VISUALIZATION EXPERIMENT AND NUMERICAL SIMULATION OF LIQUID IMPREGNATION WITH RACE-TRACKING FOR RTM PROCESS

Masatoshi Kobayashi[1], Koji Dan[1], Tsuyoshi Baba[1], Koshiro Yamakawa[2] and Ryo Nakano[2]

[1]Automobile R&D Center, Honda R&D Co., Ltd.,
4630 Shimotakanezawa, Haga-machi, Haga-gun, Tochigi, 321-3393, Japan
Email: Masatoshi_01_Kobayashi@n.t.rd.honda.co.jp, web page : http://world.honda.com/RandD/
[2] Computer Aided Engineering Business Div., Toray Engineering Co., Ltd.
1-1, Sonoyama 1-chome,Otsu, Shiga, 520-0842 Japan
Email: koshiro_yamakawa@toray-eng.co.jp, web page : http://www.toray-eng.com/
Email: ryo_nakano@toray-eng.co.jp, web page : http://www.toray-eng.com/

Keywords: RTM, Impregnation, Race-tracking, Visualization, 3D Numerical Simulation

Abstract

In order to understand and predict complicated impregnation behavior with the race-tracking which liquid resin flow precedes through narrow gap between preform and mold in RTM (Resin Transfer Molding), we carried out impregnation experiments using unidirectional visualization device consisting of a transparent acrylic and a metal mold. In addition, we proposed a new 3D numerical simulation method using coupled analysis by the Darcy equation and variable fluid conductance (VFC) equation that can reproduce the Stokes flow. From the visualization experiments, it was found out that the impregnation behavior was remarkably changed by the race-tracking along the narrow clearance. The new simulation method could provide results with good accuracy for prediction of the complicated impregnation behavior with the race-tracking.

1. Introduction

In automotive industry, it is well known that various improvement technologies for fuel efficiency of vehicle have been developed from the standpoint of environmental conservation and so on. One of the practical ways is to reduce vehicle weight by replacing parts made from steels with those made from lighter materials such as continuous carbon fiber reinforced plastic (CFRP) [e.g.1, 2]. The high pressure resin transfer molding (HP-RTM) for the CFRP is one of candidate technologies for mass-production and has begun to be applied to vehicle body parts.

The HP-RTM has been used both upper and lower rigid molds with a press machine. In this process, preform consisting of reinforcement fiber fabric is placed in mold cavity that whole surface is surrounded by rigid tools. Shape of the preform is difficult to completely fit that of the mold cavity. The more complicated product shape becomes, the more difficult it is to fit them. Therefore, some narrow gaps exist between the preform and the mold when liquid resin with low viscosity is injected into the mold cavity. These gaps become air channel that the liquid resin flows. Accordingly, there are roughly two kinds of flows in the mold cavity during the resin injection. In this paper, one is defined as *impregnation-flow* that fluid advances through the preform. The other is defined as *general-cavity-flow* that fluid advances through the narrow gap. When the general-cavity-flow precedes the impregnation-flow significantly, this is called the race-tracking or the edge effect. The race-tracking greatly changes the impregnation-flow and complicates filling pattern. This phenomenon makes it difficult to find out suitable molding condition. Thus, the influence by the race-tracking becomes serious problems very often. However, it is difficult to solve them easily.

One reason is that the impregnation phenomena in the mold cavity are invisible and complicated. Since flow resistance into the narrow gap is very lower than that into the preform, the resin flows easily into the gap than the preform. In other words, the general-cavity-flow moves ahead, and the

Excerpt from ISBN 978-3-00-053387-7

impregnation-flow moves with a delay. An appropriate measures method may be different by product shape because this behaviors change by position and size of the gap. However, it is difficult to find out appropriate control measures. Non-transparent rigid mold for mass production disturbs understanding of the complicated impregnation phenomena.

Another reason is that numerical simulation methods are not well established for the race-tracking. Therefore, appropriate predictions are difficult. Until now, numerical simulations using finite volume method FVM (or finite element method FEM) have been examined. In the case of the preform domain only, the impregnation-flow through porous media is well predicted using the Darcy equation, assuming that average flow velocity is proportional to the pressure gradient in flow direction. In the case of the gap domain only, the general flow with flow velocity gradient in thickness direction can be computed using the Stokes equation, ordinarily. However, when both the preform and the gap domains exist, while the impregnation-flow and the general-cavity-flow should be computed as according to the Darcy equation and the Stokes equation respectively, it is not easy to simulate them because the computation has serious problems at the interface as boundary between the gap and the preform domains. Known examples of the serious problems are that analysis results near the interface indicate unatural behaviour such as oscillation of flow velocity and pressure does not occur in the actual phenomena. At present, academic mathematicians has been studied them as a mathematical interest.

In the previous study, composites researchers have examined practical simulation methods using some governing equations, in order to predict the two different flows that occur at the same time. Realistic methods to simulate the two different flows simultaneously and continuously can be roughly classified the following.

- Conventional method (I)
  - The preform domains: Darcy equation using measured permeability.
  - The gap domains: Darcy equation using constant equivalent permeability which is easily calculated on the condition that the gap cross-section shape is simplified.

- Conventional method (II)
  - The preform domains: Darcy-Brinkman equation using measured permeability.
  - The gap domains: Darcy-Brinkman equation with permeability adjusted to infinity.

As for the conventional method (I), the impregnation-flow through the preform domains is applied Darcy equation using permeability (K) measured beforehand. The general-cavity-flow through the gap domains is applied Darcy equation assigning constant equivalent permeability (Keq) [3, 4], to make it easy to flow than the preform domains. By simplifying the gap cross-sectional shape, the Keq is easily calculated so that the general-cavity-flow can be approximated the Stokes flow. It is well known as the Hagen-Poiseuille equation that represents flow in circular tube with a constant diameter, assuming laminar flow with no acceleration. This is a famous equation as exact solution that can solve the Navier-Stokes equation. By applying this equation, the Keq can be also assigned for simple cross-sectional shape such as circle (diameter $2a$: $Keq=a^2/8$). Understandably, the Keq for the gap domain is larger than the K for the preform domain. This method has no problem with the interface between the preform and the gap domains because of the exact same equation form. However, there is another problem with computation for the general-cavity-flow through complex shape gap such as diverging and converging section. An appropriate Keq must be changed by the cross-sectional shape, in order to produce flow velocity that accord with the Stokes flow. The more complicated gap shape becomes, the more likely analysis accuracy is to decrease. Furthermore, since the equation using Keq has a form same as Darcy, calculated flow velocity is mean value and there isn't flow velocity gradient in the thickness direction at the gap domains. In other words, it becomes not shear flow but uniform flow without flow velocity distribution in the thickness direction. For this reason, this method (I) should be utilized in the range that influence on flow behaviour of the gap shape simplification is small.

As for the conventional method (II), by adjusting the porosity and the permeability, Darcy-Brinkman equation [5] can represent the Stokes equation and can also approximate Darcy equation. The impregnation-flow through the preform domains is applied approximate Darcy equation using permeability measured beforehand. The general-cavity-flow through the gap domains is applied the

Excerpt from ISBN 978-3-00-053387-7

Darcy-Brinkman equation that agrees with the Stokes when the permeability is infinite and the porosity is 1. This method can also use the same governing equation at each domain. However, it is reported that flow velocity and pressure became unnatural at the interface between these domains. At present, interpolation methods for stable and natural solution at the interface have been studied [e.g.6]. On the other hand, the gap domains need very fine meshes to capture correctly the Stokes term. Since these meshes are connected at the interface, it necessarily follows that the preform domains are also set very fine meshes. This is an important problem for industrial use that requires efficient analysis.

As described above, not only the simulation methods are not established enough for industrial application, but also actual phenomena with the race-tracking are not clarified. In this paper, first, we have made actual impregnation behaviors clear by visualization model experiments that the race-tracking is generated. Second, we have proposed an effective approach to compute these two different flows simultaneously and continuously, in 3-dimensional (3D) RTM simulation. And the effect of this approach has been validated in comparison with the visualization experiments.

2. Methods of Experiment and Numerical Simulation

2.1. Visualization Experiments

2.1.1. Reinforcement

CF fabric is biaxial NCF (0°/90°) composed from carbon fiber tows and polyester threads for stitching without binder. The carbon fiber linear density (fineness) is 3300tex. An overall areal weight is about 297g/m^2 (the weigh per layer is each 145 g/m^2). This fabric is stitched by tricot-franse pattern. The textiles cut from reinforcement roll were used as the preform. The sizes are two kinds. One is about 700mm in length and 300mm in width. The other is about 700mm in length and 298mm in width. The 0° and 90° directions at the preform were respectively determined in the warp and the weft of the roll. The preform laminated by asymmetric lay-up $[0^\circ/90^\circ]_8$ was charged on visualization device.

2.1.2. Testing Fluid

As a testing fluid, polyalkylene glycol-based synthetic oil with water-soluble instead of resin was used. This is Newtonian liquid. We measured the oil viscosity in the range of 10-30°C. The viscosity at 20°C was about 0.015Pa・s.

2.1.3. Experimental Setup

As shown in Figure 1, we produced unidirectional visualization device consisting of a transparent acrylic and a metal mold with sealing rubber, in order to clarify actual impregnation behaviours with the race-tracking. Table 1 shows the experiment condition. Since CF volume fraction varies by mold cavity height and the number of layers, we actually measured the height using lead plates every experiment. It was approximately 2.95-3.05mm. We were prepared two types of tests for uniform impregnation with no race-tracking using 700x300mm preform and for non-uniform impregnation with race-tracking using 700x298mm preform. About latter test, clearance between the preform and the mold was adjusted to one side of the laminated preform. Subsequently acrylic mold was set on the

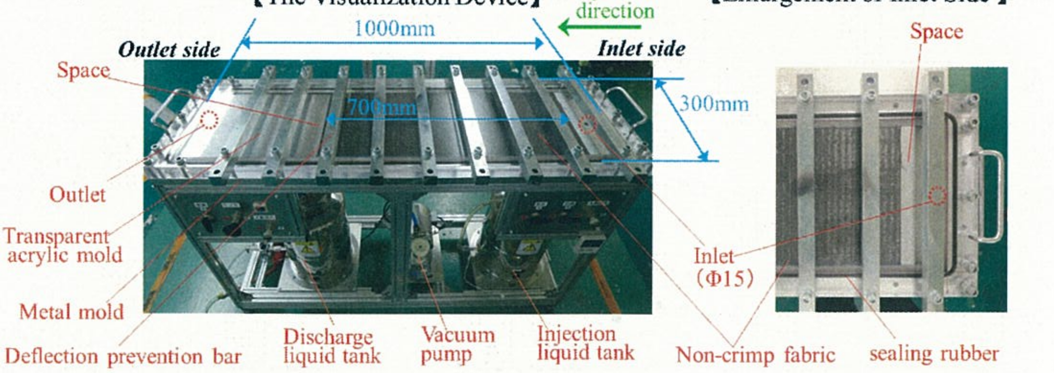

Figure 1. Unidirectional visualization device for model experiment of fluid impregnation through non-crimp fabric.

Table 1. Experiment condition for impregnation visualization.

Material Characterization and Parameters	
Testing fluid	Polyalkylene glycol-based synthetic oil •Viscosity: 0.015Pa•s (at 20℃) •Density: $0.99 g/cm^3$ (at 20℃)
CF fabric	Biaxial non-crimp fabric(NCF) 0°/90° without binder •Fineness yarn: 3300tex •Areal weight: $297 g/m^2$
Temperature	Room Temp.
Differential pressure	-95kPa

lamination and the mold cavity was decompressed by a vacuum pump. After reduce pressure level was stabilized, liquid injection was started from the inlet under constant pressure of -95kPa. Thus, we carried out the visualization experiments with the race-tracking by generating the impregnation-flow through the preform and the general-cavity-flow through the clearance at the same time.

2.2. Numerical Simulation

With regard to the simultaneous analysis of the impregnation-flow and the general-cavity-flow, we propose a new approach which also becomes effective for industrial application as follows.

- New method
  - The preform domains: Darcy equation using measured permeability.
  - The gap domains: Variable fluid conductance (VFC) equation in consideration of both flow channel shape and its internal position.

About the impregnation-flow through the preform domains, Darcy law expressing the average flow velocity in proportion to the pressure gradient is applied as the governing equation as with the other conventional methods described above. Here, the Darcy equation will be described. In its simplest form, Darcy equation is expressed using permeability as coefficients indicating the impregnation ability as follows:

$$V_x = -\frac{k_x}{\eta}\frac{\partial P}{\partial x} \qquad V_y = -\frac{k_y}{\eta}\frac{\partial P}{\partial y} \qquad V_z = -\frac{k_z}{\eta}\frac{\partial P}{\partial z} \tag{1}$$

where x, y and z are three-dimensional coordinates of space, V_x, V_y, and V_z are the apparent flow velocities in the direction of the respective coordinates axes, k_x, k_y and k_z are the permeability in the direction of the respective coordinates axes, P is the pressure, and η is the viscosity of the fluid. These permeability values are necessary to measure before. As for the pressure, substitution of the equation (1) into the continuity equation ($\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}=0$) gives as follows:

$$\frac{\mathbf{K}}{\eta}\left(\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\frac{\partial^2 P}{\partial z^2}\right)=0 \tag{2}$$

where **K** is the permeability tensor. Setting the boundary condition of the pressure, the pressure distribution in the preform domains can be solved using the equation (2). The flow computed by the equation (1) and (2) is called as the Darcy flow.

About the general-cavity-flow through the gap domains, the VFC equation with flow channel shape-dependence is applied, in order to reproduce flow velocity and pressure distribution that almost accord with the Stokes flow. Here, the VFC equation will be described in detail. The fluid conductance is a coefficient to decide fluidity and the permeability is also one kind of the conductance. It is generally thought that the conductance should depend on flow channel shape. On the other hand, we have thought the suitable conductance should be determined not only by the flow channel shape but also by its internal position, in order to reproduce almost the same pressure and shear flow as the Stokes flow. One of the authors has already suggested the VFC equation in consideration of the flow channel shape and its internal position instead of the Stokes equation, in order to efficiently analyze resin flow in conventional injection molding for thermoplastic without reinforcement [7]. In this study, this VFC

Excerpt from ISBN 978-3-00-053387-7

equation is used for the gap domains. In the same form as Darcy equation, this VFC is described as follows:

$$V_x = -\frac{c}{\eta}\frac{\partial P}{\partial x} \qquad V_y = -\frac{c}{\eta}\frac{\partial P}{\partial y} \qquad V_z = -\frac{c}{\eta}\frac{\partial P}{\partial z} \tag{3}$$

where c is the VFC for the gap domains.
Here, the force balance dominated by viscous force is expressed as follows:

$$\left.\begin{aligned} \eta\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}\right) &= \frac{\partial P}{\partial x} \\ \eta\left(\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_y}{\partial z^2}\right) &= \frac{\partial P}{\partial y} \\ \eta\left(\frac{\partial^2 V_z}{\partial x^2}+\frac{\partial^2 V_z}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2}\right) &= \frac{\partial P}{\partial z} \end{aligned}\right\} \tag{4}$$

Here, as $C_l = c/\eta$, equation(5) can be obtained by substituting the first equation of the equation (3) into the equation (4), eliminating the flow velocity V_x and neglecting the 2nd order derivative terms concerning the x, y and z of pressure P.

$$\eta\left(\frac{\partial^2 C_l}{\partial x^2}+\frac{\partial^2 C_l}{\partial y^2}+\frac{\partial^2 C_l}{\partial z^2}\right) = -1 \tag{5}$$

As for the pressure, substitution of the equation (3) into the continuity equation gives as follows:

$$\frac{c}{\eta}\left(\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\frac{\partial^2 P}{\partial z^2}\right) = 0 \tag{6}$$

By setting the boundary condition of pressure, the pressure distribution in the gap domains can be solved using the equation (6).

The VFCs are varied by the internal position of the channel. In the case of setting a boundary condition that the VFC at mold surface is zero, smaller VFC at positions closer to mold surface and lager VFC at positions far away are obtained by the equation (5). Using this procedure, the VFC distribution can be also given in complicated flow channel cross-section with shape-change. Unlike the other methods, this new method does not require the channel shape simplification. There is no need to measure anything at all, for determining the VFCs that accord with the Stokes flow. The VFCs are automatically determined by the equation (5) depending on the channel configuration and the internal position. Since flow velocity increases with the distance from mold surface, shear flow can be reproduced in the thickness direction of the gap domains.

As you can see, the equations (1) and (3) have the same form. The pressure equations (2) and (6) also have the same form. Accordingly, this new method has no problem with the interface between the gap and the preform domains. Furthermore, effective analysis for industrial applications can be expected because very fine meshes required in the other method are not necessary in this new method.

In this study, the flow computed by the equation (3) to (6) is called as the VFC flow. Using the three conservation law (the mass, the momentum and the energy), the finite volume method FVM, and the equation (1) to (6), a numerical simulation program was developed by modifying commercial resin flow analysis software 3D TIMON (Toray Engineering Co., Ltd.). This program can analyze both general-cavity-flow through the gap domains as the VFC flow and impregnation-flow through the preform domains as the Darcy flow simultaneously and continuously, in order to simulate the liquid impregnation with the race-tracking in mold cavity by 3D. To verify this new method, we compared analysis using the VFC equation (3) to (6) with that using the Stokes equation in 3D flow channel with shape-change. Since this is verification of whether it is possible to represent the Stokes flow by the VFC flow, we prepared only mesh model for the general-cavity-flow without those for the

Excerpt from ISBN 978-3-00-053387-7

impregnation-flow. The VFC flow was analyzed by the development program. The Stokes flow was analyzed by commercial general-purpose flow analysis software ANSYS Fluent (Ansys, Inc.).

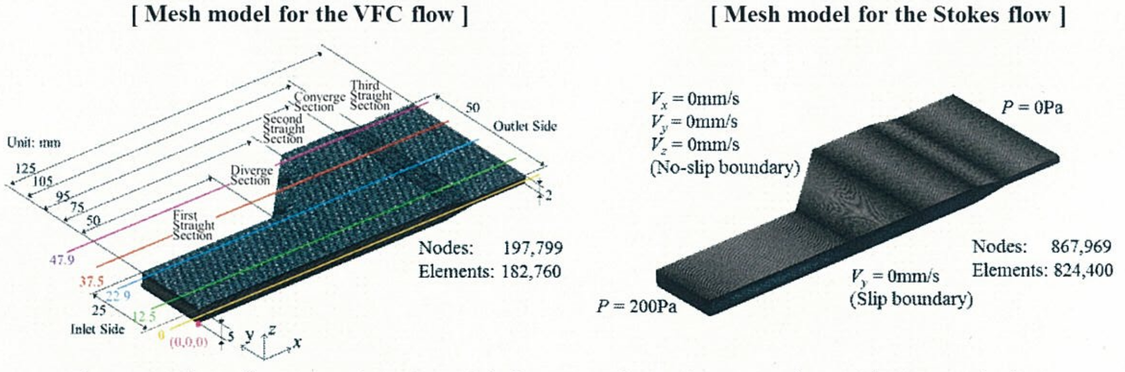

Figure 2. Three-dimensional mesh models for comparison of the VFC flow with the Stocks flow.

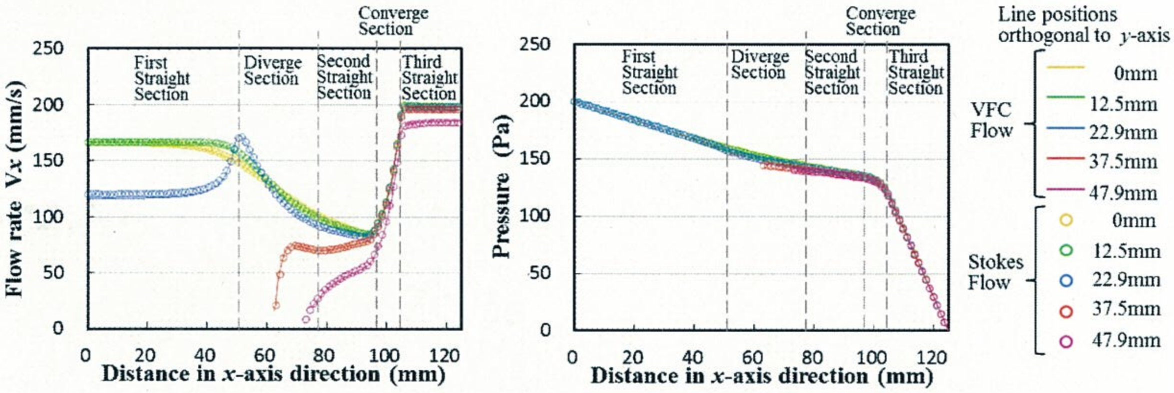

Figure 3. Computed results of flow rate and pressure at thckness direction center by steady-state analyses for the VFC and the Stocks flow.

Figure 2 shows 3D mesh models using hexahedron for this verification. These models were used the same shape and were composed of 5 sections as the first straight, the diverging, the second straight, the converging and the third straight section in order from the upstream to the downstream. Analysis for the VFC flow didn't need very fine meshes for the Stokes flow. Both side surfaces were set in slip and no-slip boundary respectively. Fluid viscosity was set to 0.015Pa・s. As boundary condition, pressure at inlet and outlet surface was set in 200 and 0Pa respectively. Initial flow velocities at inlet and outlet surface were both given 0 mm/sec. We carried out iterative computation until convergence. The computation for the Stokes flow was used the SIMPLE (Semi-Implicit Method for Pressure-Linked Equation) as solution algorithm. Figure 3 shows results of steady analyses. Each distribution of the flow velocity and the pressure by the VFC flow was well accorded with those by the Stokes flow in not only the straight sections but also the diverging and the converging section. Especially, in the diverging section, decrease and increase tendencies of flow velocity were consistent at each position enough. In the converging section, rapid increase tendencies of flow velocity were also in good agreement at every position. This result clearly indicates that the VFC flow can reproduce the Stokes flow in complicated flow field with the channel shape-change. In this paper, we tried to predict impregnation with race-tracking through constant cross-sectional gap using the new method (coupled analysis by the Darcy and the VFC flow).

3. Results and Discussion

3.1. Visualization Experiments

We visualized liquid impregnation under constant vacuum by the unidirectional visualization device showed in the Figure 1. By adjusting cut size and placement of laminated preform, the tests by two conditions were carried out. One was condition A which was not set clearance between laminated preform and mold for uniform impregnation with no race-tracking. The other was condition B which was set the clearance 2mm at only one side of the long distance direction as air channel for non-

Excerpt from ISBN 978-3-00-053387-7

uniform impregnation with race-tracking. About the condition A, to obtain stable progress of impregnation, we had to pay special attention as follows. First, cut preforms with high dimensional accuracy were carefully stacked in the device so that there are not narrow gaps in both sides. Second, the acrylic mold was strongly held by high-rigid bars made from steel for deflection prevention so that mold cavity height (distance between the acrylic and the metal mold) could be uniformed as much as possible. Though it is difficult to completely equalize this height, the difference in this height at the center and the edge position could be controlled to approximately 0.1mm or less. By these measures, we were able to stably reproduce the uniform impregnation. About the condition B, in addition to the above, we had careful control over the clearance 2mm between laminated preform and mold at one side only, using dedicated locating jig.

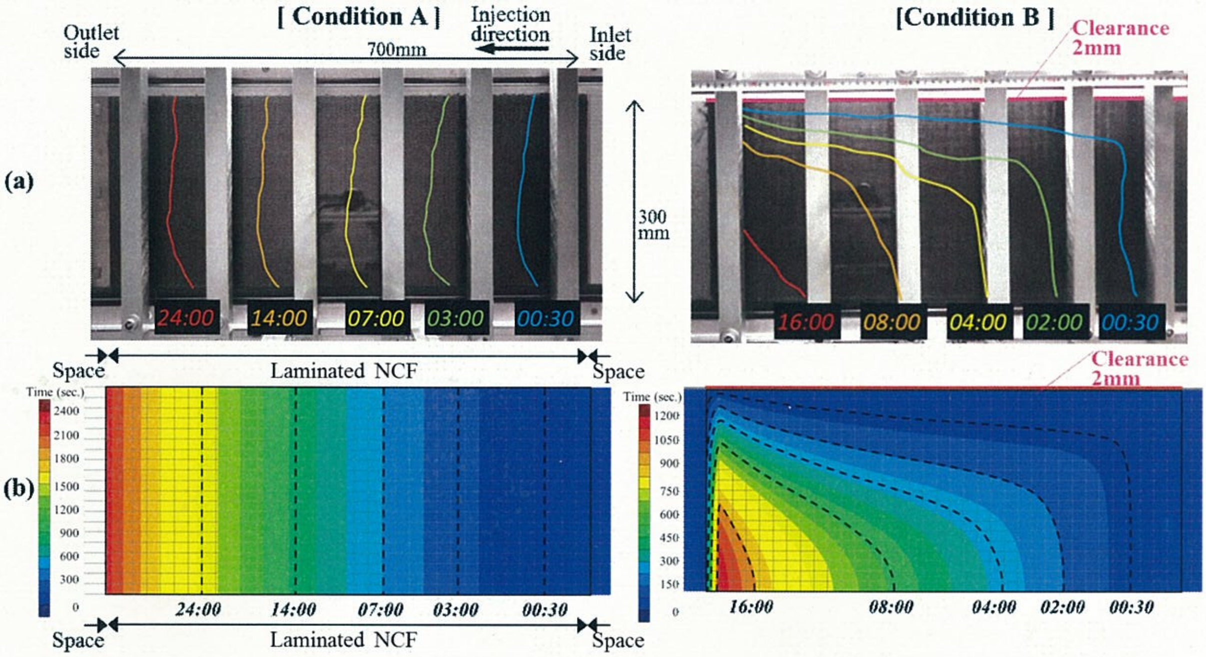

Figure 4. Flow-front arrival time comparison of visualization experimental results(a) with computed results(b) by the new 3D simulation method (coupled analysis by the Darcy and the VFC flow). The preforms were laminated by $[0^\circ/90^\circ]_S$. The condition A is for uniform impregnation with no race-tracking by not setting clearance. The condition B is for non-uniform impregnation with race-tracking by setting clearance along the top edge.

Figure 4(a) shows time change of actual flow-front pattern by the visualization experiments. The testing fluid was injected from the inlet along the longitudinal direction. The calculated fiber volume ratio (V_f) was 43.9% from the mold cavity height measured using lead plates. About result of the condition A, we could observe that the flow-front of the impregnation advanced in almost parallel. The flow-front speed was higher in the inlet side and decreases with time. Transit time from the injection start to the impregnation completion was about 35.5minutes. During this test, the pressure reduction value measured at the outlet side was -95.0kPa before the injection, but it slightly changed to about -94.0kPa after the impregnation completion. About result of the condition B, the impregnation pattern changed dramatically in comparison with that of the condition A. We could observe that the race-tracking generated at the clearance. Immediately after the injection start, the flow along the clearance advanced earlier than the impregnation through the preform. The flow-front of the impregnation curved, because the fluid impregnated through the preform from not only the longitudinal direction but also the vertical direction. Last impregnation area has been located in the opposite side of the clearance in contrast to the condition A. The flow-front speed of the impregnation decreased with time. Transit time from the injection start to the impregnation completion was about 19.5 minutes and became significantly shorter than that of the condition A. During this test, the pressure value slightly reduced as the condition A.

3.2. Numerical Simulation

In order to verify the new method (coupled analysis by the Darcy and the VFC flow), the numerical simulations by the same condition as the visualization experiments were carried out. Before the

Excerpt from ISBN 978-3-00-053387-7

simulation, we measured fiber volume fraction dependence of the in-plane permeability (k_x, k_y) for the NCF by using not only the unidirectional visualization device but also radial visualization device. From the measurements, the k_x and k_y were determined $9.19\text{x}10^{-12}$ and $8.12\text{x}10^{-12}\,\text{m}^2$, respectively. 3D mesh models using hexahedron were made for the simulations, including spaces on the inlet and the outlet side. About the condition A, the preform domain was contained 3780 nodes and 2800 elements and the each space domain was set large area enough. About the condition B, the rectangular cross-sectional clearance (2mm) domain with 900 nodes and 560 elements was added to one side of the preform domain with 3780 nodes and 2800 elements. The space domains on the inlet and the outlet side were set large area enough as the condition A.

As you can see in Figure 4(b), computed filling patterns were displayed as arrival time of flow-front by color contour. About result of the condition A, the impregnation-flow through the laminated preform that was computed by the Darcy flow advanced in parallel. The flow-front speed of the impregnation was higher in the inlet side and decreases with time, as the visualization experiment. Transit time from the injection start to the impregnation completion was about 38.6 minutes. About result of the condition B, the race-tracking could be reproduce at the clearance, that is to say, we could compute both the impregnation-flow and the general-cavity-flow using the new method (coupled analysis by the Darcy and the VFC flow) simultaneously and continuously. Computed result that was similar to the visualization experiment was indicated. First, immediately after the injection start, the general-cavity-flow through the clearance domain advanced earlier than the impregnation-flow through the preform domain. Second, fluid inflows into the preform domain from both the inlet and the clearance side were observed. Third, by these inflows, the impregnation-flow of the longitudinal and the vertical direction has occurred at the preform domain. The flow-front curved. Fourth, last impregnation area has been located in the opposite side of the clearance. Finally, the flow-front speed of the impregnation decreased with time. Transit time from the injection start to the impregnation completion was about 19.7 minutes. The transit time was shortened in comparison with the condition A. Thus, we confirmed that the new method was able to reproduce the actual behaviors well.

4. Conclusions

We have concluded as follows. First, by the visualization experiments, we observed that the race-tracking which the fluid preceded through narrow clearance changed the impregnation behavior through the preform remarkably. Second, we proposed the new numerical simulation method using the coupled analysis by the Darcy and the VFC flow. By comparison with the visualization experiment, it has become obvious that this new method could provide results with good accuracy for prediction of the impregnation behavior with the race-tracking.

References

[1] M. Kobayashi and Y. Tange. High cycle hot press process by CFRP with rapid curing epoxy resin. *Proceedings of the 12th Japan International SAMPE Symposium & Exhibition, Automobile Application-5. Tokyo, Japan,* November 9 -11, 2011.

[2] Lescheticky, J., Barnes, G., and Schrank, M. System level design simulation to predict passive safety performance for CFRP automotive structures. *SAE Technical Paper,*2013-01-0663.

[3] A. Hammami, R. Gauvin and F. Trochu. Modeling the edge effect in liquid composites molding. *Composites Part A,* 29A: 603-609,1998.

[4] S. Bickerton, S. G. Advani, R. V. Mohan and D. R. Shires. Experiment analysis and numerical modeling of flow channel effects in resin transfer molding. *Polymer Composites*, 21(1): 134-153, 2000.

[5] H. C. Brinkman. A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles. *Journal of Applied Scientific Research,* Vol. A1: 27-34, 1947.

[6] K. Fujisawa, S. Arimoto and A. Murakami. Simultaneous computation of incompressible and Darcy flows solving the Darcy-Brinkman equations. *IDRE Journal,* 81(5) :419-428, 2013.

[7] R. Nakano. Apparatus and method for analyzing process of fluid flow, an apparatus and method for analyzing an injection molding process, an injection molded product, and a production method of the injection molded product. *U.S. Patent*, No. 5,835,379, Nov. 10 1998.

Excerpt from ISBN 978-3-00-053387-7