

## Progressive failure analysis of static compression of filled-hole structure based on micro-mechanics of failure

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### Abstract

The static strength of filled-hole compressive (FHC) structure of polymer composites using the theory of micro-mechanics of failure (MMF) is analyzed. The MMF approach combined with non-linear analysis in macro-level and a linear elastic micromechanical failure analysis in micro-level is introduced. A face-centered cubic micromechanics model is constructed to analyze the stresses in fiber and matrix in micro-level. Non-interactive failure criteria are used to characterize the strength of fiber and matrix. The non-linear shear behavior of the laminate is studied experimentally, and a novel approach of cubic spline interpolation is used to capture significant non-linear shear behavior of laminate. The user-defined material subroutine UMAT for the non-linear shear behavior is developed and combined in the mechanics analysis in the macro-level using the Abaqus Python codes. The failure mechanism and the static strength of the FHC structure of UTS50/E51 polymer composites is studied based on MMF method.

### 1. Introduction

In recent years, carbon fiber reinforced polymer (CFRP) composite are used in various advanced structures due to their high strength and stiffness, outstanding design ability, long fatigue life, corrosive resistance and so on. Mechanical connection is one of the most important connection modes for the main load-carrying structure of composite material<sup>[1]</sup>. However, mechanical fasteners will introduce complex stress field in the areas around the bolt hole and reduce the bearing capacity of the components, thus joint connection become a common aircraft structure damage location.

Most aircraft composite structures are still designed based on the allowable values of the open-hole compression. The open-hole samples can provide a repeatable stress concentration, sample form is simple and the manufacturing cost is low. However, for the mechanical connection design, the assumption treating the bolt hole as open hole is too conservative<sup>[2]</sup>. So it is necessary to use the allowable values for filled-hole compression (FHC) structure which is more representative to demonstrate the local stress distribution around the fasteners. The FHC value can improve the allowable bearing capacity of structure.

This paper is based on the micro-mechanics of failure (MMF) which proposed by Ha SK<sup>[2]</sup>. It is a real physics-based strength theory for strength prediction of the structures made of CFRP laminates, in which the initial failure of constituents is analyzed in micro-level with the interactive failure criteria. In this theory, the square or hexagonal unit cell model applied with the loads, corresponding to the stress states in macro-level were used to look insight into the stresses on the fiber and matrix in micro-level.

In this paper, the failure modes and the strength and of FHC structure of UTS50/E51 with multidirectional stacking sequence have been investigated. The predicted results are compared with that predicted from other widely used failure criteria as well as experimental data.

■ 2. The MMF theory

MMF theory is different from other theories because it is an physics-based strength theory for failure prediction of the structures made of CFRP laminates. Firstly, this theory selects distinctive unit cell which include fiber, matrix and the interface between them, then selects a certain number of key points from them (this article selected 17 points on fiber, 19 points on resin) and analyze the stress state of these points. Extracting the stress amplification matrix then the stresses of laminate are converted to the stresses of fiber and matrix at micro level using the stress amplification factors. Finally failure criterion of judgment can accurately determine the position of the failure and the concrete failure mechanism. This paper chooses centered cubic microscopic model as the unit cell, stress amplification matrix is the key factor to realize the transformation from macro stress to micro stress, and stress amplification matrix is obtained by applying model of the micro unit load, extraction of each unit load (six mechanical load and a thermal load) under the stress of each key component values.

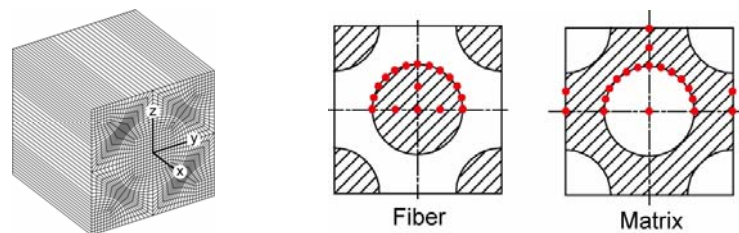


Figure 1. Microscopic stress analysis unit cell and the position of key point

Achieving the transformation from macro stress to the microscopic stress after getting the stress amplification matrix, the transformation equation is shown by(1):

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix}^{(i)} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & M_{36} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & M_{46} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & M_{56} \\ M_{61} & M_{62} & M_{63} & M_{64} & M_{65} & M_{66} \end{bmatrix}^{(i)} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix}_{\sigma} + \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{pmatrix}_{\sigma} \Delta T \quad (1)$$

Where  $\sigma_{mech}^{(i)}$  is the micro stress in fiber or matrix,  $\sigma_{mech}$  is the macro stress in CFRP laminate,  $\Delta T$  is the temperature difference between environment temperature and the curing temperature,  $M_{\sigma}^{(i)}$  is the stress amplification factors of point  $i$  either in fiber or matrix resin,  $A_{\sigma}^{(i)}$  is the thermal stress amplification factors. the enlarged stress on each key point is obtained by macro to micro transformation equation, compared with four characteristic parameters of strength in micro failure theory and the failure state of the composites were characterized. The four characteristic parameters are: fiber tensile strength of  $T_f$ , fiber compression strength of  $C_f$ , tensile strength  $T_m$  matrix, and the matrix compression strength of  $C_m$ . Four characteristic parameters of expression as shown in (2) in the equation. Characteristic parameters are obtained as follows: Firstly, according to the data in table 1 and using the numerical simulation software to apply the corresponding radial tensile stress, radial compressive stress, the transverse tensile stress and transverse compressive stress on the unit cell respectively. Extract the stress state ( $\sigma_1 \sim \sigma_6$ ) of unite cell on each key point (17 key point on the fiber and 19 key points on resin ) under the force state. Then according to the type(2), 36 key points and each corresponds to the internal stress along the fiber direction are extracted. equivalent stress on the fiber direction, the first stress invariant matrix, the matrix equivalent stress, these four types of stress intensity of each stress types maximize as characteristic parameters. Finally the strength of the component are characterized.

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**Table 1.** Material strength of the UD laminate for UTS50/51

Strength parameters	Average value /MPa
Longitudinal tensile strength $X$	2100
Longitudinal compressive strength $X'$	1932
Transverse tensile strength $Y$	54
Transverse compressive strength $Y'$	179

$$\left\{ \begin{array}{l} T_f = \max(\sigma_1^{f,i})_{i=1,2,3 \dots 17}, (\sigma_1^f > 0) \\ C_f = \max(\sigma_{eq}^{f,i})_{i=1,2,3 \dots 17}, (\sigma_1^f < 0) \\ T_m = \max(I_1^{m,i})_{i=1,2,3 \dots 19} \quad I_1^m = \sigma_1 + \sigma_2 + \sigma_3 \\ C_m = \max(\sigma_{eq}^{m,i})_{i=1,2,3 \dots 19} \quad \sigma_{eq}^m = \sqrt{0.5[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 + 6(\tau_{yz}^2 + \tau_{xz}^2 + \tau_{xy}^2)]} \end{array} \right. \quad (2)$$

Where the material axes 123 are used as coordinate direction. The  $T_f$  and  $C_f$  are extracted as the maximum stress in fiber direction. The  $T_m$  and  $C_m$  are extracted as the function of stress component. The failure index of the fibers and matrix is defined as

$$k = \max \left[ \max \left( \frac{\sigma_t^{f,i}}{T_f} \right), \max \left( \frac{-\sigma_{eq}^{f,i}}{C_f} \right), \max \left( \frac{I_1^{m,j}}{T_m} \right), \max \left( \frac{\sigma_{eq}^{m,j}}{C_m} \right) \right] \quad (3)$$

Where  $i = 1, \dots, n_1, j = 1, \dots, n_2, n_1, n_2$  representing the numbers of key points in the fiber and matrix, respectively. When the failure index  $k$  reaches 1, the failure occurs either in the fiber or in the matrix.

The progressive failure analysis model based on the MMF theory in our previous paper<sup>[9]</sup> is used to analyze the stiffness matrix degradation during loading. All these processes, including the macro to micro stress transformation, calculation of the MMF parameters for the components, material performance degradation, are performed in the platform of Abaqus, using the user material subroutine (UMAT) compiled.

### 3. Static compression test and numerical analysis model

#### 3.1 Experimental material

The experiment adopts the standard of ASTM/D6742, the sample size is 38.1 mm \* 118 mm, hole diameter is 6 mm, layer order is [45/0/-45/90]<sub>2s</sub>, molding by hot pressing technology<sup>[10]</sup>. Samples are shown in Figure 2. Material system is UTS50 / E51 carbon fiber reinforced resin matrix composites, Table 2 shows the performance of the material. All mechanical performance test are proceeded on the INSTRON1195 microcomputer control electronic universal testing machine<sup>[11]</sup>.



**Figure 2.** The filled-hole structure specimen with stacking sequence [45/0/45/90]<sub>2s</sub>

**Table 2** MMF critical parameters of constituents of UTS50/E51<sup>[17]</sup>

Strength parameters	Values /MPa
Fiber tensile strength $T_f$	3710
Fiber compression strength $C_f$	3440
Matrix tensile strength $T_m$	155
Matrix compression strength $C_m$	207

**3.2 The numerical analysis model**

The size, material and stacking sequence of the laminated plates are consistent with those in test. According to the clamping condition in the test, fixed boundary conditions on the left side of the laminated plates is applied. The element type of the laminated plates is C3D8R (eight nodes with reduced integral point)<sup>[12]</sup>. The element number of laminated plate is 64872.

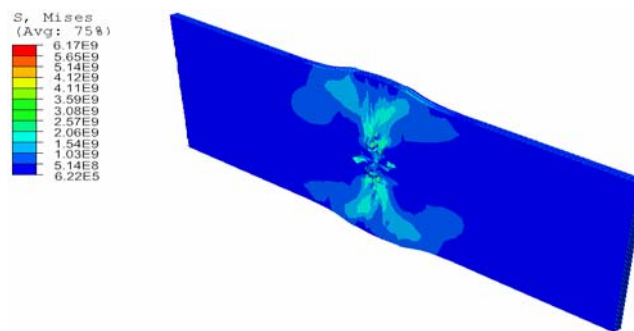


**Figure 3.** FHC specimen and mesh model with stacking sequence $[45/0/-45/90]_{2s}$

**4. The model prediction and experimental validation**

**4.1 The numerical calculation results and analysis**

This article does not use composite dialog for fast modeling when using Abaqus software. The reason is that the composite model is created based on the consideration of the overall mechanical properties, it does not highlight the practical effect of a single layer. Laminated plates, for example, the stress state of different layers but with same angle of the laminate is same in composite dialog, but in practice, due to the same angle boundaries and surface effects, the stress between different layers are different. Based on the consideration of actual effect, this paper uses the partition function to divide layers, then the direction of material and material properties are setted, the simulation results will be more real<sup>[13]</sup>. Figure 4 is the stress contour of FHC laminate which created in Abaqus.



**Figure 4.** The static compression stress contour of the FHC structure

The macro contour show that after the compression, the stress mainly concentrated around the hole, especially in the up and down parts of the hole. The overall stress distribution shape is umbrella like.

MMF theory is used to establish the nonlinear model and analyse the micro stress of FHC structure, depicting the microscopic stress distribution contour of components within each layer<sup>[16]</sup>. Figure 5 (a) shows the microscopic stress distribution contour of the first four layers of components when failure occurs. The tensile stress on the resin of the second layer is 156.76 MPa which beyond the MMF characteristics of resin tensile parameters which is 155MPa. This result suggests that the failure position of the FHC structure is in 0°layer, the failure mechanism is tensile failure of matrix.

In order to analyze the failure mechanism of the FHC structure more accurately. Using MMF theory combine with the established nonlinear model to analyze the index distribution of each component, depicting all the four failure index distribution of each layer<sup>[17][18]</sup>. Figure 5(b) shows the failure index distribution of top four layer, the figure shows that the first one which the failure index reaches to 1 is the 0° layer tensile failure of the matrix  $k_{T,m}$ . The failure of FHC structure is sparked by the hole edge.

Figure 6 (a) and (b) show the MMF parameter contour and the failure index contour of 1 to 4 layer in the final failure of FHC structure. The stress concentration around the hole had been expanded in a flake form. Specific failure mechanism can be directly observed by the failure index contour of each component. The failure zone is mainly along both sides of the holes perpendicular to the direction of the load, which are consistent with the test results. The fiber and resin of laminated plates in each layer both occurred failure, only in the 90° layer, on both sides of the holes have the fiber tensile failure. the failure mechanism of ±45° and 0° layer is the compression failure in both matrix and fiber, the predicted stress of the FHC structure is 17.1kN.

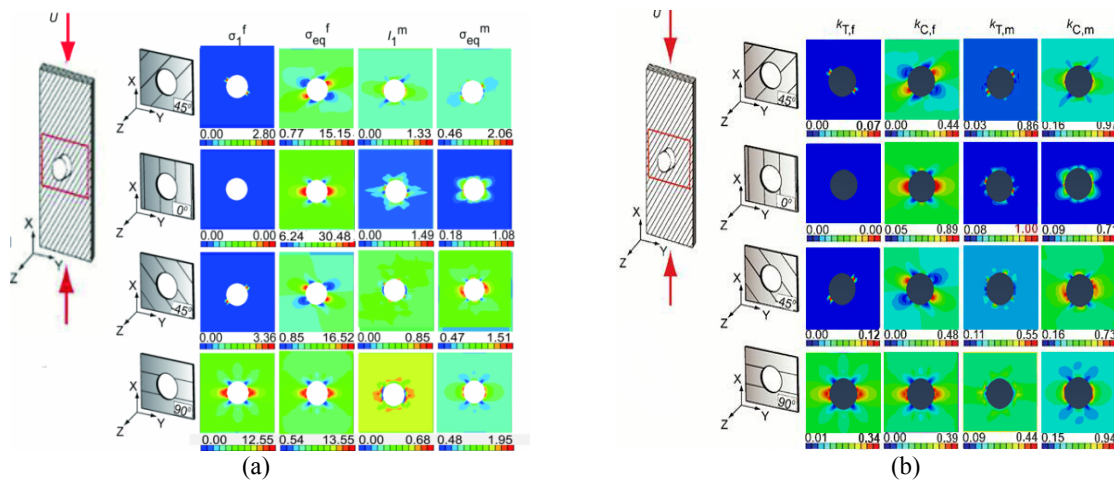


Figure 5. MMF parameter and failure index in first four layer of FHC structure with stacking sequence  $[45/0/-45/90]_{2s}$  under static compression in initial failure

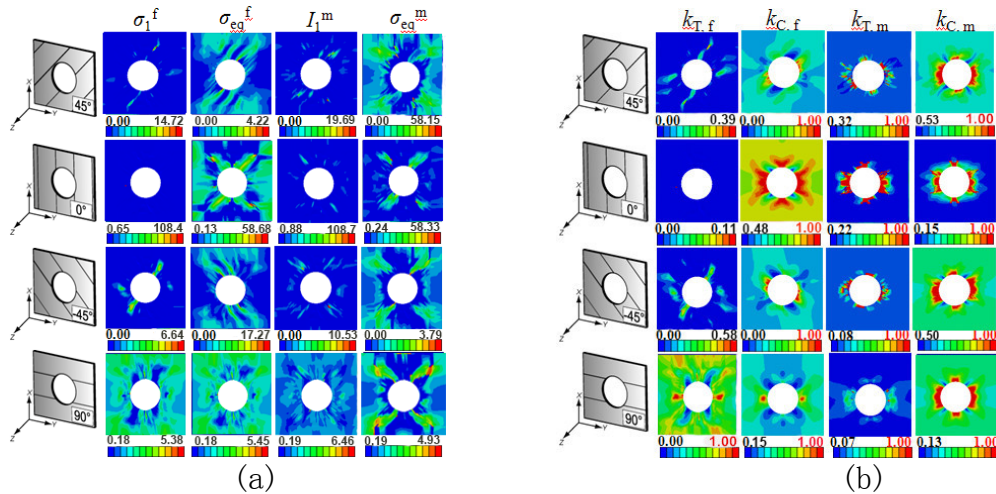
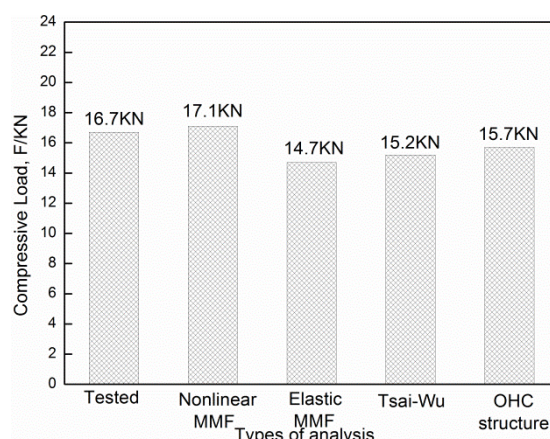


Figure 6. MMF parameter and Failure index of first four layer of FHC structure with stacking sequence  $[45/0/-45/90]_{2s}$  under static compression in final failure

Figure7 shows the nonlinear MMF predicted values and experimental values of the FHC structure, the comparison between the elastic theory of MMF, Tsai - Wu theory and OHC structure. The nonlinear MMF predicted values is relatively close to the experimental results, the error is 2.4%, and the relative error between the elastic MMF method and Tsai - Wu predicted results are 11.9% and 8.9% respectively. Besides, the deviation of FHC result from the OHC structure result is 5.9% higher, this result justifying the conservatism of OHC structure.

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**Figure7.** The static compression strength comparison between prediction and test value

Analyzing the relationship between damage failure mechanism of each layer and the press displacement with gradual failure analysis model based on the theory of the MMF. Table 3 shows that the initial failure appears in the matrix of 0° layer, then the failure appears in the resin of +45° layer and fiber of 0° layer. Subsequently is the resin of -45° and 90° layer, the final failure appears in the fiber of 90° layer and +45° layer. When the displacement is greater than 0.93 mm filled-hole structure completely destroyed.

**Table 3.** The gradual failure mechanisms of FHC structure made of UTS50/E51 laminate with stacking sequence [45/0/-45/90]2s under static compression.

Compression Displacement /mm	45° layer		0° layer		-45° layer		90° layer	
	fiber	matrix	fiber	matrix	fiber	matrix	fiber	matrix
0.57				•				
0.62		•	•	•				
0.73		•	•	•		•		•
0.79-0.93		•	•	•	•	•		•
> 0.93	•	•	•	•	•	•	•	•

#### 4.2 Test results

The static compression tests are performed by a universal testing machine INSTRON1195 according to the standard of ASTM D 6742/6742M-12 at a displacement rate of 1 mm/min. The average value of three specimens is used to characterize the static strength of FHC structure. Compression load-displacement curve is shown in Figure 8. The curve shows that there is no jumping in the stress-strain curve till final failure and the damage process from the initial to the final failure is accomplished instantaneously. The static compression strength of FHC structure characterized by the average of three groups of test, the average intensity of test is 16.76 kN, The simulation prediction is very close to the test result and the average error is 2.4%.

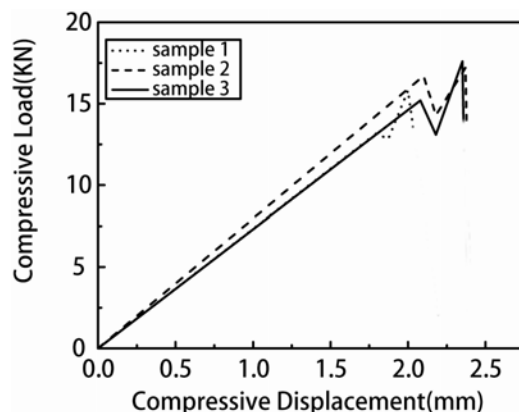


Figure 8. Compression load - displacement curve of FHC structure made of UTS50/E51 laminate with stacking sequence  $[45/0/-45/90]_{2s}$  under static compression

Figure 9 is the macroscopic morphology of final failure. It shows that after the compression, crack extends along the direction that perpendicular to the loading direction, and the most dangerous area is around the hole where fiber fracture and exfoliation appears. This is consistent with the prediction results.



Figure 9. The macro contour of FHC structure with stacking sequence  $[45/0/-45/90]_{2s}$  under static compression

## 5. Conclusions

The failure modes and the strength of filled hole compressive (FHC) with stacking sequence  $[45/0/-45/90]_{2s}$  of UTS50/E51 have been investigated using the progressive failure method based on the theory of micro-mechanics of failure (MMF).

The initial failure occurs in the matrix of  $0^\circ$  layer, subsequently the break of fiber in the  $0^\circ$  layer appears, which leads to the final failure of the FHC structure.

The predicted strength of the FHC is 17.1kN, agreeing well with the experimental result with relative error of 2.4% higher. This result is also higher than the experimental result for OHC structure of 15.7kN. This result shows the OHC experiment is a conservative method, and the FHC result is more precise one near the practical condition.

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