TORSION-INDUCED BUCKLING OF VARIABLE STIFFNESS COMPOSITE CYLINDERS

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Keywords: Fiber steering, Variable stiffness, Torsion, Buckling, Composite, Elliptical cylinder

Abstract

Advanced robotic Automated Fiber Placement (AFP) machines have made it possible to continuously change the orientation angle of the fibers/tows to manufacture composite laminates with spatially variable mechanical properties. The resulting so called variable stiffness (VS) composite laminate has spatial stiffness properties that can be used in a composite parts to create efficient path between the loading points and the supports. Using these VS laminates, the designers find more room to exploit the directional properties of composite materials for structural components with significantly higher performance and/or lower weight compared with constant stiffness structures. In this research work, variable stiffness circular and elliptical composite cylinders made by fiber steering are designed and optimized for maximum torsion-induced buckling capacity (BT_{cr}). A multi-step design optimization procedure is used to get the maximum potential improvement in buckling performance due to torsional load. In order to improve the computational efficiency of the design optimization process, the high fidelity finite element (FE) analysis is substituted by low-cost and computationally inexpensive surrogate models based on Radial Basis Functions (RBF). The torsional buckling load of the optimized VS composite cylinders are also compared with their optimum constant stiffness (CS) counterparts to evaluate the potential improvement the VS composite cylinders can offer in terms of the structural performance.

1. Introduction

With the advent of Automated Fiber Placement (AFP) machines, it has become possible to continuously change (steer) the orientation angle of the fibers/tows to manufacture composite laminates with spatially variable mechanical properties. The resulting so called variable stiffness (VS) composite component has spatial stiffness properties that can create an efficient path between the loading points and the supports. These VS laminates allow the designer to fully exploit the directional properties of composite materials. In designing components made of these laminates, the design space is extended so that structural components with significantly higher performance and/or lower weight compared with constant stiffness structures can be made.

In aerospace structures, the weight savings through reduction of skin thickness must be balanced against the requirement for structural stability; therefore, studying the buckling performance of VS composite structures for aerospace applications is very important. Since buckling is a stiffness-driven phenomenon, this is a perfect candidate for the VS design optimization problem for aerospace structures.

Cylindrical structures are very commonly used in many aerospace applications. Tatting [1] was amongst the first researchers who studied the design of VS composite cylinders for buckling. The axial variation of

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stiffness in his study resulted in a little improvement of the axial buckling load compared to the constant stiffness (CS) laminate. It was followed by extensive research works by others to increase the buckling capacity of VS composite cylinders subjected to other types of loading such as bending load [2–5]. In the bending-induced buckling load case in which the section load is inhomogeneous on the structure's skin, the optimum VS design takes the advantage of this inhomogeneity to vary the orientation angle of fibers from the compressive side to the tension side so that the section loads are redistributed more efficiently and the buckling load is increased. In general, the orientation angles are varied so that the VS design provides a more effective load path between the loading points and the supports. As a result, the directional properties of the fibers are fully exploited for better structural performance. Axisymmetric structures such as circular composite cylinders subjected to axisymmetric loading, have not been studied for buckling improvement by fiber steering. This is because of the fact that there is no inhomogeneity or non-uniformity in the loading condition or structural (geometrical) properties. Therefore, any variation of stiffness in the circumferential direction will result in a structure weaker than its CS counterpart. A different approach to possibly increase the buckling performance is to vary the orientation angle of the fibers in the axial direction of the cylinder in which the question is: Is there any improvement in the torsional buckling load for a VS composite circular cylinder compared with its optimum CS counterpart if the orientation angle varies in the axial direction? We will address this question in one part of this research work.

Due to the aerodynamic considerations in aerospace applications, noncircular cylinders may also be required. Therefore, studying the potential improvement in structural performance of noncircular cylinders has a great importance. Followed by Haynie and Hyer [6] who studied torsion-induced buckling and postbuckling behavior of elliptical quasi-isotropic (QI) composite cylinders, a few studies on elliptical VS composite cylinders [7–9] have shown considerable improvement in their axial buckling performance by stiffness tailoring (fiber steering). Their results showed that, compared with a CS case, the axial section forces in VS elliptical cylinders are redistributed more efficiently so that they are partially transferred from the flatter portions of the elliptical cross-section to the more highly curved parts that are more stable in carrying the compressive loads.

To the best knowledge of the authors, torsion-induced buckling of VS elliptical composite cylinders have not been studied yet. In this research work, as a continuation of our previous work [3–5], VS circular and elliptical composite cylinders made by fiber steering are designed and optimized for maximum buckling load due to torsion. A multi-step design optimization procedure [10] is developed to get the maximum potential improvement in torsion induced buckling. High fidelity finite element (FE) analyzes are substituted by low-cost and computationally inexpensive surrogate models based on Radial Basis Functions (RBF) to improve the computational efficiency of the design optimization process. Different RBF formulations are also studied and compared with each other in terms of their accuracy. The torsion-induced buckling capacity of the optimized VS composite cylinders are also compared with their CS counterparts to evaluate the potential improvement the VS composite cylinders can offer in terms of their structural performance.

2. Modeling

Figure 1a shows the problem being considered in this study, in which a composite cylinder with an elliptical cross section is subjected to a torsional load. The commercial software ABAQUS [11] was used for finite element analysis (FEA) of the VS composite structure. To apply the boundary conditions the multipoint constraint (MPC) option built in ABAQUS was used for both ends of the cylinders in which the only allowed rotation is about the axis of the cylinder. One end is considered fixed and the other end is allowed to move only in the axial direction. Assuming a cross-sectional aspect ratio of b/a=0.7, the elliptical composite cylinders were modeled in ABAQUS of which S8R5 shell elements were used for the

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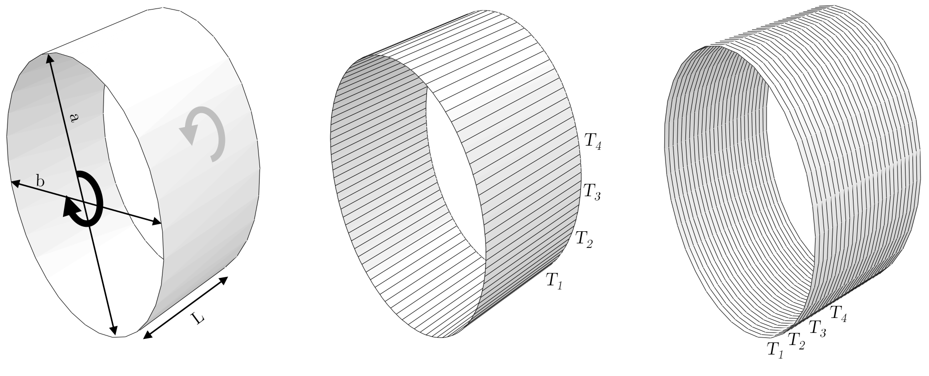
Table 1. Mechanical properties of each unidirectional composite ply made of carbon/epoxy materials.

Property	AS4D/9310
E_1 (GPa)	134
$E_2 = E_3$ (GPa)	7.71
$G_{12} = G_{13}$ (GPa)	4.31
G_{23} (GPa)	2.76
$\nu_{12} = \nu_{13}$	0.301
ν_{23}	0.396
Thickness (t) (mm)	0.127

FEA. The size of the finite elements for each elliptical cylinder was determined by a convergence study. As illustrated in Fig. 1(b,c), the axial and circumferential narrow bands with equal widths were generated on the surface of the cylinders that represent the regions in which the fiber orientation angles are assumed to be constant, but different from their adjacent ones in case of a VS laminate. This way, a piece-wise constant model [2] was used to approximate the continuous variation of the fiber orientation angles in the axial or circumferential direction on the surface of the cylinders. A balanced symmetric 8-ply laminate made of AS4D/9310 carbon/epoxy materials with $[\pm\theta_1/\pm\theta_2]_s$ stacking sequence was considered for the composite cylinders. The mechanical properties of each ply is given in Table 1. The lengths of the major and minor axes are chosen so that the circumference of the ellipse is equal to that of a circle with the diameter of 15 in. Therefore, in all cases the same amount of materials are used. Table 2 lists the geometrical properties of the cross-section of the elliptical cylinders considered for modeling, analysis, and design optimization in this study. The length are changed to study the effect of length aspect ratio (L/R) on the potential improvement in torsion-induced buckling capacity of VS cylinders.



(a) (b) (c)

Figure 1. (a) Torsion-induced buckling problem for an elliptical composite cylinder. (b) Axial narrow bands in which the orientation angles are constant and shifting direction of the curved fibers are axial. (c) Circumferential narrow bands in which the orientation angles are constant and shifting direction of the curved fibers are circumferential.

As stated above, for VS composite cylinders the orientation angle in each ply (θ_i) is allowed to axially or circumferentially vary on the surface of the cylinders. Assuming one design variable per each narrow band of the piece-wise constant model shown in Fig. 1(b,c) will result in a large number of design variables that substantially increases the computational expense of the optimization problem. Therefore,

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Table 2. Geometrical properties of the elliptical composite cylinders.

Aspect ratio b/a	Major axis a (in.)	Minor axis b (in.)	Eccentricity $e = \sqrt{1-(b/a)^2}$	Length L (in.)
0.7	17.510	12.257	0.714	15.0

a reasonable number of equally distant narrow bands were considered as design variables (T_i's). A linear interpolation between the values of the orientation angles in these points are used to calculate the orientation angles in the remaining narrow bands as formulated in [3–5]. This way, the number of design variables are considerably reduced and, as a result, the computational efficiency of the design optimization problem will significantly increase.

3. Design optimization

The design optimization problem for this study is one of finding the optimum orientation angles on a limited number of specified narrow bands, that are the design variables in each ply (T_i's) as shown in Fig. 1, so that the buckling capacity is maximum, as given in Eq. 1:

$$\begin{aligned} \text{Max.}\ \ & BT_{cr} = f(T_i\text{'}s) \\ s.t.\ \ & 0 \leq T_i \leq 90;\ \ i = 1, \ldots, NDV \end{aligned} \tag{1}$$

where BT_{cr} is the critical buckling torque in terms of the design variables (T_i's) and NDV is the number of the design variables. Since the stacking sequence considered in this study is $[\pm\theta_1/\pm\theta_2]_s$ and θ_1 and θ_2 have four design variables each, NDV=8 in this study.

There is not an analytical function of the buckling load (BT_{cr}) in terms of the design variables (T_i's). Therefore, calculation of BT_{cr} for each set of the design variables (T_i's) requires a FEA which is computationally expensive. On the other hand, because of the iterative nature of the optimization procedures, using any optimization method to find the optimum solution for Eq. 1 would require numerous function evaluations with the proportional computational cost associated with them. The computational cost will be even more challenging in case of using evolutionary optimization approaches that are population based methods and a very large number of function calls (high fidelity FEA in this study) are required. To reduce the computational expense, metamodel-based design optimization (MBDO) methods are used in which the costly high fidelity functions are replaced by low-cost analytical approximation functions so-called *surrogate models* or *metamodels* [12]. Besides the substantially improved computational efficiency, MBDO methods are also capable of alleviating the noisy behavior of the high fidelity models by replacing them with smooth analytical surrogate models [12, 13]. In its simplest form, a surrogate models is constructed by doing high fidelity modeling/simulations (FEA in this study) for a limited number of well-distributed sample or training points referred as the design of experiments (DOE), followed by a curve fitting to the responses. The resulted curve, or hyper-surface in case of higher dimensions, is a low cost approximation for the costly high fidelity model that is used for function calls in MBDO process.

Among several metamodeling techniques developed for engineering design optimization purposes [12, 13], the radial basis functions (RBF) offered accurate and reliable predictions at a relatively low computational cost for buckling response of VS composite structures [3, 4]. Therefore, RBF-based metamodels were used in this study to construct surrogate models that relate the buckling load (BT_{cr}) to the design variables (T_i's). Since metamodels are approximate functions, there are some errors associated with the responses predicted by them compared to the responses computed by the high fidelity models. These errors are calculated by comparing the responses computed from metamodels and high fidelity models in

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a number of so-called test points. In this study, the number of test points are considered to be one tenth of the DOE training points (one tenth of the sample size). To reduce the errors associated with metamodeling, a multi-step design optimization method [10] was used in which the design domain is narrowed down in multiple steps of the optimization process so that a converged optimum design emerges with negligible error when the result from metamodel is compared with the high fidelity FE model.

The same MBDO method was also used to find the optimum orientation angles for the CS designs. However, it should be noted that in the CS cases the design variables are θ_1 and θ_2 as they are not varying on the surface of the elliptical cylinders. Therefore, the computational time needed to find the best CS laminate is considerably lower than VS one because of the reduced number of the design variables.

4. Results

Before studying the elliptical cylinders, circular cylinders were studied for their potential buckling improvement. Although both the loading and geometry are axisymmetric in this case, the finite length of the cylinder made it a candidate for fiber steering to investigate the potential improvement in torsional buckling. In other words, it can address the possibility of improvement in torsion-induced buckling capacity of a finite-length circular cylinder by varying the orientation angle along the length of the cylinder (from the ends to the middle). Therefore, the fiber orientation angles are changed only in the axial direction for circular cylinders with different aspect ratios (L/R) from end parts to the middle (as shown in Fig. 1c).

The optimization results for circular cylinders are shown in Fig. 2. It is observed that the potential buckling improvement decreases by increase of the length aspect ratio of the cylinders. However, the magnitude of the improvement due to fiber steering is not considerable for circular cylinders (less than 2% for long and 6% for short cylinders).

The optimum orientation angle distribution from one end to the other end is also shown in Fig. 3. As observed, the results show that the optimum orientation angles at the end supports for VS cylinders tend to be lower than their CS counterparts while slightly higher at the middle. Although this redistribution of the orientation angle gives a better load path from the loading points to the supports, it is not enough to considerably improve the buckling performance. As stated earlier, it is because of the axisymmetry of the loading and the structure that leaves negligible scope for improvement by fiber steering.

In contrast to circular cylinders, there are two justifiable shifting directions for a curved fiber on the surface of the elliptical cylinders; (1) axial, and (2) circumferential. Figure 4 shows the torsion-induced buckling response of the CS and VS circular composite cylinders in which the orientation angles are varied in either axial and circumferential directions. It can be observed that more area is involved for carrying the buckling load in VS elliptical cylinders compared with their CS counterpart. The elliptical cylinders are considered to have the length aspect ratio of (L/R=1) where R is the radius of a hypothetical cylinder that gives the same circumferential length (perimeter) of the elliptical one.

Figure 5 also shows the orientation angle distribution of composite plies in both CS and VS cylinders. As seen, for axial variation of the orientation angles the trend is very similar to the circular cylinder (see Fig. 3). However, for the circumferential variation the orientation angles start from almost the same values as in their CS counterparts at the highly curved parts to some different values at the flatter parts that are more prone to buckle. Unlike the circular cylinders and elliptical cylinders with axial variation of orientation angles, circumferential variation results in higher increase of the area to carry the buckling load (see Fig. 4b). To see if this is always the case, the length aspect ratio (L/R) is changed and the buckling load improvement of VS cylinders with both axial and circumferential variation of the orientation angles are calculated and compared to their CS counterparts.

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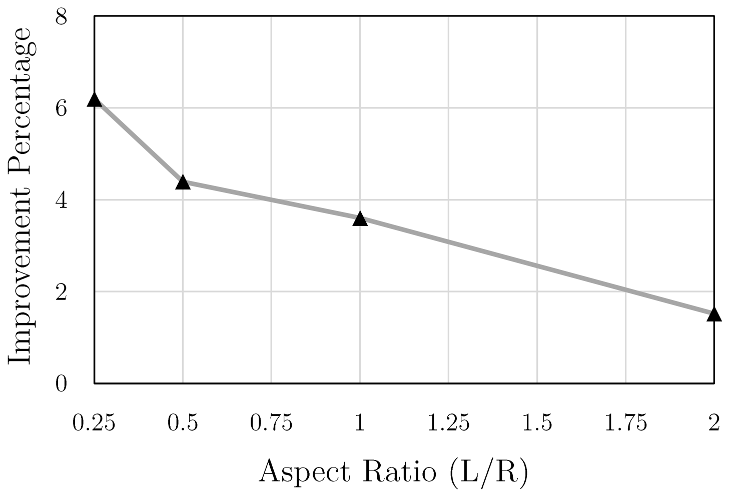

Figure 2. Torsion-induced buckling improvement of circular composite cylinders with different length aspect ratios.

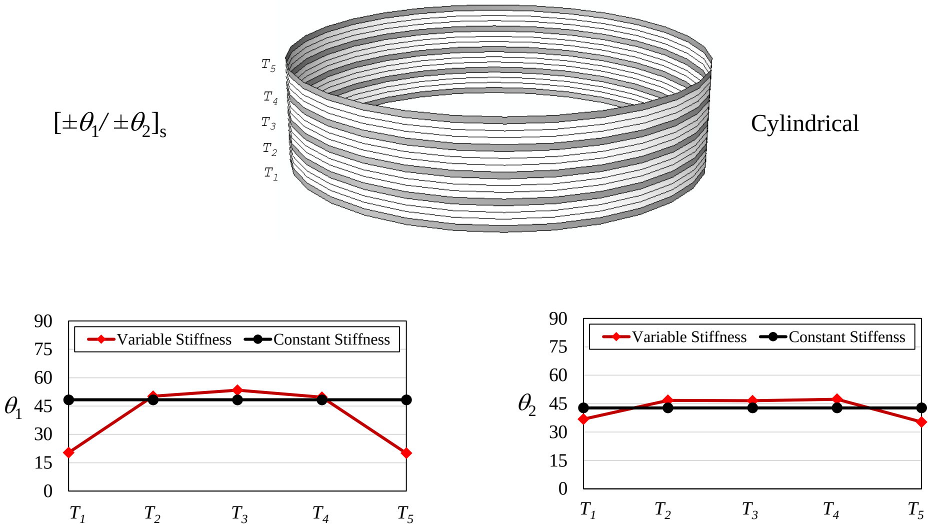

Figure 3. Optimum orientation angle distribution of composite plies in a circular composite cylinder.

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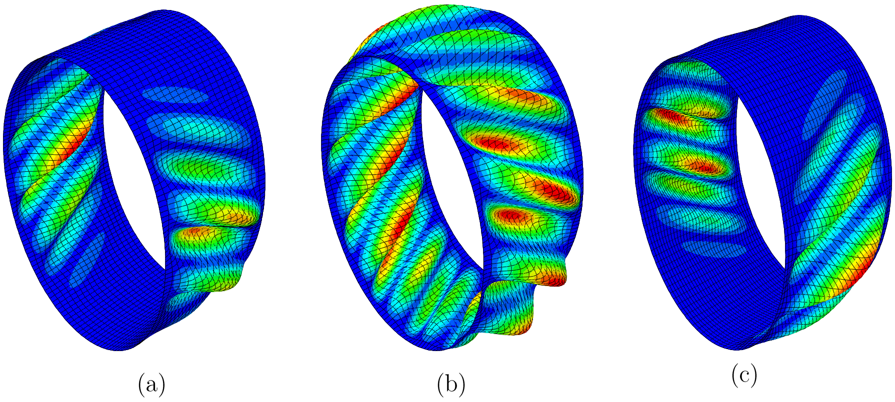

Figure 4. Torsion-induced buckling mode shape of elliptical cylinders with (a) CS, (b) VS with axial narrow bands, and (c) VS with circumferential narrow bands.

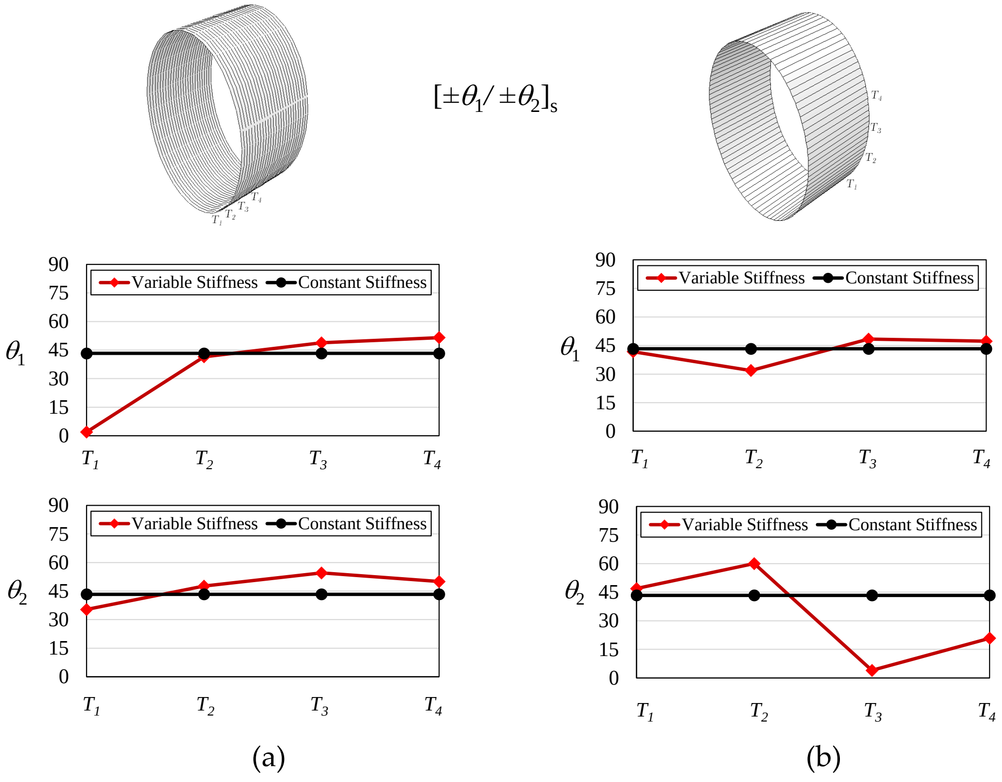

Figure 5. Optimum orientation angle distribution of composite plies in elliptical composite cylinders with (a) axially and (b) circumferentially varying orientation angles.

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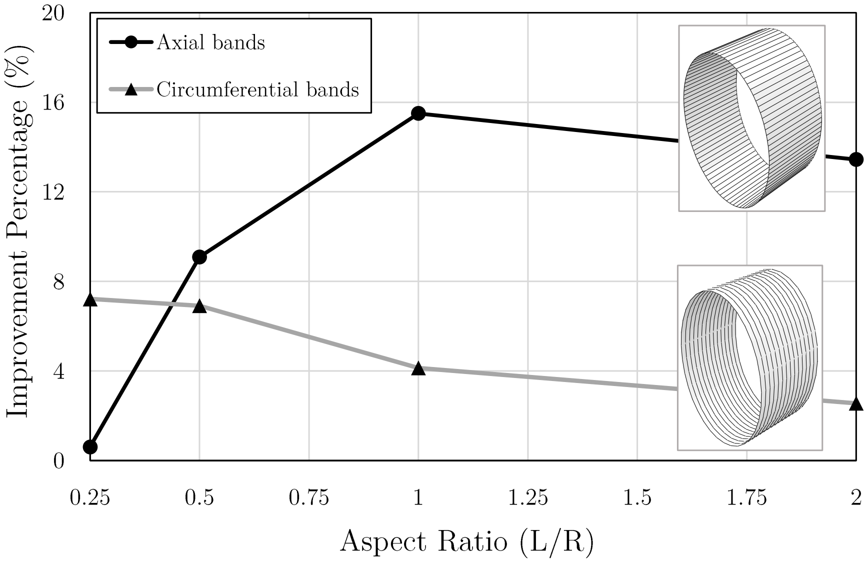

Figure 6. Torsion-induced buckling improvement of elliptical composite cylinders with different length aspect ratios for curved fibers shifted in axial (circumferential narrow bands) and circumferential (axial narrow bands) directions.

By changing the length aspect ratio (L/R), Fig. 6 shows how the maximum room for buckling improvement may change if the shifting direction of the curved fibers is considered to be axial or circumferential. It shows for long cylinders, the axial shifting direction for curved fibers in the design optimization offers more room for torsion-induced buckling improvement whereas for short cylinders it is the circumferential shifting direction that results in more improvement for buckling. This is due to the boundary regions' contribution to the buckling load which is high in short cylinders but negligible in the long ones. Therefore, the axial variation for short cylinders affect more on the buckling load improvement for short cylinders whereas in the long ones the circumferential variation gives higher improvement using the ellipticity of the structure.

5. Conclusion

Using fiber steering, variable stiffness (VS) circular and elliptical composite cylinders were designed and optimized for maximum torsion-induced buckling capacity. To this end, the orientation angles were varied in both axial and circumferential directions. The design optimization was performed by surrogate modeling (metamodeling) and genetic algorithms.

Circular composite cylinders were shown to have negligible scope for improvement in torsion-induced buckling by fiber steering. It is because of the axisymmetry of both structure and loading. Elliptical cylinders, because of the inhomogeneity of the curvature, offers some room for buckling improvement by fiber steering.

It was shown that for long elliptical cylinders, circumferential variation of the orientation angle offers more scope for buckling improvement whereas the orientation angles should vary axially in short elliptical cylinders to result in more improvement in torsion-induced buckling capacity. The maximum improvement in torsion-induced buckling capacity is about 16% that is for long VS elliptical composite cylinders with circumferentially varying orientation angles.

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Acknowledgments

The financial contributions from the Natural Sciences and Engineering Research Council of Canada (NSERC) industrial chair on Automated Composites, Bell Helicopter Textron Canada Ltd., Bombardier Aerospace, Delastek Ltd. and Concordia University are appreciated.

References

[1] B. F. Tatting. *Analysis and design of variable stiffness composite cylinders*. PhD thesis, Virg. Tech, Blacksburg, Virginia, 1998.

[2] Adriana W. Blom, Patrick B. Stickler, and Zafer Gürdal. Optimization of a composite cylinder under bending by tailoring stiffness properties in circumferential direction. *Composite Part B: Engineering*, 41(2):157–165, March 2010.

[3] M. Rouhi, H. Ghayoor, S. V. Hoa, and M. Hojjati. Effect of structural parameters on design of variable-stiffness composite cylinders made by fiber steering. *Composite Structures*, 118:472–481, 2014.

[4] M. Rouhi, H. Ghayoor, S. V. Hoa, and M. Hojjati. Multi-objective design optimization of variable stiffness composite cylinders. *Composites Part B: Engineering*, 69:249–255, 2015.

[5] M. Rouhi, H. Ghayoor, S. V. Hoa, and M. Hojjati. The effect of the percentage of steered plies on the bending-induced buckling performance of a variable stiffness composite cylinder. *Science and Engineering of Composite Materials*, 22(2):149–156, 2015.

[6] Waddy T Haynie and M Hyer. Behavior of elliptical composite cylinders in torsion. In *47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference , Newport, Rhode Island*, 2006.

[7] M. Sun and M. W. Hyer. Use of material tailoring to improve buckling capacity of elliptical composite cylinders. *AIAA Journal*, 46(3):770–782, 2008.

[8] A. Khani, M. M. Abdalla, and Z. Gürdal. Circumferential stiffness tailoring of general cross section cylinders for maximum buckling load with strength constraints. *Composite Structures*, 94(9):2851–2860, September 2012.

[9] A. Khani, M. M. Abdalla, and Z. Gürdal. Optimum tailoring of fibre-steered longitudinally stiffened cylinders. *Composite Structures*, 122:343–351, 2015.

[10] M. Rouhi, H. Ghayoor, S. V. Hoa, and M. Hojjati. Multi-step design optimization of variable stiffness composite cylinders made by fiber steering. In *Proceedings of the American Society for Composites - 29th Technical Conference, La Jolla, CA, USA*, Sep, 2014.

[11] ABAQUS Inc., Pawtucket, RI, USA. *ABAQUS Version 6.11 User's Manual*, 2011.

[12] G. G. Wang and S. Shan. Review of metamodeling techniques in support of engineering design optimization. *Journal of Mechanical Design, Transactions of the ASME*, 129(4):370–380, 2007.

[13] Nestor V. Queipo, Raphael T. Haftka, Wei Shyy, Tushar Goel, Rajkumar Vaidyanathan, and Kevin P. Tucker. Surrogate-based analysis and optimization. *Progress in Aerospace Sciences*, 41(1):1–28, January 2005.

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