DESIGN SPACE INTERROGATION FOR NEW C-PLY LAMINATE ARCHITECTURES

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Abstract

Spread tow Non-Crimp Fabric (NCF) materials, e.g. C-PlyTM, are now available in a range of aeral weights and ply architectures, including $\frac{0}{45}$, $\frac{0}{45}$, $\frac{45}{45}$ and $\frac{0}{90}$, which correspond to the standard ply orientations employed in traditional UD material lay-ups. The benefit of NCF material is generally associated with increased deposition rate, but this advantage may be offset by reduced design freedoms when a specific form of mechanical coupling behaviour is required, ply terminations must also be introduced and thermal warping distortion eliminated.

This article investigates the extent to which new NCF architectures, based on 0/45, $\frac{0}{45}$, $\frac{45}{45}$ and 0/90 NCF can be tailored to achieve warp free tapered laminates without the need for deposition with off axis alignment, and thus avoid ply discontinuities. Interrogation of the available lamination parameter design space is used to demonstrate the effect of applying ply termination constraints. Finally, buckling factor contours are mapped onto the lamination parameter design space to assess the bounds, under compression and shear loading, for infinitely long simply supported plates

1. Introduction

Recent research has demonstrated that tailored Non-Crimp Fabric (NCF) designs, based on 0/45 and 0/-45 architectures, can produce fully uncoupled laminates or laminates with *Extension-Shearing* and/or *Bending-Twisting* coupling, and that all have immunity to thermal warping distortion [1]. The extent to which tapered laminate designs can be achieved, without introducing unwanted thermomechanical coupling, was also investigated through a ply termination algorithm to introduce single-ply or, where necessary, multiple-ply terminations.

Recent research on laminate design has demonstrated that fully uncoupled laminates [3], or those with *Extension-Shearing* [2] and/or *Bending-Twisting* coupling [4] have a design spaces containing predominantly non-symmetric stacking sequences. The results presented in this article are based on these four laminate classes, illustrated under free thermal contraction in Fig. 1. All are immune to thermal warping distortions by virtue of the fact that their coupling stiffness properties are null $(B =$ **0**); as would be expected from symmetric laminate configurations. The first two classes contain balanced angle-ply layers, leading to uncoupled extensional stiffness properties. The *Simple* laminate in the first column is also uncoupled in bending, whilst the laminate class in the second column possesses *Bending-Twisting* or *B-T* coupling. The final two laminate classes possess unbalanced angle-ply layers, leading to *Extension-Shearing* or *E-S* coupling properties. The laminate class in the third column is uncoupled in bending, whereas the laminate class in the fourth column has both

Extension-Shearing and *Bending-Twisting* or *E-S;B-T* coupling, as would arise from unbalanced and symmetric laminates.

Figure 1 – In-plane thermal contraction responses (not to scale) resulting from a typical high temperature curing process. All examples shown are square, initially flat, composite laminates. The example stacking sequences are 24-ply laminates and are given in symbolic form, where symbols $+, -,$ \odot and \bullet are used in place of standard ply orientations ± 45 , 0 and 90°, respectively. The underlining highlights the NCF ply pairings.

 $C-Ply^{TM}$ (bi-angle) non-crimp fabrics consists of two plies of UD carbon fibre; one at 0° and the other at either a shallow angle, e.g. 20° or the standard 45° angle, considered here, stitched together. The new design solutions reported here, loosely follow the well known repeating bi-angle philosophy, [$0/\theta$]_{*r*T}, which possesses *Extension-Shearing* and *Bending-Twisting* dominant coupling, but now with immunity to thermal warping distortions; warping is eliminated in $[0/\theta]$ _{*rT*} laminates only when the number (*r*) of repeats becomes very large.

The four design freedoms associated with the stacking sequences used in standard UD laminate manufacture, with ply orientations 0, 90, 45 and -45, are increased to eight using 0/45 and 0/-45 NCF: by flipping (-45/0 and 45/0), rotating (90/-45 and 90/45) or both (45/90 and -45/90). However, rotating introduces ply discontinuity in the angle ply layers whenever the length of a component or structure is greater than the width of the fabric being deposited.

The main purpose of the investigation is to determine the extent to which new C-Ply architectures, based on 0/45, 0/-45, 45/-45 and 0/90 NCF, can be tailored to achieve tapered laminates with similar thermo-mechanical properties to those illustrated in Fig. 1 and thus avoid the need for deposition of NCF with off axis alignment and the ply discontinuities that this may cause. Underlining is used to highlight the NCF pairings.

Note that the four design freedoms associated with these new C-Ply architectures are also increased to eight, but only involve flipping $(-45/0, 45/0, -45/45, 90/0)$. Double underlining is used to highlight the NCF pairings which have been flipped.

Buckling factor mapping onto the lamination parameter design space, for infinitely long simply supported plates, is developed and applied to descrete sections through the 3-dimensional design space surfaces to assess the bounds under shear and compression loading for laminates with these new NCF architectures.

2. Derivation of stacking sequences

The common feature relating the *Simple*, *Extension-Shearing* and/or *Bending-Twisting* coupled laminate classes is that all are decoupled, i.e. $B_{ii} = 0$; hence in-plane and out-of-plane behaviour are independent and can therefore be treated separately. The constitutive relations simplify and the elements of the stiffness matrices are derived from the well known relationships:

$$
A_{ij} = \Sigma Q'_{ij}(z_k - z_{k-1})
$$

\n
$$
D_{ij} = \Sigma Q'_{ij}(z_k^3 - z_{k-1}^3)/3
$$
\n(1)

in which: the summations extend over all *n* plies; Q'_{ij} are the transformed reduced stiffnesses $(i, j = 1, j)$ 2, 6) and; z_k represents the distance from the laminate mid-plane of the k^{th} ply.

In the derivation of the database of stacking sequences, which assumes (but is not restricted to) combinations of standard fibre angle orientations, i.e. 0, 90 and/or $\pm 0^{\circ}$ (= $\pm 45^{\circ}$), the general rule of symmetry is relaxed. Neither cross plies nor angle plies are constrained to be symmetric about the laminate mid-plane. The derivation of the NCF laminates involved the added restrictions that each layer in the two-ply pairing: has identical orthotropic material properties; has identical thickness, *t*, and; differs only by its orientation, representing any combination of the eight pairings: 0/45, 45/0, 0/- $45, -45/0, 45/45, -45/45, 0/90$ and $90/0$.

Non-dimensional parameters allow the extensional and bending stiffness properties to be readily calculated for any fibre/matrix system and angle-ply orientation and provide a compact data set alongside each laminate stacking sequence derived.

The development of non-dimensional parameters involves the summations, for each ply orientation, of $(z_k - z_{k-1})$, $(z_k^2 - z_{k-1}^2)$ and $(z_k^3 - z_{k-1}^3)$, relating to the **A**, **B** and **D** matrices, respectively. Here, the distance from the laminate mid-plane, *z*, is expressed in terms of ply thickness *t*; assumed to be unit value.

These non-dimensional parameters, together with the transformed reduced stiffness, Q'_{ij} , for each ply orientation of constant ply thickness, *t*, facilitate simple calculation of the elements of the extensional, coupling and bending stiffness matrices from:

$$
A_{ij} = \{n_+Q'_{ij+} + n_-Q'_{ij-} + n_0Q'_{ij_0} + n_{\bullet}Q'_{ij_{\bullet}}\}t D_{ij} = \{\zeta_+Q'_{ij+} + \zeta_-\overline{Q'}_{ij-} + \zeta_0Q'_{ij_0} + \zeta_{\bullet}Q'_{ij_{\bullet}}\}t^3/12
$$
\n(2)

The *Extension-Shearing* and *Bending-Twisting* coupled laminate satisfies the following nondimensional parameter criteria:

$$
n_+ \neq n_- \tag{3}
$$

whilst $n_+ = n$ and/or $\zeta_+ = \zeta$ are the conditions giving rise to the *Bending-Twisting* coupled and *Simple* or *Extension-Shearing* coupled laminate classes in Fig. 1, respectively.

The transformed reduced stiffnesses, Q'_{ij} , are defined by: $Q'_{11} = Q_{11} cos^4\theta + 2(Q_{12} + 2Q_{66})cos^2\theta sin^2\theta + Q_{22}sin^4\theta$ $Q'_{12} = Q'_{21} = (Q_{11} + Q_{22} - 4Q_{66})\cos^2\theta \sin^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta)$ $Q'_{16} = Q'_{61} = \{ (Q_{11} - Q_{12} - 2Q_{66})\cos^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^2\theta \}\cos\theta \sin\theta$ $Q'_{22} = Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{22} \cos^4 \theta$ $Q'_{26} = Q'_{62} = \{ (Q_{11} - Q_{12} - 2Q_{66}) \sin^2 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \cos^2 \theta \} \cos \theta \sin \theta$ $Q'_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\cos^2\theta \sin^2\theta + Q_{66}(\cos^4\theta + \sin^4\theta)$ (4)

and the reduced stiffness terms, Q_{ij} , are calculated from the orthotropic material properties: $Q_{11} = E_1/(1 - v_{12}v_{21})$ $Q_{12} = v_{12}E_2/(1 - v_{12}v_{21})$ $Q_{22} = E_2/(1 - v_{12}v_{21})$ $Q_{66} = G_{12}$ (5)

Note that orthotropic material properties imply that $Q_{16} = Q_{26} = 0$.

2.1 Lamination parameters

Lamination parameters, originally conceived by Tsai and Hahn [5] offer an alternative set of nondimensional expressions when ply angles are a design constraint. They were first applied to optimum design by Miki [6] and presented in graphical form by Fukunaga and Vanderplaats [7]. Optimized lamination parameters may be matched against a corresponding set of stacking sequences. Graphical representations help with this design process, since arguably the greatest challenge to the composite laminate designer, is the inverse problem of generating practical laminate configurations, which satisfy the optimized lamination parameters.

Elements of the extensional (**A**) and bending (**D**) stiffness matrices are each related to lamination parameters, ξ_i , and laminate invariants, U_i , respectively, by:

where the laminate thickness H (= number of plies, n , \times constant ply thickness, t) and the laminate invariants, U_i , are calculated from the reduced stiffness terms, Q_{ij} , of Eq. (5): $U_1 = \{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\}/8$ $U_2 = {Q_{11} - Q_{22}}/2$ $U_3 = {Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}}/8$ $U_4 = {Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}}/8$ $U_5 = {Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}}/8$ (8)

These ply orientation dependent lamination parameters are also related to the non-dimensional parameters, used in Eq. (2), by the following expressions:

 $\xi_1 = \{n_+ \cos(2\theta_+) + n_- \cos(2\theta_-) + n_0 \cos(2\theta_0) + n_0 \cos(2\theta_0)\} / n$ $\xi_2 = \{n_+ \cos(4\theta_+) + n_- \cos(4\theta_-) + n_0 \cos(4\theta_0) + n_0 \cos(4\theta_0)\} / n$ $\xi_3 = \{n_+ \sin(2\theta_+) + n_- \sin(2\theta_-) + n_{\text{o}} \sin(2\theta_{\text{o}}) + n_{\text{o}} \sin(2\theta_{\text{o}})\} / n$ (9)

$$
\xi_9 = {\zeta_+ \cos(2\theta_+) + \zeta_- \cos(2\theta_-) + \zeta_0 \cos(2\theta_0) + \zeta_{\bullet} \cos(2\theta_{\bullet})}/n^3
$$

\n
$$
\xi_{10} = {\zeta_+ \cos(4\theta_+) + \zeta_- \cos(4\theta_-) + \zeta_0 \cos(4\theta_0) + \zeta_{\bullet} \cos(4\theta_{\bullet})}/n^3
$$

\n
$$
\xi_{11} = {\zeta_+ \sin(2\theta_+) + \zeta_- \sin(2\theta_-) + \zeta_0 \sin(2\theta_0) + \zeta_{\bullet} \sin(2\theta_{\bullet})}/n^3
$$
\n(10)

Noting that for standard ply orientations 0, 90 and $\pm 45^{\circ}$, assumed here, $\xi_4 = \xi_{12} = 0$.

3. Laminate Tapering Algorithm

Tapered laminates are certified for symmetric laminate construction, the majority of which possess *Bending-Twisting* coupling, but such designs have a severe design constraint, i.e., a single angle-ply termination requires a further three angle-ply terminations to maintain balanced and symmetric construction. This section investigates the extent to which this restriction can be overcome by employing NCF or balanced plain weave material without changing the mechanical behaviour or introducing undesirable warping distortions.

Tapered laminate designs have been developed in a two stage process: The first stage of the termination scheme is not shown, but involves: *m* ply terminations, applied in turn to specific ply combinations in every stacking sequence with *n*-ply layers of the NCF; comparison with all stacking sequences with *n*-*m* plies and; recording exact matches. The first (or upper surface) and last (or lower surface) plies are assumed to be continuous throughout the tapering process; this represents a practical design constraint to prevent surface ply delamination.

Repeated sequences are removed from the data when multiple matches arise from different ply terminations within a single stacking sequence. This forms a starting point for the second stage of the tapering algorithm.

The second stage of the tapering algorithm can be described as a bottom up process, and begins with compatible stacking sequences representing the minimum ply number grouping (*n*) of interest. These sequences are then algorithmically filtered through higher ply number groupings, in turn, but now only sequences compatible with the minimum ply number grouping are retained. As in the first stage, the *m* ply termination scheme involves the removal of specific ply combinations, in turn, for each stacking sequence with $n+m$ plies, and a comparison made against each stacking sequence with n plies, and matches recorded. This procedure ensures that all solutions reported here will be compatible with higher ply number groupings beyond those reported here.

4. Results

The Number of NCF laminate solutions for *Simple*, *Bending-Twisting* or *B-T* coupled *Extension-Shearing* or *E-S* coupled and *Extension-Shearing* and *Bending-Twisting* or *E-S*;*B-T* coupled warp-free laminate classes are reported in Table 1. Each row of the table corresponds to a particular ply number grouping, n_{NCF} with C-Ply layers and the equivalent ply number grouping, n_{UD} with UD layers.

Table 1: Number of NCF laminate solutions.

By generating the lamination parameters for each of these solutions permits interrogation of the extent of resulting design space for the definitive listing of laminate designs, since individual laminate stacking sequences can now be represented by a single point in a 3-dimensional space for the extensional stiffness properties and the bending stiffness properties. Each point, within the resulting point cloud, defines a co-ordinate set, from which the stiffness properties may be readily determined using Eq. (6) or Eq. (7). The 3-dimensional point cloud representing the bending stiffness lamination

parameters for the *Extension-Shearing* and *Bending-Twisting* or *E-S*;*B-T* coupled laminates of Table 1 is illustrated as an orthographic projection in Fig. 1. As a result of the constraint imposed by the NCF architecture, the design space for all laminate classes is found to be substantially reduced in comparison to the equivalent UD design space, reported previously [4]. The shear buckling strength can be assessed through a buckling factor mapping procedure for (infinitely) long plates, discussed elsewhere [8], indicates that designs with the new NCF architectures occupy an unfavourable region.

Figure 1. Lamination parameter design space for: (a) – (c) bending stiffness in *Extension-Shearing* and *Bending-Twisting* coupled NCF laminates with $n_{NCF} = 4 - 12$ and; (d) shear buckling factor, $k_{xy,\infty} =$ $N_{xy,\infty}b^2/\pi^2D_{\text{Iso}}$ (with $D_{\text{Iso}} = U_1H^3/12$), for laminate designs at cross section $\xi_{11} = 0.2$.

4.1 Tapered designs.

Table 2 give the number of *tapered* solutions for *Extension-Shearing* and *Bending-Twisting* or *E-S*;*B-T* coupled warp-free laminates, with even ply number groupings, after apply the taper algorithm, where each row correspond to different ply number groupings, n_{NCF} , with NCF layers. The equivalent number, n_{UD} , with UD layers is also indicated. The number of stacking sequences in column (2) is repeated from Table 1.

Note that ply contiguity ≤ 2 is a constraint by virtue of the NCF architecture, i.e., the number of adjacent plies with the same orientation never exceeds 2. Column (3) corresponds to the number of laminates from column (2) that match laminates with n_{NCF} –2 after top-down termination scheme, i.e., the number of compatible sequences with those immediately below in the list. The number of laminates matching $n_{NCF}+2$ ply laminates are shown in parentheses, and these represents the number of

compatible sequences with those immediately above in the list. Column (4) represents the number of laminates from column (3) matching laminates with $n_{\text{NCF}}+2$ after *continuous* bottom-up termination scheme. Here the bottom-up process begins with the lowest ply number grouping. Note that whilst all 11 sequences with $n_{NCF} = 4$ are compatible with $n_{NCF} = 6$, not all sequences with $n_{NCF} = 6$ are compabitible with $n_{NCF} = 8$ or those of higher ply number groupings. The design space is therefore constrained by the lowest ply number grouping of interest. Finally, column (5) indicates the number of tapered solutions between adjacent ply number groupings n_{NCF} and n_{NCF} –2. The number of tapered solutions is always equal or greater than the number of laminates from which they are derived, given that there may be several ply termination options for a given stacking sequence.

Table 2 – Number of *tapered* solutions for *Extension-Shearing* and *Bending-Twisting* or *E-S*;*B-T* coupled warp-free laminates corresponding to, n_{NCF} , with NCF layers and equivalent number, n_{UD} , with UD layers.

\perp	\sim رك	د).	(4	(5)
$n_{NCF} (\equiv n_{UD})$				
8(16)	1,625	1,347(1,625)	1,127	3,912
6(12)	124	110 (124)	110	232
4(8)		$- (11)$		

 (a) (d)

N

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An example of the bending stiffness lamination design space for odd ply laminates with *Extension-Shearing* and *Bending-Twisting* coupling is given in Fig. 2, demonstrating *tapered* C-Ply NCF laminates from 5 to 9 NCF layers (*or 10 to 18 UD layers*). All solutions arise from a single (n_{NCF} =) 5 ply design, compatible with 7 stacking sequences with $(n_{NCF} =)$ 7 plies, giving rise to 28 tapered designs. These, in turn, are compatible with 109 stacking sequences with $(n_{NCF} =)$ 9 plies, giving rise to 739 tapered design combinations. Tapered designs can be identified from the lamination parameter design spaces, as illustrated in Fig. 2, by strings originating from the single $(n_{NCF} =) 5$ ply design through all 7 stacking sequences with $(n_{NCF} =)$ 7 plies and on to compatible sequences with $(n_{NCF} =)$ 9 plies. For any tapered design, the change in lamination parameter can be related to a change in compression buckling strength by visual inspection of the contour map at the appropriate section through the design space, as illustrated in Fig. 2, which is discussed in more detail elsewhere [8].

5. Conclusions.

A two-ply termination algorithm has been employed to develop permissible tapered designs for new Non-Crimp Fabric (NCF) laminates in which consistent mechanical coupling characteristics and immunity to thermal warping distortion are preserved and the need for off-axis deposition is eliminated.

Lamination parameter design spaces help to indicate the extent to which the available NCF designs are severely reduced, including the effect of the additional design contraints for laminate taper. These have been illustrated for *Extension-Shearing* and *Bending-Twisting* coupled laminates and related to buckling strength through a buckling factor mapping procedure for (infinitely) long plates.

Acknowledgments

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