München, 2. 8. 2019,

PART 2: Novel Fatigue Lifetime Prediction for Brittle Materials

by using

Strength Failure Mode-linked Modeling of Loading,
a Mode-linked Basic (master) S-N Curve,
the Application of a Strength Mode-linked variation of
Kawai's 'Modified Fatigue Strength Ratio' for estimating further S-N curves

- brittle material behavior such as isotropic grey cast iron, transversely-isotropic UD material

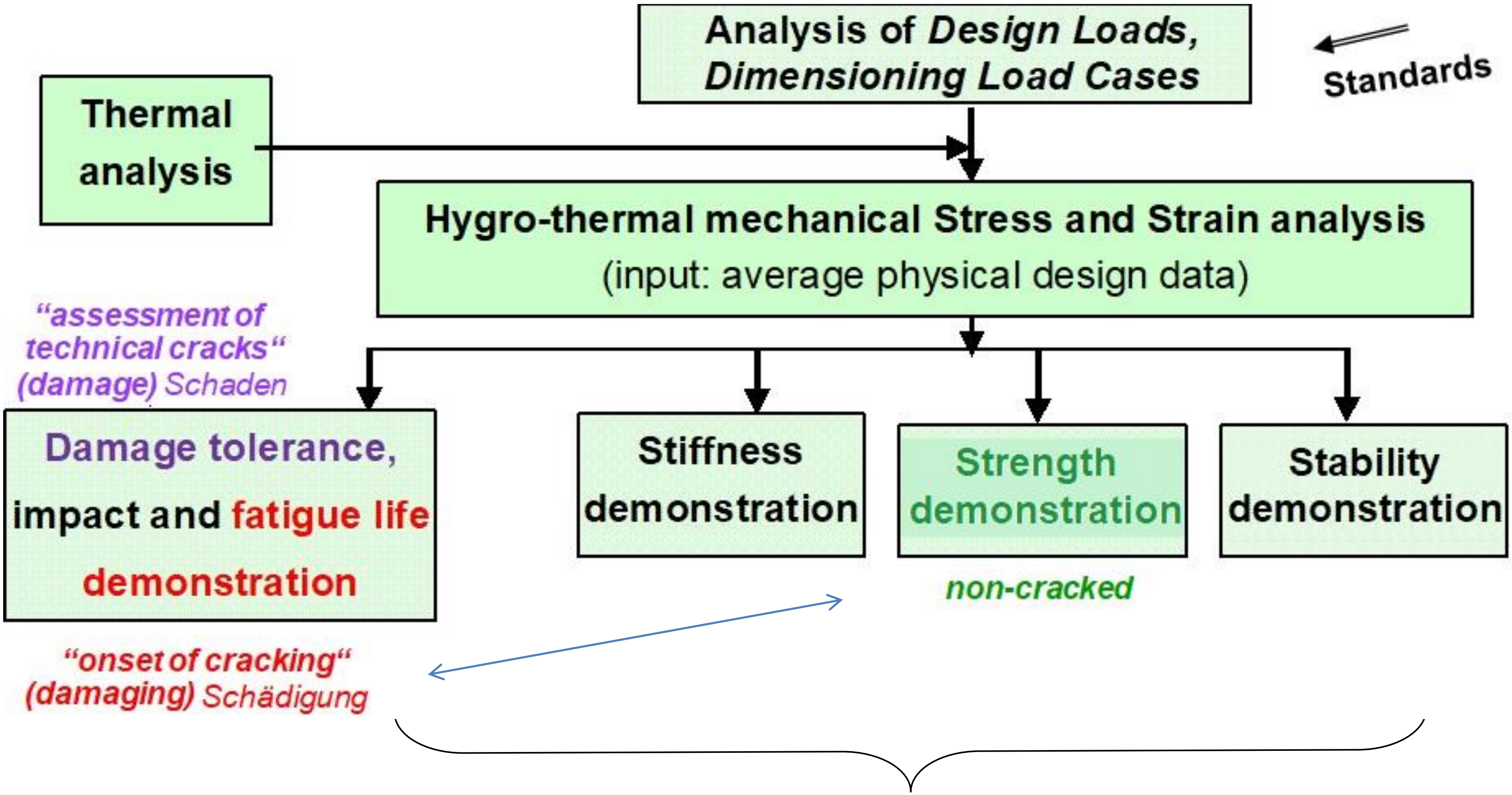
1 Introduction to Static and Fatigue Design
2 Cuntze's ***Failure Mode Concept-based*** Strength Criteria
3 Cuntze's ***Fatigue Lifetime Prediction Concept***
4 Generation and ***Novel Interpretation of UD Haigh Diagrams***
5 Steps of the Proposed *Fatigue Lifetime Prediction Concept*

Presentation of a never funded hobby-investigation in fatigue.

Prof. Dr.-Ing. habil. Ralf Cuntze VDI, formerly MAN-Technologie AG
linked to Carbon Composite e.V. (CCeV) Augsburg

Flow Diagram: Structural Design and Design Verification

Nachweise



Analysis of *Design Loads, Dimensioning Load Cases*

Standards

Thermal analysis

Hygro-thermal mechanical Stress and Strain analysis

(input: average physical design data)

"assessment of technical cracks"
(damage) Schaden

Damage tolerance, impact and fatigue life demonstration

"onset of cracking"
(damaging) Schädigung

Stiffness demonstration

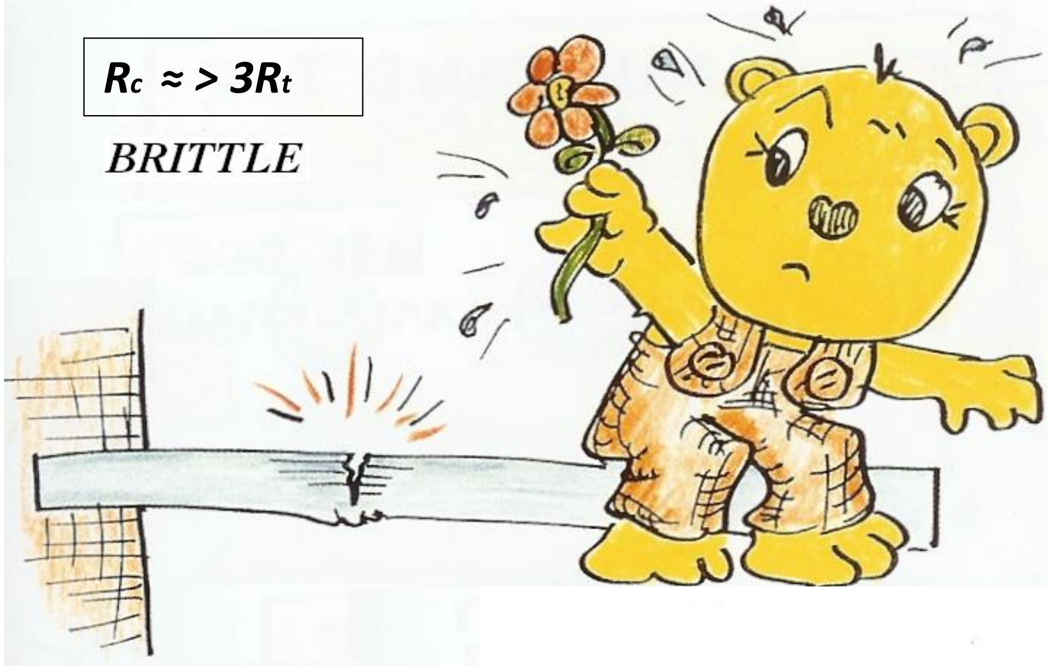
Strength demonstration

non-cracked

Stability demonstration

'Resistances', to be demonstrated
by a Reserve factor *RF* ≥ 1 or a *positive Margin of Safety MoS ≥ 0*
in order to achieve Structural Integrity *!*

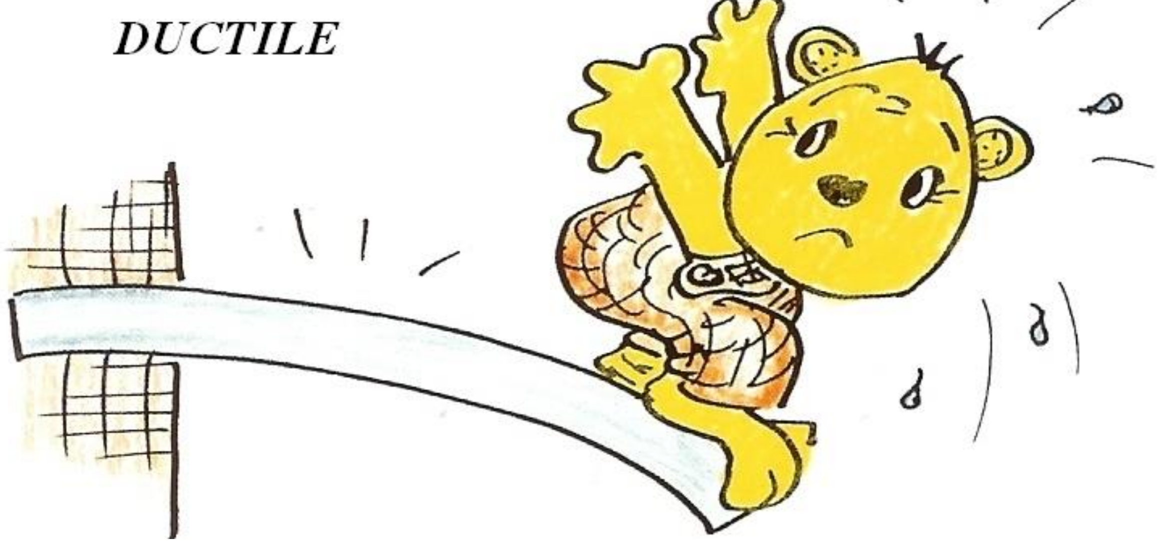
How may one principally discriminate *Material Behaviour* ?



$R_c \approx > 3R_t$

One feels good until sudden fracture occurs

Courtesy: Prof. C. Mattheck



Ductile Fracture =
type of failure in a material or a structure generally preceded by a large amount of plastic deformation

Design Verification by theoretical prediction

STATIC :

- Reserve Factor is load-defined :

$$RF = \frac{\text{Predicted Failure Load}}{j \cdot \text{Design Limit Load}} > 1.$$

Material Reserve Factor :

$$f_{Res} = \frac{\text{Strength Design Allowable}}{\text{Stress at } j \cdot \text{Design Limit Load}} > 1.$$

Werkstoffanstrengung: Eff = 1 / f_{Res} < 100%

CYCLIC :

- *$RF_{life, \text{ Predicted Lifetime}}$*
- Determination of **Inspection time**
- Determination of **Replacement time**

$$RF_{life} \approx \frac{\text{Predicted Lifetime}}{j_{life} \cdot \text{Design Limit Lifetime}} > 1.$$

Eff := accumulated static damaging portions under increased loading.
is applicable in linear and non-linear analysis.
j := design factor of safety

Some Definitions needed for Modeling *What is ?*

Material : homogenized (smeared) model of the envisaged complex material which might be a material combination

Failure : structural part does not fulfil its functional requirements such as FF = fiber failure, IFF = inter-fiber-failure (matrix failure), leakage, deformation limit (tube widening, delamination size limit, ..) ⇨ **= a project-defined ‘defect‘**

Fatigue : process, that degrades material properties

Fatigue Life Stages (1) accumulation of damaging until initiation of a critical damage size (classical fatigue life prediction domain), (2) damage growth until onset of final fracture (domain of damage tolerance concepts), (3) separation (not of interest)

Damaging (not also damage, as used in English literature) **:** process wherein the results, the damaging portions, finally accumulate to a damage size such as a macro-scopic delamination. Accumulation tool usually used is *Palmgren-Miner‘s Damaging Accumulation Rule* (= model)

Damage : sum of the accumulated damaging or an impact failure, that is judged to be critical. Then, *Damage Tolerance Analysis* is used to predict damage growth under further cyclic loading or static failure under Design Ultimate Load

Haigh Diagram : involves all S-N curves required for fatigue life prediction.

Some Notions

Stress Life Fatigue **(preferred for brittle bahaviour)** ← approaches → **Strain Life fatigue** **(ductile behaviour)**

Fatigue limit, endurance limit, and fatigue strength:

expressions, used to describe a cyclic property of a material

Behaviour:

- brittle : max stress (Oberspannung), σ_{max} is responsible for damaging
- ductile: amplitude stress σ_a *is* responsible for damaging (slip)

S-N curve (Wöhlerkurve): $R = \sigma_{min} / \sigma_{max}$

σ_a *and* σ_{max} (if brittle) are used

stress-life fatigue curve of a material, in terms of fracture cycles *N*, for a distinct applied stress $S \equiv \sigma(N)$. *(Note: Renders the weakening of a repeatedly loaded material)*

Haigh Diagram:

stress amplitude $\sigma_a(\sigma_m, R)$ is used

Mean stress σ_m influence f_M *of isotr. materials: prediction on basis of 2 test points* $(\sigma a_{R=-1}, 0), \sigma a_{R=0}, \sigma a_{R=0})$,

$f_M = \sigma a_{R=-1} / \sigma a_{R=0}$, *represents the slope*

Global versus Modal Strength Failure Conditions SFCs (criteria)

Example: UD

zwangsverheiratet

1 Global *strength* **failure condition** : F ({σ}, {*R*}) = 1 (usual formulation)

Set of Modal **strength failure conditions** : F ({σ}, R^{mode}) = 1 (addressed in FMC)

vector of 6 stresses (general)

$$\{\sigma\}=(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{31},\tau_{21})^T$$

vector of 5 strengths

$$\{R\}=(R_{||}^t,R_{||}^c,R_{\perp}^t,R_{\perp}^c,R_{\perp||})^T$$

needs an Interaction of Failure Modes:
performed here by a *series failure system model*

each failure mode is reigned by one associated strength

Understanding the terms **Material Stressing Effort and Equivalent Stress**

Helpful "To turn the right screw" in design is the delivery of

equivalent stresses and of **material stressing efforts *Eff***

mode **material stressing effort * (in German "Werkstoffanstrengung")**

The relationship is

$$Eff^{\text{mode}} = \sigma_{eq}^{\text{mode}} / R^{\text{mode}}$$

anology to 'Mises'

$$Eff^{\text{fracture mode}} = \sigma_{eq}^{\text{fracture mode}} / R_m$$

$$Eff^{\text{Mises}} = \sigma_{eq}^{\text{Mises}} / R_{po.2}$$

mode equivalent stress

mode associated average strength ***(in test data mapping bar over $\bar{R}$)***

- **material stressing effort** *Eff* = *artificial technical term , created together with QinetiQ, UK, during the World-Wide-Failure-Exercises*

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Notes:

* ***Cuntze's Failure-Mode-Concept-based Static Strength Criteria*** **are applicable to isotropic, transversely-isotropic UD and orthotropic woven materials**

* all model parameters used are measurable: the strengths R_m and friction values μ

* the interaction exponent m is estimated due to mapping experience

* an artificial english denotation for the self-explaining German term Werkstoffanstrengung was created with QinetiQ during the WWFE reading material stressing effort *Eff*

Basic Features of the author‘s Failure-Mode-Concept (FMC), 1995

plus a confirmation that transversely-isotropic UD Materials exhibit a 5-fold material symmetry characteristic = 5 Strengths, 5 Failure Modes, 5 elasic properties

---

- **Each failure mode represents 1 independent failure mechanism**

  and thereby 1 piece of the complete *failure surface*

- **Each failure mechanism is governed by 1 basic strength** (is observed!)

- **Each failure *mode* can be represented by 1 failure *condition.***

Therefore, equivalent stresses can be computed for each mode !! This is of advantage when deriving S-N curves and Haigh diagrams with minimum test effort.

Consequently, the FMC-approach requires :

the *interaction of all 5 Modal (fracture) Failure Modes !*

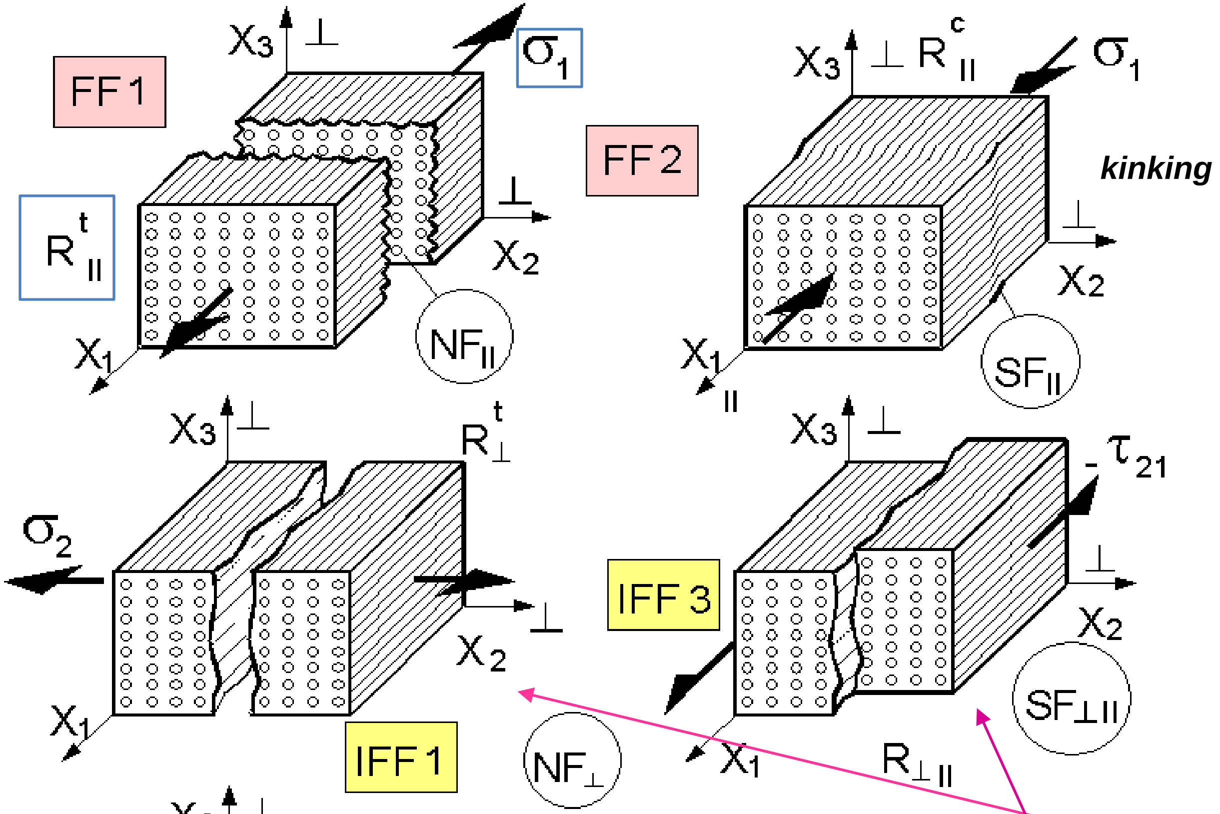
This modal thinking in Static Loading can be kept for Cyclic Loading !

Observed ! Fracture Morphology of Transversely-isotropic UD Material

[Cun04]

t = tension

c = compression



▶ 5 Fracture modes exist

= 2 FF (Fiber Failure)

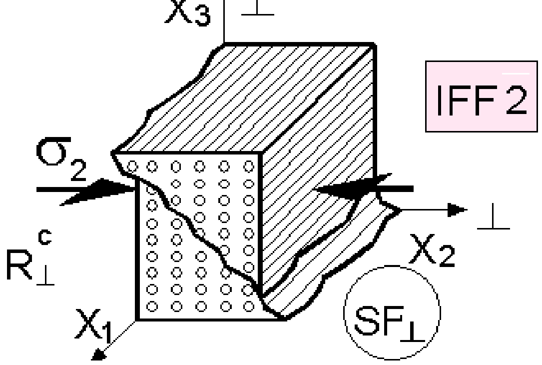
\+ 3 IFF (Inter Fiber Failure)

critical in a loaded laminate:
FF1, FF2 + possibly IFF2 !

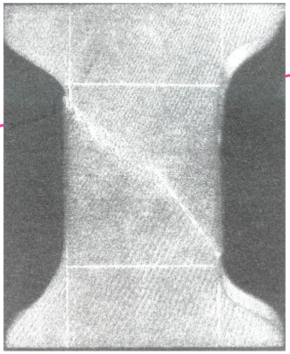
from the observations follow ..

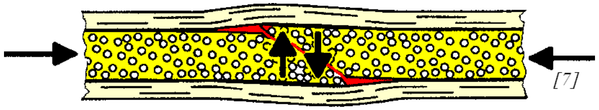
NF := Normal Fracture

SF := Shear Fracture



wedge type





Cuntze's **Set** of **Modal 3D UD Strength Failure Conditions** ('criteria')

Cuntze = 'simple Mises ' amongst the UD criteria

Invariants, replaced by their stress formulations !!

strains from FEA

[Cun04, Cun11]

FF1 $Eff^{\|\sigma} = \breve{\sigma}_1 / \bar{R}^t_{\|} = \sigma^{\|\sigma}_{eq} / \bar{R}^t_{\|}$, $\breve{\sigma}_1 \cong \varepsilon^t_1 \cdot E_{\|}$ *

FF2 $Eff^{\|\tau} = -\breve{\sigma}_1 / \bar{R}^c_{\|} = +\sigma^{\|\tau}_{eq} / \bar{R}^c_{\|}$, $\breve{\sigma}_1 \cong \varepsilon^c_1 \cdot E_{\|}$

2 filament modes

IFF1 $\boxed{Eff^{\perp\sigma}} = [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}] / 2\bar{R}^t_{\perp} = \boxed{\sigma^{\perp\sigma}_{eq} / \bar{R}^t_{\perp}}$

IFF2 $Eff^{\perp\tau} = [(\frac{\mu_{\perp\perp}}{1-\mu_{\perp\perp}}) \cdot (\sigma_2 + \sigma_3) + \frac{1}{1-\mu_{\perp\perp}} \sqrt{(\sigma_2 - \sigma_3)^2 + 4\tau_{23}{}^2}] / \bar{R}^c_{\perp} = +\sigma^{\perp\tau}_{eq} / \bar{R}^c_{\perp}$

3 matrix modes

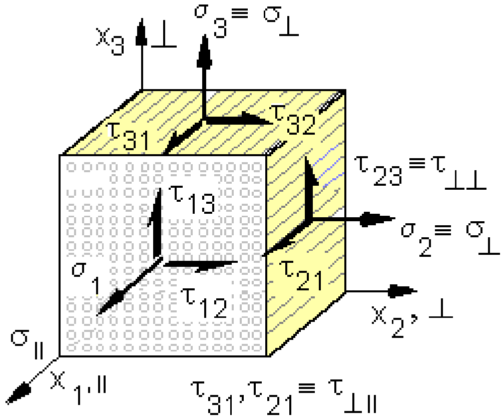
IFF3 $Eff^{\perp\|} = \{[\mu_{\perp\|} \cdot I_{23-5} + (\sqrt{\mu_{\perp\|}{}^2 \cdot I_{23-5}{}^2 + 4 \cdot \bar{R}_{\perp\|}{}^2 \cdot (\tau^2_{31} + \tau^2_{21})^2}] / (2 \cdot \bar{R}_{\perp\|}{}^3)\}^{0.5} = \sigma^{\perp\|}_{eq} / \bar{R}_{\perp\|}$

with $I_{23-5} = 2\sigma_2 \cdot \tau^2_{21} + 2\sigma_3 \cdot \tau^2_{31} + 4\tau_{23}\tau_{31}\tau_{21}$

Interaction of modes:

$$Eff^m = (Eff^{\|\tau})^m + (Eff^{\|\sigma})^m + (Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$

with mode-interaction exponent $2.5 < m < 3$ from mapping tests data



Typical friction value data range: $0.05 < \mu_{\perp\|} < 0.3, \quad 0.05 < \mu_{\perp\perp} < 0.2$

see [Pet16] for measurement

Poisson effect * : bi-axial compression strains the filament without any σ_1

t:= tensile, c: = compression, || : = parallel to fibre, ⊥ := transversal to fibre

Modal treatment *requires* an **Interaction of the Single SFCs**

In the FMC:I

Interaction of adjacent Failure Modes in the mode transition zones

= by a '*series failure system*' model that considers an

'Accumulation' of interacting mode– associated failure danger portions Eff^{mode}

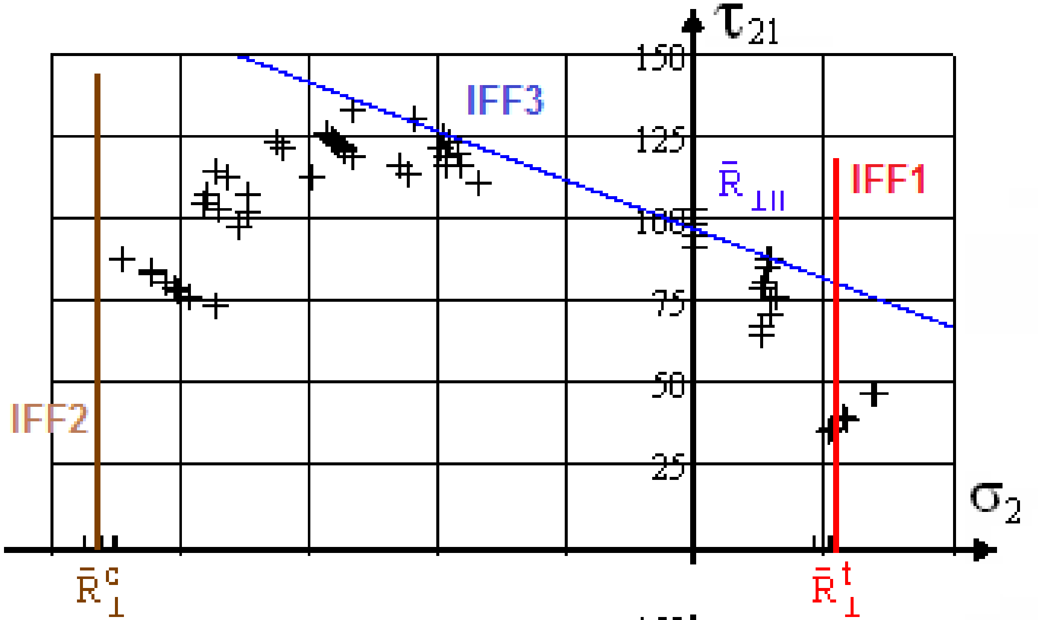
$$Eff = \sqrt[m]{(Eff^{\text{mode 1}})^m + (Eff^{\text{mode 2}})^m + \ldots.} \quad = 1 = 100\%\text{, if failure}$$

with mode-interaction exponent ***m ,*** *from mapping experience*

It is assumed engineering-like: m takes the same value for all

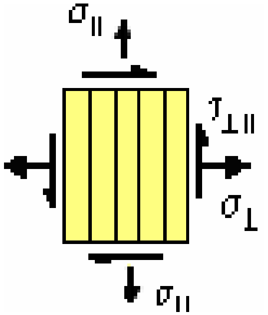
mode transition zones captured by the interaction formula above

Interaction Visualization of UD Failure Modes $\tau_{21}(\sigma_2)$ $\{\sigma\}=(0,\sigma_2,0,0,0,\tau_{21})^T$



Mapping of course of IFF test data in a pure mode domain by the *single Mode Failure Condition.*

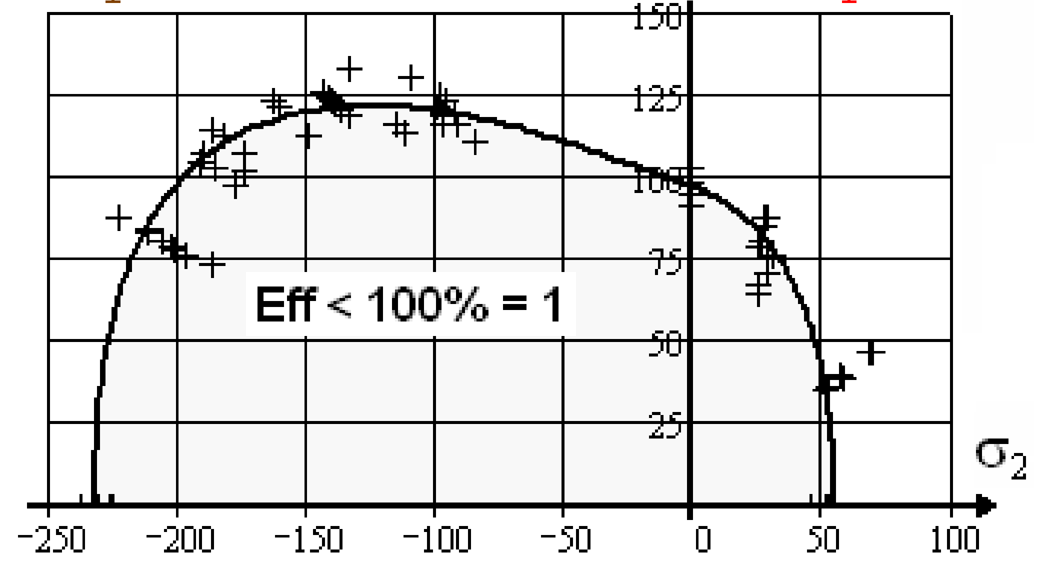
3 IFF pure modes = straight lines !.



IFF 1 : $$\frac{\sigma_2}{\bar{R}_{\perp}^{t}}=1$$

IFF 2 : $$\frac{-\sigma_2}{\bar{R}_{\perp}^{c}}=1$$

IFF 3* *(2D simplified) : $$\frac{|\tau_{21}|}{\bar{R}_{\perp\|}-\mu_{\perp\|}\cdot\sigma_2}=1$$



Mapping of course of test data by *Interaction Model*

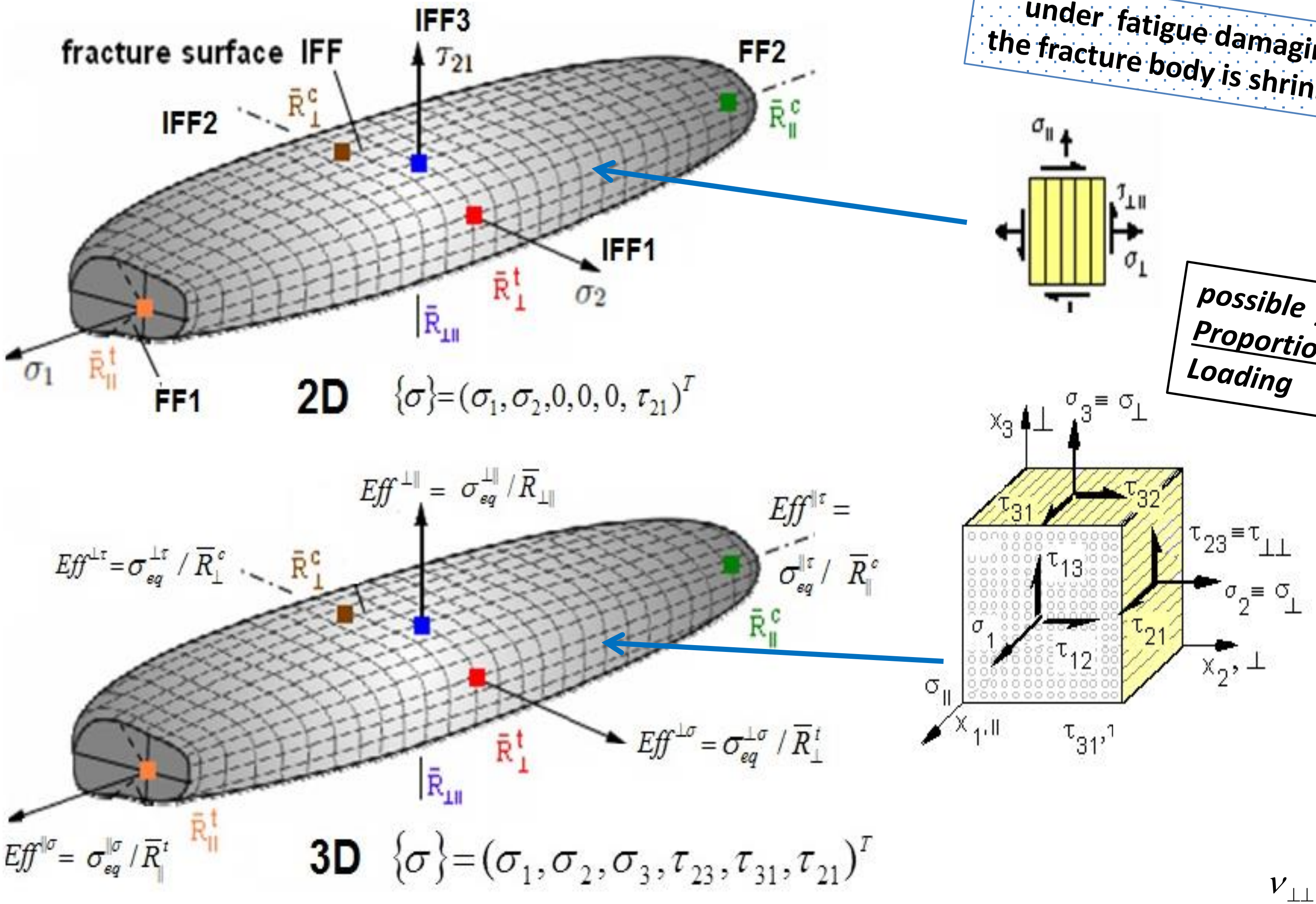
$$(Eff^{\perp\sigma})^m + (Eff^{\perp\tau})^m + (Eff^{\perp\|})^m = 1$$

$$m = 2.5, \quad \mu_{\perp\|} = 0.3$$

2D ⇨ 3D Fracture Surface by replacing stresses σ, τ by $\sigma_{eq, modal}$

2D σ

3D σ_{eq}



under fatigue damaging the fracture body is shrinking

possible for ***Proportional*** Loading

$$\nu_{\perp\perp} = 0.4,$$
$$\nu_{\perp\parallel} = 0.3$$

R = general strength and statistically reduced ‘strength design allowable

Bar over R, $\bar{R}$, means average strength, applied when mapping.

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General Matters in Fatigue Life Estimation

1 Input

Operational Loading: Load time curves (modeling rain flow, ..)

Time domain: Cycle-by-cycle or block-by-block (less computation effort)

Frequency dom.: Load spectra (loss of Reihenfolge)

Safety Concept: Design safety factor Life $j_{Life} = 3 – 10$, or

an Inspection interval, or an Replacement time approach

2 Transfer of operational loadings into stresses by using structural analysis

3 Domains of Fatigue Analysis

LCF: high stressing,

HCF: intermediate stressing 10.000 < n < 1.000.000, *rotor tube*

VHCF: low stressing and strains (SPP1466) > 10.000.000 *centrifuges, wind rotor blades*

4 Provision of Haigh Diagrams which involve all necessary S-N curves with generation of ‘Constant Fatigue Life (CFL) curves‘

.

State of the Art : Cyclic Strength Analysis of UD-ply composed Laminates

- **No Lifetime Prediction Method available, applicable to any Laminate**
- **Procedures base – as with metals – on stress amplitudes and mean stress correction f_M**
- **Procedures base on specific laminates and therefore cannot be generally applied**
- **Up to now: Engineering Approach for UD materials**

  ***Static Design Limit Strain* of $\varepsilon < 0.3\%$, practically means** negligible matrix-microcracking.

  Design experience proved: No fatigue danger given

- **Future : *Design Limit Strain* shall be increased** (EU-project: MAAXIMUS)

  Beyond $\varepsilon \approx 0.5\%$ *first filament breaks , diffuse matrix-microcracking*

  ***changes to a discrete localized* one.**

Often, *fiber-dominated laminates* are used in high performance stress applications.

UD-material should be better exploited !

> ***„Fatigue is the black art,***
>
> ***to produce financial black holes“***

[J. Draper]

Therefore, in order to reduce very costly cyclic laminate test programs

the German Academic Research Group BeNa (proof of service strength),

founded by the author in 2010,

aims at :

A failure mode-based Lifetime Prediction Method,
lamina-oriented on the embedded lamina
in order to capture in-situ effects
and using <u>failure mode-based S-N curves</u>.

for UD material,
some S-N curves were available from
Dr. Clemens Hahne, AUDI

Ermüdungsmodelle, Schädigungs‘portion‘, Schädigungsakkumulation

- ***Ermüdungmodelle für Composites***

  Nach *[Degrieck-Paepegem*] eingeteilt in 3 Kategorien:

1.) ***Ermüdungsleben-Modelle***

Gebrauch von Wöhlerkurve und Versagensbedingung für Ermüdung

(angewendet vom Autor, aber versagensmodusweise)

2.) ***Phenomenologische Modelle***

Gebrauch von Steifigkeitsverlust oder Restfestigkeit

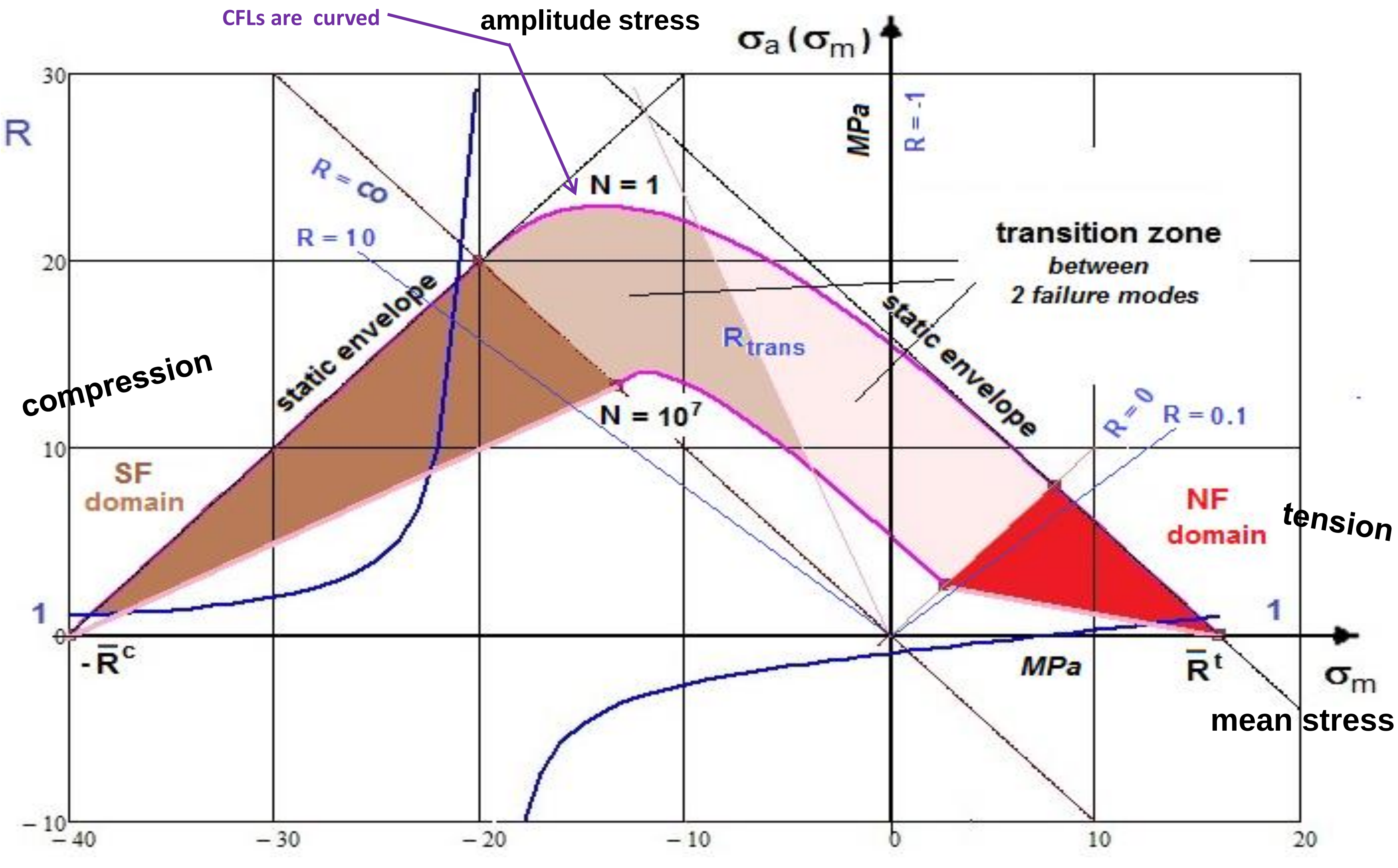
3.) ***Progressive Schädigungsmodelle***

Gebrauch von Schädigungsvariablen in Kombination mit meßbaren Schädigungsgrößen wie Matrixrisse.

- ***Bestimmung von Schädigungs’portion’*** *und*
  - ***Determination of damaging portions*** *(from diffuse and discrete damaging)*

- ***Schädigungsakkumulation***
  - ***Accumulation of damaging portions*** *(cycle-wise, or block-wise, or … ?)*

Haigh Diagram of a Brittle behaving Isotropic Material



R **:= stress ratio** $\sigma_{min}/\sigma_{max}$

NF = Normal Fracture, SF = Shear Fracture, *N* = fracture cycle number

Haigh Diagrams, brittle-behaving Materials as $\sigma_a(\sigma_m, R)$ and $\sigma(\sigma_m, R)$

Fatigue limit, **endurance limit**, and **fatigue strength**
are all expressions used to describe a cyclic limit property of materials:

Stress Life Fatigue ⇦ **approaches** ⇨ **Strain Life fatigue (*ductile behaviour*)**

Mean stress sensitivity *M of isotropic materials*:

$$M = [\sigma_{endurance}(R = -1,\ \sigma_m = 0) - \sigma_{endurance}(R = 0,\ \sigma_m = \sigma_a)] / \sigma_m(R = 0) \quad \text{for} \quad n > 10^6\ cycles$$

Wechselfestigkeit Schwellfestigkeit

- Brittle behavior: *M* ⇨ 1, max stress (Oberspannung) σ_{max} is responsible for damaging
- Ductile behavior *M* ⇨ 0, amplitude stress σ_a is responsible for damaging (slip)

AIM:
Automatic Establishment of the Curved Constant Life Curves (CFL) in Haigh Diagrams
on basis of a mode-linked, measured
Master S-N curve plus a model.

N := failure cycle number, n := running cycle number

Rendering Fatigue Test Results of Brittle Materials

Ductile Materials:

Driver of damaging process (Mises, yielding): $\Delta\sigma = \sigma_{max} - \sigma_{min} = \sigma_{max} \cdot (1 - R)$

$\Delta\sigma$ as stress amplitude. Mises considers the multi-axial stress states

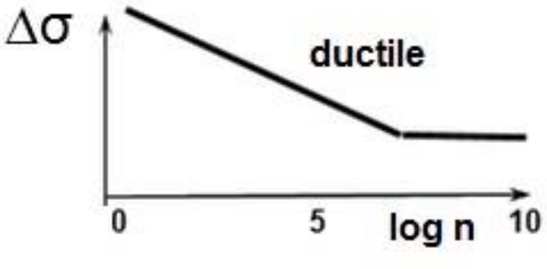
Mathematically correspond (Mohr stresses): $\sigma_{max} = \sigma_{I}$, $\sigma_{min} = \sigma_{III}$

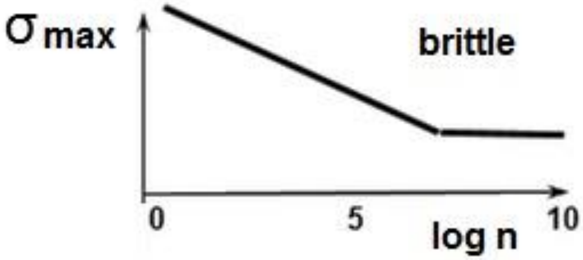
$\sigma_{I} > \sigma_{II} > \sigma_{III}$

Brittle Materials:

Driver of damaging process: tension NF σ_{I} , compression SF σ_{III}

Modal approach advantageous !





Damaging Determination in Brittle behaving cyclically loaded Composites

Assumption:

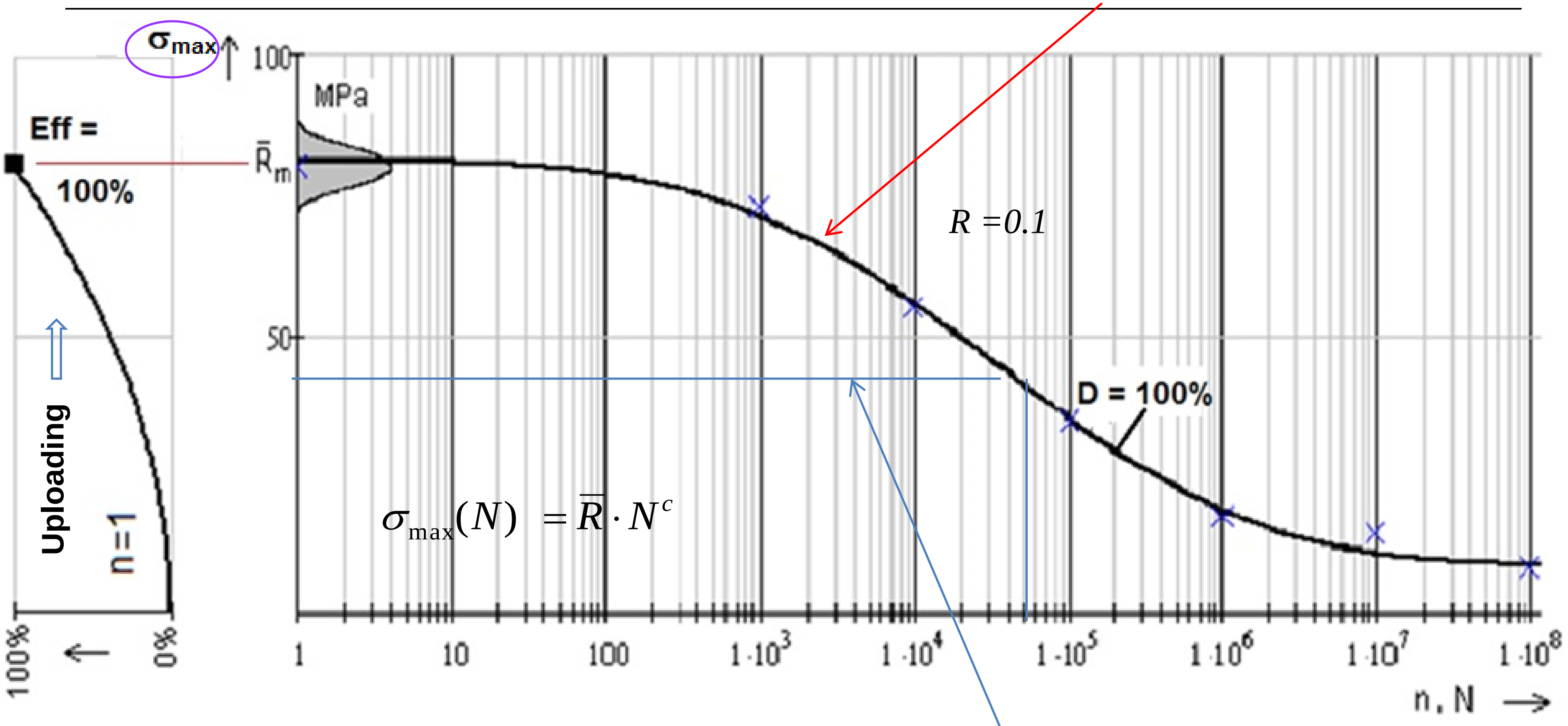
“If the failure mechanism (mode) cyclically remains the same

as in the static case t*hen*

- the fatigue damaging driving failure parameters are the same and

- the applicability of static SFCs is allowed for quantifying damaging portions !“

Cyclic development of damaging, average S-N-curve, brittle material



Static

R_m basic strength

Cyclic

residual strength $\sigma(R,N)$

Eff (static) = 100%
→ D = 100%

Analogous limits of the material capacities :

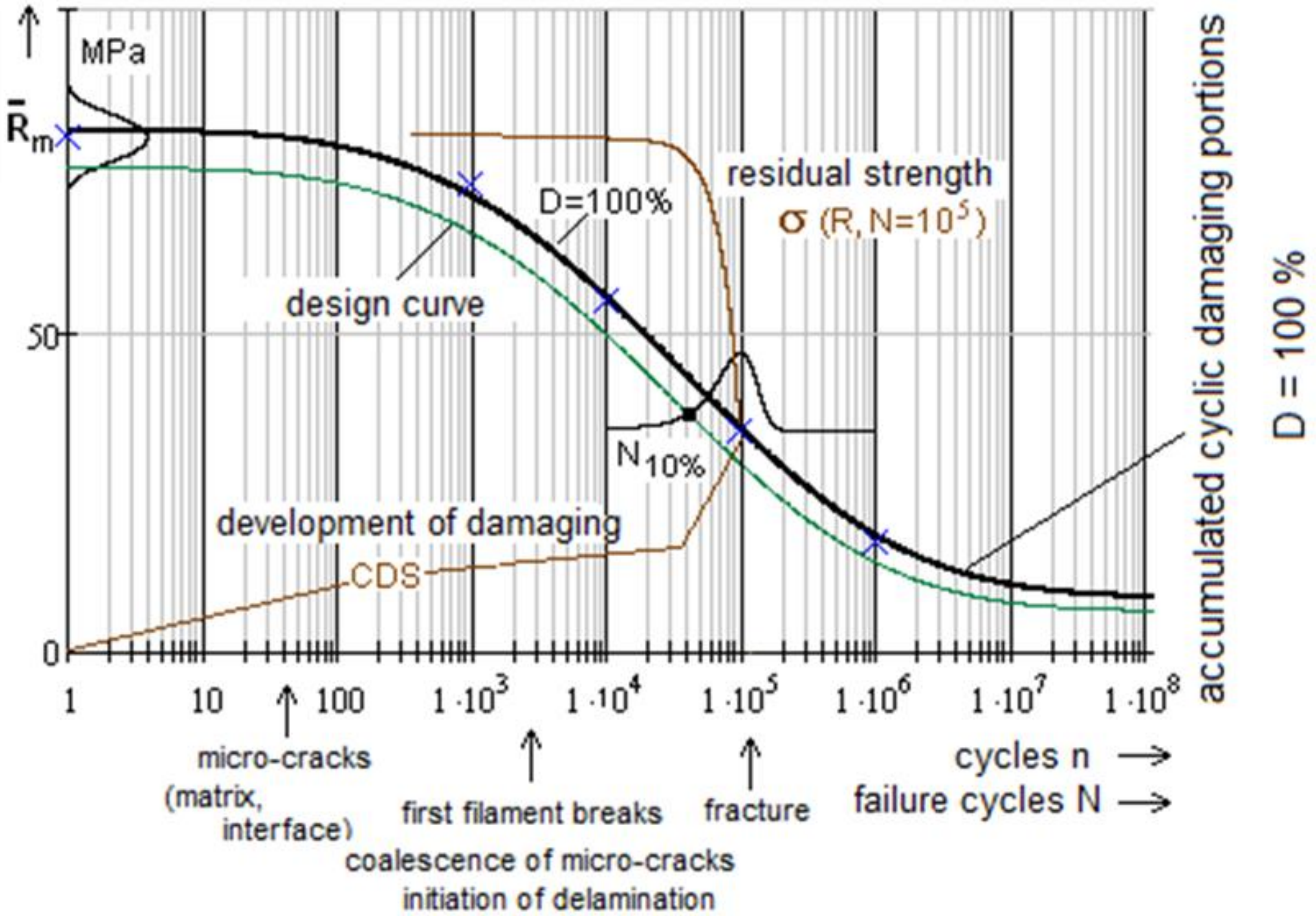
- Static : material stressing effort Eff (N=1) = 100 %
- Cyclic : material damaging sum D (N) = 100 %

= *sum of damaging portions*

S-N-mapping of brittle materials: Use of σ_{max} in the S-N curve is advantageous compared to the amplitude σ_a !

Lin-Log S-N Curve: Average Curve (mapping) and Design Curve (verification)

σ_{max} ⇨ $\sigma_{eq,\,max}$ **if more stresses act together within the envisaged failure mode (like Mises)**



FF:= fibre failure. IFF:= Inter Fibre Failure, CDS:= characteristic damage state at the end of diffuse damaging

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Es ist an der Zeit, wieder Gas zu geben
UND   für spröde Werkstoffe
Ermüdung neu zu denken!

AIM Cuntze:

Applicable to brittle isotropic and UD-materials

Automatic Establishment of the curved Constant Fatigue Life Curves (CFL) in Haigh Diagrams – for each single mode - *on basis of*

- a measured mode-decisive S-N curve as Master or basic curve, non-linear over the full range, plus
- a model [M. Kawai] to predict other necessary S-N curves on basis of above **Master S-N curve**

➢ Original Kawai used all R-curves, independent of the inherent failure mode. Therefore, had to be further developed by Cuntze, *failure-mode-linked*

In practice, usually ,
Haigh Diagrams
are partly composed by straight lines.

Question?
Is this reasonable, especially for UD materials ?
So many straight lines!?
Or can we generate curved lines ? [C. Hahne]

⇨ see examples later

Kawai's Modified Fatigue Strength Ratio Ψ

$\sigma_{\max} = \Delta\sigma/(1-\mathrm{R}) \equiv [2\cdot\sigma_a/(1-\mathrm{R})]$ with $\Delta\sigma = $ stress range ,

$R_m = R^t = \sigma_{\max}\ (n = N = 1)$

R = $(\sigma_m - \sigma_a)/(\sigma_m + \sigma_a)$, $\sigma_a = 0.5\cdot\sigma_{max}\cdot(1\text{-R})$, $\sigma_m = 0.5\cdot\sigma_{max}\cdot(1\text{+R})$.

R = - 1 *means fully reversed alternating stress*

3 free parameter-Weibull + strength point: model goes through the strength point and maps the endurance limit.

$$\sigma_{\max}(N) = c1 + \frac{\bar{R} - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$$

Assumption: The 'Modified Fatigue Strength Ratio' is prediction-capable

The application of his model is as follows. Kawai first normalizes the fatigue strength σ_{max} by the static strength $R_{||}^{t}$, which means the use of the (static) *material stressing effort* (bar over *R* skipped here, due to simpler writing)

$$Eff^{||\sigma} = \sigma_{max} / R_{||}^{t}\ .$$

Eff corresponds to Kawai's ψ, termed *'fatigue strength ratio'*. Using the amplitude σ_a and the mean stress σ_m , then the static failure condition above can be expressed as

$$Eff^{||\sigma} = (\sigma_a + \sigma_m)/ R_{||}^{t} \equiv \psi. \qquad . \qquad (3)$$

In the fracture case, meaning $\psi = 1 = 100\% \equiv Eff^{||\sigma}$, this reads for the stress case tension

$$\psi = 1 = \sigma_{max} / R_{||}^{t} = (\sigma_a + \sigma_m)/ R_{||}^{t} \quad \text{or} \quad 1 = \sigma_a / (R_{||}^{t} - \sigma_m),$$
$$\textit{where-in} \quad \sigma_a = 0.5\cdot\sigma_{max}\cdot(1\text{- R}),\ \sigma_m = 0.5\cdot\sigma_{max}\cdot(1\text{+R}).$$

Analogically to ψ, Kawai defines the also non-dimensional '*<u>modified</u> fatigue strength ratio*'

$\underline{\sigma > 0}$: $\Psi_t = \sigma_a / (R_{||}^{t} - \sigma_m) = 0.5\cdot(1\text{- R})\cdot \sigma_{max} / [R_{||}^{t} - 0.5\cdot(\mathbf{1}\text{+R})\cdot \sigma_{max}]$ or (4)

$= 0.5\cdot(1\text{-R})\cdot Eff^{||\sigma} / [1 - 0.5\cdot(1\text{+R})\cdot Eff^{||\sigma}]$ with $\sigma_{max} > \sigma_{min}$

$\underline{\sigma < 0}$: $\Psi_c = \sigma_a / (R_{||}^{c} - \sigma_m) = 0.5\cdot(1\text{-R})\cdot \sigma_{min} / [R_{||}^{c} - 0.5\cdot(1\text{+R})\cdot \sigma_{min}]$ with $|\sigma_{min}| > |\sigma_{max}|$,

(corresponding to a *cyclic material stressing effort*) as a scalar quantity and thereby he could introduce the stress ratio *R*. According to being a material quantity Ψ is positive.

Fitting the course of test data, Kawai obtains a Master Ψ-curve. Based on the chosen mapping function for σ_{max} the FF-linked S-N curves can be estimated by the resolved equations

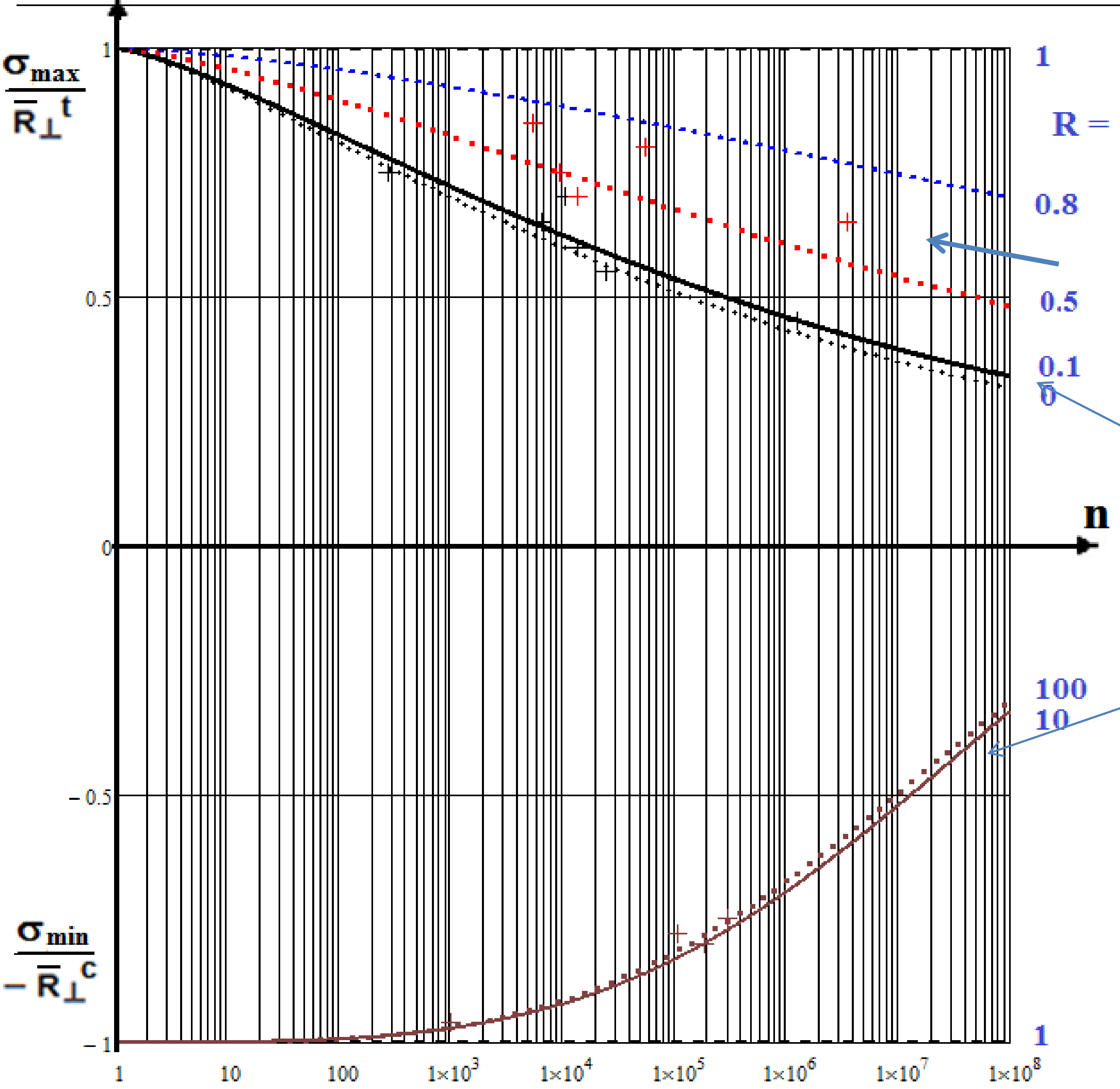
$$\sigma_{max}(R) = (2\cdot R_{||}^{t}\cdot \Psi_{master}) / [\Psi_{master} - \mathrm{R} + \mathrm{R}\cdot\Psi_{master} + 1], . \qquad (5)$$

$$\sigma_{min}(R) = (2\cdot R_{||}^{c}\cdot \Psi_{master}) / [\Psi_{master} - \mathrm{R} + \mathrm{R}\cdot\Psi_{master} + 1],$$

<u>Application results:</u>

- **Limit Curves are mapped.**
- **Intermediate Curves must be physically-based improved**
- **Model looked very promising, after 'failure mode-adapted , by the author**

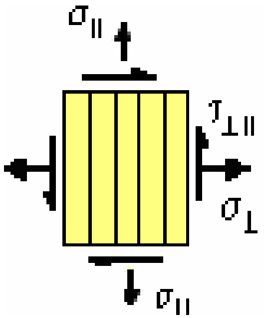
How look the 'Kawai model-predicted ' Mode S-N curves ? *Example IFF1, IFF2'*



In the intermediate R-regime, the Kawai model-predicts 'too' conservative S-N curves.

Improvable.

on basis of the two given 'Master mode S-N curves' 0.1, 10



t := tension
c := compression

Kawai's Modified Fatigue Strength Ratio Ψ mode-linked applied to predict further necessary S-N curves on basis of one measured **Master S-N curve** R = 0.1 (NF), = 10 (SF)

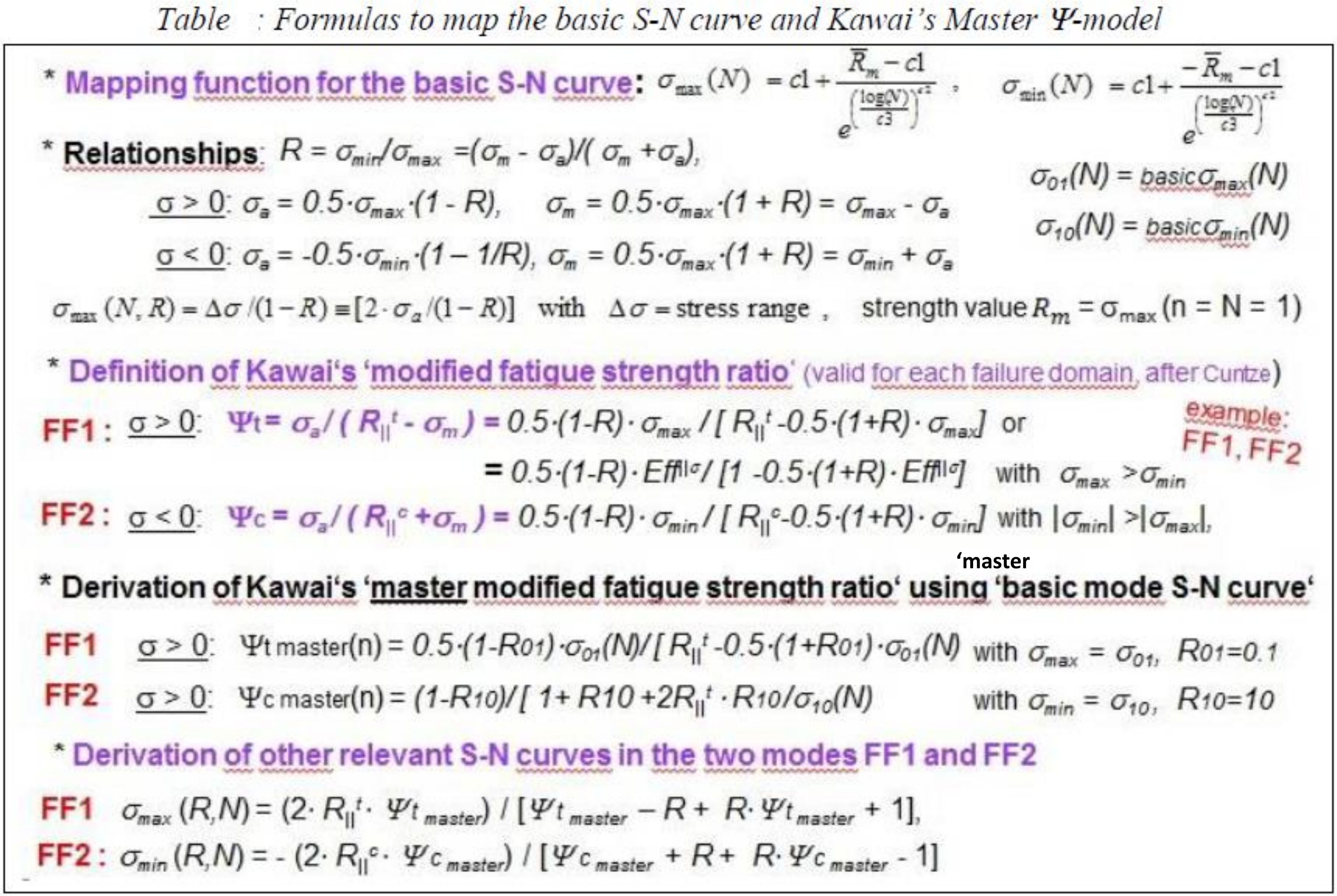


Table : Formulas to map the basic S-N curve and Kawai's Master Ψ-model

* **Mapping function for the basic S-N curve:** $\sigma_{max}(N) = c1 + \frac{\overline{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$, $\sigma_{min}(N) = c1 + \frac{-\overline{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$

* **Relationships**: $R = \sigma_{min}/\sigma_{max} = (\sigma_m - \sigma_a)/(\sigma_m + \sigma_a)$,

$\underline{\sigma > 0}$: $\sigma_a = 0.5 \cdot \sigma_{max} \cdot (1 - R)$, $\sigma_m = 0.5 \cdot \sigma_{max} \cdot (1 + R) = \sigma_{max} - \sigma_a$

$\underline{\sigma < 0}$: $\sigma_a = -0.5 \cdot \sigma_{min} \cdot (1 - 1/R)$, $\sigma_m = 0.5 \cdot \sigma_{max} \cdot (1 + R) = \sigma_{min} + \sigma_a$

$\sigma_{01}(N) = basic\,\sigma_{max}(N)$

$\sigma_{10}(N) = basic\,\sigma_{min}(N)$

$\sigma_{max}(N, R) = \Delta\sigma/(1-R) \equiv [2 \cdot \sigma_a/(1-R)]$ with $\Delta\sigma$ = stress range , strength value $R_m = \sigma_{max}$ (n = N = 1)

* **Definition of Kawai's 'modified fatigue strength ratio'** (valid for each failure domain, after Cuntze)

FF1 : $\underline{\sigma > 0}$: $\Psi t = \sigma_a / (R_{||}^t - \sigma_m) = 0.5 \cdot (1-R) \cdot \sigma_{max} / [R_{||}^t - 0.5 \cdot (1+R) \cdot \sigma_{max}]$ or

$= 0.5 \cdot (1-R) \cdot Eff^{||\sigma} / [1 - 0.5 \cdot (1+R) \cdot Eff^{||\sigma}]$ with $\sigma_{max} > \sigma_{min}$

example: FF1, FF2

FF2 : $\underline{\sigma < 0}$: $\Psi c = \sigma_a / (R_{||}^c + \sigma_m) = 0.5 \cdot (1-R) \cdot \sigma_{min} / [R_{||}^c - 0.5 \cdot (1+R) \cdot \sigma_{min}]$ with $|\sigma_{min}| > |\sigma_{max}|$,

'master

* **Derivation of Kawai's 'master modified fatigue strength ratio' using 'basic mode S-N curve'**

FF1 $\underline{\sigma > 0}$: $\Psi t\,\text{master}(n) = 0.5 \cdot (1-R01) \cdot \sigma_{01}(N) / [R_{||}^t - 0.5 \cdot (1+R01) \cdot \sigma_{01}(N)$ with $\sigma_{max} = \sigma_{01}$, $R01 = 0.1$

FF2 $\underline{\sigma > 0}$: $\Psi c\,\text{master}(n) = (1-R10)/[1 + R10 + 2R_{||}^t \cdot R10/\sigma_{10}(N)$ with $\sigma_{min} = \sigma_{10}$, $R10 = 10$

* **Derivation of other relevant S-N curves in the two modes FF1 and FF2**

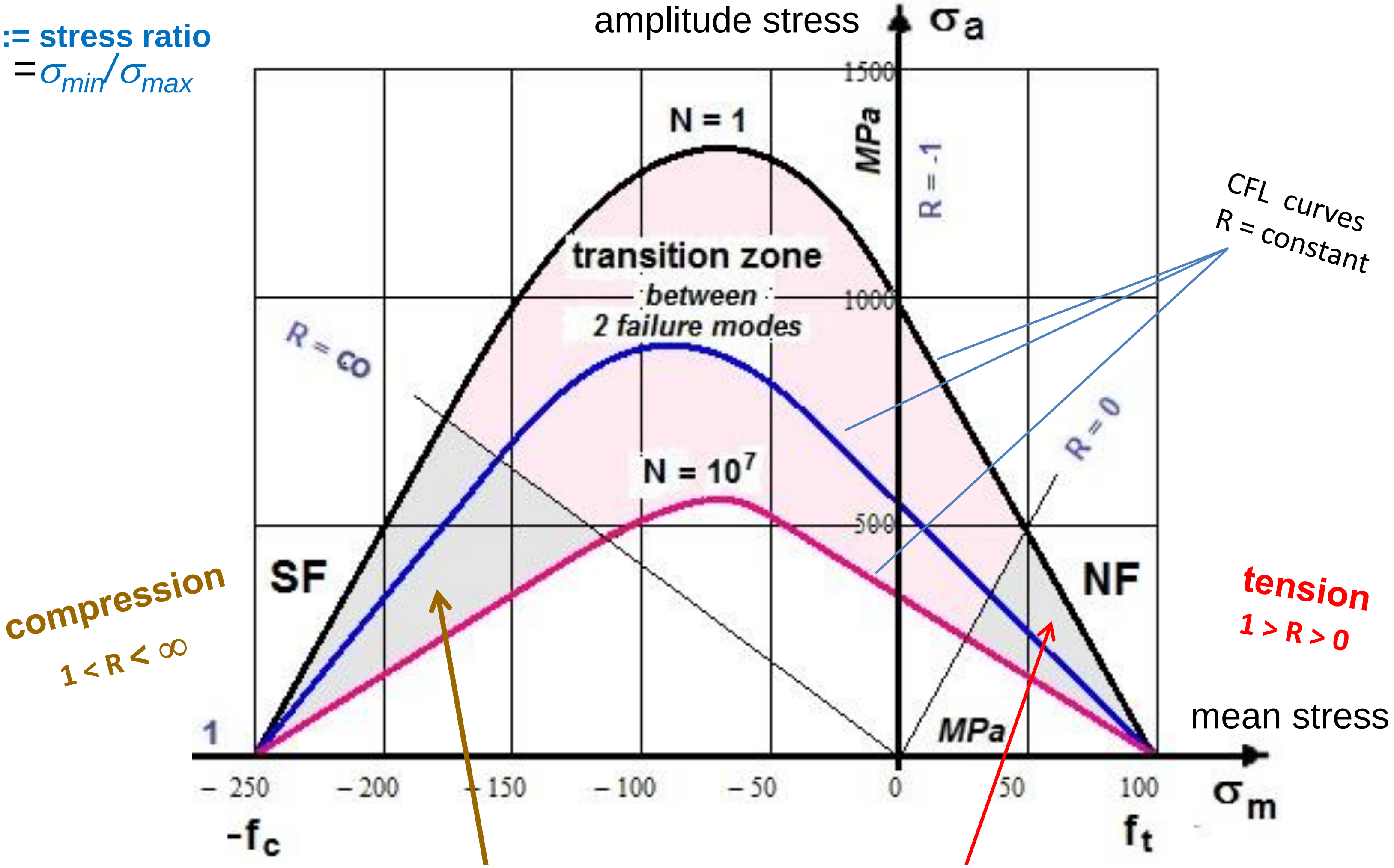
FF1 $\sigma_{max}(R,N) = (2 \cdot R_{||}^t \cdot \Psi t_{master}) / [\Psi t_{master} - R + R \cdot \Psi t_{master} + 1]$,

FF2 : $\sigma_{min}(R,N) = -(2 \cdot R_{||}^c \cdot \Psi c_{master}) / [\Psi c_{master} + R + R \cdot \Psi c_{master} - 1]$

S-N curve can be modelled: linearly, non-linearly in semi-log, log-log diagrams

Haigh Diagram of a pretty Brittle Isotropic Material, Scheme

***R* := stress ratio**
$= \sigma_{min} / \sigma_{max}$



concrete under compression

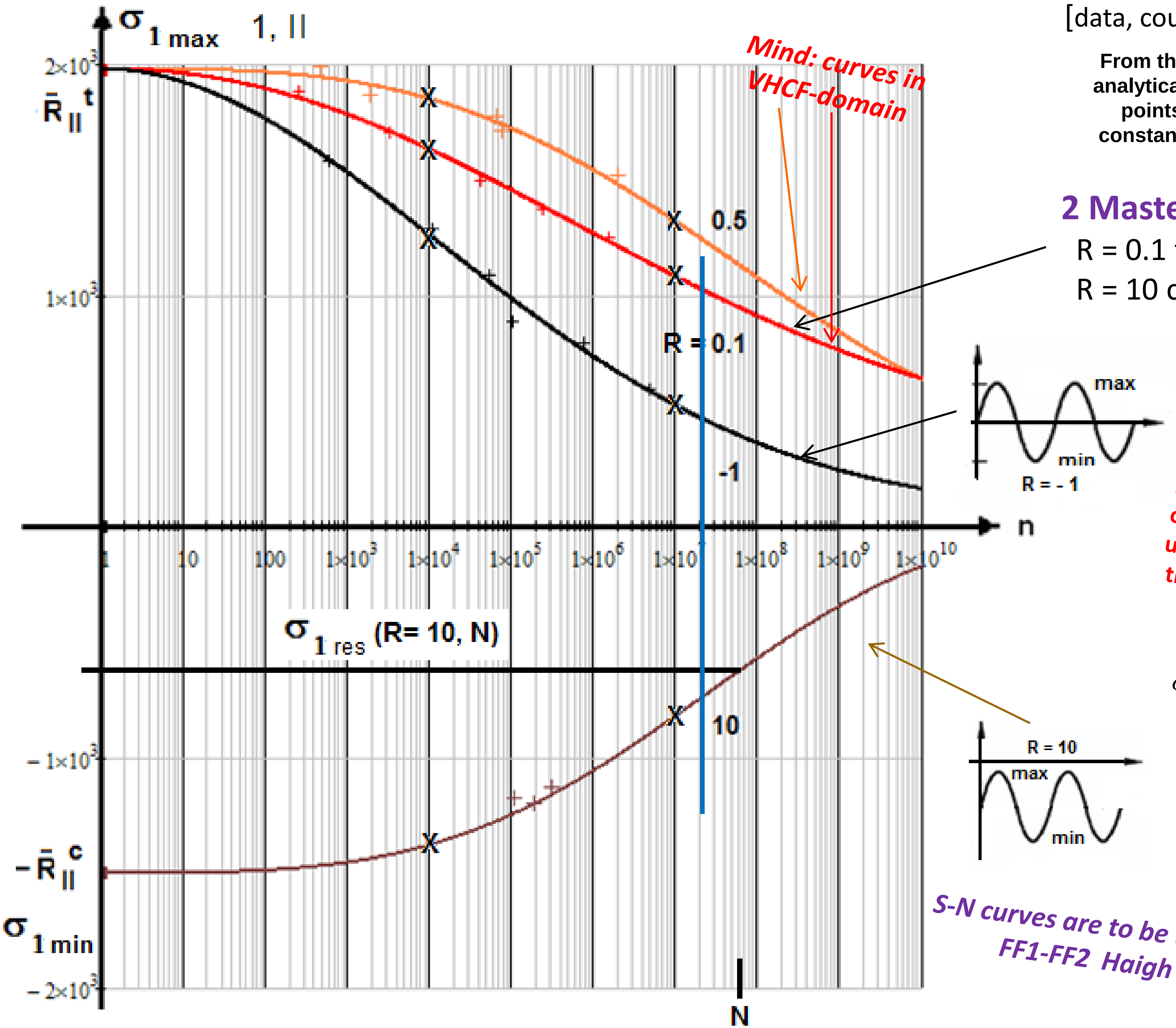
schematic example

lamella under tension

pure failure mode domains

NF = Normal Fracture, SF = Shear Fracture, *N* = fracture cycle number

Example UD: Individually lin-log mapped <u>FF1-FF2</u>-linked S-N curves



[data, courtesy Kawai-Suda]

From the mapped test data curve analytically determined are anchor points X for the prediction of constant fatigue life (CFL) curves

2 Master S-N-Curves:

R = 0.1 tension (FF1)

R = 10 compression

high N prediction depends on the mapping curve used for the mapping of the given fatigue test data

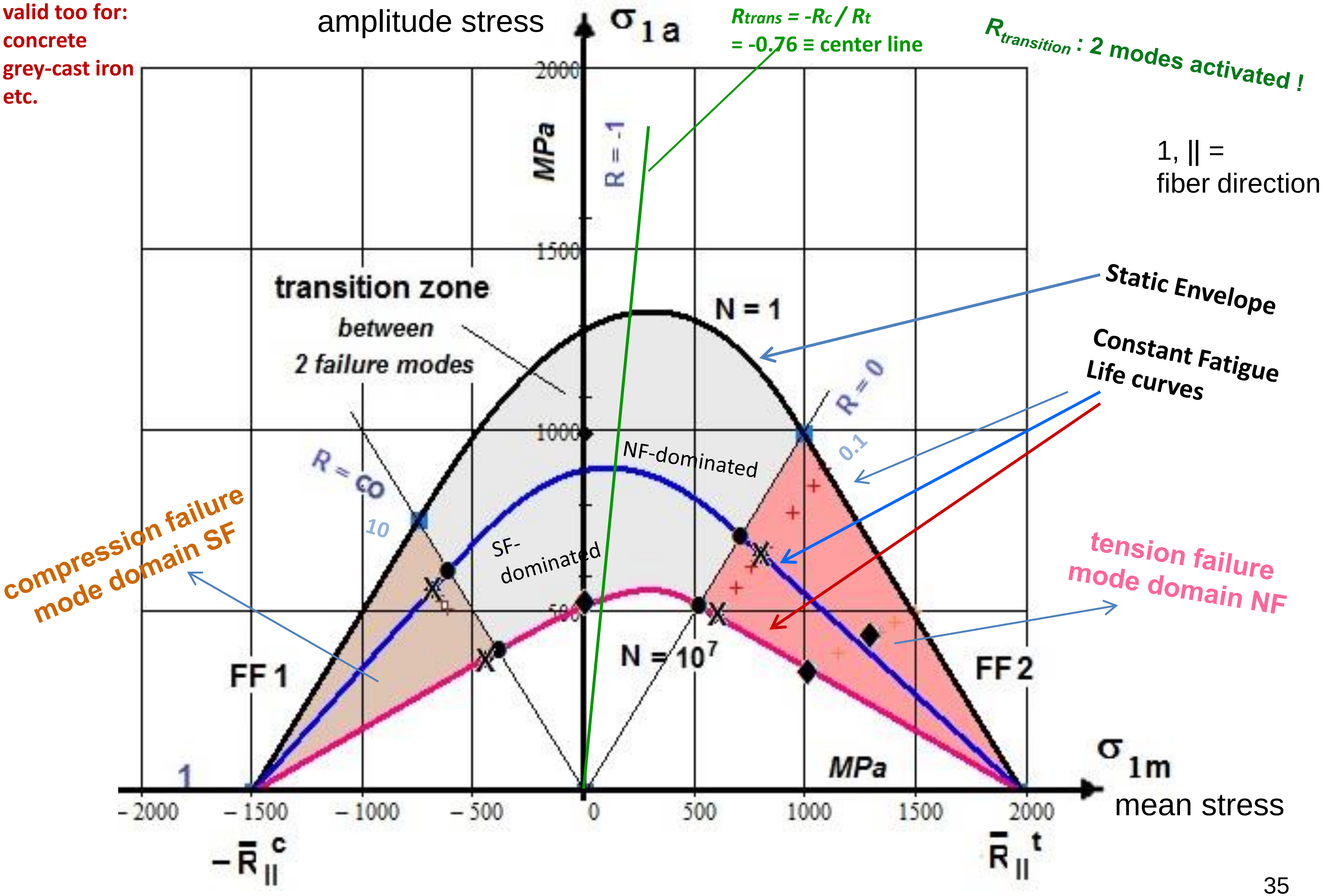
$$\sigma_{\max}(N) \;= c1 + \frac{\overline{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}} \;,$$

S-N curves are to be transferred into the FF1-FF2 Haigh Diagram

Rigorous **Interpretation of the Haigh diagram:** example FF1-FF2 UD

displaying failure mode domains and transition zone, test data [Hah14]

valid too for:
concrete
grey-cast iron
etc.



$R_{trans} = -R_c / R_t$
$= -0.76 \equiv$ center line

$R_{transition}$: 2 modes activated !

1, || =
fiber direction

Static Envelope

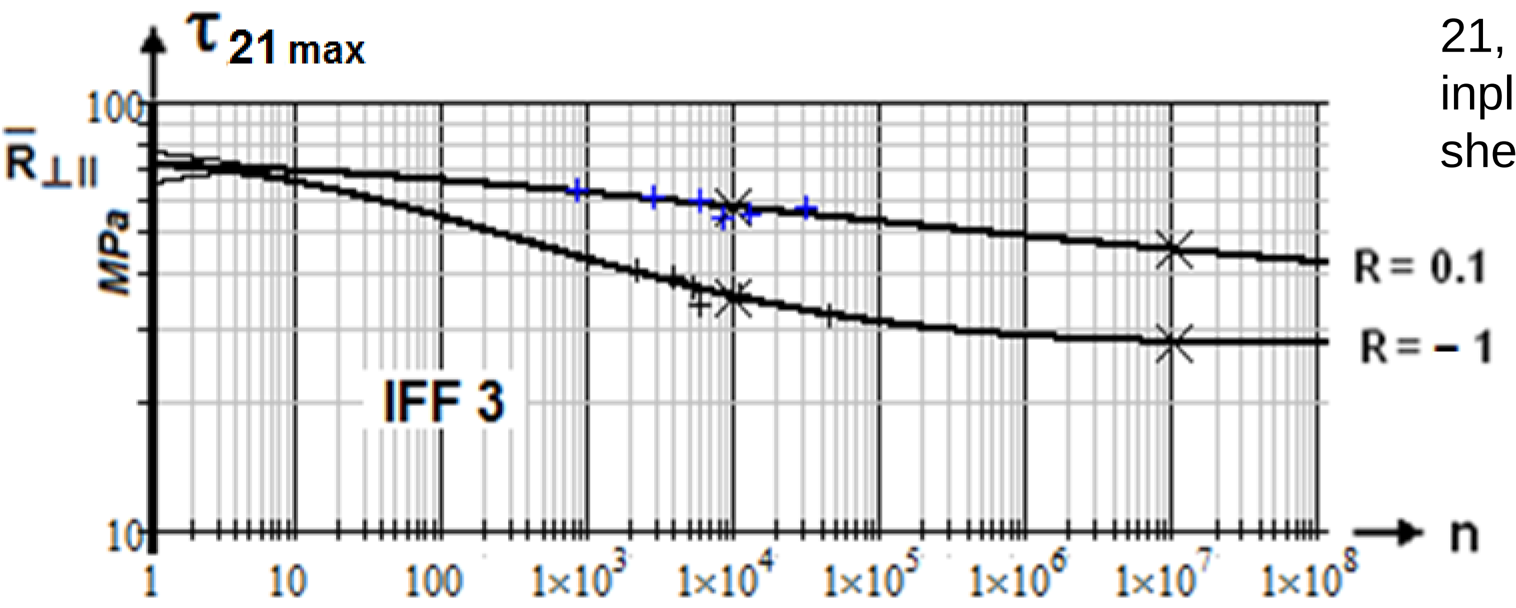
Constant Fatigue Life curves

compression failure mode domain SF

tension failure mode domain NF

NF = Normal Fracture, SF = Shear Fracture, *N* = fracture cycle number, CFL = Constant Fatigue Life

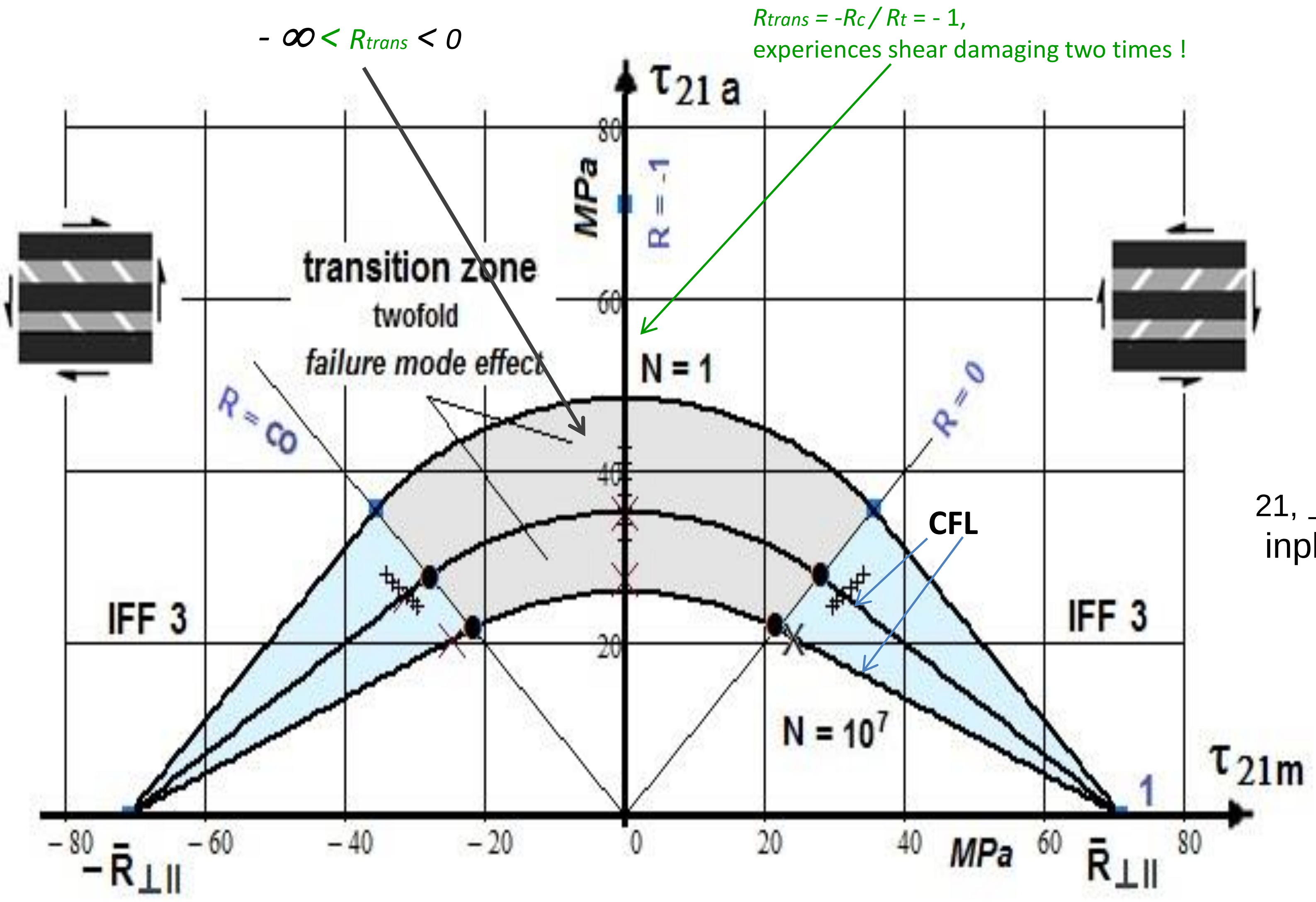
Log-log IFF3-linked S-N curves [data, courtesy C. Hahne]



21, ⊥|| =
inplane
shear

IFF3 UD Haigh diagram,

Display of a two-fold mode effect (a:= amplitude, m:= mean, N := number of fracture cycles, R := strength and R := $\sigma_{min}/\sigma_{max}$). Test data CF/EP, courtesy [Hahne14]

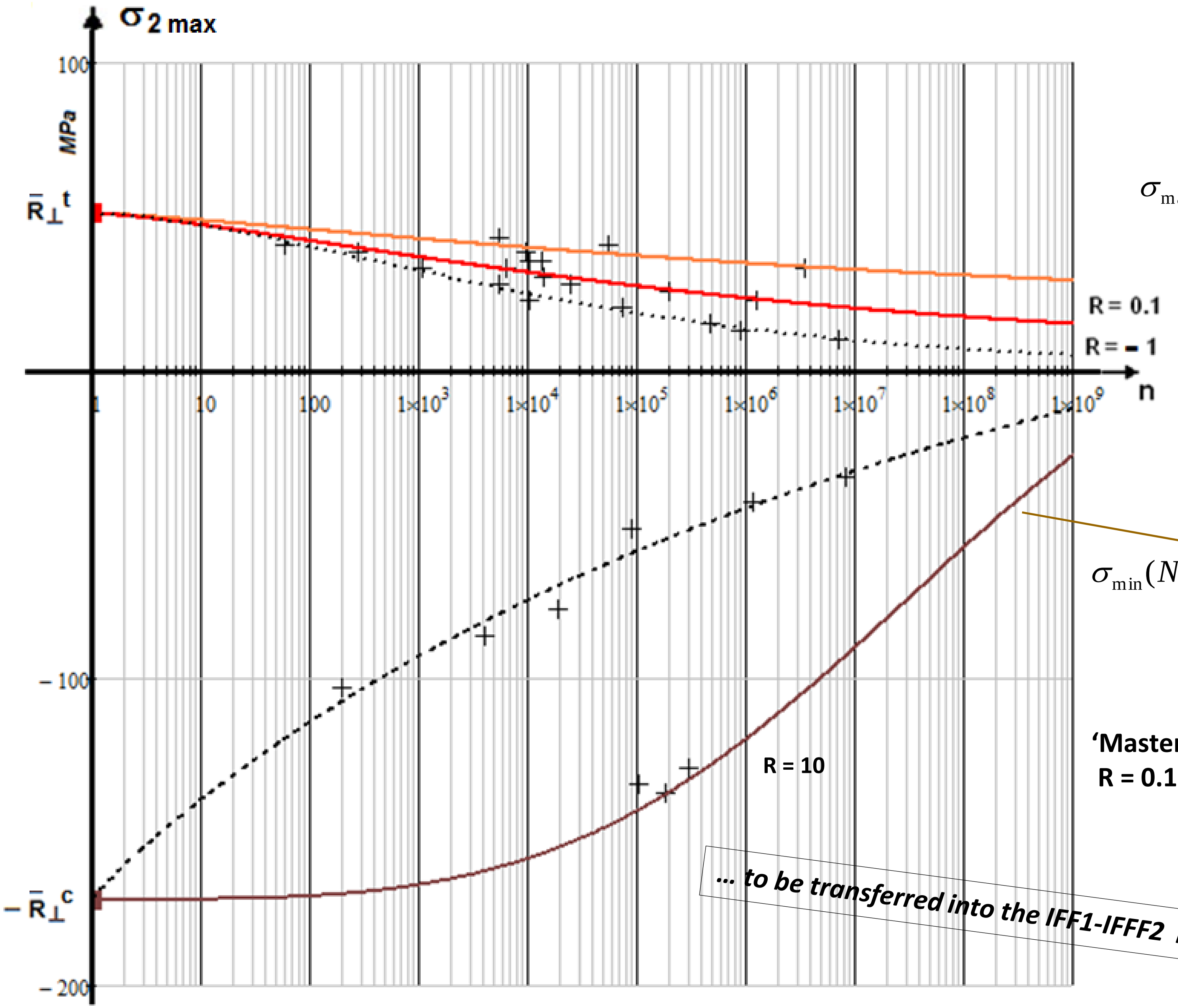


$- \infty < R_{trans} < 0$

$R_{trans} = -R_c / R_t = -1$, experiences shear damaging two times !

21, ⊥|| =
inplane shear

Mapped lin-log IFF1-IFF2-linked S-N curves *[data, courtesy C. Hahne]*



2, ⊥ = across fiber direction

$$\sigma_{\max}(N) = c1 + \frac{\bar{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$$

$$\sigma_{\min}(N) = c1 + \frac{-\bar{R}_m - c1}{e^{\left(\frac{\log(N)}{c3}\right)^{c2}}}$$

‘Master mode S-N curves‘ :
R = 0.1 and R = 10

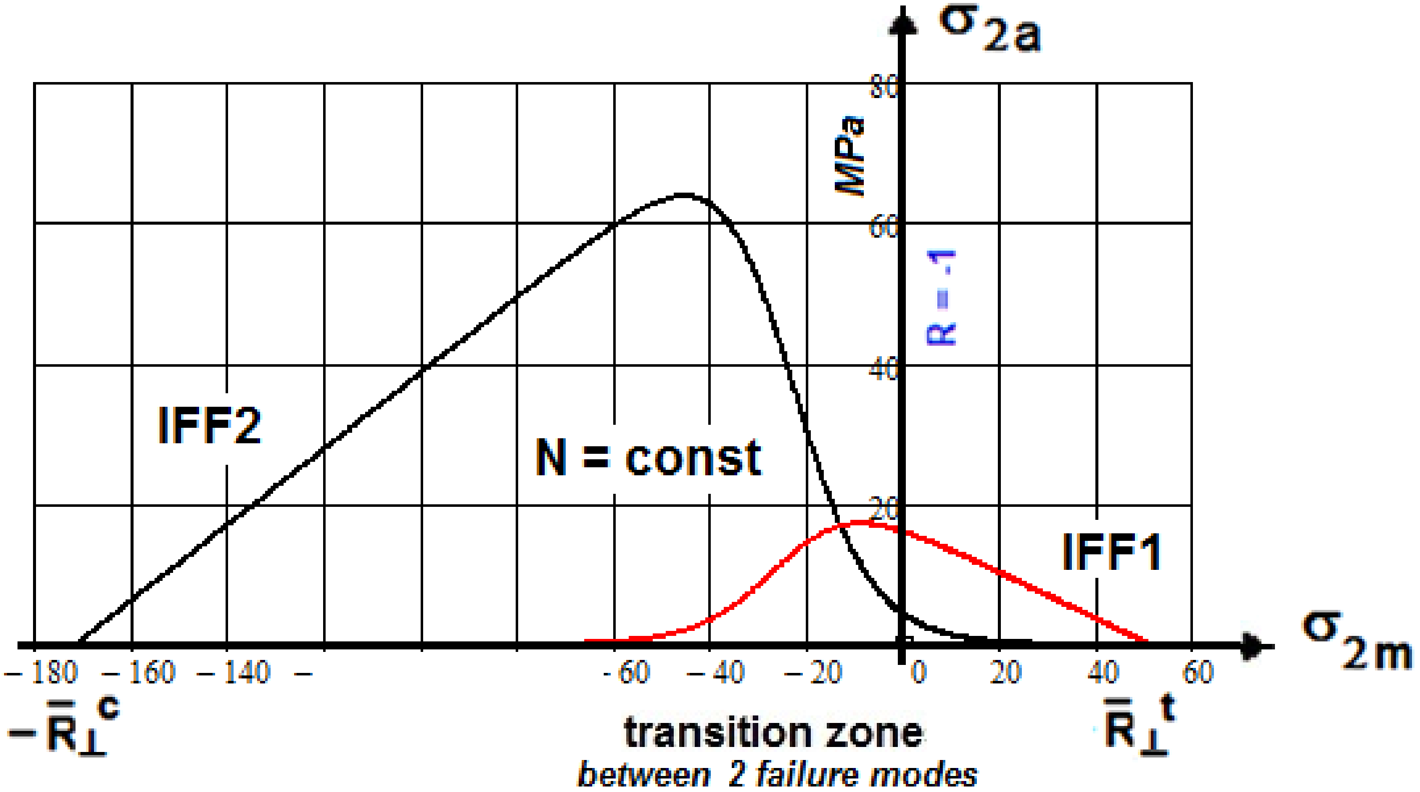
… to be transferred into the IFF1-IFFF2 Haigh Diagram

How to obtain <u>CFL</u> curves in the Transition Domain ?

** There is no problem to establish the Haigh diagrams FF and IFF3 due to the strength values being of similar size in each case: The static interaction formula was sufficient.*

** For a Haigh Diagram for really brittle materials, **when R_{trans} is very different to -1**, a new solution procedure had to be used.*

Chosen was an exponentially decaying function, that practically ends where the other mode reigns.



$$f = \left[\frac{1}{1+e^{\frac{c_1+\sigma_m}{c_2}}}\right].$$

Example:
2, ⊥ = across fiber direction

→ failure domain-linked <u>constant fatigue life (CFL)</u> curves: ***σ_a (σ_m, R, N=constant)***

Solution procedure , IFF1-IFF2 Haigh Diagram

Static strength failure $Eff = [(Eff^{NF})^m + (Eff^{SF})^m]^{m^{-1}} = 100\%$

$$\left(\frac{-(\sigma_{2m} - \sigma_{2a}) + |\sigma_{2m} - \sigma_{2a}|)}{2 \cdot \overline{R}_{\perp}^{\,c}} \right)^m + \left(\frac{\sigma_{2m} + \sigma_{2a} + |\sigma_{2m} + \sigma_{2a}|)}{2 \cdot \overline{R}_{\perp}^{\,t}} \right)^m = 1$$

The used static procedure still works for N = 1 with the interaction formula above delivering the CFL curve for N = 1 cycle, activating both NF + SF .

For higher N the interaction formula is engineering-like simplified. *It reads*:

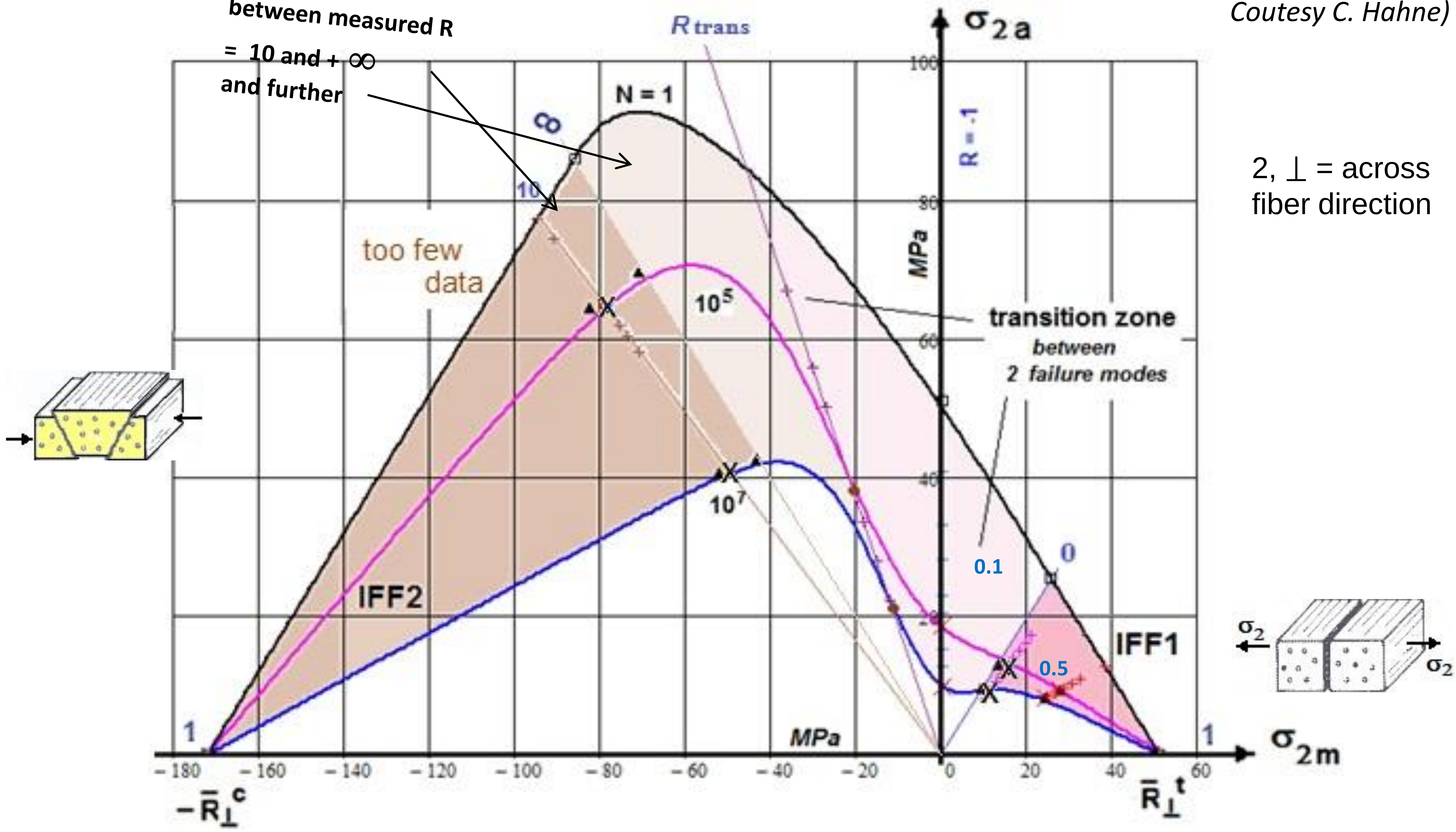
$$\sigma_a(\sigma_m) = \left[\left(\frac{\sigma_{aSF}}{1 + e^{\frac{c_{1SF} + \sigma_m}{c_{2SF}}}} \right)^m + \left(\frac{\sigma_{aNF}}{1 + e^{\frac{c_{1NF} + \sigma_m}{c_{2NF}}}} \right)^m \right]^{1/m}$$

Thereby, an exponential decay function of the SF mode CFL curve for SF from R = ∞ down to zero at the end of the NF CFL curve at R = 0 is applied:

$$f = \left[\frac{1}{1 + e^{\frac{c_1 + \sigma_m}{c_2}}} \right] .$$

IFF1- IFF2 UD Haigh diagram

displaying the failure mode domains, transition zone



test-free area between measured R = 10 and + ∞ and further

$R_{trans} = -R_c / R_t = -3.4$

(Test data: Coutesy C. Hahne)

2, ⊥ = across fiber direction

- *Curve in the* IFF1 *domain looks non-linear !*
- *Check points from Ψ-prediction lie higher than points from S-N test data evaluation*

(The computed S-N curve X-points are anchor (checking) points for the to be predicted CFL curves)

1 Introduction to Static and Fatigue Design

2 Cuntze's Failure-Mode-Concept-based Strength Criteria

3 Cuntze's Fatigue Life Prediction Estimation Concept

4 Generation and Novel Interpretation of UD Haigh Diagrams

5 Steps of the Full Fatigue Life Prediction Method Proposed

Keep in mind !

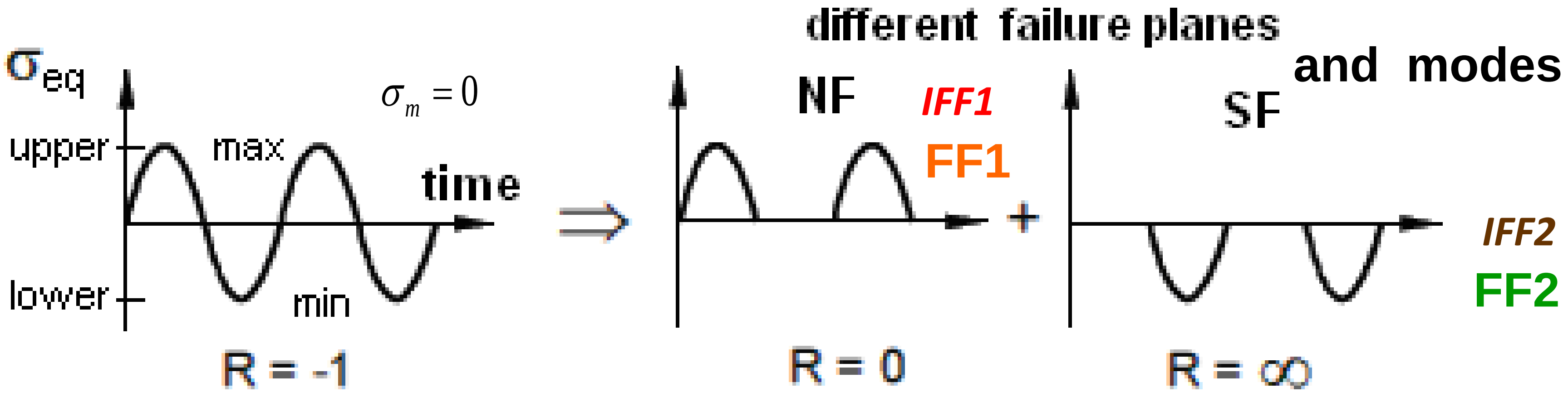
All is difficult prior to becoming simple!

[Moslik Saadi]

Novel ? idea: Failure mode-wise modelling of Loading Cycles for high-performance ‘fiber-dominated designed’, UD laminas-composed laminates

For simply displaying the approach *it is chosen* : a *loading R = -1*

Separation due to the activated inherent different failure modes



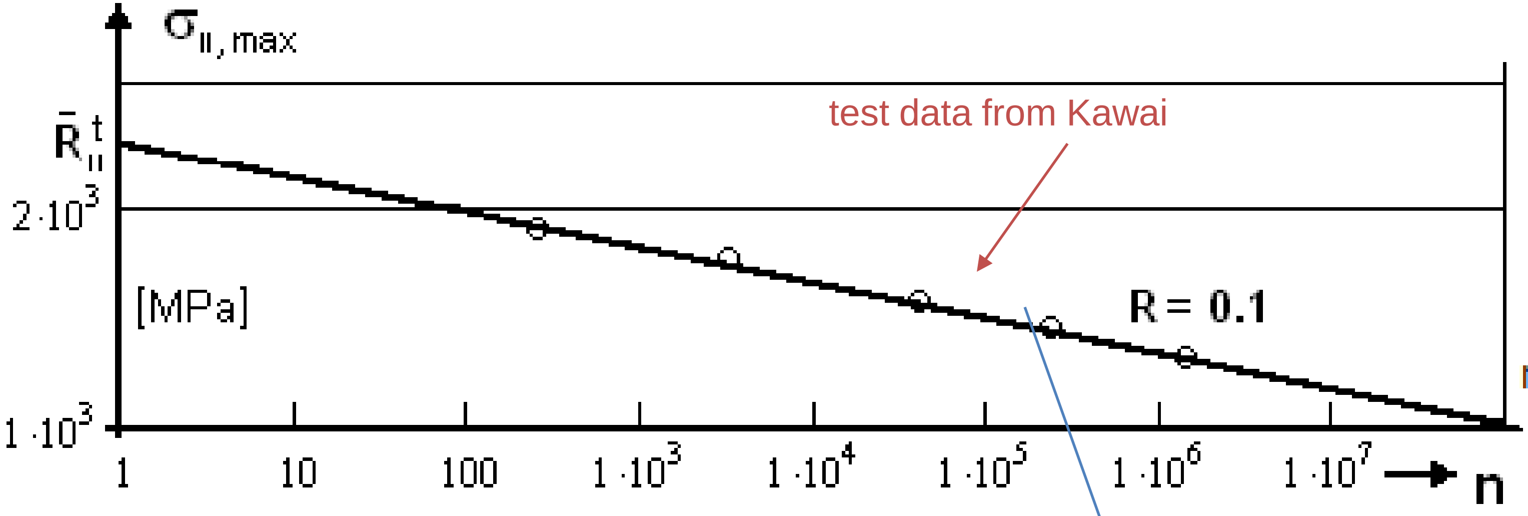
NF := Normal Fracture, SF := Shear Fracture

Step 1 : Failure mode-linked apportionment of cyclic loading (novel)

A specific **rain-fall** procedure must be applied

Mapping of S-N data and mode-representative 'basic' *S-N curve*

Example:* FF1 *failure mode



applied as
master S-N curve
measured

Step 2 : S-N curve can be mapped, e.g. by a straight * line in a log-log graph

$$\sigma_{\parallel,\ \max}^{Master}(n) \approx \bar{R}_{\parallel}^{t} \cdot n^{c_{Master}} \; {}^{*}$$

Measured curve used

as mode-representative ***Basic S-N curve*** **for FF1**

FF1 strength

In the general case of variable loading: Several S-N-curves are needed !

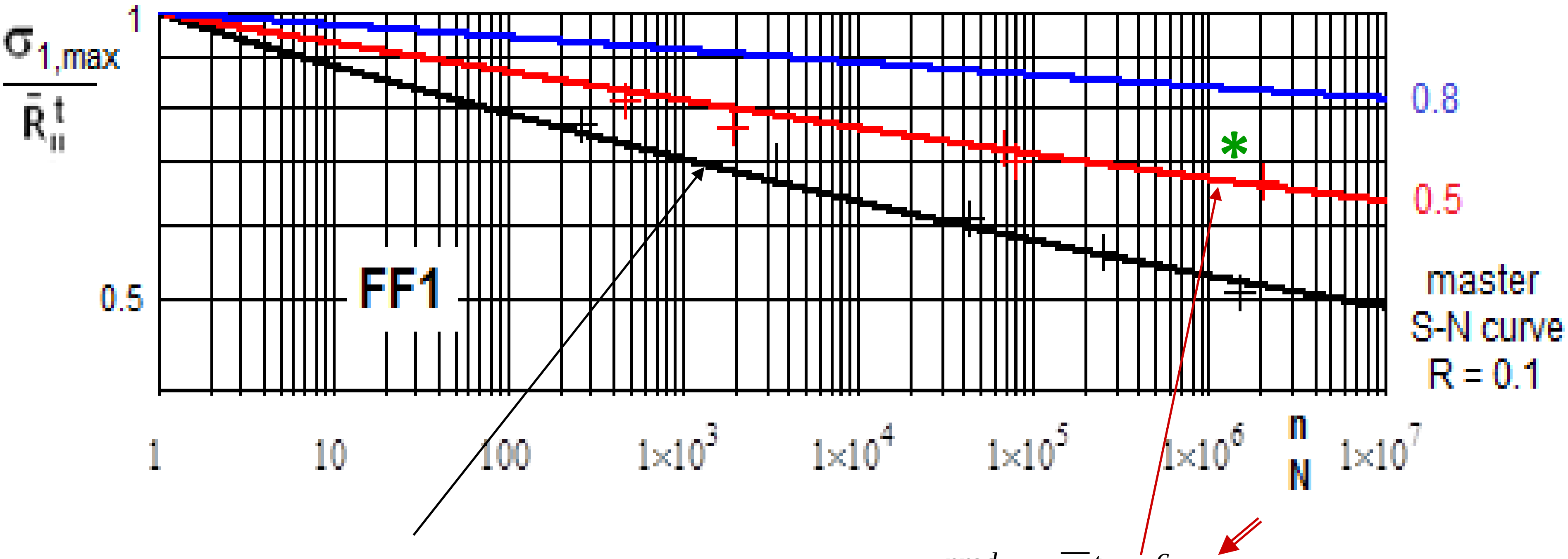
Master

---

* *more complicated S-N models may be also applied !*

Prediction of needed other FF1 S-N curves from

Basic Mode S-N curve and Kawai model (Ψ Curve)



Given : $\sigma_{\|,\ \max}^{Basic}(n) \approx \overline{R}_{\|}^{t} \cdot n^{c_{Basic}}$

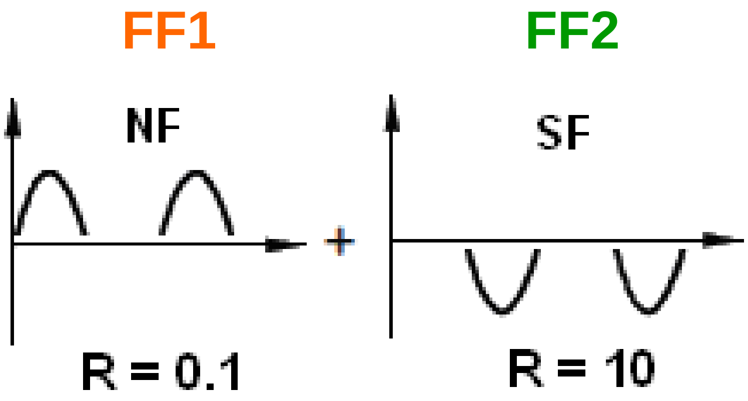
Searched : $\sigma_{\|,\max}^{pred} = \overline{R}_{\|}^{t} \cdot n^{c_{pred}}$

Slope of R = 0.5 ?

Step 3: Application of Kawai's 'Modified Fatigue Strength Ratio'.

* **improvable in the intermediate R-range**

Application of Miner-‘Rule‘ , for the *simple* loading *example* *R* = -1



$$D\ (FF1, FF2) = NF : (n_1 / N_1 + n_2 / N_2 + n_3 / N_3) + SF : (n_4 / N_4)$$

$$\Rightarrow D = D\ (FF1, FF2) + D\ (IFF1, IFF2, IFF3)\ \leq\ D_{feasible}$$

from test experience

Step 4: Determination of Damaging Portions by Static Strength Criteria

Step 5: Mode-wise Accumulation of Damaging Portions (novel)

What was the main Objective of this Investigation ?

*on basis of the ‘**rigorous failure -mode thinking**‘*

Fatigue pre-dimensioning of

‘well-designed’, UD laminas-composed laminates

just by single lamina-dedicated, mode-representative Master S-N curves,

derived from *sub-laminate* test specimens,

which capture the embedding (in-situ) effects,

and

on S-N data from

automatically derivable (curved) Constant-Fatigue–Life curves or

numerically constructed Haigh diagrams, respectively.

Some progress is reached.
Further investigations are necessary !

“ Scientists would rather use someone else's toothbrush than someone else's terminology! “ … or *theory*

(Nobel laureate Murray Gell-Mann)

Hopefully,
this will not be the same
with my new idea
on Lifetime Prediction !?

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Damaging Drivers of Ductile and Brittle behaving Materials

- **Ductile Material Behavior** (example: isotropic metal materials)

  1 damaging mechanism *acts = "slip band shear yielding"*

  drives damaging under cyclic tensile, compressive, shear and torsional stresses:

  Therefore, this single mechanism can be described by one single strength formulation:

  such as the Mises Yield failure condition!

- **Brittle Behaving Material Behavior :** isotropic Materials

  2 damaging driving mechanisms act = *Normal Fracture failure mode (NF), Shear Fracture failure (SF)*

- **Brittle Behaving Material Behavior : transversely-**isotropic UD Materials

  5 damaging driving Fracture failure mechanisms act ≡ 5 Fracture failure modes

Some Lessons Learnt to Tackle Uncertainties

1. ***Model physics accurately***

   The choice of the task-corresponding stress-strain curve has to be carefully performed (min or mean or max value).

2. ***Recognize the design driving parameters and reduce their scatter*** *(uncertainty)*

   Increasing mean value and decreasing *standard deviation* lower failure probability

3. ***Design robust*** (tolerant) **for robustness to later changes of design parameters** with identification of the most sensitive design driving parameters

4. ***Transfer uncertainty (fuzziness) into stochastic uncertainty***

   This makes a quantitative assessment possible in design and highly pays off !

5. ***Do not overreact by re-design if the MoS turns slightly negative***

   Reduce scatter where possible !

   The *Failure probability* p_f does not dramatically increase!

   A *MoS* value does not outline risk or failure probability!