VARIABLE STIFFNESS COMPOSITE LAMINATES FOR ROTATING PRE-TWISTED PLATES

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Abstract
A methodology for the design and optimisation of non-symmetric variable angle tow (VAT) laminates for pre-twisted plates subjected to centrifugal force is discussed and the results shown. The use of VAT concepts allows for a laminate to fulfil different criteria throughout a highly stressed structure. It is shown that by using a non-symmetric laminate which intrinsically has stiffness coupling such as extension-twist and bend-twist coupling, the untwist of a rotating pre-twisted can be passively increased while maintaining a desirable cross-sectional shape. It is shown that the combination of non-symmetric and VAT laminates can improve structural efficiency of the pre-twisted-plate.

1. Introduction

Rotating pre-twisted plates are used for many applications within engineering, most notably within turbomachinery. They are exposed to a complex combination of centrifugal, gas and temperature loading and it is important that they retain their shape as the aerodynamic profile is critical to their performance. The combination of loading coupled with the plate geometry causes a pre-twisted plate to deform in an undesirable manner, in particular with centrifugal loading causing the plate to untwist. This can be thought of as a geometric coupling between initial plate geometry and loading. The use of composite materials such as carbon fibre reinforced plastics are ever increasing, with a rapid interest in their use for turbomachinery applications in recent years. A useful property of composite materials is the ability to tailor the ply stacking sequence such that they exhibit stiffness coupling effects such as extension-twist and bend-twist coupling. Using stiffness coupling the pre-twisted plate shape change can be passively controlled.

The use of variable angle tow (VAT) non-symmetric and/or unbalanced laminates with stiffness coupling has been extensively researched over the past few decades, most notably for wind turbine blades [1] and aeroplane wings [2]. There have been a few studies and investigations into spinning pre-twisted

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unidirectional laminate composite plates, although with small values of pre-twist. Bhumbla [3] and Kosmatka [4] investigated the behaviour of spinning pre-twisted composite plates with non-symmetric laminates. They showed that using anti-symmetric laminates, the amount of untwist of the plate can be increased or decreased by varying the antisymmetric ply angle. They also showed that the optimum ply angle for decreasing or increasing untwist varies with the pre-twist of a plate.

As the loading magnitude and direction varies throughout the structure, so too should the fibre direction. This can be accomplished by using VAT [2, 5, 6] laminates which vary fibre angle dependent upon location. The use of VAT composite has recently received considerable interest, but has yet to be utilised in turbomachinery applications. This paper demonstrates a methodology for the design of non-symmetric VAT laminates with the aim of increasing the amount of tip untwist of a pre-twisted plate while satisfying maximum strain allowables. In this paper the effect that four different unidirectional (UD) antisymmetric laminates has on the untwist of a pre-twisted plate was investigated in section 3. A cost function that takes mean tip twist, cross sectional shape and strain allowables into account was developed and implemented for the optimisation of a VAT laminate with a genetic algorithm (GA) optimiser in section 4.

2. Twisted Plate Model

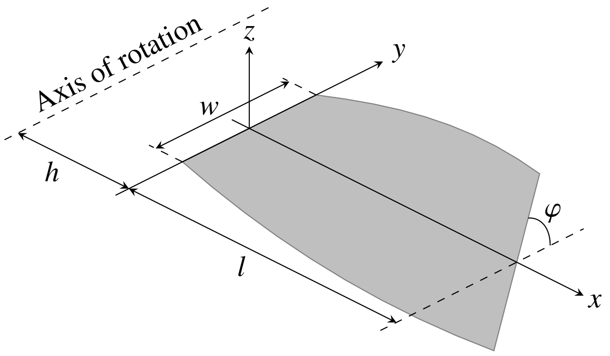

Figure 1. Pre-twisted plate geometry.

The pre-twisted plates were assumed to be a rectangle of span length l and chord width w, where x_0, y_0 and z_o are the flat plate Cartesian coordinates shown in figure 1. The flat plate coordinates satisfy $0 \leq x_0 \leq l$, $-0.5w \leq y_0 \leq 0.5w$ and $z_0 = 0$. The flat plate can then be transformed to its pre-twisted equivalent coordinates using

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi' x_0) & \sin(\varphi' x_0) \\ 0 & \sin(\varphi' x_0) & \cos(\varphi' x_0) \end{bmatrix} \begin{Bmatrix} x_0 \\ y_0 \\ z_0 \end{Bmatrix} \tag{1}$$

where linear pre-twist rate φ' is

$$\varphi' = \frac{\varphi}{L} \tag{2}$$

where φ is the tip angle. The pre-twisted plate had dimension $l = 1000$mm, $w = 500$mm, $\varphi = 70°$ and a disc-hub radius (distance from plate root to centre of rotation) $h = 415$mm. The pre-twisted plate was subjected to centrifugal force equivalent to a rotational velocity of 2660 rpm. Typical material properties for carbon fibre reinforced epoxy plastic were used and are shown in table 1.

Abaqus finite element (FE) software input files were directly created using Matlab. The FE models used S4 elements for a linear analysis. The model was fully restrained at all nodes at the root i.e. at $x = 0$. Once the FE analysis was performed the results were then post-processed in Matlab. The amount of tip untwist is measured as the difference in angle of the tip nodes with their chordwise adjacent neighbour, between the unloaded and loaded states.

3. Unidirectional Laminates

A study into the effect four different anti-symmetric unidirectional (UD) laminates has on the untwist of the pre-twisted plate was conducted. This provided an understanding of the relationship between ply angle and twist while scoping the design space used as a target for improvement in the later optimisation.

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Table 1. Material properties for carbon fibre reinforced epoxy plastic lamina.

Stiffness		Strain Allowables	
E_1 (GPa)	140	ε^*_{1t} ($\mu\varepsilon$)	4000
E_2 (GPa)	10	ε^*_{1c} ($\mu\varepsilon$)	-4000
G_{12} (GPa)	5	ε^*_{2t} ($\mu\varepsilon$)	4000
ν_{12}	0.3	ε^*_{2c} ($\mu\varepsilon$)	-4000
ρ (kg/m^3)	1600	γ^*_{12} ($\mu\varepsilon$)	2600
Ply thickness t_0 (mm)	0.125		

Table 2. Anti-symmetric laminates used with unidirectional laminate investigation.

Laminate Number	Stacking Sequence
1	$[\ \theta_{32}\ /\ -\theta_{32}\ ]$
2	$[\ \theta_{16}\ /\ 0^\circ_{32}\ /\ -\theta_{16}\ ]$
3	$[\ \theta_{16}\ /\ 90^\circ_{32}\ /\ -\theta_{16}\ ]$
4	$[\ \theta_{16}\ /\ -\theta_{16}\ /\ \theta_{16}\ /\ -\theta_{16}\ ]$

The four laminates are shown in table 2, with each laminate having 64 plies. The value of θ was varied in multiples of 5° from -90° to 90°.

Figure 2 shows the mean twist for the laminates in table 2 with plus or minus one standard deviation plotted either side. A negative value of twist indicates an untwisting of the plate. There is a large variation

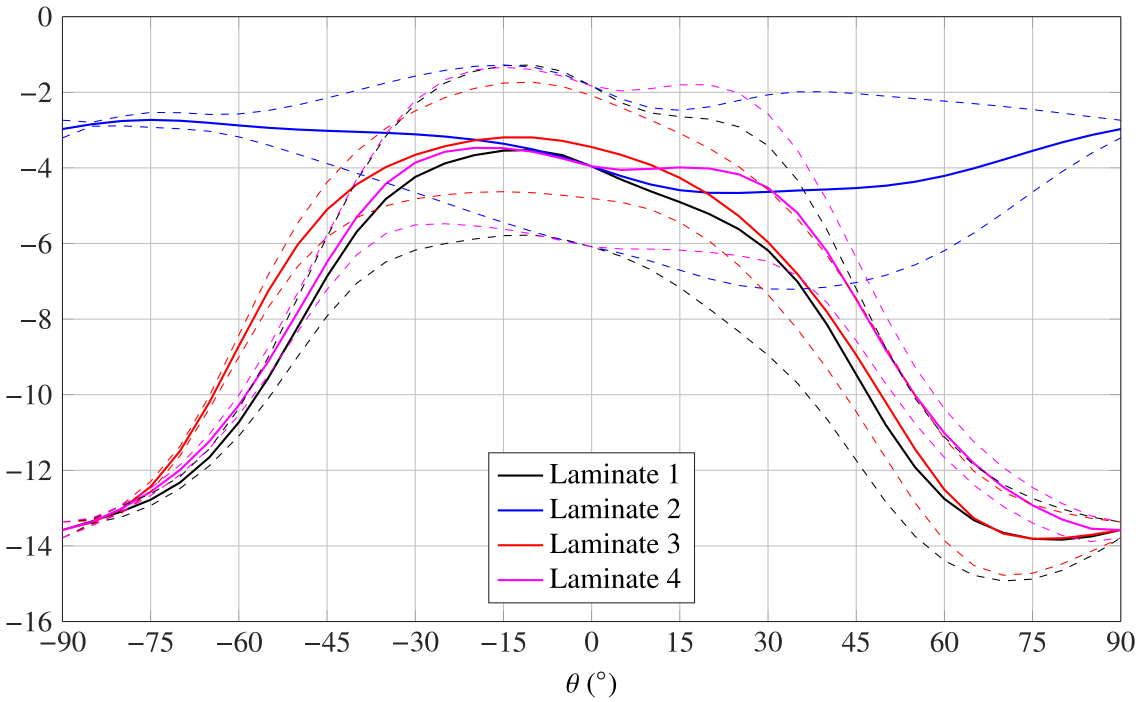

Figure 2. Mean tip twist $\bar{\psi}$ against variation of ply angle θ for various antisymmetric laminates shown in table 2. Dashed line indicating mean twist plus/minus one standard deviation ($\bar{\psi} \pm \sigma$).

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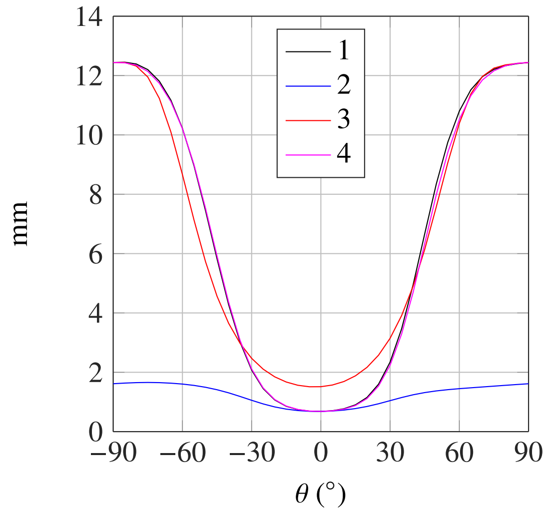

Figure 3. Tip mid chord displacement in x direction for different ply angle θ for various antisymmetric laminates in table 2.

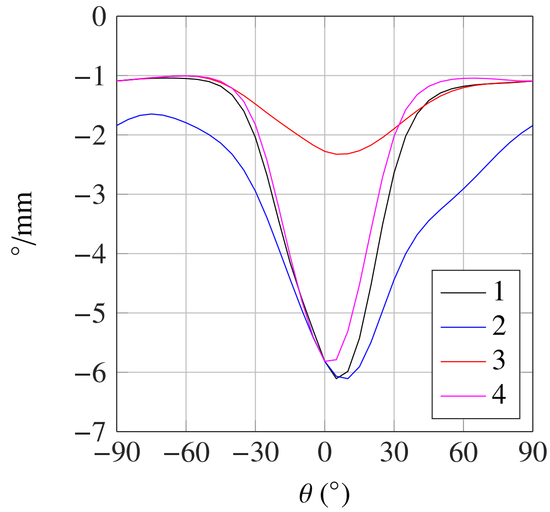

Figure 4. Mean tip twist to tip mid chord displacement in x direction ratio for different ply angle θ of various antisymmetric laminates in table 2.

in mean untwist for the different laminates, with positive angles of θ generally producing greater amounts of untwist due to the extension-twist coupling acting in that direction. The mean tip twist is used as it was noticed that when the plates untwist they exhibit some chordwise bending. The standard deviation (σ) of the tip elements was used to quantify the amount of chordwise bending, a low σ equates to a small amount of chordwise bending and conversely a high σ leads to a large amount of chordwise bending. For plates used in turbomachinery applications it is important for aerodynamic reasons that the plate retain its initial cross sectional shape, therefore small values of σ are desired. From figure 2 it would seem that for laminate 2-4, when $\theta = \pm 90°$ is a very good laminate for increasing untwist. This can be explained when taking into account the mid chord extension of the plate in the spanwise direction, shown in figure 3. When θ is of a high angle i.e. ±90°, the plate is mostly made of soft matrix materials in the spanwise direction and therefore extends the greatest. Due to the untwisting a pre-twisted plate exhibits when rotated, it can be shown that the greater the extension in the spanwise direction the greater the untwist will be. This can be considered as geometric extension-twist coupling between rotational velocity and twist. It would however be detrimental to use a plate of mainly 90 ° plies as this would be weak and fail easily. When considering the amount of chordwise bending it is clear from the relationship between standard deviation and ply angle θ that large magnitude ply angles i.e. ±90° provide the smallest standard deviation and therefore the least amount of chordwise bending. This result is because of the increased chordwise bending stiffness (D_{22}) of the laminate. The effect the central group of plies has on the overall characteristics of the plate is most evident when comparing laminate 2 and 3. As laminate 2 has a central group of 0° plies, the laminate does not produce as much untwist due to the geometric extension-twist coupling discussed earlier. Laminate 2 exhibits a similar relationship between the value of θ and untwist to that of extension-twist (B_{16}) coupling, with the contribution of chordwise bending (D_{22}) and torsional (D_{66}) stiffness accounting for deviations from the relationship between B_{16} and θ. In most cases of turbomachinery, a casing would surround the plates, therefore the amount of extension of the plate would be limited by the clearance between the two. It may be necessary to therefore compare the extension-twist coupling of the geometric and laminate stiffness coupling together, figure 4 shows the mean tip twist to extension ratio. From figure 4 it can be seen that low values of θ i.e. $0° \leq \theta \leq 10°$ provide the greatest amount of untwist to extension, but as previously discussed this is caused somewhat by the chordwise bending of the plate.

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Analysis of the four laminates ply strains revealed all exceeded one or more of the allowable strains criterion shown in table 1. The dominant failure criteria for all laminates was shear. For most laminates this can be attributed to the amount of untwist that occurred in the plate. To avoid exceeding the allowable amount of shear strain, the difference in ply grouping interface angle needs to be reduced. Using a large number of intermediate ply groups would also reduce the amount of shear. As the plate is subjected to a rotational body force, the most critically stressed region occurs at the root. Laminates 1 and 3 exceed the maximum ply matrix strain at the root of the plate for $-30° \gtrsim \theta \gtrsim 30°$, as the laminate is mainly matrix material at the root. Therefore in terms of developing a methodology for a antisymmetric VAT laminate under this loading it is important to use low magnitude fibre angles at the root. Alternatively laminate 2 failed in matrix tension for $5° < \theta < 50°$ towards the tip end of the plate due to chordwise bending.

4. Variable Angle Tow Laminates

The use of VAT laminates allows for the laminate to be functionally and elastically tailored throughout the plate. For example from section 3 it was discussed that the laminate needed to be sufficiently strong in spanwise tension at the root while sufficiently strong in chordwise bending towards the tip of the plate while providing as much stiffness coupling as possible to increase untwist. It is for this reason that VAT laminates are particularly useful for this application.

4.1. Variable Angle Tow Ply Definition

Defining VAT geometries can be categorised into two distinct methods, shifted [5] and parallel [6], both use a baseline tow path and it is the offset of the subsequent tows that differ in each method. The shifted method simply shifts each tow in a certain direction dictated by the shifting angle in relation to a datum tow path. The parallel method places the adjacent tows exactly parallel to one another so no overlap occurs. Each method has its advantages and disadvantages, with shifted being the most commonly used due to its simplicity. This investigation uses a method and notation for VAT path definition developed by Kim et al [5], which uses a shifted piecewise quadratic Bézier curve. A two segmented piecewise quadratic Bézier curve to allow for greater design freedom. Each ply can be defined using 6 variables with notation

$$\phi \pm < \alpha_0(\gamma_1)\alpha_1(\gamma_2)\alpha_2 > \tag{3}$$

where ϕ is the angle perpendicular to the shifting direction, $\alpha_{i=0,1,2}$ are the fibre angles at the start, centre and end of the plate respectively and where $\gamma_{i=1,2}$ are the angle variation coefficient. As the plate geometry is simplified to that of a rectangle of which pre-twisted is then applied using using equation 1, the VAT paths and angle variation use the same method. As the notation for a VAT ply in equation 3 can become quite long, ζ_i will be used as shorthand notation for a VAT ply. To avoid large changes in fibre angles values of α were limited to $-60° \leq \alpha \leq 60°$ and $-90° \leq \phi \leq 90°$. Values of γ were limited to $0.05 \leq \gamma_1 \leq 0.45$ and $0.55 \leq \gamma_2 \leq 0.95$.

4.2. Optimisation

Matlab's genetic algorithm (GA) integer optimiser was used to find the optimal VAT ply definition values in equation 3. A cost function was used that applied penalties [7] to laminates that did not satisfy strain and chordwise bending allowables, given by

$$\textbf{Minimise} \qquad f = \bar{\psi} + \sum_{i=1}^{n_{con}} p_i \tag{4}$$

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Table 3. Laminate definitions used for optimisation. ζ_i indicates a VAT laminate defined using 6 variables each, with $-\zeta_i$ indicating a ply with an antisymmetric tow path i.e. with the additive inverse of the α_i values. n_i are variables that define the quantity of the ply.

Laminate	Stacking Sequence	No. Variables
1_V	$[\ (\zeta_1)_{32}\ /\ (-\zeta_1)_{32}\ ]$	6
2_V	$[\ (\zeta_1)_{n_1}\ /\ (\zeta_2)_{(32-n_1)}\ /\ (-\zeta_2)_{(32-n_1)}\ /\ (-\zeta_1)_{(n_1)}\ ]$	13
3_V	$[\ (\zeta_1)_{(n_1)}\ /\ (\zeta_2)_{(32-n_1-n_2)}\ /\ (\zeta_3)_{(n_2)}\ /\ (-\zeta_3)_{(n_2)}\ /\ (-\zeta_2)_{(32-n_1-n_2)}\ /\ (-\zeta_1)_{(n_1)}\ ]$	20

Table 4. VAT laminate optimisation results.

Laminate	Optimal Variables	Cost	Twist (°)
1_V	ζ_1 = -27<-6(0.45)34(0.7)-59>	9.5005	-4.0956
2_V	ζ_1 = 84<37(0.31)-2(0.69)-7> ζ_2 = 40<-22(0.17)-31(0.59)-1> n_1=17	-3.1086	-3.1086
3_V	ζ_1 = 74<-11(0.2)23(0.77)13> ζ_2 = 45<-5(0.1)-27(0.73)22> ζ_3 = -44<50(0.22)19(0.8)34> n_1=15, n_2=4	-3.3442	-3.3442

where $\bar{\psi}$ is the mean tip twist and n_{con} is the number of penalty terms p_i. Six penalty terms were used, one for exceeding chordwise bending and five for strain allowables. Laminates were penalised for chordwise bending when the value of standard deviation σ was greater than 5% of $\bar{\psi}$, with the penalty term given by

$$p_i = \begin{cases} 0 & \text{if } \left|\frac{\sigma}{\bar{\psi}}\right| \leq 0.05 \\ \bar{\psi}_{ref}|g_i| + (\bar{\psi}_{ref} \times 0.02) & \text{if } \left|\frac{\sigma}{\bar{\psi}}\right| > 0.05 \end{cases} \quad \text{where} \quad g_i = \frac{\left|\frac{\sigma}{\bar{\psi}}\right| - 0.05}{0.05} \tag{5}$$

were $\bar{\psi}_{ref}$ is a reference twist, which in this case was 5°. A small fixed penalty of 2% of the reference twist $\bar{\psi}_{ref}$ is used to ensure the optimiser did not converge towards a solution that has a small violation of the constraint to simulate a hard constraint. Similarly for strain allowables ε^* shown in table 1

$$p_i = \begin{cases} 0 & \text{if } |\varepsilon| < \varepsilon^* \\ \bar{\psi}_{ref}|g_i| + (\bar{\psi}_{ref} \times 0.02) & \text{if } |\varepsilon| \geq \varepsilon^* \end{cases} \quad \text{where} \quad g_i = \left|\frac{\varepsilon - \varepsilon^*}{\varepsilon^*}\right| \tag{6}$$

Three different laminate designs were used and are shown in table 3. All three laminates are designed so that they are antisymmetric in terms of tow path, but locally can be non-symmetric. Laminate 2_V and 3_V have 1 and 2 variables n_i respectively that vary the percentage of each ply while the total number of plies remains the same at 64 for all laminates.

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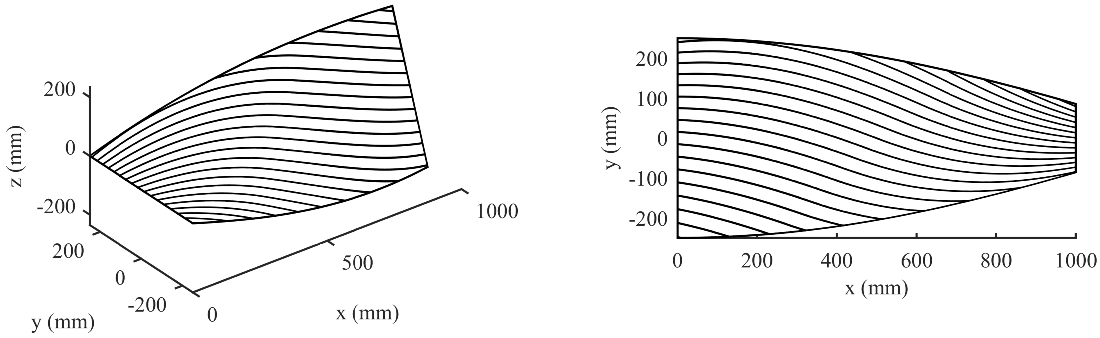
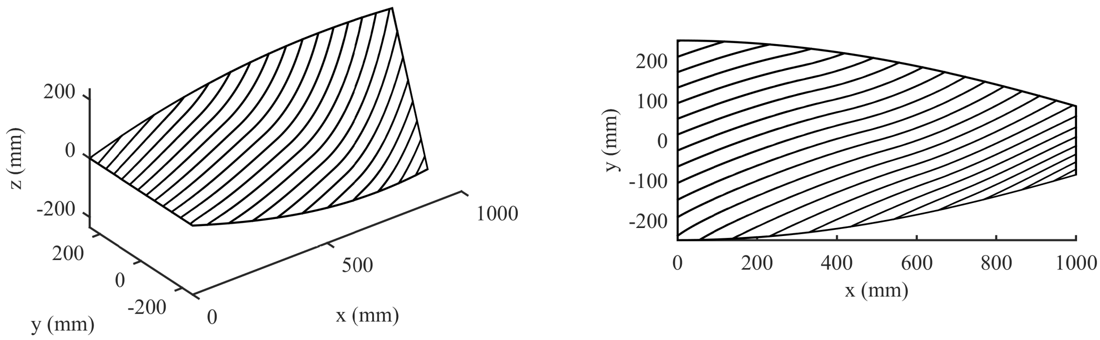
4.3. Optimisation Results

The optimisation results for the laminates shown in table 3 are shown in table 4. Laminate 3_V produced the lowest cost function and therefore the best design. Although 1_V produced the greatest amount of untwist its it did not satisfy all of the strain allowables and was therefore penalised as reflected by the difference between the cost and twist values. The GA did not manage to find any design for 1_V that satisfied all strain allowables, and only the optimal design for 2_V satisfied them all. Laminate 3_v produced



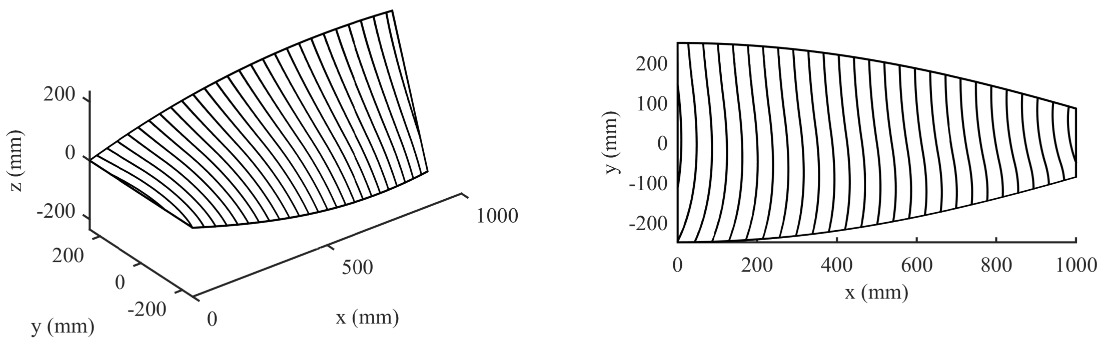
(a) Isometric view. (b) Plan view.

Figure 5. $\zeta_3 = -44<50(0.22)19(0.8)34>$ for laminate V_3.



(a) Isometric view. (b) Plan view.

Figure 6. $\zeta_2 = 45<-5(0.1)-27(0.73)22>$ for laminate V_3.



(a) Isometric view. (b) Plan view.

Figure 7. $\zeta_1 = 74<-11(0.2)23(0.77)13>$ for laminate V_3.

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many designs that satisfied all constraints, showing that strain allowables, most importantly shear, are best satisfied by increasing the number of different VAT plies through the thickness of the laminate. By looking at the tow paths for the optimal VAT plies for 3_V shown in figures 5-7, it can be seen that the GA has converged on a design that appears to assign different roles to the different plies. The inner most ply ζ_3 in figure 5 is approximate 0° at the root, it is assumed this it to provide enough stiffness in the x direction (A_{11}) as this is the most critically stressed region. Ply ζ_2 in figure 6 is approximately 40° at the root before changing to approximately 60° at the tip, providing the majority of the extension-twist (B_{16}) and bend-twist (D_{26}) coupling as it is the most non-symmetric group of plies in the laminate. Ply ζ_1 in figure 7 is approximately 90° and its primary role appears to be provide enough chordwise bending stiffness (D_{22}) to provide a low twist standard deviation.

5. Conclusion

A methodology for the design of non-symmetric VAT laminates to increase untwist of a pre-twisted plate subjected to centrifugal force has been demonstrated. The study has shown that non-symmetric laminate can be used to increase the untwist of rotating pre-twisted plate while VAT composites can be used to change the role of a ply in different locations. It has been shown that increasing the quantity of different VAT plies throughout the laminate will increase the performance of the laminate.

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