Bei diesem Beitrag handelt es sich um einen wissenschaftlich begutachteten und freigegebenen („reviewten") Fachaufsatz.

R. Cuntze

Is a Costly Re-Design Really Justified if Slightly Negative Safety Margins are Encountered?

Teure Umkonstruktion bei leicht negativen Sicherheitsmargen?

Abstract *The paper intends to contribute to a general discussion: How safe is a positive Margin of Safety (MS)? What is the real meaning of a slightly negative MS value because a MS value itself does neither outline the size of risk nor specify the probability of failure? A value of MS = +4% in the one case may incorporate a higher risk than –2% in another case.*
The author's intention is to make those engineers, who are responsible for 'approving and releasing' structural designs, aware of the real sources of risk. There is no compulsory need to reject a design if a Margin of Safety becomes slightly negative, e.g. –0.5%. No part of the structure will necessarily loose its functionality or will fracture, but an outright rejection could result in high additional costs as well as in a delay of the whole project due to re-designing efforts needed. A numerical example from the aerospace industry is used here to illustrate this topic.

Inhalt *Der Aufsatz beabsichtigt einen Beitrag zur generellen Diskussion zu liefern: Wie sicher ist eine positive Sicherheitsmarge (MS)? Was ist die wirkliche Bedeutung eines leicht negativen MS-Wertes, weil dieser weder die Größe des Risikos widerspiegelt noch die Versagenswahrscheinlichkeit spezifiziert? Ein Wert von MS = +4% kann im einen Fall ein höheres Risiko beinhalten als –2% in einem anderen Fall.*
Des Autors Absicht ist, diejenigen Ingenieure auf die wirklichen Risiko-Quellen aufmerksam zu machen, die für Genehmigung und Freigabe des Bauteils verantwortlich sind. Nichts zwingt uns ein Design zu verwerfen, falls die Sicherheitsmarge leicht negativ wird, z.B. –0.5%. Kein Teil der Struktur wird zwangsläufig seine Funktionalität verlieren oder wird brechen. Aber, eine völlige Zurückweisung kann sowohl in hohen zusätzlichen Kosten resultieren als auch in einer Verzögerung des ganzen Projektes infolge des benötigten Aufwandes für die Umkonstruktion. Ein numerisches Beispiel aus der Luft- und Raumfahrtindustrie soll diesen Punkt mit Zahlenwerten beleuchten.

Author
Prof. Dr.-Ing. habil. Ralf G. Cuntze VDI,
formerly MAN Technologie AG in
D-86153 Augsburg, Germany ,
Matthaeus-Guenther-Strasse 32
D 85229 Markt Indersdorf
Phone/Fax: +49 (08136) 7754
E-mail: Ralf_Cuntze@t-online.de

1 Introduction

Engineers, especially those working in the aerospace industry, tend to start to shiver once they notice minor negative safety margins (*MS*) in strength analysis or when verifying the design strength for a so-called 'design verification'. This has often led in the past and still leads today to an over-reaction attempting to get rid of the negative value, and this is not necessarily a rational engineering response. Engineers should always bear in mind that a *MS* value is the result of *modelling* a structural problem, and modelling as such is always affected by uncertainties. Here, the term 'uncertainties' should be understood to include inaccuracies as well as any simplifications in the modelling.

Uncertainties can be found in the area of data input and in the analysis of the whole modelling process, a flow diagram of which is depicted in *Fig. 1*. Stages with uncertainties comprise load analysis, testing and test data evaluation, choice of non-linear stress-strain curve and safety concept, choice of yield condition and fracture conditions, structural analysis procedure, and finally the determination of the *MS* value itself. All these sources contribute to the overall structural **risk**, which let's define here arbitrarily as ***amount of costs*** (*direct and consequential*) ***incurred in the case of later failure times the probability that the distinct failure occurs.***

In the *stress* analysis itself as well as in the failure criterion used in the *strength* analysis there are design parameters, which can exhibit relative large uncertainties. The nature of uncertainty of these parameters (loads, strength properties, geometry, elasticity properties, etc.) is either of *mechanical* one or of *statistical* one (e.g. the way measurements are performed, lack of accurate information due to insufficient sample size in measurements of a specific design parameter). Besides this, there is always some uncertainty in the *calculation model* (e.g. solution procedure, mesh, ...) as well as in the *results* provided by *testing* and *evaluation* of 'raw test data'. Usually, loads are those design parameters with highest uncertainty. Therefore, the design parameter *load* will be looked into more details in this paper in order to emphasise the real structural risks.

For obtaining 'Product Certification' in the aerospace industry, three (quasi-static) '*Design Verifications*' have to be performed in order to demonstrate that the design possesses sufficient strength. This is achieved if *analytical* and/or *test* methods reveal both MS_{yield}, and MS_{ult} to be positive and if damage tolerance is proven (see *Fig. 2*). However, a design engineer should not make the mistake to just look at the two margins, where

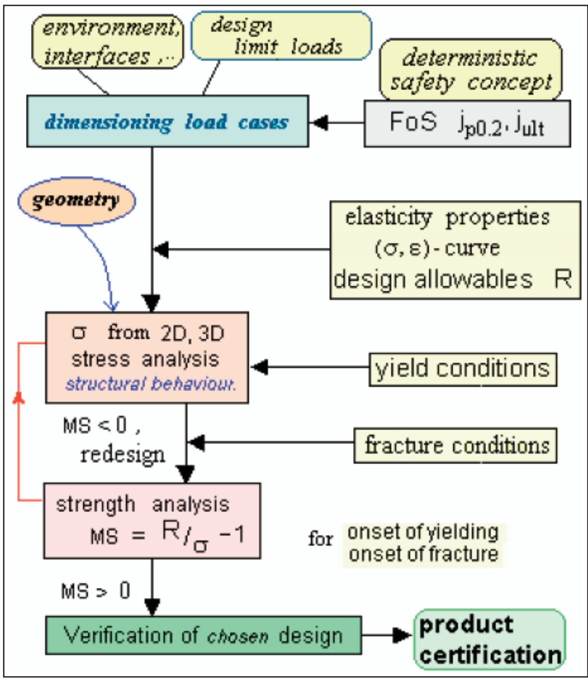


Figure 1

Flow diagram of structural analysis.
FoS:= (design) factor of safety,
***MS*:= margin of safety,**
***R*:= strength design allowable,**
σ:= stress

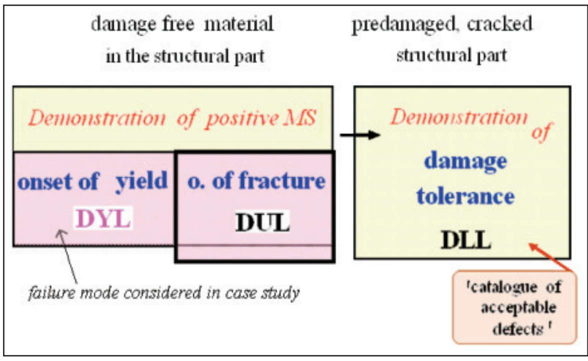


Figure 2

Static Design Verification.
***DLL*: = maximum anticipated load or combination of loads the structure is expected to experience.**
***DYL*: = Design Yield Load = *DLL* · $j_{p0.2}$, *DUL*: = Design Ultimate Load = *DLL* · j_{ult}**

one might get a slightly negative *MS* value, and forget damage tolerance. If a damage tolerance *demonstration* is to be performed, then MS_{yield} (linear analysis, low effort) together with damage tolerance could well compensate the MS_{ult} (non-linear analysis, large effort) verification.

In some standards, for example the pressure vessel code ISO14623, MS_{ult} is considered to be the *design driver*, even in the case of *ductile* materials. However, past experience with ductile materials is in contradiction with such consideration as here MS_{yield} should be the design driver (with few exceptions depending on the required *Factor of Safety*). Until now, there is no general agreement in industry on a unique procedure how to determine MS_{ult} accurately and the various approximation procedures applied tend to *under*estimate the real value by many percents.

*Note: Why should it not be possible to directly accept a slightly negative (≤ **1%**) MS_{ult} margin from an underestimating procedure in the case of ductile behaviour as it is known from mechanics that MS_{yield} < MS_{ult} applies? In case of a negative MS_{yield} one has to treat this situation differently, as will be shown.*

Special aim of the paper is 'the sensitization of the engineer for the actual risk'. This will be hopefully achieved by presenting quantitative margin of safety values within a case study which compares the usual deterministic and the stochastic processing way.

2 The Design Parameter "Load"

Taking the aerospace industry as an example, the first task in any load analysis is to establish all load events the structure is likely to experience in later application. This includes estimating all induced thermal, mechanical (static and dynamic) and acoustical environment of the structure as well as the corresponding lifetime requirements (duration, number of cycles), as specified by an authority or a common standard. In the next stage the so-called Limit Load (*LL*) values are determined, usually derived from mission simulations utilizing the so-called mathematical models (at first on basis of the preliminary design) of the full structure. The ARIANE 5 General Specification A5-SG-1-10, e.g., defines the *LL* as follows: a) if a statistic dispersion of the single load is known, the *LL* is at

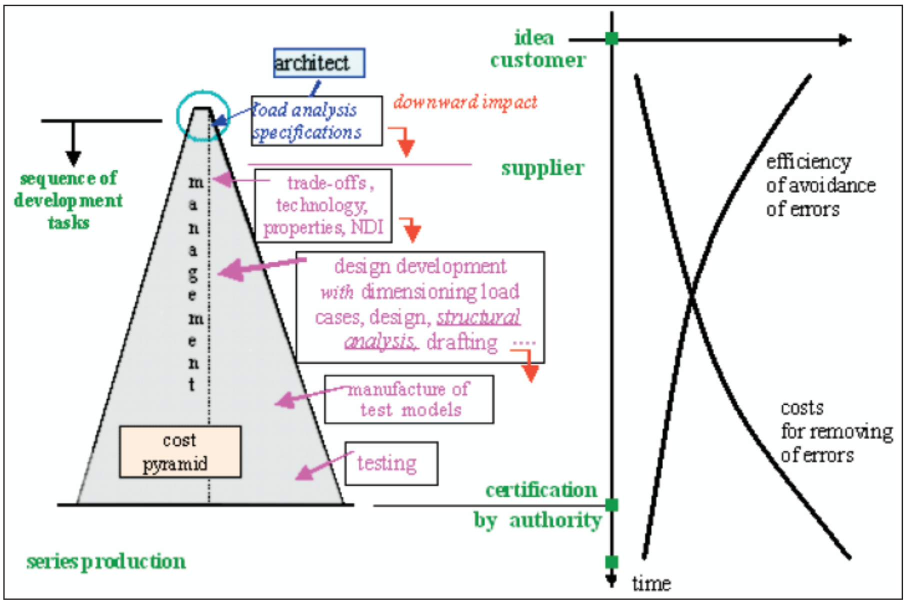
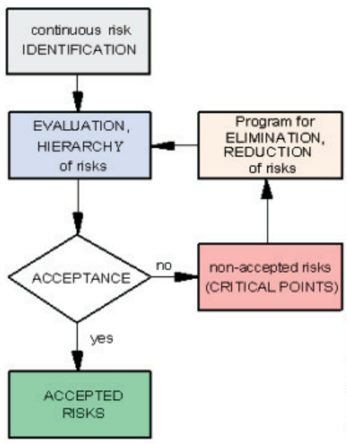


Figure 3

'Cost pyramid' in structural analysis respecting the effect of re-design



effect on:		safety		mission objective	performance availability	no effects
Severity		catastrophic 0_A	fatal 0_B	major 1	minor 2	negligible 3
Probability	P = / S =	16	8	4	2	1
very probable	16	256	128	64	32	16
probable	8	128	64	32	16	8
less probable	4	64	32	16	8	4
improbable	2	32	16	8	4	2
very improbable	1	16	8	4	2	1

Figure 4

Process of risk control and risk assessment in Structural development. Scheme. Risk = P x S [Gri04]

least equal to the load having a 99 % probability of not being exceeded in service with a confidence level of 90 %.; b) if a statistic dispersion of the load is unknown, the *LL* is determined on the basis of a rational conservative estimate.

Following that, the *LL* values may have to be multiplied by an assumed factor *K* (of the magnitude of the Factors of Safety *j*) which represents the uncertainty of the *LL* estimate. Additional project factors, due to uncertainties in the project targets, have to be included here. The uncertainty factor *K* should decrease with maturity of the design of the structure. The product *LL* • *K* becomes the so-called Design Limit Load (*DLL*).

The *DLLs* are the input for the design engineer and they are the largest anticipated loads, which a structure may have to sustain during its service life in association with the applicable operating conditions. In case of load combinations the statistic definition above is applicable for the resulting load combination too, however it is not used in the usual deterministic design due to the higher complexity compared to single loads. In the probabilistic design this statistic definition can be considered It has a mass saving effect.

In the next stage, from all potentially design-relevant load combinations all those have to be extracted which are finally really design decisive. They are derived from the probably hundreds or thousands of load combinations by engineering judgment or by Finite Element Analysis (in case of linear behaviour possible) achieved by computing all combinations.

In practise often fast decisions have to be made during the design development phases and for these only a reduced number of really essential design driving load cases are required. These cases are referred to as *Dimensioning Load Cases*.

The higher the uncertainty is the more likely errors will be made and errors will result in additional costs. *Fig. 3* depicts a cost pyramid, which illustrates the individual tasks in structural analysis, their impact, shares in costs, and which effects potential errors can have. The obvious conclusion with respect to the load parameter is that reducing the load uncertainty pays off most and it also improves the accuracy of the MS values.

Note: Very often the estimate of loads is the design parameter which carries the highest uncertainty of all. Improving accuracy of load estimation could eliminate slightly negative safety margins. Here, for the sake of simplicity the scattering load is modelled by a normal distribution.

3 Basics on Safety Concepts

3.1 General on Risk and Reliability

In aerospace industry RAMS (Reliability, Availability, Maintainability, Safety) activities support engineering and management decisions to identify and evaluate technical risks. In the last two decades a major trend in structural design has been the promotion of structural reliability principles to account for the uncertain nature of the design parameters. However, a number of practical difficulties, including lack of suitable design data and the inability to undertake design optimisation in a rational and efficient way, preclude their widespread application.

Reliability comprises "Reliability in Operation" and "Integrity". "Reliability in Operation" itself covers the characteristic, which refers to the functional behaviour of the structure over a given time of operation. "Integrity" covers the capability of the structure to withstand the functional, operational, and environmental loads during a given duration of use.

In order to achieve structural integrity, adequate non-destructive inspection and control procedures need to be applied during manufacturing of the hardware. This is essential to ensure that deviations or imperfections in excess of those being initially tolerable are reliably excluded.

As far as safety is concerned, major efforts have to focus on risk identification, risk assessment and management with the understanding that risk is linked to the probability of loss of human life and/or damage to or loss of equipment/property. Risk management is ensured through a so-called critical points list, which categorises items with respect to the consequences of their worst-case effects. *Fig. 4* depicts the main elements of a risk control process. These are the elements identification, evaluation/hierarchy, reduction/elimination, and acceptance [Gri03]. Finally, *Fig. 4* illustrates the interdependence of the risk (criticality) level on the severity of the failure and its probability of occurrence. This picture demonstrates that one should always consider the real risk entities and not view just the margins of safety.

Note: The essential question with respect to all uncertainties is whether these increase the risk to an unacceptable level or not. This has to be evaluated and could result in avoiding additional costs [Sar97].

3.2 Choice of Factors of Safety (FoS)

In the design process the scatter of individual values and parameters is usually dealt with by using fixed deterministic *Factors of Safety*, which act as load increasing multiplying factors and should be called, more correctly, *design FoS*. According to the uncertainties (in structural analysis, manufacturing process, modelling, material properties, failure criteria etc. but not loads anymore if the above mentioned K-factor principle is applied) suitable *FoS* 'j' are taken in the design input to ensure reliability of the structural part and, these *FoS* are chosen based on long term experience with structural testing. Depending on the risk consequences different classes of *FoS* are applied, e.g. for manned spacecrafts higher *FoS* are used than for unmanned spacecrafts.

The previously mentioned *DLLs* have to be multiplied by the *FoS* in order to obtain the Design Ultimate Load (*DUL*) or the Design Yield Load (*DYL*). All three loads are distinct, but different design loads: *DLL* is the load level for fatigue analyses, *DYL* for onset of yielding (= global deformation limit), and *DUL* for ultimate fracture analyses. Dimensioning aspects require that the structure has to withstand

- Design Yield Load = $DYL = j_{p0.2} \cdot DLL$ without detrimental deformation and
- Design Ultimate Load = $DUL = j_{ult} \cdot DLL$ without collapse.

The *FoS* $j_{p0.2}$ and j_{ult} determine the design yield limit and the design ultimate limit. E.g. in aerospace, a relatively low value of $j_{ult} = 1.25$ is taken if a static qualification test is foreseen. In case of untested hardware or 'design by analysis' a higher value is required, e.g. a value of *1.5*.

In case of high pressure vessels the so-called Maximum Design Pressure (*MDP*) (it should be better termed Design Limit Pressure (*DLP*)) is treated similarly [ECSS].

Deterministic	Probabilistic	
det R-S problem	**prob R-S problem**	**general probabilistic**
$\sigma_{eq}(p_{int}, d, t)$, $daR_{p0.2}$	COMREL-reduced problem $2X_j$: $\sigma_{eq}, R_{p0.2}$ distributions	$4X_j$: $p_{int}, R_{p0.2}, d, t$ distributions
worst case scenario *values for* determination of σ_{det}	probability of the stochastic design parameters to act together is considered by utilizing the *distributions for* determ. of σ^*_{prob}, σ_{prob}	
comparison of σ_{det} with $daR_{p0.2}$	comparison of σ^*_{prob}, σ_{prob} with $da\,R_{p0.2}$	--
MS_{det}	p_f, $2\alpha_j$; MS^*_{prob}, MS_{prob}, from handbook, MATHCAD, or	p_f, $4\alpha_j$ COMREL-computed

Table 1

Scheme for deterministic and probabilistic *MS* determination

R = strength, S = applied stress

Example: Cylindrical part of a pressure vessel

Note: FoS are based on long engineering experience. The reliability assessment of the FoS, applied e.g. in aerospace, outlined that they are located in the failure probability regime of 10^{-7} through 10^{-9}. Different industry, however, has different risk acceptance attitudes [Rac04].

4 Margin of Safety (MS) in Relation to Structural Reliability

In the design process any structure is first being dimensioned and then checked for each *single* strength failure mode, thereby distinguishing various cases, e.g. *onset of yielding* and *ultimate fracture*. Each of the required two *MS* computations has to demonstrate a positive margin, $MS \geq 0$, otherwise the structure is not considered to have met the strength design requirement.

In the following, a probabilistic analysis is used to evaluate the deterministic value *MS*. Onset of yielding is the driving failure mode (because the usually applied materials are ductile behaving ones) and is considered as the only failure mode in this example.

The various computations necessary to evaluate $MS = MS_{det}$ and compare it to an MS_{prob} and a failure probability p_f are delineated in *Table 1*. The derivation of the probabilistic data is performed in *Annex 1*.

4.1 Theoretical Background

4.1.1 The Multi-stochastic parameter Problem

Conservatism in the traditional design is taken into account by assuming a worst case scenario with respect to loading, temperature and moisture, and undetected damage. Furthermore, design parameters are not treated as stochastic design parameters, which can have an influence for the actual failure mode considered, [Cru94, Cun88, Guo97, Gri89, Rac86, Ver92]. As the scattering design parameters, e.g. loads and strengths, are stochastic, structural reliability analyses have to be carried out in order to verify, that the 'possibility for failure', quantified by the *failure probability* p_f, does not dramatically increase if *MS* turns slightly negative. This requires to compute p_f or to make a close estimate of it using the equation [Tho82, Sch81]

$$p_f = P(R \leq S) = P(R - S \leq 0) = \\ = P\left[\left(g(X_j)\right)\right] \leq 0 = \int_{Vol} f_{\underline{x}}(\underline{x})\,dx, \qquad (1)$$

a multi-dimensional (convolution) integral over the multidimensional density function $f_{\underline{x}}$, which is to be integrated in the failure domain $g \leq 0$ (means failure state), i.e. when the stress *S* exceeds the strength *R* (*see Annex 2*). The function $g(X_j) = g(\underline{X}) = 0$ is called limit state function. The vector $\underline{X}$ includes all stochastic design parameters, frequently also denoted as basic random variables.

For this computation task an effective numerical method is necessary. Best known amongst the analytical methods is the first-order reliability method (FORM, see [Com87]), which uses a linear approximation to the limit state surface g = 0 and computes the *most likely failure point*, the so called x^*-point or β-point, [Tho82, Has74]. This is the point of the limit state that marks the minimum distance from the origin in the so-called standard normal space, see [Sch81]. This minimum distance, as defined by Hasofer and Lind [Has74], is termed β, the so-called *reliability index*, being the percentile of the standard normal distribution Φ of Gauß

$$p_f = \Phi(-\beta) = 1 - \Phi(\beta) \quad \text{and} \\ \Re = 1 - p_f = \Phi(\beta) \qquad (2)$$

with $\Re$ the survival probability. For any given β the corresponding value for Φ can be taken from a statistical handbook such as [Gra66] or can be determined by a commercial mathematical code like MATHCAD.

The influence of the stochastic design parameters, also termed uncertain basic variables, on the failure mode is best shown in terms of the co-called sensitivity measures, the *α-values*. An *α-values* is a normalised $\left(\sum \alpha_i^2 = 1\right)$ value showing the influence of the individual basic variable on the failure event. The α_i are derived from the gradient of the limit state function g with respect to the uncertain basic variables at the most likely failure point x^*, and may be interpreted as an effective percentage, see *Annex 1*. A positive sign indicates an increase of $\Re$ if the mean value μ of the basic variable is increased and a negative sign (typically found for load variables) indicates a negative influence on $\Re$ for an increasing mean load value.

4.1.2 The Two-stochastic-parameter Problem or Strength-Stress Problem

In case of the most simple limit state, $g(X_j) = R - S \equiv$ *load resistance R – load S* or, if linear analysis is permitted, $\equiv$ *strength R – stress S*, one has to treat just two basic variables, one *R* and one *S* variable. Assuming a normal distribution for both of these basic variables *R* and *S*, the measures β and p_f can be derived by utilizing the formulation

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{\mu_Z}{\sigma_Z}, \quad \text{and}$$
$$\Phi(\beta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta} e^{-t^2/2} dt, \tag{3}$$

with- μ:= mean value, σ:= standard deviation of parent distribution. Details for the development of this derivation are given in [Sch81, S. 95 and Tho82, page 72]. For this simple '(*R – S*) problem' the desired comparison of deterministic *MS* values with probabilistic values of β or p_f is possible.

MS can be formulated on stress level and on load level (correct version)

$$\begin{aligned} MS &= \frac{\mu_R - k_R \cdot \sigma_R}{j \cdot (\mu_S + k_S \cdot \sigma_S)} - 1 = \\ &= \frac{\textit{minimum load resistance}}{\textit{design load}} - 1 = \\ &= \frac{\textit{load resistance 'design allowable'}}{\textit{design load}} - 1 = \\ &= \frac{daLR}{\textit{design load}} - 1. \end{aligned} \tag{4a}$$

According to the high measurement cost, the design allowable *daLR* (abbreviated *da* to discriminate for instance $daR_{p0.2}$ from the equally termed distribution of the stochastic parameter $R_{p0.2}$) for the load resistance of a structure is barely determined. A test series on pressure vessels may be an exception. Therefore, the load resistance 'design allowable' is defined to be that load when the associated equivalent stress σ_{eq} in the structure achieves the level of the design allowable $R_{p0.2}$ or R_m, respectively.

A stress level formulation may be utilized if a linear structural analysis is permitted:

$$\begin{aligned} MS &= \frac{\mu_R - k_R \cdot \sigma_R}{j \cdot (\mu_S + k_S \cdot \sigma_S)} - 1 = \\ &= \frac{\textit{strength 'design allowable'}}{\textit{stress at design load}} - 1. \end{aligned} \tag{4b}$$

This is the case, when high *FoS* are required such as for pressure vessels in nuclear industry.

To achieve a reliable design the design allowable has to be applied. This is a value, above which at least 99 % ("A" value, application of military Safe Life Concept) or 90 % ("B" value, application of Damage Tolerance Concept in case of multiple load paths, redundancy) of the population of values is expected to fall, with a 95 % confidence level.
The factors k_R and k_S respect this. They are to be chosen according to reliability goals, set for the actual project, and extracted from statistical tables (see e.g. [HSB]). They further depend on the stochastic model (normal distribution, Weibull distribution, etc.), the statistical basis (sample size n, one- or two-sided tolerated), and on the desired confidence level for the transfer of the sample population to the basic population. Specifics may be extracted from the following numerical example.

Note 1: The new possibilities to disclose the risk characteristics must be of interest for a design engineer and he should apply structural probabilistic tools, whenever reasonable and possible with respect to the needed input.

4.2 Numerical Example: Cylindrical part of a pressure vessel

4.2.1 Modelling

In the above risk-disclosing context a simple case study shall be employed: the cylindrical part of a safety critical pressure vessel. In order to demonstrate the effect of the scatter of the stochastic design parameters X_i (the associated vector reads $\underline{X}$) on the MS value a probabilistic computation is performed by computing the previously mentioned convolution integral applying the code COMREL [Com87], see *Table 1*.

The input consists of the data for the *mechanical* model, the *stochastic* model, and of the associated modelling equations:

– ***Mechanical model:***
– The cylindrical part represents a thin-walled tube. It is, due to its membrane state of stress, a non-redundant element. For the derivation of the stress the so-called 'vessel formula' is applied (linear analysis shall be permitted for reasons of simplification)

$$\sigma_{hoop} = p_{int} \cdot d/2t, \; \sigma_{ax} = \sigma_{hoop}/2, \tag{5}$$

with the internal pressure p_{int}, the diameter *d*, and the thickness *t* as input quantities.
– 'Onset of Mises Yielding' was considered as single failure mode. This delivers

$$\begin{aligned} \sigma_{eq}^{Mises} &= \sqrt{\sigma_{ax}^2 + \sigma_{hoop}^2 - \sigma_{ax}\sigma_{hoop}} = \\ &= \sigma_{hoop} \cdot \sqrt{3/4} = \left(\frac{p_{int} \cdot d}{2t}\right) \cdot \frac{\sqrt{3}}{2}, \end{aligned} \tag{6}$$

– Limit state function

$$\begin{aligned} g(\underline{X}) &= R_{p0.2} - \sigma_{eq} = \\ &= R_{p0.2} - \frac{p_{int} \cdot d}{4t}\sqrt{3} = 0. \end{aligned} \tag{7}$$

Stochastic model:
– The four $X_j = R_{p0.2}, p_{int}, d, t$ are assumed to be statistically independent. For these stochastic design parameters their means μ_j, standard deviations σ_j, and density distributions f_j are to be provided (see *Table 2*). For simplicity, all these uncertain basic variables are assumed to follow a normal distribution (abbreviated ND).
– The statistically based *k*-factors shall be specified (intentionally arbitrarily) as $k_R = 3$ and $k_S = 2.3$. The numbers for k correspond to a so-called one-sided percentile and a probability of survival of *99.86 %* in case of k_R and of *98.9 %* in case of k_S. For details the reader is referred to handbooks of statistics, *e.g.* Graf/Henning/Stange or HSB.
– Geometrical tolerances are distributions, often truncated by the applied non-destructive inspection method. For reasons of simplicity they are taken here as a full normal distribution. This means that the tails are accounted for in the probabilistic calculation. It is common practice to assume the tolerance band width equals four standard deviations σ.

Safety Requirements:
Usually assigned values of failure probability – for such a structural element that may fail – are on the level $p_f = 10^{-7}$. Assuming that the technical specification requires a failure probability of $p_f < 2 \cdot 10^{-7} \equiv p_f^{admissible}$, a probabilistic analysis of the cylinder can be conducted for an assigned target value for the reliability of

j	X_j	μ_j	σ_j	f_j	CoV_j
1	p_{int}	6.00 bar	0.36 bar	ND	6 %
2	$R_{p0.2}$	442 MPa	22.1 MPa	ND	5 %
3	d	5000 mm	50 mm	ND	1 %
4	t	4.40 mm	0.05 mm	ND	0.1 %
		assumed basic populations			

input

α_j	x_j of x^*	p_f
-0.620	7.1 bar	$1.8 \cdot 10^{-7}$
0.762	356 MPa	
-0.122	5031 mm	
0.141	4.36 mm	

output COMREL

Table 2

Input and solution for the 4 parameter problem. Input for determination of *MS*, p_f (or β), and output of the coordinates x_j and of sensitivity measures α_j of the COMREL-computed most likely failure *point* x^*. *CoV:= Coefficient of Variation* = σ/μ. *ND := normal distributed*

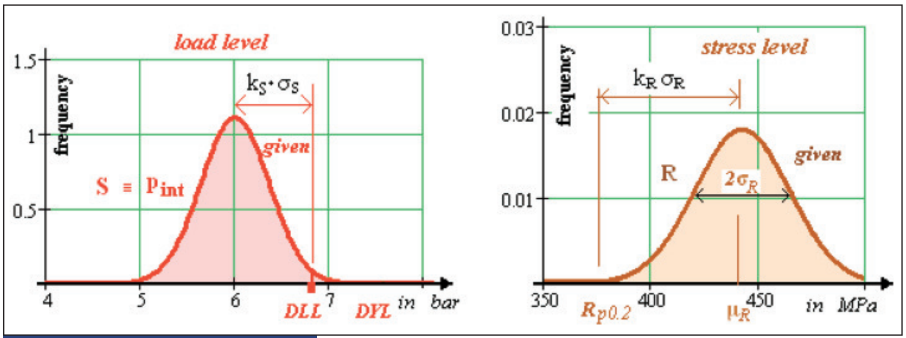


Figure 5

Deterministic Design Verification
Measured density distributions of a) load S = 'internal pressure', and of b) strength R.
Here, σ = standard deviation

– probabilistic:
$\Re = 1 - p_f = 1 - 2 \cdot 10^{-7}$. (8)
The probabilistic approach above is to be compared with the conventional deterministic one. The latter one is defined by (*CoV*:= coefficient of variation)

– conventional: $DLL = \mu_S + k_S\,\sigma_S$,
$daR_{p0.2} = \mu_R - k_R\sigma_R = \mu_R(1 - k_R\,CoV_R)$,
$j_{p0.2} = 1.1$ (*assumed here to be required by a standard*). (9)

Note: There is no 100 % safety existing. However, can we imagine what the difference is to an assigned reliability value of for instance 99.999 %?

4.2.2 Solution and Results

As the objective here is to compare the results of a deterministic determination of the MS with those of a probabilistic one, it is necessary to point out the essential differences between the two procedures. While the deterministic procedure chooses the worst combination of design parameters (e.g., minimum strengths and maximum loadings), the probabilistic procedure considers all combinations of the stochastic design parameters taking into account their respective probability.

One has to bear in mind, that not all design parameters are necessarily stochastic. For example, an elasticity modulus may be a constant and such a constant design parameter (no combinations have to be respected) will affect the results similar as $R_{p0.2}$ would do, if kept constant in *Eq. 7*.

Deterministic Solution:

Figure 5 shows the density distributions of the load 'internal pressure' p_{int} and of the strength *R*. Input data for the computations (*1 bar = 0.1 MPa*) are the

- design loads:

$DLL = \mu_L + 2.3\,\sigma_L = 6.0 + 2.3 \cdot 0.36 = 6.83$ bar,
$DYL = j_{p0.2} \cdot DLL = 1.1 \cdot 6.83 = 7.51$ bar $= 0.751$ MPa $\equiv p_{int}$

- yield strength design allowable:

$daR_{p0.2} = \mu_R - 3\,\sigma_R = 442 - 3 \cdot 22.1 = 376$ MPa,

- geometry:

$maxd = 5000 + 2\sigma = 5000 + 50 = 5100$ mm,
$mint = 4{,}40 - 2\sigma = 4.40 - 0.10 = 4.30$ mm

- k-factors, chosen: $k_S = 2.3$, $k_R = 3$.

This yields the deterministic worst case-based 'Mises equivalent stress'

worst case *det*

$$\sigma_{eq}^{Mises} \equiv \sigma_{det} = \left(\frac{p_{int} \cdot d}{4t}\right) \cdot \sqrt{3} = \left(\frac{DLL \cdot \cdot j_{p0.2} \cdot max\,d}{4 \cdot mint}\right) \cdot \sqrt{3} = 386 \text{ MPa}, \quad (10)$$

and a margin of safety of

$$MS_{det} = \frac{\text{strength 'design allowable'}}{\text{stress at design load}} - 1 = \frac{daR_{p0.2}}{\sigma_{det}} - 1 = \frac{376}{386} - 1 \equiv -2.6\,\%. \quad (11)$$

Mind: Results are
$\Rightarrow \sigma_{det} = 386$ MPa and $MS_{det} = -2.6\,\%$.

The second part of this paper will be published in the next edition of the journal.