Evaluation of Impact Damage in Circular Laminates Subjected to a Transverse Load

Hiroshi Suemasu

Department of Engineering and Applied Sciences, Sophia Uiversity, Chiyodaku Tokyo, Japan
Email: suemasu@sophia.ac.jp, Web Page: http://www.me.sophia.ac.jp

Keywords: Laminate, impact damage, energy release rate, multiple delaminations, axisymmetric plate

Abstract
Responses of nonlinear axisymmetric plates with multiple delaminations subjected to a transverse concentrated load are approximately solved to evaluate the significance of low velocity foreign object damage in quasi-isotropic composite laminates. Relationships between applied load and the displacement of the loading point are given based on a mechanical consideration. Effects of transverse shear deformation and local indentation are included and the geometrical nonlinearity is also considered. The linear terms are derived based on the Mindlin plate theory. The damaged portion is modelled as equally spaced multiple circular plates without delamination opening. A superposition technique is used to derive an approximated closed-form formula; a global response of intact plate and a local problem that is an additional displacement due to the introduction of the multiple delaminations. An explicit expression of the energy release rate is derived as a function of geometrical and material parameters. The solution agrees well with finite element solutions.

1. Introduction

Composite laminates having weak interfaces compared to their superior inplane performances are vulnerable to damage due to local bending when subjected to transverse impact and transverse concentrated loads [1]. The damage causes significant compressive strength reduction even when the damage is in a barely visible state (CAI) [2-5]. There are a number of numerical and/or experimental works to study impact damage problems of laminated composites due to its importance in the design of aeronautical structures [5-9], such as the relation between the damage and impact energy (or the impact force), scale effects, interlaminar toughness, stacking sequence, coupling between the delamination and matrix cracks, etc.

It is very helpful to estimate the various effects of material and structural parameters on impact damage problems of the composite laminates by some closed form expression. However, a limited number of such analytical works on the topic have been reported due to the geometrical complexity and its nonlinear nature. Suemasu and Majima obtained a closed form solution on the delamination propagation and quasi-static concentrated force for the linear problem [10] and Rayleigh-Ritz approximated solution for the nonlinear problem [11]. It is shown that large nonlinear effect must be considered to predict the impact damage. Olsson [12] obtained an analytical expression by separately considering bending and membrane components based on a similar idea to the present analysis. Suemasu et al. proposed a simple mathematical expression to estimate the significance of the impact damage in terms of impact load and impact energy [13]

A simple and more accurate form of the energy release rate is given for nonlinear plates in terms of applied force, damage size and various geometrical and material parameters for both simply-supported and fixed boundary conditions in the present paper. Then, the solution is compared with finite element solutions to demonstrate the applicability of the present theory to the real problem.

Excerpt from ISBN 978-3-00-053387-7

2 ANALYSYS

2.1 Linear Solution

Firstly, we consider a linear problem of circular axisymmetric laminates of radius *R* and thickness *h* as shown in Figure 1(a). Midlin plate theory is used instead of Kirchhoff's plate theory to consider the effect of transverse shear. The loading condition at the loading point is illustrated in Figure 2. The problem is expressed as the sum of the indentation of hemispherical indenteor on the rigid foundation (Figure 1b) and the deformation due to the locally distributed load equal to the reaction of the foundation of the indentation problem(Figure 1c). The contact area changes according to the load, while the distribution of the reaction force changes a little. As the effect of the distribution profile of the load on the total displacement of the laminate is not significant, uniform distributed force is assume at a small portion of a radius $b \approx h$ as shown in Figure 1(d).

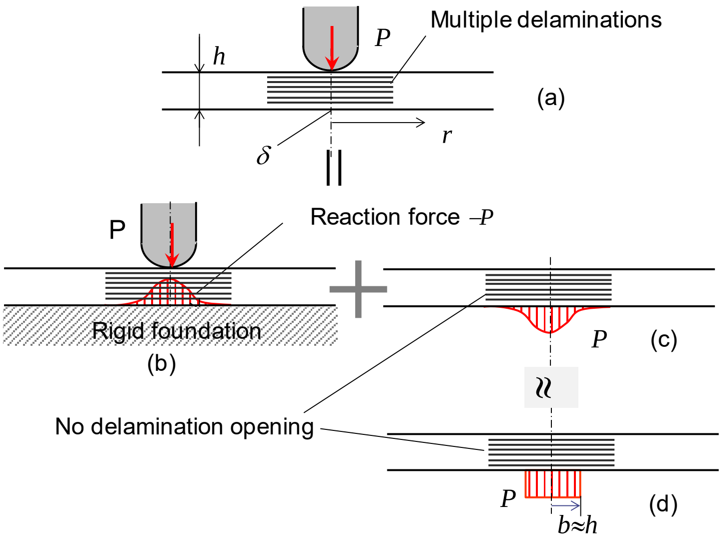

Figure: 1 Axisymmetric plate with multiple delaminations loaded at its center by hemispherical indenter and its modelling

Relations between the internal forces (bending moment M_r and M_θ and shear force Q_r) and out-of-plane displacement *w* and rotation of cross-section ϕ are

$$\begin{aligned} M_r &= -D\left(\frac{\partial \phi}{\partial r} + \nu r^{-1}\phi\right) \\ M_\theta &= -D\left(r^{-1}\phi + \nu\frac{\partial \phi}{\partial r}\right) \\ Q_r &= kGh\gamma_{rz} = kG_{rz}h\left(\frac{\partial w}{\partial r} - \phi\right) \end{aligned} \tag{1}$$

where *r* is radial coordinate, G_{rz} is out of plane shear modulus, k=5/6 is shear coefficient and *D* is bending stiffness. The equilibrium equations are

$$\begin{aligned} &\frac{\partial Q_r}{\partial r} + r^{-1}Q_r = 0 \\ &\frac{\partial M_r}{\partial r} + r^{-1}(M_r - M_\theta) - Q_r = 0 \end{aligned} \tag{2}$$

Following nondimensional expressions are introduced.

Excerpt from ISBN 978-3-00-053387-7

$$W = \frac{w}{h},\ \varphi = \frac{R}{h}\phi,\ p_0 = \frac{PR^2}{16\pi Dh} \tag{3}$$

$$\rho = \frac{r}{R},\ \alpha = \frac{a}{R},\ \ \beta = \frac{b}{R},\ \ \zeta = \frac{8D}{kG_{rz}h^3}\left(\frac{h}{R}\right)^2 \tag{4}$$

where a and b (<<R) are the radii of delaminations and load-distributing area.

Fixed and simply-supported boundary conditions are written as

$$\left.\begin{array}{l} \begin{cases} \varphi = 0 & \text{for fixed plates} \\ \varphi' + \nu\rho^{-1}\varphi = 0\ \ (M_r = 0) & \text{for simply-supported plates} \end{cases} \\ W = 0 \end{array}\right\} \quad \text{at } \rho = 1 \tag{5}$$

$$\varphi = \text{finite} \quad \text{at } \rho = 0$$

$$2\pi bQ_z = P \quad \text{at } \rho = \beta$$

Considering β^2<<1, the linear solution satisfying the boundary and the continuity conditions for the deformations and internal forces at the connecting position ($\rho = \alpha$) is approximately obtained as

$$W = p_0\left\{c_b + \left(N^2 - 1\right)\alpha^2 + c_s\right\} \tag{9}$$

$$c_b = \begin{cases} \dfrac{3+\nu}{1+\nu} & \textit{Simply-Supported} \\ 1 & \textit{Fixed} \end{cases}$$

$$c_s = \tfrac{1}{2}\zeta\left(1 - 2\log\beta\right)$$

2.2 Local Indentation

The analytical result of the local indentaion of a stiff hemispherical head indentor is introduced here neglecting the existence of delaminations[14].

$$q_I = A_0 {p_0}^{2/3} - A_1 p_0 \tag{10}$$

where $q_I=\delta_I/h$ and

$$A_0 = \left(\frac{16\pi DB}{R^2\sqrt{hR_I}}\right)^{2/3}, \qquad A_1 = \frac{8DK_0}{hR^2}\ln 2$$

$$B = \frac{3}{8}\sqrt{\frac{1-\nu_{rz}\nu_{zr}}{G_{rz}E_z}}\left(1+\nu\right)\left[1 + 2\frac{G_{rz}}{\sqrt{EE_z}}\left\{\sqrt{\frac{\left(1-\nu_{rz}\nu_{zr}\right)\left(1-\nu\right)}{1+\nu}} - \sqrt{\nu_{rz}\nu_{zr}}\right\}\right]^{1/2}$$

$$K_0 = \frac{C_{zz}}{\sqrt{\mu_1\mu_2}C_{rr}}\frac{k_1\sqrt{\mu_1} - k_2\sqrt{\mu_2} + k_1k_2\left(\sqrt{\mu_1} - \sqrt{\mu_2}\right)}{\sqrt{\mu_2}\left(1+k_2\right)\left(k_1C_{zz} - \mu_1C_{rz}\right) - \sqrt{\mu_1}\left(1+k_1\right)\left(k_2C_{zz} - \mu_2C_{rz}\right)}$$

$$C_{rr} = \frac{E\left(1-\nu_{rz}\nu_{zr}\right)}{\left(1+\nu\right)\left(1-\nu-2\nu_{rz}\nu_{zr}\right)}, \quad C_{zz} = \frac{E_z\left(1-\nu\right)}{1-\nu-2\nu_{rz}\nu_{zr}}$$

$$C_{rz} = \frac{\nu_{zr}E}{1-\nu-2\nu_{rz}\nu_{zr}}, \qquad C_{44} = G_{rz}$$

$$\left.\begin{array}{l}\mu_1\\ \mu_2\end{array}\right\} = \frac{C_{rr}C_{zz} - C_{rz}\left(C_{rz} + 2C_{44}\right) \pm \sqrt{\left\{C_{rr}C_{zz} - C_{rz}\left(C_{rz} + 2C_{44}\right)\right\}^2 - 4C_{rr}C_{zz}{C_{44}}^2}}{2C_{rr}C_{44}}$$

$$k_j = \frac{C_{rr}\mu_j - C_{44}}{C_{rz} + C_{44}}$$

Excerpt from ISBN 978-3-00-053387-7

The local indentation for delaminated laminates can be roughly estimated by this equation as a function of the applied load and the elastic constants of the laminates as well as its thickness and the radius of the indenter head.

2.3 Nonlinear Solution

Low velocity and large mass impact response may be replaced by a quasi-static concentrated load problem [13]. Load-displacement histories of impact damaged plates can be well expressed by those of the plate with an equivalent number of equally spaced multiple circular delaminations. Considering the facts, expresions to roughly estimate the significance of the impact damage will be derived. Circular transversely-isotropic laminates of radius R and thickness h constrained at their boundary are considered. The plates have N-1 multiple circular equally-spaced delaminations of radius a. The bending stiffness of the damaged portion devided into N equal thickness ligaments reduces to $1/N^2$. The plates are loaded by a quasi-static load P at its center as shown in Figure 2(a), the delaminated portion deforms significantly and shows very large nonlinearity, while the intact portion usually deflects a little and shows slight nonlinearity. Membrane and shear stiffness reduces only little even by the introduction of multiple delaminations. The approximated response can be given by superposing three problems (b), (c) and (d). The sum of the applied load of three problems is same as that of problem (a).

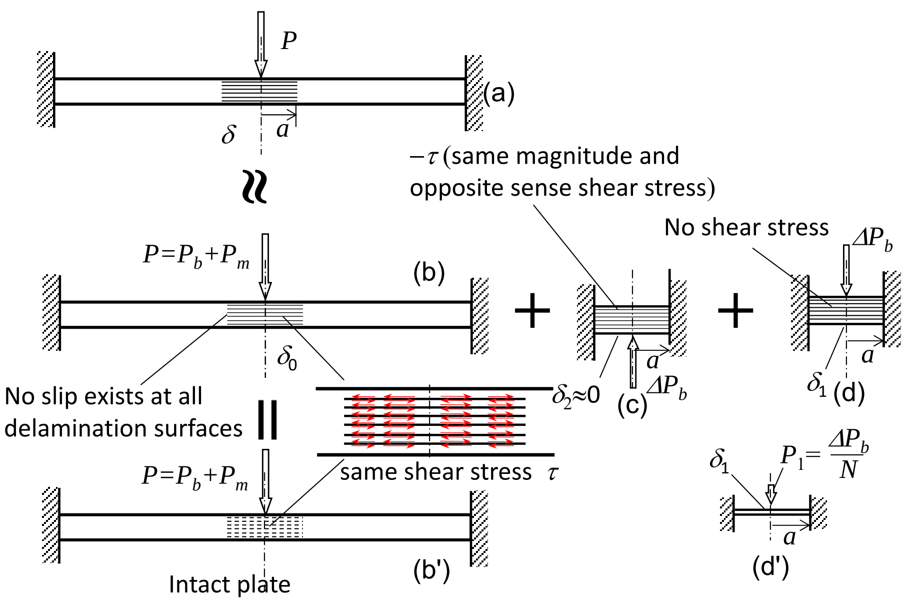

Figure 1: A circular plate with multiple circular delaminations subjected to a concentrated load at its center can be expressed as a Superposition of three problems

In the problem (b) an equal magnitude distributed shear stress to that existing at the corresponding interfaces in the intact plate is assumed at the delaminated surface. The solution of the problem (b) is same as that of the intact plate (b'). The second and the third problems are piled N circular panels fixed at the delamination boundary. The opposite direction same magnitude loads are applied for the Problems (c) and (d). All the delaminated ligaments are assumed to deflect together. Since membrane and shear stiffness are unchanged due to the introduction of the multiple delaminations, the necessary load to produce same deflection as the intact plate reduces same rate as the bending stiffness reduction. In problem (c) an opposite sense shear stress to that of problem (b) is given at all the delamination surfaces. Linearized solution of this problem causes no deflection at the loading point. No constraint exist to the relative slipping at the delaminated surface for problem (d). Then, the solutions of the load displacement relations of the two nonlinear plate problems are needed, that is, a global base plate (b') and a thin delaminated plate (d').

Excerpt from ISBN 978-3-00-053387-7

The following nondimensinal relation between the load p_0 and a normalized displacement q_0 is assumed for intact plates (case of Figure 2(b')) [13].

$$p_0 = k_{L0}\, q_0 + k_{N0} q_0^{\gamma} \tag{11}$$

where $k_{L0}=(c_b+c_s)^{-1}$. The coefficients of the nonlinear terms k_{N0} and γ_0 depending on the boundary conditions and dimensions of the plates are numerically determined. (If the deformed shape of the plate did not change during the deflection, the parameter γ_0 would equal three.) The bending compliance of the delaminated portions is $N^2(=N\times N^{-3})$ times larger.

Relation between the load and displacement for the case Figure (d') is also derived following same rule as the global plate. Since the additional deformation starts from the equilibrium state of the global deformation (Figure b), the boundary of the local additional deformation is assumed to be fixed at its periphery and the nonlinear effect of inplane stress before the introduction of the delaminations must be incorporated in the load displacement relation.

$$q + k_N \left\{ (q+s)^{\gamma} - s^{\gamma} \right\} = p \tag{12}$$

where

$$q = \delta_1 / t$$
$$p = \frac{\Delta P / N}{16\pi D_d t / a^2} = N^3 \alpha^2 \Delta p_{0B} = N\left(N^2-1\right)\alpha^2 k_L q_0 \tag{13}$$

The equivalent initial normalized displacement s must be a function of the global normal displacement q_0. It is not clear how to determine the relation between s and q_0. In the present paper we determined to use the relative displacement from the damage boundary to the centre $\tilde{\delta}/t$ of the damaged portion of the intact plate as illustrated in Figure 3. Then,

$$s = \frac{\tilde{\delta}}{t} = \frac{\delta_0 \mid_{r=0} - \delta_0 \mid_{r=a}}{t} = Nm(\alpha) q_0 \tag{14}$$
$$m(\alpha) \approx 1 - \frac{c_b\left(1-\alpha^2\right) - 2\alpha^2 \log\alpha - \zeta \log\alpha}{c_b + c_s}$$

Present expression coincides with the linear solution Eq. 9 when the nonlinear term is neglected.

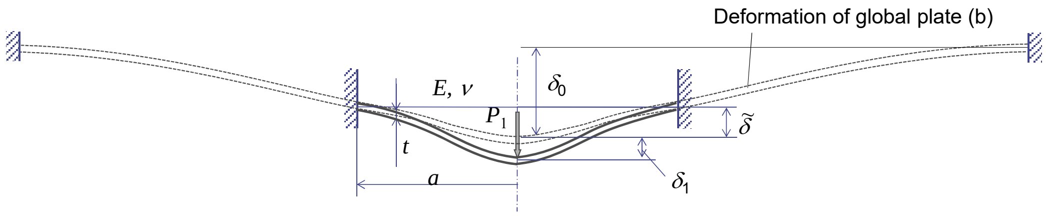

Figure 3: Local model of damaged portion.

2.4 Energy Release Rate

When the damages ize is constant, a stored complimentary energy is obtained by integrating the displacement δ of the loading point by the applied load P. The displacement of the loading point δ is a sum of the global displacement δ_0 and local additional displacement δ_1. Considering Eqs. 11 and 13, the complimentary energy is given as

$$\Pi_C\left(P|\alpha\right) = \int_0^P \delta(P)\,dP = \int_0^P (\delta_0 + \delta_1)\,dP = \Pi_{C0} + \Pi_{C1} \tag{15}$$

$$\Pi_{C0} = \int_0^P \delta_0(P)dP = \frac{16\pi D_0 h^2}{R^2}\int_0^{q_0} q_0 \frac{dp_0}{dq_0} dq_0 = \frac{16\pi D_0 h^2}{R^2}\left(\frac{1}{2}k_{L0}{q_0}^2 + \frac{\gamma_0}{\gamma_0+1}k_{N0}{q_0}^{\gamma_0+1}\right)$$
$$\Pi_{C1} = \int_0^P \delta_1(P_b)dP = \frac{16\pi D_0 h^2}{NR^2}\int_0^{q_0} q \frac{dp_0}{dq_0} dq_0 = \frac{16\pi D_0 h^2}{NR^2}\int_0^{q_0} q\left(k_{L0} + \gamma_0 k_{N0}{q_0}^{\gamma_0-1}\right)dq_0 \tag{16}$$

where the local displacement q is a function of q_0 and the damage size α. As Π_{C0} is independent of the damage size α, the energy release rate of uniform simultaneous growth of all delaminations can be given by differentiating the energy Π_{C1} with respect to delamination area.

$$G = \left[\frac{\partial \Pi_{C1}}{\partial A}\right]_{P=const} = \left[\frac{\partial \Pi_{C1}}{2\pi(N-1)a\partial a}\right]_{P=const} = \left[\frac{\partial \Pi_{C1}}{2\pi(N-1)R^2\alpha\partial\alpha}\right]_{p_0=const}$$
$$= \frac{1}{N(N-1)}\frac{8Dh^2}{R^4}\int_0^{q_0}\frac{1}{\alpha}\left[\frac{\partial q}{\partial \alpha}\right]_{P=const}\left(k_{L0} + \gamma_0 k_{N0}{q_0}^{\gamma_0-1}\right)dq_0 \tag{17}$$

Differentiating both sides of Eq. 13 by α under the condition P=constant, the following relation is derived after some manipulations.

$$\frac{1}{\alpha}\frac{\partial q}{\partial \alpha} = \frac{2N\left(N^2-1\right) - \gamma k_N N\left\{(q+s)^{\gamma-1} - s^{\gamma-1}\right\}\frac{1}{\alpha}m'}{1+\gamma k_N (q+s)^{\gamma-1}}q_0 \tag{18}$$

$$\tfrac{1}{\alpha}m'(\alpha) \approx \frac{2(c_b - 2\log\alpha - 1) + \alpha^{-2}\zeta}{c_b + c_s}$$

Substitution of Eq.18 into Eq.17 yields a normalized energy release rate Γ as

$$\Gamma = \frac{R^4}{8Dh^2}G = 2(N+1)\int_0^{q_0}\frac{\frac{1}{c_b} - \frac{1}{2}\frac{1}{N^2-1}\frac{1}{\alpha}m'(\alpha)\gamma k\left\{(q+s)^{\gamma-1} - s^{\gamma-1}\right\}}{1+\gamma k(q+s)^{\gamma-1}}\left(k_{L0}q_0 + \gamma_0 k_0 {q_0}^{\gamma_0}\right)dq_0 \tag{19}$$

The normalized energy release rate Γ is a function of q_0, that is, the applied load p_0. The equilibrium path of P and δ when $\Gamma = \Gamma_{cr}$ can be obtained numerically with increasing the parameter α.

3. RESULTS AND DISCUSSIONS

Owing to the limit of space we only show the case of simply-supported plates. The center deflection of simply-supported intact plate is plotted against the applied load in Figure 4. The effects of the shear deformation, local indentation and nonlinearity are well expressed by the present approximation(k_0=0.115 and γ_0=2.80). The following analyses are conducted by using the values and the values k_1=0.450, γ_1=2.57 are chosen for fixed boundary and the local deformation of damaged portion. The effects of local indentation and the deformation due to the trasverse shear force are not very small but some effect on the impact load. Both effects increase with the thickness of the plate.

The relationships between the nondimensional applied load p_0 and the normalized deflection δ/h are plotted in Figure 5. The present solution showed good agreement with the finite element solution for N=4 even for large α (=0.4). Though the present definition of the equivalent initial normalized displacement s is not rigorous, the additional displacement can be obtained well and the present solution is sufficient for the rough estimate of the damaged plate response, while the effect s had better be obtained based on a rigorous mechanical development to obtain more physically reliable solution.

The square root of the nondimensional energy release rate $\sqrt{\Gamma}$ is plotted against the applied load for several cases of α when N=8 and compared with finite element results as shown in Figure 6. The present results excellently agree with the finite element solutions for a wide range of the load. The present analysis is appropriate to evaluate the stability of delaminations during the indentation loading.

Excerpt from ISBN 978-3-00-053387-7

The larger the delamination radius α is, the less the energy release rate increases with the load. It is because the membrane term becomes dominant with the increase of the delamination size and number when the load is constant. As the energy release rate decreases with the growth of the delaminations, the load must be increased to keep the delaminations to grow.

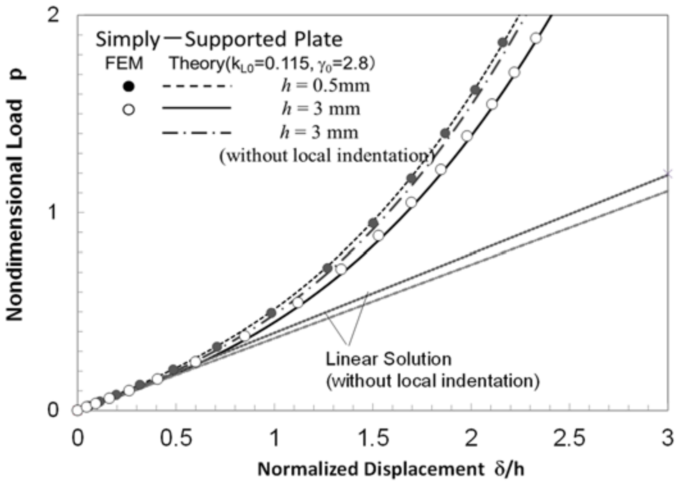

Figure 4: Nonlinear relation between the load and deflection for the fixed circular plate

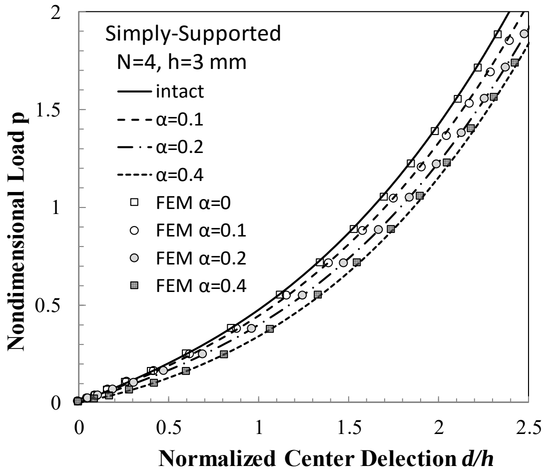

Figure 5: Load-displacement relation for simply-supported plate with three delaminations(N=4)

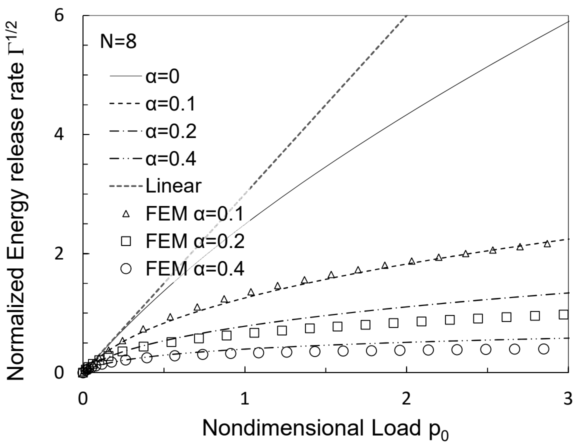

Figure 6: Relationship between load and energy release rate for simply-supported plate with three delaminations(N=8)

Excerpt from ISBN 978-3-00-053387-7

4. CONCLUSIONS

An analytical solution is proposed for the impact damage problem, where the energy release rate for the simultaneous growth of multiple circular delaminations is given as an integral form of applied displacement for simply-supported and fixed boundary conditions. The results showed the local indentation of the indenter and the effect of the shear are usually small for the rough estimate of the damage. The present solutions agree well with finite element results. The expression can be used for the rough estimate of the significance of impact damage in terms of applied energy, interlaminar toughness and dimensions of the laminates. The effect of stacking sequence may be taken into account through the number of the delaminated ligaments *N* in the thickness direction.

This paper is based on results obtained from a project commissioned by the New Energy and Industrial Technology Development Organization (NEDO).

References

[1] T.E. Tay, Characterization and analysis of delamination fracture in composites: An overview of developments from 1990 to 2001", *Appl. Mech. Rec.*, 56(1), 2001, pp. 1-32.
[2] E.F. Dost, L.B. Ilcewicz, and W.B. Avery, The effects of stacking sequence on impact damage resistance and residual strength for quasi-isotropic laminates, *ASTM STP* 1110, pp. 476-500 1991.
[3] S.M. Lee, Compression-after-impact of composite with toughened matrices, SAMPE J., 22: 64–8, 1986.
[4] T. Ishikawa, S. Sugimoto, M. Matsushima and Y. Hayashi, Some experimental findings in CAI test of CF/PEEK and conventional CF/epoxy flat plates, *Composite Science Technology,* **55**, 1995, pp. 349–362,
[5] Y. Aoki, H. Kondo, H. Hatta, Effect of Specimen Thickness on Impact-Induced Delamination and CAI Behavior, 39th International SAMPE, CD–ROM, Paper No. TM1-03, 2007.
[6] J.P. Hou, N. Petrinic and C. Ruiz. 2001. "A delamination criterion for laminated composites under low-velocity impact", *Compos. Sci. Tech.*, 61(14): 2069-2074.
[7] O. Allix, D. Guedra-Degeorges, and A. Vinet. 2002. "A 3D damage analysis of low-velocity impacts on laminated composite, *Compos. Sci. Tech.*, 62(4): 585-589.
[8] Y. Aoki and H. Suemasu, Damage analysis in composite laminates by using interface element, *Advanced Composite Materials*, **12**, 1, 2003, pp. 13-22.
[9] Y. Aoki, H. Suemasu and T. Ishikawa, Damage propagation in CFRP laminates subjected to low velocity impact and static indentation, *Advanced Composite Materials*, **16**, 2007, pp. 45–61,
[10] H. Suemasu and O. Majima, Multiple delaminations and their severity in circular axisymmetric plates subjected to transverse loading, *J. Compos. Mat.*, 30, 4, 1996, pp. 441-453.
[11] H. Suemasu and O. Majima, Multiple delaminations and their severity in nonlinear circular axisymmetric plates subjected to transverse loading, *Journal of Composite Materials*, **32**, 2, 1998, pp. 123-140.
[12] R. Olsson, Analytical prediction of large mass impact in composite laminates, *Composites A*, **32**, 2001, pp. 1207-1215,
[13] H. Suemasu, M.R. Wisnom, X.C. Sun and S.R. Hallett, An analytical study on multiple delaminations and instability in nonlinear plate subjected to transverse concentrated load, 13th Japan International SAMPE Symposium and Exibition, November 11-13, 2013.
[14] H.Suemasu, S. Kerth and M. Maier, Indentation of spherical head indentors on Transversely isotropioc Composite plates, Journal of Composite Materials, Vol. 28, No. 17, 1723-1739, 1994.

Excerpt from ISBN 978-3-00-053387-7