

# OPTIMISING THE PLY DROPPING ORDER IN VARIABLE STIFFNESS, VARIABLE THICKNESS LAMINATES USING STACKING SEQUENCE TABLES

Daniël M.J. Peeters<sup>1</sup>, François-Xavier Irisarri<sup>2</sup>, Mostafa M. Abdalla<sup>1</sup>

<sup>1</sup>Faculty of Aerospace Engineering, Aerospace structures and computational mechanics, Delft University of Technology, Kluyverweg 1, Netherlands

Email: d.m.j.peeters@tudelft.nl

<sup>2</sup>ONERA (French Aerospace Lab), Chatillon, France

**Keywords:** optimisation, stacking sequence table, variable thickness, variable stiffness

## Abstract

Modern composite structures offer two avenues of optimising performance. One, optimising a single stacking sequence over the structure leading to constant stiffness designs. Two, varying the stiffness over the structure. This may be achieved by dropping plies, changing the thickness, or by steering the fibres, changing the fibre angles. Optimising ply drops involves two decisions: the selection of ply drop boundaries, and of the ply drop order. Previous work considered the problems of optimising the fibre angle distribution and ply drop boundaries but ply drop order was pre-specified. This paper extends the work to simultaneously optimise the fibre angle distribution, the ply drop boundaries, and order. The optimisation of fibre angle distribution lends itself to gradient-based methods. The ply drop boundary optimisation is formulated using topology optimisation techniques and is thus also solvable using gradient-based methods. The ply drop order optimisation requires discrete variables and is hence approached using an evolutionary algorithm based on stacking sequence tables. In this paper an efficient multi-step algorithm is developed combining the optimisation of all aspects of variable stiffness laminates. The results indicate that significant improvements may be obtained by including the ply drop order in the optimisation at a relatively modest computational cost.

## 1. Introduction

Composite materials are attractive due to their high stiffness-to-weight and strength-to-weight ratio. It has been shown that by spatially varying the stiffness, even better performance can be obtained without adding extra weight. Varying the stiffness can be done in two ways: either by changing the fibre angles, by steering the fibres, or by changing the number of plies from one point to the next by dropping plies. To develop constant thickness, steered laminates, a three-step optimisation approach has been developed. [1, 2] In the first step the optimal stiffness distribution in terms of lamination parameters is found, in the second step the optimal fibre angles are obtained, and in the third step the optimal fibre paths are retrieved. A thickness variation has already been implemented in the first step of the optimisation, and showed that significant improvements could be obtained, however, no information about fibre angle or number of plies is available at the first step, so the physical construction of the laminate remains to be found. A method to optimise the fibre angle and ply drop locations has recently been developed, but the ply drop order was pre-specified. [3]

Thickness variation in a laminate is described using two variables: the ply drop location and the ply drop order. The most popular approach is to use an evolutionary algorithm, typically a genetic algorithm, to optimise the number of layers per 'patch', while also optimising the ply drop order and stacking sequence, limited to a discrete set of angles (e.g.,  $0^\circ$ ,  $\pm 45^\circ$ , and  $90^\circ$ ). This area of research is referred to as laminate blending [4–8] and assumes that potential ply drop locations (i.e., patch boundaries) are pre-specified by the user. A technique where the fibre angle is not restricted to a discrete set has also been developed, however, no manufacturing constraints, limiting the change in fibre angle from one element to the next, are posed. [9]

Other techniques where the ply drop locations are not pre-specified use continuous optimisation. Shape optimisation [10] is used to determine the shape and hence ply coverage and ply drop locations of the different layers. The optimisation is performed using a level-set approach with fibre angles limited to a discrete set. Another continuous method is the discrete material and thickness optimisation method, where the fibre angles belong to a discrete set and fictitious density variables are used to select the ply angles at any given location. This has been done for compliance and buckling optimisation. [11, 12] For this method, also a thickness filter has been implemented to get to physically feasible designs. [13] Thickness optimisation for buckling load under uni- and bi-axial compression has also been performed. This work showed that large improvement in buckling load could be made without affecting the in-plane stiffness. [14]

The easiest ply drop orders are inner or outer blending, where the layers are dropped from the symmetry plane, or from the outside respectively. To determine the optimal ply drop order, guide-based designs can be used: [5] a stacking sequence for the thickest laminate, called the guide laminate, is defined and the number of layers per patch. The stacking sequence is then derived by dropping layers from the inside or outside, depending whether inner or outer blending is used, from the guide laminate. A method that offers more possible ply drop orders and takes into account industrial guidelines is using stacking sequence tables. [15] A ply drop order and guide laminate are optimised.

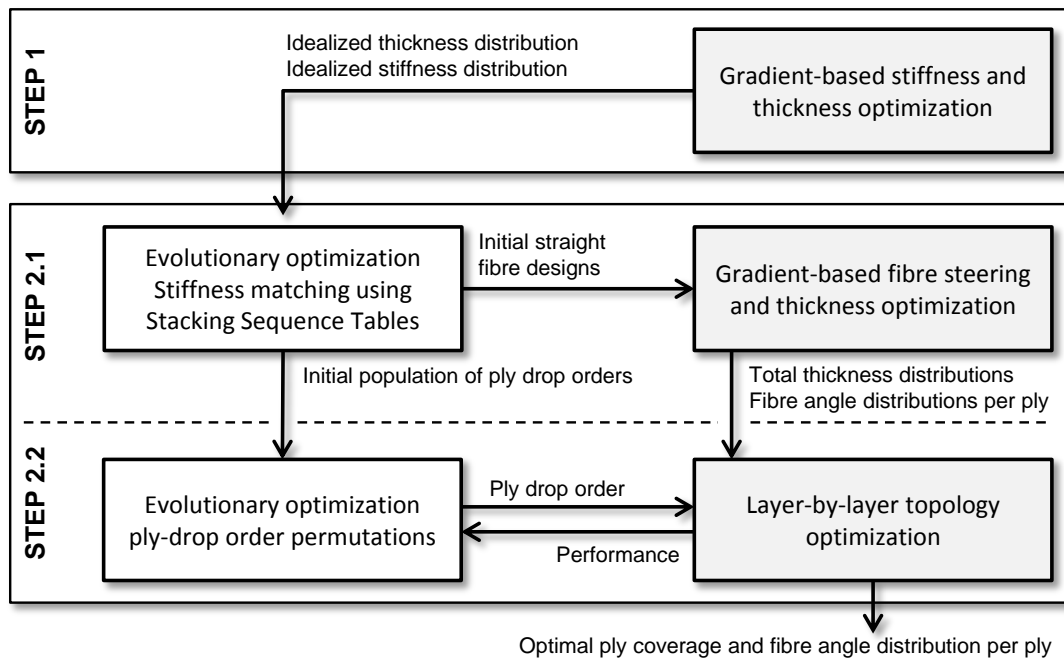
This paper aims to combine the ideas behind the optimisation of optimising ply drop locations and the stacking sequence tables. [3, 15] The outcome will be an optimal steered, variable thickness laminate: both the ply drop location and ply drop order are optimised. The remainder of the paper is organised as follows: first the general approach is explained in section 2, next a description of the stacking sequence tables is given in section 3, some numerical examples in section 4 and finally a conclusion in section 5.

## 2. Approach

The overall optimisation strategy is based on the three step optimisation approach developed by IJsselmuide [1]. The present work focuses on the second step of the optimisation and combines the optimisation of the spatial distribution of the fiber angles and the optimisation of the ply drop location and ply drop order. Figure 1 gives an overview of the proposed optimisation approach. Step 1 (see [2, 16, 17]) returns an *idealized design* defined as an optimal stiffness and thickness distribution over the structure. The idealized design gives an upper bound of the performance of the structure.

In the proposed approach, Step 2 is subdivided into two successive optimisation phases referred to as Step 2.1 and Step 2.2 respectively. Both phases combine an evolutionary optimiser and a gradient-based optimizer for their complementary efficiency in solving combinatorial problems and continuous problems respectively.

Step 2.1 aims at providing a relevant initial guess of the fibre angle distributions per ply, total thickness distribution and ply drop order for the subsequent optimisation phase. Step 2.1 takes as input the ide-



**Figure 1.** Overview of the optimisation strategy.

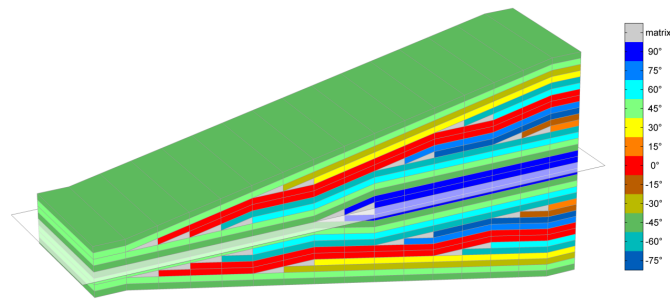
alized design obtained from Step 1. The thickness distribution is set to the *rounded* idealized thickness distribution and a stiffness matching optimisation is performed targeting the idealized stiffness distribution. A Pareto multi-objective evolutionary optimiser is used to match the membrane stiffness distribution and the bending stiffness distribution of the idealized design. The optimiser returns a set of straight fiber solutions, each one defined by a Stacking Sequence Table (SST), meaning a guide laminate and a ply drop order. The non-dominated solutions from the EA are used as starting points for a subsequent gradient-based optimisation. Here the spatial distribution of the variation of fibre angles in each ply, and a *continuous* thickness distribution are found over the structure. At this stage, the designs have realistic fibre angle and ply drop designs but non-manufacturable continuous thickness.

Step 2.2 aims at converting the laminate thickness distributions into properly defined ply drop locations. The optimisation is initialized with the non-dominated front issued from Step 2.1. An evolutionary algorithm (EA) specialized for ply drop order optimisation is hybridized with a gradient-based method devised for ply drop location optimisation. For each ply drop order generated by the EA, a topology-like optimisation is performed using a fictitious density distribution for each ply. The densities are forced to converge to either one or zero which defines the ply coverage and the ply drop locations. The gradient based optimisation alternates between density optimisation and fiber angle optimisation in each ply. The optimisation of the ply drop locations is not discussed in this paper, the interested reader is referred to Peeters and Abdalla [3].

### 3. Stacking sequence table optimisation

The Evolutionary Algorithms (EA) devised in the present work for Step 2.1 and Step 2.2 (see Figure 1) share the same architecture based on the algorithm proposed in [15]. The main features of the algorithm that are preserved in the present work are briefly summarized in the following. The algorithm uses two populations: a regular population and an archive population. The archive population contains a fixed-sized subset of the best solutions encountered during the evolutionary process. The optimization starts with an initial population and an empty archive. The following steps are then iteratively repeated.

Excerpt from ISBN 978-3-00-053387-7



**Figure 2.** Schematic of a unidirectional symmetrical thickness transition.

- The solutions composing the current populations are evaluated.
- The archive is updated using the environmental selection method proposed in [18].
- Solution fitness values are assigned to the union of the current population and the archive population.
- The fitness function aggregates a density estimate and the Pareto-rank of the solution to enable multi-objective optimization.
- Optimization constraints formulated on the mechanical performances of the solutions are enforced through a modified binary tournament selection scheme [19].
- The current population is replaced after mutation and recombination of the selected solutions.
- Design guidelines whose evaluation require no mechanical analysis of the solutions, for example symmetry and balance of the laminates, are enforced by construction of the solutions, using specific variation operators.

### 3.1. Step 2.1. SST-based stiffness matching

A Stacking Sequence Table (SST) describes the sequence of ply insertions ensuring the transition between a thin laminate and a thicker one. The SST is encoded within the EA using two integer vectors. The vector  $SST_{lam}$  represents the stacking sequence of the thickest laminate. The  $SST_{ins}$  corresponds to the rank of insertion of the plies, and reciprocally defines the ply drop sequence. Rank 0 is attributed to the plies that form the thinner laminate for the SST. The first ply inserted is given rank 1, the second ply inserted is given rank 2 and so on. For example, if  $SST_{ins} = [0\ 3\ 6\ 1\ 5\ 7\ 2\ 0\ 4]$  then the ply drop order is  $[6/3/5/9/2/7/4/8/1]$ . In the present work, laminates are symmetrical and composed of balanced pairs of plies which allows to encode only one fourth of the SST. Additionally, the covering guideline and internal continuity guideline are enforced during the optimization (see [15]) for strength concerns. The covering guideline imposes that the plies on the surface of the laminates are never dropped in the SST. The internal continuity guideline states that a continuous ply ( $rank = rank_{min} = 0$  in  $SST_{ins}$ ) should be kept every three consecutive dropped plies ( $rank > 0$ ). Here, these guidelines are evaluated on the vectors  $SST_{ins}$ , thus the guidelines are transposed to pairs of plies. The internal continuity guideline implies a dependency of the maximal number of plies  $n_{max}$  to the minimal number of plies  $n_{min}$  within the SST:  $n_{max} \leq 3 \times n_{min} - 2$ . When reached, the limitation is relaxed by incrementing the value of  $rank_{min}$ . For instance, Figure 2 shows a unidirectional thickness transition between a 12-ply laminate and a 36-ply laminate compatible with the following encoding:  $SST_{lam} = [45\ 30\ 60\ 0\ -75\ -15\ -60\ 45\ 90]$  and  $SST_{ins} = [0\ 3\ 6\ 1\ 5\ 7\ 2\ 0\ 4]$ .

For stiffness matching the thickness distribution in the structure is set to the thickness distribution of the idealized design rounded to the nearest upper number of plies. Subsequently, the thickness distribution is kept unchanged. The stiffness distribution of the idealized design is targeted. Proximity in stiffness space is evaluated using the distance proposed in [20]. The optimization aims at minimizing both the

**A**-distance in membrane space and the **D**-distance in bending space as defined by Equation 1:

$$d(\mathbf{C}_1, \mathbf{C}_2) = \mathbf{C}_1^{-1} : \mathbf{C}_2 + \mathbf{C}_1 : \mathbf{C}_2^{-1} - 6. \quad (1)$$

where the operator  $:$  stands for the trace of the product of two square matrices and  $\mathbf{C}_1$  and  $\mathbf{C}_2$  are either two membrane stiffness matrices or two bending stiffness matrices. The objective functions correspond to the sum of the distances computed at the nodes and weighted by the nodal areas. The optimization returns a set of non-dominated solutions. Each solution represent a trade-off between proximity to the membrane stiffness distribution of the idealized design and its bending stiffness distribution.

### 3.2. Step 2.2. Ply-drop order optimization

At Step 2.2, the design variables are distributed between the overall evolutionary algorithm and the gradient-based solver used during the ply drop location optimiser. [3] The EA is used to solve the combinatorial optimization problem of the ply drop order and calls for each solution the gradient-based solver to perform local improvement of the topology and fiber steering of the plies. Thus, the EA operates on the ply drop sequence only, which is encoded by the vector  $SST_{ins}$ . The permutation operator for the vector  $SST_{ins}$  proposed in [15] is reused in the present work. The operator permutes the insertion ranks of two plies within the vector  $SST_{ins}$  for plies with non-zero ranks. A specific crossover operator is devised to operate on insertion rank vectors  $SST_{ins}$ , inspired by the ordered crossover operator [21]:

1. Let  $SST_{ins}^{01}$  and  $SST_{ins}^{02}$  be two insertion rank vectors.
2. A random subset of ranks strictly superior to  $rank_{min}$  is selected.
3. In the proposed example  $rank_{min} = 1$ , and the selected ranks are underlined:  $SST_{ins}^{01} = [0 \ 2 \ \underline{3} \ \underline{5} \ 1 \ \underline{6} \ 7 \ 0 \ 4]$  and  $SST_{ins}^{02} = [0 \ \underline{6} \ 7 \ 1 \ 4 \ \underline{5} \ 2 \ 0 \ \underline{3}]$ .
4. To create the offspring vector  $SST_{ins}^{11}$ , vector  $SST_{ins}^{01}$  is copied and the selected subset is reordered according to  $SST_{ins}^{02}$ . Vector  $SST_{ins}^{12}$  similarly is a copy of  $SST_{ins}^{02}$  with the ordering of  $SST_{ins}^{01}$  for the selected subset.
5.  $SST_{ins}^{11} = [0 \ 2 \ \underline{6} \ \underline{5} \ 1 \ \underline{3} \ 7 \ 0 \ 4]$  and  $SST_{ins}^{12} = [0 \ \underline{3} \ 7 \ 1 \ 4 \ \underline{5} \ 2 \ 0 \ \underline{6}]$ .

As it does not operate on ranks lower than  $rank_{min}$ , the proposed crossover operator preserves the internal continuity guideline, as defined in Section 3.1.

## 4. Results

The proposed method is applied to the buckling load maximisation of a simply supported square plate under longitudinal compression. The first two buckling modes are considered in the optimisation. Furthermore, a stiffness constraint is used: all solutions are required to present higher or equal longitudinal stiffness than the 24-ply quasi-isotropic constant thickness laminate. The side length of the plate is 500mm. The base ply properties are the following:  $E_{11} = 177GPa$ ,  $E_{22} = 10.8GPa$ ,  $G_{12} = 7.6GPa$  and  $\nu_{12} = 0.27$ . For the optimisation, the number of plies can locally vary from 8 plies to 36 plies. The maximum volume allowed corresponds to a 24-ply constant thickness laminate. A global steering constraint is enforced that corresponds to a minimal steering radius of 333 mm.

Table 1 shows shows the highest buckling loads obtained after optimisation Step 1, Step 2.1 and Step 2.2 of the proposed method. The results are compared with two constant thickness solutions: the constant thickness with straight fibers, and the steered fibre laminate. The steered fiber variable thickness solution was obtained in [3] with  $SST_{ins}$  defined as  $[0 \ 6 \ 4 \ 2 \ 1 \ 3 \ 7 \ 5 \ 0]$ . All buckling loads are normalized with respect to the critical buckling load of the 24-ply quasi-isotropic “black metal” solution (i.e., all lamination parameters equal to zero). The reference solution to assess the performance improvement is the steered fibre constant thickness solution. The idealized design obtained at Step 1 is defined as

Optimisation	Stiffness	Buckling load 1	Buckling load 2	Difference w.r.t. constant thickness
Constant thickness, straight fibres	1.000	1.223	1.840	-40.0%
Constant thickness, steered fibres [3]	1.000	2.042	2.473	0 %
Ply drops, steered fibres [3]	1.000	4.229	4.233	+ 107.1 %
Lamination parameters, variable thickness (step 1) [3]	1.000	6.236	6.344	+ 205.4 %
Variable thickness, steered fibres (step 2.1)	1.000	4.021	4.026	+ 96.9 %
Ply drops, steered fibres (step 2.2)	1.000	5.117	5.128	+ 150.6 %

**Table 1.** Overview of the highest buckling load obtained after each optimisation step, compared with results presented in [3] and constant thickness solutions.

a combined stiffness (i.e., lamination parameters) and thickness distribution and gives an upper bound on the achievable performances. The constant thickness with straight fibers solution is the following:  $[\pm 41 \pm 41 \pm 40 \pm 35 \pm 19 \pm 6]_s$ .

The performances of the best solution found are detailed in Table 1. The corresponding ply insertion chromosome  $SST_{ins}$  is [0 5 4 1 7 2 3 0 6]. The fibre paths are presented in Figure 3 for each design layer composed of a ply and its adjacent balanced counterpart. The solution forms large stiffeners along the longitudinal sides of the plate to increase the in-plane stiffness.

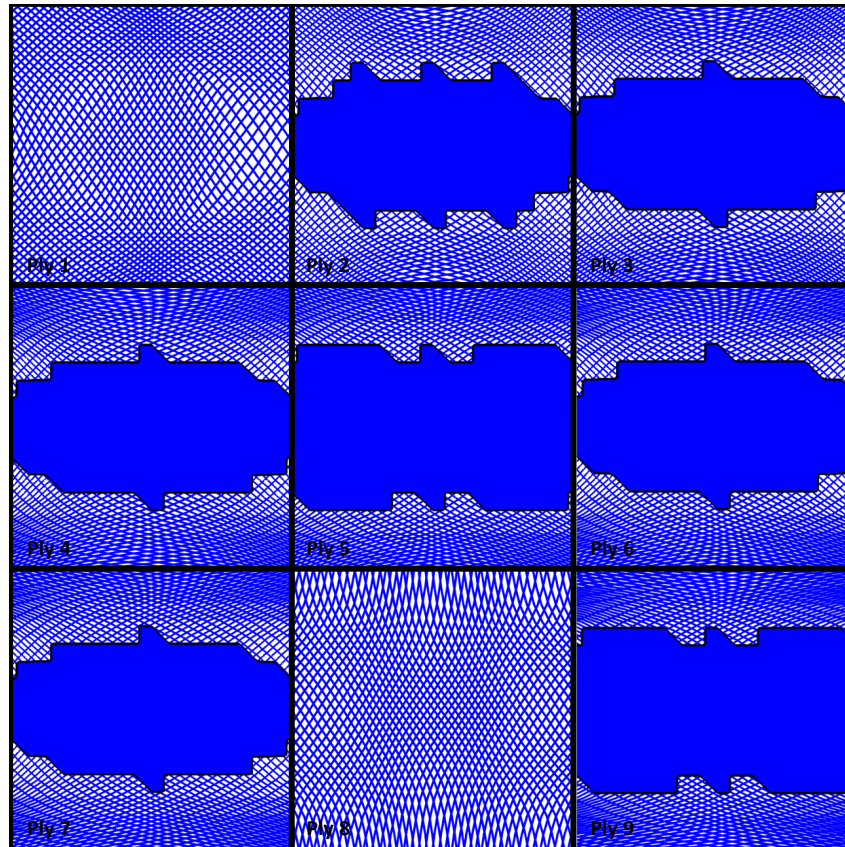
In order to quantify the influence of the ply drop order on the buckling performance of the plate, a buckling minimization was run at Step 2.2 with the same initialization of the EA. The minimal critical buckling load obtained 2 is 4.449 (i.e., 117.8% improvement with respect to the best constant thickness design). Hence 15% lower than the best buckling load found.

## 5. Conclusion

In this paper, a method to optimise the fibre angle distribution, ply drop location and ply drop order is presented. The optimisation is a combination of an evolutionary and gradient based optimiser. The ply drop order is optimised using a permutation genetic algorithm, while the ply drop locations and fibre angle distributions are optimised using the method of successive approximations. The ply drop locations are optimised using topology-like ideas: each layer is given a fictitious density distribution and this is optimised in two steps: in the first step the overall thickness is optimised, in the second step the exact ply drop locations are determined at places where the fictitious density changes from near one to near zero.

Initial results indicate that large improvements in performance are possible by also optimising the drop order. A flat, square plate under uni-axial compression has been optimised for buckling with stiffness constraint. The stiffness is constrained to be at least the stiffness of a quasi-isotropic laminate with the same weight without ply drops. The influence of the dropping order on the buckling load is large: up to 32.8% of the buckling load of the best steered constant stiffness laminate. These results show that checking just one possible drop order during the ply drop location optimisation can have a significant influence on the final result.

While for the fibre angle distribution some manufacturing constraints, such as a maximum change in fibre angle between adjacent nodes, is taken into account, the effect of gaps and overlaps occurring due to the steering is not taken into account. Furthermore, the minimal distance between consecutive ply drops is not constrained, meaning multiple ply drops can occur at the same point, or within a short distance from each other. While this may be the subject of future research, a post-processing step can also be implemented to adhere to this minimal distance.



**Figure 3.** Outcome of Step 2.2. Ply fiber orientation shown with their balance counterpart.

### Acknowledgements

This work is supported by the CANAL (CreAting Non-conventionAl Laminates) project, part of the European Union Seventh Framework Program.

### References

- [1] S. T. IJsselmuiden. *Optimal design of variable stiffness composite structures using lamination parameters*. Ph.D. thesis, Delft University of Technology, 2011.
- [2] Samuel T. IJsselmuiden, Mostafa M. Abdalla, and Zafer Gürdal. Optimization of variable-stiffness panels for maximum buckling load using lamination parameters. *AIAA Journal*, 48(1):134–143, 2010.
- [3] Daniël Peeters and Mostafa Abdalla. Optimization of ply drop locations in variable-stiffness composites. *AIAA Journal*, pages 1–9, 2016.
- [4] Chun-Gon Kim Jung-Seok Kim and Chang-Sun Hong. Optimum design of composite structures with ply drop using genetic algorithm and expert system shell. *Composite Structures*, 46(2):171 – 187, 1999.
- [5] Genetic algorithm optimization and blending of composite laminates by locally reducing laminate thickness. *Advances in Engineering Software*, 35(1):35 – 43, 2004.

- [6] Multi-step blended stacking sequence design of panel assemblies with buckling constraints. *Composites Part B: Engineering*, 40(4):329 – 336, 2009.
- [7] Søren N. Sørensen and Mathias Stolpe. Global blending optimization of laminated composites with discrete material candidate selection and thickness variation. *Structural and Multidisciplinary Optimization*, pages 1–19, 2015.
- [8] Dianzi Liu, Vassili V. Toropov, Osvaldo M. Querin, and David C. Barton. Bilevel optimization of blended composite wing panels. *Journal of Aircraft*, 48(1):107–118, 2015/07/01 2011.
- [9] D.M. De Leon, C.E. de Souza, J.S.O. Fonseca, and R.G.A. da Silva. Aeroelastic tailoring using fiber orientation and topology optimization. *Structural and Multidisciplinary Optimization*, 46(5):663–677, 2012.
- [10] G. Delgado. *Optimization of composite structures: A shape and topology sensitivity analysis*. Theses, Ecole Polytechnique X, June 2014.
- [11] Søren N. Sørensen, René Sørensen, and Erik Lund. Dmto a method for discrete material and thickness optimization of laminated composite structures. *Structural and Multidisciplinary Optimization*, 50(1):25–47, 2014.
- [12] E. Lund. Buckling topology optimization of laminated multi-material composite shell structures. *Composite Structures*, 91(2):158 – 167, 2009.
- [13] Ren Sørensen and Erik Lund. Thickness filters for gradient based multi-material and thickness optimization of laminated composite structures. *Structural and Multidisciplinary Optimization*, pages 1–24, 2015.
- [14] M. G. Joshi and S. B. Biggers Jr. Thickness optimization for maximum buckling loads in composite laminated plates. *Composites Part B: Engineering*, 27(2):105 – 114, 1996.
- [15] F.-X. Irisarri, A. Lasseigne, F.-H. Leroy, and R. Le Riche. Optimal design of laminated composite structures with ply drops using stacking sequence tables. *Composite Structures*, 107(0):559 – 569, 2014.
- [16] Design of variable-stiffness composite panels for maximum buckling load. *Composite Structures*, 87(1):109 – 117, 2009.
- [17] A. Khani, S.T. IJsselmuiden, M.M. Abdalla, and Z. Gurdal. Design of variable stiffness panels for maximum strength using lamination parameters. *Composites Part B: Engineering*, 42(3):546 – 552, 2011.
- [18] Eckart Zitzler, Marco Laumanns, and Lothar Thiele. SPEA2: Improving the Strength Pareto Evolutionary Algorithm. Technical Report 103, Computer Engineering and Networks Laboratory (TIK), Swiss Federal Institute of Technology (ETH) Zurich, Gloriastrasse 35, CH-8092 Zurich, Switzerland, May 2001.
- [19] K. Deb and H. Gupta. A constraint handling strategy for robust multi-criterion optimization. Technical Report 2005001, Indian Institute of Technology Kanpur, Kanpur, India, 2005.
- [20] Francois-Xavier Irisarri, Mostafa M. Abdalla, and Zafer Gürdal. Improved Shepard’s Method for the Optimization of Composite Structures. *AIAA Journal*, 49(12):2726–2736, December 2011.
- [21] David E. Goldberg. *Genetic algorithms in search, optimization, and machine learning*. Addison-Wesley Pub. Co, Reading, Mass, 1989.