Intra–laminar Progressive Damage of General Configuration Laminated Composites

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Abstract

A new analytical model for progressive damage in form of matrix cracking in continuous fiber reinforced composites is developed, based on the extension of the energy principle from fracture mechanics to the particular behavior of matrix crack multiplication in laminated composites. The model is applicable to general laminate stacking sequence, and is able to consider the effect of both membrane and flexural deformation. The increase of material fracture toughness during matrix crack multiplication specific to fiber reinforced laminates (*R–curve* behavior) is attributed to crack fiber bridging, and is included into the present model. New parameters describing the R–curve behavior, and the experimental procedure for their determination, are proposed. By its energy based formulation, the model is able to predict the scale effects specific to damage initiation and progression in laminated composites (e.g., the ply thickness dependence of the damage process), which is not possible by using traditional strength based criteria.

Key words: matrix cracking, fracture toughness, resistance curve, fiber bridging

1 Introduction

Two main aspects are addressed by the *progressive damage model*. First, the degradation in thermo–elastic material properties as a function of damage level can be described by the equation $[C] = [C(\lambda^{(k)})]$, where $[C]$ is the generic notation for the material thermo–elastic properties, and λ is the damage state variable (i.e., the metric of damage level inside of the material); the superscript (k) represents the ply index, and it is used for the case when individual damage state variables $\lambda^{(k)}$ are assigned to individual plies (k) of the laminated material. Second, damage onset and progression due to external

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thermo–mechanical loading can be described by the equation $\lambda^{(k)} = \lambda^{(k)}(\varepsilon)$, where ε is the generic notation for the composite laminate deformation. Here, onset and evolution criteria are required in order to detect the conditions for matrix crack multiplication in individual plies of the laminate. Limitations of the strength of materials based criteria to accurately describe the progressive damage process in laminates have been presented in [1, 2]. Alternatively, fracture mechanics based criteria can be used [3, 4], as generically described by the equation:

$$F(G_i, G_{Ci}) \geq 1 \tag{1}$$

where G_i is the *energy release rate (ERR)* (defined as the strain energy released for the formation of a new unit surface of crack), and G_{Ci} is the *critical energy release rate* (or the *fracture toughness*), which is a material parameter and represents the material resistance to crack formation or extension; the subscript i in (1) is used to denote modes I, II or III crack formation. The criterion proposed in [3] is selected in the present model, for the mixed modes transverse crack formation in individual plies of laminated composites of general stacking sequence.

However, there is indication that suggests that the fracture toughness G_C could feature an increase with the damage level (R–curve behavior). First observations in this sense where made based back–calculated G_C values through analytical damage models [5, 6], and experimental evidence was presented in [7]; additional evidence can be found in damage micromechanics studies based on new imaging methods like micro-CT [8, 9]. Due to the observed R–curve behavior, even the idea that a G_C based damage evolution law is inappropriate to describe the progressive matrix cracking in laminated composites has been formulated in some studies [7, 10].

The work presented in the present manuscript puts an accent on the later aspect of the progressive damage model, as described by (1); the former aspect, of the degraded material properties, is presented elsewhere [11, 12]. The model proposed here includes the R–curve behavior as expected to be encountered in laminated composites, and then the damage evolution law will consequently have this effect built–in. Based on current evidence, the model is implemented only for the critical parameter for mode I of cracking formation G_{CI}. Even if it is possible that a similar behavior would be encountered for mode II crack formation, there is no available data in support of this assumption. This is why G_{CII} is considered constant in the present model, and G_C actually denotes G_{CI} in the following, for the simplicity of notations.

2 Laminate fracture toughness resistance curve

The phenomenological explanation of the straightening effect $G_C = G_C(\lambda, t)$ is based on additional energy dissipation mechanisms that take place during transverse matrix crack formation: isolated fiber breaks and delaminations in the plies neighboring the cracking ply, and especially *fiber bridging* of the separated faces of the cracks. The load–damage mechanism for tunneling cracks formation and multiplication in a laminated composite is different compared to the one encountered for splitting crack formation and growth in an UD composite.The tunneling crack suddenly spans the whole width of the laminated

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composite specimen, for a brittle composite system. However, if we regard the moment of the first crack occurrence, there will be bridging fibers in between the two faces of the first crack, Fig. 1 (c). Function of some conditions, among which we can assume the elastic and strength properties of the fiber and matrix constituents, the strength of the fiber–matrix interface, the thickness t of the cracking ply, the crack opening displacement (COD), and especially the fibers misalignment inside the composite which basically governs the fibers bridging effect, there might be fractured and debonded fibers from the very first crack occurrence in the laminate. There will be a corresponding critical ERR value for the formation of the first crack, which is noted here as G_{C0}. When the crack

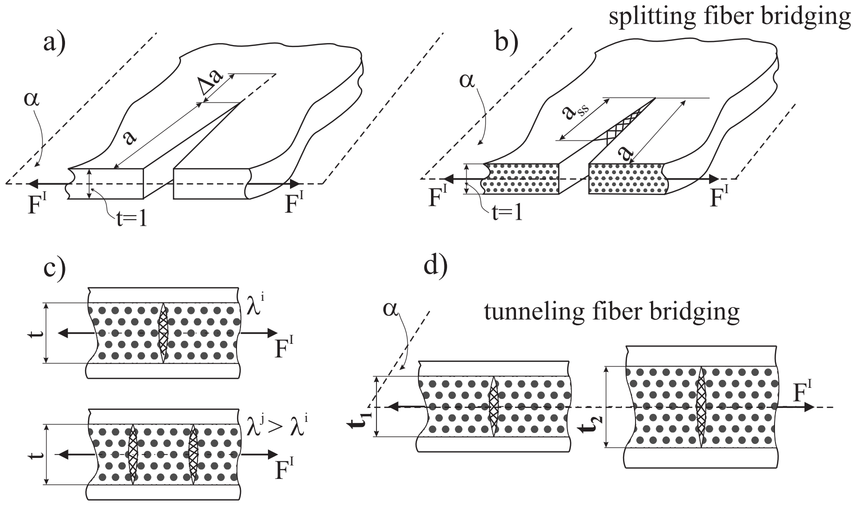

Fig. 1. Mode I crack extension: a) isotropic; b) UD FRP splitting crack; c) Laminated FRP tunneling cracks: crack multiplication at constant ply thickness; d) Laminated FRP tunneling cracks at different plies thicknesses.

density increases under loading (let's say, from state λ_i to state λ_j in Fig. 1 (c)), new bridging cracks corresponding to the λ_j damage state are formed, while new fibers from the previous λ_i bridging cracks will be broken and/or debonded. The result is a cumulative energy dissipation effect due to cracks fiber bridging (and most probable other additional dissipation mechanisms, like isolated fiber breaks and delaminations) at an overall scale of the laminated composite. The resistance to crack multiplication is continuously increasing due to the fact that increasing part of the stored elastic energy is released through the aforementioned cumulative dissipation effects, from the previously formed cracks.

The bridging fibers in an existing crack will never be totally broken and the two faces of the crack will never be totally stress free, due to the small COD. It is thus expected that there will be no self similar crack multiplication at the laminate level, and there will probably be no steady state value of G_C; there is instead a continuous increase $G_C = G_C(\lambda)$, denoted as the *R–curve* behavior of matrix cracking, as depicted in Fig. 2. The $G_C(\lambda)$ variation can be modeled based on this phenomenological explanation. This is achieved here based on an assumed linear variation (see Fig. 2), which is in agreement with the back–calculated data in [13] and with the experimental data in [7], but it is not in a very good agreement with the back–calculated data in [14]. While the linear characteristic is implemented in the present analytical model, a higher order polynomial or an exponential variation might better describe some material systems and processing conditions. Another aspect to be considered is the ply thickness effect on the R–curves

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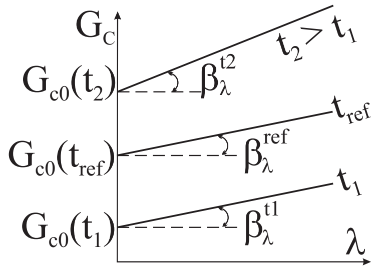

Fig. 2. *In–situ, R–curve* model for transverse crack multiplication in laminated FRP.

[7, 10]. It can be explained based on the fact that the plain strain assumption in the case of traditional fracture mechanics (which is the plane α in Fig. 1 (a) for isotropic material and in Fig. 1 (b) for orthotropic composite) is not satisfied any more for the case of a laminated composite; this is because of the constraining effect of the neighboring plies on the COD of the cracking ply. The plane α in Fig. 1 (d) can not be regarded as a plain strain plane any more. There is no self–similarity over the thickness of the cracking ply. There will probably be more broken and pulled off fibers at the middle of the cracking ply than at the plies interface (based on higher COD at the middle), or in the thicker ply than in the thinner one (based on higher COD at higher ply thickness). This would have the consequence of a ply thickness influence on the aforementioned dissipative mechanisms, regarded here as the *in–situ* effect $G_C = G_C(\lambda, t)$, as schematically presented in Fig. 2. It is expected that the ply thickness t has an influence on both the critical ERR for the formation of the first crack G_{C0}, and the slope β of the subsequent crack evolution $G_C(\lambda)$: $G_{C0}(t_2) \geq G_{C0}(t_1)$ and $\beta_\lambda^{t2} \geq \beta_\lambda^{t1}$ for $t_2 > t_1$.

a) The R–curve effect on G_C at $t = t_{ref}$

A first set of experimental data for matrix crack onset and multiplication, $\lambda = \lambda(\varepsilon)$, at an arbitrary reference thickness of the cracking ply $t = t_{ref}$, is needed for the calibration of the *R–curve* behavior of a given composite material. The needed $\lambda = \lambda(\varepsilon)$ experimental data is preferable to come from a $[S/90_n]_S$ laminate configuration under axial loading, such that only mode I crack formation exists. The reference thickness t_{ref} will be further used as a base for the calibration of the *in–situ* behavior of the material at $t \neq t_{ref}$.

The R–curve behavior at the reference thickness is modeled by the linear equation:

$$G_C(\lambda, t = t_{ref}) = G_C^{ref} = G_{C0}^{ref} + \beta_\lambda^{ref} \cdot \lambda \tag{2}$$

The values of the critical ERR for the matrix crack onset (formation of the first crack) G_{C0}^{ref}, and the slope parameter β_λ^{ref} (see Fig. 2) are selected such that the set of experimental data $\lambda = \lambda(\varepsilon)$, at this arbitrary reference thickness $t = t_{ref}$, is best fit; the parameter $\beta_\lambda^{ref} \geq 0$ further affects the inclination of the $\lambda = \lambda(\varepsilon)$ curve for the matrix cracking evolution at the reference thickness, as depicted in Fig. 3 (a).

b) The in–situ effect on G_{C0} at $t \neq t_{ref}$

As stated before, the first parameter that can be influenced by the thickness of the cracking ply is the critical ERR for the onset of matrix cracking G_{C0} (see Fig. 2).

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This influence can be described in a linear manner by the equation:

$$G_{C0}(t) = G_{C0}^{ref} \cdot \Big[1 + \beta_0^t \cdot (t/t_{ref} - 1)\Big] \tag{3}$$

It can be noticed that the in–situ effect on the onset value $G_{C0}(t)$ is established by the fit parameter β_0^t, which is considered to be bounded by the interval $0 \leq \beta_0^t \leq 1$:
– for the bounding value $\beta_0^t = 0$: there is no in–situ influence on the critical ERR for damage onset: $G_{C0}(t) = G_{C0}^{ref} = \text{ct.}$;
– for the bounding value $\beta_0^t = 1$: there is a direct proportionality between the critical ERR for damage onset at $t \neq t_{ref}$ and the critical ERR for damage onset at $t = t_{ref}$, with the proportionality factor given by the ratio t/t_{ref}: $G_{C0}(t) = t/t_{ref} \cdot G_{C0}^{ref}$;
– for intermediate values $0 \leq \beta_0^t \leq 1$: the proportionality factor between the critical ERR for damage onset at $t \neq t_{ref}$ and the critical ERR for damage onset at $t = t_{ref}$ becomes $[1 + \beta_0^t \cdot (t/t_{ref} - 1)]$.

The in–situ effect on the critical ERR for damage onset G_{C0} is presented in $(G_C - \lambda)$ coordinates in Fig. 2 (see the vertical axis of the graph) and in $(\lambda - \varepsilon)$ coordinates in Fig. 3 (b); here, it can be noticed that the influence of the β_0^t parameter is on the damage onset strain ε_0.

It has to be noted that for a given material, even if the equality $G_{C0}(t) = G_{C0}^{ref}$ is assured for $\beta_0^t = 0$ in eq. (3), this does not imply that $\varepsilon_0(t \neq t_{ref})$ in Fig. 3 (b) becomes equal to ε_0^{ref} in Fig. 3 (a); this is because the onset of matrix cracking is influenced not only by the critical ERR for onset G_{C0}, but it is also influenced by the laminate configuration.

One extra experimental determination of the moment of damage onset (ε_0 in Fig. 3 (b)), at an arbitrary thickness of the cracking ply $t \neq t_{ref}$, is needed in order to evaluate the fit parameter β_0^t.

c) The combined in–situ and R–curve effect on G_C at $t \neq t_{ref}$

As stated before, the second parameter that can be influenced by the thickness of the cracking ply is the slope of the $G_C(\lambda)$ variation at $t \neq t_{ref}$ (see Fig. 2). This influence can be captured in a linear manner by the equation:

$$G_C(\lambda, t) = G_{C0}(t) + \Big[1 + \beta_\lambda^t \cdot (t/t_{ref} - 1)\Big] \cdot \beta_\lambda^{ref} \cdot \lambda \tag{4}$$

The new parameter β_λ^t in eq. (4) is considered bounded by the limit values $0 \leq \beta_\lambda^t \leq 1$:
– for the bounding value $\beta_\lambda^t = 0$: the slope of the $G_C(\lambda)$ variation at $t \neq t_{ref}$ remains equal to the slope value at $t = t_{ref}$, β_λ^{ref}; there is no in–situ effect on the slope of the $G_C(\lambda)$ variation;
– for the bounding value $\beta_\lambda^t = 1$: there is a direct proportionality between the slope value of the $G_C(\lambda)$ variation at $t \neq t_{ref}$ and the slope at $t = t_{ref}$ (β_λ^{ref}); the proportionality factor is given by the ratio t/t_{ref};
– for intermediate values $0 \leq \beta_\lambda^t \leq 1$: the proportionality factor between the slope value of the $G_C(\lambda)$ variation at $t \neq t_{ref}$ and the slope at $t = t_{ref}$ (β_λ^{ref}) becomes $[1 + \beta_\lambda^t \cdot (t/t_{ref} - 1)]$.

The in–situ effect on the slope of the $G_C(\lambda)$ variation is shown in $(G_C - \lambda)$ coordinated in

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Fig. 2, and in $(\lambda - \varepsilon)$ coordinates in Fig. 3 (c); here, it can be noticed that the influence of the β_λ^t parameter is on the inclination of the $\lambda(\varepsilon)$ crack evolution curve.

A full experimental curve $\lambda = \lambda(\varepsilon)$ at a cracking ply thickness $t \neq t_{ref}$ is needed in order to evaluate the fit parameter β_λ^t.

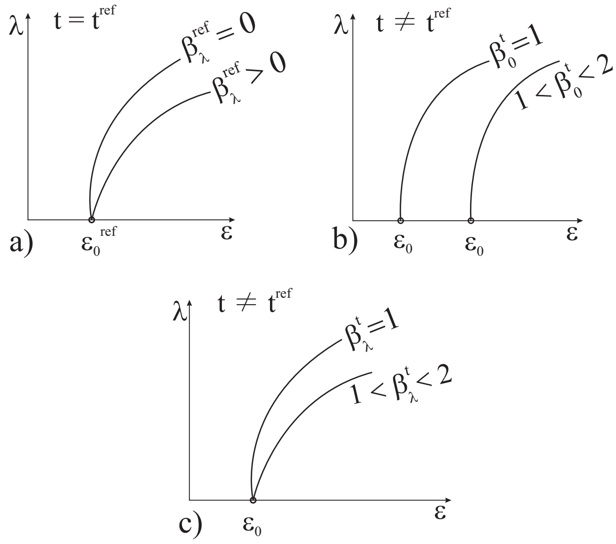

Fig. 3. Influence of the model parameters on the model predictive behavior.

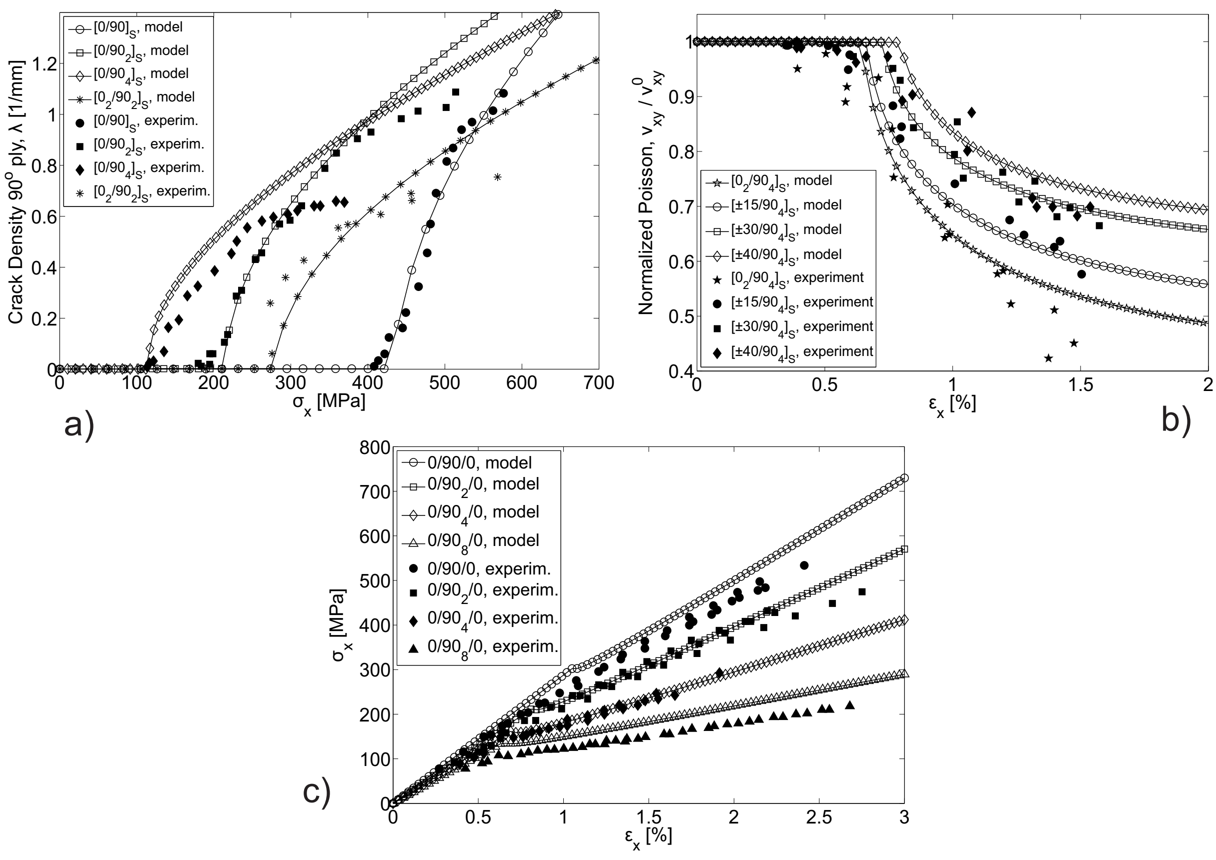

Fig. 4. a) Cracking density evolution, model vs. experiment [15, 16]; b) Poisson coeff. variation, model vs. experiment [17]; c) Stress–strain curves, model vs. experiment [10].

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As a final remark, if the fit parameters in eq, (2), (3), (4) are set to the values $\beta_\lambda^{ref} = \beta_0^t = \beta_\lambda^t = 0$, then a model with no R–curve behavior and no in–situ influence is retrieved.

Validation of the model predictions against experimental data for various materials and laminate configurations is shown in Fig. 4

3 Conclusions

A physically meaningful damage variable is used by the present model, namely the crack density in individual plies of the laminate. The selection of such damage metric offers additional information compared to phenomenological models based on stiffness reduction scalar damage variables. By calculating and tracking the crack density during loading, the model offers a higher resolution of the damage process for those applications where the gas/liquid permeability of the structure is a limiting design factor (e.g., pressure vessels). The effect of thermal residual stresses is also included into the present analytical model, which extends the range of the applicability of the model to structural applications at very low (cryogenic) temperatures.

Two experimental sets of matrix cracking onset and multiplication, at two different thicknesses of the cracking ply, are needed in order to extract the additional parameters of the analytical model. It is clear that additional non–standard material parameters is not an attractive idea for engineers. However, it is also clear that additional physics of the material behavior can not be captured, and additional high resolution information can not be obtained, without paying the price of the need for additional material parameters. In the current model, these parameters will account not only for the properties of the fiber and matrix constituents, but also for the whole set of additional processing factors, which might be different from one manufacturing site to another.

The current implementation corresponds to mode I transverse matrix cracking in laminated composites. Additional experimental investigation and research is needed in order to understand and decide if a similar material behavior and modeling approach ca also be applied to the mode II crack formation.

Acknowledgments

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