INTERLAMINAR STRESSES AND EDGE EFFECTS IN QUASI-ISOTROPIC LAMINATES SUBJECT TO TENSILE LOADS

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Abstract
Interlaminar stresses in quasi-isotropic symmetric laminates subject to tension are analyzed analytically and numerically. The coupling effects of the upper and lower halves balance each other and the total behavior is similar to an isotropic material, from a deformation point of view. This balance is due to the resultant bending and twisting moments that correspond to the in-plane stress distribution of each sublaminate. According to the analytic approach, in-plane stresses have an hyperbolic distribution with zero values at the edges. Consequently, interlaminar stresses are maximum at the edges of the strip. The analytic approach is checked with a numerical approach that includes the submodeling technique at the edges. According to classical laminated theory, the strength of a quasi-isotropic laminate do not depend on tensile direction or on the stacking sequence. Nevertheless, the new approach show that edge stresses depend on both, load direction and stacking sequence.

1. Introduction

In a unidirectional lamina subject to pure tension where the fibers have an oblique orientation with respect to the load axis, beside normal strains, shear strains are present [1]. If the tensile load is applied in a way where those strains are constrained, shear forces and bending moments appear [2,3]. This phenomenon is known as normal-shear coupling. In the case of a multidirectional composite subjected to pure tensile loads, in the most general case coupling effects induce shear strains, bending curvatures in two planes and twisting curvatures [4]. Thus, it is said that membrane and plate behaviors are coupled. If the laminate is symmetric with respect to the middle plane, membrane and plate behaviors are not coupled. Thus, if such a laminate is subjected to tensile loads, only in-plane shear deformations can appear. In the particular case of balanced laminates, normal shear coupling does not occur. Moreover, if the laminate is quasi-isotropic, according to Classical Laminated Plates Theory (CLPT) the elastic behavior in membrane does not depend on the orientation [5]. Therefore, the strains and stresses in the laminate do not depend on the orientation of the load. Nevertheless, Turon et al. [6] observed that failure depend on the orientation of the load in tensile tests of quasi-isotropic laminates of non-crimp-fabrics.

If the two halves of a symmetric laminate are not symmetric, each of them would suffer bending and twisting curvatures under tension if it was loaded alone. Then, each half of the laminate constraint the other half to deform freely. If the constrained deformations are bending and twisting curvatures, the constraint is carried out by interlaminar shear stresses.

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The aim of this work is to deepen the analytic approach developed to determine interlaminar stresses in antisymmetric laminates, checked with numeric results obtained from Abaqus [7]. The analytic model is applied to the case of a quasi-isotropic laminate subject to uniform tension in order to analyze the effect of the direction of the applied load and the effect of the laminate layup. According to CLPT, those factors do not affect the mechanical behavior of a quasi-isotropic laminate.

2. Analytic approach

2.1. Displacement and strain field

The upper half of a symmetric laminate with strip geometry subject to tension is analyzed. The following displacement field has been assumed:

$$\begin{aligned} u &= u_0(x,y) + z\theta_x(x,y) \\ v &= v_0(x,y) + z\theta_y(x) \\ w &= w_0(x) - y\theta_y(x) \end{aligned} \tag{1}$$

where w is the normal deflection of the laminate and thus the laminate has been considered to be inextensible in z-direction. u and v are the in-plane displacements of the laminate which have been assumed to be linear functions of the coordinate z. θ_y is the twisting angle of the laminate. According to Eq. $(1)_3$, it is assumed that specimen remains straight along the width. This assumption is based on the fact that the length-to-width ratio of the specimen is considered to be great. From Eq. (1), using the notation of Daniel and Ishai [5] for shear components, the strains are:

$$\begin{aligned} \varepsilon_x &= u_{,x} = \varepsilon_x^0 + z\kappa_x \\ \varepsilon_y &= v_{,y} = \varepsilon_y^0 + z\kappa_y \\ \varepsilon_z &= 0 \\ \gamma_{xy} &= \gamma_s = u_{,y} + v_{,x} = \gamma_{xy}^0 + z\kappa_{xy} \\ \gamma_{zx} &= \gamma_r = u_{,z} + w_{,x} = w_0' + \theta_x - y\theta_y' \\ \gamma_{yz} &= \gamma_q = v_{,z} + w_{,y} = \theta_y - \theta_y = 0 \end{aligned} \tag{2}$$

where,

$$\begin{aligned} &\varepsilon_x^0 = u_{0,x} \qquad \varepsilon_y^0 = v_{0,y} \qquad \gamma_s^0 = u_{0,y} + v_{0,x} \\ &\kappa_x = \theta_{x,x} \qquad \kappa_y = \theta_{y,y} = 0 \qquad \kappa_{xy} = \kappa_s = \theta_{x,y} + \theta_{y,x} = \theta_{x,y} + \theta_y' \end{aligned} \tag{3}$$

According to Eq. $(2)_5$ it results that $\theta_x = \gamma_r + y\theta_y' - w_0'$. Differentiating with respect to y and replacing in Eq. $(3)_6$, the twisting curvature is:

$$\kappa_{xy} = \kappa_s = \theta_{x,y} + \theta_{y,x} = \gamma_{r,y} + 2\theta_y' \tag{4}$$

2.2. Stresses, resultant forces and resultant moments

The stress-strain relations for in-plane components without taking into account hygrothermal effects is:

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$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_s \end{Bmatrix}_k = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix}_k \left[\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_s^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{Bmatrix} \right] \tag{5}$$

where $\{\sigma\}_k$ are in-plane stresses at lamina *k*, $[Q]_k$ are the reduced stiffness coefficients of lamina *k*, $\{\varepsilon^0\}$ are strains of the middle plane and $\{\kappa\}$ are the curvatures of the middle plane. Force and moment resultants concerning in-plane stress components are given by:

$$\begin{Bmatrix} N_x \\ N_y \\ N_s \\ \hline M_x \\ M_y \\ M_s \end{Bmatrix} = \left[\begin{array}{ccc|ccc} A_{xx} & A_{xy} & A_{xs} & B_{xx} & B_{xy} & B_{xs} \\ A_{xy} & A_{yy} & A_{ys} & B_{xy} & B_{yy} & B_{ys} \\ A_{xs} & A_{ys} & A_{ss} & B_{xs} & B_{ys} & B_{ss} \\ \hline B_{xx} & B_{xy} & B_{xs} & D_{xx} & D_{xy} & D_{xs} \\ B_{xy} & B_{yy} & B_{ys} & D_{xy} & D_{yy} & D_{ys} \\ B_{xs} & B_{ys} & B_{ss} & D_{xs} & D_{ys} & D_{ss} \end{array} \right] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_s^0 \\ \hline \kappa_x \\ \kappa_y \\ \kappa_s \end{Bmatrix} \tag{6}$$

The laminate stiffness matrices are:

$$\begin{aligned} [A] &= \sum_{k=1}^{n} [Q]_k \left(z_k - z_{k-1} \right) \\ [B] &= \tfrac{1}{2} \sum_{k=1}^{n} [Q]_k \left(z_k^2 - z_{k-1}^2 \right) \\ [D] &= \tfrac{1}{3} \sum_{k=1}^{n} [Q]_k \left(z_k^3 - z_{k-1}^3 \right) \end{aligned} \tag{7}$$

The inverse relation of Eq. (6) is:

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_s^0 \\ \hline \kappa_x \\ \kappa_y \\ \kappa_s \end{Bmatrix} = \left[\begin{array}{ccc|ccc} a_{xx} & a_{xy} & a_{xs} & b_{xx} & b_{xy} & b_{xs} \\ a_{yx} & a_{yy} & a_{ys} & b_{yx} & b_{yy} & b_{ys} \\ a_{sx} & a_{sy} & a_{ss} & b_{sx} & b_{sy} & b_{ss} \\ \hline c_{xx} & c_{xy} & c_{xs} & d_{xx} & d_{xy} & d_{xs} \\ c_{yx} & c_{yy} & c_{ys} & d_{yx} & d_{yy} & d_{ys} \\ c_{sx} & c_{sy} & c_{ss} & d_{sx} & d_{sy} & d_{ss} \end{array} \right] \begin{Bmatrix} N_x \\ N_y \\ N_s \\ \hline M_x \\ M_y \\ M_s \end{Bmatrix} \tag{8}$$

In the case of out-of-plane shear strain components, constitutive relation at ply *k* is:

$$\begin{Bmatrix} \gamma_q \\ \gamma_r \end{Bmatrix}_k = \begin{bmatrix} S_{qq} & S_{qr} \\ S_{qr} & S_{rr} \end{bmatrix}_k \begin{Bmatrix} \tau_q \\ \tau_r \end{Bmatrix}_k \tag{9}$$

As it can be seen in Eqs. $(2)_{5,6}$ γ_r and γ_q do not depend on *z* and therefore, they are the same as the mean values along the thickness $\overline{\gamma}_r$ and $\overline{\gamma}_q$. Consequently, the out-of-plane constitutive equations have been expressed in terms of average values through the thickness:

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$$\begin{Bmatrix} \bar{\gamma}_q \\ \bar{\gamma}_r \end{Bmatrix} = \begin{bmatrix} \bar{S}_{qq} & \bar{S}_{qr} \\ \bar{S}_{qr} & \bar{S}_{rr} \end{bmatrix} \begin{Bmatrix} \bar{\tau}_q \\ \bar{\tau}_r \end{Bmatrix} = \frac{1}{2h} \begin{bmatrix} \bar{S}_{qq} & \bar{S}_{qr} \\ \bar{S}_{qr} & \bar{S}_{rr} \end{bmatrix} \begin{Bmatrix} V_q \\ V_r \end{Bmatrix} \tag{10}$$

where $\bar{\tau}_r$ and $\bar{\tau}_q$ are mean shear stresses; V_r and V_q are stress resultants induced by τ_r and τ_q, respectively; $\bar{S}_{ij}$ are equivalent compliance coefficients; and $2h$ is the thickness of the sublaminate. Since according to Eq. $(2)_6$ $\gamma_q = 0$, it results that

$$V_q = -\frac{\bar{S}_{qr}}{\bar{S}_{qq}} V_r \quad \text{and} \quad \bar{\gamma}_r = \frac{V_r}{2h}\left(-\frac{\bar{S}_{qr}^2}{\bar{S}_{qq}} + \bar{S}_{rr} \right) = \frac{\bar{S}_{rr}^*}{2h} V_r \tag{11}$$

2.3. Forces and moments of the upper half of a symmetric laminate subject to tension

Equilibrium equations in terms of resultant forces and moments can be obtained integrating along the thickness the equilibrium equations concerning stresses [8]. In the general case of any sublaminate of thickness $2t$ obtained of a laminate, equilibrium equations are:

$$\begin{aligned} & N_{x,x} + N_{s,y} + \tau_r(t) - \tau_r(-t) = 0 \\ & N_{s,x} + N_{y,y} + \tau_q(t) - \tau_q(-t) = 0 \\ & V_{r,x} + V_{q,y} + \sigma_z(t) - \sigma_z(-t) = 0 \\ & M_{x,x} + M_{s,y} + h\left[\tau_r(t) - \tau_r(-t)\right] - V_r = 0 \\ & M_{s,x} + M_{y,y} + h\left[\tau_q(t) - \tau_q(-t)\right] - V_q = 0 \end{aligned} \tag{12}$$

In Eq. (12) forces and moments correspond to the general sublaminate. It is worth noting that interlaminar stresses appear in equilibrium equations. It would be desirable to isolate sublaminates with null stresses in the upper and lower faces.

In general, the upper half of a symmetric laminate subject to tension has all kind of coupling effects. It has not stresses applied on the upper face. With respect to the lower face, interlaminar shear stresses are null due to the symmetry. Then, Eqs. (12) can be applied assuming that $\tau_q(\pm h)$and $\tau_r(\pm h)$are null.

Under uniform tension, variations in x are null and transverse loads have not been applied. Then, according to Eq. $(12)_1$ N_s is uniform and as $N_s = 0$ at the edges, N_s vanishes along the width. By analogous reasoning in Eqs.$(12)_2$ it results that $N_y = 0$. Then, the laminate is supposed to be under the action of the N_x known force and M_x, M_y and M_s unknown moments. These moments can be determined from from Eq. (8) after imposing the deformation conditions of the whole laminate, namely $\kappa_x = \kappa_y = \theta'_y = 0$. Then, according to Eq. (4) it results $\kappa_s = \bar{\gamma}_{r,y}$ and thus:

$$\begin{aligned} & d_{xx} M_x + d_{xy} M_y + d_{xs} M_s = -c_{xx} N_x \\ & d_{xy} M_x + d_{yy} M_y + d_{ys} M_s = -c_{yx} N_x \\ & d_{xs} M_x + d_{ys} M_y + d_{ss} M_s = \bar{\gamma}_{r,y} - c_{sx} N_x \end{aligned} \tag{13}$$

From Eq. (13) M_x, M_y and M_s are:

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$$\begin{aligned} M_x &= \Delta^{-1}\left(N_x\Delta_1 + \overline{\gamma}_{r,y}\Delta_x\right) \\ M_y &= \Delta^{-1}\left(N_x\Delta_2 - \overline{\gamma}_{r,y}\Delta_y\right) \\ M_s &= \Delta^{-1}\left(N_x\Delta_3 + \overline{\gamma}_{r,y}\Delta_s\right) \end{aligned} \tag{14}$$

Where

$$\Delta_1 = -\begin{vmatrix} c_{xx} & d_{xy} & d_{xs} \\ c_{yx} & d_{yy} & d_{ys} \\ c_{sx} & d_{ys} & d_{ss} \end{vmatrix} \quad \Delta_2 = -\begin{vmatrix} d_{xx} & c_{xx} & d_{xs} \\ d_{xy} & c_{yx} & d_{ys} \\ d_{xs} & c_{sx} & d_{ss} \end{vmatrix} \quad \Delta_3 = -\begin{vmatrix} d_{xx} & d_{xy} & c_{xx} \\ d_{xy} & d_{yy} & c_{yx} \\ d_{xs} & d_{ys} & c_{sx} \end{vmatrix} \quad \Delta = \begin{vmatrix} d_{xx} & d_{xy} & d_{xs} \\ d_{xy} & d_{yy} & d_{ys} \\ d_{xs} & d_{ys} & d_{ss} \end{vmatrix}$$
$$\Delta_x = \begin{vmatrix} d_{xy} & d_{xs} \\ d_{yy} & d_{ys} \end{vmatrix} \quad \Delta_y = \begin{vmatrix} d_{xx} & d_{xs} \\ d_{xy} & d_{ys} \end{vmatrix} \quad \Delta_s = \begin{vmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{vmatrix} \tag{15}$$

With respect to Eq. $(12)_3$ $\sigma_z(-h)$ is unknown. Otherwise, Eqs.$(12)_{4,5}$ reduce to:

$$\begin{aligned} M_{s,y} &= V_r \\ M_{y,y} &= V_q \end{aligned} \tag{16}$$

Differentiating Eq. $(14)_1$ with respect to y, assuming that N_x is uniform along the width $M_{x,y}$ can be extracted, being:

$$M_{x,y} = \Delta^{-1}\Delta_x\overline{\gamma}_{r,yy} \tag{17}$$

Differentiating Eq. $(8)_6$ with respect to y, replacing Eq. (16) and (17):

$$\overline{\gamma}_{r,yy}(1-f) = d_{ys}V_q + d_{ss}V_r \qquad f = d_{xs}\Delta^{-1}\Delta_x \tag{18}$$

Taking into account Eq. (11):

$$\overline{\gamma}_{r,yy}\left(1-f\right) = \left(d_{ss} - d_{ys}\frac{\overline{S}_{qr}}{\overline{S}_{qq}}\right)\frac{2h}{\overline{S}^*_{rr}}\overline{\gamma}_r = 2h\frac{d^*_{ss}}{\overline{S}^*_{rr}}\overline{\gamma}_r \tag{19}$$

Eq. (19) can be written as:

$$\overline{\gamma}_{r,yy} - k^2\overline{\gamma}_r = 0 \quad \text{where} \quad k^2 = 2h\frac{d^*_{ss}}{\overline{S}^*_{rr}\left(1-f\right)} \tag{20}$$

The general solution of Eq. (20) is:

$$\overline{\gamma}_r(y) = C_1\sinh ky + C_2\cosh ky \tag{21}$$

Being $2b$ the width of the laminate, replacing Eq. (21) in the expression of M_s obtained from Eq. (14) and imposing that $M_s = 0$ at the edges $y = \pm b$, C_1 and C_2 are determined:

$$C_1 = -N_x \frac{\Delta_3}{\Delta_s}\left(k\cosh\lambda\right)^{-1} \quad \lambda = kb$$
$$C_2 = 0 \tag{22}$$

Replacing Eq. (22) and (21) in Eq. (14) bending and twisting moments are:

$$\begin{aligned} M_x &= N_x \Delta^{-1}\Delta_1\left(1-\frac{\Delta_x\Delta_3}{\Delta_s\Delta_1}\frac{\cosh ky}{\cosh\lambda}\right)\\ M_y &= N_x \Delta^{-1}\Delta_2\left(1-\frac{\Delta_y\Delta_3}{\Delta_s\Delta_2}\frac{\cosh ky}{\cosh\lambda}\right)\\ M_s &= N_x \Delta^{-1}\Delta_3\left(1-\frac{\cosh ky}{\cosh\lambda}\right)\end{aligned} \tag{23}$$

The terms out of parenthesis in Eq. (23) correspond to CLPT. Replacing the moments given in Eq. (23) and the applied axial force N_x in Eq. (8), the strains and curvatures of the reference plane are obtained. Replacing those terms in Eq. (5) the in-plane stress components σ_x, σ_y and τ_s are determined for each lamina.

2.4. Interlaminar stresses

Equilibrium equations of stresses are given by:

$$\begin{aligned} \sigma_{x,x} + \tau_{s,y} + \tau_{r,z} &= 0\\ \tau_{s,x} + \sigma_{y,y} + \tau_{q,z} &= 0\\ \tau_{r,x} + \tau_{q,y} + \sigma_{z,z} &= 0\end{aligned} \tag{24}$$

In Eq. (24) the derivatives with respect to x are 0. The derivatives with respect to y of σ_y and τ_s are function of the derivatives of the moments given in Eq. (23). Integrating Eq. $(24)_1$ and $(24)_2$ interlaminar shear stresses $\tau_r = \tau_r(y,z)$ and $\tau_q = \tau_q(y,z)$ in each lamina are determined, respectively. Integration constants are calculated by the continuity condition of interlaminar stresses. Finally, integrating Eq. $(24)_3$ interlaminar normal stresses $\sigma_z = \sigma_z(y,z)$ are determined. In this case, integration constants are determined by continuity condition of normal stresses.

2.5. Equivalent shear stiffness

Equivalent shear stiffness coefficients are obtained equating the strain energy of the actual interlaminar stresses with the energy corresponding to averaged values. In a differential surface element of the laminate, the energy per unit surface area is:

$$\tfrac{1}{2}\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\left(\tau_q\gamma_q + \tau_r\gamma_r\right)dz = \tfrac{1}{2}\left(\overline{\tau}_q\overline{\gamma}_q + \overline{\tau}_r\overline{\gamma}_r\right) \tag{25}$$

According to Eq. (2) shear strains are the mean values in thickness, as they do not vary in z. Then, Eq. (25) can be written as:

$$\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\tau_r\gamma_r dz = \overline{\tau}_r\overline{\gamma}_r \tag{26}$$

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3. Quasi-isotropic laminate

A quasi-isotropic $[0/45/90/-45]_s$ laminate is subjected to a tensile axial load of N100 N/mm in the longitudinal direction of the laminate as it can be seen in Fig. 1. The material properties correspond to the carbon epoxy composite AS4/3501-6 [5], the width is 10 mm and the ply thickness is 0.125 mm.

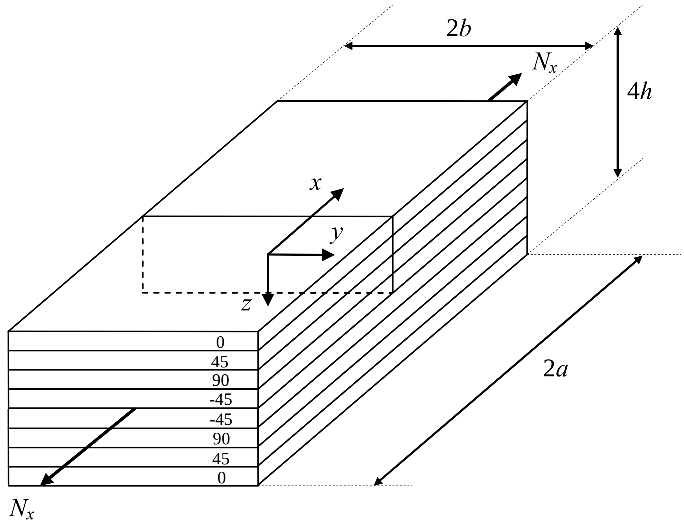

Figure 1. Quasi-isotropic laminate under tensile load.

When the upper sublaminate is analyzed, only one half of the axial load is taken into account. This part would present a twisting curvature and two bending curvatures if it were alone. The sublaminate does not actually present any curvature as it is constrained by the twisting moment M_s and the bending moments M_x and M_y. Table 1 shows the moments that corrrespond to CLPT after rotating some angle all the plies, which is equivalent to change the tensile load direction [6].

Table 1. Influence of the load direction in the moments of the sublaminate.

Rotation	Sublaminate	M_x (N·mm)	M_y (N·mm)	M_s (N·mm)
0	[0/45/90/-45]	-4.77	0.01	-1.31
-22.5	[-22.5/22.5/67.5/-67.5]	-5.22	-1.51	1.08
-45	[-45/0/45/90]	-2.61	-2.14	1.31
-67.5	[-67.5/-22.5/22.5/67.5]	0.00	0.00	0.78
-90	[90/-45/0/45]	2.61	2.14	1.31

Table 2. Influence of ply layup in the moments of the sublaminate.

Permutation	Sublaminate	M_x (N·mm)	M_y (N·mm)	M_s (N·mm)
1 2 3 4	[0/45/90/-45]	-4.77	0.01	-1.31
4 1 3 2	[-45/0/90/45]	-1.85	-0.53	1.97
1 4 2 3	[0/-45/45/90]	-5.54	-1.60	0.66
3 1 4 2	[90/0/-45/45]	-0.31	2.69	0.66

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Table 2 shows the moments according to CLPT obtained for different ply layups of the original sublaminate [0/45/90/-45]. The values of Table 1 and Table 2 correspond approximately to the value corresponding to y = 0 in the hyperbolic distribution given in Eqs. (23). It is worth noting that the maximum stresses at each ply do not change according to CLPT and then, the security factor is the same for any of the laminates in Tables 1 and 2. Nevertheless, as moments change, also change V_r and V_q and consequently, the distribution of interlaminar stresses.

4. Conclusions

In a general symmetric laminate subject to tension, the upper and the lower halves have a stress state which resultant bending and twisting moments avoid curvatures. As these moments depend on the width coordinate, there are interlaminar shear stresses that are maximum at the edges.

In the case of a quasi-isotropic symmetric laminate subject to tension, the mechanical behavior does not depend on the direction of load application nor the ply layup. Nevertheless, as moments depend on these factor, interlaminar stresses too.

For a quasi-isotropic laminate constituted by a series of plies, it is possible to obtain the permutation of plies that minimize the effect of interlaminar stresses. Similarly, for a given laminate layup, it is possible to get a load orientation where the effect of interlaminar stresses is minimum.

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