COMBINED FE/STATISTICAL APPROACH FOR THE STRENGTH OF COMPOSITE FIBRE BUNDLES CONSIDERING HIERARCHICAL FAILURE

Gaël Grail[1], Marie Coq[1,2], Charlotte Guesdon[1], Soraia Pimenta[3], Silvestre T. Pinho[1], Paul Robinson[1]

[1]The Composites Centre, Dept. of Aeronautics, Imperial College London, South Kensington Campus, SW7 2AZ London, UK
Emails: g.grail@imperial.ac.uk, charlotte.guesdon15@imperial.ac.uk, silvestre.pinho@imperial.ac.uk, p.robinson@imperial.ac.uk Web Page: http://www.imperial.ac.uk/composites-centre/
[2]Currently working at Messier-Bugatti-Dowty, Cheltenham Road East, GL2 9QH Gloucester, UK
Email: marie.coq@safranmbd.com
[3]The Composites Centre, Dept. of Mechanical Engineering, Imperial College London, South Kensington Campus, SW7 2AZ London, UK
Email: soraia.pimenta@imperial.ac.uk, Web Page: http://www.imperial.ac.uk/composites-centre/

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Abstract
This paper presents an overview of a combined FE/statistical approach for predicting the full strength distribution of unidirectional Composite Fibre Bundles (CFBs). FE models of CFBs with differently sized pre-fractured clusters of fibres were built to predict the stress concentration field around the respective broken cluster, and this was then used in an independent statistical model to predict failure of the surrounding fibres. The latter considers a hierarchical crack propagation from a single fibre break, with strength following a Weibull distribution, to its closest neighbour, hence forming a bundle of two broken fibres; the process is then repeated hierarchically, until final failure of the bundle. A key attribute of the statistical model for fibre failure is that it uses accurate full stress fields *for each* size of the broken cluster of fibres. Results show that, if a hierarchical approach (with separation of hierarchies) is used in both the FE and statistical models, there is no need to include detailed stress fields to obtain a good correlation with experiments. However, if no separation of hierarchies is imposed in the FE models, it is necessary to consider the full stress fields to achieve more realistic predictions and to capture the correct trends in size effects.

1. Introduction

Reliably predicting the tensile failure of UniDirectional (UD) Fibre-Reinforced Polymers (FRPs) presents a significant challenge for the design of large composite structures [1]. To achieve that, two key features need to be considered: the stochastic strength distribution of individual fibres, and the stress redistribution around a broken fibre/cluster.

The failure of high-performance technical fibres is governed by defects, which makes their strength stochastic and subject to statistical size effects, well explained by the Weakest Link Theory (WLT) and described by a Weibull law [2]. However, the Weibull distribution does not reflect the strength distribution of FRPs [3, 4]; this creates a need for the development and validation of full scaling models. Fibre Bundle Models (FBMs) have this potential [5, 6]; the key is to calculate the strength distribution of a

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Composite Fibre Bundle (CFB) with a given characteristic length, which can be then rescaled to any length using the WLT [7, 8]. Due to the exponential increase of the complexity of most FBMs with the number of considered fibres, much work focused on asymptotic analyses and Monte-Carlo simulations [9].

In FRPs, the local stress redistribution has been extensively investigated at the fibre scale in the literature, either by shear-lag analyses [10, 11] or Finite Element (FE) methods [12, 13]. However, there are barely any studies of local stress redistributions near clusters of broken fibres, where complex interactions between individual fibre breaks may occur.

Combining the complexity of fibre strength distributions and realistic 3D stress fields corresponding to broken fibre clusters of various size in full scaling models is very challenging. This paper presents an overview of a combined FE/statistical approach for predicting the full strength distribution of CFBs. A FE Model (FEM) was built to predict the full stress fields near a broken cluster of any size, which is then used in a statistical model. The objective is to understand the effect of considering the full 3D stress field around a broken cluster on the strength prediction of CFBs, to provide guidance for further development of reliable and computationally efficient models for the strength of UD composites.

2. Pre-fractured composite fibre bundle FEM

2.1. Geometry and mesh

CFBs are modelled hierarchically, assuming axisymmetry around the longitudinal z-axis, by considering a *coordination number* of 2, *i.e.* by pairing 2 individual fibres (of level–[0]) into a level–[1] bundle, and then sequentially grouping 2 level–$[i-1]$ bundles into a level–$[i]$ bundle, where i is the hierarchical level of the bundle [14, 6]. Fig. 1a shows the cross-section of a level–$[i+1]$ CFB used in the FEM, where $\mathrm{A}^{[i]}$ is the homogenized central level–$[i]$ bundle, and $\mathrm{B}^{[i]}$ is the homogenized level–$[i]$ neighbouring bundle, both forming a level–$[i+1]$ bundle with 2^{i+1} fibres; $\mathrm{A}^{[i]}$ is initially broken in the plane $z=0$, and a cohesive zone is introduced between $\mathrm{A}^{[i]}$ and $\mathrm{B}^{[i]}$, to model the interfacial behaviour. A Surrounding Composite (SC) is added, whose volume is calculated for each level–$[i]$ such that the overall CFB always contains a given number of fibres n^{f} ($n^{\mathrm{f}} > 2^{i+1}$).

Figs. 1b and c show the longitudinal section and the structured mesh of the FEM. One quarter of the section is modelled explicitly, and a fractured plane is introduced in $\mathrm{A}^{[i]}$ at $z=0$, exploiting symmetry and the boundary conditions represented in Fig. 1b. Standard linear full integration elements were used, and convergence studies were performed to verify that the mesh and the model's length (L) led to converged results.

The objective of the FEM is to predict the full non-linear stress field $\sigma_{\mathrm{G}}^{[i]}$ in $\mathrm{B}^{[i]}$, and the length of the process zone $l_{\mathrm{pz}}^{[i]}$, which will then be used in the statistical part of the model (Section 3) for predicting the strength distribution of a level–$[i^{\max}]$ bundle (where $i^{\max} = log_2(n^{\mathrm{f}})$).

2.2. Mechanical behaviour and properties

The FRP bundles $\mathrm{A}^{[i]}$, $\mathrm{B}^{[i]}$ and SC were assumed linear-elastic, with transversly isotropic constitutive behaviour; a bilinear traction-separation law was defined for the cohesive zone (see Fig. 1), with extremely high values of fracture toughness in mode I and II in order to mimic a perfectly-plastic interface. The mechanical properties of the FRP and cohesive elements are summarised in Table 1.

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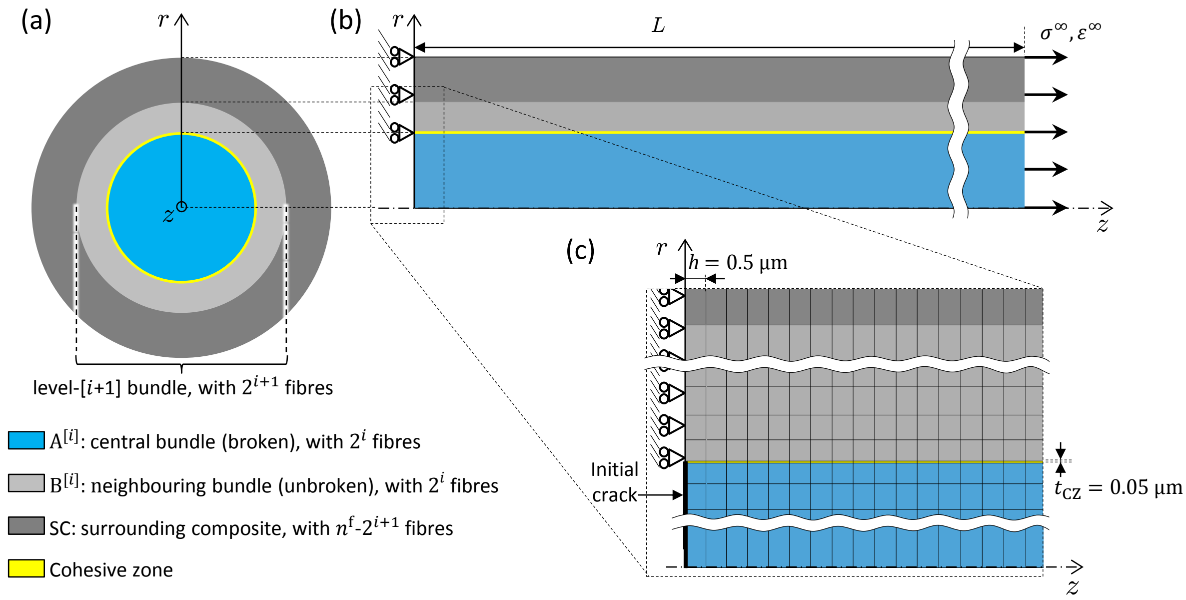

Figure 1: Axisymmetric CFB geometry considered in the FEM: (a) cross-section, (b) longitudinal section and (c) mesh at the crack location.

Table 1: Mechanical properties of T800H/3631. Longitudinal and transverse directions are indicated respectively by "l" and "t" in subscript. E and G are the tensile and shear elastic moduli respectively, ν is the Poisson's ratio, and T_{I} and T_{II} are the cohesive tractions in mode I and II respectively respectively.

AS4/3501-6 fibre bundles					Perfectly plastic interface	
E_{l} (GPa)	E_{t} (GPa)	G_{lt} (GPa)	ν_{lt} (-)	V^{f} (%)	T_{I} (MPa)	T_{II} (MPa)
177.$^{(\star)}$	9.57 [15]	4.5 [15]	0.36 [15]	60 [15]	60. [15]	79.6$^{(\dagger)}$

$^{(\star)}$ Obtained with rule of mixture from [16]
$^{(\dagger)}$ Assumed equal to the matrix shear-lag strength used in the statistical model, see Table 2

2.3. Loading and computation

A uniform displacement was applied to the bundle along the fibre direction at $z = L$. Each FE model was solved in ABAQUS 6.13 using its implicit and linear geometric formulation. The full stress field in B$^{[i]}$ and the length of the cohesive process zone $l^{[i]}_{\mathrm{pz}}$ were exported for each load step of the calculation, to be used as inputs in the statistical model. This process was repeated for all level–[i] and n^{f}, with $1 \le 2^i \le n^{\mathrm{f}}/2$ and $2 \le n^{\mathrm{f}} \le 1024$.

3. Overview of the statistical model

3.1. Hierarchical scaling law for the strength of CFBs

As discussed in the previous section, a level–[$i+1$] bundle is made of two concentric level–[i] sub-bundles in contact; when sub-bundle A$^{[i]}$ breaks, sub-bundle B$^{[i]}$ experiences stress concentrations, accompanied by matrix (or matrix/fibre interface) damage along a process zone with length $l^{[i]}_{\mathrm{pz}}$. The failure of a level–[$i+1$] bundle is assumed to occur when the two level–[i] bundles break within the same *con-*

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trol region – defined in a length $l_{\mathrm{c}}^{[i+1]} = 2 \cdot l_{\mathrm{pz}}^{[i]}$ – in which case the two process zones induced by the level–[i] breaks overlap [6]. Consequently, a long level–[$i+1$] bundle can be decomposed into a series of statistically independent segments of length $l_{\mathrm{c}}^{[i+1]}$; following the Weakest Link Theory (WLT), the tensile failure of the bundle under uniform stress occurs when the weakest element fails. The survival probability $\mathcal{S}_{\mathtt{U,c}}^{[i+1]}$ of a level–[$i+1$] bundle under uniform stress σ^{∞} (subscript U) within the control length $l_{\mathrm{c}}^{[i+1]}$ (subscript c) can then be recursively calculated from that of level–[i] as [6]

$$\mathcal{S}_{\mathtt{U,c}}^{[i+1]}(\sigma^{\infty}) = \mathcal{S}_{\mathtt{U,pz}}^{[i]}(\sigma^{\infty})^{4} + 2\cdot\left[1 - \mathcal{S}_{\mathtt{U,pz}}^{[i]}(\sigma^{\infty})^{2}\right] \cdot \mathcal{S}_{\mathtt{U,pz}}^{[i]}(\sigma^{\infty}) \cdot \mathcal{S}_{\mathtt{G,pz}}^{[i]}(\sigma^{\infty}), \tag{1}$$

where:

- $\mathcal{S}_{\mathtt{U,pz}}^{[i]}$ is the survival probability under uniform stresses σ^{∞} of a level–[i] bundle within the length $l_{\mathrm{pz}}^{[i]}$ (subscript pz). $\mathcal{S}_{\mathtt{U,pz}}^{[i]}$ can be scaled from $\mathcal{S}_{\mathtt{U,c}}^{[i]}$ using the WLT. For $i = 0$, the strength distribution of a bundle containing one single fibre follows a Weibull distribution with parameters given in Table 2;
- $\mathcal{S}_{\mathtt{G,pz}}^{[i]}$ is the survival probability of a level–[i] bundle in $l_{\mathrm{pz}}^{[i]}$ under the non-linear stress field $\sigma_{\mathtt{G}}^{[i]}$ (subscript G).

The survival probability $\mathcal{S}_{\mathtt{G,pz}}^{[i]}$ was calculated by describing $\mathrm{B}^{[i]}$ as a chain of elements of infinitesimal length dz along the z–axis (see Fig. 1c), where each of them experiences a different loading. Applying the WLT on this chain of elements gives the survival probability of $\mathrm{B}^{[i]}$ under stress concentrations. If we were to ignore stress gradient in the radial direction, the homogenized stress field $\hat{\sigma}_{\mathtt{G}}^{[i]}$ would be considered to calculate $\mathcal{S}_{\mathtt{G,pz}}^{[i]}$ rather than the full field $\sigma_{\mathtt{G}}^{[i]}$. In this case, $\mathcal{S}_{\mathtt{G,pz}}^{[i]}$ can be expressed as [6, 17]

$$\ln\left[\mathcal{S}_{\mathtt{G,pz}}^{[i]}(\sigma^{\infty})\right] = \frac{2}{l_{\mathrm{r}}}\int_{0}^{l_{\mathrm{pz}}^{[i]}(\sigma^{\infty})/2} \ln\left[\mathcal{S}_{\mathtt{U,r}}^{[i]}\left(\hat{\sigma}_{\mathtt{G}}^{[i]}(z,\sigma^{\infty})\right)\right] \mathrm{d}z, \tag{2}$$

where $\mathcal{S}_{\mathtt{U,r}}^{[i]}$ is the survival probability under uniform stresses σ^{∞} of a level–[i] bundle within an arbitrary reference length $l_{\mathrm{r}}^{[i]}$ (subscript r). This accounts for the stress gradient in the neighbourhood of a broken cluster in the longitudinal direction but not in the radial one. However, considering the stress gradient in the radial direction, the full field $\sigma_{\mathtt{G}}^{[i]}$ (which includes the longitudinal stress gradient too) is considered to calculate $\mathcal{S}_{\mathtt{G,pz}}^{[i]}$. In this case,

$$\ln\left[\mathcal{S}_{\mathtt{G,pz}}^{[i]}(\sigma^{\infty})\right] = \frac{2}{l_{\mathrm{r}}}\int_{0}^{l_{\mathrm{pz}}^{[i]}(\sigma^{\infty})/2} \ln\left[\mathcal{S}_{\mathtt{R,r}}^{[i]}\left(\sigma_{\mathtt{G}}^{[i]}(r,z,\sigma^{\infty})\right)\right] \mathrm{d}z, \tag{3}$$

where $\mathcal{S}_{\mathtt{R,r}}^{[i]}\left(\sigma_{\mathtt{G}}^{[i]}(r,z,\sigma^{\infty})\right)$ is the survival probability of a level–[i] bundle with a reference length l_{r} under the non-uniform loading $\sigma_{\mathtt{G}}^{[i]}(r,z,\sigma^{\infty})$ (subscript R). The calculation of $\mathcal{S}_{\mathtt{R,r}}^{[i]}\left(\sigma_{\mathtt{G}}^{[i]}(r,z,\sigma^{\infty})\right)$ required significant developments, summarised in next section and fully presented in the related paper [18] (in preparation).

3.2. Strategy for calculating the survival probability of a level–[i] bundle under non-uniform loading

Consider an infinitesimally short slice of $\mathrm{B}^{[i]}$, with length dz and at a distance z from the plane of the broken cluster in $\mathrm{A}^{[i]}$; this slice experiences the non-uniform stress field $\sigma_{\mathtt{G}}^{[i]}(r,z,\sigma^{\infty})$. Using the WLT, this slice can be rescaled to the chosen reference length l_{r} in Eq. 3, as shown in Fig 2.

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Table 2: Properties of T800H fibres and 3631 matrix used to calculate the cumulative strength distribution of CFBs, where m and $\sigma_{\mathrm{r}}^{\mathrm{f}}$ are respectively the shape and the scale parameters of the Weibull distribution of a fibre of length l_{r}, from which are calculated the average strength of fibres $X_{\mathrm{m,r}}^{\mathrm{f}}$ and the coefficient of variation of the strength distribution $\mathrm{CoV}_X^{\mathrm{f}}$.

T800H fibres							**3631 matrix**
Young's modulus	Strength distribution					Diameter	Shear lag strength
E^{f} (GPa)	l_{r} (mm)	$\sigma_{\mathrm{r}}^{\mathrm{f}}$ (GPa)	m (–)	$X_{\mathrm{m,r}}^{\mathrm{f}}$ (GPa)	$\mathrm{CoV}_X^{\mathrm{f}}$ (%)	ϕ^{f} (μm)	τ_{SL} (MPa)
294 [16]	10.	5.46(⋆)	3.57 [16]	4.93	29.4	5 [16]	79.6(†)

(⋆) calculated from m and $\sigma_{\mathrm{r}}^{\mathrm{f}} = 3.57$ GPa, both measured for a length of 50 mm [16]

(†) Okabe and Takeda [16] provided a value of 52.4 MPa, which is low compared to the value of 100 MPa found in [15] (from Nishikawa, Okabe and Takeda) and [19] (independently measured; a compromise with an intermediate value was used in this work

The resulting level–$[i]$ bundle is then recursively sub-divided into level–$[i,j]$ (with $j<i$) sub-bundles $\mathrm{A}^{[i,j]}$ and $\mathrm{B}^{[i,j]}$, where the radial stress gradient in each sub-bundle $\mathrm{A}^{[i,j]}$ (closer than $\mathrm{B}^{[i,j]}$ to the level–$[i]$ broken cluster) is considered, but for $\mathrm{B}^{[i,j]}$ it is homogenised, as shown in Fig. 2. The non-uniform loading $\sigma_{\mathrm{G}}^{[i]}(r,z,\sigma^{\infty})$ is therefore sub-divided into sub-loadings $\sigma_{\mathrm{A}}^{[i,j]}(r,z,\sigma^{\infty})$ and $\hat{\sigma}_{\mathrm{B}}^{[i,j]}(z,\sigma^{\infty})$ (see Fig. 2). Similarly to Section 3.1, a control length is defined for each sub-bundle $\mathrm{A}^{[i,j+1]}$ by $l_{\mathrm{c}}^{[i,j+1]} = l_{\mathrm{a}}^{[i,j]} + l_{\mathrm{b}}^{[i,j]}$, where $l_{\mathrm{a}}^{[i,j]}$ and $l_{\mathrm{b}}^{[i,j]}$ are respectively the length of the process zones induced by the failure of sub-bundle $\mathrm{A}^{[i]}$ (subscript a) and sub-bundle $\mathrm{B}^{[i]}$ (subscript b) respectively.

It is considered that each sub-bundle $\mathrm{A}^{[i,j+1]}$ survives within the control length $l_{\mathrm{c}}^{[i,j+1]}$ if (i) both sub-bundles $\mathrm{A}^{[i,j]}$ and $\mathrm{B}^{[i,j]}$ survive, (ii) $\mathrm{A}^{[i,j]}$ fails and $\mathrm{B}^{[i,j]}$ survives the resulting stress field, or (iii) $\mathrm{B}^{[i,j]}$ fails and $\mathrm{A}^{[i,j]}$ survives the resulting stress field. For the sake of simplicity, it is assumed that the stress concentrations induced by the breaks of $\mathrm{A}^{[i,j]}$ or $\mathrm{B}^{[i,j]}$ are linear and calculated using shear-lag theory. Extending Eq. 1 to this case, the survival probability of the $\mathrm{B}^{[i]}$ bundle $\mathcal{S}_{\mathrm{R,c}}^{[i]}(\sigma_{\mathrm{G}}^{[i]})$ within the control length $l_{\mathrm{c}}^{[i,j+1]}$ (with $\mathcal{S}_{\mathrm{R,c}}^{[i]}(\sigma_{\mathrm{G}}^{[i]}) = \mathcal{S}_{\mathrm{R,c}}^{[j+1=i]}(\sigma_{\mathrm{A}}^{[i,j+1=i]})$) is calculated recursively from that of sub-bundles $\mathrm{A}^{[i,j]}$ and $\mathrm{B}^{[i,j]}$ with (omitting r, z, and σ^{∞} for readability):

$$
\begin{aligned}
\mathcal{S}_{\mathrm{R,c}}^{[j+1]}(\sigma_{\mathrm{A}}^{[i,j+1]}) = \\
&\mathcal{S}_{\mathrm{R,a}}^{[j]}(\sigma_{\mathrm{A}}^{[i,j]})\cdot\mathcal{S}_{\mathrm{R,b}}^{[j]}(\sigma_{\mathrm{A}}^{[i,j]})\cdot\mathcal{S}_{\mathrm{U,a}}^{[j]}(\hat{\sigma}_{\mathrm{B}}^{[i,j]})\cdot\mathcal{S}_{\mathrm{U,b}}^{[j]}(\hat{\sigma}_{\mathrm{B}}^{[i,j]}) && (i)\\
&+\left[1-\mathcal{S}_{\mathrm{R,a}}^{[j]}(\sigma_{\mathrm{A}}^{[i,j]})\cdot\mathcal{S}_{\mathrm{R,b}}^{[j]}(\sigma_{\mathrm{A}}^{[i,j]})\right]\cdot\mathcal{S}_{\mathrm{U,b}}^{[j]}(\hat{\sigma}_{\mathrm{B}}^{[i,j]})\cdot\mathcal{S}_{\mathrm{K,a}}^{[j]}(\hat{\sigma}_{\mathrm{B}}^{[i,j]}) && (ii)\\
&+\left[1-\mathcal{S}_{\mathrm{U,a}}^{[j]}(\hat{\sigma}_{\mathrm{B}}^{[i,j]})\cdot\mathcal{S}_{\mathrm{U,b}}^{[j]}(\hat{\sigma}_{\mathrm{B}}^{[i,j]})\right]\cdot\mathcal{S}_{\mathrm{R,a}}^{[j]}(\sigma_{\mathrm{A}}^{[i,j]})\cdot\mathcal{S}_{\mathrm{R,K,b}}^{[j]}(\sigma_{\mathrm{A}}^{[i,j]}) && (iii)\ ,
\end{aligned}
\tag{4}
$$

where:

- $\mathcal{S}_{\mathrm{U,a}}^{[j]}$ and $\mathcal{S}_{\mathrm{U,b}}^{[j]}$ are the survival probabilities under uniform stress of the sub-bundle $\mathrm{B}^{[i,j]}$ within the lengths $l_{\mathrm{a}}^{[i,j+1]}$ and $l_{\mathrm{b}}^{[i,j]}$ respectively, calculated with Eq. 1;
- $\mathcal{S}_{\mathrm{K,a}}^{[j]}$ is the survival probability of sub-bundle $\mathrm{B}^{[i,j]}$ within the length $l_{\mathrm{a}}^{[i,j]}$ and under linear stress concentrations (subscript K) due to $\mathrm{A}^{[i,j]}$ failing, calculated from $\mathcal{S}_{\mathrm{U,r}}^{[j]}$ using the relation established by Pimenta and Pinho [6];
- $\mathcal{S}_{\mathrm{R,K,b}}^{[j]}$ is the survival probability of sub-bundle $\mathrm{A}^{[i,j]}$ within the length $l_{\mathrm{b}}^{[i,j]}$ and under linear stress

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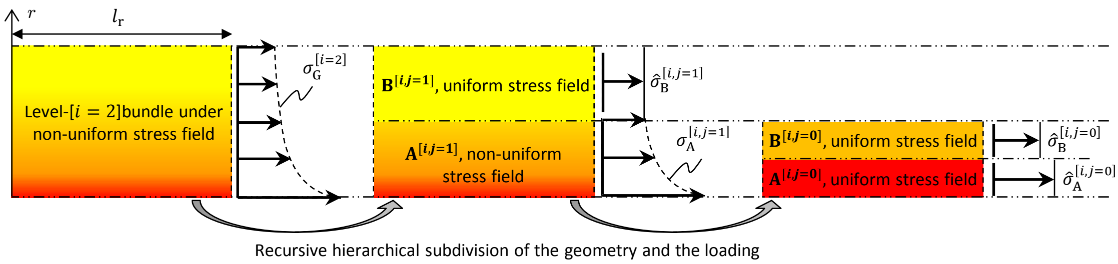

Figure 2: Hierarchical subdivision of a level–[$i = 2$] bundle $\mathrm{B}^{[i]}$ into sub-bundles, and corresponding subdivision of the non-uniform stress field $\sigma_{\mathrm{G}}^{[i=2]}$.

concentrations due to $\mathrm{B}^{[i,j]}$ failing, considering the radial stress gradient due to bundle $\mathrm{B}^{[i]}$ failing. Its expression is derived from the one used to calculate $\mathcal{S}_{\mathrm{K,a}}^{[j]}$.

The process described above is repeated recursively down to level–[$j = 0$]. As at this level sub-bundles $\mathrm{A}^{[i,j=0]}$ and $\mathrm{B}^{[i,j=0]}$ contain only one single fibre, it is considered that $\mathcal{S}_{\mathrm{R,r}}^{[j=0]}(\sigma_{\mathrm{A}}^{[i,j=0]})$ is equal to $\mathcal{S}_{\mathrm{U,r}}^{[j=0]}(\hat{\sigma}_{\mathrm{A}}^{[i,j=0]})$, where $\hat{\sigma}_{\mathrm{A}}^{[i,j=0]}$ corresponds to $\sigma_{\mathrm{A}}^{[i,j=0]}$ homogenized in the radial direction, and $\mathcal{S}_{\mathrm{U,r}}^{[j=0]}$ follows a Weibull distribution with parameters given in Table 2.

The implementation of the statistical model requires the right tail asymptotic behaviour (RTA, $\sigma^{\infty} \to \infty$) of the two hierarchical scaling laws given by Eq. 1 and 4 to be defined. This was done by assuming that the right tail of the strength distribution of level–[i] bundles and level–[j] sub-bundles under non-uniform stress tends asymptotically to a Weibull distribution, as demonstrated by Pimenta and Pinho [6] in the case of bundles under uniform stress. This assumption was verified for the cases studied in this work.

4. Key results

Fig. 3 shows the distribution of mean strength $X_{\mathrm{C}}^{[i]}$ in level–[$0 \leq i \leq 10$] CFBs obtained with the present approach; results obtained if we were not considering either the surrounding composite (see Fig 1) or the radial stress gradient (Eq. 2) are also indicated. These are compared with predictions from Pimenta and Pinho's model [6] (no radial stress gradient and assuming a linear stress field with maximum SCF equal to 2), and experimental measurements from Okabe and Takeda [16].

Not considering the surrounding composite leads to an homogenized SCF of 2 in the closest neighbouring level–[i] bundle of a broken one. Pimenta and Pinho [6] showed that, with such large SCFs, the strength distributions of level–[$i \geq 4$] bundles are bounded by the WLT, thus failure of one bundle leads to the immediate failure of its neighbour. In this case, taking into account the more realistic full stress fields obtained with FE does not have any meaningful impact on the mean strength predictions, as shown by the similarity between the results obtained with this model and Pimenta and Pinho's in Fig. 3.

When the surrounding composite is considered, the predicted homogenised SCFs are lower than 2; this leads (i) to an overestimation of the strength by the present model, as shown in Fig. 3 by comparing the results against the experimental data, and (ii) to predicted strength distributions no longer being bounded by the WLT. In this case, taking into account the radial stress gradient is necessary as shown in Fig. 3: if this would not be done, the predicted strengths would be unreasonably high; moreover, the model would predict the opposite size effetc (*i.e.* increasing strength for larger bundles) to that widely observed experimentally (*i.e.* decreasing strength for larger bundles).

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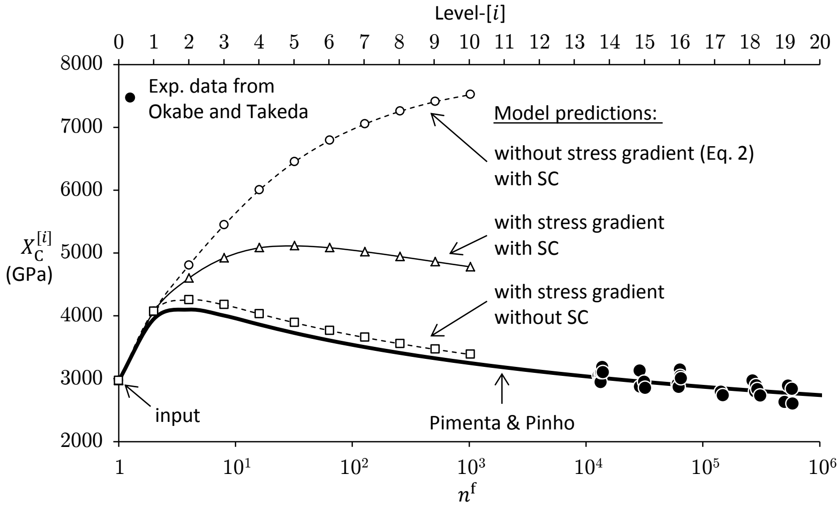

Figure 3: Mean strength of level–[$0 \leq i \leq 10$] CFBs predicted with the present combined FE/statistical model vs. Pimenta and Pinho's analytical model [6] and experimental measurements from [16].

5. Conclusion

The model presented in this work combines accurate values of SCF obtained via FE with the statistical hierarchical approach proposed by Pimenta and Pinho [6]. Results show that, if a hierarchical approach (with separation of hierarchies) is used in both the FE and statistical models, there is no need to include detailed stress fields to obtain a good correlation with experiments. However, if no separation of hierarchies is imposed in the FE models, it is necessary to consider the full stress fields to achieve more realistic predictions and to capture the correct trends in size effects. However, even considering the full stress gradients predicted by FE, the present model still predicts higher bundle strengths than those observed experimentally; further work is focusing on extending the present approach to higher coordination numbers, to make the statistical and FE models more compatible.

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