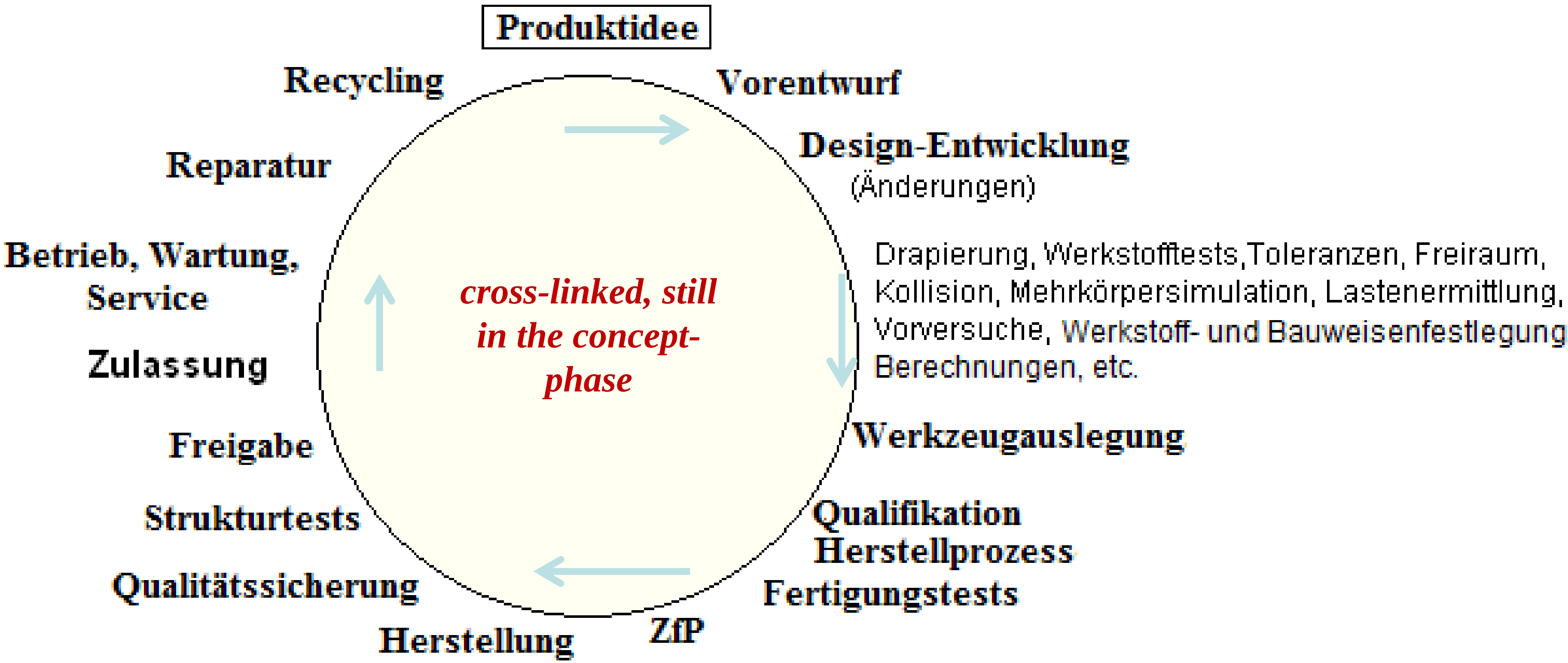
Process Chain in Product Development



Prüfbarkeit des Bauteils!

Simulationsgebundene Aufgaben ?
Überall

Ziele dabei:

* robuste Prozesse
* klare Verantwortlichkeiten
* durchgängiges Prozeßdatenmanagement (Monitoring, Haltung,Weitergabe)

Now*: Certification of structural parts is *dominated by tests, aided by analysis
Future:* Certification of struct. parts is *driven by models, substantiated by tests!

… in this context

Gute Bemessung und Nachweis,

dass eine Festigkeits-Grenze noch nicht erreicht ist

verlangt die Anwendung

validierter Festigkeitsbedingungen.

Dazu gehören

Fließbedingungen für nicht-lineare Analyse und Fließgrenzennachweis (duktiles Verhalten)

sowie

Bruchbedingungen für den Bruchnachweis

= Festigkeitsbedingungen *F* für Bruch.

Zugehörige Bruch-Festigkeitsbedingungen und deren Visualisierung als Bruchkörper ist Gegenstand des Vortrags !

Es gibt viele Bruchmodelle im Maschinenbau und im Bauwesen.

- **Doch welches soll man im isotropen oder transversal-isotr. Fall nehmen?**
- **Gibt es Nachteile bei den bekannten ‘isotropen‘ Werkstoff-Modellen Tresca, Drucker-Prager, oder den Betonmatrix-Modellen wie Ottosen, Drucker-Prager, Willem-Warnke etc. ?**

*** Wie bewertet man im Bauwesen anisotrope CFK-Lamellen (sheets besser tapes), wenn diese auch quer zur Faserrichtung beansprucht werden?**

Is there any Strength Failure Condition one may apply with fidelity for fracture?

Some well-known Developers which formulated isotropic 3D Strength Failure Conditions (SFCs)

Willam-Warnke, Ottosen etc.

Hencky-Mises-Huber

Richard von Mises
1883-1953
Mathematician

Eugenio Beltrami
1835-1900
Mathematician

Otto Mohr
1835-1918
Civil Engineer

Charles de Coulomb
1736-1806
Physician

'Onset of Yielding'

Henri Tresca, 1884-1885
Mechanical engineer

'Onset of Cracking'

Hence again, a civil engineer may proceed

CARBON COMPOSITES

Bruchversagenskörper nach Cuntzes 'Failure Mode Concept (FMC)' und ihre Anwendung bei der Auslegung von Bauteilen aus 'spröden' isotropen und UD-Composite-Werkstoffen

1 Introduction to Strength Failure Conditions (SFCs) **criteria**

2 Global SFCs versus Modal SFCs

3 Short Derivation of the Failure-Mode-Concept (FMC)

4 Materials and Material Properties

5 Application: Grey-cast Iron, Glass

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7 Application: Normal Concrete and Ultra High Performance Concrete (UHPC)

8 Application: Transversely-isotropic UD-CFRP

Conclusions

Results of a time-consuming, never funded „hobby"

Prof. Dr.-Ing. habil. Ralf Cuntze VDI, linked to Carbon Composite e.V.(CCeV) Augsburg

Formerly MAN-Technologie AG

What do we speak about ? *Definitions*

Material: homogenized (macro-)model of the envisaged solid

Failure: structural part does not fulfil its functional requirements such as onset of yielding, brittle fracture, Fiber-Failure FF, Inter-Fiber-Failure IFF, leakage, deformation limit, delamination size limit, frequency bound

= *project-fixed* Limit State with *F* = Limit State Function

Failure Criterion: **F** >=< 1, Failure **Condition** : *F* = 1= 100%

Failure Theory: general tool to predict failure of a structural part

Strength Failure Condition: subset of a strength failure theory

tool for the assessment of a

'multi-axial failure stress state ' in a critical location of the material.

→ **Stresses** are judged by **Strengths !**

Was beschreibt den Bruchkörper? Beispiel: *Isotroper Werkstoff*

<u>Bruchkörper</u>: Menge aller Spannungskombinationen = Beanspruchungszustände, die gerade noch nicht zum Bruch führen. Der Bruchkörper begrenzt den sich mit wachsender Beanspruchung vergrößerten Fließkörper (z.B. Mises-Zylinder)

<u>Bruchkörper-Oberfläche</u>: Fläche, auf der alle Bruch-Spannungskombinationen liegen. Sie wird mathematisch durch eine Bruch-Festigkeitsbedingung

$F(\underline{\sigma}, \underline{f})$ **beschrieben (f ist Festigkeit im Bauwesen = R Resistance, Widerstand).**

Trifft die Vektorspitze des anliegenden Beanspruchungszustands die Oberfläche, so ist einerseits $RF = 1$ und andererseits die Werkstoffanstrengung $Eff = 100\%$ (= 1).

Für jeden ‘Dimensionierenden Lastfall‘ mit seinen diversen Versagensmoden ist nachzuweisen, dass an beanspruchungskritischen Stellen des Werkstoffs gilt

Festigkeit > Beanspruchung bzw. $RF > 1$.

Liegt die Spitze des den Spannungszustand beschreibenden Vektors, gebildet aus den 3 Hauptspannungen, noch innerhalb des Bruchkörpers, so liegt noch eine Reserve vor und die Belastung kann noch um den sog. Reservefaktor RF gesteigert werden bis schließlich Bruch eintritt.

Was war meine Haupt-Motivation für die Untersuchungen?

Die erlebte Erfahrung :

Man soll nicht auf den Werkstoff schauen sondern auf das Werkstoff-Verhalten !

Damit ist es möglich, die mathematische Beschreibung der Form eines Bruchkörper-Modells von einem sich ähnlich verhaltenden, bereits *mehr-axia*l test-erprobten Werkstoffes zu übernehmen.

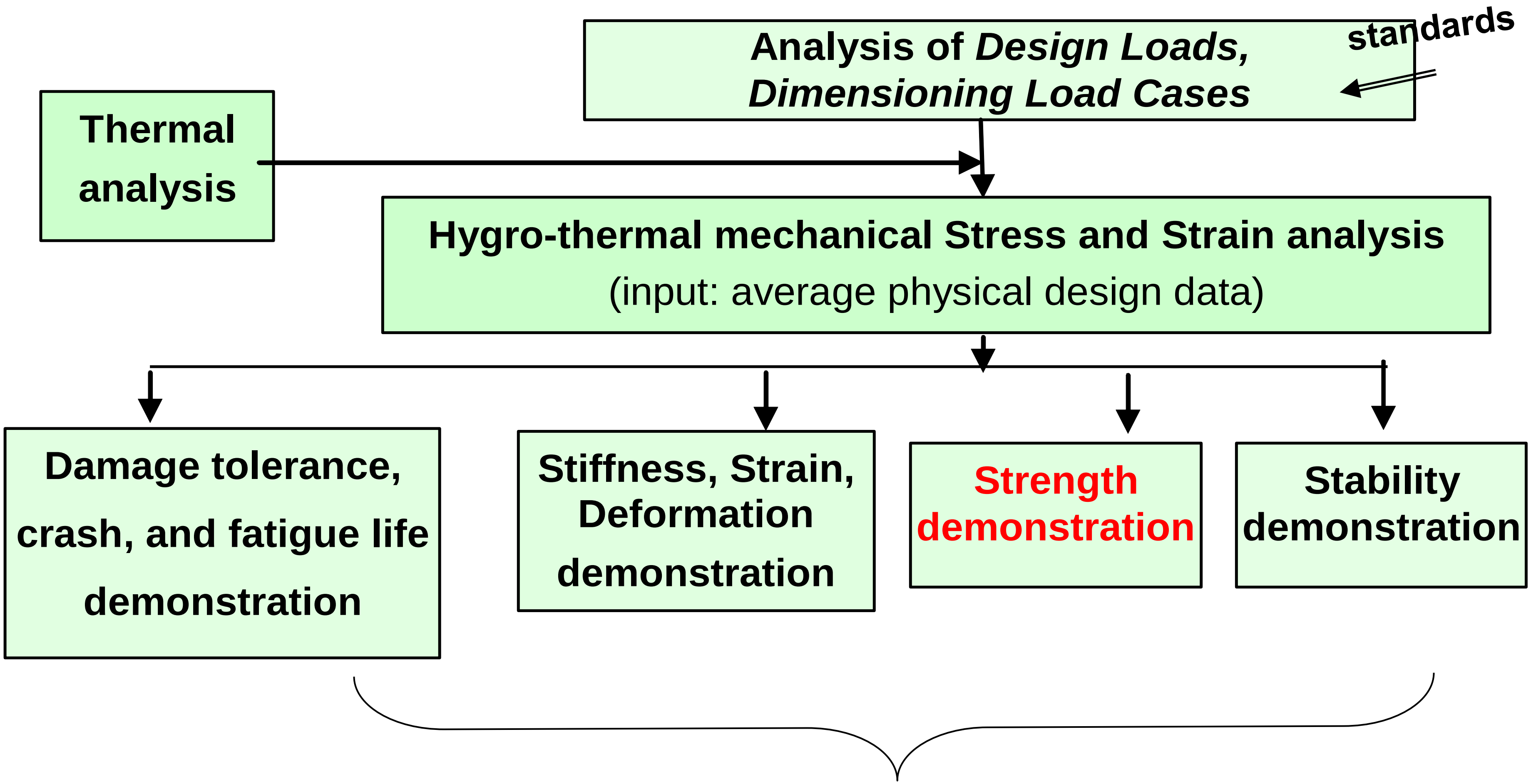
Beispiel, hier genutzt:

voll poröser Hebel-Baustein (Ytong) ähnlich porösem Sandwich-Schaum.

Ist das Bruchmodell bekannt, so wird die Größe des Bruchkörpers des sich ähnlich verhaltenden Werkstoffs - für den Tragfähigkeitsnachweis – nur noch mit dessen einfach zu messenden *ein-axialen* Festigkeiten festgelegt.

Which Design Verifications are mandatory in Structural Design ?

Nachweise



**Structural Resistances*, to be demonstrated*
by a *positive* Margin of Safety *(MoS),*
to proof Design Verification
***for achieving* Structural Integrity**

Design Verification = Achievement of a Reserve against a Limit State

For each designed structural part - for each distinct ‘Load Case‘ with its single Failure Modes - must be computed:

<u>R</u>eserve <u>F</u>actor (is load-defined) : ***RF = Failure Load / applied Design Load***

Material Reserve Factor : f_{Res} = ***Strength / Applied Stress***

if linear analysis: $f_{Res} = RF = 1 / Eff$

Material Stressing Effort : ***Eff = 100% if RF = 1*** **(Anstrengung)**

(Kunstwort, entspricht Werkstoff-Anstrengung)

is applicable in linear and non-linear analysis.

Eff **= 100% (n =1 cycle) →** ***D*** **= 100% (Wöhlerkurve, n >> 1) !!**

Eff : = accumulated static damaging portions under increased loading
D : = accumulated cyclic damaging portions (Schädigungen)

Test Data Mapping versus **Design Verification**

- **<u>Validation of SFC-Models with many Failure Test Data</u>** by

  mapping their course by an average Failure Curve (surface) based on very many experimental data

- **<u>Verification of the Design</u>** for the various Dimensiong Load Cases by

  calculation of a Margin of Safety or **a (load) Reserve Factor**

  $$MoS > 0 \quad oder \quad RF = MoS + 1 > 1$$

  on basis of a statistically reduced material failure curve

  and sometimes

  on one experiment ***(in civil engineering usually not)*****.**

Strength Failure Conditions are for homogenized materials

2K-Kleber ?

Prediction of ***Onset of Yielding*** + ***Onset of Fracture*** for non-cracked materials

Assessment of multi-axial stress states in a critical material location,

by **utilizing the uniaxial strength values R** and an
equivalent stress σ_{eq}, representing a distinct actual multi-axial stress state.

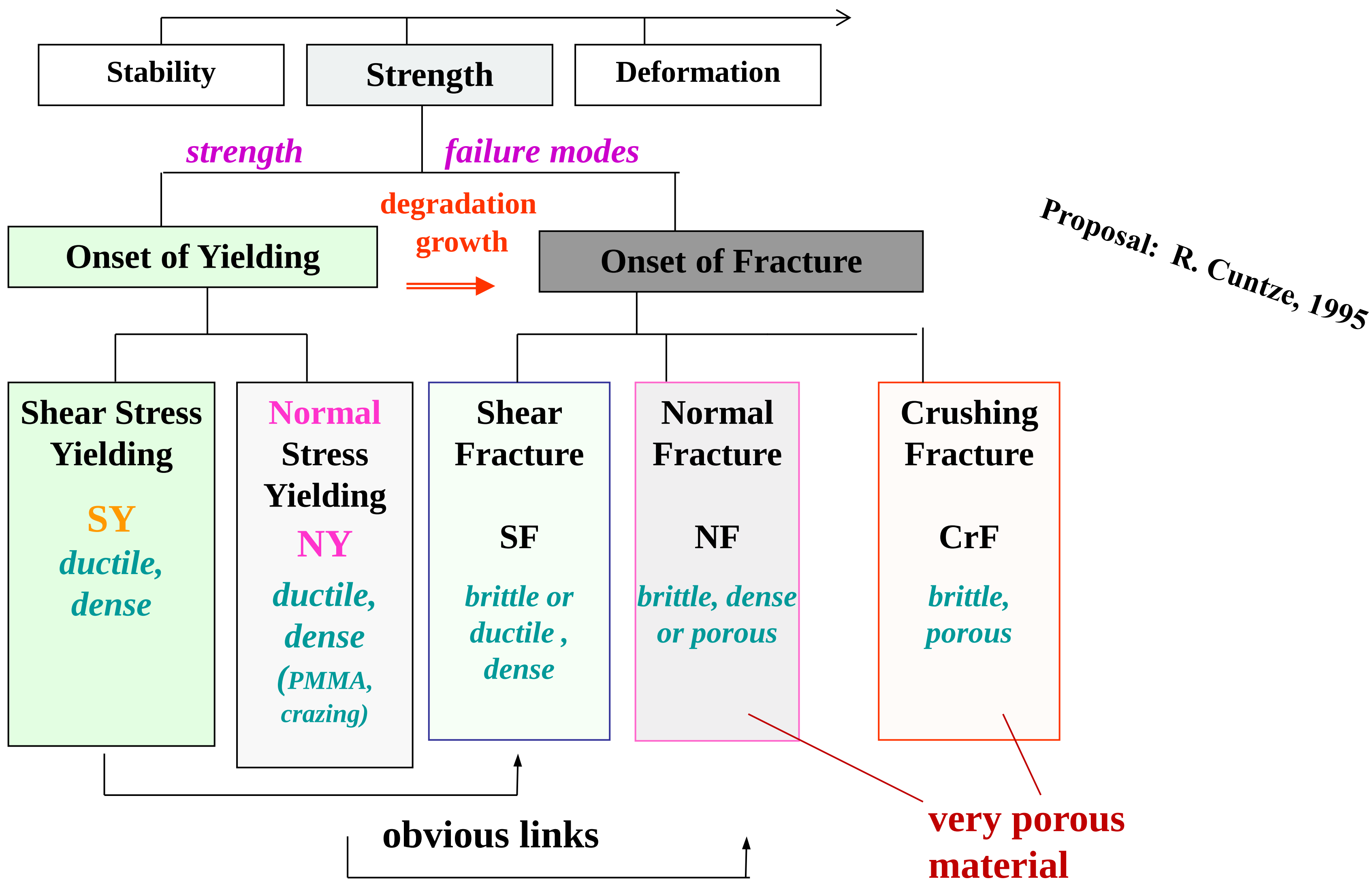
for *** dense & porous,**
*** ductile & brittle behaving materials,**

ductile : $R_{p0.2} \cong R_{c0.2}$ brittle, dense : $R_m^{\ c} \geq 3R_m^{\ t}$

for *** isotropic material**
*** transversally-isotropic material (UD := uni-directional material)**
*** rhombically-anisotropic material (fabrics) + ‘higher‘ textiles etc.**

Shall allow for inserting stresses from the utilized various coordinate systems into stress-formulated failure conditions - and if possible - invariant-based ones.

Scheme of Strength Failures Types for *isotropic materials*



Note: The growing yield body (SY or NY) is confined by the fracture surface (SF or NF)!

Assumptions for Material Modelling (Example: UD**)**

- **The UD-lamina is macroscopically homogeneous.**

  It can be treated as a homogenized (‘smeared‘) material

- **The UD-lamina is transversely-isotropic:**

  On planes, parallel to the fiber direction it behaves orthotropic and on planes transverse to fiber direction isotropic (quasi-isotropic plane)

- **Uniform stress state about the critical stress ‘point‘ (location)**

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.. .. da war noch eine weitere Motivation ?

Drucker-Prager, Ottosen, Willam-Warnke, Tsai u.a.

Globale verheiraten mathematisch alle Bruchmoden im Ansatz.
Nachteil: falls ein Festigkeitswert zu ändern ist, dann trifft es den ganzen Bruchkörper, wobei Teile des Bruchkörpers un-konservativ werden können, falls man den Verlauf aller Testdaten nicht wieder neu abbildet.

$R = f$

1 Globale Festigkeitsbedingung **:** **F ({σ}, {R}) = 1 (übliche Formulierung)**

Satz von Modalen Festigkeitsbed. **:** **F ({σ}, R^{mode}) = 1 (hier gewählt)**

Mises, Cuntze

Interaktion notwendig

Isotrop: $\{\sigma\} = (\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy})^T = (\sigma_I, \sigma_{II}, \sigma_{III})^T$

UD: $\{\sigma\} = (\sigma_1, \sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21})^T$

Modale Festigkeitsbedingungen betrachten alle Modi getrennt:

Nachteil (klein): bedingt dann natürlich eine Interaktion aller Modi

Vorteile : Festigkeitswert-Änderung betrifft nur einen Modus

+ Vergleichsspannungen σ_{eq} berechenbar !

Facts of so-called Global SFCs

Global SFCs (one failure surface)

- **Regard all failure modes of the material by one single mathematical formulation. This might even capture a (simplified view)**

  *** 2-fold acting failure mode (such as $\sigma_I = \sigma_{II}$: *is a joint failure probability*) or a**

  *** 3-fold acting failure mode (such as $p_{hyd} = \sigma_I = \sigma_{II} = \sigma_{III}$)**

- **Requires a re-calculation of all model parameters in the case that a test data change must be performed in a distinct failure mode domain of the multi-fold failure surface (body).**

  Consequence: A change in one failure domain deforms the failure surface in all other – physically independent – failure domains. There is a big chance that a Reserve Factor, to be determined in the independent domain, might be not on the conservative side

- **There are global SFCs that just use basic strengths *R* as model parameters. This is physically not permitted because Mohr-Coulomb friction requires - in the case of compression loaded, brittle behaving materials – a friction value μ.**

***Note*: a distinct failure mode can cause different failure “planes“ , which is maximum flaw driven!**

Facts of so-called Modal SFCs

Modal SFCs (multi-suface domains)

- **Describe one single failure mode in one single mathematical formulation (= one part of the full failure surface)**

  *** determine all mode model parameters in the respective failure mode domain**

  *** capture a twofold acting failure mode separately, such as $\sigma_I = \sigma_{III}$ (isotropic) or $\sigma_2 = \sigma_3$ (transversely-isotropic UD material), mode-wise by the well-known Ansatz f (J_2, J_3)**

- **Re-calculation of the model parameters just in that failure mode domain where the test data must be replaced. Only one RF_{mode} must be freshly determined.**

Equivalent Stress σ_{eq}:

(1) A stress value, combining effects of those stresses that are active in a distinct failure mode.
Examples: von Mises equivalent stress in case of the shear yielding failure mode and
(maximum principal stress) in case of a brittle tensile fracture failure mode NF.

(2) The uni-axial equivalent stress σ_{eq}-value (in German termed 'Vergleichsspannung') can be compared to a mode-associated basic strength R of the of the activated failure mode.

Bildung von Vergleichsspannungen

Hilfreich für den Ingenieur ist die Bereitstellung von *Vergleichsspannungen* mit Nutzung der ***Werkstoffanstrengung Eff***

moduszugehöriger Mittelwert der Festigkeit

$f_{cm} = \bar{R}_c$ ***f ≡ R*** **(*resistance*) zu nehmen fürs 'mapping'**

$$Eff^{mode} = \sigma_{eq}^{mode} / \bar{R}^{mode}$$

$$Eff^{fracture\ mode} = \sigma_{eq}^{fracture\ mode} / R_m$$

$$Eff^{Mises} = \sigma_{eq}^{Mises} / R_{po.2}$$

Bei modalen Festigkeitsbedingungen immer - klar definierbar - möglich !

Interaction of Single Strength Failure Modes in the modal FMC

Interaction of adjacent Failure Modes by a *series failure system* model

= 'Accumulation' of interacting *failure danger portions* Eff^{mode}

$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots} \quad = 1 = 100\%, \text{ if failure}$$

with mode-interaction exponent *2.5 < m < 3 from mapping experience*

***as modal* material stressing effort * (in German Werkstoffanstrengung)**

and

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / \bar{R}^{\text{mode}}$$

equivalent mode stress — mode associated average strength

Later: example

Interaktion der (Bruch-)Versagensmodi

= 'Akkumulation' der Anstrengungen = Summe der Bruchgefahranteile

*** artificial technical term, created together with QinetiQ during the World-Wide-Failure-Exercise**

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Failure Theory and Failure Conditions

A **3D Failure Theory** has to include:

1. Failure Conditions to ***assess multi-axial states of stress***
2. Non-linear Stress-strain Curves of a material as input
3. Non-linear Coding for structural analysis

A Failure Condition is the mathematical formulation of the failure surface !

Pre-requisites for the establishment of failure conditions are:

- simply formulated, numerically robust,

- physically-based, and therefore, need only few information for pre-dimensioning

- shall allow for a simple determination of the design driving reserve factor.

Physically-based Choice of Invariants when generating invariant-based Strength Failure Conditions

* Beltrami : "At 'Onset of Yielding' the material possesses a distinct *strain energy* composed of *dilatational energy* (I_1^2) and *distortional energy* ($J_2 \equiv Mises$) ".

* So, from Beltrami, Mises (HMH), and Mohr / Coulomb (friction) can be concluded:

Each invariant term in the *failure function F* may be dedicated to one physical mechanism in the solid = cubic material element:

- volume change : I_1^2 … *(dilatational energy)* — relevant if porous
- shape change : J_2 (Mises) … *(distortional energy)* — relevant if brittle behaving

and - friction : I_1 … *(friction energy)*

Mohr-Coulomb (→ friction)

relevant if material element shape changes (→ distortional energy)

- J_3 … als mathematisch elegante Ansatzfunktion, um die bekannten Dellen oder Auswölbungen des Bruchkörpers einfach beschreiben zu können.

Material Symmetry Requirements Aspects *(helpful, when generating SFCs*

1 If a material element can be homogenized to an ideal (= frictionless) crystal,

then, material symmetry demands for the transversely-isotropic UD-material

- 5 *elastic 'constants' , 5 strengths, 5 fracture toughnesses* (CF-lamellen) and

- *2 physical parameters (such as CTE, CME, material friction, etc.)*

(for isotropic materials the respective numbers are 2 and 1)

2 Mohr-Coulomb requires for the real crystal another inherent parameter,

- the *physical parameter 'material friction' : UD* $\mu_{\perp\parallel}$, $\mu_{\perp\perp}$, *Isotropic* μ

3 Fracture morphology witnesses:

- Each strength corresponds to a distinct *failure mode*

and to a *fracture type* such as Normal Fracture (NF) or Shear Fracture (SF).

Above Facts and Knowledge gave reason

why the FMC strictly employs single *independent* failure modes

by its failure mode–wise concept.

Basic Features of the author's Failure-Mode-Concept (FMC)

- **Each failure mode represents 1 independent failure mechanism and thereby 1 piece of the complete *failure surface***
- **Each failure mechanism is governed by 1 basic strength (is observed !)**
- **Each failure *mode* can be represented by 1 failure *condition.***

  Therefore, equivalent stresses can be computed for each mode !!

- **In consequence, this separation of failure modes requires :**

  An interaction of the Modal Failure Modes !

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Isotropic Material (for FOAM**)** *brittle behaviour, dense consistency*

Which failure types are observed ?

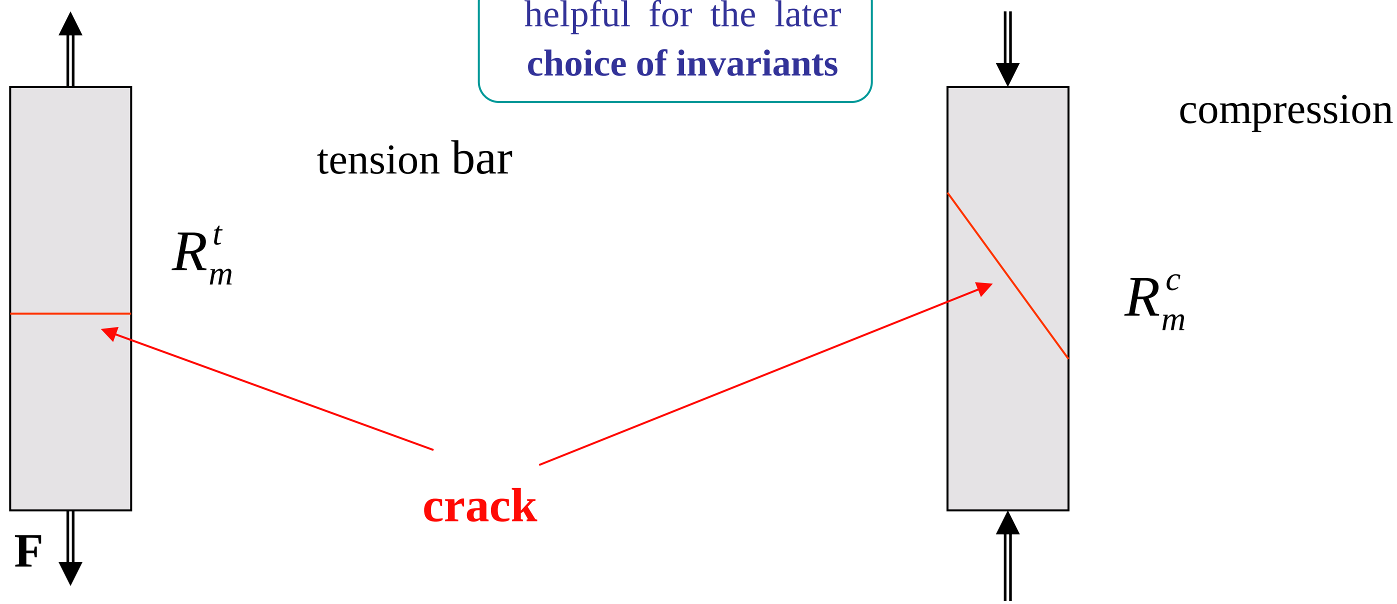
Cleavage fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- ‘smooth’ fracture surface

Shear fracture (SF) :

- shear deformation before fracture

knowledge is helpful for the later **choice of invariants**



conclusion: ▶ **2 strengths** to be measured

if brittle: failure = fracture failure

Isotropic Material *brittle*, *porous* *material*

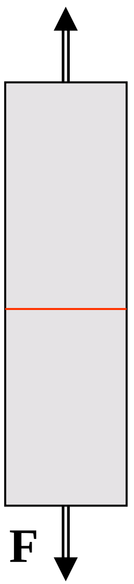
Normal Fracture (NF) (Spaltbruch, Trennbruch) :

- poor deformation before fracture
- rough fracture surface

Crushing Fracture (CrF): ⇐ **SF**

- volumetric deformation before fracture

helpful for the l
choice of invariants



Tension

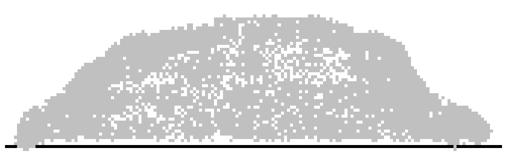
R_m^t

F

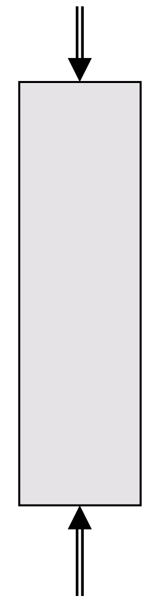
Compression

result of the compression test

= *hill of fragments* (crumbs)



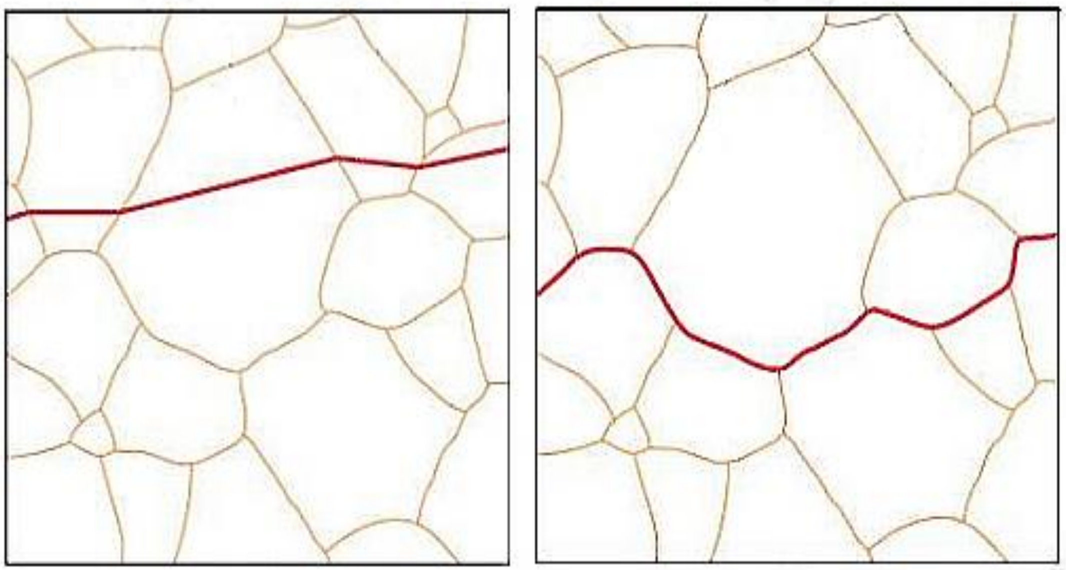
= decomposition of texture



R_m^c

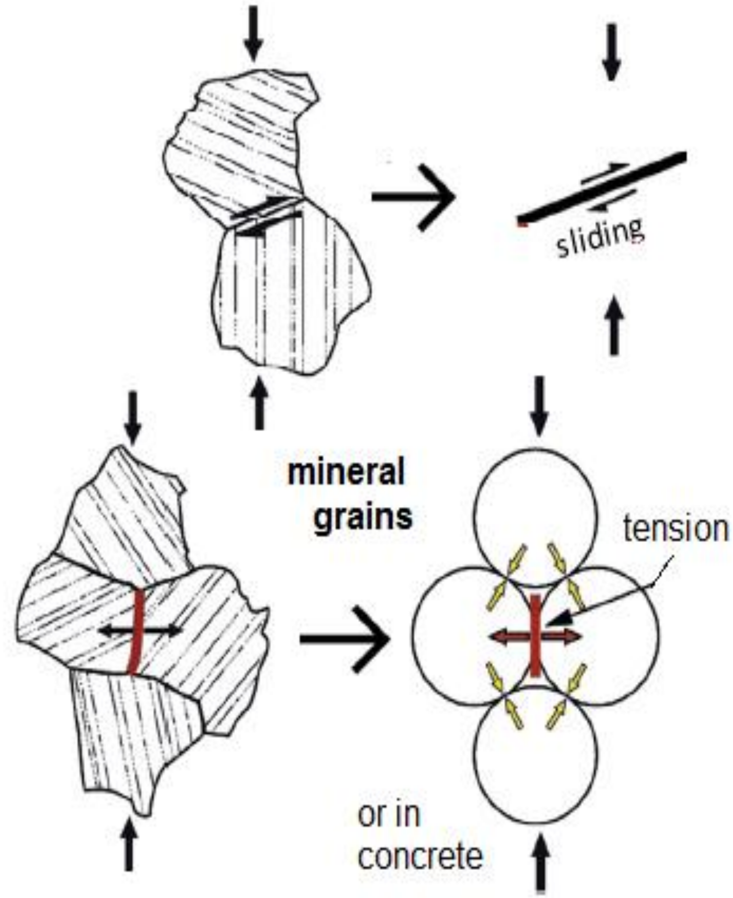
▶ **2 strengths** to be measured

<u>Texture Influence</u>: Inter-granular and trans-granular fracture under tension; fracture in mineral grains under compression

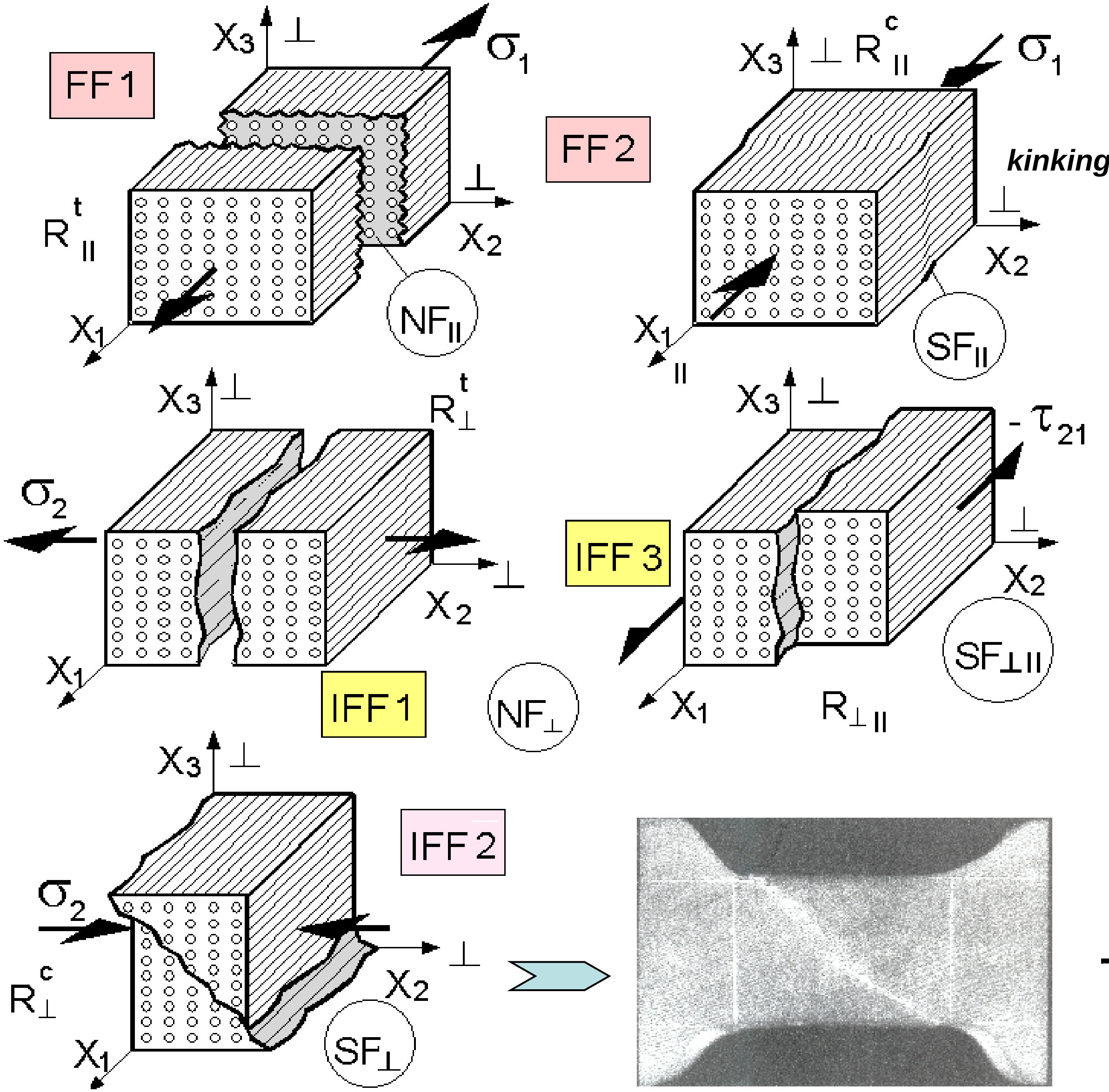


transgranular fracture

intergranular fracture



Observed Strength Failure Modes with Strengths of brittle UD Materials

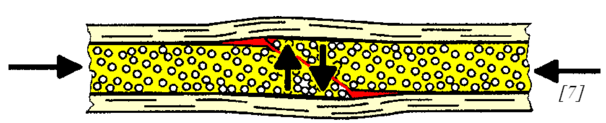


t = tension
c = compression

► **5 Fracture modes exist**

= 2 FF (Fibre Failure)
+ 3 IFF (Inter Fibre Failure)

Fracture Types:
NF := Normal Fracture
SF := Shear Fracture



wedge failure type

Material Properties *(self-explaining denotations)*

Elasticity Properties of the *homogenised material*

		Elasticity Properties									
	direction or plane	1	2	3	12	23	13	12	23	13	
9	***general orthotropic***	E_1	E_2	E_3	G_{12}	G_{23}	G_{13}	ν_{12}	ν_{23}	ν_{13}	**comments**
5	***UD, ≅ non-crimp fabrics***	$E_{\|\|}$	$E_{\perp}$	$E_{\perp}$	$G_{\|\|\perp}$	$G_{\perp\perp}$	$G_{\|\|\perp}$	$\nu_{\|\|\perp}$	$\nu_{\perp\perp}$	$\nu_{\|\|\perp}$	$G_{\perp\perp} = E_{\perp} /(2+2\nu_{\perp\perp})$ $\nu_{\perp\|\|} = \nu_{\|\|\perp} \cdot E_{\perp} / E_{\|\|}$ *quasi-isotropic 2-3-plane*
6	***fabrics***	E_W	E_F	E_3	G_{WF}	G_{W3}	$G_{\|W3}$	ν_{WF}	ν_{W3}	ν_{W3}	*Warp = Fill*
9	***fabrics general***	E_W	E_F	E_3	G_{WF}	G_{W3}	G_{F3}	ν_{WF}	ν_{F3}	ν_{W3}	*Warp ≠ Fill*
5	***mat***	E_M	E_M	E_3	G_M	G_{M3}	G_{M3}	ν_M	ν_{M3}	ν_{M3}	$G_M = E_M/(2+2\nu_M)$ *1 is perpendicular to quasi-isotropic mat plane*
2	***isotropic*** *for comparison*	E	E	E	G	G	G	ν	ν	ν	$G=E/(2+2\nu)$

Lesson Learned: - *Unique, self-explaining denotations are mandatory*
- *Otherwise, expensively generated test data cannot be interpreted and go lost*

Hygrothermal Properties of *homogenised material*

		Hygro-thermal properties						
	direction	1	2	3	1	2	3	
9	general orthotropic	α_{T1}	α_{T2}	α_{T3}	α_{M1}	α_{M2}	α_{M3}	comments
5	UD, ≅ non-crimp fabrics	$\alpha_{T\|\|}$	$\alpha_{T\perp}$	$\alpha_{T\perp}$	$\alpha_{M\|\|}$	$\alpha_{M\perp}$	$\alpha_{M\perp}$	
6	fabrics	α_{TW}	α_{TW}	α_{T3}	α_{MW}	α_{MW}	α_{M3}	*Warp = Fill*
9	fabrics general	E_W	E_F	E_3	α_{MW}	α_{MF}	α_{M3}	*Warp ≠ Fill*
5	mat	α_{TM}	α_{TM}	α_{TM3}	α_{MM}	α_{MM}	α_{MM3}	
2	isotropic for comparison	α_T	α_T	α_T	α_M	α_M	α_M	

NOTE: Despite of annoying some people, I propose to rethink the use of α for the CTE and β for the CME.
Utilizing α_T and α_M automatically indicates that the computation procedure will be similar.

Self-explaining, symbolic Notations for Strength Properties

		Fracture Strength Properties									
	loading	**tension**			**compression**			**shear**			
	direction or **plane**	1	2	3	1	2	3	12	23	13	
9	**general orthotropic**	R_1^t	R_2^t	R_3^t	R_1^c	R_2^c	R_3^c	R_{12}	R_{23}	R_{13}	**friction**
5	**UD**	$R_{\parallel}^t$ NF	$R_{\perp}^t$ NF	$R_{\perp}^t$ NF	$R_{\parallel}^c$ SF	$R_{\perp}^c$ SF	$R_{\perp}^c$ SF	$R_{\parallel\perp}$ SF	$R_{\perp\perp}$ NF	$R_{\parallel\perp}$ SF	$\mu_{\perp\perp}, \ \mu_{\perp\parallel},$
6	**fabrics**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	*Warp = Fill*
9	**fabrics general**	R_W^t	R_F^t	R_3^t	R_W^c	R_F^c	R_3^c	R_{WF}	R_{F3}	R_{W3}	$\mu_{W3}, \ \mu_{F3}, \ \mu_{WF}$
5	**mat**	R_{1M}^t	R_{1M}^t	R_{3M}^t	R_M^c	R_{1M}^c	R_{3M}^c	R_M^τ	R_M^τ	R_M^τ	*(UD, turned direction)*
2	**isotropic matrix**	R_m SF	R_m SF	R_m SF	*deformation-limited*			R_M^τ	R_M^τ	R_M^τ	μ
		R_m NF	R_m NF	R_m NF	R_m^c SF	R_m^c SF	R_m^c SF	R_m^σ NF	R_m^σ NF	R_m^σ NF	μ

<u>NOTE</u>: *As a consequence to isotropic materials (European standardisation) the letter R has to be used for strength. US notations for UD material with letters X (direction 1) and Y (direction 2) confuse with the structure axes' descriptions X and Y . *Effect of curing-based residual stresses and environment dependent on hygro-thermal stresses. *Effect of the difference of stress-strain curves of e.g. the usually isolated UD test specimen and the embedded (redundancy) UD laminae. R_m := 'resistance maximale' (French) = tensile fracture strength (superscript t here usually skipped), R:= basic strength. Composites are most often brittle and dense, not porous! SF = shear fracture

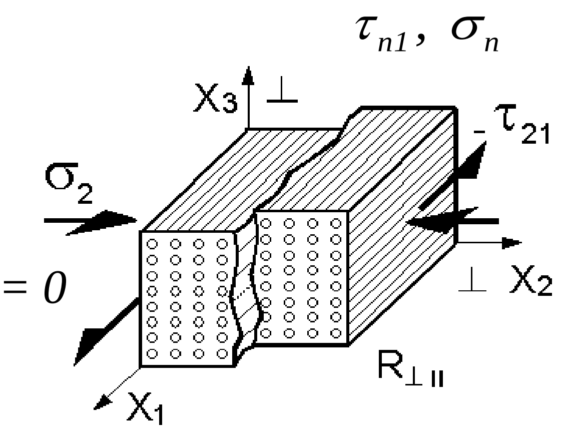
Additionally Required Material Information

Example UD: 2 Material internal Friction Parameters (brittle behaviour)

$$\tau_{21} = R_{\perp\parallel} - b_{\perp\parallel} \cdot \sigma_2$$: *FMC corresponds*

IFF 3 : $$\tau_{n1} = R_{\tau}^{\perp||} - \mu_{\perp||} \cdot \sigma_n$$: *Mohr*

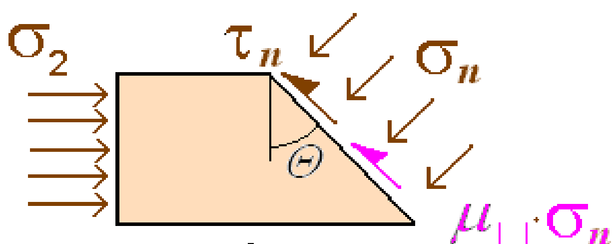
cohesion strength — *material internal friction coefficient*



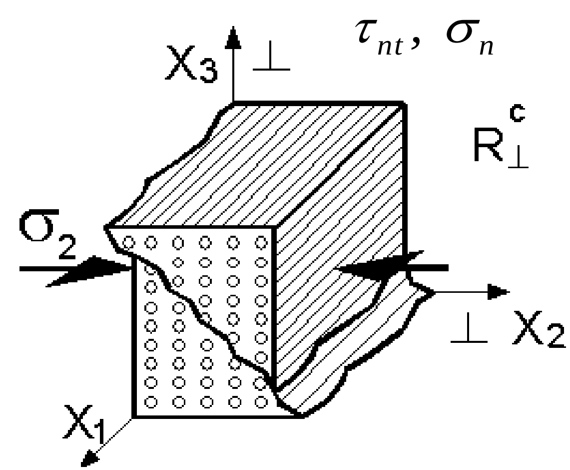
$\Theta_{fp} = 0$

Linear **Mohr-Coulomb approach + denotation**

IFF 2 : $$\tau_{nt} = R_{\tau}^{\perp\perp} - \mu_{\perp\perp} \cdot \sigma_n$$



$$\sigma_n = \cos^2\Theta \cdot \sigma_2$$
$$\tau_n = -\sin\Theta \cdot \cos\Theta \cdot \sigma_2$$

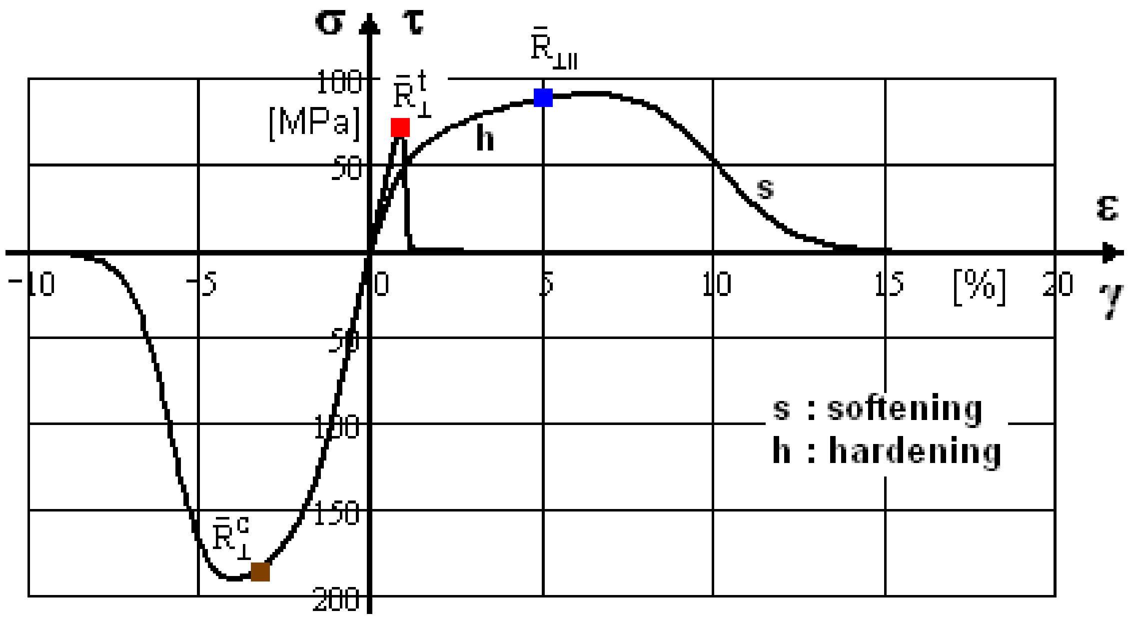


$\Theta_{fp} \geq 45°$

real material = crystal + friction

UD material: 2; isotropic material: 1

UD lamina (ply): Isolated and embedded Properties



3D-analysis,

non-linear

Test results from :

'Isolated' lamina test specimens = weakest link results (series failure system)
(hardening domain) **= used as material property *input* !**

'Embedded' laminas experience in-situ effects = redundancy result (parallel fail. system)
(softening domain) **= used in non-linear analysis**

Additionally Required Material Information

Example UD: Micro-mechanical Properties

Some lamina analyses require a micro-mechanical input, but not all micro-mechanical properties can be measured :

Solution: *Micro-mechanical equations are calibrated by macro-mechanical test results (lamina level) = an inverse parameter identification*

Condition: *micro-mechanical properties can be used only together with the equations they have been determined with.*

Micro-mechanical formulas applied in:

Elasticity domain: may be helpful tools (new formulas)

Strength domain : attempted, but not yet successful.

Alle benötigten Werstoffkennwerte und Modellparameter sollten physikalisch erklärbar und eindeutig messbar sein.

Messergebnisse,

als das Ergebnis einer Prüfvereinbarung (Norm, Standard), dienen der Vergleichbarkeit verschiedener Untersuchungen.

Die Prüfvereinbarung besteht aus Prüfeinrichtung, Prüfvorschrift, Probekörper und Auswerteverfahren.

Strength Design Values & Strength Design Allowables (Airbus, LTH)

Material Supplier	**Customer**			
Manufacturer *1* raw data, T99 / T90 data	**In-house tests** raw data, T99 / T90 data	**Pooling of T data,** **S-value adjustment, Material Procurement** **Determination of *Strength Design Values***	**Determination of *Strength Design Allowables*** *(A-, B-values)* based on statistical rules in MMPDS Hdbk (formerly MIL Hdbk 5)	**approval** by handbook committee , agency etc.
Manufacturer *2* raw data, T99 / T90 data				
Manufacturer *n* raw data, T99 / T90 data				
AIRBUS/IASB-Discussion HSB		for **design + analysis**	for **design verification**	

S-value: Procurement value
A-, B-value: Strength *Design Allowables. Statistically defined like T99/T90 –values. Number of different batches is required, on top.*
T99/T90-values: *Material* strength allowables. The determination follows the same statistical procedure as with the Strength Design Allowables. However, the data volume and batch requirements are less stringent.
A > S, only allowed if premium selection of material is applied. Normally ***A* < *S*.**

Material Homogenizing (smearing) + **Modelling**

Material symmetry shows:

Number of strengths ≡ number of elasticity properties !

Application of material symmetry knowledge:

- *Requires that homogeneity is a valid assessment for the task-determined model ,*

but, if applicable

- A *minimum number of properties has to be measured, only* (cost + time benefits) !

Übersicht Normungsaktivitäten im Bereich Composites Germany

DIN – Deutsches Institut für Normung e.V.

Nationale Gremien

CEN – European Committee for Standardization / Comité Européen de Normalisation / Europäisches Komitee für Normung

Europäische Gremien

ISO – International Organization for Standardization

Internationale Gremien

Gesamtstruktur der Normungsgremien im Bereich Composites Germany

Europäische Normung und nationales Spiegelgremium

CEN/TC 138
Zerstörungsfreie Prüfung
Sekretariat : AFNOR

CEN/TC 138/WG 2 Ultraschallprüfung Sekretariat : DIN	**NMP** **NA 062-08-23 AA** Ultraschallprüfung
CEN/TC 138/WG 7 Schallemissionsprüfung Sekretariat : ASI	**NMP** **NA 062-08-23 AA** Ultraschallprüfung
CEN/TC 138/WG 8 Sichtprüfung Sekretariat : BSI	**NMP** **NA 062-08-27 AA** Visuelle und thermografische Prüfung
CEN/TC 138/WG 11 Infrarot- und thermografische Prüfung Sekretariat : DIN	**NMP** **NA 062-08-27 AA** Visuelle und thermografische Prüfung

CEN/TC 184
Hochleistungskeramik

Internationale Normung und nationales Spiegelgremium

ISO/TC 61
Kunststoffe
Sekretariat : SAC

ISO/TC 61/SC 2 Mechanische Eigenschaften Sekretariat : AENOR	**FNK** **NA 054-01-02 AA** Mechanische Eigenschaften und Probekörperherstellung
ISO/TC 61/SC 2/WG 1 Allgemeine Eigenschaften Sekretariat : DIN	**FNK** **NA 054-01-02 AA** Mechanische Eigenschaften und Probekörperherstellung
ISO/TC 61/SC 2/WG 2 Mechanische Eigenschaften von Folien Sekretariat : DSM	**FNK** **NA 054-01-02 AA** Mechanische Eigenschaften und Probekörperherstellung
ISO/TC 61/SC 2/WG 3 Zugeigenschaften Sekretariat : ANSI	**FNK** **NA 054-01-02 AA** Mechanische Eigenschaften und Probekörperherstellung
ISO/TC 61/SC 2/WG 4 Eindruckhärte Sekretariat : ANSI	**FNK** **NA 054-01-02 AA** Mechanische Eigenschaften und Probekörperherstellung

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Conclusions

Ziele und Ausführung (Beispiel isotrop, anisotrop analog)

1. Aufstellung einer geschlossenen Ansatzfunktion für die Bruchkörper-Oberfläche beschreibende Festigkeitsbedingung F = 1
2. Kein „Fitten“ der Testdaten auf Zugmeridian und Druckmeridian. *Die Meridian-Kurven ergeben sich aus der geschlossenen Ansatzfunktion*
3. Signifikante Modellparameter seien klassisch messbare Größen. Diese sind Festigkeiten *f (= R)* und bei sprödem Verhalten Reibung(en) *μ*

- ✓ Aufstellung von Bruch-Festigkeitsbedingungen unter Verwendung von Invarianten (analog zu *v. Mises*), die einem physikalischen Mechanismus des Werkstoffelementes zuordenbar sind.

$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = f(\sigma) \,, \qquad 6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = f(\tau)$$

$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III}) \cdot (2\sigma_{II} - \sigma_I - \sigma_{III}) \cdot (2\sigma_{III} - \sigma_I - \sigma_{II})$$

Invariante: *Kombination von Spannungen – potenziert oder nicht-potenziert – dessen Wert sich bei Änderung des Koordinatensystems nicht ändert.*
σ sind die Hauptspannungen

Wie baut man nach Cuntzes „Failure-Mode-Concept“ Festigkeitsbedingungen auf ?

Cuntzes 3D-Ansatz im Druckbereich *I1 < 0*

Ansatzfunktion komplett für isotrope Betonmatrix:

Gestaltänderung — Reibung μ — Volumenänderung

$$F = c_{1\tau} \cdot \frac{3J_2 \cdot \Theta_\tau}{\overline{R}^{c\,2}} + c_{2\tau} \cdot \frac{I_1}{\overline{R}^{c}} + c_{3\tau} \cdot \frac{I_1^{\,2}}{\overline{R}^{c\,2}} = 1$$

Hier auf Druckfestigkeit normiert !

koaxialer ‘Mises‘-Zylinder

$$c_{2\tau} = f(\mu)$$

$\nu \approx 0.2$ NB-Gefahr

$\nu \cong 0$ Porosität, Bröselbruch

Nicht-Koaxialitätsparameter
(120°-Symmetrie isotroper Werkstoffe, 3 gleichwertige Hauptspannungsrichtungen)

$$\Theta_\tau = \sqrt[3]{1 + D_\tau \cdot \sin(3\theta)} = \sqrt[3]{1 + D_\tau \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}} \qquad \theta = \text{meridian angle}$$

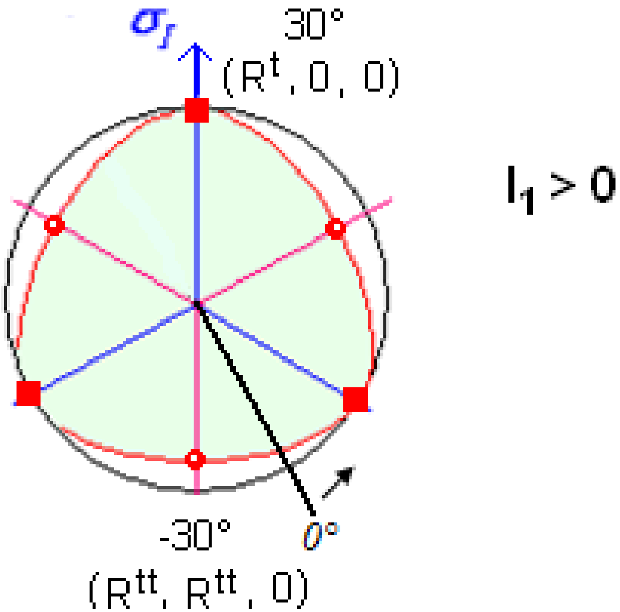
$$I_1 = (\sigma_I + \sigma_{II} + \sigma_{III})^T = f(\sigma)\,, \qquad 6J_2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = f(\tau)$$

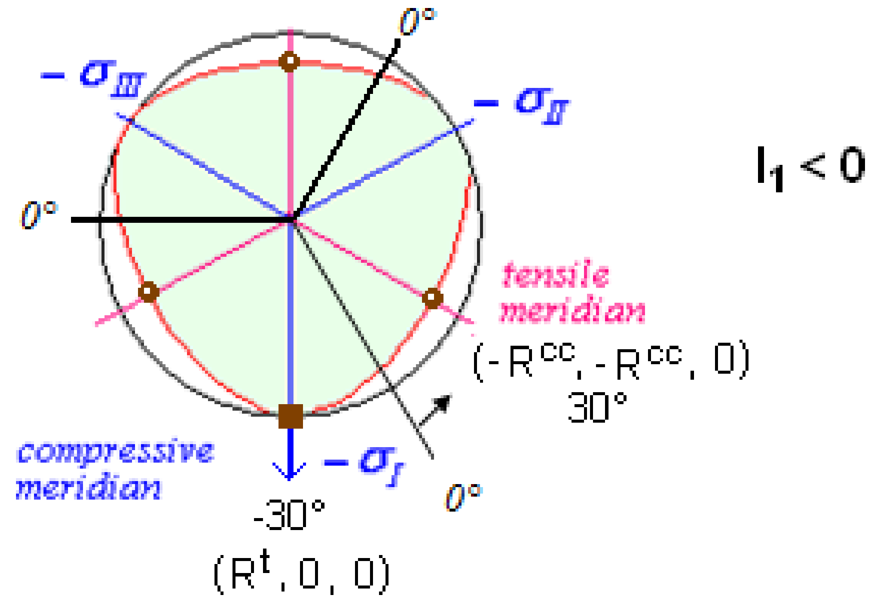
$$27J_3 = (2\sigma_I - \sigma_{II} - \sigma_{III}) \cdot (2\sigma_{II} - \sigma_I - \sigma_{III}) \cdot (2\sigma_{III} - \sigma_I - \sigma_{II})$$

Bruchwinkel θ_{fp} liefert: duktil 45° (μ=0, Gleitebenen), spröd 45°< 50° (μ= 0.174, Bruchebene), 55° (μ=0.309).
Empfohlen : 0.1 < μ < 0.2 (der kleinere Wert ist auf der konservativen Seite)

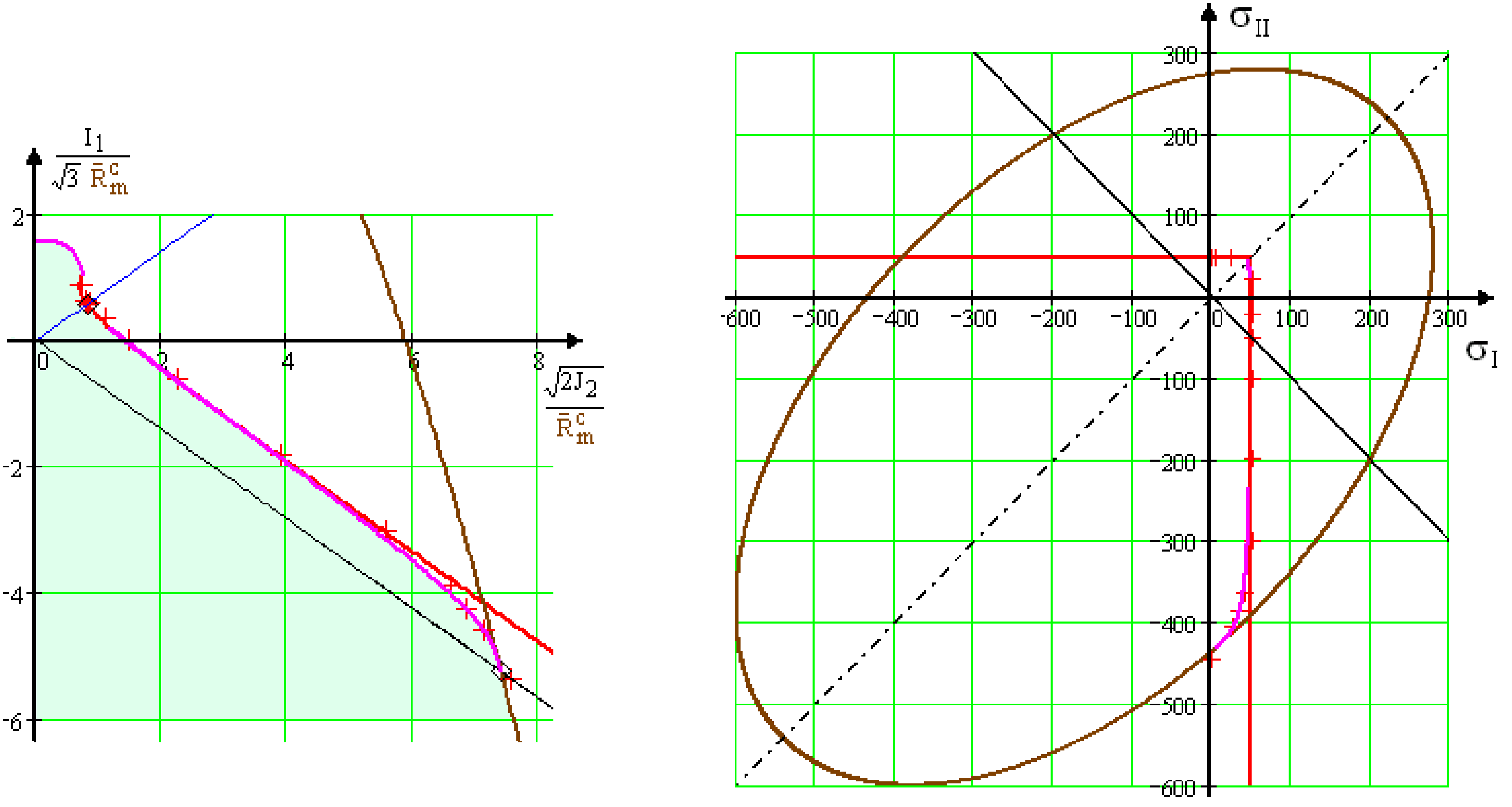
“A SCF principally describes a one-fold occurring failure mode” !

Meridians, dents and Lode (meridian) angles around the 120°-hoop

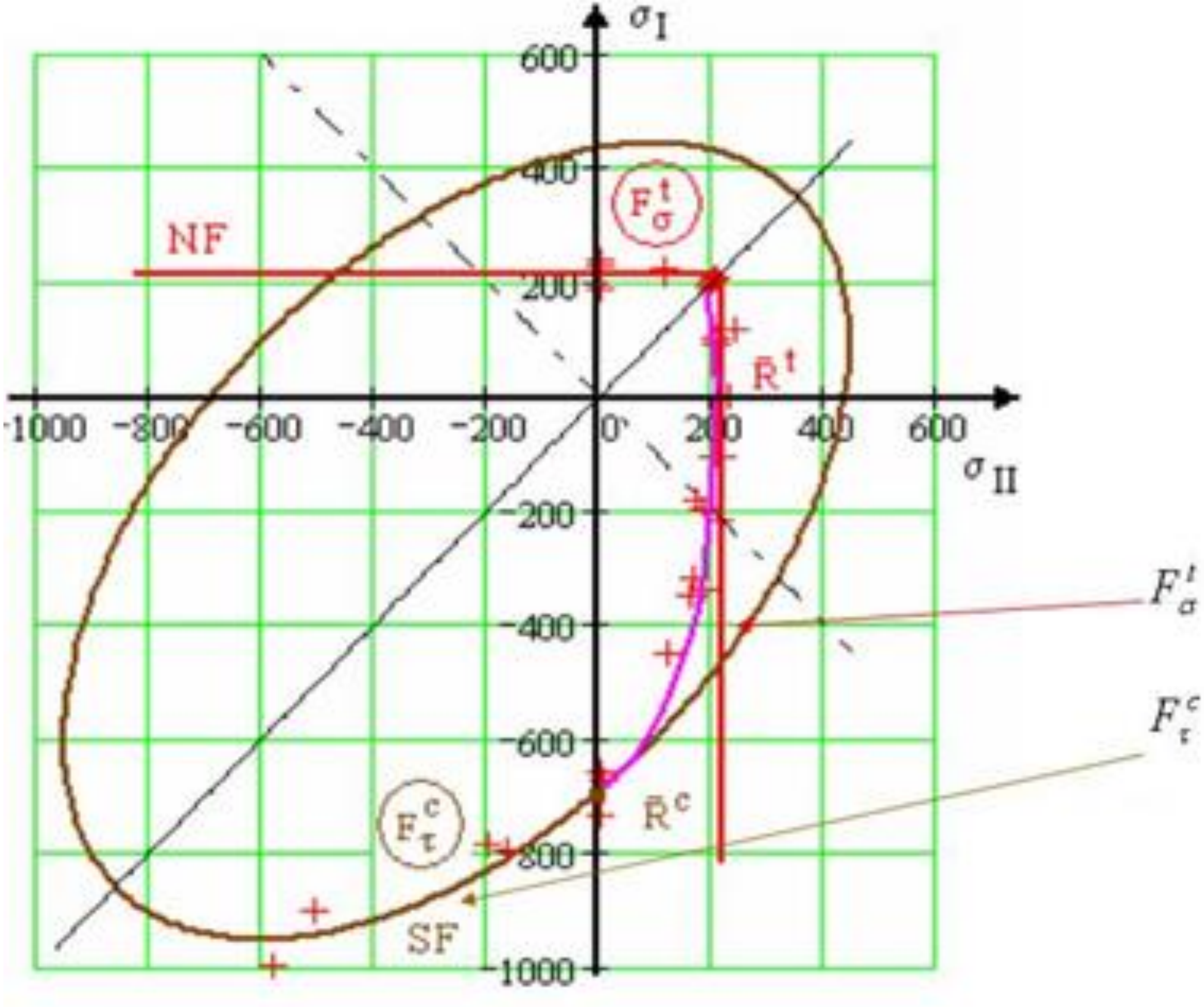




Glass, 2D principal stress plane and meridional cross section (3D)



Grey-cast Iron, principal stress plane



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Cuntzes 3D Festigkeitsbedingungen für isotrope poröse Werkstoffe

Ansätze: **Zug** $F^{NF} = \dfrac{\sqrt{4J_2 - I_1^{\,2}/3} + I_1}{2 \cdot \bar{R}_t} = 1$ **Druck** $F^{CrF} = \dfrac{\sqrt{4J_2 - I_1^{\,2}/3} - I_1}{2 \cdot \bar{R}_c} = 1$ **(Schaum, Ytong)**

Berücksichtigung bi-axialer Festigkeit (Versagensmodus zweifach): ***in Effs***

$$Eff^{NF} = c_{NF} \cdot \frac{\sqrt{4J_2 \cdot (\Theta_{NF}) - I_1^{\,2}/3} + I_1}{2 \cdot \bar{R}_t} = \sigma_{eq}^{NF} / \bar{R}_t \qquad Eff^{CrF} = c_{CrF} \cdot \frac{\sqrt{4J_2 \cdot (\Theta_{CrF}) - I_1^{\,2}/3} - I_1}{2 \cdot \bar{R}_c}$$

Zweifache Versagenswahrscheinlichkeit mit der Invariante J_3 erfassbar,
D_{NF} und D_{CrF} sind die Nicht-Koaxialitätsparameter für die beiden Bruchmoden:

$$\Theta_{NF} = \sqrt[3]{1 + D_{NF} \cdot \sin(3\theta)} = \sqrt[3]{1 + D_{NF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}} \qquad \Theta_{CrF} = \sqrt[3]{1 + D_{CrF} \cdot \sin(3\theta)} = \sqrt[3]{1 + D_{CrF} \cdot 1.5 \cdot \sqrt{3} \cdot J_3 \cdot J_2^{-1.5}}$$

Interaktion der Versagensmoden: $Eff^{NF} = [(Eff^{NF})^m + (Eff^{CrF})^m]^{m^{-1}}$

Abschluß der Versagensoberfläche durch Paraboloid-Kappen oben und unten:

$$\frac{I_1}{\sqrt{3} \cdot R_t} = s_{cap} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta_{NF}}}{R_t}\right)^2 + \frac{\max I_1}{\sqrt{3} \cdot R_t}$$

auf die R_t-normierten Lodekoordinaten bezogen

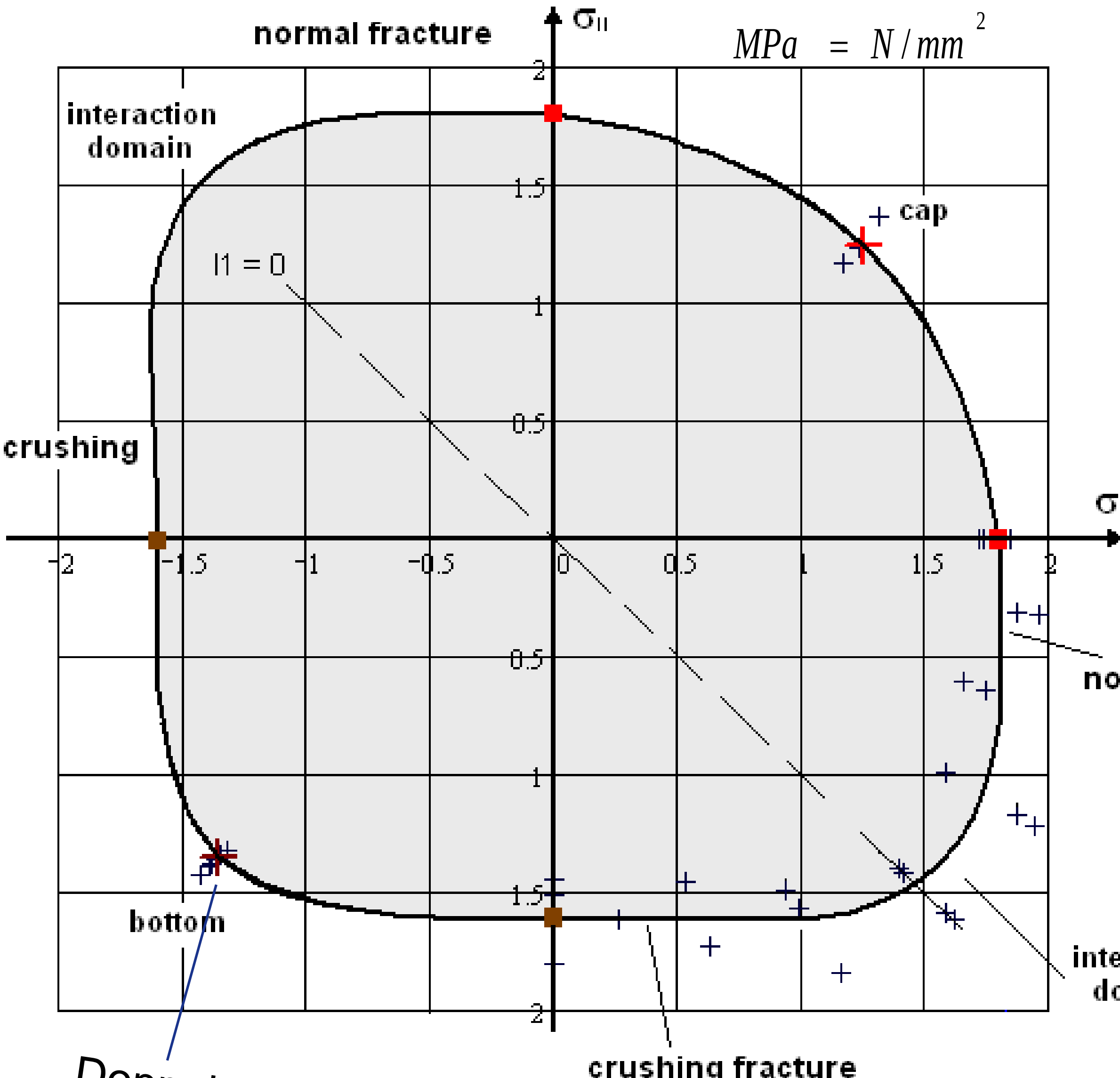
$$\frac{I_1}{\sqrt{3} \cdot R_t} = s_{bot} \cdot \left(\frac{\sqrt{2J_2 \cdot \Theta_{CrF}}}{R_t}\right)^2 + \frac{\min I_1}{\sqrt{3} \cdot R_t}$$

Zur Bestimmung der Steigungsparameter s müssen die hydrostatischen Werte bekannt sein: *$maxI_1$ kann nur abgeschätzt werden, $minI_1$* könnte gemessen werden.

Eff = material stressing effort = Werkstoff-Anstrengung (< 1 = 100%)

Normierung auf Zugfestigkeit

2D – Testdaten mit Abbildung in der Hauptspannungsebene *(brittle, porous)*



Principal Plane Cross-section of the Fracture Body *(oblique cut)*

Rohacell 71 IG

Testdaten Courtesy:
LBF-Darmstadt (DKI),
Dr. Kolupaev

… als sich ähnlich verhaltendes Material

kompressibel

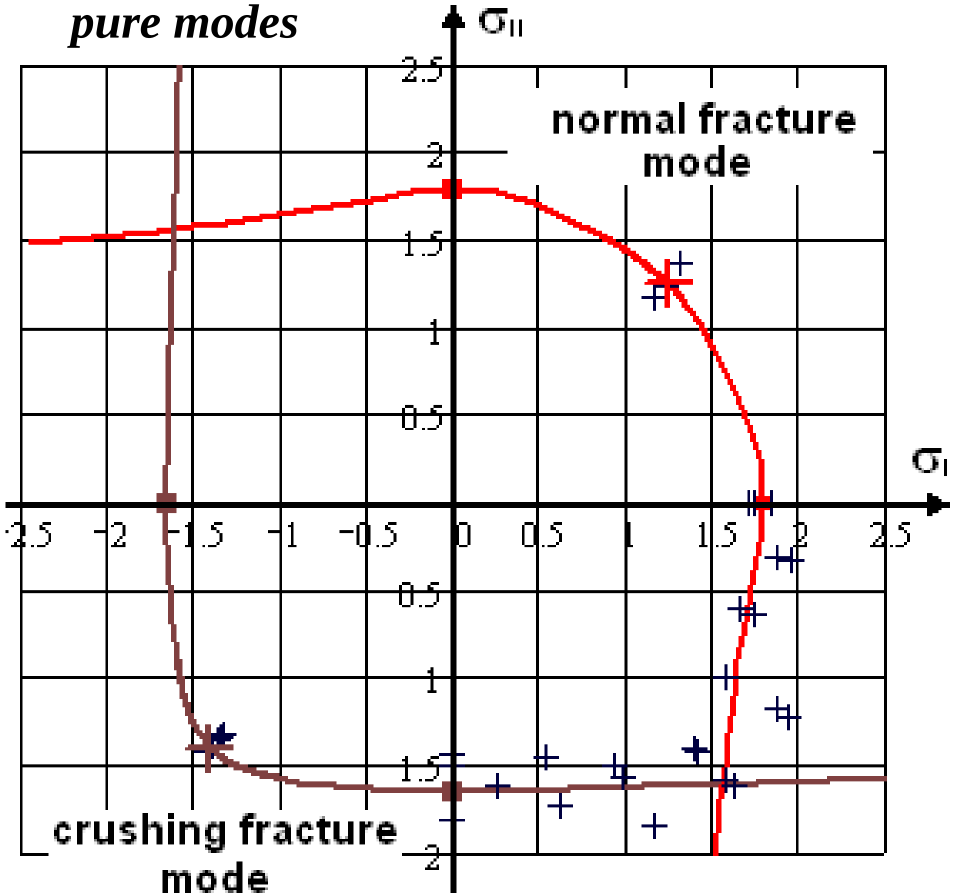
$\nu = 0$

Doppel-Bruchmodus !
Erfassung durch $\Theta(J_3)$

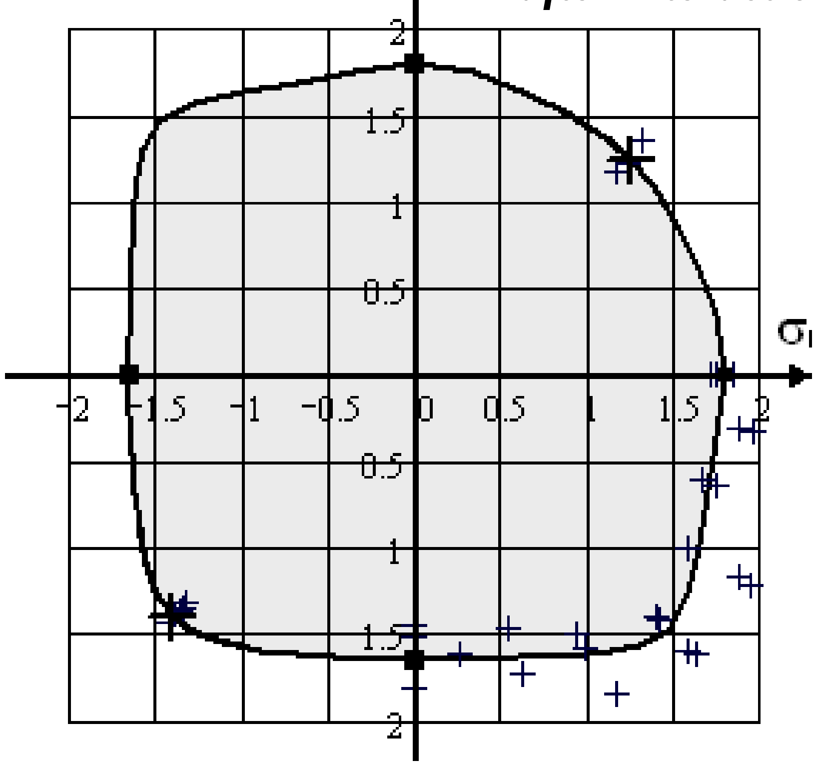
as similarly behaving material

Principal Plane Cross-section of the Fracture Body (oblique cut)

pure modes



after interaction

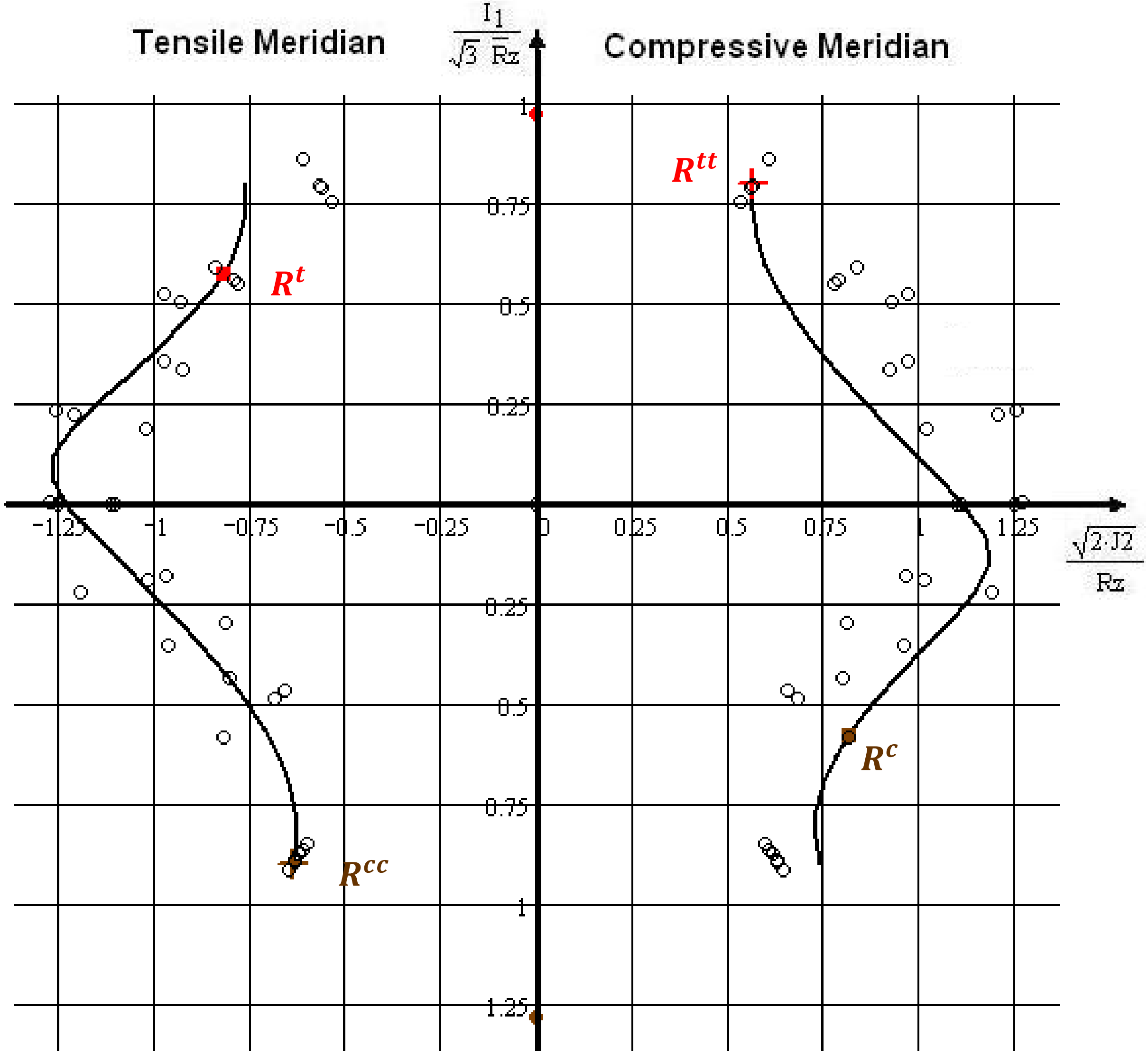


- **Mapping <u>must</u> be performed in the 2D-plane because fracture data set is given there**
- **The 2D-mapping uses the 2D-subsolution of the 3D-strength failure conditions**
- **The 3D-fracture failure surface (body) is based on the 2D-derived model parameters.**

Courtesy: LBF-Darmstadt, Dr. Kolupaev

Generic Lines of Tensile and of Compressive Meridian

Rohacell 71 IG



Meridional cross-sections of the Fracture Body

in Lode-Haigh-Westergaard coordinates

bi-axial = +

z = tensile,

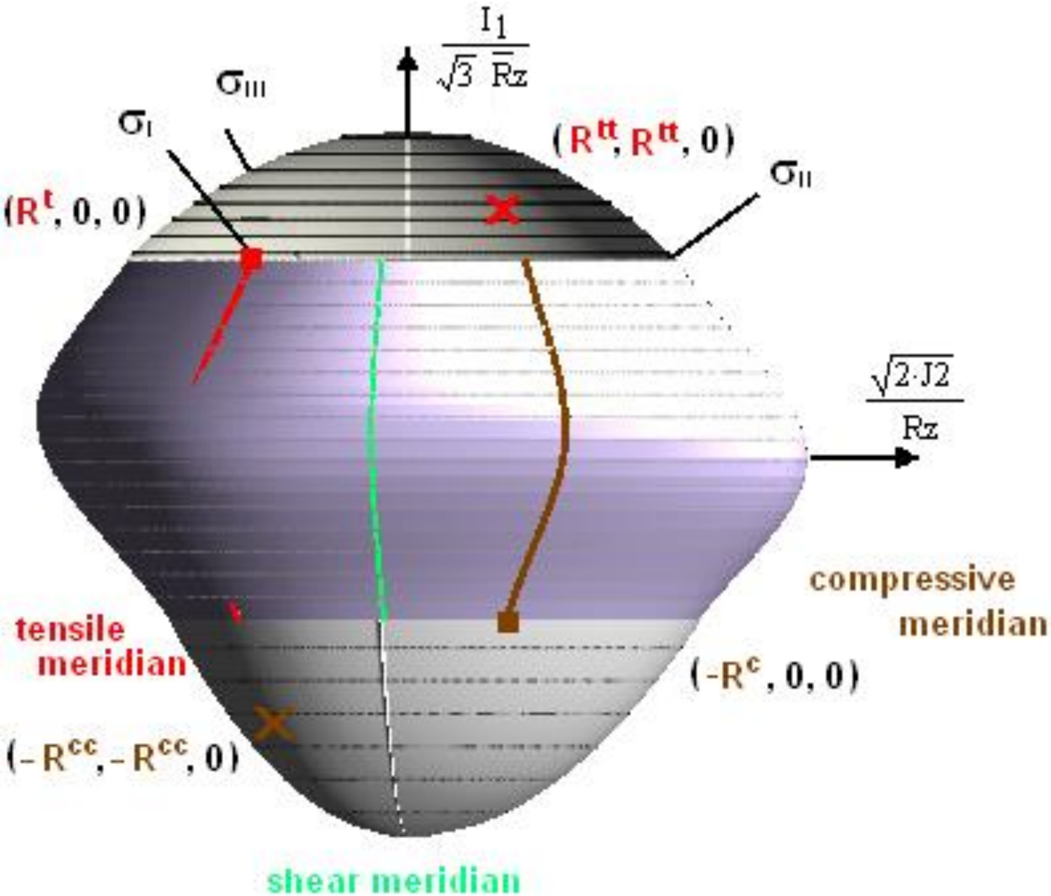
d = compressive

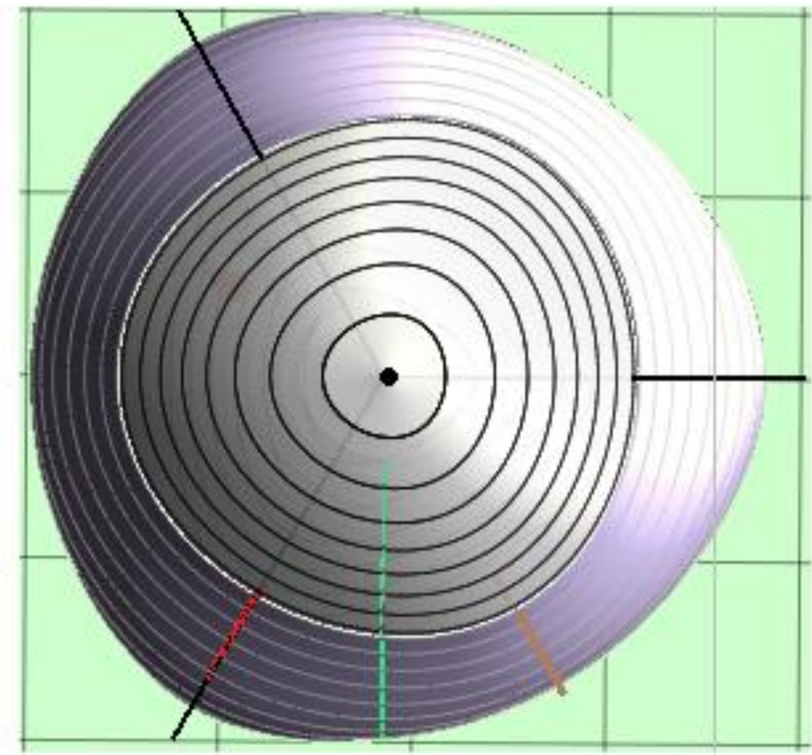
The fracture test data are located at a distinct Lode angle of its associated ring o, 120°-symmetry of the isotropic failure surface (body) .

Cap and bottom are closed by a cone-ansatz, a shape being on the conservative side.

Fracture Failure Surface of *Rohacell 71 IG*

The dent turns !

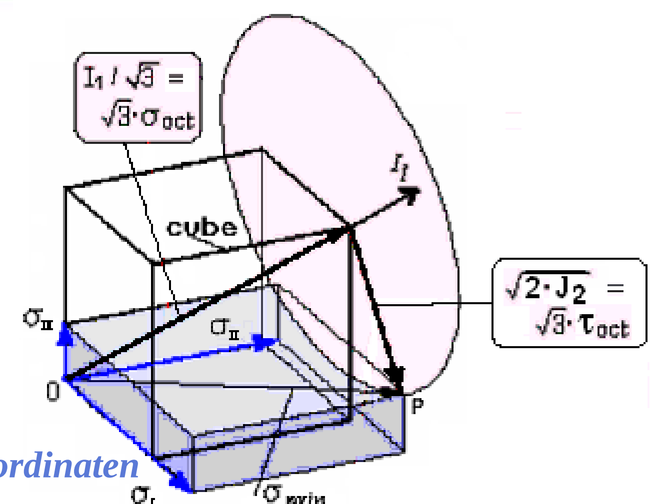




Man kann die 3 Achsen austauschen wegen der 120°-Symmetrie isotroper Körper !

Schubmeridian gewählt als COS-Ursprung

$$\Theta = \sqrt[3]{1 + D \cdot \sin(3\theta)}$$



Visualisierung der Lode-(Haigh-Westergaard) Koordinaten

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Cuntzes 3D-Festigkeitsbedingungen für Normalbeton und UHPC

Normal-Beton ::

$$F^{NF} = c_{NF} \frac{\sqrt{4J_2 \cdot \Theta_{NF} - I_1^{\,2}/3} + I_1}{2 \cdot \overline{R}^t} = 1 \qquad F^{SF} = c_{1\tau} \cdot \frac{3J_2 \cdot \Theta_\tau}{\overline{R}^{c\,2}} + c_{1\tau} \cdot \frac{I_1}{\overline{R}^c} = 1$$

$$c_{NF} = \ldots \qquad c_{1\tau} \cdot \Theta\tau_d = 1 + c_{1\tau}\,, \quad c_{2\tau} = \frac{1+3\cdot\mu}{1-3\cdot\mu}\,, \quad \mu = Cd\,, \; Cd = \cos\,(2\cdot\theta_{fpc}{}^\circ \cdot \pi/180)$$

Bruchwinkel θ_{fpc} liefert: 45° (μ=0), 50° (μ= 0.174), 55° (μ=0.309).
Empfohlen : 0.1 < μ < 0.2 (der kleinere Wert ist auf der konservativen Seite)

UHPC:

$$F^{NF} = c_{NF} \frac{\sqrt{4J_2 \cdot \Theta_{NF} - I_1^{\,2}/3} + I_1}{2 \cdot \overline{R}^t} = 1 \qquad F^{SF} = c_{1\tau} \cdot \frac{3J_2 \cdot \Theta_\tau}{\overline{R}^{c\,2}} + c_{2\tau} \cdot \frac{I_1}{\overline{R}^c} + c_{3\tau} \cdot \frac{I_1^{\,2}}{\overline{R}^{c\,2}} = 1$$

Berücksichtigung der Volumenveränderung unter hydrostatischem Druck

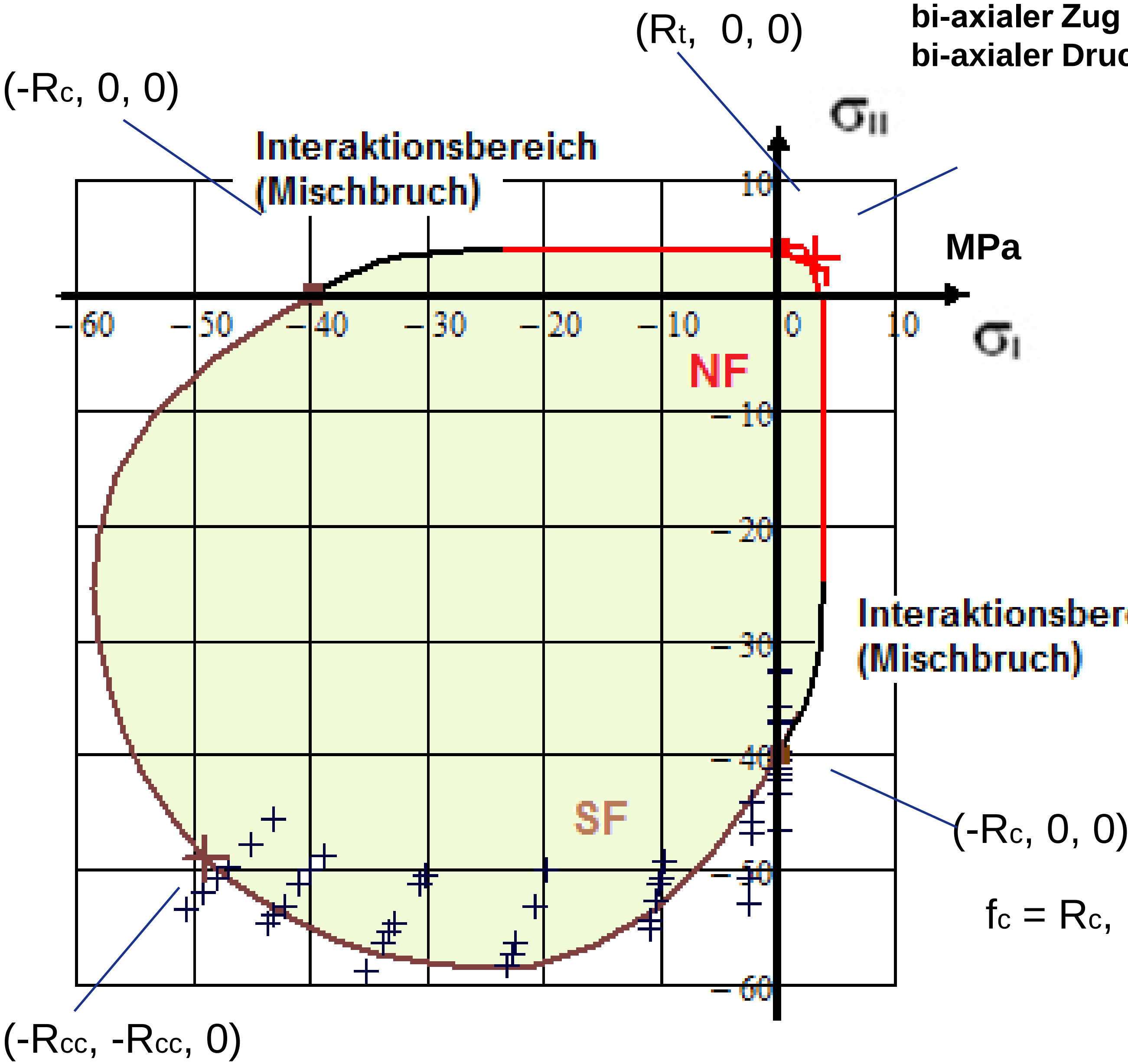
Ein Festigkeitsansatz beschreibt in der Regel nur das einmalige Auftreten eines Modus = Versagensmechanismus !

Abbildung der 2D-Testdaten in Hauptspannungsebene, Normalbeton

$\sigma_{II}(\sigma_I)$

Schiefer Schnitt durch den Bruchkörper (s. a. von Mises-Ellipse)

bi-axialer Zug = ‘schwächstes Glied‘-Versagen
bi-axialer Druck = redundantes Versagen (Stützwirkung)



$v = 0.2$

etwas elastisch kompressibel

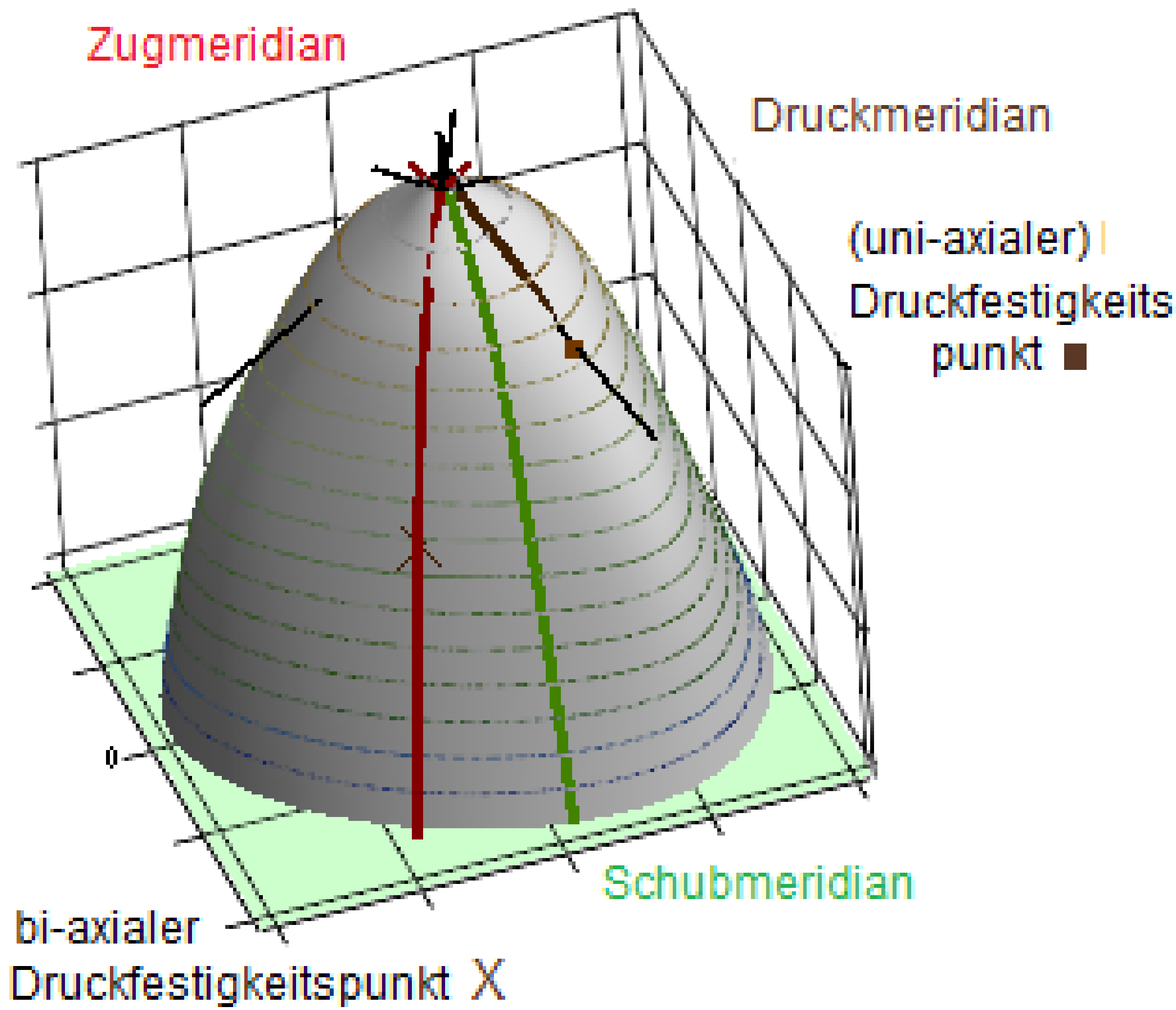
Testdaten: Dr. Scheerer, IfM, TU-Dresden

$f_c = R_c$,

Bruchkörper, Normalbeton

Nur kleine Beulen

und Dellen hier !

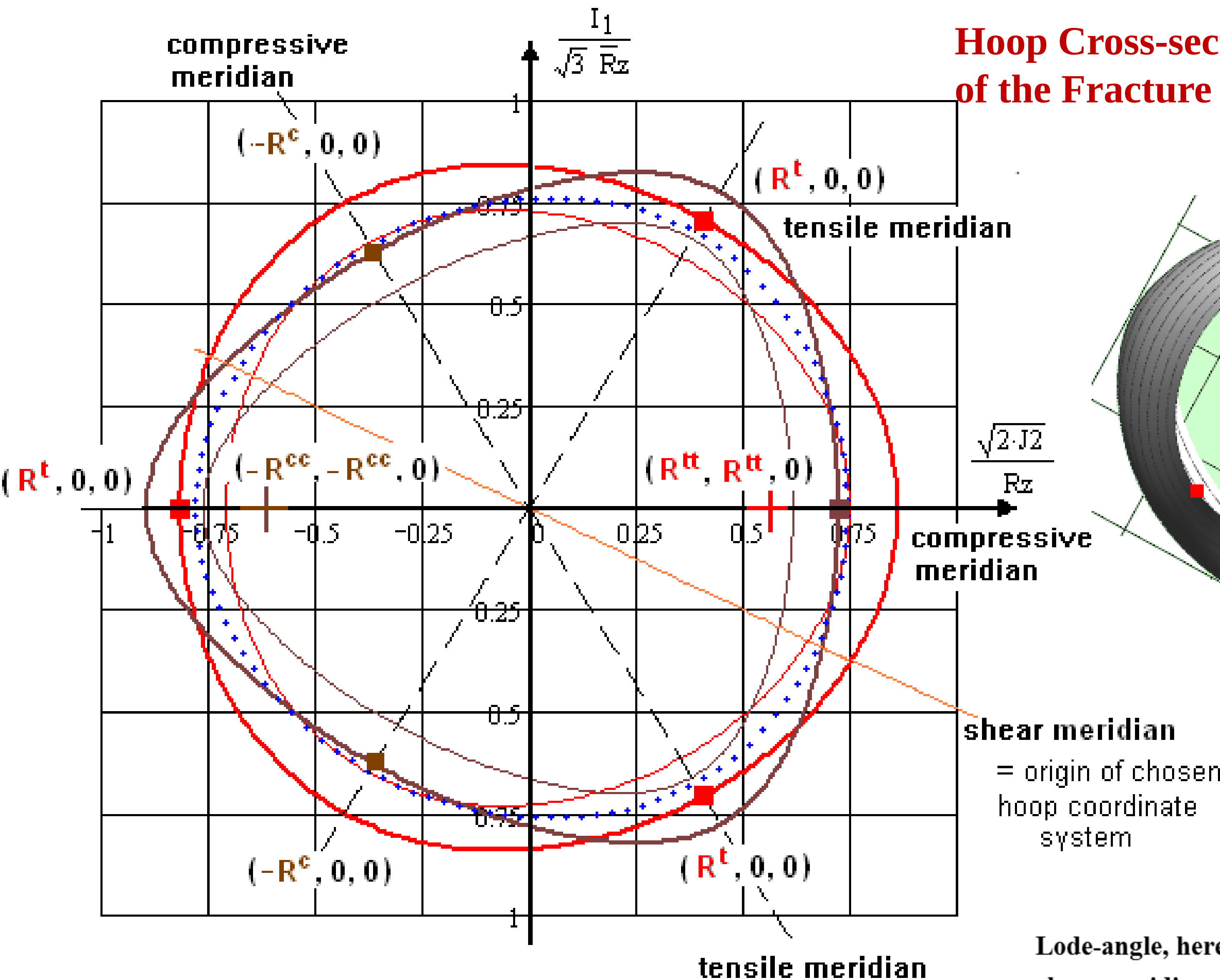


Bi-axialer Druck:
Neben **Schubbruch**
Gefahr durch Normalbruch
falls $\varepsilon_{ax} > \varepsilon_{zug}$

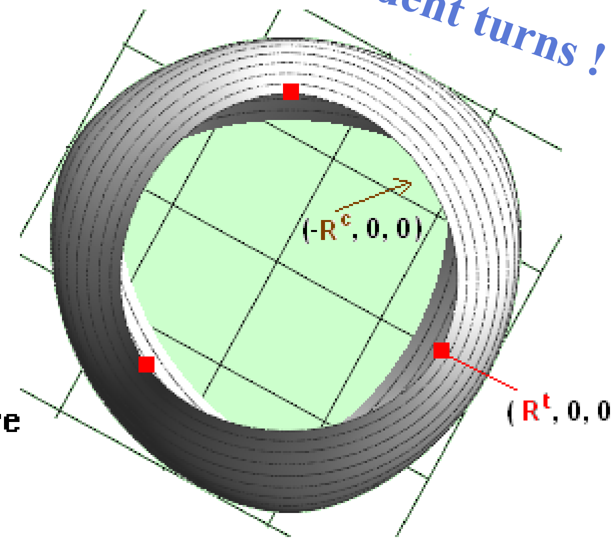
Die Ein- und Aus-Beulungen werden mit wachsendem I_1^c kleiner, oder anders, der Querschnitt wird kreisförmiger !

Unmfangsverläufe in verschied. Orthogonalen Spannungsebenen (spröd, porös)

Hoop Cross-sections of the Fracture Body



The dent turns !



Caps: No test data, shape chosen

Rohacell 71 IG

Lode-angle, here set as $\sin(3\ \theta)$: +

shear meridian angle = 0°

tensile meridian +30°

compressive meridian -30°

with characteristic uni-axial and bi-axial strength points +

crushing I1 = 0, interaction domain: Is about a circle.

Determination of the Load-defined Reserve Factor RF (foam)

Linear elastic problem for this brittle behaving material

Residual stresses = 0

$\mathbf{RF} = \boldsymbol{f}_{Res}$ (material reserve factor) $= \boldsymbol{Eff}^{-1}$

estimated from given average value

Stress state:

$\sigma I := 0.9 \quad \sigma II := -0.4 \quad \sigma III := 0.5$

Statistically reduced Strengths:

$Rz := 0.9 \cdot \bar{R}z \quad Rd := 0.85 \cdot \bar{R}d$

Shape parameters: $D\sigma = -0.71 \quad D\tau = 0.21 \quad c1\Theta\sigma = 1.15 \quad c1\Theta\tau = 1.03$

$$I1 := \sigma I + \sigma II + \sigma III \qquad J2 := \frac{\left[(\sigma I - \sigma II)^2 + (\sigma II - \sigma III)^2 + (\sigma III - \sigma I)^2\right]}{6} \qquad J3 := \frac{\left[(2\cdot\sigma I - \sigma II - \sigma III)\cdot(2\cdot\sigma II - \sigma I - \sigma III)\cdot(2\cdot\sigma III - \sigma II - \sigma I)\right]}{27}$$

$I1 = 1 \qquad J2 = 0.44 \qquad J3 = -0.07$

$$Eff\Theta\sigma := c1\Theta\sigma \cdot \frac{\sqrt{4\cdot J2\cdot\sqrt[3]{1 + D\sigma\cdot 1.5\cdot 3^{0.5}\cdot J3\cdot J2^{-1.5}} - \frac{1}{3}\cdot I1^2} + I1}{2Rz}$$

$$Eff\Theta\tau := c1\Theta\tau \cdot \frac{\sqrt{4\cdot J2\cdot\sqrt[3]{1 + D\tau\cdot\left(1.5\cdot 3^{0.5}\cdot J3\cdot J2^{-1.5}\right)} - \frac{1}{3}\cdot I1^2} - I1}{2\cdot Rd}$$

$$Eff := \sqrt[9]{Eff\Theta\sigma^{m_{int}} + Eff\Theta\tau^{m_{int}}} \qquad Eff = 0.802 \qquad RF := \frac{1}{Eff} \qquad RF = 1.25$$

The loading may be still monotonically increased by the factor RF !

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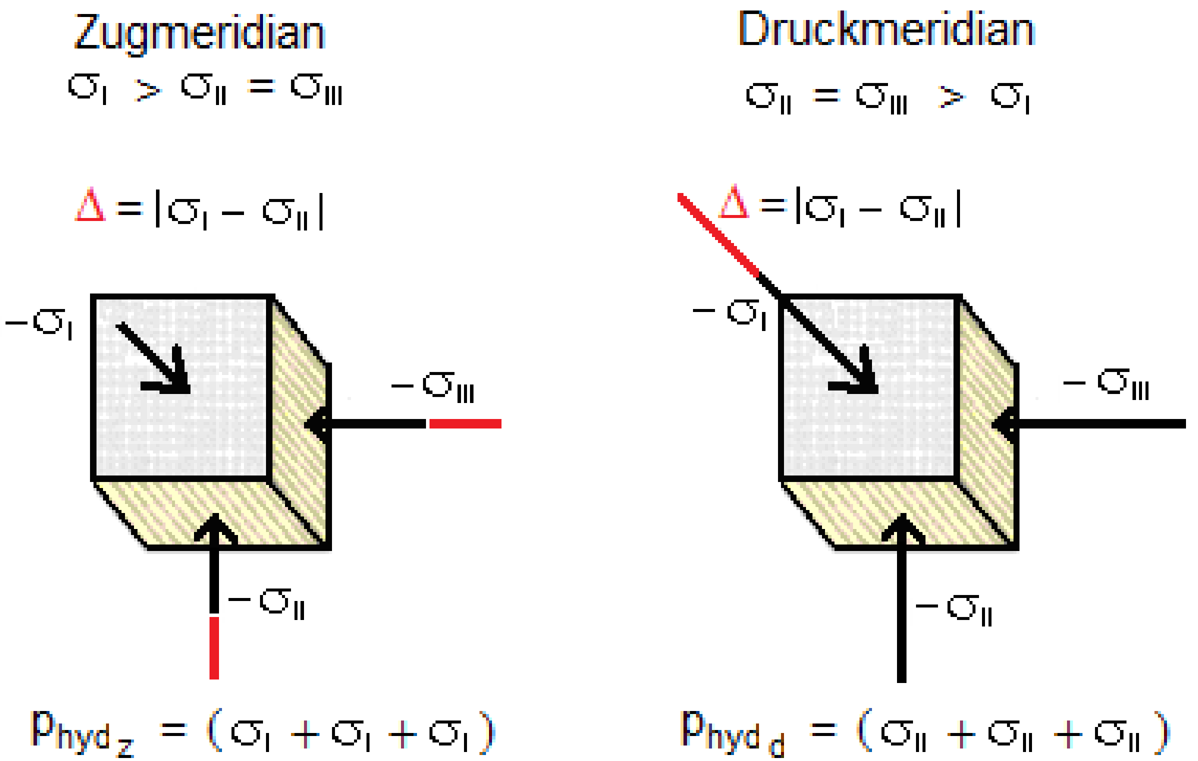
5 Application: Grey-cast Iron, Glass
6 Application: Isotropic Foam (Rohacell 71 G)
7 Application: Normal Concrete and **Ultra High Performance Concrete (UHPC)**
8 Application: Transversely-isotropic UD-CFRP
Conclusions

Gelten im hydrostatischen Druckbereich noch die Ansätze, die im 2D-Spannungsbereich erfolgreich einsetzbar sind ?

Neben der Nicht-Koaxialität tritt als weiteres Phänomen eine *Volumenveränderung* *auf.*
Damit muss neben der durch in I_1 berücksichtigten Reibung
auch noch I_1^2 in den Gesamtansatz eingefügt werden !



Spannungsdifferenz Δ bewirkt Gestaltänderung,
hydrostatische Druckspannung p_{hyd} bewirkt Volumenänderung

+ under multi-axial compression a redundancy (healing) effect occurs !

1. **1D = Festigkeitswerte *f* *(= R):* Ergebnis aus einachsigen Versuchen mit ‘freien‘ Probekörpern (‚isolated‘ test specimens) mit der Versagensart ‚Schwächstes Glied-Versagen‘**

2. **3D-Festigkeitswerte im Druckbereich: Ergebnis der Versagensart ‘Redundantes Versagen‘, Stützwirkung liegt vor, anderes Versagens-Verhalten.**

***Beispiel UHPC-Testergebnisse*:**

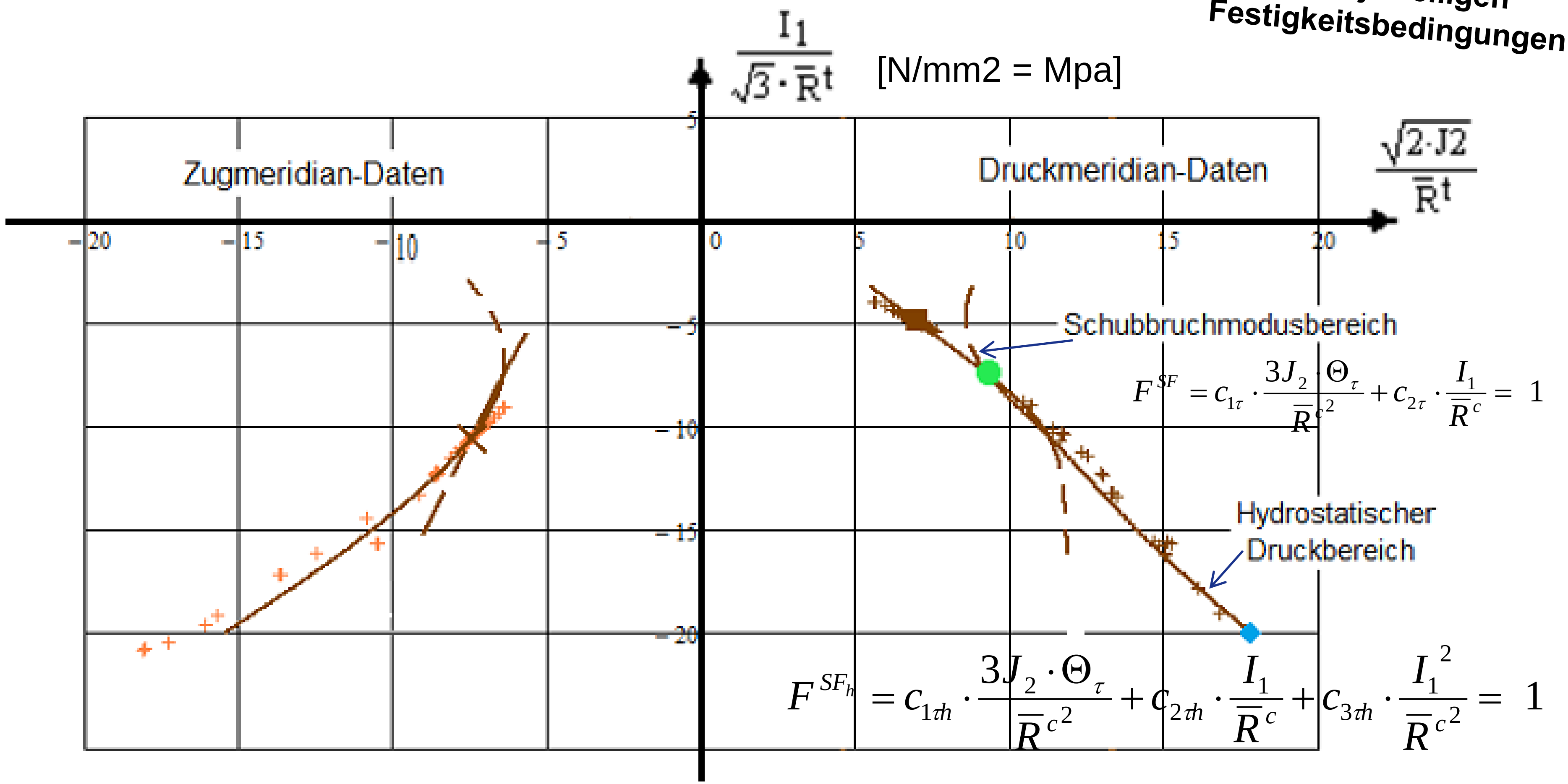
$$\sigma_{bruch} = (\sigma_I,\ \sigma_{II},\ \sigma_{III})_{bruch}{}^T \ :\ (-160,\ 0,\ 0)^T \Rightarrow (-230,\ -6,\ -6)^T$$

1D **3D**

Es sind also bei hydrostatisch hochbeanspruchtem UHPC die Festigkeitsbedingungen für Normalbeton nicht ausreichend !

Porosität beim UHPC geringer als beim Normalbeton, da dichter aufgrund der Sieblinie

Testdaten auf Zugmeridian + und Druckmeridian + mit Abbildung durch die jeweiligen Festigkeitsbedingungen



Schubbruchmodus — **Stützpunkte für die Ermittlung der Kurvenparameter** für **I1 < 0**

Ermittlung von

Druckfestigkeitspunkt ■ und Punkt ●: Größeparameter $c_{1\tau}$ und Reibungs-Parameter (Formparameter) $c_{2\tau}$

Druckfestigkeitspunkt und bi-axialer Festigkeitspunkt X : Nicht-Koaxialitäts-Parameter Θ_τ

Hydrostatischer Druckmodus

Ermittlung von hydrostatischem Druckeffekt-Parameter $c_{3\tau}$

vorläufiger Ansatz in Verwendung der Punkte X ● ◆

Testdaten: Dr. Speck, IfM, TU-Dresden

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Cuntze's Set of Modal 3D UD Strength Failure Conditions (criteria)

Invariants replaced by their stress formulations

strains from FEA [Cun04, Cun11]

FF1 $Eff^{\parallel\sigma} = \breve{\sigma}_1 / \bar{R}^t_{\parallel} = \sigma^{\parallel\sigma}_{eq} / \bar{R}^t_{\parallel}$, $\breve{\sigma}_1 \cong \varepsilon^t_1 \cdot E_{\parallel}$ *

FF2 $Eff^{\parallel\tau} = -\breve{\sigma}_1 / \bar{R}^c_{\parallel} = +\sigma^{\parallel\tau}_{eq} / \bar{R}^c_{\parallel}$, $\breve{\sigma}_1 \cong \varepsilon^c_1 \cdot E_{\parallel}$

2 filament modes

IFF1 $Eff^{\perp\sigma} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / 2\bar{R}^t_{\perp} = \sigma^{\perp\sigma}_{eq} / \bar{R}^t_{\perp}$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}\,] / \bar{R}^c_{\perp} = +\sigma^{\perp\tau}_{eq} / \bar{R}^c_{\perp}$

3 matrix modes

IFF3 $Eff^{\perp\parallel} = \{[\mu_{\perp\parallel} \cdot I_{23-5} + (\sqrt{\mu_{\perp\parallel}{}^2 \cdot I_{23-5}{}^2 + 4 \cdot \bar{R}_{\perp\parallel}{}^2 \cdot (\tau^2_{31} + \tau^2_{21})^2}\,]/(2 \cdot \bar{R}_{\perp\parallel}{}^3)\}^{0.5} = \sigma^{\perp\parallel}_{eq} / \bar{R}_{\perp\parallel}$

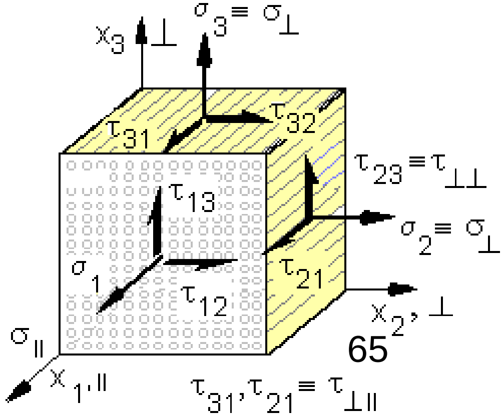
with $I_{23-5} = 2\sigma_2 \cdot \tau^2_{21} + 2\sigma_3 \cdot \tau^2_{31} + 4\tau_{23}\tau_{31}\tau_{21}$

Modes-Interaction :

$$Eff^{m} = (Eff^{\parallel\tau})^m + (Eff^{\parallel\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

with **mode-interaction exponent** $2.5 < m < 3$ **from mapping tests data**

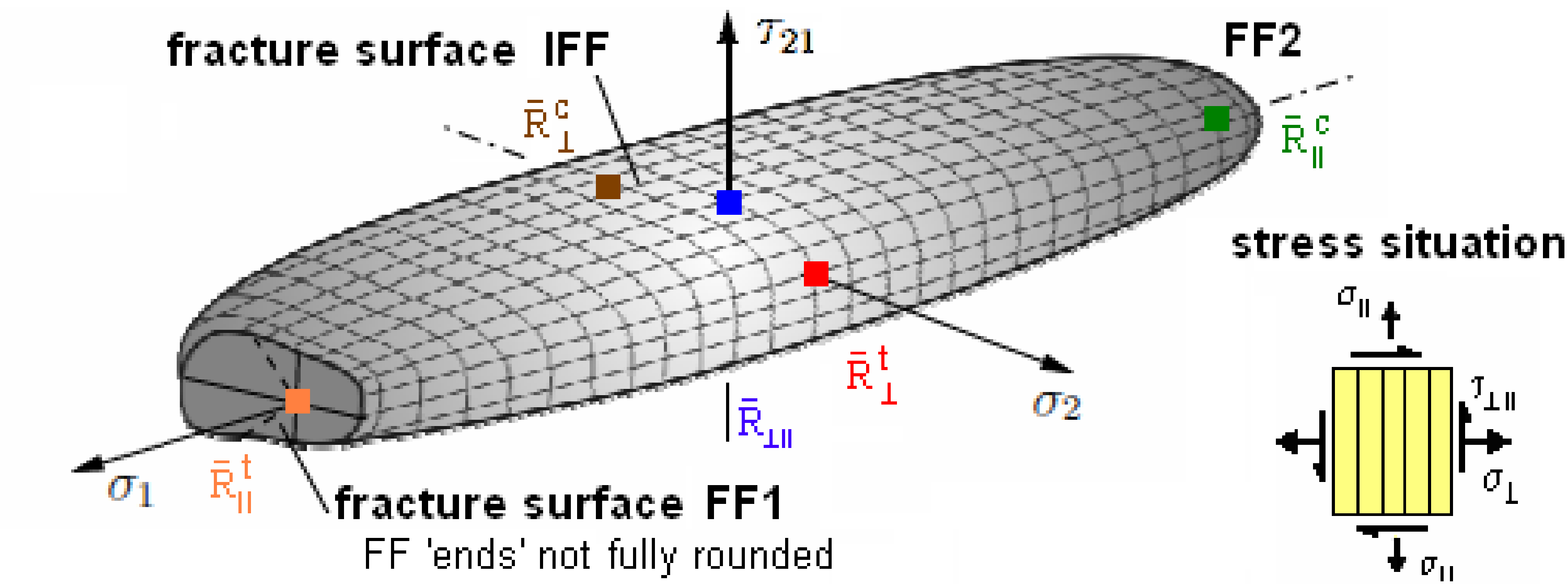
Typical friction value data range: $0.05 < \mu_{\perp\parallel} < 0.3$, $0.05 < \mu_{\perp\perp} < 0.2$



Poisson effect * : bi-axial compression strains the filament without any σ_1

t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre

Visualization of 2D UD SFCs as Fracture Failure Surface (Body)



$$\{\sigma\} = (\sigma_1, \sigma_2, 0, 0, 0, \tau_{21})^T$$

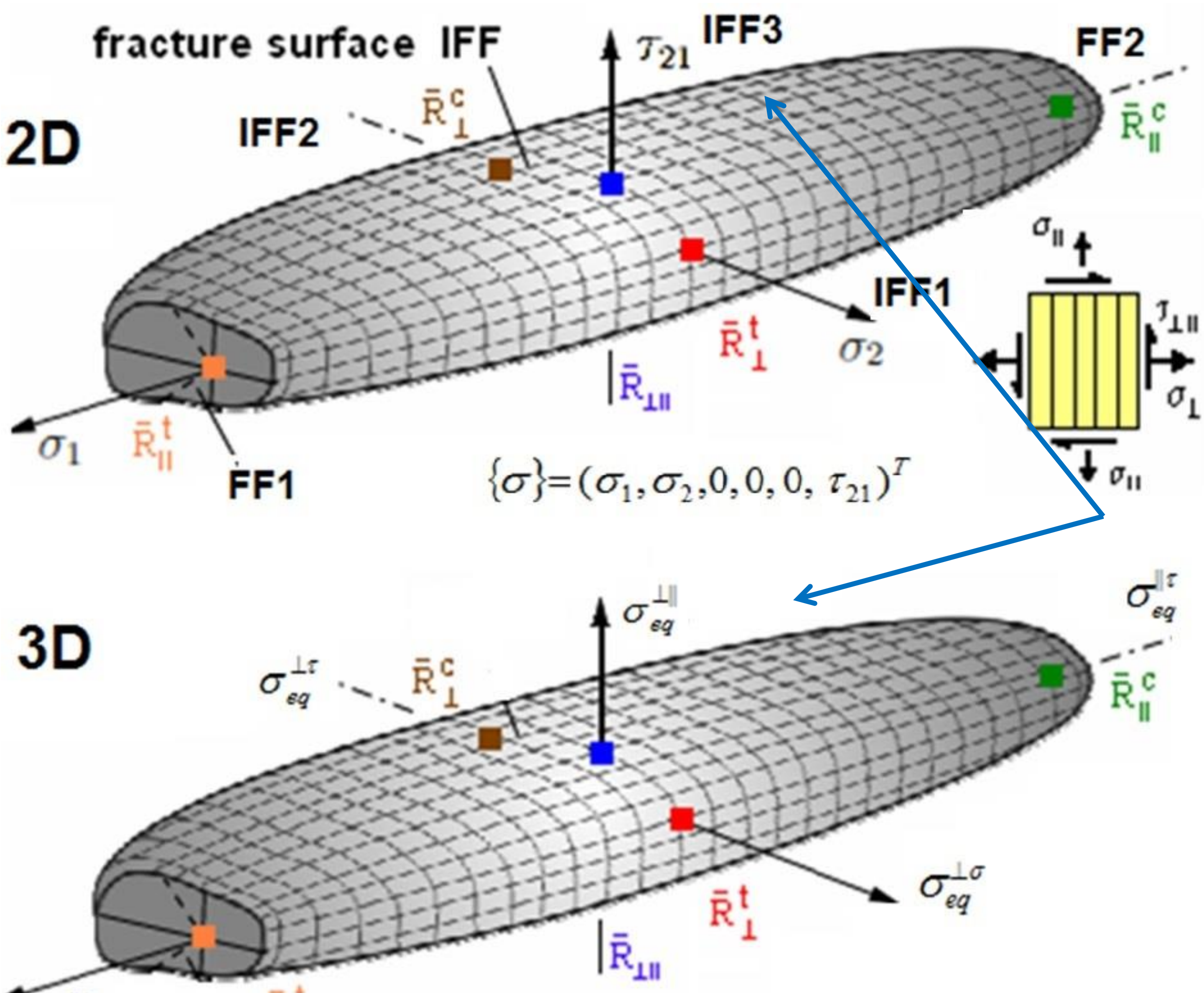
Mode interaction fracture failure surface *of FRP UD lamina*

$$Eff^{\,m} = (Eff^{\,\parallel\tau})^m + (Eff^{\,\parallel\sigma})^m + (Eff^{\,\perp\sigma})^m + (Eff^{\,\perp\tau})^m + (Eff^{\,\perp\parallel})^m = 1$$

(courtesy W. Becker) .

Mapping: Average strengths indicated

2D ➡ 3D Bruchkörper der Lamelle, nach Ersetzen von σ durch $\sigma_{eq}^{\text{mode}}$



$$\{\sigma\}=(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{31},\tau_{21})^T$$

$\nu_{\perp\perp} = 0.4,$
$\nu_{\perp\parallel} = 0.3$

wenig kompressibel

Bruchkörper = Oberfläche aller Bruchversagens-Spannungszustände

Cuntze's Pre-design Input for 3D UD SFCs

Test Data Mapping | **Design Verification**

- **5 strengths :** $\{\overline{R}\} = (\overline{R}_{\parallel}^{t}, \overline{R}_{\parallel}^{c}, \overline{R}_{\perp}^{t}, \overline{R}_{\perp}^{c}, \overline{R}_{\perp\parallel})^T$ $\{R\} = (R_{\parallel}^{t}, R_{\parallel}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\parallel})^T$

  average (typical) values | **strength design allowables**

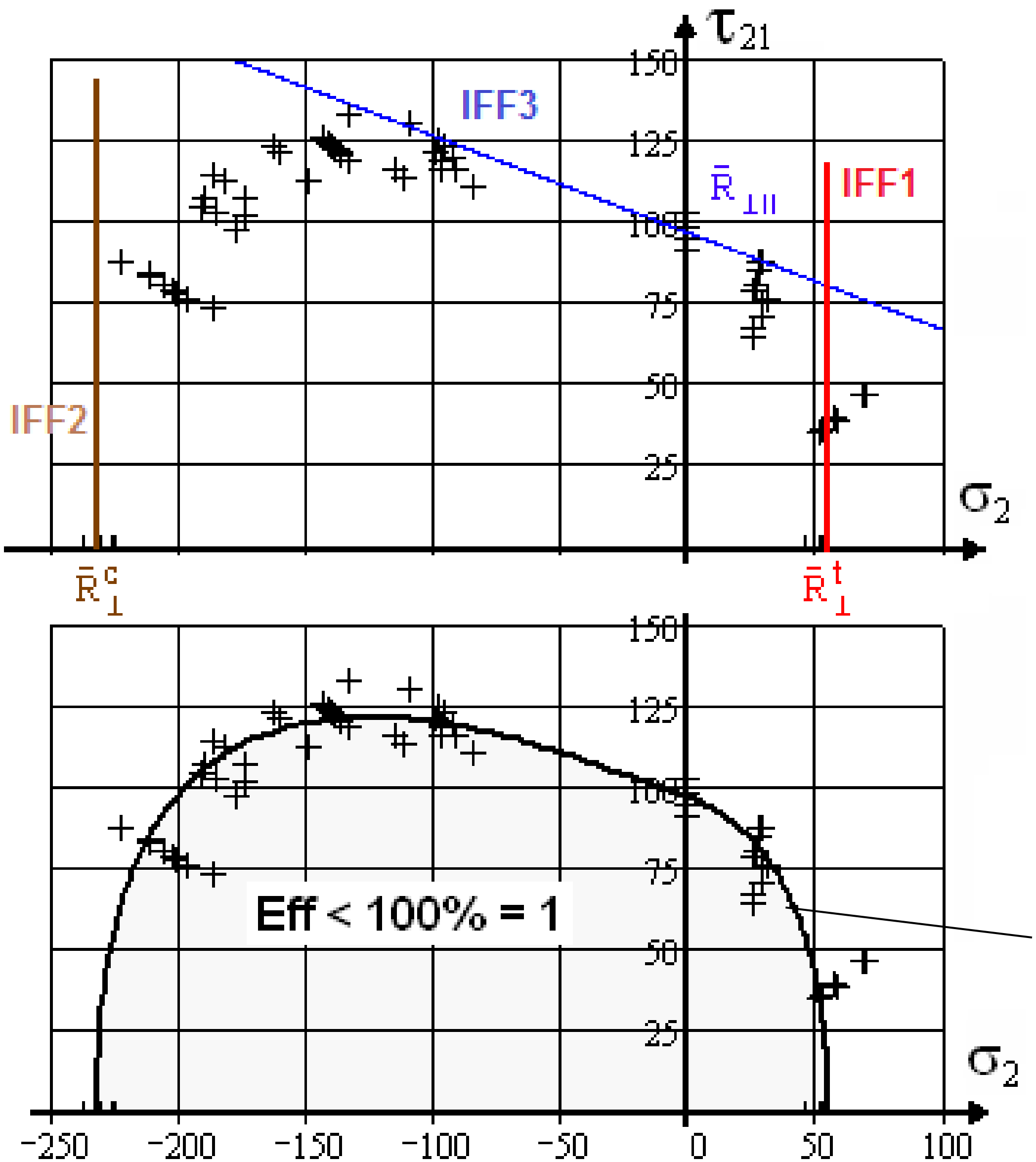
- **2 friction values :** **for 2D** $\mu_{\perp\parallel}$ **, for 3D** $\mu_{\perp\parallel}, \ \mu_{\perp\perp}$

  $\mu_{\perp\parallel} = 0.1$ $\mu_{\perp\perp} = 0.1$

- **1 mode-interaction exponent :** ***m* = *2.6* .**

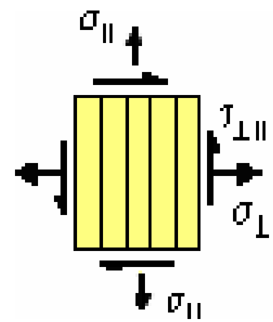
values, recommended for pre-design

Demonstration IFF Domain: Interaction of Failure Modes $\tau_{21}(\sigma_2)\ ,\ \breve{\sigma}_1 = 0$



Mapping of course of IFF test data in a pure mode domain by the associated *Mode Failure Condition.*

3 IFF pure modes = straight lines !.



IFF 1 : $\dfrac{\sigma_2}{\bar{R}_\perp^t} = 1$

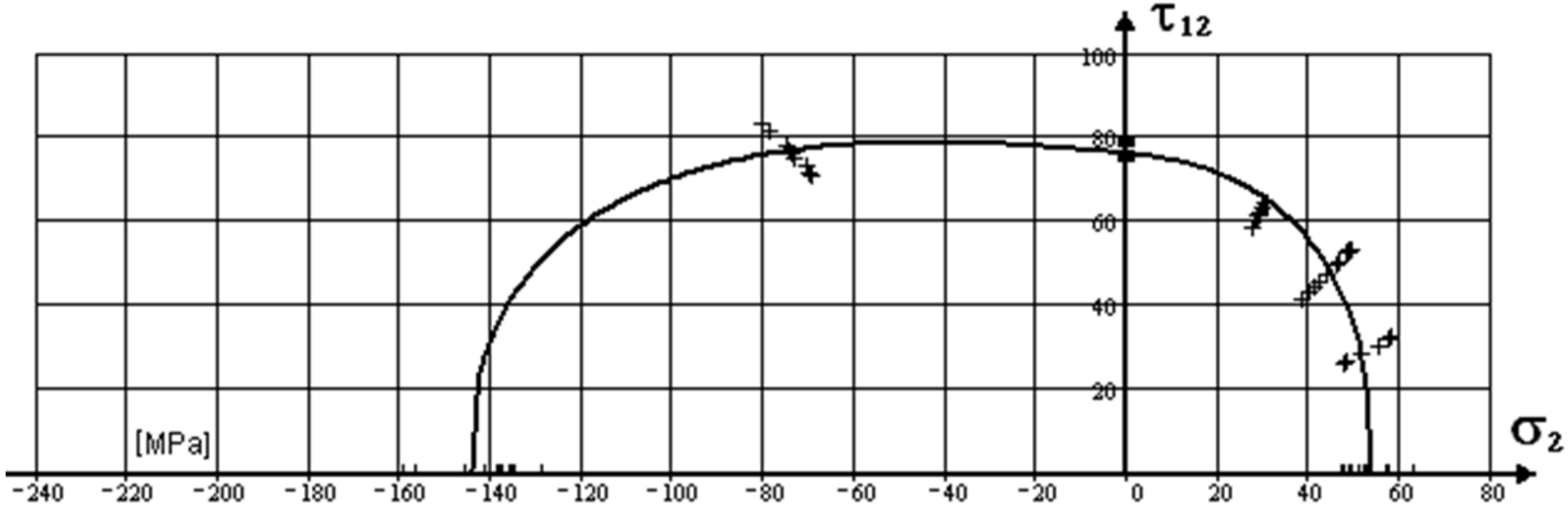
IFF 2 : $\dfrac{-\sigma_2}{\bar{R}_\perp^c} = 1$

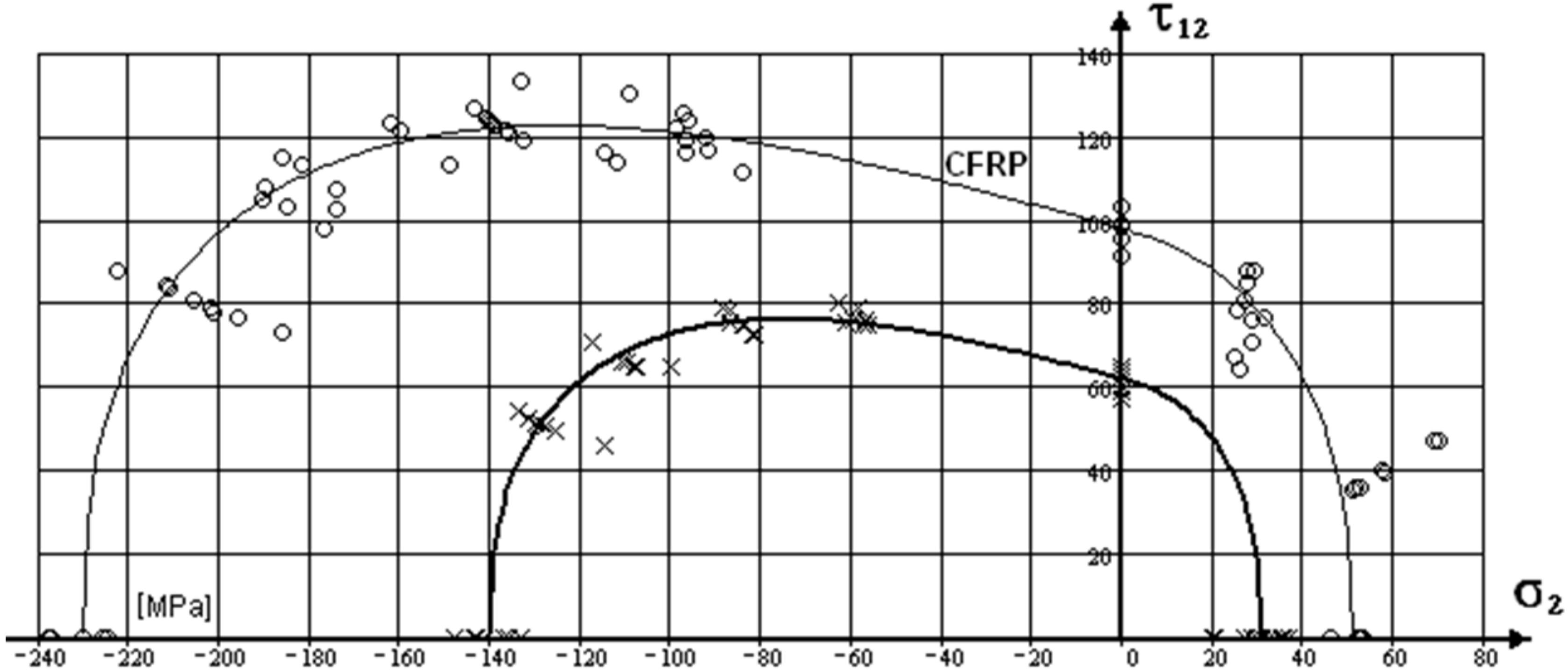
IFF 3 *(2D simplified)* : $\dfrac{|\tau_{21}|}{\bar{R}_{\perp\parallel} - \mu_{\perp\parallel} \cdot \sigma_2} = 1$

Mapping of course of test data by *Interaction Model*

$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\parallel})^m = 1$$

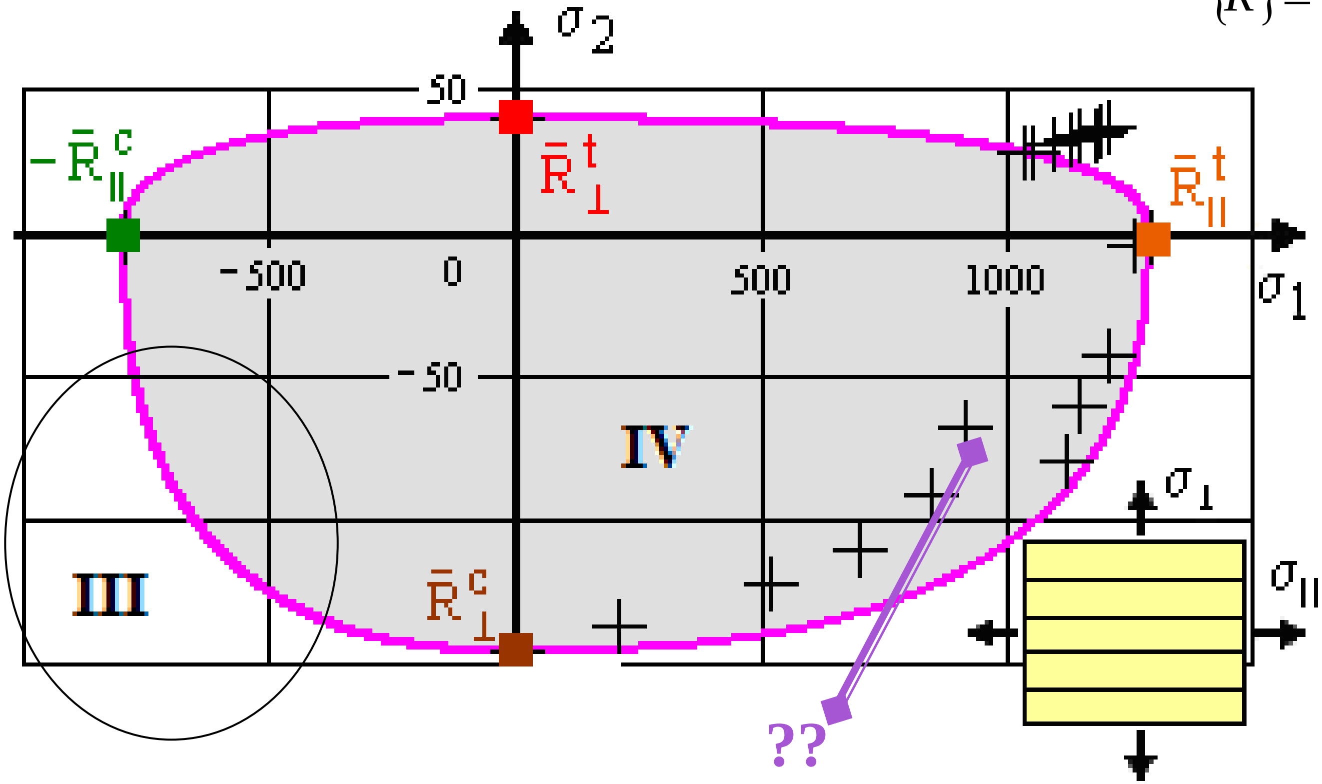
$$m = 2.5,\quad \mu_{\perp\parallel} = 0.3$$


τ12
σ2
[MPa]
-240 -220 -200 -180 -160 -140 -120 -100 -80 -60 -40 -20 0 20 40 60 80
20 40 60 80 100


τ12
σ2
CFRP
[MPa]
-240 -220 -200 -180 -160 -140 -120 -100 -80 -60 -40 -20 0 20 40 60 80
20 40 60 80 100 120 140

Test Case 3, WWFE-I $\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$

$\{\bar{R}\} = (1280,\ 800,\ 40,\ 145,\ 73)^T$



Hoop wound tube UD-lamina. E-glass/MY750epoxy +

$\sigma_1 = \sigma_{hoop}$

$\sigma_2 = \sigma_{axial}$

Part A: Data of strength points were provided, only

Part B: Test data in quadrant IV show discrepancy , testing?

No data for quadrants II, III was provided ! But, ..

Remark on gaps between theory and experiment

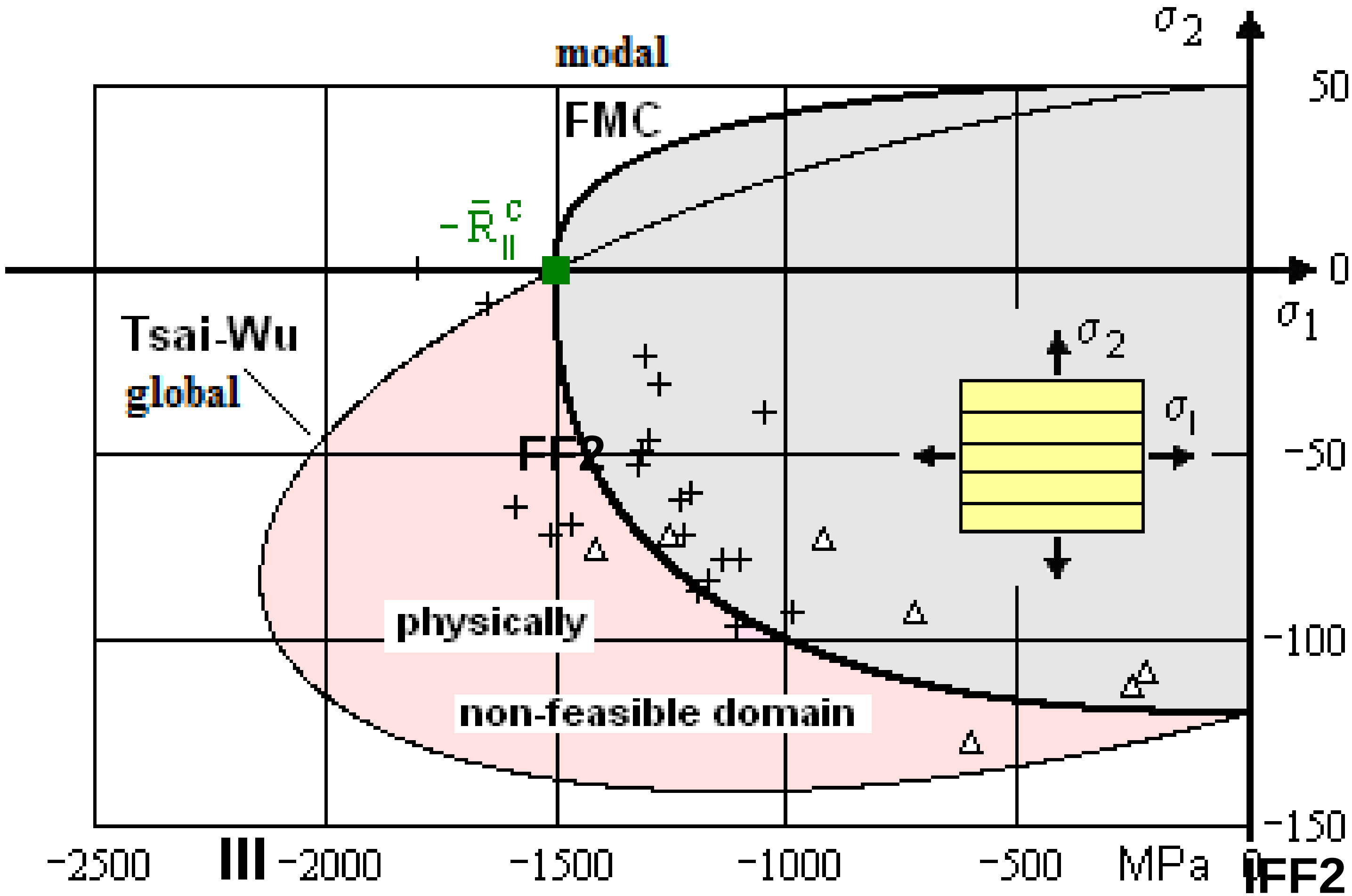
- Experimental results can be far away from the reality like a bad theoretical model.

- Theory creates a model of the reality, ‘only’,

  and

  1 Experiment is ‘just’ 1 realisation of the reality.

Mapping in the ‘Tsai-Wu non-feasible domain‘ (quadrant III)



$$\sigma_2(\breve{\sigma}_1 \equiv \sigma_1)$$

Data: courtesy IKV Aachen, Knops

Lesson Learnt: The <u>modal</u> FMC maps correctly, the <u>global</u> Tsai-Wu formulation predicts a non-feasible domain !

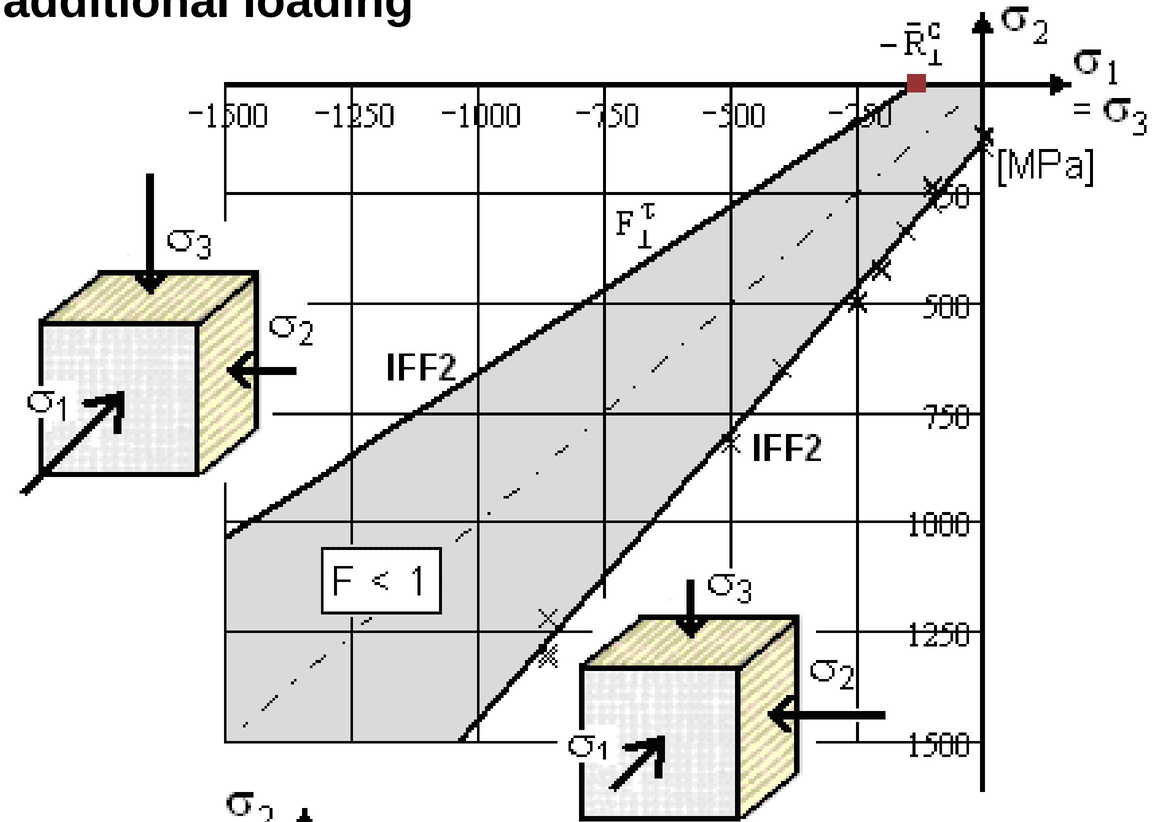
Test Case 5, WWFE-II, UD test specimen, 3D stress state $\sigma_2(\sigma_1 = \sigma_3)$

= hydrostatic pressure with additional loading

UD E-glass/MY750epoxy.

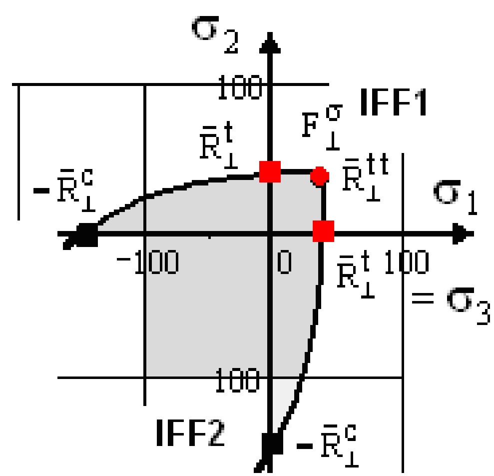
$\nu_{\perp\|} = 0.28, \quad \mu_{\perp\perp} = 0.14, \quad m = 2.8\,,$

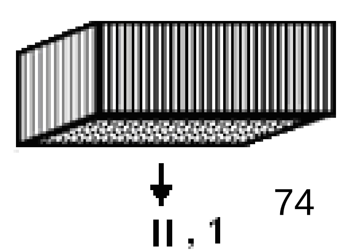
$\{\bar{R}\} = (1280,\ 800,\ 40,\ 132,\ 73)^T\ MPa$



Good Mapping, <u>after</u> QinetiQ re-evaluation of the lower branch test data
Then, the upper branch was fitting other test data, too !

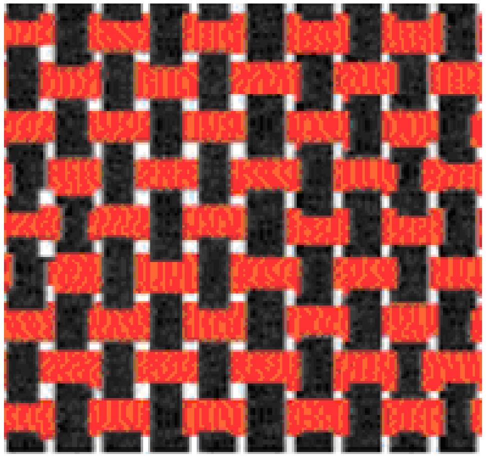
Result: ***Both branches were then reliable and could be used for model validation***



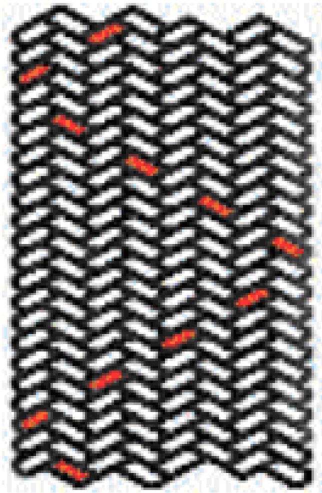


Modelling of Composites: Some Types of Fabrics (textiles)

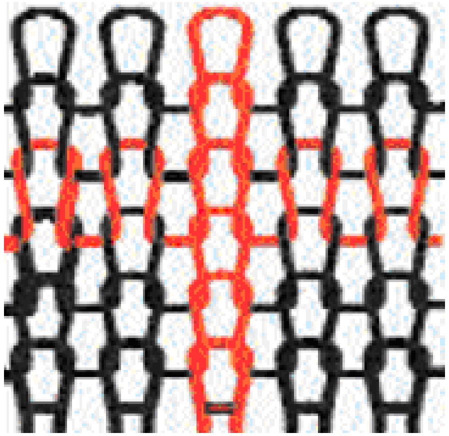
UD simpler



plain weave



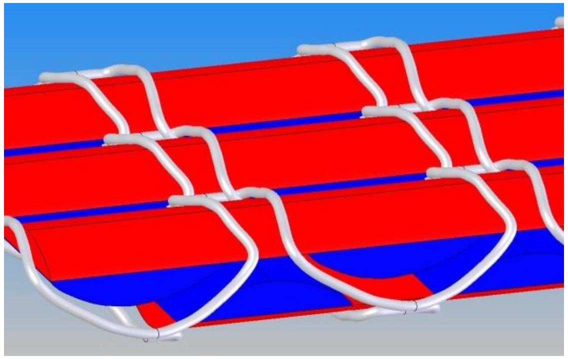
braid



weft knit

warp knit

non-crimp fabrics



Drehergewebte NCFs

Conclusions wrt Failure Mode Concept

- **The FMC is an efficient concept,**

  that improves prediction + simplifies design verification

  is applicable to brittle and ductile, dense and porous,
  isotropic, transversely-isotropic and orthotropic materials

  if clear failure modes can be identified and if the material element can be homogenized.

  Formulation basis is whether the material element experiences
  a *volume* change, a *shape change* and *friction* .
  Builds* not on the *material* but on *material behaviour !

- **Delivers a combined formulation of *independent modal failure modes*,**

  without the well-known drawbacks of global SFC formulations
  (which *mathematically combine in-dependent failure modes)* .

- **The FMC-based Failure Conditions are simple but**
  describe physics of each single failure mechanism pretty well.

- **Mapping of brittle behaving materials was successful, SFC models became validated**
  Some new findings were provided !

- **Cuntze‘s FMC enables to determine equivalent stresses, desired for design decisions!**

Conclusions wrt Isotropic Strength Failure Conditions (SFCs)

- **A SFC shall and can only describe a 1-fold occurring failure mode.**
- **A multi-fold occurrence must be additionally considered in the formulas:**

  2-fold $\sigma_{II} = \sigma_I$ **(probabilistic effect) is elegantly solved with** J_3

  3-fold $\sigma_{II} = \sigma_I = \sigma_{III}$ **(prob. effect) hydrost. compression, by closing**

- **120°-located dents of the failure body are the probabilistic result of a 2-fold acting of the same failure mode. This shape is usually described by replacing** J_2 **through** $J_2 \cdot \Theta(J_3, J_2)$**. They may be oppositely located in the *I1<0* domain to those in the *I1>0* domain**
- **The Poisson effect, generated by a Poisson ratio *v,* may cause tensile failure under bi-axially compressive stressing (dense concrete; analogous to UD material, where filament tensile fracture may occur without any external tension loading** σ_1**)**
- **Hoop Planes = deviatoric planes =** $\pi -$ **planes: *convex***
- **Meridian Planes : *not convex !***

Some Lessons Learnt w.r.t. Reliable Strength Design Verification

- **Validation of SFCs: this requires a uniform stress field at the failure-critical location**
- **All SFC-model parameters must be measurable**
- **Prediction of compressive failure (SF) of brittle behaving materials is not possible, if the physically necessary friction value μ is not available. Some global SFCs do not consider friction and therefore have a significant bottleneck when determining RFs.**
- **MIND: Failure is generated pretty locally on a lower scale than the material was homogenized on** (e.g. micro-scale)**, as we try to capture failure engineering-like by higher scale formulations !**
- **For pre-design: Exploit the knowledge from similar behaving materials !**
- **The achievement of a reliable design: This needs an <u>equally well quality</u> of**

  reliable analytical tools, solvers, test data <u>and</u> evaluating engineers !
- **Determination of modal SFC-parameters is performed in each associated pure mode domain. Global SFC-parameters are determined by a global fit over all modes**

-

Abschließend: Wozu sind die gezeigten Bruchkörper gut ?

- Zur Verständlichmachung/Bewertung mehrachsiger Bruchspannungszustände: Dazu benötigt werden (Bruch-)Festigkeitsbedingungen, die die Oberfläche des Bruchkörpers beschreiben. Diese Oberfläche definiert sich als *Einhüllende Fläche der Vektorpfeile aller Bruchspannungskombinationen*
- Belastungsreserve liegt vor (Reservefaktor RF < 1), wenn der Vektor der vorliegenden Last-Spannungen - *multipliziert mit dem Sicherheitsfaktor für die Auslegung* - kleiner als der zugehörige Bruchspannungsvektor ist.
- Visualisierung des Auftretens von Nicht-Koaxialität verursacht durch 2-fach-Modus
- "Unten" offener Bruchkörper: *Glas, Grauguß, Normalbeton, UHPC*

  (reißt gefüge-abhängig ferner noch axial, unter bi-axialem Querdruck!)
- Geschloss. Bruchkörper: *Ytong, Hebel-Stein, Lamelle*

  (Faser reißt sogar unter tri-axialem Druck !)
- Beliebig hohe 3D-Druck-Beanspruchungszustände sind auch bei porösem Werkstoff möglich, aber dann ist der Werkstoff zerbröselt.

Beliebig hohe Druck-Beanspruchungszustände, auch bei Ihnen, veranlassen ..

Für die Überlassung der aufwendig zu ermittelnden 2D/3D-Testdaten meinen besten Dank an:

Dr. Kolupaev (Fraunhofer LBF, Darmstadt) : Schaumdaten
Dr. Scheerer (IfM, TU-Dresden) : Normalbeton
Dr. Speck (IfM, TU-Dresden) : UHPC
Prof. Curbach

Theory is the Quintessence of all Practical Experience

A. Föppl

Dazu meine Erfahrung:

„Die Erzeugung zuverlässiger 3D-Testdaten ist herausfordernder als die Aufstellung einer zugehörigen zuverlässigen, auf physikalischen Überlegungen beruhenden Theorie“

Dank fürs Zuhören und Zusehen.
Es wäre schön, falls ich Sie etwas
für neue Ansätze Ihrerseits begeistern konnte.

Ihr Ralf Cuntze

Some Literature

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[Cun15b] Cuntze, R.: *Reliable Strength Design Verification – fundamentals, requirements and some hints.* 3rd. Int. Conf. on Buckling and Postbuckling Behaviour of Composite Laminated Shell Structures, DESICOS 2015, Braunschweig, March 26 -27, extended abstract , conf. handbook, 8 pages (see CCeV website)

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