VIBRATIONAL CHARACTERISTICS OF FUNCTIONALLY GRADED CARBON NANOTUBE-REINFORCED COMPOSITE PLATES UNDER UNCERTAINTY

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Abstract
In this study natural frequency of carbon nanotube-reinforced composite plates under various kinds of uncertainty is investigated. Here, geometrical configuration and mechanical properties of the carbon nanotubes and the matrix are assumed to be uncertain but bounded parameters. The uniform and functionally graded distributions of carbon nanotube through the thickness of nanocomposite plates are considered. The mechanical properties of plates are predicted on the basis of the modified rule of mixture and by using the Kirchhoff's plate theory, the equations of motion are derived. To handle uncertainty propagation, a non-probabilistic approach well-known as interval analysis method is used. The deterministic and uncertain frequencies of nanocomposite plates are calculated by the use of Galerkin's method. The suggested model is justified by a good agreement between the present results and available deterministic data in the literature. In numerical results, propagation of geometrical and mechanical uncertainty are investigated. Furthermore, variations of the lower and upper bounds of natural frequencies with respect to aspect ratio and thickness ratio are also elucidated.

1. Introduction

In the last decade, due to exceptional mechanical characteristics of carbon nanotubes (CNTs), polymer matrices reinforced by CNTs have attracted considerable attention as a state-of-the-art composite material [1, 2]. Nanocomposites made of a polymer as matrix and CNTs as reinforcement, are well-known as carbon nanotube-reinforced composites (CNTRCs). Based on high strength and stiffness to weight ratios, nanotube-based composites have used in the wide range of engineering applications. CNT Distributions along the thickness of the CNTRCs can be either uniform or functionally graded (FG). According to the concept of conventional FGMs, the mechanical properties of FG-CNTRC change gradually from one side to the other side. Due to substantial mechanical properties, nanocomposite plates attract a great deal of attention and dynamic behavior of them become subject of primary interest in recent studies. Formica et al. [3] studied vibration of CNTRC plate by employing the Eshelby–Mori–Tanaka approach. Wang and Shen [4] investigated large amplitude vibration of CNTRC plate resting on an elastic medium in thermal environments. The natural frequency FG-CNTRC annular sectorial plates resting on Pasternak foundation was studied by Hedayati and Sobhani

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Aragh [5]. Moreover, large amplitude vibration and the nonlinear bending of a sandwich CNTRC plate in thermal environment were analyzed [6]. Based on the Mori-Tanaka approach, free vibration of FG-CNTRC plate resting on an elastic foundation was studied utilizing the generalized differential quadrature method [7]. Lei et al. [8] investigated the natural frequency of functionally graded nanocomposite plates via the element-free kp-Ritz method. The three-dimensional free vibration of CNTRC rectangular plates with different boundary conditions was analyzed using Ritz method [9]. Malekzadeh and Zarei [10] examined frequency of quadrilateral laminated CNTRC plates using the first-order shear deformation theory. Aeroelastic characteristics of CNTRC plates under a supersonic flow were investigated via the Galerkin's method [11]. Furthermore, Zhang et al. [12] examined vibration characteristics and mode shapes of FG-CNTRC skew plates using the IMLS-Ritz method.

Experimental researches reveal challenging measurements at the nano-scale and defects in molecular structures led to variations of mechanical properties (such as Young's modulus and Poisson's ratio) of CNTs, polymer matrix and consequently CNTRC in considerable ranges [13, 14]. In addition to the inherent uncertainties in the mechanical properties, uncertainties in the geometry, loading and boundary conditions may be observed. As a result of emergence of various non-deterministic quantities, reducing capability of nanostructure reinforcement can be observed and uncertainties propagate through the mechanical behavior of CNTRC. The deterministic frequencies of CNTRC plates may possibly be lower than the uncertain natural frequencies, related to a non-deterministic model. To deal with uncertainty, either statistical or non-probabilistic approaches can be applied. Due to lack of the knowledge of the probabilistic distributions in nanostructures, proper non-probabilistic approaches for instance interval analysis method can be applied to handle uncertainties [15].

Despite extensive investigations in the area of the mechanical behavior of CNTRC plates, there has been no attempt to tackle the problem described in the present paper. The main purpose of this paper is to analysis vibrational characteristics of CNTRC plates under uncertainty. The geometrical dimensions and mechanical properties of the CNTs and the polymer matrix are considered as uncertain parameters. In this study, uniform and functionally graded distributions of CNT through the thickness of nanocomposite plates are considered. Via the modified rule of mixture, the mechanical properties of CNTRC plates are estimated. The governing equations are derived by means of the Kirchhoff's plate theory. A non-probabilistic approach well-known as interval analysis method is used to tackle the uncertainty problem. In order to solve coupled equations of motions simultaneously, Galerkin's method is applied and deterministic and non-deterministic natural frequencies of CNTRC plates are computed. In order to confirm the accuracy of the suggested model, the results are compared with the existing data in the literature. The propagation of uncertainty to the natural frequencies with the variations of various parameters such as aspect ratio and thickness ratio are also studied.

2. Material properties

Consider a rectangular FG-CNTRC plate of length a, width b and thickness h in the Cartesian coordinate system (x, y, z). The CNTRC plate is consisted of a polymer matrix and the single-walled CNTs as reinforcements. In order to enhanced mechanical characteristics of CNTRC plate, the CNTs can be distributed functionally graded in thickness of plate. Here, uniform distribution of CNTs known as UD and functionally graded distributions of the CNTs (FG-A, FG-V, FG-O and FG-X) are considered. In the case of FG-A contrary to FG-V, the bottom surface of panel is CNT-rich. In addition, in FG-X case, the bottom and top surfaces of panel are CNT-rich, contrary to FG-O. The CNT volume fraction of these five types are defined as follows

$$\begin{gathered}\text{UD}: V_{CNT}(z) = V^*_{CNT} \qquad \text{FG-A}: V_{CNT}(z) = (1-\frac{2z}{h})V^*_{CNT} \qquad \text{FG-V}: V_{CNT}(z) = (1+\frac{2z}{h})V^*_{CNT} \\ \text{FG-X}: V_{CNT}(z) = \frac{4|z|}{h}V^*_{CNT} \qquad \text{FG-O}: V_{CNT}(z) = 2(1-\frac{2|z|}{h})V^*_{CNT}\end{gathered} \tag{1}$$

where V^*_{CNT} indicates the overall CNT volume fractions and is defined as the following.

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$$V_{CNT}^{*}=\frac{w_{CNT}}{w_{CNT}+(\rho^{CNT}/\rho^{m})(1-w_{CNT})} \tag{2}$$

where ρ^{CNT} and ρ^{m} are denoted by the densities of CNTs and matrix. Furthermore, w_{CNT} is the mass fraction of the CNTs. In order to estimate the overall mechanical characteristics of CNTRC plates, the modified rule of mixture is applied widely. Experimental studies reveal that flawless load transfer between the CNTs and the polymer matrix don't occur. Hence, CNT effectiveness parameters, η_i (i = 1, 2 and 3) are defined. The effective properties of the FG-CNTRC plates can be obtained using the extended rule of mixture as follows [16]

$$\begin{gathered} E_{11}=\eta_1 V_{CNT}E_{11}^{CNT}+V_m E^m \qquad \frac{\eta_2}{E_{22}}=\frac{V_{CNT}}{E_{22}^{CNT}}+\frac{V_m}{E^m} \qquad \frac{\eta_3}{G_{12}}=\frac{V_{CNT}}{G_{12}^{CNT}}+\frac{V_m}{G^m} \\ \rho=V_{CNT}\rho^{CNT}+V_m\rho^m \qquad \nu_{12}=V_{CNT}^{*}\,\nu_{12}^{CNT}+(1\text{-}V_{CNT}^{*})\nu^m \end{gathered} \tag{3}$$

where E_{11}^{CNT} and E_{22}^{CNT} are Young's moduli of the CNTs in directions 1 and 2, respectively, and G_{12}^{CNT} represents shear modulus of the CNTs. Furthermore, E_{11}, E_{22}, G_{12}, ρ and ν_{12} are the Young's moduli, shear modulus, density and Poisson's ratio, respectively. In addition, E^m, G^m, ν^m and V_m represent Young's modulus, shear modulus, Poisson's ratio and volume fraction of the polymer matrix, respectively. V_{CNT} and ν_{12}^{CNT} are the volume fraction and Poisson's ratio of the CNTs, respectively.

3. Governing equations

Here, the CNTRC plates are modeled by the Kirchhoff's plate theory and the in-plane and out-of-plane displacements of an arbitrary point of nanocomposite plate can be expressed as

$$u_x=u(x,y,t)-z\frac{\partial w}{\partial x} \qquad u_y=v(x,y,t)-z\frac{\partial w}{\partial y} \qquad u_z=w(x,y,t) \tag{4}$$

where u, v and w are displacements of the point (x, y, 0) along x, y and z directions, respectively. Hence, the strain-displacement relations are expressed as

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}-z\frac{\partial^2 w}{\partial x^2}+\frac{1}{2}(\frac{\partial w}{\partial x})^2 \qquad \varepsilon_{yy}=\frac{\partial v}{\partial y}-z\frac{\partial^2 w}{\partial y^2}+\frac{1}{2}(\frac{\partial w}{\partial y})^2 \qquad \varepsilon_{xy}=\frac{1}{2}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2z\frac{\partial^2 w}{\partial y\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}) \tag{5}$$

By applying the Hamilton's principle, the governing differential equations in terms of the transverse and in-plane displacements can be obtained [11].

$$\begin{gathered} A_1\frac{\partial^2 u}{\partial x^2}+C_1\frac{\partial^2 u}{\partial y^2}+(\nu_{12}B_1+C_1)\frac{\partial^2 v}{\partial x\partial y}-A_2\frac{\partial^3 w}{\partial x^3}-(\nu_{12}B_2+2C_2)\frac{\partial^3 w}{\partial x\partial y^2}=I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^3 w}{\partial x\partial t^2} \\ C_1\frac{\partial^2 v}{\partial x^2}+B_1\frac{\partial^2 v}{\partial y^2}+(\nu_{12}B_1+C_1)\frac{\partial^2 u}{\partial x\partial y}-B_2\frac{\partial^3 w}{\partial y^3}-(\nu_{12}B_2+2C_2)\frac{\partial^3 w}{\partial x^2\partial y}=I_0\frac{\partial^2 v}{\partial t^2}-I_1\frac{\partial^3 w}{\partial y\partial t^2} \\ A_3\frac{\partial^4 w}{\partial x^4}+2(\nu_{12}B_3+2C_3)\frac{\partial^4 w}{\partial x^2\partial y^2}+B_3\frac{\partial^4 w}{\partial y^4}-A_2\frac{\partial^3 u}{\partial x^3}-B_2\frac{\partial^3 v}{\partial y^3}-(\nu_{12}B_2+2C_2)(\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 v}{\partial x^2\partial y}) \\ =-I_0\frac{\partial^2 w}{\partial t^2}-I_1(\frac{\partial^3 u}{\partial x\partial t^2}+\frac{\partial^3 v}{\partial y\partial t^2})+I_2(\frac{\partial^4 w}{\partial x^2\partial t^2}+\frac{\partial^4 w}{\partial y^2\partial t^2}) \end{gathered} \tag{6}$$

where I_0, I_1 and I_2 are the normal, coupled normal-rotary and rotary inertial coefficients, respectively. A_i, B_i, C_i (i = 1, 2, 3) and the inertial coefficients are defined by

$$\begin{gathered} \{A_1\quad A_2\quad A_3\}=\int_{-h/2}^{h/2}\left(\frac{E_{11}}{(1-\nu_{12}\nu_{21})}\right)\{1\quad z\quad z^2\}dz \qquad \{B_1\quad B_2\quad B_3\}=\int_{-h/2}^{h/2}\left(\frac{E_{22}}{(1-\nu_{12}\nu_{21})}\right)\{1\quad z\quad z^2\}dz \\ \{C_1\quad C_2\quad C_3\}=\int_{-h/2}^{h/2}G_{12}\{1\quad z\quad z^2\}dz \qquad I_0=\int_{-h/2}^{h/2}\rho\,dz \qquad I_1=\int_{-h/2}^{h/2}\rho\,z\,dz \qquad I_2=\int_{-h/2}^{h/2}\rho\,z^2dz \end{gathered} \tag{7}$$

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4. Solution procedure

Here, the movable simply supported CNTRC plate is considered. The Galerkin's method is applied to discretized partial differential equations into a set of ordinary differential equations. Hence, the following displacements are assumed [11]:

$$u(x,y,t)=\mathbf{\Phi}_u^T\,\mathbf{q}_u \qquad v(x,y,t)=\mathbf{\Phi}_v^T\,\mathbf{q}_v \qquad w(x,y,t)=\mathbf{\Phi}_w^T\,\mathbf{q}_w \tag{8}$$

where $\mathbf{q}_\text{u}$, $\mathbf{q}_\text{v}$ and $\mathbf{q}_\text{w}$ are vectors of generalized coordinates, and $\mathbf{\Phi}_\text{u}$, $\mathbf{\Phi}_\text{v}$ and $\mathbf{\Phi}_\text{w}$ are shape functions. By applying the Galerkin's method and the state space form, the governing equations can be expressed as

$$\mathbf{A}\,\dot{\mathbf{q}}=\mathbf{B}\,\mathbf{q} \tag{9}$$

where $\mathbf{q}$ is overall vector of generalized coordinates as

$$\begin{gathered}\{\mathbf{q}\}=\{\mathbf{q}_{u1}^T\,\mathbf{q}_{u2}^T\,\mathbf{q}_{v1}^T\,\mathbf{q}_{v2}^T\,\mathbf{q}_{w1}^T\,\mathbf{q}_{w2}^T\}^T\\ \mathbf{q}_{u1}=\mathbf{q}_u \qquad \mathbf{q}_{u2}=\dot{\mathbf{q}}_{u1} \qquad \mathbf{q}_{v1}=\mathbf{q}_v \qquad \mathbf{q}_{v2}=\dot{\mathbf{q}}_{v1} \qquad \mathbf{q}_{w1}=\mathbf{q}_w \qquad \mathbf{q}_{w2}=\dot{\mathbf{q}}_{w1}\end{gathered} \tag{10}$$

The overall vector of generalized coordinates can be written as a function of the natural frequency of CNTRC plates, ω. Consequently, the vibrational characteristics of nanocomposite plates are investigated by solving the eigenvalue problem (Eq. (9)).

$$\mathbf{q}=\bar{\mathbf{q}}e^{i\omega t} \tag{11}$$

5. Uncertainty propagation procedure

Since the presence of uncertainties leads to propagate them to the mechanical behavior of CNTRC plates, understanding the influence of uncertainties play a significant role in design of CNTRC plates. The propagation of uncertainties can stem from geometrical configuration, mechanical properties, boundary conditions and etc. To tackle this problem, one can employ probabilistic approaches such as the Monte Carlo simulation method. Despite efficiency and accuracy of these methods, the adequate knowledge about the variability of uncertain parameters are absent mostly. Based on lacking the experimental statistics informations including probabilistic distribution density, non-probabilistic approaches can be employed. The interval analysis method is a well-known non-probabilistic approach utilized in a wide range of engineering applications [15, 17]. Applying this method requires only bounds of uncertain parameters and variations of uncertain parameters are enclosed by multi-dimensional rectangle. The goal of this study is to determine the upper and lower bounds of the frequency under uncertain-but-bounded parameters. Here, geometrical configuration and mechanical properties of the matrix and CNTs are considered as the sources of uncertainty and can be expressed as

$$\boldsymbol{\alpha}=\begin{pmatrix}\alpha_1 & \alpha_2 & \ldots & \alpha_p\end{pmatrix}^T \tag{12}$$

where p is number of sources and $\boldsymbol{\alpha}$ is a p-dimensional vector of uncertain parameters well-known as uncertain parameter vector. It is seen that natural frequency, **A** and **B** matrices are functions of uncertain parameter vector and hence, the overall vector of generalized coordinates can be expressed as a function of uncertain parameter vector.

$$\mathbf{A}(\boldsymbol{\alpha})\,\dot{\mathbf{q}}(\boldsymbol{\alpha},t)=\mathbf{B}(\boldsymbol{\alpha})\,\mathbf{q}(\boldsymbol{\alpha},t) \tag{13}$$

In the deterministic analysis, nominal vector, $\boldsymbol{\alpha}_0$, of uncertain parameters is employed. However, in the uncertainty analysis, the uncertain parameters can be expressed by an interval vector according to the interval mathematical theory [17, 18].

$$\boldsymbol{\alpha}^I=[\underline{\boldsymbol{\alpha}},\bar{\boldsymbol{\alpha}}]=\{\boldsymbol{\alpha}:\boldsymbol{\alpha}\in R^p,\underline{\boldsymbol{\alpha}}\le\boldsymbol{\alpha}\le\bar{\boldsymbol{\alpha}}\} \tag{14}$$

where $\underline{\boldsymbol{\alpha}}$ and $\bar{\boldsymbol{\alpha}}$ are upper and lower bound of uncertain parameter vector, respectively. Moreover, one can expressed interval vector in component form as

$$\alpha_i^I=[\underline{\alpha}_i,\bar{\alpha}_i]=\{\alpha_i:\underline{\alpha}_i\le\alpha_i\le\bar{\alpha}_i\} \quad i=1,2,\ldots,p \tag{15}$$

Based on the interval arithmetic, an arbitrary interval vector are obtained as summation of the nominal or midpoint vector, $\boldsymbol{\alpha}_0$, and deviation amplitude vector, $\Delta\boldsymbol{\alpha}$.

$$\boldsymbol{\alpha}^I=\boldsymbol{\alpha}_0+\Delta\boldsymbol{\alpha}^I \tag{16}$$

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where

$$\boldsymbol{\alpha}_0 = \frac{(\bar{\boldsymbol{\alpha}} + \underline{\boldsymbol{\alpha}})}{2} \qquad \Delta\boldsymbol{\alpha} = \frac{(\bar{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}})}{2} \qquad \Delta\boldsymbol{\alpha}^I = [-\Delta\boldsymbol{\alpha}, \Delta\boldsymbol{\alpha}] \tag{17}$$

Here, the zero subscript indicates the nominal or midpoint value of specific parameter. Hence, an arbitrary uncertain parameter vector can be written as following.

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}_0 + \delta\boldsymbol{\alpha}, \; |\delta\boldsymbol{\alpha}| \le \Delta\boldsymbol{\alpha} \quad \text{or} \quad \alpha_i = \alpha_{0i} + \delta\alpha_i, \; |\delta\alpha_i| \le \Delta\alpha_i \quad i = 1, 2, \dots, p \tag{18}$$

Using the first order of Taylor series expansion about nominal vector, variations of the frequency is

$$\omega(\boldsymbol{\alpha}) = \omega(\boldsymbol{\alpha}_0 + \delta\boldsymbol{\alpha}) = \omega(\boldsymbol{\alpha}_0) + \sum_{i=1}^{p} \frac{\partial\omega(\boldsymbol{\alpha}_0)}{\partial\alpha_i}\delta\alpha_i \tag{19}$$

where $\omega_0 = \omega(\boldsymbol{\alpha}_0)$ reveals nominal or midpoint value of the natural frequency of CNTRC plates. Based on the interval mathematical theory, interval frequency, ω^I, can be obtained as

$$\omega^I = [\underline{\omega}, \bar{\omega}] = \omega(\boldsymbol{\alpha}_0) + \begin{pmatrix} \frac{\partial\omega(\boldsymbol{\alpha}_0)}{\partial\alpha_1} & \frac{\partial\omega(\boldsymbol{\alpha}_0)}{\partial\alpha_2} & \dots & \frac{\partial\omega(\boldsymbol{\alpha}_0)}{\partial\alpha_p} \end{pmatrix} \Delta\boldsymbol{\alpha}^I \tag{20}$$

Applying a mathematical procedure and definition of interval deviation amplitude vector lead to [18]

$$\underline{\omega} = \omega(\boldsymbol{\alpha}_0) - \sum_{i=1}^{p} \left| \frac{\partial\omega(\boldsymbol{\alpha}_0)}{\partial\alpha_i} \right| \Delta\alpha_i \qquad \bar{\omega} = \omega(\boldsymbol{\alpha}_0) + \sum_{i=1}^{p} \left| \frac{\partial\omega(\boldsymbol{\alpha}_0)}{\partial\alpha_i} \right| \Delta\alpha_i \tag{21}$$

Hence, the upper and lower bounds of the natural frequency under uncertain-but-bounded parameters are computed. For better understanding, non-dimensional parameters are defined as

$$r = \frac{a}{b} \qquad H = \frac{h}{a} \qquad \Omega = \omega\left(a_0^2/h_0\right)\sqrt{\rho_0^m/E_0^m} \tag{22}$$

6. Numerical results

To confirm the accuracy of the present results, the non-dimensional natural frequency of isotropic simply supported plate are compared with those available in the literature [19-22]. The square plate with thickness ratio, H = h/a, equal to 0.001 and Poisson's ratio ν= 0.3 are considered. In comparison study, non-dimensional frequency is defined as $\Omega = \omega\left(a^2/h\right)\sqrt{12\left(1-\nu_{12}^2\right)\rho/E}$. The results are listed in Table 1 and the accuracy of the current procedure is verified.

Table 1. Validation for isotropic square plate with simply supported boundary condition.

	Mode sequences							
	1	2	3	4	5	6	7	8
Ref. [19]	19.7392	49.3480	49.3480	78.9568	98.6960	98.6960	128.3049	128.3049
Ref. [20]	19.7392	49.3480	49.3480	78.9568	98.6951	98.6951	128.3029	128.3029
Ref. [21]	19.739	49.348	49.348	78.957	98.696	98.696	128.305	128.305
Ref. [22]	19.7391	49.3475	49.3475	78.9557	98.6943	98.6934	128.3019	128.3019
Present	19.7392	49.3479	49.3479	78.9566	98.6956	98.6956	128.3042	128.3042

Here, PmPV is considered as the polymer matrix. The CNTs efficiency parameters and the nominal values of mechanical properties of the matrix and the CNTs are mentioned in Ref. [16]. Unless otherwise mentioned, we take H_0 = 0.02 and r_0 = 1. The uncertainty propagation in the geometry of CNTRC plate with various nominal values of aspect ratio is depicted in Fig. 1. Here, thickness ratio, H, and aspect ratio, r, are considered as uncertain parameters. In Fig. 1(a) the frequency of UD plate with different uncertainty levels (5% and 8%) is investigated. It is seen that by increasing r_0, the deviation amplitude of natural frequency increases. Moreover, with the increase of uncertainty level, the propagated uncertainty in the frequency increases obviously. Fig. 1(b) reveals the deviation

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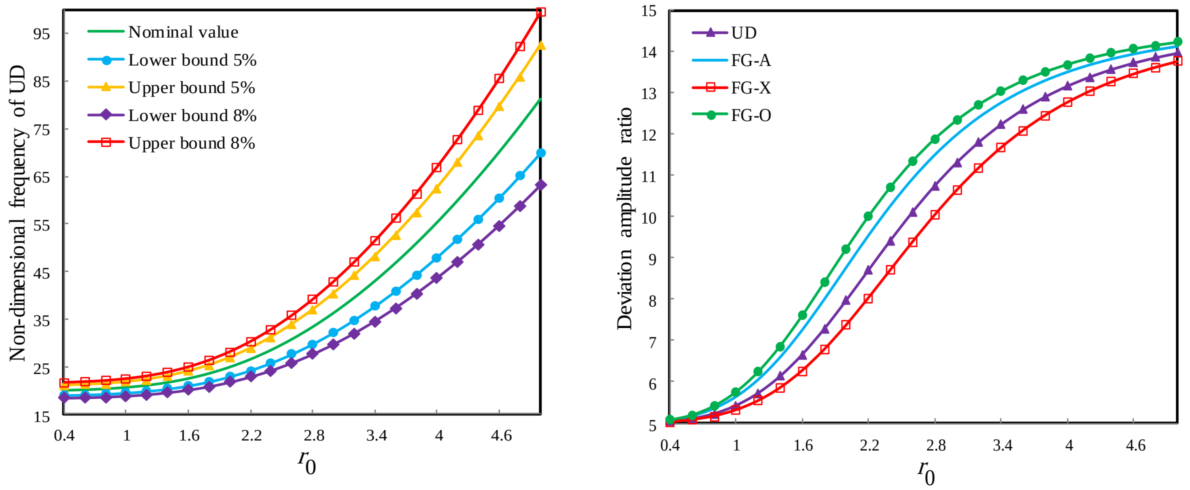
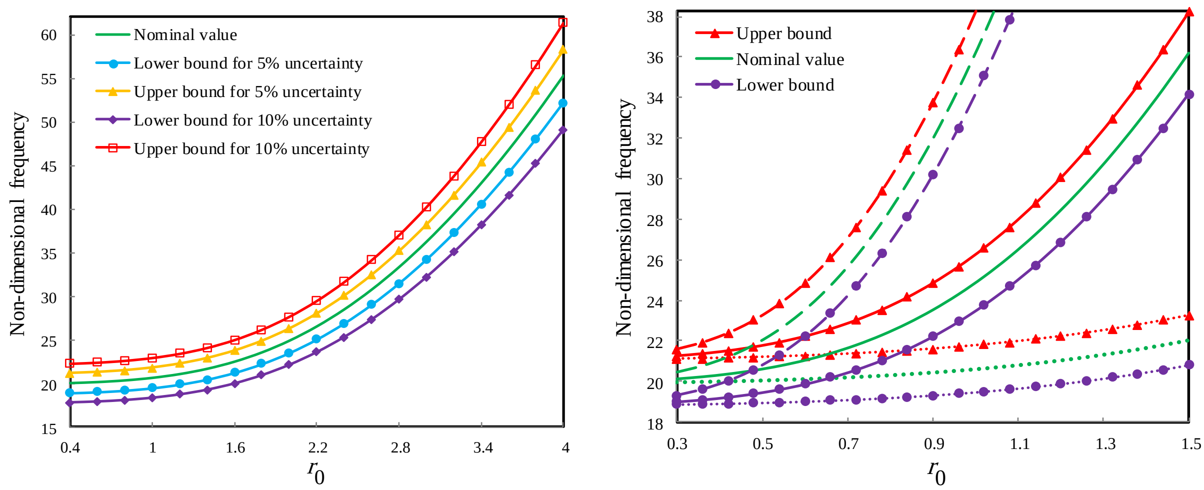
amplitude-to-nominal value ratio (deviation amplitude ratio) of various FG-CNTRC plate under 5 percent uncertainty. With the increase of r_0, the deviation amplitude ratio increases. In the low and high aspect ratio FG-CNTRC plates, distinct values of deviation amplitude ratio don't occur.



(a) (b)

Figure 1. Effect of the nominal values of aspect ratio: (a) on the frequency with different uncertainty levels, (b) on the deviation amplitude ratio.

The variation of frequency of UD-CNTRC plate versus the nominal values of aspect ratio under mechanical characteristic uncertainties is illustrated in Fig. 2. Here, mechanical properties of the CNTs and the matrix including E^m, ρ^m, ν^m, E_{11}^{CNT}, E_{22}^{CNT}, G_{12}^{CNT}, ν_{12}^{CNT} and ρ^{CNT} are considered as uncertain parameters. In Fig. 2(a) the uncertain frequency with various uncertainty levels is studied. All uncertain parameters have the same degree of uncertainty equal to 5% and 10%. The deviation amplitude increases with the increase of uncertainty level. Moreover, Fig. 2(b) reveals change of frequency modes against r_0 under 5% uncertainty. Fig. 2(b) point out that the deterministic frequencies of CNTRC plates may possibly be lower than the uncertain natural frequencies. For low aspect ratio plates, the uncertain second and third frequencies may be less than the deterministic first frequency.



(a) (b)

Figure 2. The frequency of UD plate versus the nominal values of aspect ratio: (a) different uncertainty level, (b) first (dotted line), second (solid line) and third (dashed line) modes.

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The frequency of UD plate versus the nominal values of thickness ratio, H_0, is depicted in Fig. 3. The corresponding properties of CNTRC plate are the same as the aforementioned study. To examine effects of thickness ratio, the non-dimensional frequency is redefined as $\Omega = \omega a_0 \sqrt{\rho_0^m / E_0^m}$. As it is expected, by increasing of H_0, the deviation amplitude increases for different degrees of uncertainty.

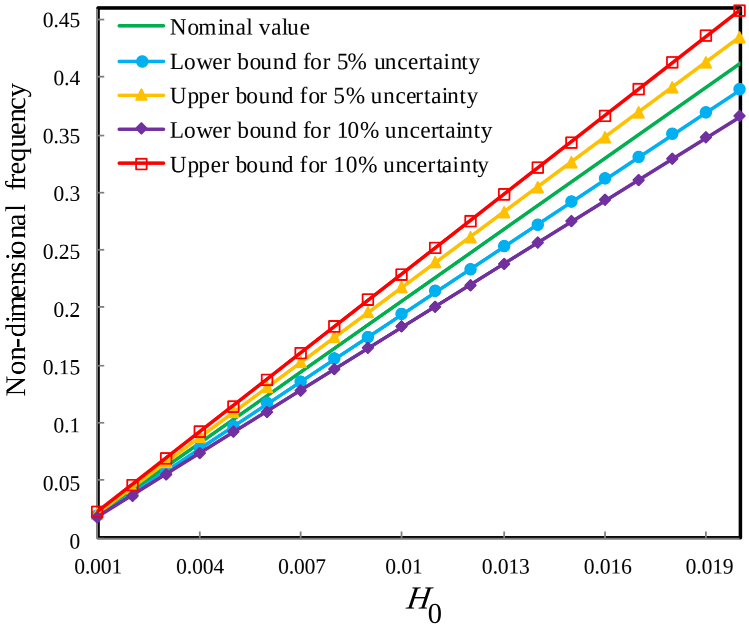

Figure 3. The natural frequency of UD plate against the nominal values of thickness ratio.

7. Conclusion

Frequency analysis of FG-CNTRC plates under uncertainty is the main contribution of the present paper. The geometrical dimensions and mechanical properties of the CNTs and the polymer matrix were considered as uncertain parameters. The mechanical properties of CNTRC plates were estimated via the modified rule of mixture. Using the Kirchhoff's plate theory, the governing equations were derived. The interval analysis method was applied to handle uncertainty propagation. The accuracy of the present model was verified by comparing with the existing data in the literature. Variations of the lower and upper bounds of frequencies with respect to aspect ratio and thickness ratio were investigated. It was seen that with uncertain mechanical properties, the uncertain second and third frequencies of low aspect ratio plates may be less than the deterministic first frequency. Under geometrical uncertainty, the deviation amplitude ratio increased with the increase of aspect ratio.

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