AUTOMATED DESIGN APPROACH AND POTENTIAL ASSESSMENT OF COMPOSITE STRUCTURES: FAST ANALYTICAL ENGINEERING TOOL FOR MULTIPLE LOAD CASES

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Abstract
Several approaches have been proposed to find suitable material orientations for fiber-reinforced components. Most of them face this task with numerical optimization. Thereby, computational time increases by number of load cases, size of the part and complexity of the simulation model. An alternative analytical evaluation of preferable fiber orientations is presented in this paper. The algorithm computes best fit material orientations based on principal stresses from more than one or two load cases. Thus, component potential for fiber-reinforced composites can be evaluated. Especially large assemblies with multiple load cases can be analyzed fast.

1. Introduction

Engineers nowadays face the challenge to design cost-efficient, high performance lightweight structures. Therefore, anisotropic materials like fiber-reinforced composites (FRC) gain importance. Fig. 1 presents

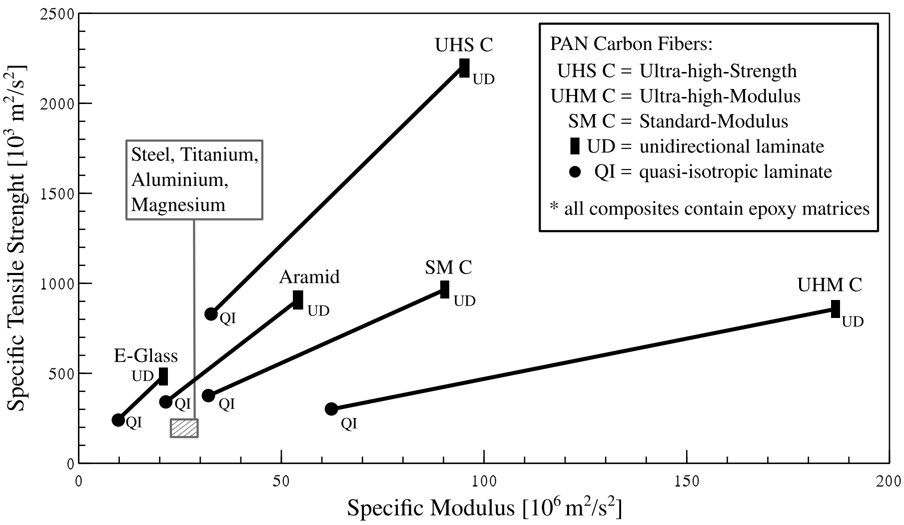

Figure 1. Specific material properties for standard FRCs and isotropic metals cf. [1]

material properties for FRCs and isotropic materials like steel or aluminum. The y-axis indicates density specific strength subjected to density specific elastic modulus on the x-axis. FRCs outperform isotropic materials in both categories. However, there is a remarkable loss of performance depending on the cho-

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sen FRC material orientation. Mechanical properties of unidirectional (UD) laminates are superior to quasi-isotropics (QI) if loading and fiber directions are aligned, regardless of the used fiber material. In 2012 Roland Berger Strategy Consultants and VDMA published an outlook on serial production of FRC components ([2]). They identify costs for an automotive part as function of material. Accordingly FRC designs are 600% more expensive than steel variants. Higher costs for FRC components can only be justified with advantages in mass specific performance. Consequently, to trade off costs, performance and mass, fibers aligned to resulting load paths (UD) should dominate cost-efficient part designs. Hence, the crucial question results: which fiber orientation is preferable regarding multiple load cases occurring over the life cycle. This preferable orientation finally leads to an assessment whether higher costs for a FRC variant could be beneficial.
A common way to find appropriate orientations is numerical optimization. Schumacher ([3]) introduces different approaches to solve optimization problems. For convex problems mathematical optimization algorithms will derive a solution. These gradient-based methods reach their limits for complex tasks. On the one hand they tend to find local optima on the other, computing time increases. Consequently, approximation methods like lamination parameters [4] were developed reducing complexity to the disadvantage of accuracy. Furthermore, suitable objective functions, such as maximum strain energy, should be defined. Stochastic search methods are mentioned as alternative approaches. They provide possibilities to find global optima for complex objective functions. Evolutionary and genetic algorithms are established in this context. An attempt to find optimized fiber paths through genetic algorithm is proposed by Legrand ([5]). Although results are good, these methods show a tendency to increase computing time. In early stages of the development topology optimization can be useful. A distribution of material within a given design domain for all applied loads and boundary conditions is determined hereby. This, in contrast to mathematical optimization algorithms and stochastic search methods, changes part design and not local fiber orientations. Combinations of topology and mathematical optimization for fiber orientations give promising results ([4], [6]). Ghiasi et al. conducted a review of different publications for optimized fiber orientations. They separate between methods for constant and variable stiffness design ([7], [8]) and categorize related optimization approaches. Whereby, constant stiffness design considers the optimization of laminate stacking sequences and variable stiffness design defines changing fiber orientations or material distributions within one structural element. Computation time of all these methodologies increases by number of load cases, size of the part and complexity of the simulation model.
In the following sections an alternative analytical evaluation for preferable fiber orientations will be presented. For most aerospace and automotive applications multiple load cases must be taken into account. The proposed evaluation of such parts refers to typical development processes of composite design engineers without optimizations. At first the engineer would construct a substitute model with isotropic material. Material tests or finite element (FE) simulations identify stress distributions and principal stress orientations for each load case. Depending on homogeneity of stress distributions and orientations the engineer makes a decision whether this part is applicable for FRCs including the preferable laminate stacking sequence. According to this development process Durst ([9]) introduces an evaluation which is based on a fast analytical post-processing of principle stresses. Durst defines three evaluation criteria (all values range from 0 to 1):

- Principal Stress Criterion: Ratio between first and second principal stress
- Orientation Criterion: Deviation of principal stress orientation
- Weighting Criterion: Load percentage (related to the sum of all loads)

These criteria are calculated within one FE and detect UD stresses. For each load case all criteria are calculated for every FE separately. Afterwards, they are multiplied together whereby a coefficient (ranging from 0 to 1) for each element and load case is determined. Finally the sum of all element/load case coefficients rates the FE's stress status. Values close to 1 suggest a homogeneous stress orientation and hence potential for UD laminate designs. The presented research follows Durst's methodology. It supplements an evaluation of the potential for biaxial (BIAX) fiber designs and improvements of the existing evaluation criteria.

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2. Basic idea of the approach

The proposed algorithm evaluates the stress status of potential FRC components. Therefore, we compute best-fit material orientation angles (Section 3.1). Deviation of principal stresses from the best-fit angle lead to an assessment of the element's stress status. In Fig. 2 two preferable stacking sequences, UD

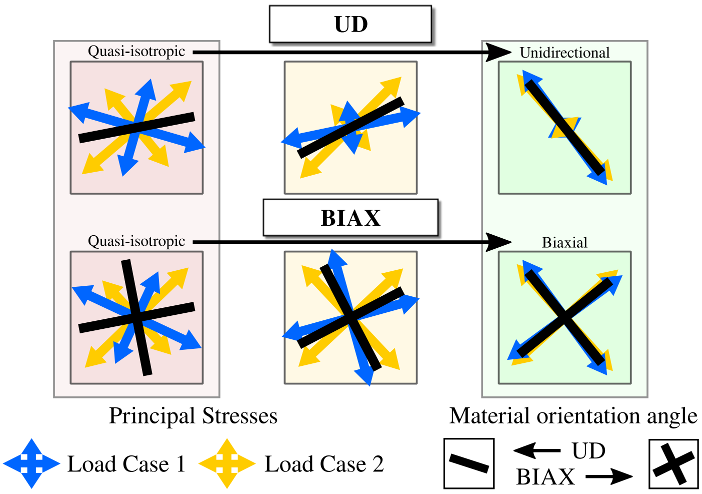

Figure 2. Best fit approach for UD and BIAX stacking sequences

and BIAX, are illustrated for one FE. The introduced BIAX evaluation provides better performance of orthogonal stacking sequences compared to quasi-isotropics. If shear or perpendicular stresses dominate the stress status, a BIAX stacking sequence follows. One dominating principal stress direction over all load cases lead to UD stacking sequences. If neither tends to fit, the stress status can be described as QI.

3. Analytical methods for the evaluation

At the very beginning a FE simulation of the examined part with isotropic material properties has to be performed. A subroutine reads out generated solution files and sorts the computed data (principal stresses and stress orientations, von Mises stresses). Below, we address basic calculation methods of the algorithm.

3.1. Calculation of the orientation angle

The material orientation angle is fundamental for the method. The key question: "What is the best fit angle for more than one load case, or rather for more than two load cases ?", arises. Among others, projections of principal stresses to an orientation angle prove to be most promising (Fig. 3). For the best fit angle the maximum projection should arise. This hypothesis was validated iteratively with discrete rotating orientation angles, but became time consuming for larger models. Hence, we solve the computation analytically. The projection P of all principal stresses to the orientation angle $\varphi_{\varnothing}$ is described in (1). With m as total number of load cases, L_i for the maximum principal stress and $\alpha_{max,i}$ for the maximum absolute principal

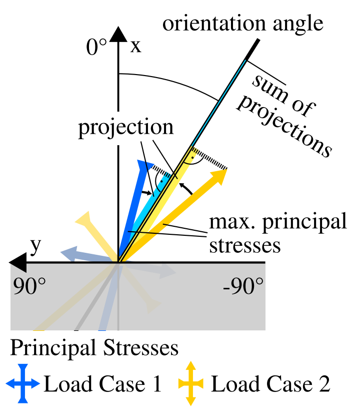

Figure 3. Projection approach

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stress orientation of load case i. Equating the derivative P' to zero yields the maximum projection.

$$P\left(\varphi_\varnothing\right) = \sum_{i=1}^{m} \left|L_i \cos\left(\alpha_{max,i} - \varphi_\varnothing\right)\right| \tag{1}$$

$$\frac{dP}{d\varphi_\varnothing} = P'\left(\varphi_\varnothing\right) = \sum_{i=1}^{m} L_i \left|\sin\left(\alpha_{max,i} - \varphi_\varnothing\right)\right| = 0 \tag{2}$$

where

$$\left|\sin\left(\alpha_{max,i} - \varphi_\varnothing\right)\right| = \sin\left(\alpha_{max,i} - \varphi_\varnothing\right) \frac{\cos\left(\alpha_{max,i} - \varphi_\varnothing\right)}{\left|\cos\left(\alpha_{max,i} - \varphi_\varnothing\right)\right|} \tag{3}$$

$$\frac{\cos\left(\alpha_{max,i} - \varphi_\varnothing\right)}{\left|\cos\left(\alpha_{max,i} - \varphi_\varnothing\right)\right|} = \epsilon_i\left(\varphi_\varnothing\right) = \pm 1 \tag{4}$$

The change of sign of $\epsilon_i\left(\varphi_\varnothing\right)$ is related as:

$$\epsilon_i\left(\varphi_\varnothing\right) = \begin{cases} +1, & 0 < \alpha_{max,i} - \varphi_\varnothing < \frac{\pi}{2} \\ -1, & \frac{\pi}{2} < \alpha_{max,i} - \varphi_\varnothing < \frac{3}{2}\pi \\ +1, & \frac{3}{2}\pi < \alpha_{max,i} - \varphi_\varnothing < 2\pi \end{cases} = \begin{cases} +1, & \alpha_{max,i} - \frac{\pi}{2} < \varphi_\varnothing < \alpha_{max,i} \\ -1, & \alpha_{max,i} - \frac{3}{2}\pi < \varphi_\varnothing < \alpha_{max,i} - \frac{\pi}{2} \\ +1, & \alpha_{max,i} < \varphi_\varnothing < \alpha_{max,i} - \frac{3}{2}\pi \end{cases} \tag{5}$$

Thus P' can be formulated with:

$$P'\left(\varphi_\varnothing\right) = \sum_{i=1}^{m} L_i \epsilon_i\left(\varphi_\varnothing\right) \sin\left(\alpha_{max,i} - \varphi_\varnothing\right) = 0 \tag{6}$$

Linear combination for more than two sinusoids can be written as:

$$\textstyle\sum_i a_i \sin\left(x + \delta_i\right) = a \sin\left(x + \delta\right) \tag{7}$$

$$\text{with} \qquad a^2 = \textstyle\sum_{i,j} a_i a_j \cos\left(\delta_i - \delta_j\right) \tag{8}$$

$$\text{and} \qquad \tan\left(\delta\right) = \frac{\sum_i a_i \sin(\delta_i)}{\sum_i a_i \cos(\delta_i)} \tag{9}$$

Applying equation (7) to equation (6) results in:

$$P'\left(\varphi_\varnothing\right) = \sum_{i=1}^{m} L_i \epsilon_i\left(\varphi_\varnothing\right) \sin\left(\alpha_{max,i} - \varphi_\varnothing\right) = a \sin\left(\alpha - \varphi_\varnothing\right) = 0 \tag{10}$$

(10) is true for $\alpha - \varphi_\varnothing = n\pi$ $(n = 0, 1, 2 \ldots)$. Angles in FE coordinate systems can be described sufficiently by values from $-90°$ to $90°$. Consequently rotating by $n\pi$ has no effect.

$$\varphi_\varnothing = \alpha - n\pi \rightarrow \varphi_\varnothing = \alpha \tag{11}$$

with (9)

$$\varphi_\varnothing = \alpha = \arctan\left(\frac{\sum_i L_i \epsilon_i\left(\varphi_\varnothing\right) \sin\left(\alpha_{max,i}\right)}{\sum_i L_i \epsilon_i\left(\varphi_\varnothing\right) \cos\left(\alpha_{max,i}\right)}\right) \tag{12}$$

$\epsilon_i\left(\varphi_\varnothing\right)$ $(= \pm 1)$ is still unknown. Thus, we compute $\epsilon_i\left(\varphi_\varnothing\right)$ in respect of given limits in (5). More precisely, preferable $\varphi_\varnothing$ are calculated for ranges between all sign changes of $\epsilon_i\left(\varphi_\varnothing\right)$. Graphs of equations (1) or (2) are composed of piecewise functions whereby calculated $\varphi_\varnothing$ represents the maximum projection of each function. A comparison of each resulting projection leads to the most preferable angle $\varphi_\varnothing$.

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3.1.1. Adjustment for BIAX orientation angles

The proposed computation for preferable orientation angles can not be applied without adjustments for BIAX orientation angles. Hence, we define two rectangular sectors from 0° to 90° (green) and −45° to 45° (blue) (Fig. 4). They intersect within 0° and 45°. By calculating orientation angles in both sectors, major and orthogonal minor principal stress of one load case are considered separately for one of these angles. Furthermore, the maximum absolute principal stress is used for both principal stress directions, as the regarded BIAX material consists of equal warp and weft yarns. In order to avoid confusion, it should be mentioned that orientation angle 1 and 2 don't have to be perpendicular. At last, the angle with maximum projection of stresses and it's orthogonal equivalent (dashed lines in Fig. 4) are preferable material orientations for a BIAX fiber design. Presented load cases in Fig. 4, as an example, would yield to a green orientation angle (sector 1).

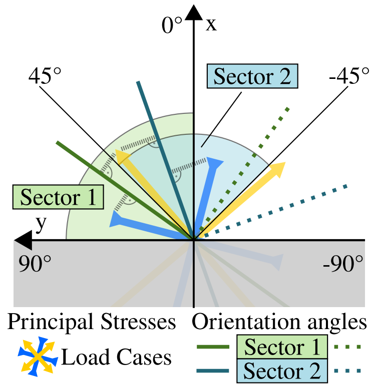

Figure 4. BIAX orientation angles

3.2. Stacking sequence identification

Suggested stacking sequences should be motivated by optimized mechanical performance, especially stiffness. A common way to describe stiffness properties of anisotropic materials are polar diagrams. They show stiffness as a function of loading angles. Polar diagrams for UD, BIAX and QI stacking

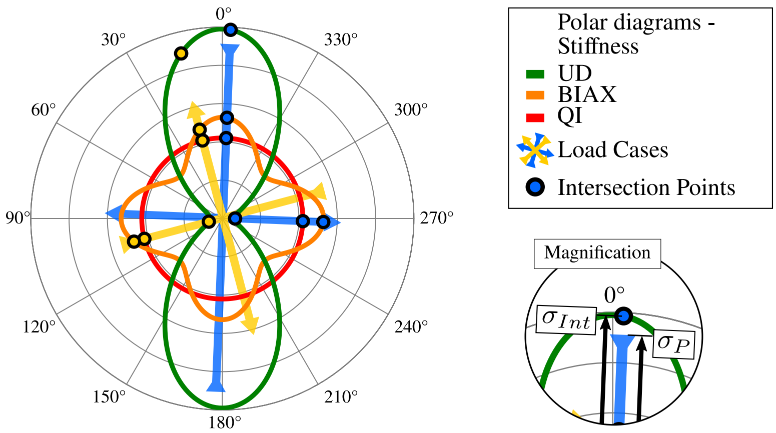

Figure 5. Polar diagrams for stiffness - stacking sequence identification

sequences are shown in Fig. 5. Presented figure is representative for one FE of examined part. We suppose that UD and BIAX orientation angles are equal in this case (0°). All three laminates have the same thickness and material. For this reason stiffness differs between UD, BIAX and QI based on material orientation. Principal stresses are added to evaluate which stiffness distribution fits best for all load cases applied on the structure. For each polar diagram, intersection points between polar curve and absolute major/minor principal stress of each load case are calculated. Subsequently, we compute reserve factors R_i as quotients between principal stress P_i and stress at intersection point Int_i for every stress vector i:

$$R_i = \frac{\sigma_{P_i}}{\sigma_{Int_i}} \tag{13}$$

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The higher R_i the more critical the stress. For every polar diagram most critical R_i factors remain stored. Minimal most critical R_i among UD, BIAX and QI yields to best fitted stacking sequence S.

$$S = \min\Big(\max\left(R_{i\ UD}\right), \max\left(R_{i\ BIAX}\right), \max\left(R_{i\ QI}\right)\Big) \tag{14}$$

In Fig. 5 the first load case (blue) with 270° orientation is most critical for UD ($\max(R_{i\ UD})$). For BIAX and QI second load case (yellow) in about 15° orientation follows for $\max(R_{i\ BIAX})$ and $\max(R_{i\ QI})$. $\max(R_{i\ BIAX})$ turns out to be the lowest reserve factor. Consequently, a BIAX stacking sequence fits best for all applied load cases.

4. Validation

For validation and traceability a simple FE example with three load cases is investigated (Fig. 6). Neither tension force nor direction is equal. Thus, resulting principal stress orientations differ in most FEs. Fig. 7 shows resulting stacking sequence suggestions. Different tension forces yield to asymmetrical solutions. Most elements have a BIAX stacking sequence except elements around the hole. Stress orientations are similar here for each load case because of geometrical circumstances. Four elements are chosen for a closer look. Element 1, at the edge of the hole, is classified undoubtedly as UD. All principal stresses have almost the same orientation. For Element 2 two perpendicular stress orientations dominate. Therefore, a BIAX stacking sequence is suggested. Element 3 and 4 are neighbors with different results. The principal stress distribution is almost the same for both elements. However, second principal stress of load case *left* (yellow), which is perpendicular to

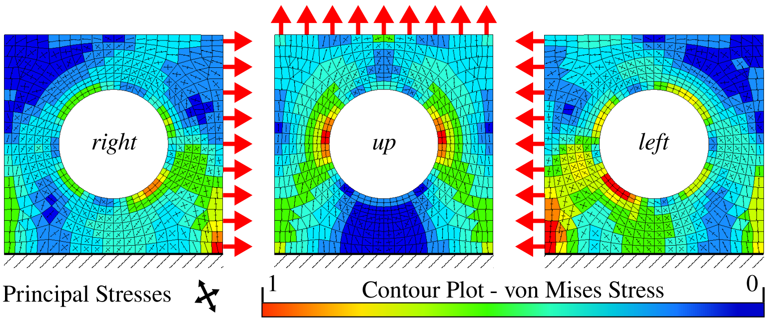

Figure 6. FE example for validation - load cases

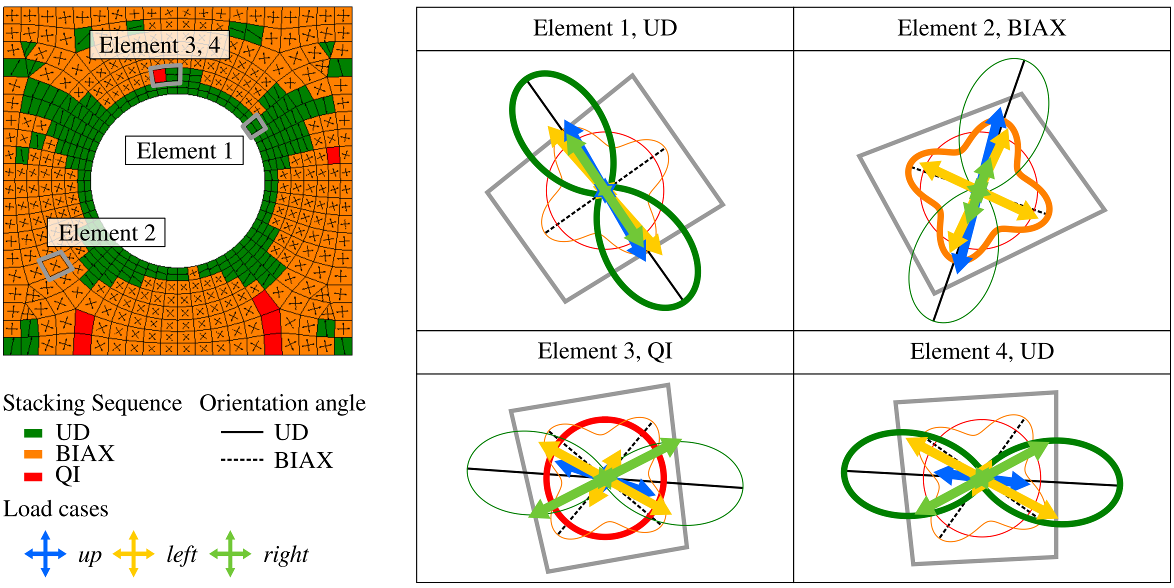

Figure 7. FE example for validation - results

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the UD material orientation, is higher in Element 3 than in 4. On that account, UD is not the best fit for this element. QI and BIAX are nearly the same but QI fits slightly better for load case *right* (green).

5. Results

In the next step an automobile with 1.092.682 Elements and 12 different load cases is evaluated. The whole computation takes 139 min[1] without parallelization. Fig. 8 presents an overview of possible utilizations. To detect the potential for FRC designs, the percentage share of stacking sequence types

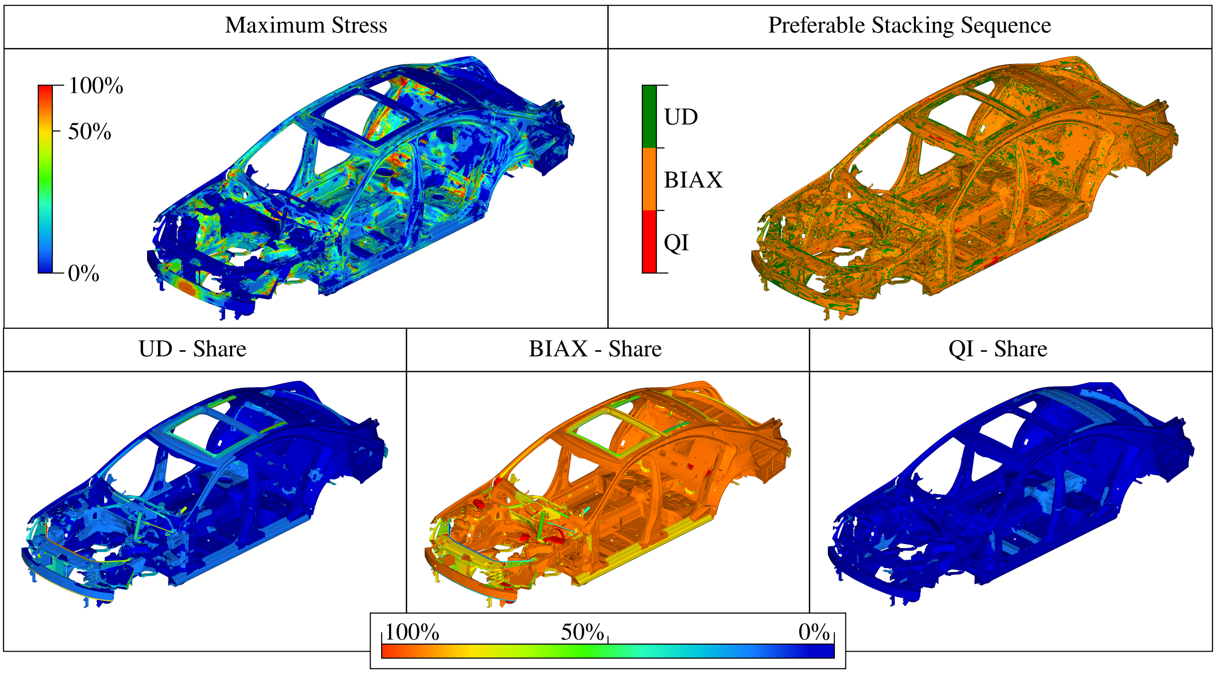

Figure 8. Automobile example (source: DaimlerAG) - Application of the approach

within one component is informative. Parts with many UD stacking sequences are potentially better for FRC designs based on their stress status. BIAX Elements are more suitable than QIs. The example in Fig. 8 consists mostly of BIAX elements. Only few struts or rods tend to have many UD elements. To ease identification of preferable stacking sequences, the percentage share of UD, BIAX or QI elements of each component is shown with contour plots (three sub-figures on the bottom of Fig. 8). Finally, maximum stress can be plotted (picture top left Fig. 8). The maximum von Mises stress of all load cases is saved for each element. QI elements often occur in areas with low stress. Whereas UD elements align tangentially to high stress paths. Informations about stacking sequence types are less important in low stress areas. We developed an additional tool to illustrate possible load paths resulting from computed stacking sequences (Fig. 9). It can be used to make preliminary conclusions considering the design of tailored FRC

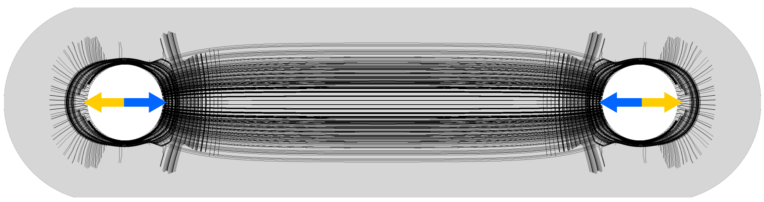

Figure 9. Illustration of load paths

[1]Intel Xeon 8-Core E5-2650v2 (2.6 GHz)

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components. Moreover, we established connections between proposed algorithm and numerical optimizations. They show good and fast results because of better material orientations in the initial state. These approaches and full results will be published elsewhere.

6. Conclusion

An analytical evaluation of preferable stacking sequences for fiber-reinforced composites components is presented in this paper. The proposed method computes whether finite elements of a part should have an unidirectional, biaxial or quasi-isotropic design. Preferable stacking sequences yield to conclusions if composite material designs are beneficial. Preliminary designs can be developed using material orientations or fiber paths can be derived for techniques like tailored fiber placement. Due to analytical methods, computation time of the approach proposed in this paper is less than for numerical optimizations, for assemblies with many finite elements and multiple load cases.

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