OPTIMIZATION OF FIBER-STEERED COMPOSITES BY USING THE ISO-CONTOUR METHOD WITH MAXIMUM CURVATURE CONSTRAINT

A.L. Arsenyeva[1], F. Duddeck[2]

[1]Department of Civil, Geo and Environmental Engineering, Technical University of Munich (TUM), Arcisstr. 21, 80333, Munich, Germany
Email: anna.arsenyeva@tum.de, Web Page: https://www.cm.bgu.tum.de/index.php?id=63&L=1
[2]Department of Civil, Geo and Environmental Engineering, Technical University of Munich (TUM), Arcisstr. 21, 80333, Munich, Germany
Email: duddeck@tum.de, Web Page: https://www.cm.bgu.tum.de/index.php?id=58&L=1

Keywords: Fibre steered laminate, Composite material optimization, Variable stiffness laminates, Iso-contour Method, Manufacturing constraints

Abstract

The paper presents a novel method for the optimization of fiber-steered composite shell structures, considering manufacturability constraints, expressed in terms of maximum allowed curvature - maximum fiber curvature constraint (MFCC). Unlike the average fiber curvature constraint (AFCC) [1], the method is able to capture local effects, guaranteeing the manufacturability of the optimal design. In this research, the previously introduced iso-contour method [2], where the fiber paths are represented as iso-contour lines of an artificial surface, defined over the 2D geometry domain, is extended by a technique to compute the maximum fiber curvature. The method is successfully tested with exemplary problems, including the optimization of the clamped plate stiffness and design of the fuselage panel for maximum buckling force. Results show, that the introduced method can precisely control the maximum curvature constraint during the fiber-steered composite optimization, and obtained designs provide significant improvement over simple laminates.

1. Introduction

Fiber-steered composites can provide significantly better performance in comparison with simple laminates, thanks to being specifically optimized for a particular structural role, e.g., for maximum stiffness under a certain loading conditions or for buckling resistance. It can be viewed as a topology optimization, but for the distribution of material mechanical properties within a fixed geometry domain, rather then the shape of the domain. Various methods are proposed in the literature aimed at finding the optimal fiber orientation distributions. Several methods propose special ways of fiber path parametrization, for example in [3] proposed a parametrization for linear fiber orientation variation. Another way was introduced by [4], where contour lines of cubic polynomials are used to define the fiber paths over a 2D domain. Unlike the method presented in the current work, where an arbitrary hyper-surface with multiple control points is used, this approach provides limited flexibility of the path parametrization and directly manipulates polynomial coefficients. A different approach is proposed in [1, 5, 6], starting from the optimization of the stiffness distribution by using Lamination Parameters (LP), followed by finding corresponding local stacking sequences at each node of the finite element (FE) model, and finally obtaining corresponding optimal fiber paths.

Excerpt from ISBN 978-3-00-053387-7

The fiber tows can be individually placed to produce an optimal laminate design by an Automated Fiber Placement (AFP) [7]. However, when the tows are steered individually, the inner and outer fiber radii differ, which can cause local placement imperfections, such as local fiber buckling and ply wrinkling. This imposes limitations in the maximum manufacturable fiber curvature, which should be taken into account during the fiber placement optimization. Several techniques for imposing curvature constraint are proposed in recent research papers.

For example, in [8] the manufacturing constraint is applied to the curvature of the reference fiber path (defined following [3]), which is replicated in a certain direction to create the composite ply. A similar way is used by other researchers [9, 10] for the fiber turning radius, again parametrized following [3]. Another popular approach consists in replacing MFCC by the AFCC. For example in [1] the AFCC is calculated from the optimal fiber angles orientation in finite-element model as the rate of change in fibre angles, expressed as the norm of the divergence of the fibre angle. In [11] this method was coupled with the Iso-contour method employed here. The drawback of element-based computation is the dependency of the obtained curvature on the finite-element discretization. AFCC is also used by [4], by averaging the iso-contour lines curvatures, obtained exactly from the cubic control surface. In general, AFCC approach cannot guarantee that designs are manufacturable w.r.t. maximum curvature, but can provide an improvement over an unconstrained design while being less complicated as maximum curvature constraint. To summarize, there is still a need for improved fiber curvature control.

This research provides a method to combine the Iso-contour [2] method with the MFCC, extended to the case of multiple-layered composites. In the following section, a brief description of the Iso-contour is given, followed by the MFCC computation technique.

2. Iso-contour Method

The key idea of the Iso-contour method is to steer fiber paths at each point of the design domain by iso-contour lines of an artificial surface. The artificial surface is defined by a number of control points and corresponding ”height” values, interpolating the given height values over the design domain. For example, Kriging, Radial-basis Functions (RBF) or splines can be used. In some sense this is similar to the method of [4], but provides much more flexible fiber paths parametrisation. In case of Kriging and RBF, the control points can be placed freely, covering the design domain, while splines interpolation requires points to be placed in a grid.

In general the number and location of the points can be varied, allowing to define a parameterization flexible enough for various problem and geometry complexities. By varying the control points ”heights”, the artificial surface and its iso-contours are changed, providing various steered-fiber layouts. General black-box optimization techniques can be used to find optimal layout, preferably global-search methods, which can handle possibly multi-modal problems. In particular, a combination of Evolutionary Algorithm [12] with the COBYLA [12] method is employed in this work.

The proposed method is implemented using various hyper-surface methods, available in Python 2.7 scipy [13] and sklearn packages, together with ANSYS 14.5 for structural finite-element (FE) analysis and DAKOTA 5.4 software for the optimization methods. The general workflow of the method is illustrated in Fig. 1.

Excerpt from ISBN 978-3-00-053387-7

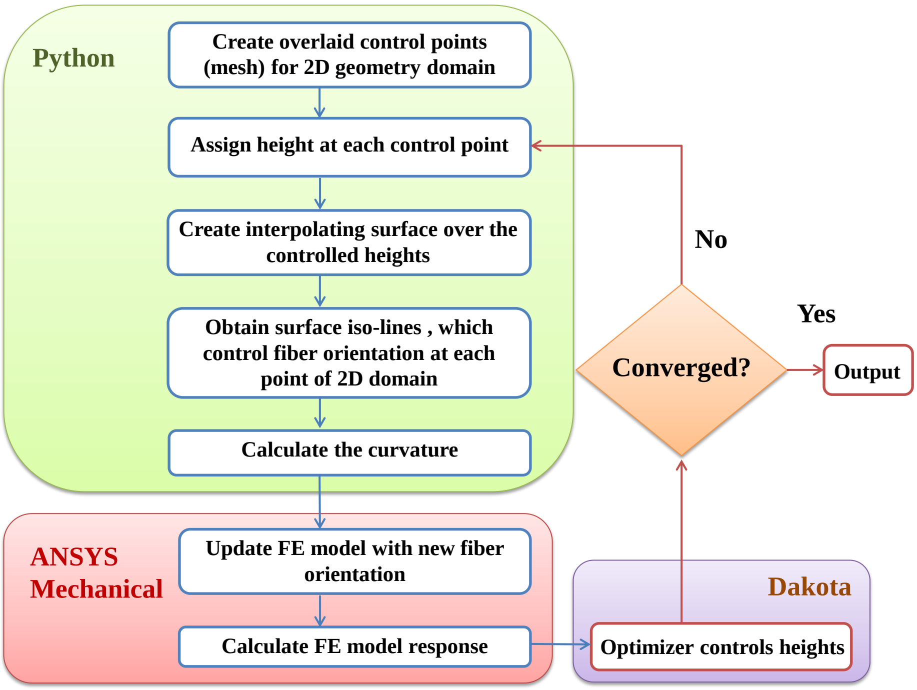


Figure 1. Process flow for Iso-contour based method.

3. Maximum curvature constraint

The distribution of the fiber angles in the design domain can be viewed as a vector field V, defined as follows:

$$V = vx(x, y)\underline{i} + vy(x, y)\underline{j}, \tag{1}$$

where vx and vy are projections of fiber tangent direction. With this definition, the absolute curvature of the vector field V can be expressed following [14]:

$$k_{x,y} = \left| \frac{vx^2 vy_x - vy^2 vx_y + vx\, vy(vy_y - vx_x)}{(vx^2 + vy^2)^{3/2}} \right| \text{ where: } \frac{\partial}{\partial x} =< \cdot >_x, \frac{\partial}{\partial y} =< \cdot >_y . \tag{2}$$

On the other hand, the fiber directions are tangent to the iso-contours of the artificial surface $\varphi(x, y)$, which means that:

$$vx = \frac{\partial \varphi}{\partial y} = \varphi_y; \qquad vy = -\frac{\partial \varphi}{\partial x} = -\varphi_x, \tag{3}$$

with $\varphi(x, y) = C$ defining the iso-contour. Moreover, because of the smoothness of the chosen artificial surfaces, which define $\varphi(x, y)$, second mixed derivatives of $\varphi(x, y)$ are symmetric. Taking this into account, and substituting (3) into (2), the expression of the fibers curvature, depending on the $\varphi(x, y)$ is obtained, which is actually the determinant of the Bateman-Reiss operator [15]:

$$k_c = \left| \frac{\varphi_{xx}\varphi_y^2 - 2\varphi_{xy}\varphi_x\varphi_y + \varphi_{yy}\varphi_x^2}{(\varphi_x^2 + \varphi_y^2)^{3/2}} \right| \tag{4}$$

Excerpt from ISBN 978-3-00-053387-7

In the case of multiple layers, different layer designs are possible, e.g. reflected layer, when the fiber angle α is changed to $-\alpha$. In this case:

$$vx^{[-\alpha]} = vx = \frac{\partial\varphi}{\partial y};\ vy^{[-\alpha]} = -vy = \frac{\partial\varphi}{\partial x} \Rightarrow k_c^{[-\alpha]} = \left|\frac{\varphi_{\mathrm{y}}^2\varphi_{\mathrm{xx}} - \varphi_{\mathrm{x}}^2\varphi_{\mathrm{yy}}}{(\varphi_{\mathrm{x}}^2 + \varphi_{\mathrm{y}}^2)^{3/2}}\right| \tag{5}$$

Another option to construct new layers is to rotate fibers by $\pi/2$: $\alpha = \alpha + \pi/2$. In this case:

$$vx^{[\pi/2]} = -vy = \frac{\partial\varphi}{\partial x};\ vy^{[\pi/2]} = vx = \frac{\partial\varphi}{\partial y} \Rightarrow k_c^{[\pi/2]} = \left|\frac{-\varphi_{\mathrm{xx}}\varphi_y^2 + \varphi_{yy}\varphi_x^2 + \varphi_x\varphi_y(\varphi_{yy} - \varphi_{xx})}{(\varphi_{\mathrm{x}}^2 + \varphi_{\mathrm{y}}^2)^{3/2}}\right| \tag{6}$$

In general, for the given rotation angle γ, the curvature can be calculated following [14]:

$$k_c^{[\gamma]} = |k_c \cos\gamma + k_c^{[\pi/2]} \sin\gamma| \tag{7}$$

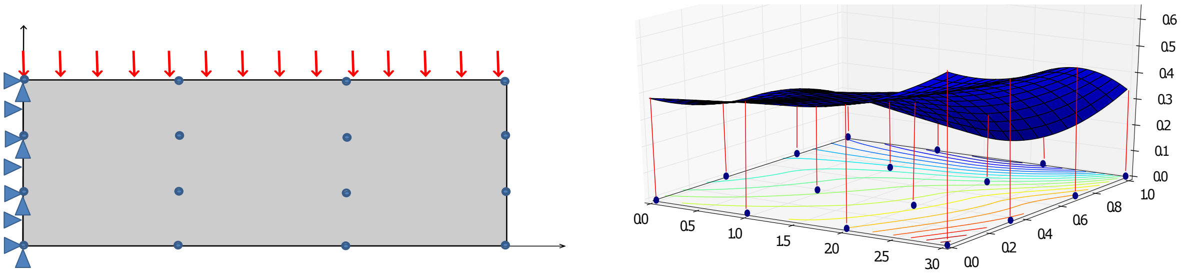
Using the derived expressions for the curvature of the various layer designs, the MFCC for each layer is obtained by finding the maximum of curvature computed for a large number of sample points in the design domain. It this work, a 200×200 grid of points is used. Due to extremely cheap surface function evaluations, the time for the MFCC computation is negligible w.r.t. FE analysis. Additionally, the obtained MFCC value is independent from the FE discretization.

4. Testing of algorithm

Two exemplary problems, including the optimization of the clamped plate stiffness and design of the fuselage panel for maximum buckling force are used to illustrate the capabilities of the introduced method. For both problems, orthotropic material represent homogenized fiber-steered composite properties: elastic moduli $E_1 = 130$ GPa, $E_2 = 10$ GPa, shear modulus $G_{12} = 5$ GPa and Poisson's ratio equals $\nu_{12} = 0.35$. The MFCC is formulated as $max[k_c] \leq k_0$.

4.1. Plate in bending

A simple plate in a bending problem with uniformly distributed top in-plane loading (as shown in Fig.2) is used to illustrate the method and possibilities of the curvature constraint. Dimensions of the plate are defined similar to Setoodeh [16], with the sides aspect ratio equal to 3:1. The left side of the plate is clamped, and the upper side is loaded with uniformly distributed pressure. Single-layer composite material is considered for this example.



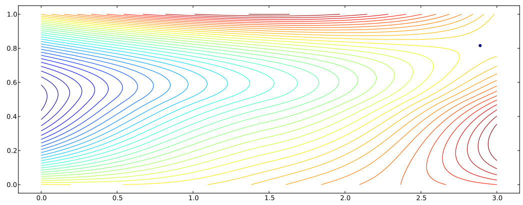
a) b)

Figure 2. a)The test problem with overlaid 4×4 control points, b) Spline surface over controlled heights.

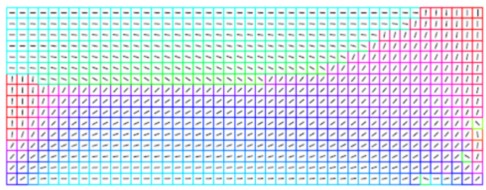
Excerpt from ISBN 978-3-00-053387-7

The number of design variables is equal to 16 (4×4) control points of the hyper surface (see Fig. 2). The objective is to minimize the average displacement on the top line, which approximately corresponds to compliance minimization. The aim of this test problem is comparison of fiber paths design obtained by the proposed method and results obtained by using a Cellular Automata method to vary fiber direction at each cell independently [16].

Comparison of the fiber distributions for both methods are shown in Fig. 3, and results are very similar. Main differences are found mainly in the right part of the plate, while in the left part fiber paths are almost the same. The reason for this is that the overall structural stiffness is mainly affected by the plate design near the clamped side, while the right part of the plate does not contribute much to the stiffness, which means that the optimizer is more sensitive to the design trends of the area near the clamp. Another important point is that paths, obtained with the proposed method, are much smoother than in the optimal solution from Setoodeh, even without the constraint.



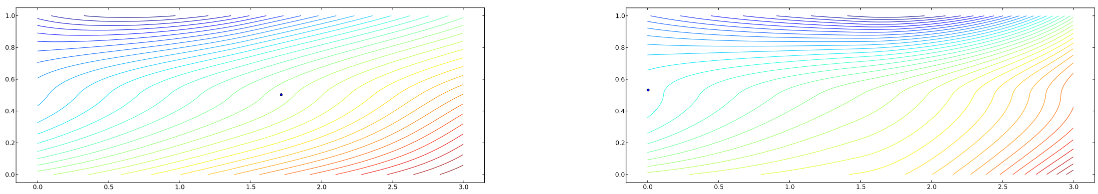
a)



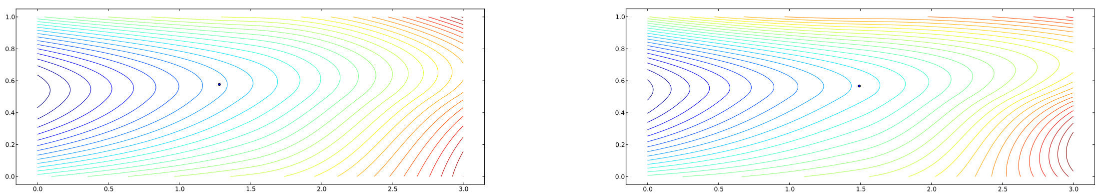
b)

Figure 3. a) Optimal iso-contours for unconstrained case, b) Results obtained by Setoodeh [16]

In Fig. 4 the results obtained for different maximum curvature constraints are shown. As can be seen, very different optimal layouts correspond to the different values of MFCC, which might indicate the need of global optimization methods for this kind of problems. The constraint is active for all optimal solutions. Moreover, less strict MFCC results in a better design performance, which might indicate existence of a Pareto front between the displacement and maximum curvature.



$k_0 = 5$ $k_0 = 15$



$k_0 = 20$ $k_0 = 30$

Figure 4. Iso-lines obtained with different MFCC, k_0 $[m^{-1}]$.

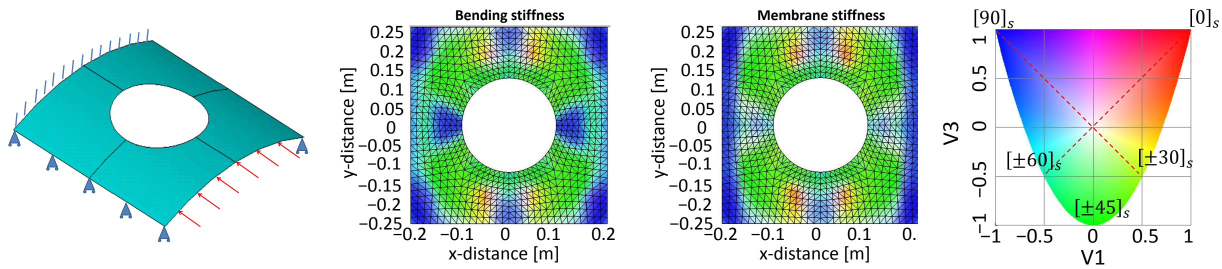
Table 1 lists average displacements δ_{avg} for the optimized fiber placement. The displacements are normalized with respect to the average displacement of constant stiffness laminate which has 0° optimal ply orientation. As can be seen, fiber-steered laminates significantly improve the design, compared to a simple constant stiffness laminate.

Table 1. Average top line displacements for optimal fiber placement w.r.t. 0° simple laminate solution.

	0°	Fiber-steered laminates				
$k_0[m^{-1}]$	-	∞	30	20	15	5
δ_{avg}	1.0	0.557	0.559	0.574	0.706	0.812

4.2. Uni-axial buckling of the cylindrical plate with a hole

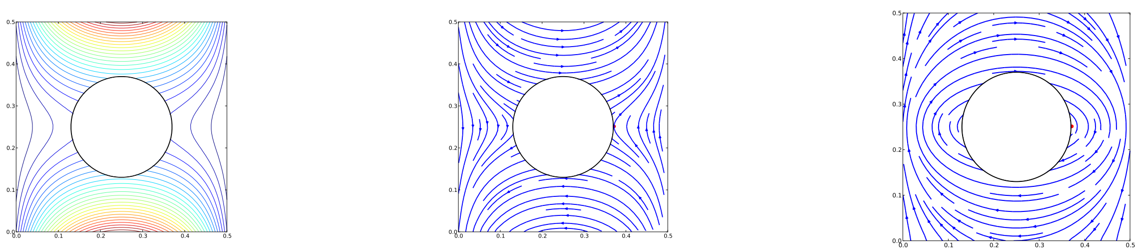
The second example is taken from earlier work [11]: a cylindrical plate with a hole is regarded, which is simply supported on the straight edges and subjected to uni-axial compression at the curved edges, as shown in Fig. 5a. This example represents a simplified fuselage section with a window. Dimensions of the plate are 0.5 m x 0.5 m, radius of the curvature is 0.75 m, the radius of the hole is 0.12 m. 4-layers laminates are designed as $[\pm\alpha]_s$ and $[\alpha, \alpha + 90°]_s$, where $[\alpha]$ is the local fibre angle in the design layer. The total ply thickness equals 1 mm. The objective of the optimization is to maximize the critical buckling load. The aims of this problem are: comparison of fiber paths design obtained by the proposed method to results obtained by using lamination parameters [17], as shown in Fig. 5b,c, and testing the ability of the method to handle highly non-linear MFCC for multiple layers simultaneously.



a) b) c) d)

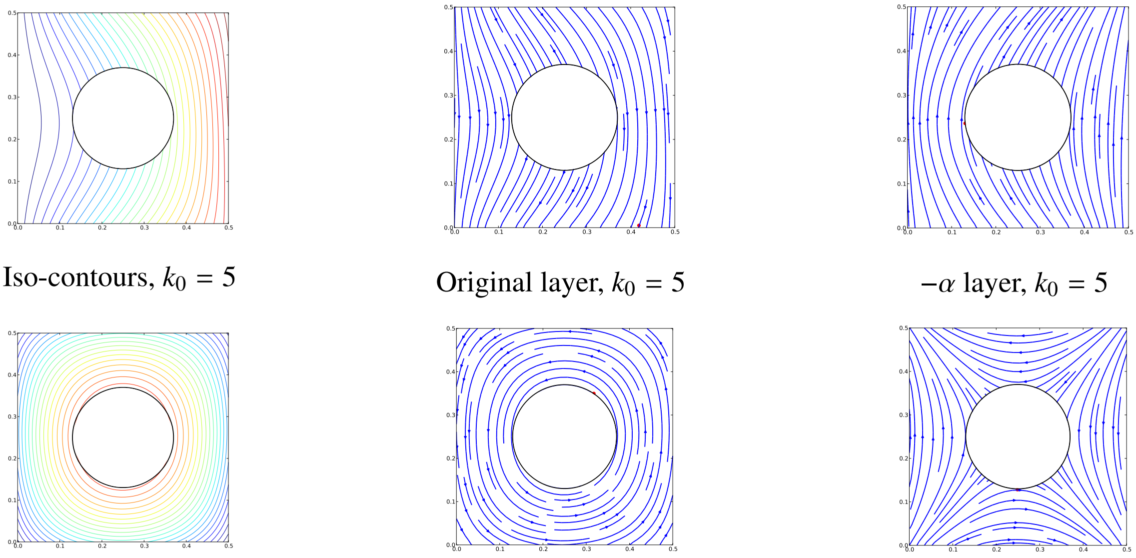
Figure 5. a) Problem example, b-c) Stiffness distributions displayed by using lamination parameters [17], d) Miki diagram of the angles distribution for b-c).

The fiber distributions for the unconstrained problem obtained by the proposed Iso-contour method for $[\pm\alpha]_s$ layers and 3×3 control points are shown in Fig. 6, with the maximum curvature location indicated by a red point for each layer. As can be seen, very similar results were obtained by Hesse [17], the fibers are aiming to be straight near the straight edges and at 45° near the middle (green areas in Fig. 5c,d). Results for the constrained case are shown in Fig. 7. It can be seen, that depending on the MFCC, very different optimal designs are obtained.



Iso-lines, $k_0 = \infty$ Original layer, $k_0 = \infty$ $-\alpha$ layer, $k_0 = \infty$

Figure 6. Optimal iso-contours for unconstrained $[\pm\alpha]_s$ laminate.

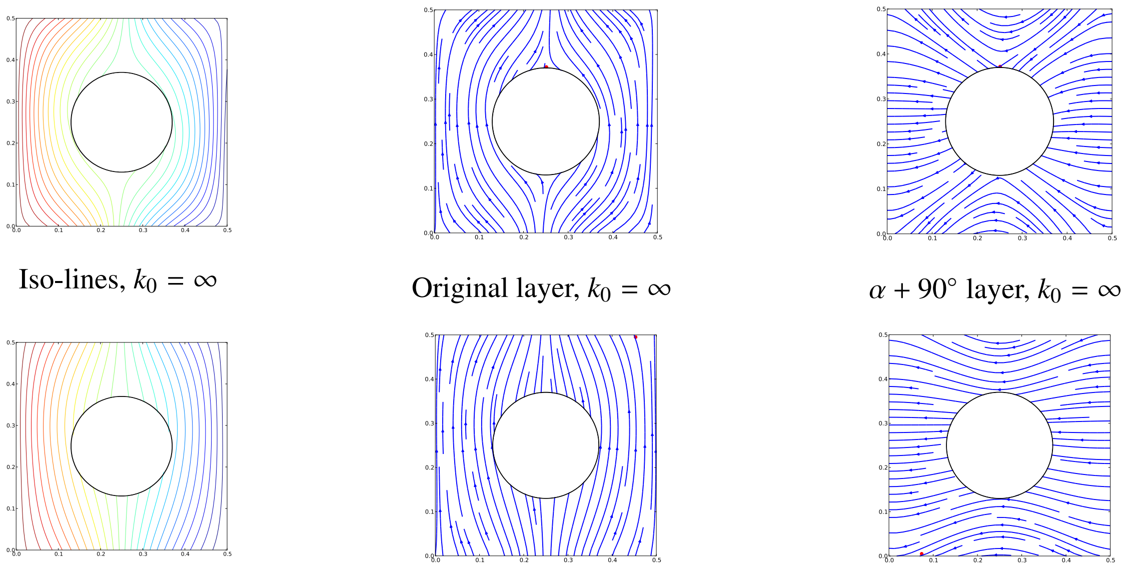


Iso-contours, $k_0 = 5$ | Original layer, $k_0 = 5$ | $-\alpha$ layer, $k_0 = 5$

Iso-contours, $k_0 = 10$ | Original layer, $k_0 = 10$ | $-\alpha$ layer, $k_0 = 10$

Figure 7. Iso-lines obtained with different MFCC for $[\pm\alpha]_s$ laminate, k_0 $[m^{-1}]$.

Even better optimal layouts can be obtained for laminate with the $[\alpha, \alpha + 90°]_s$ layer design. For this case, 4×4 control points are used. As can be seen, here also the straight fiber directions near the middle can be obtained (see Fig. 8).



Iso-lines, $k_0 = \infty$ | Original layer, $k_0 = \infty$ | $\alpha + 90°$ layer, $k_0 = \infty$

Iso-lines, $k_0 = 5$ | Original layer, $k_0 = 5$ | $\alpha + 90°$ layer, $k_0 = 5$

Figure 8. Iso-lines obtained with different MFCC for $[\alpha, \alpha + 90°]_s$ laminate, k_0 $[m^{-1}]$.

The critical loads for the buckling problem are normalized here with respect to the critical loads of a structure with ±45° ply orientation. For the constant stiffness laminate, an optimal ±92° ply orientation was obtained from optimization. Optimal fiber-steered laminates are obtained for different number of control points, different MFCC values k_0 and different layer designs, see Table 2. As can be seen $[\alpha, \alpha + 90°]_s$ layer design gives better results than $[\pm\alpha]_s$ design. Also the variation of the number of control

Excerpt from ISBN 978-3-00-053387-7

points has a clear influence on optimal results; more control points provide higher fiber path flexibility to improve the objective while satisfying complicated maximum curvature constraints.

Table 2. Comparison of optimization results for the critical buckling load.

			Fiber-steered laminates							
	$[\pm 45°]_s$	$[92°]_s$	$[\alpha + 90°]_s$, 4×4 points		$[\alpha + 90°]_s$, 3×3 points		$[\pm\alpha]_s$ 3×3 points			
$k_0[m^{-1}]$	-	-	∞	5	∞	5	∞	20	10	5
P_{cr}	1.0	1.387	1.997	1.726	1.982	1.56	1.89	1.83	1.766	1.502

5. Conclusion

In this work, the manufacturing maximum curvature constraint for the Iso-contour optimization method of the fiber-steered composite materials was presented. The method was tested with several 2D problems and produced reasonable results, which are in an agreement with the solutions obtained using alternative techniques and reported earlier in the literature. The obtained designs provide significant improvements over constant stiffness laminate, e.g. the critical buckling force for the fuselage panel can be almost doubled while keeping the same weight. The method is able to accurately handle maximum curvature constraint for multi-layered fiber-steered composites, e.g. $[\alpha, \alpha + 90°]_s$, $[\pm\alpha]_s$, where the maximum curvature can differ significantly for different layers. Moreover, as the curvatures are computed directly from the artificial surface function, obtained values are independent from the FE discretization and are calculated within almost negligible time, compared to the FE analysis.

Further research will be focused in taking the effect of changing fibers density for non-parallel fiber tows designs. A more sophisticated optimization technique is another important method improvement area, which was not the focus of this paper. In general, the method can be extended to the case, when each layer has an independent artificial surface, to allow independent fiber paths placement. The potential of this approach will be studied in the future.

Acknowledgments

The authors would like to thank the Aerospace Multidisciplinary Enabling Design Optimisation project (AMEDEO), Marie Curie Actions - Research Fellowship Programme, Seventh Framework Programme (FP7).

References

[1] D. M. J. Peeters, S. Hesse and M. M. Abdalla,*"Stacking sequence optimisation of variable stiffness laminates with manufacturing constraints"*, Composite Structures, vol. 125, pp. 596-604, 2015

[2] A. Arsenyeva and F. Duddeck, *"Iso-contour method for optimization of steered-fiber composites"*, In: 10th ASMO UK/ISSMO Conference on Engineering Design Optimization, Delft, The Netherlands, 2014.

[3] Z. Gürdal, R. Olmedo,*"Composite laminates with spatially varying fiber orientations: variable*

Excerpt from ISBN 978-3-00-053387-7

stiffness panel concept." In: Proceedings of the 33rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Dallas,Texas,USA, 1992.

[4] S. Honda, T. Igarashi and Y. Narita, *"Multi-objective optimization of curvilinear fiber shapes for laminated composite plates by using NSGA-II."*, Composites Part B: Engineering, vol. 45, no. 1, pp. 1071-1078, Feb 2013.

[5] S.T. Ijsselmuiden, M.M. Abdalla, S. Setoodeh, Z. Gürdal, *"Design of Variable Stiffness Panels for Maximum Buckling Load Using Lamination Parameters."*, In: Proceedings of the 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Schaumburg, Illinois, USA, 2008.

[6] S.T. Ijsselmuiden, *"Optimal design of variable stiffness composite structures using lamination parameters"*, PhD Thesis, Delft University of Technology, Delft, The Netherlands, 2011.

[7] D. H.-J. Lukaszewicz, C. Ward and K. D. Potter, *"The engineering aspects of automated prepreg layup: History, present and future"*, Composites Part B: Engineering , vol. 43, no. 3, pp. 997-1009, 2012.

[8] A.W. Blom, *"Structural Performance of Fiber-Placed, Variable-Stiffness Composite Conical and Cylindrical Shells"*, PhD Thesis, Delft University of Technology, Delft, The Netherlands, 2010.

[9] Z. Zamani, H. Haddadpour, M.R. Ghazavi, *"Curvilinear fiber optimization tools for design thin walled beams"*, Thin-Walled Structures, vol. 49, pp. 448454, 2011.

[10] C.S. Lopes, Z. Gürdal, P.P. Camanho, *"Variable-stiffness composite panels: Buckling and first-ply failure improvements over straight-fibre laminates"*, Composite Structures, vol. 86, no. 9, pp. 897-907, 2008.

[11] S.H. Hesse, A. Arsenyeva, D.H.-J.A. Lukaszewicz, F. Duddeck, *"A Parameterized Surface Method to Determine Optimal Variable Stiffness Layup by Global Search"*, In: 20th International Conference on Composite Materials, Copenhagen, Denmark, 2015

[12] B. M. Adams, K. R. Dalbey, M. S. Eldred and L. P. Swiler, *"DAKOTA, a multilevel parallel object-oriented framework for design optimization, parameter estimation, uncertainty quantification, and sensitivity analysis"*, Version 5.2 Reference Manual, Sandia National Laboratories Albuquerque, New Mexico, 2011.

[13] *"SciPy Reference Guide"*, Release 0.12.0, 2013

[14] H. Theisel, *"Vector Field Curvature and Applications"*, PhD Dissertation, Rostock, Germany, 1995

[15] M.G. Katz, *"Analytic and Differential geometry"*, Techn. Report, Department of Mathematics, Bar Ilan University, Ramat Gan, Israel, 2015

[16] S. Setoodeh, *"Optimal Design of Variable-Stiffness Fiber-Reinforced Composites Using Cellular Automata"*, PhD Dissertation, Virginia, USA, 2005

[17] S.H. Hesse, *"Multi-Objective Design Optimization of Steered Composite Structures"*, Master thesis, Delft University of Technology, Delft, The Netherlands, 2012

Excerpt from ISBN 978-3-00-053387-7