SUITABILITY OF STIFFNESS AND STRENGTH BASED CONCEPTS FOR THE FATIGUE-LIFE PREDICTION OF COMPOSITES

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Keywords: continuous-fibre reinforced plastics, fatigue, lifetime prediction

Abstract

Continuous-fibre reinforced plastics offer extraordinary potential for the use in lightweight structures. However, in applications where composite materials are most superior, e.g. in vehicles or aircrafts, the occurrence of cyclic loads and changing environment are very likely. Mechanical load and temperature changes over time are known to induce fatigue, which is furthermore connected to the decrease of mechanical properties such as stiffness or strength. In order to offer reliable tools for fatigue-life prediction of parts in service, theories being able to take a range of influences on the degrading material behaviour into account are necessary.

In this paper, the authors critically review the applicability of fatigue-life prediction tools available at the moment in the scientific and industrial community. Potentials and weaknesses of the different theories are discussed in detail, always having the applicability to structural parts under complex load situations in mind.

1. Introduction

If the application of a material to an engineering component is contemplated, it is essential to answer not only the fundamental questions regarding strength and stiffness, but also the question of how long the material will last under the conditions applied [1]. In general, fatigue is concerned with the loss of properties and performance caused by external influences such as mechanical loading, thermal loading or chemical environments [2]. The reduction of strength and the subsequent failure of materials subjected to repeated loading has been identified as a fundamental problem of material science since the early 1800s [1–4]. However, unexpected material failure caused by fatigue can still occur as shown in figure 1 or have catastrophic consequences as in the train accident 1998 in Eschede, Germany [4]. The applications for which composite materials are most superior (e.g. high performance structures such as power systems, aircrafts or other vehicles) are precisely those situations in which degradation of strength and life by fatigue processes is most likely to occur [2].

The loads inducing fatigue in materials may be deterministic, periodic, aperiodic or stochastic. Cyclic loads with constant amplitudes are often described in relation to a constant or changing mean load by introducing $R = (minimum load / maximum load)$

Figure 1 Fatigue failure of a composite rotor blade in Conisholme, Lincolnshire, 2009 [5].

2. Fatigue and damage of composite materials

The viscoelastic effects within composite materials, especially with resins as matrix materials, are not as significant as for other polymeric materials as a result of the high stiffnesses and small deformations usually obtained in composite materials. However, the fatigue behaviour is decisively dominated by a series of complex damage mechanisms responsible for the macroscopically observed loss of stiffness and strength.

For the description and analysis of damage development in composite materials under both quasi-static and cyclic loading it is necessary to consider the determination of damage development in composite laminates including damage mechanisms and their chronology and duration of damage events. The general understanding of fatigue as complex combination of damage modes and damage mechanisms resulting in property degradation proposed by Reifsnider, Talreja and others since the 1980s as illustrated in figure 2 is well-accepted e.g. [7–14].

Figure 2 Schematic representation of the development of damage and damage modes in fatigue or quasi-static tests with composite laminates according to [16,19,20].

The initial state consists of primary matrix cracking along fibres in the plies inclined to the principal load direction. The cracking occurs in a multiple mode while the density of cracks increases. The state of damage given by the saturated and stable matrix cracking pattern in a laminate has been termed the characteristic damage state (CDS) [15,16]. As the primary matrix cracks forms, fibre failure initiates in regions of stress concentration created by the primary cracks. The primary matrix cracks also initiate small secondary cracks which extend short distances away from the interface between the ply. The next event that may occur is delamination in the interior of the laminate. This delamination is caused by a mixed mode of growth of interlaminar cracks which is driven by the strong interlaminar stresses in regions where primary and secondary cracks cross. In compression loading the out-of-plane displacement of plies also influences the delamination growth [15]. The later stage of damage development is typified as a rapidly increasing rate of progression of all damage modes. The terminal event of separation consists usually of large-scale fibre failures in plies aligned with or most nearly aligned with the principal tensile stress [15,16]. However, it is a well-known that the damage development caused by the respective stress states is significantly dependent on the unique properties of the material (i.e. fibre and matrix materialj, stacking sequence) [9,17,18].

3. Theories describing composite fatigue

Many theories describing the fatigue behaviour and the fatigue processes within composite materials have been studied during the last decades. Different approaches have been suggested. While some of them are motivated by the physical progresses occurring on different scales from microscopic damage mechanisms to macroscopic decrease of properties during fatigue life, others characterise fatigue by failure of entire specimens or structures. In the following, four of the most widely spread approaches towards the description of fatigue behaviour of composite materials are introduced. Stiffness and residual strength, energy based approaches, fracture mechanics and traditional fatigue strength based S-N curves are discussed. Of course, this listing does neither assert the claim to be complete nor to discuss all aspects in detail.

3.1 Stiffness and residual strength based fatigue-life approaches

In some cases a laminate could be considered to have functionally failed before separation as illustrated in figure 2 due to substantial loss of mechanical properties such as residual strength or stiffness [6,15,16]. As figure 3 illustrates, both stiffness and residual strength decrease during fatigue life. The fatigue strength, which was added to the chart, represents the strength at failure of a permanently loaded specimen, which is often used to create the well-known Wöhler lines or S-N curves [3,4].

Figure 3 Schematic diagram of strength change and stiffness progress during fatigue life of an unnotched laminate referring to [6].

Change in stiffness of composite laminates under long-term, cyclic loading is significant for two important reasons. First, many engineering structures made of composite materials are deformationlimited, or stiffness-critical, structures. Aircraft and aerospace components are aero-elastically designed and tailored for optimum performance. The ability of compressively loaded columns and shell structures to carry load without buckling is directly related to the stiffness of the structure. Changes in stiffness caused by damage alter the response of the component to loads and reduce the performance level of the structure. Fatigue life is thereby determined by a specified stiffness change in a performance-critical component rather than by fracture [6,21]. The second reason for interest in

stiffness change in composite laminates is that change in stiffness provides a non-destructive technique for monitoring damage throughout a loading history. Furthermore, changes in stiffness are directly related to the severity of the damage by the mechanics associated with the subsequent response of the material. Thus, stiffness change can be used, along with appropriate models, to anticipate and predict remaining strength and fatigue-life [6]. The characteristic stages I to III (figure 3) of stiffness degradation, which correlate with the damage mechanisms already described in figure 2, depend on the cyclic loading and on the respective material. However, the general form of the stiffness change expended life relationship has been verified for many material systems, including metal matrix composites [6].

Residual strength of laminates is determined by using a two-stage test. The first stage is a cyclic test in order to create the desired condition in the specimen. The condition could be exposure to a specified number of load cycles, a selected change in a stiffness component, a prescribed state or damage or some condition related to a specific service application [6]. In the second stage, a monotonic test tensile or compressive test is performed. The strength measured in these monotonic tests represents the residual strength of specimens including the applied load history. Various models using the residual strength as parameter for fatigue-life description have been published [6,22–24].

3.2 Energy based approaches

Beyond stiffness and residual strength based models to describe the fatigue-life of composites, another approach – among many others which are not going to be included herein – has been discussed by many researchers. Thermodynamic assumptions or energy based approaches are used to describe the fatigue life of composite materials e.g. [25,26]. In these theories, stress-strain fields and the related anticipated energies are taken into account by analysing the material's hysteresis loops caused by cyclic loads [25]. Shokrieh and Taheri-Behrooz proposed a fatigue life model based on the strain energy concept by assuming an elastic stress-strain relation which can be normalised to the maximum of monotonic strain energy, i.e. the product of maximum monotonic stress and strain [26]. It has to be noted that linear elastic concepts overestimate the actual, usually non-linear material behaviour [25]. Although these concepts are based on the stress-strain behaviour of the material, the possibility of correlating anticipated energies with physical damage mechanisms, as possible for stiffness decreases described by Talreja, Reifsnider, Stinchcomb or many others e.g. [6,15,16], is limited.

3.3 Fracture mechanical approaches

Fracture mechanical approaches are based on the assumption that crack growth, initiated by a defined starting crack, void or other imperfection, leads to fracture. Different theories describing the crack tips, plastic zones around the crack tips and evolution laws exist. For polymeric pipe materials for example, fracture mechanical tools can be successfully applied for life-time estimation e.g. [27–29]. With regard to composite materials, fracture mechanics is a useful approach for the understanding and description of delamination, which is a separation of the individual plies weakening the performance of the entire composite e.g. [30–33]. Composite delamination in structures subjected to in-plane loading can be considered as subcritical failure mode whose effect may be a stiffness loss, a local tensile strain concentration in the load bearing plies that causes tensile failure or a local instability that causes further growth which ends in compressive failure. Some potential delamination sites are common design details that result in discontinuities in the load path such as free edges, hole boundaries, dropoff of the interior plies of a laminate to taper thickness, bonded or co-cured joints or bolted joints [30]. However, in composite structures many possibilities for crack or delamination initiation exist making the fatigue-life prediction based on fracture mechanics a very comprehensive task. Nevertheless, delaminations are one decisive failure mode and have to be taken into account.

3.4 Approaches based on traditional S-N curves

While the approaches of stiffness, residual strength, energy and fracture mechanics discussed so far are based on material specific parameters such as physical damage mechanisms and their effect on measurable mechanical properties, the description of fatigue-life by using S-N curves does not take damage mechanisms during fatigue life into account as such. This results mainly from the fact that for the creation of S-N curves one single event (e.g. failure) is used as criterion to assess a specimen and the history prior to this event e.g. stiffness and strength reduction are not evaluated. The creation of S-N curves is schematically illustrated in figure 4. For the illustration, stress controlled fatigue tests are chosen and results are drawn in a double-logarithmic way as applied nominal stress amplitude *σ^a* versus number of cycles *N*. Specimens are tested experimentally in constant amplitude tests on different stress levels until a previously defined event occurs, e.g. failure of the specimen. Numbers of cycles of each test are recorded. If specimens do not fail within reasonable test times, these specimens may be stopped and marked as run-outs. Depending on the applied load amplitude and consequently on the scales of reached numbers of cycles, S-N curves may be portioned in different regimes. High applied load amplitudes denote the low cycle fatigue (LCF) regime whereas most fatigue tests are performed in the high cycle fatigue (HCF) area. Very low amplitude tests reaching numbers of cycles in scales of 10^8 or 10^9 are usually called ultra high cycle fatigue (UCHF). The illustrated S-N curve can be described by two data pairs of σ_a and *N* in combination with the slope *k* in the doublelogarithmic diagram (figure 4).

Figure 4 Schematic illustration of a S-N curve or Wöhler line. The entity of fatigue life can be divided into the low cycle fatigue (LCF), high cycle fatigue (HCF) and ultra high cycle fatigue (UHCF) regime.

S-N curves are a popular way to describe the fatigue behaviour of materials due to their simplicity in terms of mechanical testing and data evaluation. Since this approach was developed for classical metallic fatigue, a lot of engineering experience for conducting fatigue characterisation properly and many studies on the rating of different influence factors are already available [3,4]. However, a lot of time-consuming material testing is needed especially if anisotropic material behaviour has to be reflected in the fatigue data.

3.5 Prediction of composite fatigue failure

Although failure criteria for assessing quasi-static composite failure and also many theories about the progresses during fatigue-life are available, failure criteria for fatigue failure are difficult to define. On the one hand, this is based on the complexity of damage mechanisms depending on the inner structure of each composite and influencing the fatigue behaviour decisively as already discussed. The variety of physical damage mechanisms makes fatigue failure criteria which are able to reflect the actual material damage and damage history way more complex than the introduced quasi-static criteria. Furthermore, since the damage progress is influenced by the specific characteristics such as fibre and matrix material stacking sequence etc., a general failure criterion based on physical damage mechanisms valid for all composite types is not realistic. On the other hand, the overall objective of each fatigue-life prediction should be to meet the real application of a composite structure. However, this aspired aim will probably not be realisable with theories built on effects on microscopic scales due to the quantity of effects which would have to be included into the predictions. Furthermore, the upscaling of theories describing the physical behaviour on microscopic level to prediction of the behaviour of entire structures might not be valid due to different effects on these scales.

As a result of these uncertainties, many theories focus on building on rather technological predictions. Therefore, the prediction of composite fatigue failure nowadays is often based on theories known from metallic materials which are intended to be adapted for composites [34–36]. A lot of effort has been put into accurate description of Haigh or constant life diagrams for composite materials. In general, Haigh diagrams are used to illustrate the correlation between the applied load amplitude σ_a and the mean stress of the cyclic load σ_m in fatigue tests. It is known from metallic materials that the bearable load amplitude σ_a decreases with increasing mean stress σ_m . This effect is usually described as "mean" stress sensitivity" *M* [3,4]. The Haigh diagram is created based on experimentally evaluated fatigue strengths and usually valid for one defined number of cycles. Different theories describing the shape of the Haigh diagram between the experimentally measured points have been developed for metallic materials. A common approach for the extension of the Haigh diagram to unknown load ratios *R* is to use quasi-static values such as tensile strength σ_M or stress at 0.2 % strain $\sigma_{0.2\%}$ as borders for the diagram [4].

Among others, Kawai [34,35,37] and Vassilopoulos [38–40] published theories and formulations for accurate description of Haigh diagrams for different composite materials. For composite materials, the shape of the Haigh Diagram may appear different from the traditional Haigh diagram as illustrated in figure 5 as a result of the significant influences of the e.g. composite type, the fibre architecture and the applied load angle. Furthermore, temperature, load ratio or multiaxial loads determine the material fatigue behaviour. One representative example of experimental data evaluated at different load ratios *R* and predictive formulations for constant life diagrams of a $[90/0/\pm 45/0]$, E-glass/polyester laminate from Vassilopoulos, Manshadi et al. [40] is illustrated in figure 5. Accurate predictions of Haigh or constant life diagrams consequently allow statements about the performance of composite materials under unknown load situations.

Figure 5 Representative example of experimental data and predicted constant life diagrams of a [90/0/±45/0]^s E-glass/polyester laminate from Vassilopoulos, Manshadi et al. [40]. The formulations were based on S-N curves with *R*=-1 and *R* =10.

To assess the effect of load time history, well-known approaches for metallic materials such as rainflow counting may be used [3]. One simple but effective way to validate the damage history is the Miner rule, initially developed for metallic materials. The Miner rule accumulates the damage distribution of each single cycle. Therefore, the number of cycles of each load block n_i is set into relation to the theoretically possible number of cycles *Nⁱ* limited by the S-N curve with the slope *k*. If the damage sum reaches 1, failure is assumed to occur (equ. 2). Different ways of counting the applied cycles have been proclaimed: Miner original, Miner modified and Miner elementary according to [4]

(figure 6). Of course, resulting from the simplicity of theory, crack initiation, crack development or sequence influences in non-constant amplitude fatigue tests are not taken into account [3]. However, a technological estimation of expected number of cycles to failure even under multiaxial load channels is possible with this theory. Some studies have shown that the Miner's rule can be applied to composite materials as well [41,42]. For composite materials, the modification Miner elementary (figure 6) refers best to the usually observed material behaviour.

Figure 6 Damage calculation developed for metallic materials based on S-N curves according to "Miner original", "Miner Elementary" and "Miner modified according to Haibach" according to [4].

However, even if strength based fatigue approaches offer the advantage of experience from metallic materials in complex applications, adaptions to meet the unique behaviour of composite materials have to be made. In order to include the damage history, not on microscopic but on macroscopic scale reflected by the mechanical properties, fatigue-life predictions will have to take the decreasing mechanical properties discussed in this chapter into account. Solely strength and damage accumulation based approaches will not provide satisfying results because within these approaches usually only the event of failure is taken into account.

Therefore, tools being able to calculate the stiffness decreases in composite materials based on input parameters will have to be developed in order to decrease the experimental time effort. Although theories being able to calculate properties of composites based on the properties of the single plies such as the classical laminate theory work well for quasi-static loads, fatigue-induced stiffness decreases describing the events prior to failure can hardly be predicted so far. Additionally, the prediction of the event of failure has to be adapted for fatigue failure. New studies try to combine the benefits of quasi-static failure criteria based on physical considerations with S-N curve based theories developed for metallic fatigue [43].

4 Outlook

The authors have applied a concept based on S-N curves and damage accumulation, well-experienced for metals and also for short fibre reinforced composites, on the fatigue-life prediction of multiaxial laminates made of carbon fibre reinforced composites in the last years [44]. Although characteristics such as the anisotropic material behaviour or fracture modes according to Puck were implemented, the results show clearly the limitation of purely strength based concepts for continuously fibre reinforced composites. The main reasons for this observation are first that damage mechanisms, their interactions and possible change during fatigue life are not taken into account at all in such concepts. A detailed inclusion of micro-mechanical damage is hardly feasible, but damage mechanisms need to be taken into account in a qualitative way nevertheless. The second reason for the limitation of strength based fatigue concepts is that the degradation of other important material characteristics, such as stiffness, is not included in the concept at all. Therefore, the authors have developed a tool based on the classical laminate theory for the prediction of fatigue-induced stiffness degradation of laminates based on the properties of plies [45]. First results of this study are very promising, however, the application to fatigue-life prediction of real-life parts, which should be the overall goal, is not in the immediate future.

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