STACKING SEQUENCE OPTIMIZATION OF CURVED UD-CFRP LAMINATES FOR IMPROVING UNFOLDING STRENGTH CONSIDERING THERMAL RESIDUAL STRESSES

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Abstract
Highly curved laminates are prone to the unfolding failure. The unfolding failure consists in a delamination which appears when the curved component is loaded under an opening bending moment. This failure is associated with the interlaminar stresses which appear due to the curvature. They can be calculated by several methods such as Lekhnitskii's equations. Therefore, the unfolding failure criteria and other delamination criteria are mainly based on the normal and shear interlaminar stresses. The present study shows results of a new analytical model to analyse the effect of residual stresses, due to the cooling after solidification in the manufacturing process, on the unfolding failure. These results can be used for carrying out an optimization of the stacking sequence from the residual stresses point of view. This result coincides with the optimum from the bending moment loading point of view. Results show the convenience of positioning the plies oriented in the curved direction in radii greater than those corresponding to the plies oriented in the straight direction of the single-curved beam.

1. Introduction

Highly curved composite laminates are usually sized with the interlaminar stresses due to the unfolding failure (see [1]). The curvature induces the appearance of this kind of stresses even when the laminates are loaded only under axial loads. However, classical calculation methods are developed only for flat or lowly curved laminates and they do not predict the interlaminar stresses (see [2, chap. 4]). Some components prone to this kind of failure are angles, T-sectioned beams, corrugated laminates, etc. (see [3, 4]).

Lekhnitskii developed the analytical solution for calculating stresses in curved beams under end loads in a homogeneous material (see [5, chap. 3]). These equations were extended to the non-homogeneous laminate case by Ko and Jackson [6] as a system of equations, the number of equations being proportional to the number of plies. These equations are those most used in the bibliography to calculate the interlaminar stresses used to predict the unfolding failure.

However, these equations are not accurate enough under certain loading states and geometries. It is due mainly to non-regularized effects, e.g., the Saint-Venant effect in the vicinity of a change of curvature (see [7]). Furthermore, the previously mentioned equations are extremely difficult to extend to the calculation of these non-regularized effects. The authors developed a simplified model [8] that approximates the Ko and Jackson equations, and this model was extended to the non-regularized effects, the results being shown in [9].

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However, the non-regularized effects are not the only reason for the lack of representativity in the aforementioned stresses calculation methods. The manufacturing process can induce also residual stresses in the laminate (see [10]), which are mainly due to the differential thermal expansion, the cure shrinkage (chemical shrinkage) and the tool part interaction.

The residual stresses and deformations caused by differential thermal expansion, the chemical shrinkage and the tool part interaction in composite laminates have been highly studied in the literature. Intralaminar residual stresses are usually analysed. However, when the laminate is highly curved, interlaminar residual stresses can be significant too, especially taking into account the low values of the interlaminar strengths compared to the intralaminar strengths.

The authors have developed a novel model consisting of an extension of the Ko and Jackson's equations to the 3D case including the thermoelastic problem. The present document shows the results of the model corresponding to curved laminates, showing a high potential to determine interlaminar residual stresses and to optimize the stacking sequence from this point of view.

Chemical shrinkage effect has been demonstrated to be of the same order than the thermal one. Therefore, residual stresses presented in this paper may be much higher when considering the other effects.

Once stresses have been calculated, a failure criterion must be applied. The most used criteria are the Hashin criterion [11] and the Kim and Soni criterion [12]. These criteria stablish a quadratic relation between the interlaminar normal and shear stresses.

2. Theoretical basis

The model is developed by extending the Ko and Jackson's equations given in [6] to a three-dimensional problem and including residual stresses due to the manufacturing.

Residual stresses are usually provoked by strains due to thermal or chemical effects, these strains being restricted by any part or component of the laminate. In that way, this kind of problem is usually modelled in the constitutive equations of an individual ply as increments in the strains. Therefore, the constitutive law in the orthotropic axes yield:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{12} & C_{22} & C_{23}\\ C_{13} & C_{23} & C_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}-\Delta\varepsilon_{11}^{MP}\\ \varepsilon_{22}-\Delta\varepsilon_{22}^{MP}\\ \varepsilon_{33}-\Delta\varepsilon_{33}^{MP}\end{bmatrix} \tag{1}$$

$$\gamma_{23}=C_{44}\varepsilon_{23}\ ,\quad \gamma_{13}=C_{55}\varepsilon_{13}\ ,\quad \gamma_{12}=C_{66}\varepsilon_{12} \tag{2}$$

Where, in the case of a homogeneous change of temperature from the curing temperature, the strains increments are modelled as follows:

$$\Delta\varepsilon_{ii}^{\mathrm{MP}}=\alpha_{\mathrm{i}}\Delta\mathrm{T}\ ,\quad i=1,2,3 \tag{3}$$

Where α_i is the thermal expansion coefficient in the ply direction i and ΔT is the temperature increment. The strain increment has also other components depending on the curing process and the tool configuration which may be also significant. However, the modelling of these strains is not the objective of the present paper (see [13]). The viscoelastic effects in the curing process are neither considered.

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The development to determine the analytical equations will be presented soon in another paper. The present model lets to evaluate with a closed form system of equations, similar to the Ko and Jackson ones, the stresses and strains in the curved laminate for a 3D state.

3. Residual stresses due to the curing temperature

The matrix has a much higher thermal expansion coefficient than the fibre and, therefore, due to the stacking sequence, when cooling the laminate from the solidification temperature the fibre direction (0º direction) of the plies is compressed and the 90º direction is tensioned. By using the equilibrium equation in the radial direction of the laminate, considering that shear interlaminar stresses are null, the in-plane stresses due to the previously mentioned effect have to be equilibrated with a normal interlaminar stress, σ_r:

$$\frac{\partial(r\sigma_r)}{\partial r} = \sigma_\theta \tag{4}$$

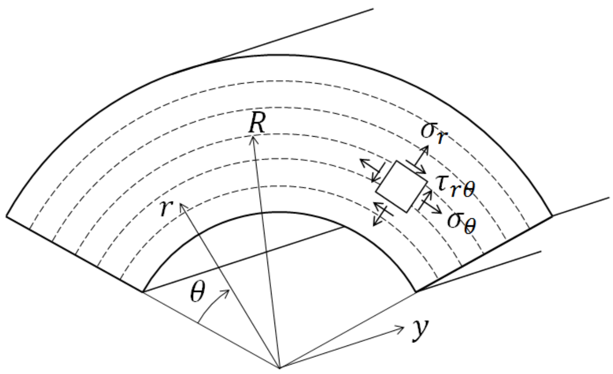

Figure 1. Coordinate system definition and stresses.

A cylindrical coordinate system is used as can be seen in Figure 1, where θ is the circumferential coordinate, r is the radial coordinate and y is the axial coordinate.

Therefore, from (4), it can be directly obtained that tensioned plies in θ-direction should have a positive slope of the interlaminar stresses with the radial coordinate, and that compressed plies should have a negative slope.

Unfolding failure is associated with tensile interlaminar stresses, so by using the previous concept it is important to avoid residual tensile stresses in the critical zones (which are typically located in a radial coordinate value lower than the mean radius) and to introduce compressive stresses.

In order to evaluate some examples, the following material properties are being used: $E_{11} = 150$ GPa, $E_{22} = E_{33} = 10$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.3$ and $G_{12} = G_{13} = G_{23} = 4.8$ GPa. Furthermore, the thermal expansion coefficients are $\alpha_1 = -1 \cdot 10^{-6}$ K^{-1} and $\alpha_2 = \alpha_3 = 3 \cdot 10^{-5}$ K^{-1}. The 0º direction is defined as the curved direction.

3.1. Single-ply laminates

First, a single-ply laminate can be approximated by a homogeneous beam, so it can be assumed to expand and contract without any restriction and, therefore, without this kind of residual stresses. However, this only occurs for $[0º]_n$ and $[90º]_n$ laminates, since in other cases the curvature constrains

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the free deformation of the component and provokes torsional deformation and residual stresses. This behaviour has been used to validate the model by comparison with FE results. In that way, Figure 2 shows the results obtained with the analytical model for a 45º single-ply laminate with $\Delta T = -160$ºC and $R/t = 2$ compared with the FE results. The FE model consists of a 3D open cylindrical surface with rigid solid displacements constrained and with a cubic mesh of 10 quadratic elements in the thickness. Stresses are measured far enough from the free edges (free-edge effect is not considered in the present model).

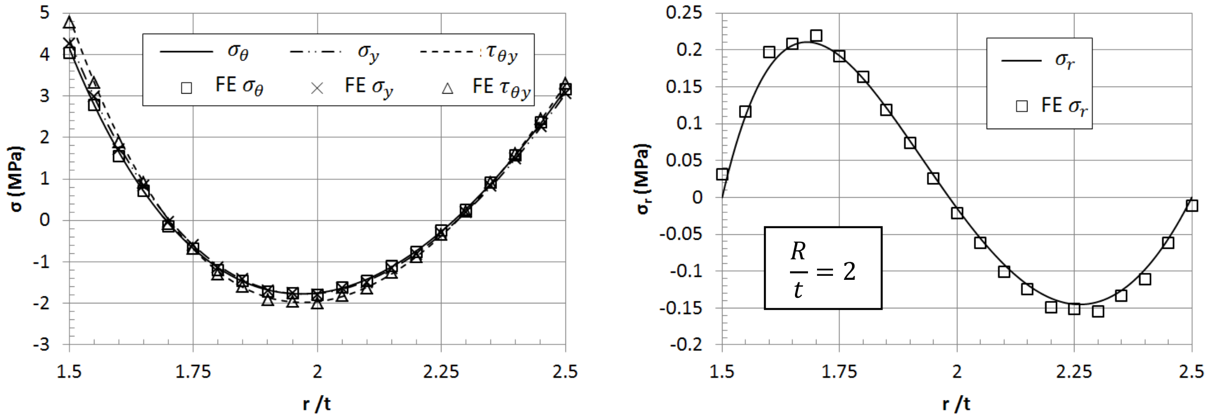

Figure 2. 45º single-ply laminate results of the present model (continuous line) compared with FE results (asterisks).

It can be observed that the model agrees in a very accurate way with the FE results. Furthermore, the previous stresses expressed in the local ply coordinate system shows that the ply is mostly loaded in the fibre direction. However, residual interlaminar stresses in this case are very small respect to its allowable. The maximum of this stress is about 0.2 MPa while the allowable is typically between 30-50 MPa.

Figure 3 shows the maximum radial stress for $R/t = 1$ depending on the ply orientation:

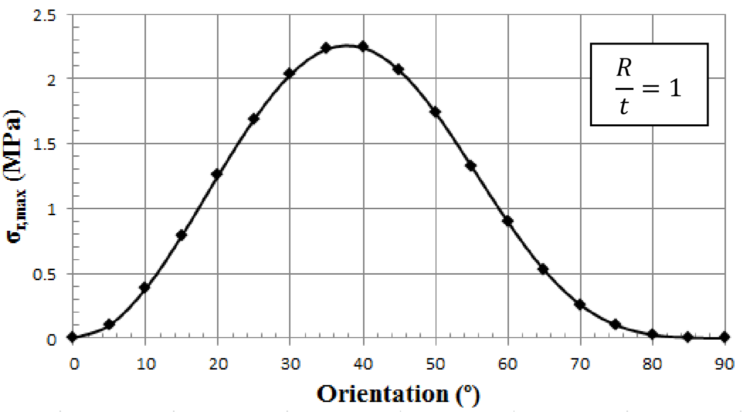

Figure 3. Maximum radial stress for a single-ply laminate depending on the ply orientation.

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First of all, notice that the maximum radial stress for a 45º ply has increased its value in one order of magnitude when the R/t ratio has decreased from 2 to 1. The maximum value of the normal interlaminar stress is given for an orientation of approximately 38º.

3.2. Composite laminates

When applying the model to different stacking sequences, the differential thermal expansion of the plies provokes the appearing of higher residual stresses. Figure 4 shows the interlaminar residual stresses in two different stacking sequences for $R/t = 1$.

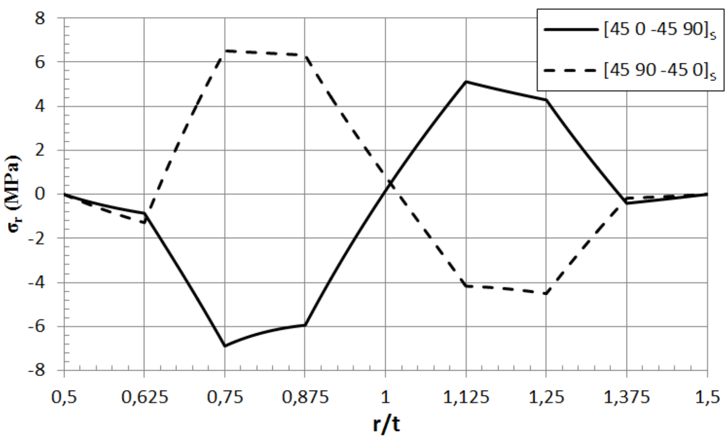

Figure 4. Interlaminar residual stresses due to the differential thermal expansion.

As was previously commented, 0º plies (which are compressed in the circumferential direction) have a negative slope of the normal interlaminar stress with respect to the radial coordinate, and 90º plies (which are tensioned) have a positive one. Therefore, from a residual stresses point of view, it is more interesting to locate the 0º plies outer than the 90º, and in this way compressive interlaminar stresses can be obtained in the critical zone.

Furthermore, from a structural point of view, when the laminate is under an opening bending moment radial stresses can be minimized mainly by locating the 0º plies outer in the stacking sequence than the 90º plies. Therefore, both points of view converge to a common design criterion. However, this design criterion may not coincide with the design requirements, a more detailed analysis in each case being necessary.

Notice that the maximum radial stress in the case showed in Figure 4 has an absolute maximum value of approximately 7 MPa. Allowables in this direction have typical values of 30-50 MPa depending on the material, so the maximum stresses are around a 14-20% of the allowable. This shows the importance of considering the manufacturing process in the unfolding calculation, which nowadays is not usually taken into account.

Furthermore, the lesser isotropic the stacking sequence the higher the maximum of the absolute value of the interlaminar residual stresses.

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5. Conclusions

Results of an extension to a three-dimensional case of the Ko and Jackson's equations, used to calculate the complete stress state in a bi-dimensional curved beam, have been shown. The manufacturing process effects are considered in this extended model as strain increments in every ply.

This model lets to estimate the residual stresses present in a curved laminate, especially the interlaminar ones. Results show the convenience of positioning the 0º plies outer than the 90º ones in the stacking sequence (independently of the position of the 45º ones, whose effect is much lighter), in order to obtain compressive interlaminar stresses in the critical unfolding zone. This zone is located in the most cases in radial positions lower than the mean radius.

The model is a powerful tool for optimization of the stacking sequence, which lets to obtain a solution of the interlaminar stresses in curved laminate once the in-plane problem has been solved and, therefore, the strain increments are known. This lets to avoid using FE software, which requires many elements in the thickness to estimate accurately the stresses in the interlaminar direction.

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