RESEARCH OF MECHANICAL PROPERTIES OF HIGHLY-FILLED POLYMER COMPOSITES UNDER COMPLEX HARMONIC LOADINGS

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Abstract

In this work, nonlinear representations of stress and strain under two-frequency loadings were presented, and it was proposed to describe dynamic modules and loss angles by polynomials and to use a time-temperature superposition for determining dependencies of the viscoelastic parameters on the temperature; to determine viscoelastic parameters, it was proposed to use the Fourier series.

1. Introduction

The topic relevance is due to: the use of highly-filled polymer composites in important aerospace structures and other industries; the action of complex harmonic loadings on structures where highly-filled polymers are used; the need to develop methods of experimental research and to define deformation properties of materials and calculation methods for structures working in extreme conditions.

The aim of this research is to develop methods for conducting the dynamic experiment, to define viscoelastic parameters of highly-filled polymer composites under stationary two-frequency loadings, and to identify the mathematical model for calculating the stress-strain state of viscoelastic aerospace structures.

2. Nonlinear representations of stress and strain under two-frequency loadings

Filled polymers are typically characterized by non-linear beahviour even in relatively small deformations [1-8]. The general description of a method to mathematically model nonlinear viscoelastic behavior was accomplished by Volterra using an earlier representation developed by Frechet. The Volterra-Frechet equation [9-11] for one dimension is

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$$\begin{gathered}\sigma(t)=\int_{-\infty}^{t} E_1\left(t-\tau_1\right) d\varepsilon(\tau_1)+\int_{-\infty}^{t}\int_{-\infty}^{t} E_2\left(t-\tau_1 ; t-\tau_2\right) d\varepsilon(\tau_1) d\varepsilon(\tau_2)+ \\ +\int_{-\infty}^{t}\int_{-\infty}^{t}\int_{-\infty}^{t} E_3\left(t-\tau_1 ; t-\tau_2 ; t-\tau_3\right) d\varepsilon(\tau_1) d\varepsilon(\tau_2) d\varepsilon(\tau_3)+\ldots+ \\ +\int_{-\infty}^{t}\ldots\int_{-\infty}^{t} E_n\left(t-\tau_1 ; \ldots ; t-\tau_n\right) d\varepsilon(\tau_1) \ldots d\varepsilon(\tau_n)+\ldots\end{gathered}, \quad (1)$$

where τ is the time before the observation moment t, $E(t)$ is the relaxation modulus (function, kernel), σ is stress, ε is strain. The lower limit of the integral is -∞, because all events over the history of a viscoelastic material contribute to the current state of stress and strain. Hereinafter we will use only the first three terms of the series (1).

Now, we decompose $E_1(t)$, $E_2(t)$ and $E_3(t)$ from Eq. (1) into two parts

$$E_n(t)=\overset{\circ}{E}_n+\hat{E}_n(t) . \quad (2)$$

The strain dependence on the time under stationary two-frequency loadings is

$$\varepsilon(t)=\varepsilon_{a1}\cdot e^{i\omega_1 t}+\varepsilon_{a2}\cdot e^{i\omega_2 t}, \quad (3)$$

where ε_{a1}, ε_{a2} are the strain amplitudes, ω_1, ω_1 are the angular frequencies. Next, we insert Eq. (2) into Eq. (1), and then Eq. (3) in Eq. (1). As a result, the first term of the series (1) can be written as

$$\begin{gathered}\int_{-\infty}^{t} E_1\left(t-\tau_1\right) d\varepsilon(\tau_1)=\overset{\circ}{E}_1\,\varepsilon_{a1}\cdot e^{i\omega_1 t}+i\omega_1\varepsilon_{a1}\int_{-\infty}^{t}\hat{E}_1\left(t-\tau_1\right) e^{i\omega_1\tau_1} d\tau_1+ \\ +\overset{\circ}{E}_1\,\varepsilon_{a2}\cdot e^{i\omega_2 t}+i\omega_2\varepsilon_{a2}\int_{-\infty}^{t}\hat{E}_1\left(t-\tau_1\right) e^{i\omega_2\tau_1} d\tau_1\end{gathered}. \quad (4)$$

After changing the variable $t-\tau_1=\eta_1$ Eq. (4) can be transformed as

$$\begin{gathered}\int_{-\infty}^{t} E_1\left(t-\tau_1\right) d\varepsilon(\tau_1)=\varepsilon_{a1} e^{i\omega_1 t}\left[\overset{o}{E}_1+\omega_1\int_0^{\infty}\hat{E}_1(\eta_1)\sin\omega_1\eta_1\,d\eta_1+i\omega_1\int_0^{\infty}\hat{E}_1(\eta_1)\cos\omega_1\eta_1\,d\eta_1\right]+ \\ +\varepsilon_{a2} e^{i\omega_2 t}\left[\overset{o}{E}_1+\omega_2\int_0^{\infty}\hat{E}_1(\eta_1)\sin\omega_2\eta_1\,d\eta_1+i\omega_2\int_0^{\infty}\hat{E}_1(\eta_1)\cos\omega_2\eta_1\,d\eta_1\right]\end{gathered}. \quad (5)$$

$$E'_{1/1}(\omega_1;T)=\overset{o}{E}_1+\omega_1\int_0^{\infty}\hat{E}_1(\eta_1)\sin\omega_1\eta_1\,d\eta_1, \quad (6)$$

$$E''_{1/1}(\omega_1;T)=\omega_1\int_0^{\infty}\hat{E}_1(\eta_1)\cos\omega_1\eta_1\,d\eta_1, \quad (7)$$

$$\tilde{E}_{1/1}(\omega_1;T)=E'_{1/1}(\omega_1;T)+iE''_{1/1}(\omega_1;T), \quad (8)$$

$$E'_{1/2}(\omega_2;T)=\overset{o}{E}_1+\omega_2\int_0^{\infty}\hat{E}_1(\eta_1)\sin\omega_2\eta_1\,d\eta_1, \quad (9)$$

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$$E''_{1/2}(\omega_2;T)=\omega_2\int_0^\infty \hat{E}_1(\eta_1)\cos\omega_2\eta_1\,d\eta_1 \ , \tag{10}$$

$$\tilde{E}_{1/2}(\omega_2;T)=E'_{1/2}(\omega_2;T)+iE''_{1/2}(\omega_2;T)\ . \tag{11}$$

After similar solutions (4-11) for second and fourth terms of the series (1), Eq. (1) can be written as

$$\begin{gathered}\sigma(t)=\varepsilon_{a1}e^{i\omega_1 t}\tilde{E}_{1/1}(\omega_1;T)+\varepsilon_{a1}^2e^{i2\omega_1 t}\tilde{E}_{2/1}(\omega_1;T)+\varepsilon_{a1}^3e^{i3\omega_1 t}\tilde{E}_{3/1}(\omega_1;T)+\\+\varepsilon_{a1}e^{i\omega_1 t}\varepsilon_{a2}e^{i\omega_2 t}\tilde{E}_{2/3}(\omega_1;\omega_2;T)+\varepsilon_{a1}^2e^{i2\omega_1 t}\varepsilon_{a2}e^{i\omega_2 t}\tilde{E}_{3/3}(\omega_1;\omega_2;T)+\varepsilon_{a1}e^{i\omega_1 t}\varepsilon_{a2}^2e^{i2\omega_2 t}\tilde{E}_{3/4}(\omega_1;\omega_2;T)+.\\+\varepsilon_{a2}e^{i\omega_2 t}\tilde{E}_{1/2}(\omega_2;T)+\varepsilon_{a2}^2e^{i2\omega_2 t}\tilde{E}_{2/2}(\omega_2;T)+\varepsilon_{a2}^3e^{i3\omega_2 t}\tilde{E}_{3/2}(\omega_2;T)\end{gathered} \tag{12}$$

$$E'_{2/1}(\omega_1;T)=\overset{o}{E}_2-\omega_1^2\int_0^\infty\int_0^\infty \hat{E}_2(\eta_1;\eta_2)\cos(\omega_1\eta_1+\omega_1\eta_2)d\eta_1\,d\eta_2 \ , \tag{13}$$

$$E''_{2/1}(\omega_1;T)=\omega_1^2\int_0^\infty\int_0^\infty \hat{E}_2(\eta_1;\eta_2)\sin(\omega_1\eta_1+\omega_1\eta_2)d\eta_1\,d\eta_2 \ , \tag{14}$$

$$\tilde{E}_{2/1}(\omega_1;T)=E'_{2/1}(\omega_1;T)+iE''_{2/1}(\omega_1;T)\ , \tag{15}$$

$$E'_{2/2}(\omega_2;T)=\overset{o}{E}_2-\omega_2^2\int_0^\infty\int_0^\infty \hat{E}_2(\eta_1;\eta_2)\cos(\omega_2\eta_1+\omega_2\eta_2)d\eta_1\,d\eta_2 \ , \tag{16}$$

$$E''_{2/2}(\omega_2;T)=\omega_2^2\int_0^\infty\int_0^\infty \hat{E}_2(\eta_1;\eta_2)\sin(\omega_2\eta_1+\omega_2\eta_2)d\eta_1\,d\eta_2 \ , \tag{17}$$

$$\tilde{E}_{2/2}(\omega_2;T)=E'_{2/2}(\omega_2;T)+iE''_{2/2}(\omega_2;T)\ , \tag{18}$$

$$E'_{2/3}(\omega_1;\omega_2;T)=2\overset{o}{E}_2-\omega_1\omega_2\int_0^\infty\int_0^\infty \hat{E}_2(\eta_1;\eta_2)\left[\cos(\omega_1\eta_1+\omega_2\eta_2)+\cos(\omega_2\eta_1+\omega_1\eta_2)\right]d\eta_1\,d\eta_2 \ , \tag{19}$$

$$E''_{2/3}(\omega_1;\omega_2;T)=\omega_1\omega_2\int_0^\infty\int_0^\infty \hat{E}_2(\eta_1;\eta_2)\left[\sin(\omega_1\eta_1+\omega_2\eta_2)+\sin(\omega_2\eta_1+\omega_1\eta_2)\right]d\eta_1\,d\eta_2 \ , \tag{20}$$

$$\tilde{E}_{2/3}(\omega_1;\omega_2;T)=E'_{2/3}(\omega_1;\omega_2;T)+iE''_{2/3}(\omega_1;\omega_2;T)\ , \tag{21}$$

$$E'_{3/1}(\omega_1;T)=\overset{o}{E}_3-\omega_1^3\int_0^\infty\int_0^\infty\int_0^\infty \hat{E}_3(\eta_1;\eta_2;\eta_3)\sin(\omega_1\eta_1+\omega_1\eta_2+\omega_1\eta_3)d\eta_1\,d\eta_2\,d\eta_3 \ , \tag{22}$$

$$E''_{3/1}(\omega_1;T)=-\omega_1^3\int_0^\infty\int_0^\infty\int_0^\infty \hat{E}_3(\eta_1;\eta_2;\eta_3)\cos(\omega_1\eta_1+\omega_1\eta_2+\omega_1\eta_3)d\eta_1\,d\eta_2\,d\eta_3 \ , \tag{23}$$

$$\tilde{E}_{3/1}(\omega_1;T)=E'_{3/1}(\omega_1;T)+iE''_{3/1}(\omega_1;T)\ , \tag{24}$$

$$E'_{3/2}(\omega_2;T) = \overset{o}{E}_3 - \omega_2^3 \int_0^\infty\int_0^\infty\int_0^\infty \hat{E}_3(\eta_1;\eta_2;\eta_3)\sin(\omega_2\eta_1 + \omega_2\eta_2 + \omega_2\eta_3)\,d\eta_1\,d\eta_2\,d\eta_3\ , \tag{25}$$

$$E''_{3/2}(\omega_2;T) = -\omega_2^3 \int_0^\infty\int_0^\infty\int_0^\infty \hat{E}_3(\eta_1;\eta_2;\eta_3)\cos(\omega_2\eta_1 + \omega_2\eta_2 + \omega_2\eta_3)\,d\eta_1\,d\eta_2\,d\eta_3\ , \tag{26}$$

$$\tilde{E}_{3/2}(\omega_2;T) = E'_{3/2}(\omega_2;T) + iE''_{3/2}(\omega_2;T)\ , \tag{27}$$

$$\begin{gathered}E'_{3/3}(\omega_1;\omega_2;T) = 3\overset{o}{E}_3 - \omega_1^2\omega_2 \int_0^\infty\int_0^\infty\int_0^\infty \hat{E}_3(\eta_1;\eta_2;\eta_3)\Big[\sin(\omega_2\eta_1 + \omega_1\eta_2 + \omega_1\eta_3) + \\ + \sin(\omega_1\eta_1 + \omega_2\eta_2 + \omega_1\eta_3) + \sin(\omega_1\eta_1 + \omega_1\eta_2 + \omega_2\eta_3)\Big]d\eta_1\,d\eta_2\,d\eta_3\end{gathered}\ , \tag{28}$$

$$\begin{gathered}E''_{3/3}(\omega_1;\omega_2;T) = -\omega_1^2\omega_2 \int_0^\infty\int_0^\infty\int_0^\infty \hat{E}_3(\eta_1;\eta_2;\eta_3)\Big[\cos(\omega_2\eta_1 + \omega_1\eta_2 + \omega_1\eta_3) + \\ + \cos(\omega_1\eta_1 + \omega_2\eta_2 + \omega_1\eta_3) + \cos(\omega_1\eta_1 + \omega_1\eta_2 + \omega_2\eta_3)\Big]d\eta_1\,d\eta_2\,d\eta_3\end{gathered}\ , \tag{29}$$

$$\tilde{E}_{3/3}(\omega_1;\omega_2;T) = E'_{3/3}(\omega_1;\omega_2;T) + iE''_{3/3}(\omega_1;\omega_2;T)\ , \tag{30}$$

$$\begin{gathered}E'_{3/4}(\omega_1;\omega_2;T) = 3\overset{o}{E}_3 - \omega_1\omega_2^2 \int_0^\infty\int_0^\infty\int_0^\infty \hat{E}_3(\eta_1;\eta_2;\eta_3)\Big[\sin(\omega_1\eta_1 + \omega_2\eta_2 + \omega_2\eta_3) + \\ + \sin(\omega_2\eta_1 + \omega_1\eta_2 + \omega_2\eta_3) + \sin(\omega_2\eta_1 + \omega_2\eta_2 + \omega_1\eta_3)\Big]d\eta_1\,d\eta_2\,d\eta_3\end{gathered}\ , \tag{31}$$

$$\begin{gathered}E''_{3/4}(\omega_1;\omega_2;T) = -\omega_1\omega_2^2 \int_0^\infty\int_0^\infty\int_0^\infty \hat{E}_3(\eta_1;\eta_2;\eta_3)\Big[\cos(\omega_1\eta_1 + \omega_2\eta_2 + \omega_2\eta_3) + \\ + \cos(\omega_2\eta_1 + \omega_1\eta_2 + \omega_2\eta_3) + \cos(\omega_2\eta_1 + \omega_2\eta_2 + \omega_1\eta_3)\Big]d\eta_1\,d\eta_2\,d\eta_3\end{gathered}\ , \tag{32}$$

$$\tilde{E}_{3/4}(\omega_1;\omega_2;T) = E'_{3/4}(\omega_1;\omega_2;T) + iE''_{3/4}(\omega_1;\omega_2;T)\ , \tag{33}$$

$$\tilde{E}_n = E'_n + iE''_n\ , \tag{34}$$

$$E'_n = E^*_n \cos\varphi_{En}\ , \tag{35}$$

$$E''_n = E^*_n \sin\varphi_{En}\ , \tag{36}$$

$$E^*_n = \sqrt{E'^2_n + E''^2_n}\ , \tag{37}$$

$$\tan\varphi_{En} = E''_n \big/ E'_n\ , \tag{38}$$

where $\tilde{E}$ is the complex modulus, *E'* is storage modulus, *E''* is loss modulus, φ_E is phase lag between stress and strain (phase angle, loss angle), E^* is dynamic modulus.

After inserting Eqs. (35) and (36) into Eq. (34) we obtain

$$\tilde{E}_n = E_n^*(\cos\varphi_{En} + i\sin\varphi_{En}) \,. \tag{39}$$

Using Euler's formula [12], Eq. (39) can be transformed as

$$\tilde{E}_n = E_n^* e^{i\varphi_{En}} \,. \tag{40}$$

The analysis of Eqs. (6-33) shows: $E_{1/1}(\omega_1;T)$ and $E_{1/2}(\omega_2;T)$, $E_{2/1}(\omega_1;T)$ and $E_{2/2}(\omega_2;T)$, $E_{3/1}(\omega_1;T)$ and $E_{3/2}(\omega_2;T)$ have the same graphic dependences respectively; if $K = 1$ (two-frequency loading → one-frequency loading, $\omega_1 = \omega_2$), then $E_{1/1}(T) = E_{1/2}(T)$, $E_{2/1}(T) = E_{2/2}(T) = E_{2/3}(T)/2$, $E_{3/1}(T) = E_{3/2}(T) = E_{3/3}(T)/3 = E_{3/4}(T)/3$; if $K = 0$ (two-frequency loading → one-frequency loading + preliminary static strain ε_{st}, $\varepsilon_{st} = \varepsilon_{a1}$, $\omega_1 = 0$), then $E'_{2/3}(T) = 2\overset{o}{E}_2$, $E''_{2/3}(T) = 0$, $E'_{3/3}(T) = E'_{3/4}(T) = 3\overset{o}{E}_3$, $E''_{3/3}(T) = E''_{3/4}(T) = 0$.

The relaxation functions (modules, kernels) can be described by different dependencies [11, 13], for example, an exponential function. However, the relaxation function selection from the dynamic test data is a rather difficult and time-consuming process. In the authors' opinion, it is easier to determine dynamic modules and loss angles from the experiment by determining the dependencies of stress and strain in the sample under two-frequency loadings as

$$\varepsilon(t) = \varepsilon_{a1}\sin 2\pi\nu_1 t + \varepsilon_{a2}\sin 2\pi\nu_2 t \,, \tag{41}$$

$$\nu_1 = \omega_1 / 2\pi \,, \qquad \nu_2 = \omega_2 / 2\pi \,, \qquad K = \omega_1 / \omega_2 \,, \tag{42}$$

where ν is frequency, and obtained dependencies of dynamic modules and loss angles on frequency and temperature can be described by polynomials. Polynomial models are rather simple in practical application [14]. A description of these models can be improved by increasing the polynomial degree. Time–temperature superposition can be used for describing the viscoelastic material behavior under various constant temperatures [9, 15-19]. The values of dynamic modules and loss angles can be determined like in works [20-21] by Fourier series.

3. Conclusions

As a result of this work, nonlinear representations of stress and strain under two-frequency loadings were presented, and it was proposed to use polynomials to describe dependencies of dynamic modules and loss angles on frequency. It was also proposed to use a time-temperature superposition for the accounting of the viscoelastic properties on the temperature, and to use the Fourier series to determine the viscoelastic parameters.

Future work includes defining graphic dependences of dynamic modules and loss angles on frequencies and temperature, developing optimal experimental design, determining material constants, and checking the model adequacy.

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