MODEL REDUCTION METHOD APPLIED TO 3D SIMULATION OF HONEYCOMB SANDWICHES WITH PARAMETRIZED GRADIENT OF PROPERTIES IN THE CORE

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Abstract
The goal of this study is to evaluate the effect of a gradient of properties in the core of a honeycomb composite sandwich. The considered sandwich is constituted of two composite laminate skins, and a polymer based hexagonal shaped honeycomb core. For a given stacking sequence of the two skins, we study the effect of a linear gradient of mechanical properties in the core of the sandwich. Our approach is to perform a full 3D simulation of the structure, where the microstructure (honeycomb cells) is properly discretized in order to obtain the full local strain and stress fields. If using a classical direct technique such as 3D finite elements method, resolving this type of problem with a sufficiently refined mesh would be computationally expansive. For that reason, the so called Proper Generalized Decomposition (PGD) model reduction technique is used to reduce the complexity of the 3D problem to a set of 1D and 2D finite elements subproblems. In addition, two parameters $E1$ and $E2$ representing the extreme values of the core properties are added to the simulation, resulting in a 5D simulation. The result of such parametric simulation lives in a 5 dimensions space, where $E1$ and $E2$ can take any values in a predefined range. The solution fields are explicit functions of the different coordinates, making the post-processing easy.

1. Introduction

The development of additive manufacturing techniques brings new possibilities for generating microstructures, or material properties gradation. In this paper we attempt to evaluate the potential benefits of functionally graded properties of a honeycomb core in composite sandwich structures. The objective here is not to address the manufacturing challenges to produce such parts, but to evaluate using finite elements simulations the potential benefits of those, assuming that they can be produced. For sake of simplicity, only linear variation of properties through the thickness of the honeycomb core is considered, the sandwich skins being regular composite laminates. Although extensive literature exists for plates and shells simulation, homogenization techniques can become complicated when comes the edge effects corrections [1] [2] [3] or in our case gradient of properties trough the thickness of a sandwich structure [4] [5]. For those reasons, our study will be carried using 3D Finite Element Method (FEM). The use of a 3D model gives easy access to local quantities allowing for easy post-processing or evaluation of quantities of interest (to evaluate debonding of the skin/honeycomb interface for example), however the

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discretization has to be fine enough to model the microstructure of the honeycomb, leading to high number of elements and therefore high number of degrees of freedom for the finite element problem. In order to efficiently solve for large 3D problems involved in this study, the Proper Generalized Decomposition (PGD) [6] [7] method is used in order to solve 3D and 5D multidimensional parametric problems at low computational costs [12] [9] [10] [11]. The numerical method used for solving 3D problems using a PGD based space variables separation will be described first. The method will next be extended to solve parameterized problems, where the Youngs modulus of the honeycomb core is linearly changing from a value E1 on one side to a value E2 on the other side (where E1 and E2 are within a user defined range). Finally, a 3D Finite Element model was also used as a verification tool and time reference to benchmark the method.

2. SOLVING MULTIDIMENSIONAL PROBLEMS USING THE PGD

2.1. 3D linear elastic problem

The separated representation induced by the PGD method introduced in [12] has shown to provide impressive numerical cost savings. A similar in-plane-out-of-plane representation is used here. The domain Ξ is considered, such that $\Xi = \Omega \times I$, where $(x, y) \subset \Omega$, $\Omega \in R^2$ and $z \subset$, $\in R$. The unknown displacement field for the 3D problem writes:

$$\mathbf{u}(x,y,z) = \begin{pmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{pmatrix} \approx \sum_{i=1}^{N} \begin{pmatrix} P_u^i(x,y) \cdot T_u^i(z) \\ P_v^i(x,y) \cdot T_v^i(z) \\ P_w^i(x,y) \cdot T_w^i(z) \end{pmatrix} \tag{1}$$

A more compact expression of $\mathbf{u}$ can be defined using the Hadamard product: each vector $\mathbf{P}^i(x, y)$ and $\mathbf{T}^i(z)$ contain the three components.

$$\mathbf{u}(x,y,z) \approx \sum_{i=1}^{N} \mathbf{P}^i(x,y) \cdot \mathbf{T}^i(z) \tag{2}$$

All the functions $\mathbf{P}^i(x, y)$ and $\mathbf{T}^i(z)$ are free from any hypothesis and calculated one after the other, differentiating from plates theories where kinematic hypothesis are assumed. Moreover, the number of products of function N is arbitrary, and the enrichment procedure will keep going until a convergence criterion is satisfied. The weak form for a 3D linear elasticity problem reads:

$$\int_{\Xi} \varepsilon(\mathbf{u}^*)^T \cdot \mathbf{K} \cdot \varepsilon(\mathbf{u})\, d\mathbf{x} = \int_{\Xi} \mathbf{u}^* \cdot \mathbf{f}_d \, d\mathbf{x} + \int_{\Gamma_N} \mathbf{u}^* \cdot \mathbf{F}_d \, d\mathbf{x}, \quad \forall \mathbf{u}^* \tag{3}$$

Where $\mathbf{f}_d$ are body forces, and $\mathbf{F}_d$ are surface forces. Those quantities are assumed to have a separated representation as well in order to separate all the integrals involving (x, y) to those involving z. is the symmetrical part of the gradient of the displacement field:

$$\varepsilon(\mathbf{u}) = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right) \tag{4}$$

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The method is based on the use of a greedy algorithm, where products of functions of the different variables of the problem are computed one at the time, until a criterion on the global residual of the problem is reached. Considering $n-1$ products of functions are already computed, the next product of function to be computed will be $\mathbf{P}^n(x,y)\cdot\mathbf{T}^n(z)$, such that:

$$\mathbf{u}^n(x,y,z) = \mathbf{u}^{n-1}(x,y,z)\mathbf{P}^n(x,y)\cdot\mathbf{T}^n(z) \tag{5}$$

The new functions $\mathbf{P}^n(x,y)$ and $\mathbf{T}^n(z)$ have to be computed and added to the existing basis. After substituting equations 4 and 5 in the weak form, a non-linear problem involving the new functions $\mathbf{P}^n(x,y)$ and $\mathbf{T}^n(z)$ has to be solved. Those new functions are alternatively computed using a simple fixed-point method, after one of them have been initialized to an arbitrary function. The norm of the residual of the problem is used to decide if more products of functions should be computed to enrich the already computed basis, similarly to the method described in [12].

2.2. Problem definition

The considered geometry is a sandwich structure, where each skin is a 4 plies carbon reinforced composite laminate, and the core is a polymer based supposed isotropic material. The mechanical properties are defined in Tables 1 to 3, and the geometry and boundary conditions are defined on Figure 1. The dimensions of the considered sandwich are $80\times40\times16$ *mm*. Each ply of the skin is 0.125 *mm* thick, and contains 20 linear elements in the thickness direction. The total number of hexahedral cells in the 3D reconstructed mesh is approximately 10^7,for approximately 3.10^7 degrees of freedom. The face $x = 0$ is fixed in all directions, and the loading on the top face consists of a unit pressure.

Table 1. Mechanical properties of one composite ply (1 is the direction of the fibers, 3 is the out of plane direction)

E_{11} (GPa)	140
E_{22} (GPa)	9.7
E_{33} (GPa)	9.7
ν_{12}	0.29
ν_{13}	0.29
ν_{23}	0.5
G_{12} (GPa)	5.585
G_{13} (GPa)	5.585
G_{23} (GPa)	3.3

Table 2. Mechanical properties of the core material for the homogeneous case

E (GPa)	3.5
ν	0.29

Table 3. Mechanical properties of the core material for the graded case

E_{min} (GPa)	2
E_{max} (GPa)	5
ν	0.29

One other advantage of the method is the problems involving the (x,y) coordinates are independent from the problems related to the z coordinate. Consequently, a large number of degrees of freedom can be added in the thickness direction with little extra numerical cost, since the problems to solve are only

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1D problems. This is particularly suitable in the case of composite laminates since the anisotropy jump between the plies induces very localized high gradients of stress across the laminate, which only a very fine mesh can capture.

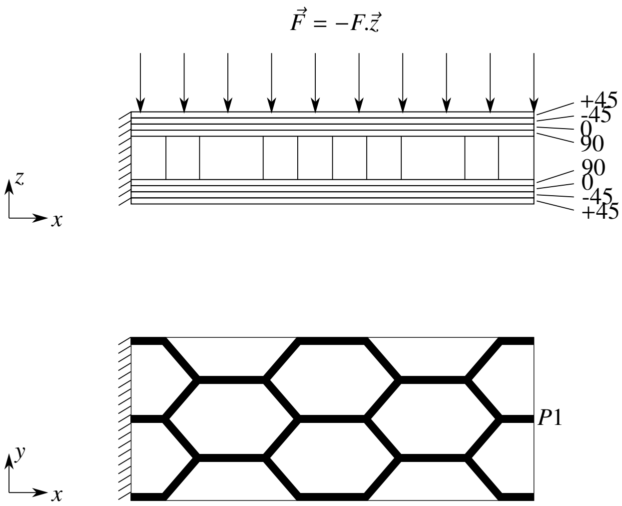

Figure 1. Geometry definition

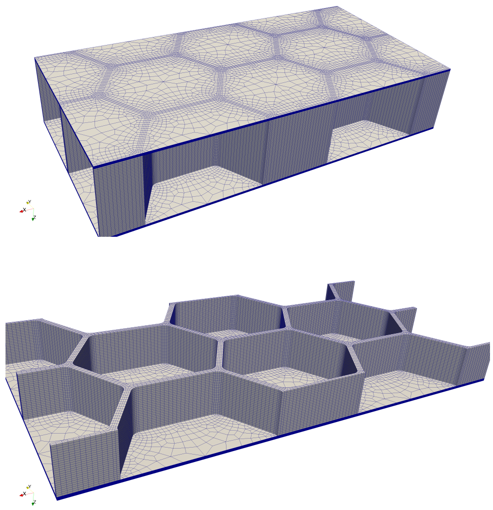

Figure 2. Representation of the meshed structure. top: uncut, bottom: hidden top skin

In that section, a structure where the properties of the core are homogenous (see Table 2.) is compared to a structure with linearly graded properties as function of the z coordinate (see Table 3.). The value for the Youngs modulus of the homogenous one is chosen to be the average value of the linearly graded one, in order to obtain a global stiffness for both cases in the same order of magnitude.

2.3. Results

As expected, the deflection for the two problems is in the same range (see Figure 3.). However, some differences arise on the strain and stress fields. Since the graded case has a lower stiffness on the top of the core, the strain amplitude is larger in that region. The stress is consequently distributed differently as well in this region.

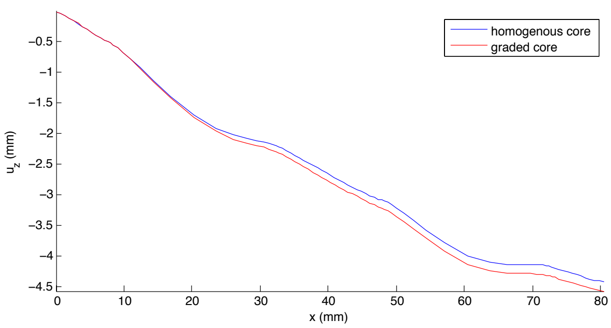

Figure 3. Deflection of the centerline, on the top of the structure as function of x

Since only one gradient of modulus have been modeled in that section, it is hard to conclude about the interest of such a properties repartition at this point. For that reason, a parameterized model has been

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developed in order to investigate further the influence of a gradient of properties (see next section).

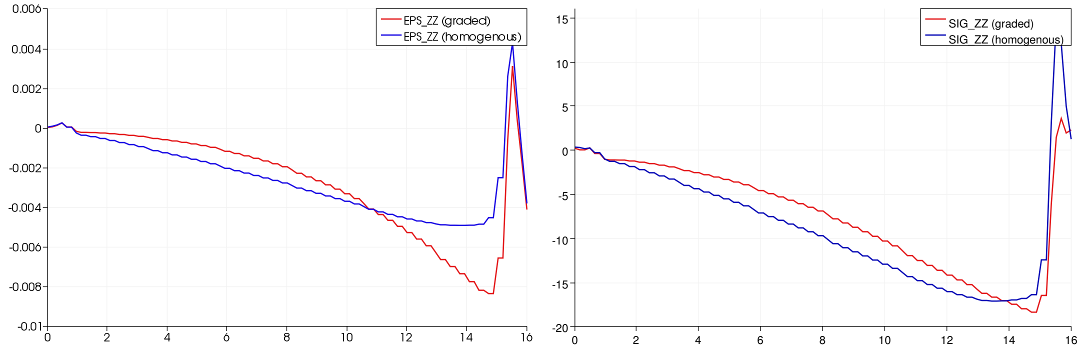

Figure 4. Strain and stress on the vertical line located under $P1$ as function of z

3. 5D parameterized problem

3.1. Motivation

Results from the previous section are encouraging, however, a parametric study should be performed to understand better the effect of the minimum and maximum values of the Youngs modulus for the functionally graded case. A systematic parametric study would require solving several 3D cases, varying the properties of the graded core in order to obtain an exploitable dataset for post processing quantities of interest. Another approach could be to define first a quantity of interest, and optimize the set $\{E1, E2\}$ using an optimization algorithm. Our approach here will take advantage of the parametric capabilities of the PGD based resolution scheme. Instead of running separately a large number of 3D simulations, using or not an optimization algorithm, only one parameterized simulation is performed once. By solving the problem on all the parameter space at once, the method takes advantage of the similarities between the solutions of the problem for different values of the parameters, allowing for large computation time saving. Later on, an optimization algorithm can be used a posteriori to find optima values for any number of cost functions in the numerical chart, without requiring solving any more 3D problem. Moreover, the finite element interpolation on the $E1$ and $E2$ spaces generate a virtually infinite number of combinations for the top and bottom values within a given minimum and maximum possible values of the Youngs modulus across the honeycomb core.

3.2. (3+2)D parameterized linear elastic problem

In this section, a more complex problem is addressed: the Youngs modulus of the core of the honeycomb is parameterized using two parameters $E1$ and $E2$. The properties in the core will be a linear variation between a user defined arbitrary value on one side and another user defined arbitrary value on the other side of the core. Consequently, the problem is now defined in a 5D space, where the solution field is expressed as follows:

$$\mathbf{u}(x, y, z, E1, E2) \approx \sum_{i=1}^{N} \mathbf{P}^i(x, y) \cdot \mathbf{T}^i(z) \cdot \mathbf{E1}^i(E1) \cdot \mathbf{E2}^i(E1) \tag{6}$$

$E1$ and $E2$ are two 1D spaces, where $E_{min} < E1 < E_{max}$, and $E_{min} < E2 < E_{max}$. Since a continuous finite element linear interpolation scheme is used for $E1$ and $E2$ values, within a predefined range, this

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5D problem is virtually equivalent an infinite number of 3D problems where $E1$ and $E2$ are fixed. For that particular problem, one key parameter is to be able to define the material properties as function of the five coordinates $(x, y, z, E1, E2)$. The separated representation of the elasticity tensor can be expressed as:

$$\mathbf{K}(x,y,z,E1,E2) = \sum_{i=1}^{N_{plies}} \mathbf{K}^i \cdot \chi_z^{i^{laminate}}(z) + \mathbf{K}1(E1)\cdot\chi_{xy}^{core}(x,y)\cdot\chi_z^{core}(z)\cdot\alpha(z) + \mathbf{K}2(E2)\cdot\chi_{xy}^{core}(x,y)\cdot\chi_z^{core}(z)\cdot\beta(z) \tag{7}$$

Where $\mathbf{K}^i$ is the elasticity tensor for the i^{th} ply of the skin, and $\chi_z^{i^{laminate}}(z)$ is a characteristic function whose value is 0 everywhere except in the i^{th} ply, where its value is 1. $\mathbf{K}1$ is the elasticity tensor field defined in the $E1$ 1D space, for which the Youngs modulus varies from E_{min} to E_{max} as function of the $E1$ coordinate. $\mathbf{K}2$ is similarly defined as function of $E2$. $\chi_{xy}^{core}(x,y)$ is a function of x and y that is 0 out of the projection of the honeycomb core on the (x,y) plane, and 1 inside. $\chi_z^{core}(z)$ is the function of z whose value is 1 in the core, and 0 in the skins, and finally α and β are two linear weight functions respectively going from 0 to 1 and 1 to 0 between the z position of the beginning and the end of the honeycomb core.

Note that the Poisson coefficient is kept constant (without loss of generality) at 0.29 in the function $\mathbf{K}1$ and $\mathbf{K}2$. The procedure for resolving the problem is exactly similar to the one described in the section 2, except that now four functions are involved in the constructor of a new set of functions.

3.3. Results

A parameterized simulation has been performed, using the same geometry and boundary conditions than in the previous section. The 1D spaces for both $E1$ and $E2$ are discretized with 201 equally space points from 1 GPa to 10 GPa. The compact representation of the solution allow to easily access to any value of any field, and easily post-process them by choosing which coordinate to particularize, and which one not to particularize. For instance, the deflection of point $P1$ at the top of the top laminate (see Figure 1.) is shown on Figure 4 as function of $E1$ and $E2$. All results on the diagonal line $E1 = E2$ correspond obviously to a homogenous core, and any deflection obtained with a homogenous core can also be obtained by a large range of values of $E1$ and $E2$ (following isovalues on Figure 4.).

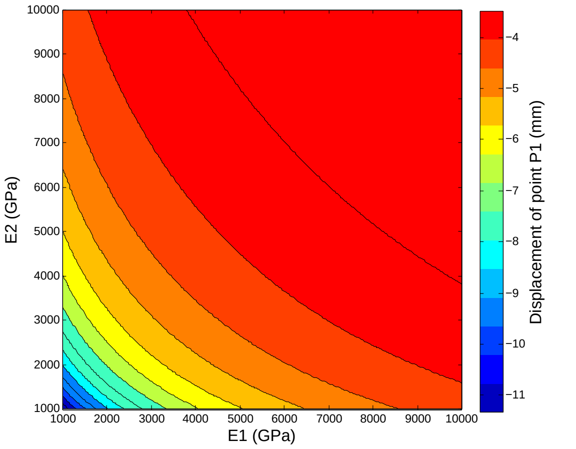

Figure 5. Displacement of the point $P1$ (see Figure 1.) as function of $E1$ and $E2$ values

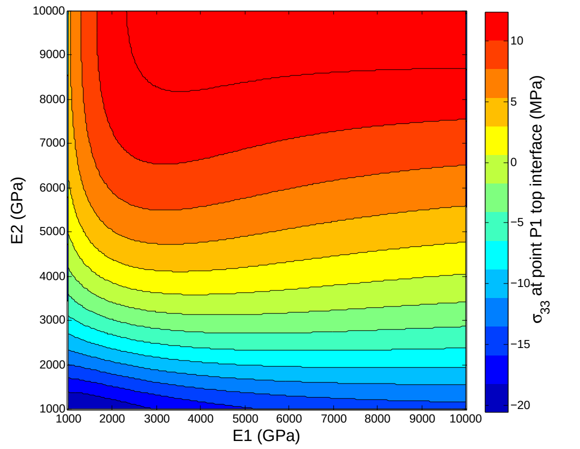

Figure 6. Out-of-plane stresses at the top interface between the honeycomb and the laminate at point $P1$

As suggested in [13], the out-of-plane stress can be used to determine the crack initiation between the core and the lamina. Thanks to explicit expression of the stress as function of all five coordinates of the problem, it is extremely easy to minimize the stress at the interface. For this simple case, it is obvious that for a given deflection, the smallest value of $E1$ will minimize the stress at the top interface. Also, any strain, stress or mixed criteria can be post-processed in the same way thanks to the explicit values of all quantities as function of the coordinates. 3D Finite element reference solution and time benchmarking A 3D finite element simulation of the same problem has been performed using Virtual Performance Solution (VPS). The results shows excellent agreement with the PGD based approach. Table 4. Shows the computing time comparison between the different solvers. The computing time is however qualitative since a different machine was used to resolve the 3D finite elements problems using VPS, and the PGD based reduced models. Also, as demonstrated in [12] and [10], the time gain using the 2D/1D PGD based approach versus the standard 3D finite element method increases with the problem size. Figure 7. shows the deflection of the central line on the top of the structure. The parameterized solution matches very well the non-parameterized one and the 3D solution from VPS. The computing time however is even more impressive considering the content of the parameterized solution. It is difficult to evaluate how long it would take to generate this kind of database using standard 3D finite elements method, since there would not be any kind of interpolation between the different solutions, but most definitely it would be orders of magnitude higher. Moreover, once the multidimensional solution is computed, any number of optimizations can be run without running any more simulation, just post-processing the existing one.

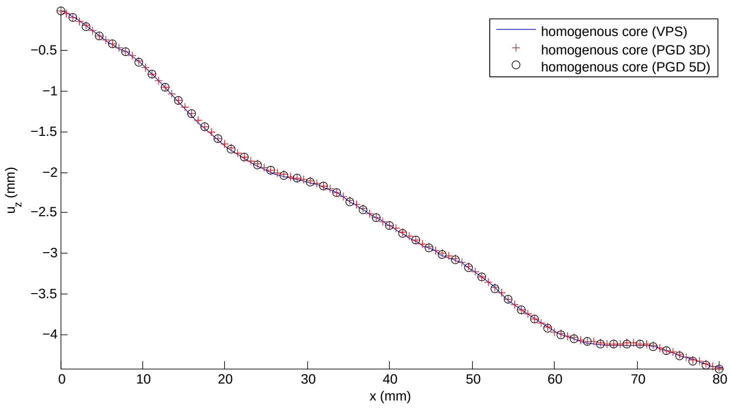

Figure 7. Deflection of the line on the top of symmetry plane for the homogenous case

Table 4. Time comparison between commercial software and research code

	VPS computing time (s)	PGD 2D/1D computing time (s)	PGD 2D/1D/1D/1D computing time (s) (parameterized problem)
Homogenous core	7096	3431	43530
Graded core	7029	3526	

4. Conclusions

In this paper, an efficient method to solve complex 3D problems on large structures has first been presented. The method is based on the separation of variables using the so-called Proper Generalized Decomposition. It is particularly adapted to solve 3D problems on a kind of structures where the quantities can be expressed as sums of products of in-plane and a out-of-plane functions. The method has next

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been extended to solve parametric problems, where for instance a linear variation of properties of a honeycomb core through the thickness is simulated using two parameters. The same method of resolution is used for the parameterized problem, but two extra sets of coordinates are added to the model, totaling five dimensions (three for space and two for parameters). The method demonstrates advantageous gains in term of computing time, and also provides an explicit solution of all coordinates, making all sorts of post-processing easy and fast. Finally, introducing a gradient of properties in the core of a sandwich seems to be profitable for the strength of the structure. Assuming any value for the Youngs modulus of the core can be generated, for a given deflection, a well-chosen gradient of properties decreases the stress at the interface between the skin and the honeycomb core, maximizing the strength of the structure by postponing the onset of damage initiation at the skin/core interface.

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