ROBUST SIZING OF LARGE THIN-WALLED, COMPOSITE STRUCTURES SENSITIVE TO BUCKLING

V. De Groof[1], H. Haller[1], M. Gratt[1]

[1]INTALES GmbH, Innsbrucker Str. 1, 6161 Natters, Austria
Email: degroof@intales.com, www.intales.com

Keywords: non-linear buckling analysis, thin-walled structures, industrial

Abstract
A nonlinear stability analysis is performed on a state of the art, industrial problem from the aerospace industry. The method used is an iterative branching analysis approach during the Newton-Raphson increments. Bifurcation points are looked for by examining eigenvalues and eigenvectors at selected load states. Once a bifurcation point is found, the analysis can be continued in the post-buckling regime using an imperfection analysis with the NULL-eigenvector. This procedure is repeated until the desired load level is attained, or global collapse is reached. This scheme leads to a computationally efficient method that can be used with complex, large (multiple millions degrees of freedom) finite element models using commercially available finite element packages. To show the power of the method, a comparison with full scale test results is presented.

1. Introduction

The development of modern aircraft structures is driven by the quest for weight saving opportunities. By reducing the weight, material and fuel consumption are reduced, thereby reducing the cost. In order to efficiently identify these weight saving opportunities, detailed and reliable structural analysis is required.

Nowadays, the sizing and validation of flight hardware is done by linear analysis. Nonlinearities in geometry, boundary conditions and material are ignored, since a complete nonlinear treatment of the problem would render the development impractical and is unnecessary. The high detail of the structural model as well as the high number of load cases to be investigated would lead to exorbitant development costs. However, as some parts of the structure are identified to be driven by highly nonlinear shell bending, a standard linear buckling analysis would lead to over-conservative results with a unnecessary weight penalty as a consequence. In order to take this mass opportunity, linear and nonlinear analysis are combined to minimize the weight penalty but still maintaining a competitive development cost.

Therefore, a two stage methodology is proposed. In the first step, a linear buckling analysis is performed as a preselection of critical load cases. Only for these specific load cases, a nonlinear buckling analysis is performed. The requirements for such a nonlinear analysis in an industrial environment are clear:

1. Use of commercially available software (verified software).
2. Accurate determination of instability points and assessment of the structural behavior around these points.
3. Possibility to automate the procedure in order to increase robustness and decrease user error.

Excerpt from ISBN 978-3-00-053387-7

In the following, a description of the proposed method is presented. A number of alternatives are described and shown to be impractical for industry. Next, the validity of the method is shown by comparison with the results of a full scale test of the structure.

2. Methodology

The investigation of stability points on the load-displacement curve is characterized by investigating the following mathematical condition:

$$K_t(u)\varphi = 0, \tag{1}$$

where $K_t(u)$ is the tangent stiffness matrix at a certain load level, and φ is the eigenvector corresponding to the zero eigenvalue. Therefore, $K_t(u)$ is singular at the critical load level. This situation is characterized by the structure possessing no stiffness in the direction of the buckling mode.

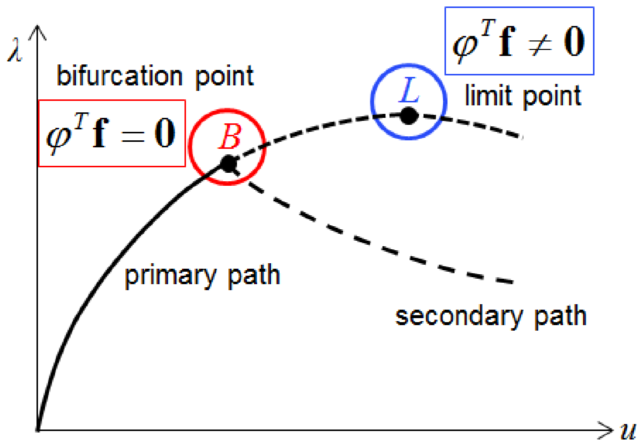

Figure 1. Load path with bifurcation and limit points, showing the primary and secondary load paths.

In order to successfully complete a nonlinear buckling analysis, the complete load path (Fig. 1) has to be derived until the desired load level is achieved. The load path can be found using an iterative solver such as the Newton-Raphson method. However, around stability points one has to pay special attention. Here, one has to differentiate between:

1. A limit point, defined by:

$$\begin{cases} \det(K_t) = 0 \\ \varphi^T f \neq 0 \end{cases} \tag{2}$$

2. A bifurcation point, defined by:

$$\begin{cases} \det(K_t) = 0 \\ \varphi^T f = 0 \end{cases} \tag{2}$$

A limit point can be overcome by switching locally to the arc length method (e.g. Riks). At a bifurcation point, special methods are needed to accurately determine the correct continuation from the primary onto the secondary load path. A bifurcation point can be determined directly by solving the following set of equations ([1], [2], [3]):

Excerpt from ISBN 978-3-00-053387-7

$$G(u,\varphi,\lambda,\mu) = \begin{Bmatrix} g(u,\lambda)+\mu\varphi \\ K_t\varphi \\ l(\varphi,\lambda) \\ \varphi^T f \end{Bmatrix} = 0 \tag{4}$$

Looking at this set of equations, Eq. 4.1 represents the equilibrium condition perturbed by the scaled buckling mode. Eq. 4.2 refers to the stability point condition of Eq. 1 and Eq. 4.4 to the bifurcation condition of Eq. 3. Finally, Eq. 4.3 is a scaling function of the buckling mode to prevent φ being the NULL vector. Eq. 4 can be solved directly, immediately returning the bifurcation point. This procedure should then be followed by a branch switching analysis to determine the secondary path [4].

Using a commercial Finite Element package, e.g. ABAQUS, this approach is not possible. Accessing or modifying the stiffness matrix and its extension is not possible, or infeasible for systems of equations with multiple millions degrees of freedom. Some packages provide methods to improve the convergence behavior which are believed to be useful to overcome bifurcation points. These method add viscous forces to the equilibrium equations and thereby also modify them. They require careful tuning of the viscosity parameter rendering them hard to use when no test results are available a priori. The analyzed equilibrium path is thus questionable when no test results are available. Therefore the direct method needs to be adapted in order to assess the full load path.

The methodology follows an iterative approach observing the occurrence of negative eigenvalues during the Newton-Raphson iterations:

1. A non-linear run using Newton-Raphson method.
2. Observing the occurrence of negative eigenvalues, or cut-backs during the Newton-Raphson iterations. These events are an indication for a singular state of the stiffness matrix.
3. Once a problematic region has been observed, perform multiple eigenvalue extractions in order to iteratively find the load level where the stiffness matrix is (approximately) zero. As is shown in Fig. 2, it is important to check if the found state belongs to the smallest bifurcation point. It has therefore be ensured that the load steps are not too large.
4. Once a critical point has been identified, see Fig. 3, the secondary load path can be entered by adding an imperfection (e.g. a linear combination of the lowest eigenmodes). In the case of a limit point, an arc-length method can be used to overcome it.
5. Continue the analysis on the secondary load path and continue with step 2 up to the desired load level, or the next stability point is reached.

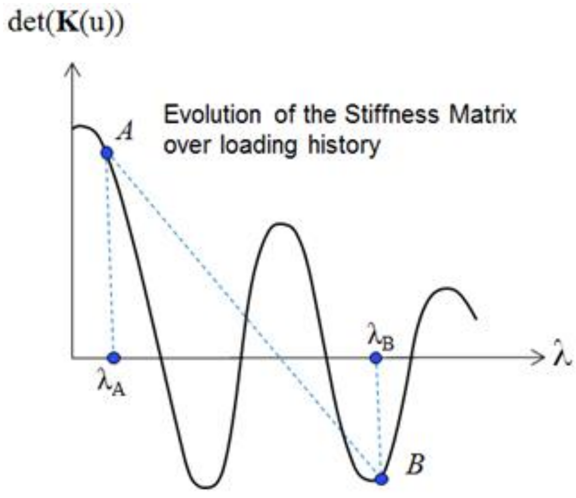

Figure 2. Change of the stiffness matrix as a function of the load level.

Excerpt from ISBN 978-3-00-053387-7

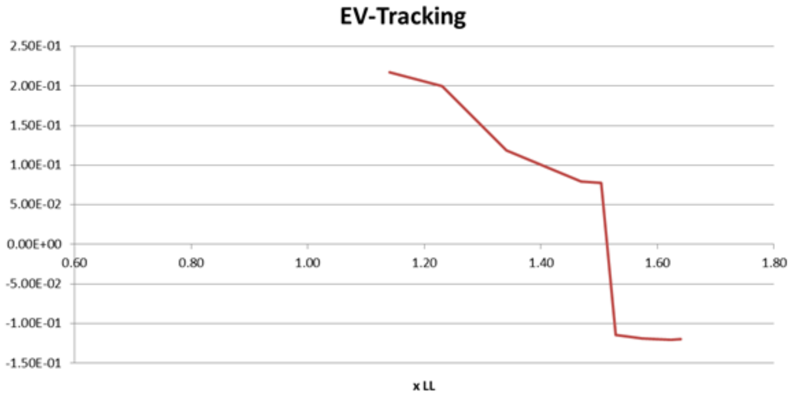

Figure 3. Example of the sign switch, occurring during sizing of the structure, of the lowest eigenvalue, indicating a singular stiffness matrix at approx. 1.5 x limit load.

When looking for the zero eigenvalue, it is important to track the eigenmode and ensure that no eigenmode switching is present. The mode tracking can be done by visually inspecting the extracted eigenmodes. But it can also be done by checking the following condition:

$$\varphi_{\lambda_n}^T \varphi_{\lambda_m} \approx 0, \tag{5}$$

where φ_{λ_n} and φ_{λ_m} represents the eigenvector extracted at a specific load level.

3. Industrial verification

Multiple steps in the analysis have to be carried out a great number of time. Without any support also this method would become too cumbersome to be considered. Therefore, a software tool has been developed to support the creation of the different input files required for the Riks analysis, eigenvalue extraction and the branch switching by perturbation. By parsing of the status file, it is easy for the user to see indications for a critical point and start the next analysis step at the desired load level.

A full scale test campaign has been performed for 3 different load cases (Max Up Bend, Max Down Bend. Max Torque). Strain gauges were installed at a large number of locations and the results were compared with linear and nonlinear predictions. While parts of the structure can accurately be sized using linear predictions, it was confirmed that parts of the structure behave highly nonlinear. Fig. 4 shows the load-strain curve of 2 strain gauges at a component of the structure that behaves highly nonlinear during the max up bend test. As can be seen from Fig. 4, the nonlinear predictions match very well, whereas the linear prediction clearly diverges from the real behavior as the loading increases.

The results of the nonlinear buckling analysis for the max up bend case are shown in Fig. 5. It shows the size of the lowest eigenvalue at different load levels. A comparison with the load level at which buckling according to the linear prediction should occur is also included. Whereas the linear buckling analysis estimates buckling at around 80% of the ultimate load level, the nonlinear analysis does not reveal any NULL eigenvalues before the ultimate load level is reached. Also during the full test campaign, no buckling has been detected. This confirms the underestimation of the buckling load by the linear analysis and affirms the correct identification of a weight reduction opportunity.

Excerpt from ISBN 978-3-00-053387-7

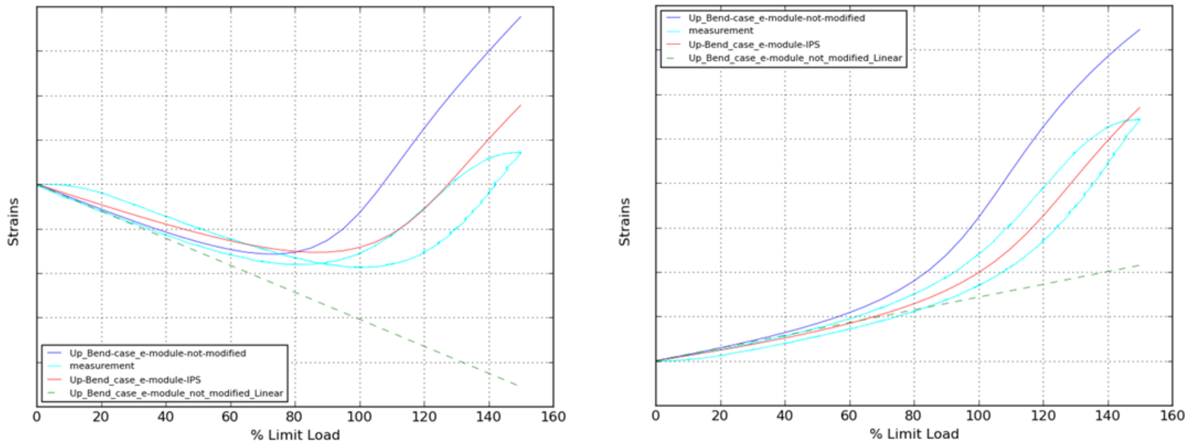

Figure 4. Comparison of test results with linear and nonlinear predictions at 2 different locations.

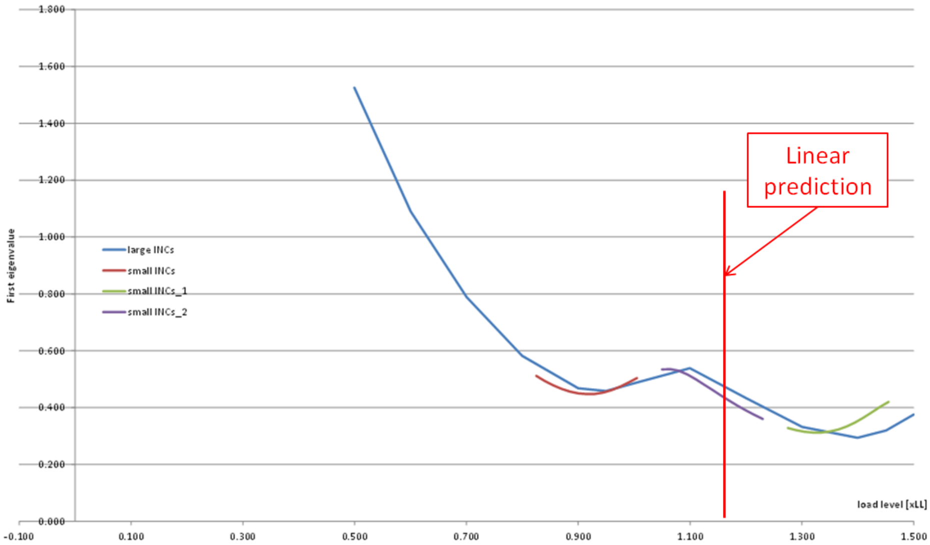

Figure 5. Change of the lowest eigenvalue of max up bend load case during sizing including a comparison with the over-conservative linear buckling estimate.

4. Conclusions

The presented method is shown to be useful for industrial applications as it can be used with commercial software. During the analysis, it is important to ensure that the identified zero eigenvalue belongs to the first critical point and no stability points are skipped. Experience and careful incrementation of the load level can be supported by tracking the eigenmodes. In order to support the engineer, software tools have been developed to automate the input file creation in ABAQUS. Further automation of the branching analysis is possible by improving the mode tracking algorithm.

The proposed method was used during the development of a highly loaded structure in the aerospace industry. During the development, it was observed that high shell bending moments lead to an underestimation of the buckling loads by linear buckling analysis. Therefore, the presented method was adopted to estimate the buckling loads more realistically and correctly dimension the structure. Verification with full scale test results confirmed the shell bending moment presumption and confirmed the validity of the results obtained with the proposed, nonlinear buckling analysis.

References

[1] P. Wriggers and J.C. Simo. A general procedure for the direct computation of turning and bifurcation points. *International Journal of Numerical Methods in Engineering,* 30:155–176, 1990.

[2] G. Moore. The numerical treatment of non-trivial bifurcation points. *Numerical Functional Analysis and Optimization.* 2(6):441–472, 1980.

[3] F.J. Falkner. Numerische Struktur- und Stabilitätsberechnungen von Schalen. *PhD Thesis, Universität Innsbruck, Denmark,* 28. April 2014.

[4] J. Shi. Computing critical points and secondary paths in nonlinear structural stability analysis by the finite element method. Computers and Structures, 58(1):203–220, 1996.

Excerpt from ISBN 978-3-00-053387-7