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TWO-SCALE DAMAGE MODELLING OF A FIBRE REINFORCED COMPOSITE BASED ON X-RAY COMPUTED TOMOGRAPHY DATA

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Abstract

A two-scale progressive damage model is constructed, aimed at taking into account stochastic fibre failure and formation of clusters of fibre breaks at micro-scale. The meso-scale model is a voxel finite element model constructed from an X-ray computed tomography image. The meso-scale model represents the geometry of the material as the distribution of the components (matrix/reinforcement), local fibre orientations, and local fibre volume fraction. The local fibre volume fraction is a parameter of the micro-scale model that determines the number of fibres in the micro-volume. The micro-scale model is a model of a fibre bundle with fibre strengths sampled from Weibull distribution, and takes into account stress enhancement due to fibre breaks. Simulations showed that the micro-scale model is able to reproduce formation of the clusters of fibre breaks and unstable propagation of fibre failure when a critical load level is exceeded.

Keywords: damage modelling, X-ray computed tomography

1. Introduction

Damage is an irreversible change of the internal structure of a material, leading to the degradation of the material's stiffness. Due to its irreversibility, damage is a cumulative process, i.e. the stiffness cannot increase with time. Modelling of the damage process is aimed at predicting the stiffness reduction of the material under an increasing load. In fibre-reinforced polymers, there are several types of damage that can be distinguished. When the strain in the direction of fibres exceeds an ultimate value for the fibre, the fibre breaks. The break occurs virtually instantaneously and releases a significant amount of the elastic energy in the form of acoustic waves, which can be registered with the acoustic emission technique. After the break, the stress in a segment of the fibre near the break drops and redistributes to the neighbouring fibres. The length of the broken fibre over which the longitudinal stress is reduced below certain value is usually referred to as ineffective length. The critical value of stress recovery, which underlies the notion of the ineffective length concept, is rather conventional, Rosen [1] suggested to choose it at 90% level of the ambient stress. The stress profile along the broken fibre can be calculated using shear lag model [2]. According to this model, the length at which the fibre is under-loaded due to the break depends on the shear modulus of the matrix, modulus of the fibre, diameter of the fibre.

After the break, a part of the load the fibre was carrying, redistributes to the nearby fibres through the matrix. This redistribution can be expressed as a stress concentration factor (SCF), which shows how much the stress in the nearby fibres is increased compared to the stress level before the break. A basic

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estimation of the SCF can be done using shear lag model, which assumes that the fibres carry only axial loads and the matrix carries only shear loads [3-5]. Improvement of this model was made in [6] by taking into account fibre and matrix stiffness and the fibre volume fraction. Their analysis showed that the SCF increases with increasing value of the ratio E_f/E_m and with increasing fibre volume fraction. A further improvement was made by Ochiai [7, 8] by adding the possibility of matrix cracking and fibre-matrix debonding. There is indeed an experimental evidence that the matrix cracking may accompany fibre breaks [9], with three modes: disk-shaped cracking of the matrix orthogonal to the broken fibre, two inclined conical cracks emanating from the break in fibre, and cracking along the interface. Analysis of Ochiai showed that matrix cracking increases the SCF, whereas fibre-matrix debonding decreases the SFC.

Experimental investigation of the stress redistribution using Raman spectroscopy [10-12] showed that (a) the model of Ochiai predicts the SCF reasonably well, although at low fibre volume fractions the SCF is overestimated; (b) at high fibre volume fractions, the SCF is large enough to cause failure of adjacent fibres; (c) with decreasing fibre volume fraction the SCF drops and the fibre failure pattern transitions from coordinated to random. The authors note that these results indicate that in FRP with high fibre volume fractions the fibre breaks are expected to be coordinated (clustered), with little debonding, i.e. similar to brittle fracture. A study of fracture in UD composites using synchrotron X-ray tomography [13] confirmed the clustered formation of fibre breaks. As the load is increased, clusters of fibre breaks appear with increasing number of broken fibres in a cluster. Following the idea of Griffith’s model for brittle fracture, it was suggested that there is a critical size of the cluster, leading to the unstable propagation of fracture when it is exceeded. The theoretical prediction for the critical cluster size yielded values of 20 or 21 depending on the fibre volume fraction. Experimental data yielded comparable values, although the variability was significant, which could be explained by the complexity of the experimental procedure.

As the picture of the fibre failure established above suggests, the fibre failure in FRP is a stochastic phenomenon, i.e. the order in which the fibres fail and the load level at which the next failure occurs are random. This randomness can be attributed to the following factors: variability of the strength of fibres, and variability in the local fibre volume fraction. Distribution of the fibre strengths is usually described by Weibull law. Experimental determination of the Weibull distribution parameters is associated with difficulties of statistical nature, as the number of fibres needed to be tested in order to accurately estimate the parameters is quite large (hundreds) [14]. Randomness in the fibre arrangements results in the scatter of the local fibre volume fraction [15]. The degree of this scatter depends on the spatial scale at which the fibre volume fraction is calculated: in a larger volume more fibres are averaged and the scatter is lower. In addition, misalignment of the fibres leads to an uneven distribution of the load between the fibres, even before the first fibre break.

This paper develops a two-scale model aimed at capturing the stochastic properties of fibre failure at micro-scale, while modelling the composite material geometry at meso-scale. The meso-scale model, constructed on the basis of an X-ray computed tomography image, contains information on the fibre orientations and the local fibre volume fraction. The fibre strengths, which cannot be determined from an image, are sampled from Weibull distribution.

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2. The two-scale damage model

The two-scale model includes meso-scale model and micro-scale model. The meso-scale represents the 3D geometry of the composite, which includes the distribution of the matrix and reinforcement in the modelled volume, and the local fibre orientations. The meso-scale model is a voxel finite element model. Each element of the meso-scale model contains an instance of the micro-scale model. The micro-scale model is a model of a fibre bundle, where fibres have random strengths sampled from a Weibull distribution. The micro-scale model takes into account stress redistributions after fibre breaks, which is aimed at capturing formation of the clusters of fibre breaks. The details of the meso-scale and micro-scale models are given in the next two sections. Section 2.3 describes the implementation of the two-scale model.

2.1. Meso-scale model

The meso-scale model is a voxel model, constructed from an X-ray computed tomography image of a 3D reinforced glass/epoxy composite using the approach developed in [16]. Each voxel of the meso-scale model contains variables indicating the material type (matrix/reinforcement) and the vector of the local fibre orientations, calculated from the image. In addition, as a further enhancement of the original approach, a variable is added that indicates the local fibre volume fraction inside the voxel. The local fibre volume fraction is calculated from the average grey value g in the region of the image, corresponding to a voxel, as follows:

$$v_f(g) = \frac{g - g_m}{g_f - g_m}$$

The parameter g_m is the grey value of the matrix, which is measured in the image using ImageJ software. The g_f parameter is the grey value of the fibre. It is estimated based on the measurement of the average grey value of the reinforcement g_R and the average fibre packing ratio $\overline{v_f}$:

$$g_f = \frac{1}{\overline{v_f}}\left[g_R - \left(1 - \overline{v_f}\right)g_m\right]$$

The local fibre volume fraction is a parameter of the micro-scale model, which determines how many fibres are present in the micro-scale model.

2.2. Progressive damage model at micro-scale

At micro-scale, where the reinforcement can be treated as a unidirectional material, the progressive reduction of the stiffness can be caused by the fibre failure, and by the formation of matrix cracks orthogonal to the fibres. Depending on the ratio of ultimate strains of the fibre and the matrix, one of these modes will be dominant. In fibre-reinforced polymers, the matrix usually exhibits plasticity and the matrix cracks orthogonal to fibres are not usually observed, which makes the fibre failure a primary mode of the longitudinal failure. Therefore, the reduction of the longitudinal stiffness will be assumed to be caused by the fibre failure only. The statistical distribution for the strength of fibres is the Weibull distribution, which specifies the probability of failure for the fibre over the length L:

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$$P(\sigma) = 1 - \exp\left[-\left(\frac{L}{L_0}\right)\left(\frac{\sigma}{\sigma_0}\right)^m\right]$$

where L_0 is the reference length, m and σ_0 are parameters of the distribution. The probability of failure grows with the increase of the fibre length. It can be shown that Weibull distribution of fibre strength follows from the model of flaws distributed throughout the fibre and the Griffith's theory of brittle fracture (Freudenthal). Sampling from Weibull distribution is straightforward:

$$\sigma = \sigma_0\left[-\left(\frac{L_0}{L}\right)\ln(1-\xi)\right]^{\frac{1}{m}}$$

where $\xi \in (0,1)$ is a uniform random variable. This allows sampling random fibre strengths. The next step is to create a random unidirectional arrangement of fibres to model the effect of stress redistributions after fibre breaks. One of the methods to generate random fibre arrangements was proposed by Melro and Camanho [17]. Their method includes three stages. The first stage is the classical hard-core algorithm, which consists in placing fibres at random, checking for the overlaps and rejecting the placement if the overlap is detected. This algorithm however is able to generate fibre arrangements with the volume fraction below ≈55% only. The last two stages of the Melro and Camanho method are aimed at stirring the fibres so that to create resin-rich areas where new fibres can be placed to achieve a higher volume fraction. Here we use the hard-core algorithm to create fibre arrangements with fibre volume fractions below the mentioned limit. For the higher volume fractions, we use an algorithm, similar to the one proposed by Wongsto and Li [18], which involves generation of a hexagonal arrangement with the required fibre volume fraction, and introducing random perturbations of this hexagonal structure. After the initial hexagonal configuration is created, each fibre is given a random shift along both *x* and *y* coordinates:

$$x_f = x_f + dx$$

$$y_f = y_f + dy$$

where *dx*, *dy* are normally distributed variables with zero mean and the standard deviation equal to the distance δ_{max} between fibre edges in hexagonal arrangement:

$$\delta_{max} = r\sqrt{\frac{\pi}{v_f \cos(\pi/6)}}\cos\left(\frac{\pi}{3}\right) - r$$

where r is the fibre radius, v_f – fibre volume fraction. After giving a fibre a random shift, an overlap check is performed and, if no overlap is found, the fibre position is updated. This procedure is repeated 10^5 times.

The stress redistribution due to the fibre breaks is modelled so that to take into account clustering of the broken fibres. A set of broken fibres is considered a cluster when all the fibres in the set are located within a distance h from each other. It is then assumed that the broken cluster of fibres redistributes their load onto the fibres surrounding the cluster within a distance of $2r/v_f$ from the cluster boundaries. The stress concentration factor is calculated as follows. Firs, an effective area of the cluster is calculated:

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$$A_{cl} = \frac{\pi r^2}{v_f} n_{cl}$$

where n_{cl} is the number of fibres in the cluster. The effective radius of the cluster is then $r_{cl} = \sqrt{A_{cl}/\pi}$. The area of the surrounding stress enhancement zone is then:

$$A_s = \pi\left(r_{cl} + 2r/v_f\right)^2 - \pi r_{cl}^2$$

The expected number of fibres in the surrounding area is:

$$n_s = \frac{A_s v_f}{\pi r^2}$$

The stress concentration factor is the ratio of the number of broken fibres in the cluster and the number of fibres surrounding the cluster:

$$SCF = 1 + \frac{n_{cl}}{n_s}$$

Figure 1 shows SCF as a function of the cluster size for clusters of up to 30 broken fibres.

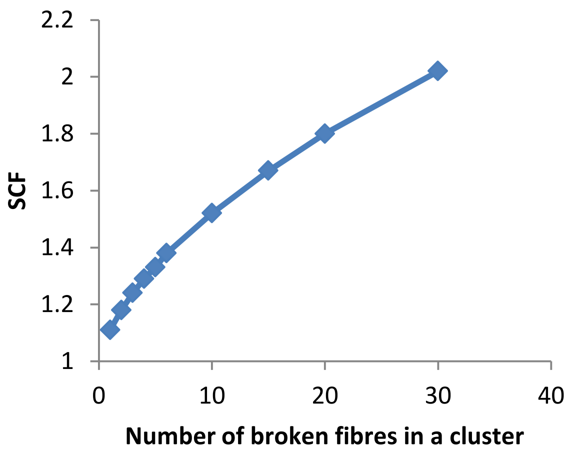

Figure 1. Stress concentration factor as a function of the cluster size

2.3. Interaction between scales and implementation

The modelling procedure involves initialization of the micro-scale models, incremental loading and equilibrium iterations at each load increment. During initialization, fibres strengths are sampled from the Weibull distribution for each micro-scale model, with the number of fibres determined by the local fibre volume fraction and the volume of the meso-scale element. A random fibre arrangement is then generated. After every load increment, micro-scale models are run with the current strain value provided from the meso-scale. For each fibre in the micro-scale model, the stress is calculated as follows:

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$$\sigma = E\varepsilon \prod_{i=1\ldots k} SCF_i$$

where SCF_i is the stress concentration factor from i-th cluster of fibre breaks nearby the current fibre; E is the fibre modulus. The calculated stress is compared to the strength of the fibre, and if the stress exceeds strength, the fibre is labelled as broken. After all fibres are evaluated, the homogenized stiffness of the micro-scale element is calculated using Chamis model, taking into account intact fibres only.

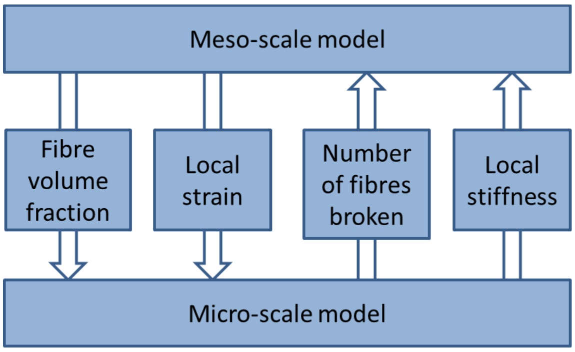

Figure 2. Interaction between the scales of the progressive damage model

3. Results

Figure 3 shows evolution of the broken fibres in a micro-model with increasing load. After the first breaks occur, further breaks tend to cluster together. The number of broken fibres (Figure 4a) increases smoothly at small loads but shows a sudden jump after a certain critical load is reached. In contrast to this, the model that does not take into account stress redistribution (Figure 4b) does not show such a sudden increase in the number of broken fibres. The number of clusters of broken fibres (Figure 5) shows two stages of cluster formation (a single broken fibre is also a cluster), followed by rapid coalescence, which appears as sudden drop in the number of clusters. This shows that the model is able to reproduce a sudden failure of a composite loaded in the direction along fibres.

4. Discussion and conclusions

The simulations with the developed micro-scale model showed that the stress redistribution lead to a qualitative change in the progression of the fibre failures, compared to the case with independent failure. This can be associated with the formation of a critical cluster of fibre breaks, after which the propagation of fibre failures becomes unstable. The model is still rather simplistic, for example, it does not take into account inefficient length. If the inefficient length is smaller than the size of a finite element of the meso-scale model, the micro-volume have to be modelled as a “chain of bundles”. The complexity of the micro-scale model, however, is limited by the computational costs, as the micro-scale model is run in every finite element of the meso-scale model.

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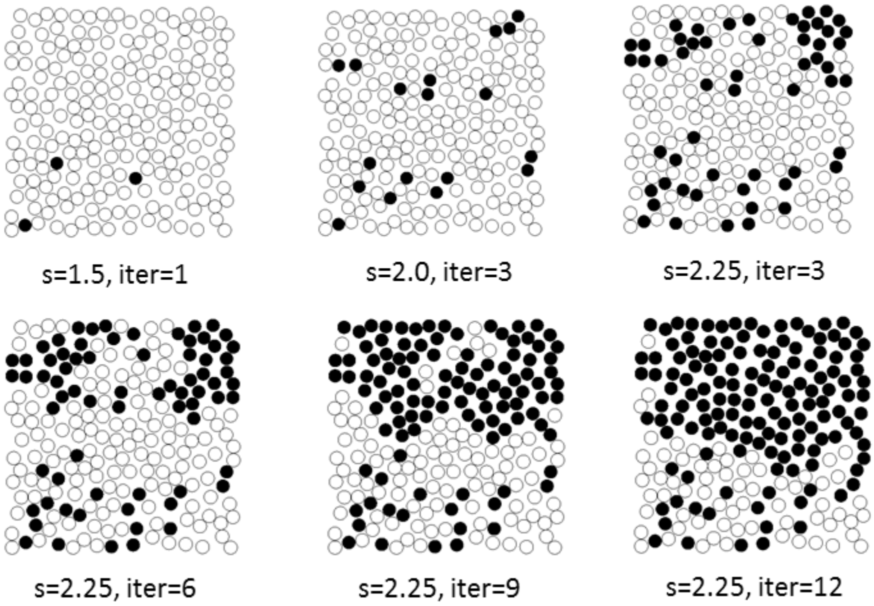

Figure 3. Formation of clusters of fibre breaks in the micro-scale model. The current stress level (in GPa) is indicated with "s", equilibrium iteration number is indicated with "iter".

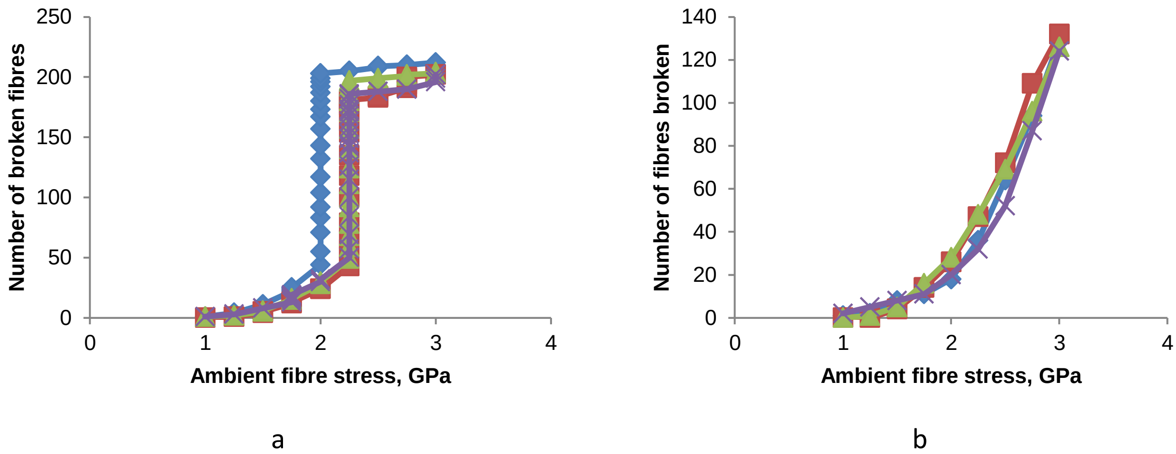

Figure 4. Evolution of the number of broken fibres in a micro-scale model with stress redistribution (a) and without stress redistribution (b).

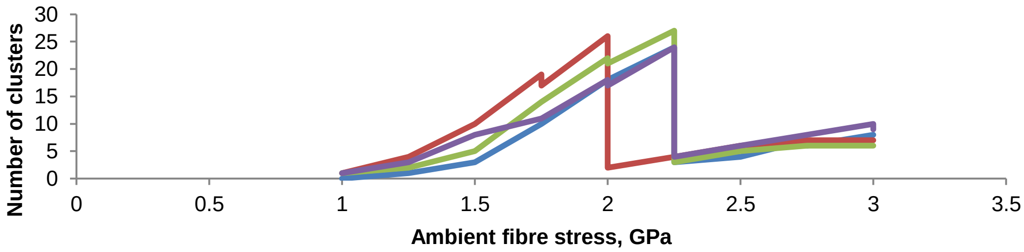

Figure 5. Number of clusters of broken fibres as a function of applied load.

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