MODELLING THE HYSTERESIS BEHAVIOR OF FABRIC CARBON COMPOSITE USING A COLLABORATIVE ELASTOPLASTO-DAMAGE MODEL WITH FRACTIONAL DERIVATIVE

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Abstract
The paper deals with the modelling of self-heating test to ensure the fast fatigue limit estimation for woven composite materials. To measure the material self-heating, the intrinsic dissipation is calculated by the collaborative model. This model is able to represent the elastoplastic damage composite behavior as well as the viscoelastic phenomena such as the appearance of hysteresis loops and the strain-rate sensitivity. The dissipation due to the in-ply damage propagation, the material hardening and the viscoelastic effects is precisely calculated.

1. Introduction

The intensive use of composite materials in industrial applications, especially in the naval and the aeronautic sectors, requires the deep analysis of the materials' durability. The composites are anisotropic and heterogeneous materials. The classical S-N fatigue test, which consists of the application of large number of cycles, elevates the time and resource costs. A method based on the self-heating tests has been developed [1] to provide the fast fatigue limit identification for the composite materials compared to conventional methods (S-N curves). The self-heating method consists of applying a sequence of constant amplitude cyclic loading blocks. The stabilized temperature of the composite specimen is measured during each block. When the value of the stabilized temperature increases significantly, it is considered that the fatigue limit is attainted. The experimental results of the self-heating method are in good agreement with the conventional fatigue tests (Wöhler curve). The numerical simulation of self-heating test would simplify the analysis and increase the potential of the experimental method. The full thermo-mechanical material analysis is required for this. The collaborative model [2] is a good tool to calculate the intrinsic dissipation which leads to the material heating.

The collaborative model is composed of two sub-models. The first one is the elastoplastic damage model [3-5] which deals during loading path. The second sub-model describes the hysteresis behavior using a fractional derivative law during unloading path. Both sub-models are strain-rate dependent and can be applied for the dynamic problems. The collaborative model has been successfully applied to represent thermoset and thermoplastic composite material response under cyclic loading. The dissipation of internal energy due to different phenomena can be easily calculated by the collaborative model.

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2. Thermodynamic problem

In order to determine the fatigue limit using self-heating method, the thermodynamic stationary problem (Eq.1) should be resolved with the corresponding limit conditions (Fig. 1). We consider that specimen is placed between two jaws at different temperatures. The two useful areas are subjected to natural convection. The radiation heat loss is neglected.

$$\begin{cases} div(\vec{q}) - S = 0 \\ \vec{q} = -\lambda \overrightarrow{grad} T \\ \vec{q} \cdot \vec{n} = h(T - T_\infty) \\ T|_{y=l} = T_{top} \\ T|_{y=-l} = T_{low} \end{cases} \tag{1}$$

where $\vec{q}$ is a heat flux, S is a heat source associated with the visco-elastoplastic damage material behavior, T_∞ is an ambient temperature, T_{top}, T_{low} are temperatures of the top and the low jaws respectively.

To model the internal source S and to determine the material self-heating, a collaborative model is developed.

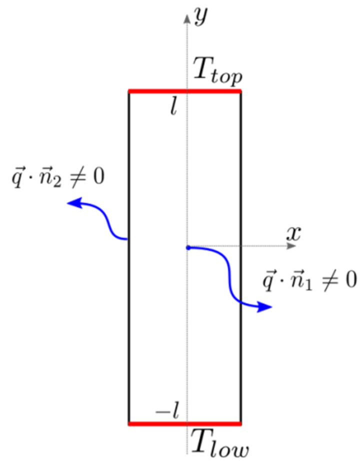

Figure 1. Thermodynamic problem limit conditions

3. Collaborative model

The collaborative model will be developed for the carbon/epoxy woven composite. The fabric is considered to be perfectly balanced and thus the longitudinal and transverse behaviors are considered to be equivalent. Typically, the experimental tests show an elastic brittle response in longitudinal (transverse) direction. However, shear behavior has a non-linear character [4-6]. The irreversible strains, the in-ply damage (which is observed from a regular decrease of shear modulus) and the hysteresis loops are presented in the stress-strain curve (Fig. 2).

The collaborative model is developed to represent the elastoplastic damage behavior as well as the hysteresis loops for elementary ply of composite fabrics under cyclic loading. This model is composed of two sub-models. The first one deals with the elastoplastic damage behavior during material loading and the second sub-model describes the hysteresis behavior using a fractional derivative law. The constitutive equations are deduced within the framework of irreversible thermodynamics processes

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using the local state method [7] under a state of plane stress. Subscripts 1 and 2 represent the warp and the weft directions, respectively.
Within the framework of thermodynamic irreversible theory, we choose Helmholtz potential depending on the internal variables [7]:

$$\rho\psi = \rho\psi(\boldsymbol{\varepsilon}^e,\ d_i, p) \tag{2}$$

where $\boldsymbol{\varepsilon}^e$, d_i, p are internal variables introduced to represent elastic strains, damage in the orthotropic directions and cumulated plasticity respectively.

To establish constitutive equations and to calculate the energy dissipation we use the Clausius-Duhem inequality:

$$\sigma\dot{\varepsilon} - \rho\dot{\psi} \geq 0 \tag{3}$$

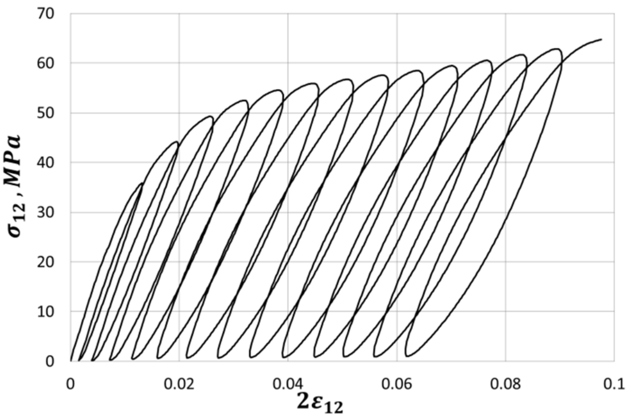

Figure 2. Experimental shear stress-strain curve for the carbon/epoxy composite

3.1. Behavior modelling during loading path

During the loading path, the following hypothesis is considered:

$$\dot{\boldsymbol{\varepsilon}}^t = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p, \quad \dot{\boldsymbol{\varepsilon}}^{ve} = 0 \tag{4}$$

Following the second principal of thermodynamics, constitutive equations are deduced from the Helmholtz potential $\rho\psi$ (Eq.2). The stress-strain relation is:

$$\boldsymbol{\sigma} = \rho\frac{\partial\psi}{\partial\varepsilon^e} \quad \Longrightarrow$$

$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sqrt{2}\sigma_{12}\end{pmatrix} = \begin{pmatrix}(1-d_{11})C_{11}^0 & \nu_{21}^0 C_{11}^0 & 0\\ \nu_{12}^0 C_{22}^0 & (1-d_{22})C_{22}^0 & 0\\ 0 & 0 & 2(1-d_{12})G_{12}^0\end{pmatrix}\begin{pmatrix}\varepsilon_{11}^e\\ \varepsilon_{22}^e\\ \sqrt{2}\varepsilon_{12}^e\end{pmatrix} \tag{5}$$

Thermodynamic forces associated with internal variable d_{ij} are defined as following:

$$Y_{ij} = -\frac{\partial W_e^d}{\partial d_{ij}} \quad \Longrightarrow$$

$$Y_{11} = \frac{1}{2}C_{11}^0(\varepsilon_{11}^e)^2 \quad ; \quad Y_{22} = \frac{1}{2}C_{22}^0(\varepsilon_{22}^e)^2 \quad ; \quad Y_{12} = \frac{1}{2}G_{12}^0(2\varepsilon_{12}^e)^2 \tag{6}$$

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These associated thermodynamic forces characterize the damage propagation. The state of damage can only grow [3, 8] and therefore, the threshold of undamaged zone is defined as a maximal thermodynamic force for all previous time (τ) up to the current time (t) [9]:

$$\overline{Y}_{ij} = sup_{\tau \leq t}\left(Y_{ij}(t)\right), \{i, j = 1{,}2\} \tag{7}$$

The damage variables $d_{ij}, \{i, j = 1{,}2\}$ represent a loss of material stiffness in different orthotropic directions. In shear, the damage variable is defined from the shear modulus diminution during experiment:

$$d_{12} = 1 - \frac{G_{12}^{i}}{G_{12}^{0}} \tag{8}$$

where G_{12}^{i} is the current shear modulus associated to each unloading-loading.

Damage evaluation law is chosen as the best approximation of experimental data. Different types of functions can be used such as linear, polynomial, logarithmic or Heaviside functions.

The experimental data shows irreversible strains appearance mainly in shear [4-6]. Thus, plastic flow is considered to be blocked in fiber directions:

$$\varepsilon_{11}^{p} = \varepsilon_{22}^{p} = 0 \quad ; \quad \varepsilon_{12}^{p} \neq 0 \tag{9}$$

The damage and plasticity coupling is made using the effective stress notation (Eq. 10).

$$\tilde{\sigma}_{12} = \frac{\sigma_{12}}{(1 - d_{12})} \tag{10}$$

The isotropic strain hardening is assumed. The elastic domain is defined by the yield function f:

$$f = \frac{|\sigma_{12}|}{(1 - d_{12})} - R(p) - R_0 \tag{11}$$

where R_0 is a yield stress and the function $R(p)$ is a material characteristic function of the cumulative plastic strain p. Generally, the hardening function $R(p)$ is approximated by a power law:

$$R = \beta p^{k} \text{ with } p = \int_{0}^{\varepsilon_{12}^{p}} (1 - d_{12}) d\varepsilon_{12} \tag{12}$$

where β and k are material parameters identified from the experimental data.

When taken into account that in fiber directions the plastic flow is blocked and that the material damage occurs instantly, the dissipated material energy during the loading Φ^{load} is deduced from the Clausius-Duhem inequality (Eq.3) as following:

$$\Phi^{load} = 2\sigma_{12}\dot{\varepsilon}_{12}^{p} - R\dot{p} + Y_{12}\dot{d}_{12} \geq 0 \tag{13}$$

3.2. Behavior modelling during unloading path. Fractional law

The hysteresis loops are associated with viscoelastic properties of composite's matrix. Hysteresis is a hereditary phenomenon, i.e. the previous loading history has to be taken into account. To model this viscoelastic behavior, fractional derivatives are introduced in the constitutive equation. The Riemann-Liouville fractional derivative [10] is defined as following:

$$D^{\alpha} f(x) = \frac{1}{\Gamma(1 - \alpha)} \frac{d}{dx} \int_{0}^{x} \frac{f(t)}{(x - t)^{\alpha}} dt, \quad 0 < \alpha < 1 \tag{14}$$

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where D^{α} is a fractional derivative of order α and Γ is the Gamma-function defined by:

$$\Gamma(z) = \int_0^{+\infty} e^{-x} x^{z-1} dx, \quad z \in \mathbb{R}_+^* \tag{15}$$

We consider that during unloading path, the plastic strain stays constant and the elastic strain is a linear function of time. The observed non-linear strain in the hysteresis loop (Fig.3) is expressed by the fractional constitutive law as following:

$$\varepsilon_{12}^{ve} = A + BD^{\alpha}\left(2\varepsilon_{12}^{e}(t)\right) \tag{16}$$

where ε_{12}^{e} is the elastic strain determined by the elastoplastic damaged model, D^{α} is the Riemann-Liouville fractional derivative (Eq.13) and A, B and α are fractional model parameters.

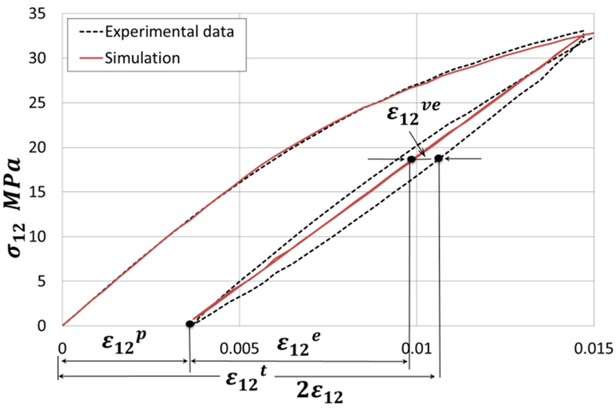

Figure 3. Total strain composition

As the plastic flow stays constant, the stress is expressed by the elastic law:

$$\sigma_{12}(t) = G_{12}^{0}(1 - d_{12})\, \varepsilon_{12}^{ve}(t) \tag{17}$$

By substitution the Eq.15 in the Eq.16, we obtain the constitutive law to represent hysteresis loop:

$$\sigma_{12}(t) = G_{12}^{0}(1 - d_{12})A + 2G_{12}^{1} D^{\alpha} \varepsilon_{12}^{e}(t) \tag{18}$$

with $G_{12}^{1} = G_{12}^{0}(1 - d_{12})B$.

Using the developed model and the Clausius-Duhem inequality (Eq.3) it is possible to calculate the dissipated energy due to the viscoelastic effects. In respect to the developed model, the damage propagation and the plastic flow stay constant during unloading: $\dot{\varepsilon}_{12}^{p} = 0$ and $\dot{d}_{12} = 0$ and the total strain coincides with the viscoelastic strain ($\varepsilon_{12}^{t} = \varepsilon_{12}^{ve}$) expressed by the Eq.16. So, the dissipated energy during unloading-loading path, which is the cause for the material self-heating, is defined such as:

$$\Phi^{unload} = \sigma_{12} \dot{\varepsilon}_{12}^{ve} \tag{19}$$

On the other hand, the dissipated energy is equal to the area of hysteresis loop, thus:

$$\Phi^{unload} = \oint \sigma_{12} d\varepsilon_{12}^{ve} \tag{20}$$

The last expression allows us to control the computation of the dissipated energy and the material self-heating by the collaborative model.

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3.3. Collaboration of two sub-models

The collaboration between the two sub-models is performed automatically, depending on the sign of the yield function (Eq.10) and its derivative. If $f = 0$ and $\dot{f} = 0$, the elastoplastic damage model is used. During unloading if $f < 0$ or $f = 0$ and $\dot{f} < 0$ and during reloading if $f < 0$ or $f = 0$ and $\dot{f} > 0$, we consider that the damage and plastic strain stay constant, and thus we switch to the fractional derivative model.

4. Results

The collaborative model is used to represent the behavior of the carbon/epoxy composite during the cyclic loading and to calculate the dissipated energy. The identification of material parameters has been made. The damage variable d_{12} is a linear function of the thermodynamic associated force $\overline{Y}_{12}$. The hardening function R is the power function of the cumulated plastic strain p. The fractional model parameters A, B and α are the linear functions of the damage internal variable d_{12}. The resulting curve is presenting in the Fig. 4.

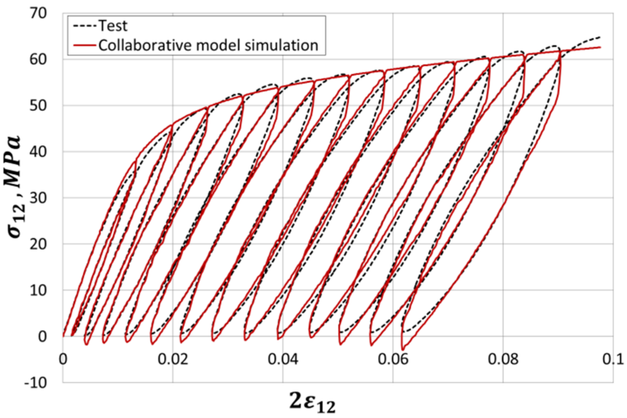

Figure 4. Experimental and numerical behavior comparison in shear for the carbon/epoxy composite

Now, the dissipated energy can be calculated by the Eq.13 and Eq.19. The dissipations due to elastoplastic damage behavior during loading path (Eq.13) and due to the viscoelastic behavior during unloading (Eq.19) are compared in the Fig. 5. From the obtained results we can conclude, that the dissipation associated with damage propagation and plastic flow appears instantly and does not vary during loading process. It means that the system of cracks appears in the matrix in the beginning of the loading and stays constant up to the rapture. The dissipation due to the elastoplastic damaged behavior is significantly smaller than the dissipation due to the viscoelastic response during unloading. So, the viscoelastic effects are dominant in the material self-heating. Moreover, the dissipation associated with the viscoelasticity effects modelling (Eq.20) is compared with the area of hysteresis loops (Eq.19) determined from the experimental data and the numerical simulation. Thus, the collaborative model is able to predict precisely the area of hysteresis loops. On the other hand, the dissipation defined by the proposed model (Eq.19) is overestimated due to hypothesis that all internal energy is turned into a heat. In the reality, some of this energy is absorbed by the material to ensure damage propagation due to fatigue.

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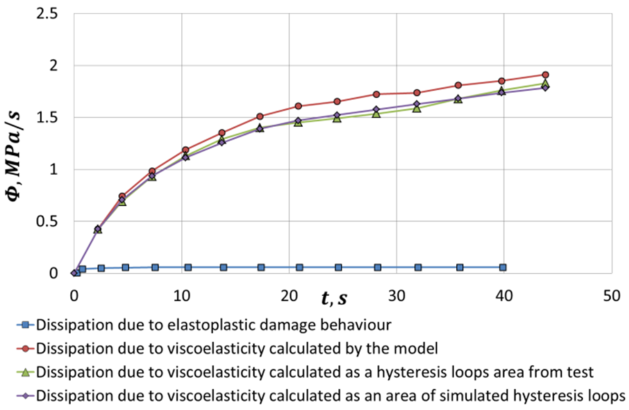

Figure 5. Dissipation comparison

When the internal heat source S is calculated, the thermodynamic problem (Eq.1) can be resolved to model the self-heating method. The self-heating curve (Fig. 6) represents the evolution of the stabilized average temperature of the specimen as a function of the applied maximum stress. The fatigue limit corresponds to the intersection point of the tangent line to the temperature curve with the σ_{max}-axis and equal to 27.1 MPa. The results are in good agreement with experimental data [11].

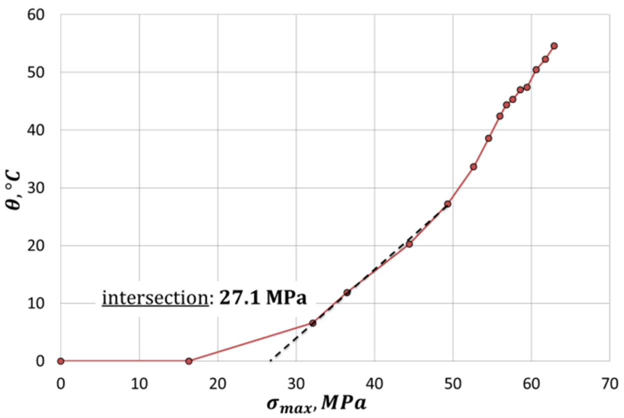

Figure 6. Simulation of the self-heating test for carbon/epoxy woven composite

5. Conclusions

In this paper, the simple method to predict the fatigue limit is presented on the example of the carbon/epoxy composite material. This method is based on the material self-heating. The thermodynamic stationary problem is resolved to determine the stabilized average temperature of the specimen. The value of the intrinsic energy dissipation, which leads to the material self-heating, has been modeled by the collaborative model. This model is able to represent elastoplastic damage behavior as well as the hysteresis loops which are associated with the viscoelastic behavior of the polymer matrix. The model is composed of two sub-models. The first one deals with the elastoplastic damaged behavior during loading path. The second sub-model is applied during unloading to represent hysteresis loops using the fractional derivative approach. The dissipated energy has been easily calculated by the collaborative model using the thermodynamic irreversible theory. From the obtained results we have noticed, that viscoelastic material properties (i.e. hysteresis loops) have the dominating

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effects on the material self-heating. The fatigue limit has been predicted successfully using the model since the obtained results are in good agreement with the experimental data.

References

[1] O. Westphal. *Analyse thermomécanique de l'endommagement en fatigue de stratifie de carbone/époxy: détermination de la limite de d'endurance à partir d'essais d'auto-échauffement.* PhD thesis, Ecole Centrale de Nantes, 2014.

[2] A. Krasnobrizha, *Modélisation des mécanismes d'hystérésis des composites tissésà l'aide d'un modèle collaboratif élasto-plastique endommageable à dérivées fractionnaires*. PhD thesis, Ecole Centrale de Nantes, 2014.

[3] P. Ladevèze and E. Le Dantec. Damage modelling of the elementary ply for laminated composites. *Composites Science and Technology*, 43:257–267, 1992.

[4] P. Rozycki, Loi de comportement pour composite et outils numériques en dynamique rapide. Habilitation thesis, Nantes, 2015.

[5] S. Marguet, P. Rozyck, L. Gornet. A rate dependence constitutive model for carbon-fiber reinforced plastic woven fabrics. *Mechanics of Advanced Materials and Structures*, 14(8), 619–631, 2007.

[6] C. Hochard, J. Payan and O. Montagnier, "Design and computation of laminated composite structures," *Composites Science and Technology*, vol. 65, p. 467–474, 2005.

[7] J. Lemaitre, J.-L. Chaboche. *Mecanique des Matériaux Solides*. Paris: Dumond, 1996.

[8] P. Ladevèze. Sur la mécanique de l'endommagement des composites. *Proceedings of the Fifth Journées Nationales sur les Composites (JNC5)*, Paris, France, 1986.

[9] A. Johnson, A. Pickett, P. Rozycki, Computational methods for predicting impact damage in composite structures. *Composites Science and Technology*, 61(15):2183–2192, 2001.

[10] S. G. Samko, A. A. Kilbas, O. I. Marichev, *Fractional Integrals and Derivatives: Theory and Applications*. Amsterdam: Gordon and Breach Science, 1993.

[11] L. Gornet, O. Westphal, M. Mateos, A. Krasnobrizha, P. Rozycki, P. Peyrac, F. Lefebvre. Propriétés mécaniques en fatigue à grands nombres des cycles des composites carbone/époxy. *Revue des Composites et des Matériaux Avancés*, 25(2):181–200, 2015.

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