AN ANISOTROPIC DAMAGE MODEL FOR CMCS UNDER NON-PROPORTIONAL LOADING CONDITIONS

E. Baranger[1]

[1]LMT Cachan, ENS-Cachan/CNRS/Université Paris Saclay, 61 avenue du président Wilson, F-94235 Cachan, France
Email: baranger@lmt.ens-cachan.fr, Web Page: http://http://www.lmt.ens-cachan.fr

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Abstract
Ceramic matrix composites have good thermo-mecanical properties at high or very high temperature. The alliance of two brittle materials (SiC fibers and SiC matrix e.g.) via an interface allows a pseudo-ductile macroscopic behavior due to crack deviation. The modeling of the crack networks using damage mechanics is not straight forward. The main reason is the presence of a crack network oriented by the loading direction, which is not known a priori. The aim of this paper is to extend an anisotropic damage model able to describe such behaviors to multi-axial loadings. For that, compliance-like tensorial damage variables are used in a thermodynamic potential able to account for crack closure effects. The damage kinematic is initially completely free and imposed by the evolution laws. The key point of the present paper is to account for an anisotropic history of damage. The results obtained are put in relation to tension-torsion tests performed on SiC/SiC tubes and richly instrumented.

1. Introduction

Ceramic matrix composites (CMC) are good candidates for the manufacturing of aeronautical engine structures or nuclear energy applications as they present very good specific properties at high temperatures and irradiations. In both cases, engineers have to dispose of mechanical models in order to design and size parts. Regarding SiC/SiC composites, several crack networks can develop depending on the densification of the material and of the fiber/matrix interface. Among them, inter-yarn cracks may develop orthogonally to the loading directions as mentioned by [1]. This one is not easy to model, accounting for crack closure effects i.e. restauration of the stiffness in compression. It has lead to the so-called anisotropic damage models in the literature. This problem is also well known in the field of concrete modeling. The difficulty resides in the obtention of continuous stress/strain relations i.e. convex potential for multi-axial non-proportional loadings. For that, two main approaches have been developed. The first one is to discretize the potential crack directions in the plane and use associated scalar damage variables in fixed directions as in [2, 3]. The second one is to use tensorial damage variables. The simplest approach is to use a second order tensor damage variable as in [4]. Note that this lead by simplification to [2]. Several difficulties associated to this model are mentioned in [5] and by the authors themselves. Another approach is to use fourth order tensors. For example, [6] used compliance tensors as damage variables. He defined associated special positive and negative parts of the stresses in order to get the good properties on the free energy potential. The damage kinematic is therefore completely free and it is imposed by the evolution laws. This model is the basis of this paper. It will be briefly presented in section 2.
Recent experiences performed at CEA by [7, 8], and richly instrumented via digital image correlation,

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confirm macroscopic observations done by ONERA [9] on tension-torsion tests on SiC/SiC tubes. Concerning the experiments of Bernachy-Barbe, the main crack network is oriented by the loading direction. Therefore, alternate torsion tests leads to the creation of two orthogonal networks oriented by the two main loading directions. It will be shown that the model developed by Ladevèze is not able to account for such a non-proportional loading as the history is isotropic. Thus, this papers aims at extending this model to account for anisotropic history using concepts developed by [10]. Note that similar concepts seems to have been developed by Chaboche while comparing the evolution his papers [11, 12], but it is not explicitly mentioned.

2. Original damage model with isotropic history

In order to introduce versatile damage kinematics, different authors have chosen to describe damage using fourth order tensors. In this part, the model from Ladevèze for SiC/SiC composites is emphasized [6, 13]. The first main idea of this model is to let the damage kinematic completely free a priori and to specify it using the evolution laws. The second idea is to separate the contributions of the different crack networks. The objective is to have a mechanical model that could be linked to physico-chemical one to treat self-healing aspects of lifetime predictions [14, 15].

The elastic potential is written in stress as the sum of three contributions: the first is a contribution only active in tension, the second is a contribution active only in compression and the third is a contribution active both in tension and compression. It reads:

$$\Psi^{el} = \frac{1}{2}\boldsymbol{\sigma}_+ : \mathbb{C} : \boldsymbol{\sigma}_+ + \frac{1}{2}\boldsymbol{\sigma}_- : \mathbb{C}_0 : \boldsymbol{\sigma}_- + \frac{1}{2}\boldsymbol{\sigma} : \Delta\mathbb{Z} : \boldsymbol{\sigma} \tag{1}$$

$\Delta\mathbb{C}$ and $\Delta\mathbb{Z}$ are the resulting damage variables of the model, they are positive and have the symmetries of a compliance tensor so that their square roots $\mathbb{H}$ and $\mathbb{H}_0$ exist. $\boldsymbol{\sigma}_+$ and $\boldsymbol{\sigma}_-$ are positive and negative parts of $\boldsymbol{\sigma}$ defined to manage unilateral contact in cracks (the shear crack closure is not taken into account in this form) and to keep a convex potential even for non-proportional loadings. For that, two spectral decompositions are used. The first is used to define the positive part:

$$\boldsymbol{\sigma}_+ = \mathbb{H}^{-1} :< \mathbb{H} : \boldsymbol{\sigma} >_+ \tag{2}$$

The second is used to define the negative part:

$$\boldsymbol{\sigma}_- = \mathbb{H}_0^{-1} :< \mathbb{H}_0 : \boldsymbol{\sigma} >_- \tag{3}$$

For diagonal compliance tensors, these positive and negative parts are equal to the classical one but not in the general case. The use of classical positive and negative parts would lead to a non-convex potential and therefor to non-continuous stress-strain relation. This is due to the presence of terms mixing eigenvalues as shown in [16].

The thermodynamical forces associated to damage variables are also fourth order tensors. One choice is to privilege the strain as in this paper while in other previous papers the stress was privileged:

$$\mathbb{Y} = \frac{1}{2}(\mathbb{C} : \boldsymbol{\sigma}) \otimes (\mathbb{C} : \boldsymbol{\sigma}) \tag{4}$$

$$\mathbb{Y}' = \frac{1}{2}(\mathbb{C} : \boldsymbol{\sigma}_+) \otimes (\mathbb{C} : \boldsymbol{\sigma}_+) \tag{5}$$

(6)

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This allows to predict correctly the crack orientations (Figure 1). Another force is introduced to deal with shear, in 2D:

$$\mathbb{Y}'' = \frac{1}{2}(\boldsymbol{R}_{\frac{\pi}{2}}.(\mathbb{C} : \boldsymbol{\sigma}_+)) \otimes (\boldsymbol{R}_{\frac{\pi}{2}}.(\mathbb{C} : \boldsymbol{\sigma}_+)) \tag{7}$$

(8)

where $\boldsymbol{R}_{\frac{\pi}{2}}$ is a $\frac{\pi}{2}$ rotation in 2D.

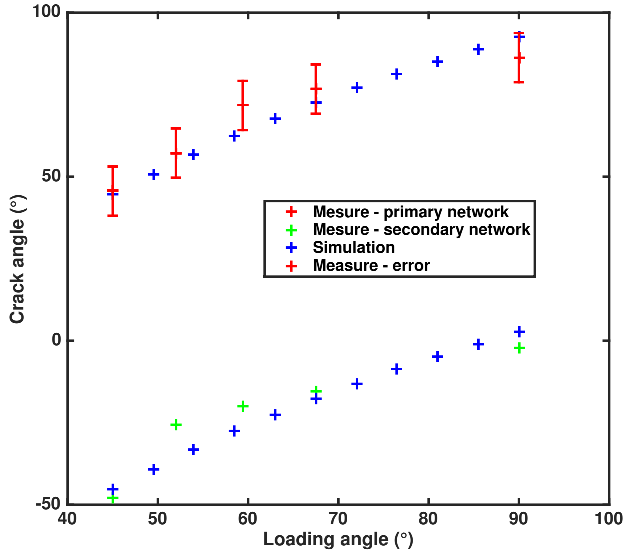

Figure 1. Crack angles vs. loading angle.

The total damage is separated in different contributions related to the different crack networks. The total damage is the sum of the different contributions. For the sake of simplicity, in the present paper, the inter-yarn cracking network is emphasized. The associated damage contributions are called: $\Delta\mathbb{C}_m$ and $\Delta\mathbb{Z}_m$. The associated driving force reads:

$$z_m = \left((1-a)Tr[\mathbb{Y}'^{n+1}] + aTr[\mathbb{Y}']^{n+1}\right)^{\frac{1}{n+1}} \tag{9}$$

(10)

a allows to pass from isotropic to anisotropic damage and n allows to emphasis the directionality of damage. The maximum force over time is defined by:

$$\bar{z}_m(t) = \sup_{\tau \leq t} z_m(\tau) \tag{11}$$

As the strain has been privileged, the evolution laws read:

$$\mathbb{C}^{-1} : \Delta\dot{\mathbb{C}}_m : \mathbb{C}^{-1} = \dot{\alpha}_m \frac{(1-a)\mathbb{Y}'^n + aTr[\mathbb{Y}']^n \mathbb{I}}{\bar{z}_m^n} \tag{12}$$

$$\mathbb{C}^{-1} : \Delta\dot{\mathbb{Z}}_m : \mathbb{C}^{-1} = \dot{\alpha}_m \frac{b\mathbb{Y}''^n}{\bar{z}_m^n} \tag{13}$$

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It has to be noted that for $a = 0$, the damage is oriented by $\mathbb{Y}'$. This choice is retained in the following.

For alternate shear loadings performed on a [±45] tube under torsion, the shear stress is plotted versus the axial strain on Figure 2. It can be noticed that is is not symmetric (regarding the abscissa axis) while the experimental one is.

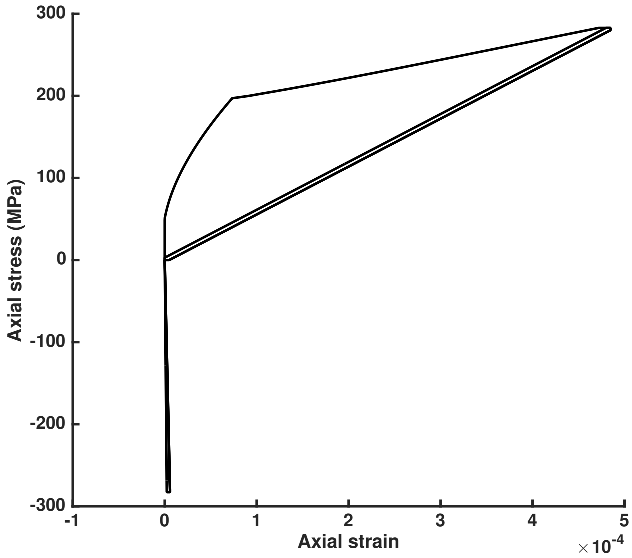

Figure 2. Simulated evolution of the shear stress vs. axial strain for a pipe loaded in torsion.

$\bar{z}_m(t)$ being a scalar, the damage history is resumed as an isotropic value even if the damage kinematic is very rich. This part of the evolution law has to be modified.

3. Enhanced damage model with anisotropic history

Equation 11 has to be modified. [10] proposed a framework for that on 2D damage tensors. The same approach is followed here. The damage flow rule is given by:

$$\mathbb{C}^{-1} : \Delta\dot{\mathbb{Z}}_m : \mathbb{C}^{-1} = \dot{\gamma}\frac{\partial z_m}{\partial \mathbb{Y}'} \tag{14}$$

Note that it is consistent with the flow rule given by equation 12. $\dot{\gamma}$ is equivalent to a plastic multiplier, some Khun Tucker conditions are associated. The criterion of damage reads:

$$f(d_{act}) = d_{act} - \alpha_m(z_m) \tag{15}$$

If $d_{act} = Tr[\Delta\mathbb{C}_m]$, then the model is equivalent to the one presented in the previous section. Thus this equivalent damage variable has to be modified, it now reads:

$$d_{act} = \Delta\mathbb{C}_m : \frac{\mathbb{Y}'}{z_m} \tag{16}$$

Here, z_m is used to normalize $\mathbb{Y}'$. The use of the scalar variable d_{act} is rather simple and conventional. Its definition via a projection on the loading direction allows to account for anisotropic history. The damage variable $\Delta\mathbb{C}_m$ stores the complete history of the degradation, its projection on the loading directions defines some kind of directional active damage contribution.

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For alternate shear loadings performed on a [±45] tube under torsion, the shear stress is plotted versus the axial strain on Figure 3. It can be noticed that is is now symmetric as for the experimental one.

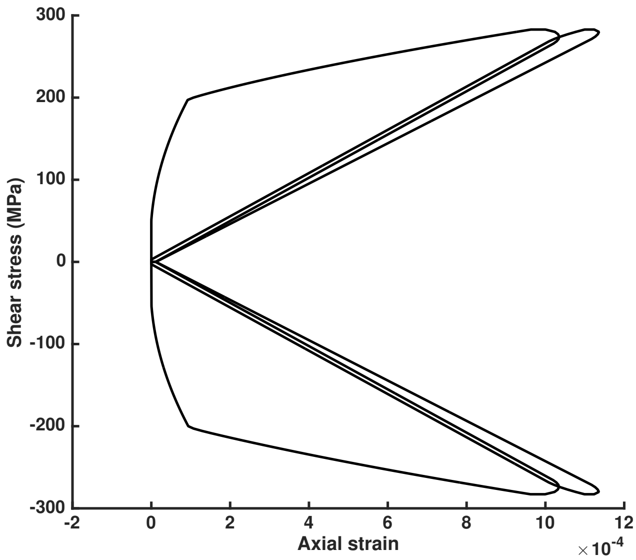

Figure 3. Simulated evolution of the shear stress vs. axial strain for a pipe loaded in torsion.

4. Conclusion

In this paper, an anisotropic damage model has been enhanced in order to account for non-proportional loadings. For that, an anisotropic history has been taken into account by projecting the fourth order damage variable on the loading direction. In order to validate the model, simulations have been compared to experimental results on SiC/SiC pipes under alternate torsion, showing a good agreement with the experimental ones. The on going work focuses on the shear damage deactivation while cracks are in compression.

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