DEVELOPMENT OF A SENSITIVITY-BASED DESIGN METHODOLOGY FOR COMPOSITE STRUCTURES

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Abstract

The aim of the developed methodology is to lead the choice of the structure designer optimizing the cost of development and ensuring a high level of confidence. The study deals with sensitivity analysis combined to uncertainty study in order to identify relevant combinations of analytical models and experimental testing to minimize uncertainty on the value of the specification requirement. The methodology was implemented for the case of a sandwich bending beam. Further analysis should be made in order to apply the methodology to more complex structures.

1. Introduction

The design of a structure is clearly linked to the confidence in the design choices that are made. These choices are often based on experimental and predictive analysis performed to characterize the studied structure by a complete or partial definition of the properties at each scale. The objective of this study is to purpose a methodology that lead the choice of the designer optimizing the cost of the experimental and predictive developments and ensuring a high level of confidence. The structure properties and the associated level of confidence at each scale could be mastered by a better understanding of uncertainties due to manufacturing process, experimental testing and numerical modeling. Indeed, uncertainty analysis is a relevant part in the design of composite material structures. Several authors treated this subject applying different statistical and computational methods to determinate the variation of the response due to errors on input parameters [1, 2]. Moreover, this analysis is often combined to sensitivity analysis at each scale of the structure to map the influence of low-scale parameters variations over properties located at a more complex level of the structure. For example, the study of Conceição et al. [1] is based on a first-order uncertainty and sensitivity analysis to study the variation of critical Tsai number and critical displacement for a cylindrical shell when the angle and elastic properties of plies vary. In their work, Noor et al. [2], use a sensitivity analysis to identify the main parameters in a multi-scale modeling of panels with cylindrical skins and T-shaped stiffeners. The sensitivity analysis coupled to uncertainty analysis allows to identify the main properties according to the specification requirements and the associated level of improvement it is possible to reach in the determination of those properties.

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2. Uncertainty and sensitivity analysis

2.1. Sensitivity analysis

For a given model, the sensitivity analysis is performed to classify the input variables according to the influence on the output parameters. Different methods achieve a sensitivity analysis and they are separated into local and global methods. Local methods are limited to small variations around the nominal value of input variables and consider linearity hypothesis. These methods provide the first-order sensitivity, the influence on output parameters of interactions between the input parameters is neglected [1]. In order to examine a larger range of values of input parameters, global methods can be used [3]. These approaches are more robust [3] and give the designer more information about the behavior of the response considering the effects of interactions between input parameters. In this study, Sobol method was used in order to perform the sensitivity analysis [4]. This method [3] uses the variance decomposition principle to analyze the variance of the model depending on the variation of one or more input variables. The total variance of the characteristic function of the model is constructed thanks to variances specific to each input parameter. Sobol sensitivity indexes are calculated by the ratio between the variance of the characteristic function when one or several input parameters vary and the total variance of the characteristic function.

2.2. Uncertainty propagation

The uncertainty propagation is used in order to define the range of values of an output variable according to the different range of values of input parameters. Considering a multi-scale model, represented by a characteristic function f, using properties from a low scale of a structure to define those of the upper one. The function f depends on several input parameters and allow the determination of an output variable. Each input parameter is characterized by a nominal value and a range of values. In this study, in order to define the range of values of the input parameters, sampling methods like Monte-Carlo or Latin-hyper cube were performed [3]. The output variable of the model was expressed as a Taylor expansion depending on the variation of the input parameters and the function evaluated at the nominal values [4]. Standard uncertainty of the output variable is assimilated to the standard deviation $\sigma = \sqrt{var(f)}$. If input variables are not correlated, the calculation of the standard uncertainty is simplified as shown in the work of Cacuci [4]. In this study, the form $Y = y \pm U$ was used. The term U is known as expanded uncertainty and it is calculated depending on the chosen level of confidence associated to the result. The level of confidence of 95 percent was required in this study, then a coverage factor of 2 was used ($U = 2 * \sigma$) [5].

3. Results and discussion

3.1. Developed methodology

The aim of this methodology is to lead the designer in its choices ensuring a high level of confidence and a low level of cost. In this study the idea of confidence is directly linked to uncertainties on specification requirements. The cost refers to the complexity of the numerical or experimental models used (number of parameters, type of parameters). Then, two new objectives are added to the specification of the product: minimize uncertainties on specification requirements and minimize the cost of the study. These objectives lead us to consider constraints as uncertainties on specification requirements. The building of this methodology was based on the definition of the macroscopic structure into different scales. Each scale was characterized by different properties related to the product requirements. The structure of the methodology was based on the principle that at each scale, the determination of the nominal value of properties can be performed by experimental testing or analytical or FEM models. We called "embedded

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path” an entity which describes for each scale the used model (experimental, FEM or analytical) to determine the nominal value of the parameters. Each embedded path crossed one or several scales of the structure to determine the uncertainty on the nominal value of product specifications thanks to models (experimental, FEM or analytical), input variables and their range of values. Each path can be related to specific level of confidence and to a path cost. The complexity of the structure increase the number of possible paths that can be identified. In order to lead the designer in its choices, sensitivity analysis is performed to identify relevant paths which could lead to reduce the cost of the study ensuring a high level of confidence.

3.2. Studied case

In a first case, the methodology was built from specification requirements defining a sandwich beam with a minimum bending stiffness of $1.8 \cdot 10^6$ N/m obtained by 3-points bending (Fig. 1) with a maximal uncertainty of 15 percent. The proposed structure is composed by AS4/3501 carbon/epoxy skins with a stacking sequence $[0_2, +45, -45, 90]_s$ and an Airex C70-200 foam core. The hypothesis of thin skins was used in this work. The Figure 1 shows the sandwich structure submitted to 3-points bending where e is the thickness of the skins, h is the thickness of the core, and L and b are respectively the length and the width of the structure.

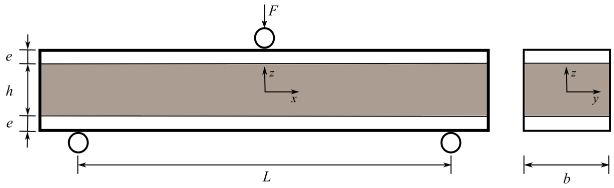

Figure 1. Studied case of a bending sandwich beam.

In order to develop the methodology, the first step consisted in an identification of the properties involved to satisfy the specifications at each scale of the studied structure. The determination of these properties can be performed by experimental testing or thanks to analytical models. Then, in this study, different models able to characterize the properties at different scales were defined. The total stiffness of the beam k_t was calculated using the principle of equivalent compliance of parallel springs representing the flexural stiffness k_f and the shear stiffness k_c. The analytical calculation of the bending stiffness of the beam deals with homogenized properties of the skins [6, 7]. First, the flexural rigidity (EI) was calculated thanks to the second moment of area (I_p and I_a) defined respectively for the skins and the core and the Young’s moduli of laminates (E_p) and core (E_a). The shear rigidity (GS) depends on both components cross-section (S_p and S_a) defined respectively for the skins and the core and transverse shear moduli of laminate (G_p) and core (G_a). At laminate level, the stiffness matrix was calculated using the first-order shear deformation theory (FSDT) [8]. In our case, symmetric laminates were used and an homogenization was done with the method showed in the work of Enie et al. [9] to calculate longitudinal Young’s modulus and transverse shear modulus of the laminate. To determinate unidirectional plies properties, models developed by Hahn [10] and Chamis [11] were used. In the same way, different experimental data were used to determine the different properties of the defined scales. The fibers, matrix and unidirectional plies properties were taken from the works of Sun et al. [12] and Ahmad [13] and they are shown in (Table 1) and (Table 2) respectively. Foam core properties (Table 3) were obtained in specifications sheets [14]. These experimental data were associated to a range of values, in his work, Chamis [15] describes the range of variation of all used elastic properties, fiber volume ratio V_f and ply thickness e_p (Table 4). In this study, it was assumed a range of variation of 1 percent on the structure length (L) and width (b) and 2 percent on the thickness of the core (h). Figure 2 summarizes the mechanical and geometrical properties involved to satisfy requirement at each scale.

Table 1. AS4 carbon fibers and 3501 epoxy resin properties [12].

Component	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	ν_{LT}
AS4	235	14	28	0.2
3501	4.8	4.8	1.86	0.34

Table 2. Unidirectional AS4-3501 properties [12, 13].

	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	$G_{TT'}$ (GPa)	ν_{LT}	V_f	e (mm)
AS4-3501	142	10.3	7.6	3.8	0.3	0.6	0.142

Table 3. C70-200 foam core properties [14].

	E (MPa)	ν	G (MPa)	h (mm)
C70-200	175	0.2	75	30

Table 4. Range of values of geometrical and mechanical parameters.

Elastic properties	V_f	e_p	L and b	h	θ (stdv)
8%	8%	5%	1%	2%	0.9°

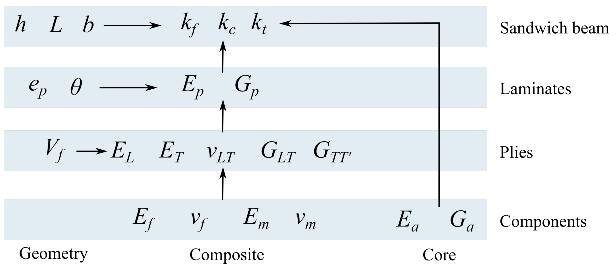

Figure 2. Mechanical and geometrical properties at each scale of the sandwich beam.

Different embedded paths can be identified from the different properties showed in (Fig. 2) depending on the range of values of the properties and the used model (analytical or experimental). For each embedded

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path, experimental values will be denoted with a superscript e and analytical values with a superscript a. For example, path 1 is showed in (Fig. 3) and in our case is one possible way to calculate the bending stiffness of the beam using only analytical models from the scale of the components.

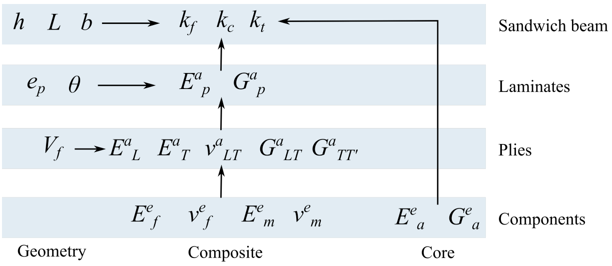
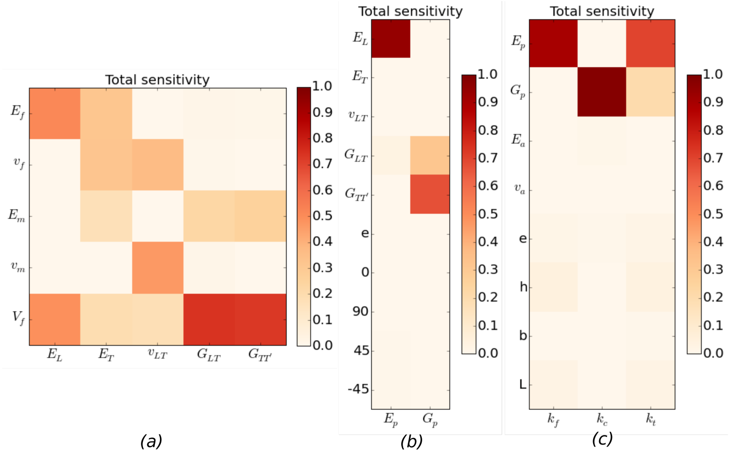


Figure 3. Path 1 to obtain beam stiffness using mechanical properties of the components determined thanks to experimental values (superscript e) or analytical values (superscript a).

This study leaned on sensitivity analysis thanks to Sobol method to determine relevant embedded paths. For that, the main properties involved in the calculation of bending stiffness of the sandwich beam were identified. The (Fig. 4) shows the sensitivity analysis of path 1 at the scales of the beam (c), of the laminate (b) and of the plies (a). For each map, input variables are on the y-axis and output parameters on the x-axis.



(a) (b) (c)

Figure 4. Sensitivity analysis of path 1 on (a) plies, (b) laminates and (c) sandwich properties.

The values of sensitivity of the parameters are restricted from 0 to 1, the value 1 corresponds to the value which have the most important effect on the output parameter. The Figure 4 shows that the fibers volume ratio (V_f) is a key parameter for ply elastic properties. At laminate scale, homogenized Young's modulus depends strongly on unidirectional ply Young's modulus. Both longitudinal and transverse shear moduli of plies have effect on transverse shear modulus of laminate. The effect of the variations in geometry and orientation angles is smaller considering their low range of values. At the last scale, there is a stronger influence of laminate Young's modulus and a not negligible effect on transverse shear modulus on the total stiffness of the beam. Once again, at this scale, geometry variations are less significant considering their smaller range of values. The sensitivity analysis identifies, at the ply scale, three main properties responsible to the major part of the uncertainty on the bending stiffness of the sandwich which are longitudinal Young's modulus of the ply (E_L), transverse shear modulus of the ply ($G_{TT'}$) and longitudinal

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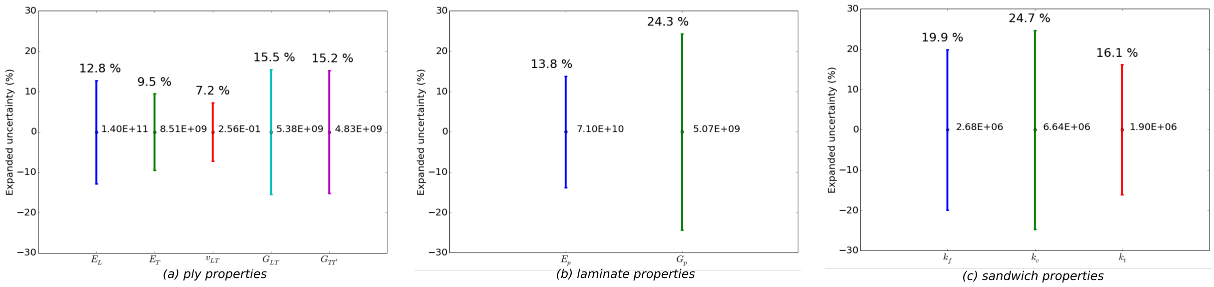
shear modulus of the ply (G_{LT}). The result of the propagation analysis is showed in (Fig. 5) for the path 1.



(a) ply properties (b) laminate properties (c) sandwich properties

Figure 5. Expanded uncertainty of properties used at ply (a), laminate (b) and sandwich scale (c).

At the scale of the ply (Fig. 5,a), uncertainty on component properties and geometrical properties lead to an uncertainty of 12.8 percent on longitudinal Young's modulus of the ply, 15.5 percent on longitudinal shear modulus and 15.2 percent on transverse shear modulus. At the scale of the laminate (Fig. 5,b), the uncertainty on ply properties and geometrical properties lead to an uncertainty of 13.5 percent on longitudinal Young's modulus and 24.3 percent on transverse shear modulus. At the sandwich beam scale (Fig. 5,c), uncertainties on laminates and core lead to an uncertainty of 20.7 percent on flexural stiffness, 25.1 percent on shear stiffness and 17 percent on total sandwich beam stiffness. The uncertainty analysis gives information about the started uncertainty on the key parameters responsible of the uncertainty on the sandwich beam stiffness. Then, improvement on these uncertainties were obtained by changing the analytical model or performing experimental testing. Sensitivity analysis demonstrate that the sandwich flexural stiffness depends on laminate longitudinal Young' modulus which depends on longitudinal Young's modulus of the ply (Fig. 5,a). Moreover, the sandwich shear stiffness depends on laminate transversal shear modulus which is mainly linked to transverse shear modulus ($G_{TT'}$) and longitudinal shear modulus (G_{LT}) of the ply. In this study, five different paths were identified to improve the results of the path 1. These paths are focused on longitudinal Young's modulus E_L, transverse shear modulus ($G_{TT'}$) and longitudinal shear modulus (G_{LT}) of the ply. The path (2) was obtained from the path (1) in which longitudinal Young's modulus of the ply (E_L) and its associated uncertainty were obtained by experimental testing (Table 2). The path (3) was obtained from the path (1) in which longitudinal shear modulus of the ply (G_{LT}) and its associated uncertainty were obtained by experimental testing (Table 2). The path (4) was obtained from the path (1) in which transverse modulus of the ply ($G_{TT'}$) and its associated uncertainty were obtained by experimental testing (Table 2). The path (5) was obtained from the path (1) in which the last three mentioned properties of the ply and their associated uncertainties were obtained by experimental testing (Table 2). Moreover, path (6) was used to demonstrate that the improvement of the uncertainty on a non-key parameter (with a small effect on the final result) is not relevant. This path was obtained from the path (1) in which transversal Young's modulus of the ply (E_T) and its associated uncertainty were obtained by experimental testing (Table 2). The results of uncertainty improvement of path 1 are shown in (Fig. 6) and expanded uncertainties of the total stiffness of the beam are shown in (Table 5) as a function of the used path.

Table 5. Expanded uncertainty of sandwich beam stiffness as a function of the chosen path.

Path	1	2	3	4	5	6
Sandwich total stiffness expanded uncertainty(%)	17	14.5	16.7	15.9	12.7	17

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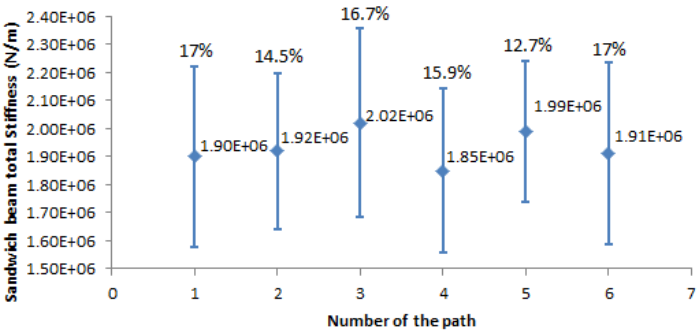

Figure 6. Evaluation of the expanded uncertainty of total sandwich stiffness versus study cost depending of the chosen path.

The Figure 6 shows that a lower uncertainty (8 percent versus 9.5 percent) on transversal Young's modulus of ply (path 6) does not change the result of total uncertainty of sandwich beam stiffness. However, decreasing the variation of longitudinal Young's modulus of the ply, for example from 12.8 percent to 8 percent through experimental testing (path 2), gives a reduction of 3 percent on the uncertainty of the sandwich beam total stiffness. The smallest value of uncertainty on the sandwich beam stiffness was obtained thanks to path 5 which was built from experimental testing at the scale of the ply. In order to take into account the cost of the study, a first evaluation of the cost was based on the complexity of the test. Then Figure 7 shows the study cost versus sandwich beam bending stiffness uncertainty as a function of the used path.

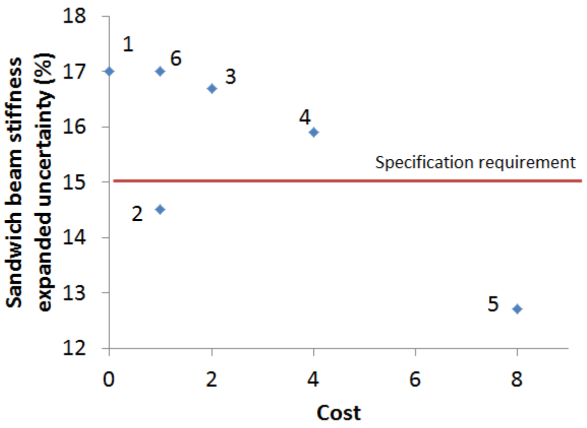

Figure 7. Evaluation of the expanded uncertainty on total sandwich beam stiffness versus study cost.

Only paths 2 and 5 respect the fixed constraint on beam stiffness uncertainty. If the designer can afford a high cost solution, path 5 could be suitable, but path 2 could be used in order to minimize the cost and ensuring the satisfaction of the constraints.

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4. Conclusion

In this study, the developed methodology allow us to identify the most significant parameters according to the uncertainty of a fixed constraint. This analysis coupled to uncertainty study enabled to highlight relevant combinations of analytical models and experimental testing to minimize uncertainty. In future work, the methodology will be able to have a larger choice of analytical and FEM models and experimental testing. This database will allow us to explore a larger space of paths that can lead the designer to minimize both uncertainty and cost of the study. A qualification model procedure will be made to define accurately the study cost.

References

[1] C. Conceição António and L. N. Hoffbauer. Uncertainty analysis based on sensitivity applied to angle-ply composite structures. *Reliability Engineering & System Safety*, 92(10):1353–1362, 2007.

[2] A. K. Noor, J. H. Starnes Jr., and J. M. Peters. Uncertainty analysis of stiffened composite panels. *Composite Structures*, 51(2):139–158, 2001.

[3] A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, and S. Tarantola. *Global Sensitivity Analysis. The Primer.* John Wiley & Sons, Ltd, Chichester, UK, 2007.

[4] D. G. Cacuci. *Sensitivity and Uncertainty Analysis, Volume II: Applications to Large-Scale Systems*. 2005.

[5] S. Salicone. *Measurement Uncertainty*. Springer Series in Reliability Engineering. Springer US, Boston, MA, 2007.

[6] S. Dariushi and M. Sadighi. A Study on Flexural Properties of Sandwich Structures with Fiber/Metal Laminate Face Sheets. *Applied Composite Materials*, 20(5):839–855, 2013.

[7] A.F. Johnson and G.D. Sims. Mechanical properties and design of sandwich materials. *Composites*, 17(4):321–328, 1986.

[8] J-M. Berthelot. *Matériaux Composites. Comportement mécanique et analyse des structures.* Lavoisier, 4 edition, 2005.

[9] R. B. Enie and R. R. Rizzo. Three-Dimensional Laminate Moduli. *Journal of Composite Materials*, 4(1):150–154, 1970.

[10] T. T. Chiao, P. L. Lien, K. L. Reifsnider, R. J. Morgan, and H. T. Hahn. Simplified Formulas for Elastic Moduli of Unidirectional Continuous Fiber Composites. *Journal of Composites Technology and Research*, 2(3):5, 1980.

[11] C. C. Chamis. Simplified composite micromechanics equations for strength, fracture toughness and environmental effects. *SAMPE Quarterly*, pages 41–55, 1984.

[12] C. T. Sun and R. S. Vaidya. Prediction of composite properties from a representative volume element. *Composites Science and Technology*, 56(2):171–179, 1996.

[13] J. Ahmad. *Machining of Polymer Composites*. Springer US, Boston, MA, 2009.

[14] Airex AG. *Airex C70. Mousse structurelle universelle*. 2011.

[15] C. C. Chamis. Probabilistic simulation of multi-scale composite behavior. *Theoretical and Applied Fracture Mechanics*, 41(1–3):51–61, 2004.

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