ELECTRIC CURRENT DISTRIBUTION BETWEEN A PAIR OF ELECTRODES ON OPPOSITE SURFACES OF CFRP LAMINATES

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Abstract

As an electric current analysis method for laminated carbon composites, orthotropic electric potential function analysis has been proposed. In the previous study, the analysis method was verified in the limited case in which the electric current source and electric grounding sink are both located in the single surface of the laminate. For actual carbon fiber reinforced composites, however, electric current flows in the through-thickness direction in some cases: delamination monitoring using electrical resistance changes or lightning strike. When electric current source and sink locate on opposite surfaces, the distribution of the electric potential difference is completely different from the case in which the source and sink locate on the single surface. In the present study, therefore, oblique flow potential analysis is newly developed, and the results of the analysis are verified by comparing with the FEM analysis and the experiment. Beam type specimens of two stacking sequences (unidirectional and cross-ply laminates) are used. The results indicate that the equivalent electric conductivity is effective for the oblique electric flow. The experimental results show that the carbon fiber reinforced composites can be treated as homogeneous material even for the oblique electric current.

1. Introduction

The use of carbon fiber reinforced polymer (CFRP) composites as structural components of aircraft has been expanded because of their excellent mechanical properties. In the toughened CFRP laminates for aircraft structure, resin-rich layers exist to prevent delamination cracking [1]. The electric conductivity of toughened CFRP composites shows strongly orthotropic due to resin-rich layers. The strongly orthotropic conductivity may result in structural damage when lightning strike occurs. As a lightning protection system, metal foils or meshes are attached to the surface of CFRP components of aircrafts [2]. It is, however, difficult to prevent eddy current or leaked current in CFRP components. Although an electric current analysis is necessary to evaluate damage caused by lightning, resin-rich layers make it difficult to analyze electric current distribution in CFRP composites.

Todoroki has previously proposed an analytical method to calculate electric current distribution using orthotropic electric potential function [3-5]. This method assumes that CFRP composites are homogeneous orthotropic materials. The toughened CFRP laminates used for aircraft, however, are not homogeneous with respect to the through-thickness direction due to their resin-rich layers. In the previous paper, the case in which a couple of electric source and grounding sink is placed on the top

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surface of the beam type specimens was studied. Electric current was experimentally measured comparing with the results of the orthotropic electric potential function analysis. The results show that this analytical method and the assumption of the orthotropic homogeneous materials in the through-thickness direction are effective for the electric current that flows near the top surface [6]. The effectiveness of the method, however, was not confirmed when the electric current flows mainly in the through-thickness direction.

The objective of the present study is, therefore, to confirm the validity of the orthotropic electric potential analysis method for the through-thickness direction electric current. In the present study, the electric current source and the electric grounding sink are placed on the opposite surfaces of a beam type specimen. The electric current flows in the oblique line between the electric current source on the top surface and the electric grounding sink on the bottom surface. A formula of an orthotropic electric potential function is derived for the oblique current path of the beam type specimen in the present study. To confirm the validity of the derived formula, comparisons with FEM analysis results and experimental results of beam type specimens are performed. Experimentally, electric current of each ply is measured and compared with the results of the orthotropic electric potential function analysis.

2. Orthotropic electric potential function analysis

2.1. Principle [3]

Let us assume that the unidirectional CFRP laminates are orthotropic homogeneous materials electrically. The Cartesian two-dimensional coordinate system is defined with the x-axis as the fiber direction and the z-axis as the through-thickness direction. Let the electric conductivities of each direction be σ_x and σ_z. From Ohm's law, the electric current density in the x and z directions (i_x, i_z) can be obtained using the orthotropic electric potential function ϕ as follows:

$$i_x = -\sigma_x \frac{\partial \phi}{\partial x}, \quad i_z = -\sigma_z \frac{\partial \phi}{\partial z} \tag{1}$$

A steady state is considered here. The conservation of charge gives the equation related to electric current density as follows:

$$\frac{\partial i_x}{\partial x} + \frac{\partial i_z}{\partial z} = 0 \tag{2}$$

Substitution of eq. (1) into eq. (2) gives

$$\sigma_x \frac{\partial^2 \phi}{\partial x^2} + \sigma_z \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{3}$$

To deal with the orthotropic coordinate, coordinate transformations as follows are adopted

$$\xi = \frac{x}{\sqrt{\sigma_x}}, \quad \eta = \frac{z}{\sqrt{\sigma_z}} \tag{4}$$

This transformation makes eq. (3) be

$$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} = 0 \tag{5}$$

Eq. (5) is the isotropic Laplace equation, which is similar to the velocity potential of perfect fluid. This means that the potential of perfect fluid can be applied for the transformed coordinate system. Using a couple of source and sink, electric potential can be analyzed for the orthotropic conductive material.
Let us consider the case which is described in Fig. 1. In this case, the electric current source is set at (a_{1x}, t_{1z}) and the sink is set at (a_{2x}, t_{2z}). Using the potential flow for this electric source-sink couple, the electric potential function ϕ_p can be formulated as

$$\phi_p = \phi_{psource} + \phi_{psink} \tag{6}$$

Here,

$$\phi_{psource} = -\frac{I}{4\pi\sqrt{\sigma_x \sigma_z}} \ln\left(\frac{(x-a_{1x})^2}{\sigma_x} + \frac{(z-t_{1z})^2}{\sigma_z} \right), \quad \phi_{psink} = \frac{I}{4\pi\sqrt{\sigma_x \sigma_z}} \ln\left(\frac{(x-a_{2x})^2}{\sigma_x} + \frac{(z-t_{2z})^2}{\sigma_z} \right) \tag{7}$$

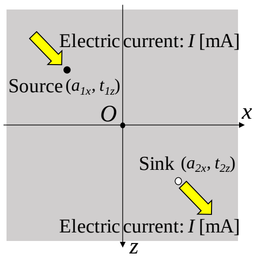

Figure 1. Infinite CFRP plate with two points; electric current source and sink.

I is an input electric current. Derivatives of eq. (6) with respect to x and z give electric current densities in each direction. The detail of the orthotropic electric potential theory can be seen in the reference [3]. When the electric current source and sink are lines as shown in Fig. 2, these lines are seen as sets of point elements. The electric potential function of line source and sink can be obtained by line integral of that of point elements expressed as eq. (6) [7].

$$\phi_l = \phi_{lsource} + \phi_{l\sin k} \tag{8}$$

Here,

$$\begin{aligned} \phi_{lsource} &= \frac{I}{b_2 - b_1} \int_{-b_2}^{-b_1} \phi_{psource} \, da_{1x} \\ &= -\frac{I}{4\pi(b_2 - b_1)\sqrt{\sigma_x \sigma_z}} \left[(x+b_2)\ln\left\{(x+b_2)^2 + \frac{\sigma_x}{\sigma_z}(z-t_{1z})^2\right\} - (x+b_1)\ln\left\{(x+b_1)^2 + \frac{\sigma_x}{\sigma_z}(z-t_{1z})^2\right\} \right. \\ &\qquad \left. + 2\sqrt{\frac{\sigma_x}{\sigma_z}}(z-t_{1z})\left\{\tan^{-1}\left(\sqrt{\frac{\sigma_z}{\sigma_x}}\frac{x+b_2}{z-t_{1z}}\right) - \tan^{-1}\left(\sqrt{\frac{\sigma_z}{\sigma_x}}\frac{x+b_1}{z-t_{1z}}\right)\right\}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} \phi_{l\sin k} &= \frac{I}{b_2 - b_1} \int_{b_1}^{b_2} \phi_{p\sin k} \, da_{2x} \\ &= \frac{I}{4\pi(b_2 - b_1)\sqrt{\sigma_x \sigma_z}} \left[(x-b_1)\ln\left\{(x-b_1)^2 + \frac{\sigma_x}{\sigma_z}(z-t_{2z})^2\right\} - (x-b_2)\ln\left\{(x-b_2)^2 + \frac{\sigma_x}{\sigma_z}(z-t_{2z})^2\right\} \right. \\ &\qquad \left. + 2\sqrt{\frac{\sigma_x}{\sigma_z}}(z-t_{2z})\left\{\tan^{-1}\left(\sqrt{\frac{\sigma_z}{\sigma_x}}\frac{x-b_1}{z-t_{2z}}\right) - \tan^{-1}\left(\sqrt{\frac{\sigma_z}{\sigma_x}}\frac{x-b_2}{z-t_{2z}}\right)\right\}\right] \end{aligned} \tag{10}$$

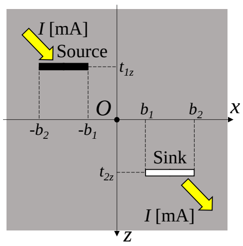

Figure 2. Infinite CFRP plate with two lines; electric current source and sink.

Electric potential difference functions, which are the derivatives of the potential ϕ_l, can be obtained as follows:

$$\frac{\partial \phi_l}{\partial x} = -\frac{I}{4\pi(a_2 - a_1)\sqrt{\sigma_x \sigma_z}} \ln \frac{\left((x+b_2)^2 + \frac{\sigma_x}{\sigma_z}(z - t_{1z})^2\right)\left((x-b_2)^2 + \frac{\sigma_x}{\sigma_z}(z - t_{2z})^2\right)}{\left((x+b_1)^2 + \frac{\sigma_x}{\sigma_z}(z - t_{1z})^2\right)\left((x-b_1)^2 + \frac{\sigma_x}{\sigma_z}(z - t_{2z})^2\right)} \tag{11}$$

$$\frac{\partial \phi_l}{\partial z} = -\frac{I}{2\pi(a_2 - a_1)\sigma_z}\left\{\tan^{-1}\sqrt{\frac{\sigma_z}{\sigma_x}}\frac{x+b_2}{z-t_{1z}} + \tan^{-1}\sqrt{\frac{\sigma_z}{\sigma_x}}\frac{x-b_2}{z-t_{2z}} - \tan^{-1}\sqrt{\frac{\sigma_z}{\sigma_x}}\frac{x+b_1}{z-t_{1z}} - \tan^{-1}\sqrt{\frac{\sigma_z}{\sigma_x}}\frac{x-b_1}{z-t_{2z}}\right\} \tag{12}$$

Let us assume that, the source is set at the top surface and the sink is set at the bottom surface of CFRP laminates whose thickness is t as shown in the Fig. 3. When the source and sink are located on surfaces, input current I [mA] needs to be doubled in order to assure that I flows inside of the laminates. This is because orthotropic electric potential theory so far has been based on the assumption that the laminates is infinite plate. Electric current density in the x direction can be expressed as below.

$$i_x = \frac{I}{2\pi(a_2 - a_1)}\sqrt{\frac{\sigma_x}{\sigma_z}} \ln \frac{\left((x+b_2)^2 + \frac{\sigma_x}{\sigma_z}z^2\right)\left((x-b_2)^2 + \frac{\sigma_x}{\sigma_z}(z-t)^2\right)}{\left((x+b_1)^2 + \frac{\sigma_x}{\sigma_z}z^2\right)\left((x-b_1)^2 + \frac{\sigma_x}{\sigma_z}(z-t)^2\right)} \tag{13}$$

Substitution of electric conductivities of the fiber σ_0 and the through-thickness direction σ_{90} into σ_x, σ_z respectively gives us the solution of electric current densities.

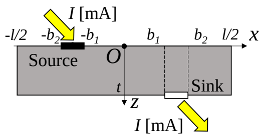

Figure 3. Finite CFRP plate with electrodes attached on the surface; Electric current source is on the top, and sink is on the bottom.

When the laminates cannot be regarded as semi-infinite plate, mirror images are installed in the longitudinal and the through-thickness direction [8]. Fig. 4 shows the schematic representation of the arrangement of mirror images. Superposition of solutions from all of the mirror images gives the electric potential of finite laminates.

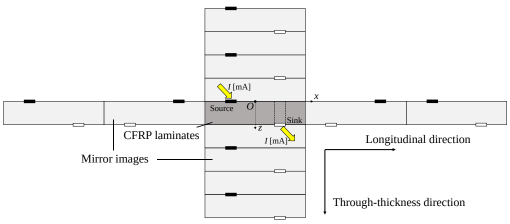

Figure 4. Finite CFRP plate with mirror images in the longitudinal and the through-thickness direction.

2.2. Equivalent electric conductivity analysis

When the laminates contain 0°-plies and 90°-plies, the distribution of electric potential difference is different from that of unidirectional laminates. Fig. 5 shows schematic representation of electric current distribution of cross-ply laminates $[0/90]_S$ using potential function of unidirectional laminates. The amount of electric current of cross-ply laminates is lower than that of unidirectional laminates due

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to low conductivity of 90°-plies. To obtain the exact potential function of cross-ply laminates, the equivalent electric conductivity in x direction is necessary. This conductivity can be obtained considering the effect of low conductivity of 90°-plies and their stacking positions. To calculate this value, an iteration method is available as follows [5]:

(1) Electric current density of cross-ply laminates in the x direction i_x at x = 0 is calculated using potential difference function of unidirectional 0°-ply laminates.

(2) i_x of cross-ply and unidirectional laminates is integrated with respect to the thickness.

(3) Equivalent electric conductivity is calculated multiplying the ratio of integrated i_x of cross-ply to that of unidirectional laminates by σ_0.

(4) Using the potential difference function of the laminates having the equivalent electric conductivity, equivalent electric conductivity is recalculated until the iterated value converges.

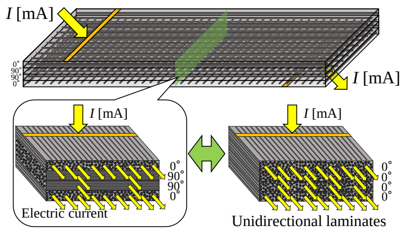

Figure 5. Electric current distribution of each ply at the cross-sectional area of CFRP laminates.

2.3. Comparison with FEM analysis

The results of the proposed analysis method are compared with the FEM results. For the FEM analysis, ANSYS Mechanical APDL ver. 15.0 was used. Type PLANE 230 (4-Node quadrilateral plane) elements were adopted for the analysis. Analytical model is shown in Fig. 6. For the FEM analysis, the beam type model is divided into 102,400 meshes. The typical mesh size is 0.5 mm in the specimen length direction and 0.019 mm in the thickness direction. 50 mA was applied in the electric source electrodes and the voltage is set to 0 V at the grounding electrode. The specimen length is 640 mm and the thickness is 1.52 mm. Two types of stacking sequences are computed: unidirectional $[0]_8$ and cross-ply $[(0/90)]_S$. Electric conductivities used for the FEM analysis is shown in Table 1 from the reference [6]. Fig. 7 shows the electric current density in the x direction at the center of the specimen (x = 0). The solid curve is the results of the proposed method and the open triangle symbols show the FEM results. The results show that the analysis results of the proposed method agree very well with the FEM results in both cases of stacking sequences.

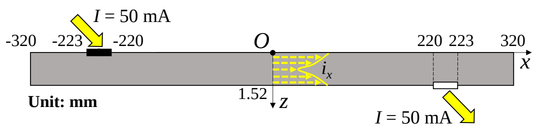

Figure 6. Analytical model of CFRP laminates under the oblique flow of electric current.

Table 1. Electric conductivities in each direction [S/m] [6].

Fiber direction	Transverse direction	Through-thickness direction	
		UD	Cross-ply $[(0/90)_2]_S$
4.93×10^{4}	6.54	3.63×10^{-3}	7.25×10^{-2}

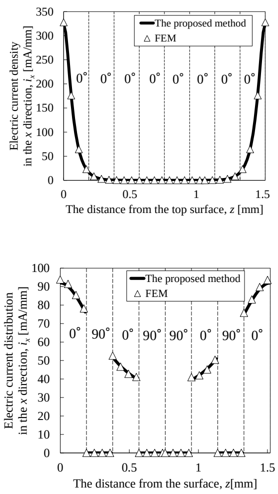

Figure 7. Analysis results of electric current density in the *x* direction calculated by the proposed method and FEM for unidirectional and cross-ply CFRP laminates.

3. Comparison with experimental results

3.1. Experimental procedure

Two laminates whose every two plies is separated at their edges are prepared, and connected through corresponding plies by lead wires as shown in Fig. 8. Electric current in each ply converges on the lead wire, which makes it possible to measure the current in each ply by the measurement of the current flowing in lead wire. In order not to disturb the electric current distribution, non-contact current sensor using Hall effect is used. As an electrode, copper plating method is adopted to reduce the contact resistance [9]. Input current *I* is set to 50 mA.

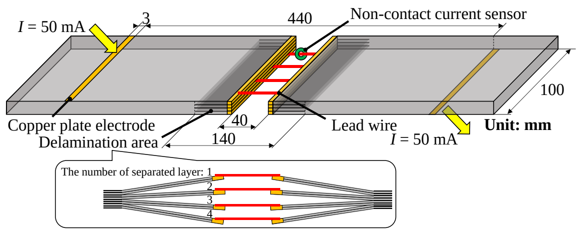

Figure 8 Experimental model for the measurement of the electric current flowing in each separated layer of CFRP laminates.

As the specimens have delamination area in which electric current cannot flow in the through-thickness direction, the effect of the electrically separated plies must be investigated using FEM analysis. Analytical model is shown in Fig. 9. The specimen size is the same as that of section 2. The only difference is the separated area at the middle of the specimen. At the separated area, nodes are doubly defined, and electric current perpendicular to the delaminated area is set to zero. Other conditions of the FEM analysis are the same as those of the section 2.

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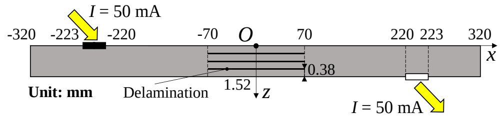

Figure 9. Analytical model with delamination at the center of CFRP laminates.

Fig. 10 shows the results of the proposed method (without delamination) and the results of the FEM analysis (with delamination). The solid curve in each figure shows the results of the proposed method, and the open triangle symbols show the FEM analysis results. The results show that FEM results of both stacking sequences agree very well with the results of the proposed method. This means that the separation of plies and connection of lead wires at the center of the specimen does not disturb the electric current distribution for the oblique electric current flow.

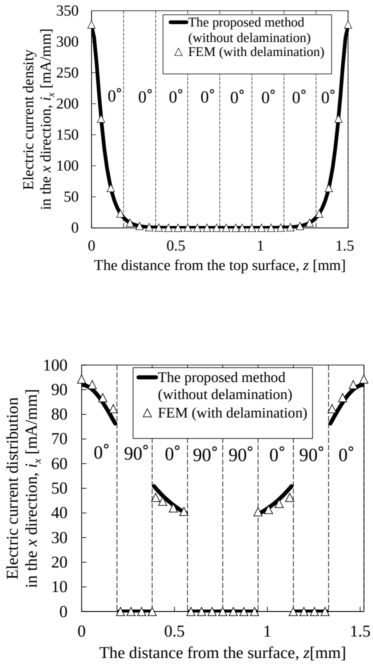

Figure 10. Comparison of analysis results of electric current density in *x* direction for unidirectional and cross-ply laminates with and without delamination.

3.2. Results and discussions

Fig. 11 shows the comparison of the results of experiments with the results of the proposed analysis method. The abscissa is the separated layer number and the ordinate is the ratio of total sum of electric current flowing in each layer to the input current. The separated layer number 1 means the upper layer and the layer number 4 means the bottom layer. Analysis values are the integral of the current density with respect to the thickness.

Solid triangle symbols represent the measured experimental results and the open square symbols represent the results of the proposed analysis method. The results of the proposed analysis method agree well with the measured electric current. This means that the assumption of the orthotropic homogeneous material is effective even for the oblique electric current of the toughened laminated composites.

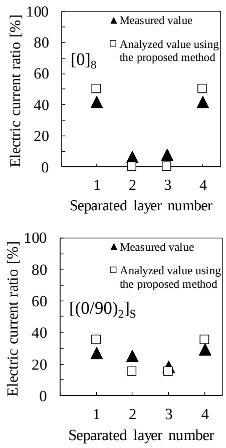

Figure 11. Comparison of experimental and analysis results of the ratio of the electric current flowing in each separated layer to the input current, in the case of unidirectional and cross-ply laminates.

4. Conclusion

1) The newly obtained orthotropic electric potential function is effective for calculation of current density of unidirectional laminated CFRP composite beams for oblique electric current flow.
2) The equivalent electric conductivity is shown to be effective for calculation of electric current distribution of cross-ply laminates even for the oblique current flow.
3) The experimental results show that CFRP laminates can be treated as orthotropic homogeneous material for the oblique electric current flow, although the electric conductivity is not uniformly distributed in the toughened CFRP.

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