NEW PHENOMENOLOGICAL FAILURE MODEL FOR COMPOSITE MATERIALS FROM A HOMOGENIZED MICROMECHANICS APPROACH

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Keywords: Multiscale modeling, computational modeling, failure prediction, long-fibre composite

Abstract

This work aims to accurately predict the response and failure of a unidirectional long-fibre composite materials using a physically based constitutive law. New constitutive laws capable of predicting failure are crucial for expanding the use of composite materials in the automotive industry. Computational modeling frameworks need to act at a component level scale to facilitate product design and development for the automotive industry. In this research, a multiscale computational model is developed where a micromechanics based approach is coupled with a macroscale phenomenological model. A user-defined material model is developed and implemented in the commercial software LS-DYNA. This model employs the Chang and Chang failure criterion, which is used to predict failure in composites. The material properties are subsequently degraded to the point where the material is no longer capable of supporting load, representing failure. This new multiscale framework is compared with experimental data available in the literature. The numerical analyses show that the proposed model can be a powerful tool for use in product development since the model is able to successfully predict failure in composites.

1. Introduction

Failure in composite materials is complex to predict, since composites fail due to multiple mechanisms, such as fibre breakage, fibre buckling and matrix cracking [1]. The ability to correctly predict the material response of a material at failure provides industry with a useful tool for manufacturing and product development, as composite materials can be tailored to provide a material with the most useful properties in a specific application. Hashin used average stress and strains for the overall material to create a 3D failure criterion [1]. Puck's model has several parameters which require calibration in the model, opposed to getting the parameters from physical test results [2]. A model with a small number of physically-based input parameters is more useful, as physically-based parameters are easier to acquire straight from mechanical testing.

Excerpt from ISBN 978-3-00-053387-7

Although unidirectional composites have been studied for many years, there are still limitations to existing models. A better understanding of failure in unidirectional composites is required in order to have confidence in model failure prediction, as Talreja states that no theory available to date is able to successfully predict behaviour for all test cases [3]. To develop a better understanding, Hinton and Soden initiated the World Wide Failure Exercise in 1998 in order to better understand failure prediction for composite laminates [4]. However, this exercise served to highlight the limitations of existing failure theories. The Edge and Rotem theories were able to provide moderate agreement for the failure envelope, but the Edge theory was unable to fully predict strength interactions in all cases, whereas the Rotem model gave slightly conservative estimates [5–7]. The Hart-Smith theories were conservative, though two of the three had good agreement with the experimental data for predicting the failure envelopes for unidirectional laminae [7, 8]. Similarly, Puck's theory also had very good agreement, as it has been developed from many experimental studies [2, 7]. Like the Rotem and the Edge theory, both Sun theories were slightly conservative, though they had good predictive capability [9]. The Tsai-Wu theory was one of the most widely used failure criterions for prediction of the failure envelopes for a unidirectional laminae, whereas the Wolfe theory gave good results apart from predicting concurrent values [7, 10]. Though these models have their strengths, the shortcomings are enough that developing a model that is able to accurately predict failure of unidirectional composites with a small number of physically-based input parameters is beneficial to the field.

A model is developed to predict failure of a unidirectional composite material. The failure in this model is based off the Chang and Chang Failure Criterion [11, 12]. This model is primarily based off the micromechanics work of Sabiston et al. along with physically relevant parameters [13, 14]. As such, the model takes in the material elastic constants for each of the constituents. In addition, the overall material strengths in tension for the longitudinal direction, and tension and compression for the transverse direction, as well as the shear strength are used for the failure calculations. This model is applied at the phenomenological scale, and is based off the micromechanical model for long-fibre composite materials developed by Sabiston et al. Consequently, the number of fibres is ignored, as the effective fibre volume fraction takes into account the effects of multiple fibres [13]. The LS-DYNA explicit dynamic software is used to implement a user-defined material model.

2. Method

This section develops the constitutive laws used in this material model as well as highlighting the failure criteria used.

2.1. Constitutive Model

In order to develop a constitutive law for a phenomenological model based off micromechanics, a total stress and strain relation is derived. The strain in the fibre, ε_f, is found by partitioning the overall strain, ε, with a strain transformation tensor including Eshelby's solution, T_{ijkl}, as shown in Eq. 1 [15].

$$\varepsilon_f = T_{ijkl}\varepsilon \tag{1}$$

Stress (σ_f) and strain in the fibre are related through an elastic constitutive law as shown in Eq. 2, such that C_f is the transversely isotropic elastic matrix for the fibre material.

$$\sigma_f = C_f\varepsilon_f \tag{2}$$

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At this point, Eq. 1 is combined into Eq. 2 to produce the stress in the fibre as a function of the strain transformation tensor and the strain (Eq. 3).

$$\sigma_f = C_f T_{ijkl} \varepsilon \tag{3}$$

Through strain partitioning, an equation for the matrix displacement gradient, G_m, as a function of strain is derived. This is taken to be equal to the matrix strain, ε_m, assuming small elastic deformation, as shown in Eq. 4.

$$\frac{\varepsilon(1 - T_{ijkl}V_{fe})}{V_{me}} = G_m = \varepsilon_m \tag{4}$$

where V_{fe} is the effective fibre volume fraction, and V_{me} is the effective matrix volume fraction.

The effective fibre volume fraction, V_{fe}, is given as shown in Eq. 5, where V_f is the fibre volume fraction, and k, l and m are material pairing constants.

$$V_{fe} = \frac{V_f}{900}\left[480m\left(l^2 - k^2\right) + 30l^2 + 360kl + 510k^2\right] \tag{5}$$

The effective matrix volume fraction is given in Eq. 6.

$$V_{me} = 1 - V_{fe} \tag{6}$$

Since the matrix material behaves as an elastic material, Eq. 7 illustrates the stress-strain relation for the matrix, where σ_m is the stress in the matrix material, and C_m is the isotropic elastic constitutive matrix for the matrix material.

$$\sigma_m = C_m \varepsilon_m \tag{7}$$

These equations are combined to create a constitutive law which relates the total stress and strain, as seen in Eq. 8.

$$\sigma = Q\varepsilon \tag{8}$$

where

$$Q = C_m(1 - T_{ijkl}V_{fe}) + C_f T_{ijkl} V_{fe} \tag{9}$$

This overall constitutive law is used to calculate the total stress in the composite part. In order to determine failure in the part, a failure criterion is defined.

2.2. Failure Criterion

The implementation of failure in this model uses the Chang and Chang Failure Criterion [11, 12]. This failure criterion is based on the Yamada and Sun model, which is in turn based on the Hashin-Rotem model [16, 17]. The Hashin-Rotem failure criterion was proposed specifically for unidirectional composites [17]. Since the Chang and Chang criteria is used for this unidirectional material model, it is said that once failure is met by one failure criterion, the overall material itself fails. Eq. 10 describes the failure condition for the matrix material, where matrix cracking occurs if $e_m \geq 1$.

$$\left(\frac{\sigma_2}{Y_t}\right)^2 + \frac{\frac{(\sigma_{12})^2}{2G_{12}} + \frac{3}{4}\alpha(\sigma_{12})^4}{\frac{(S_c)^2}{2G_{12}} + \frac{3}{4}\alpha(S_c)^4} = e_m^2 \tag{10}$$

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In Eq. 10, σ_2 and σ_{12} are the transverse tensile stress and the shear stress, Y_t is the transverse tensile strength, and G_{12} is the shear modulus. S_c is the shear strength, and α is a fitting parameter, such that $\alpha = 0$ for linear elastic behaviour.

Similarly, Eq. 11 describes the failure condition for the fibres, such that fibre breakage occurs if $e_f \geq 1$. σ_1 and X_t are the longitudinal tensile stress and strength.

$$\left(\frac{\sigma_1}{X_t}\right)^2 + \frac{\frac{(\sigma_{12})^2}{2G_{12}} + \frac{3}{4}\alpha(\sigma_{12})^4}{\frac{(S_c)^2}{2G_{12}} + \frac{3}{4}\alpha(S_c)^4} = e_f^2 \tag{11}$$

Lastly, Eq. 12 describes the compression failure condition, where Y_c is the transverse compressive strength. In this instance, failure occurs if $e_c \geq 1$.

$$\left(\frac{\sigma_2}{2S_c}\right)^2 + \left[\left(\frac{Y_c}{2S_c}\right)^2 - 1\right]\frac{\sigma_2}{Y_c} + \frac{\frac{(\sigma_{12})^2}{2G_{12}} + \frac{3}{4}\alpha(\sigma_{12})^4}{\frac{(S_c)^2}{2G_{12}} + \frac{3}{4}\alpha(S_c)^4} = e_c^2 \tag{12}$$

The failure criteria are used to determine the mode of failure. In each direction, once failure has been determined by the Chang and Chang criterion, the material properties are then degraded. For matrix failure and compression failure, the transverse modulus and the Poisson's ratios are reduced to zero, though for matrix failure, the shear modulus is also reduced to zero. For fibre failure, the longitudinal and transverse moduli, shear modulus and the Poisson's ratio are reduced to zero.

3. Results

Following the implementation of the constitutive model in LS-DYNA, material data for an APC-2/AS4 composite was used to compare the simulated results with experimental results taken from Kyriakides et al. [18]. Table 1 provides the material data used in the simulation, while Fig. 1 shows a comparison between the experimental and simulated results for tension and compression in both the transverse and the longitudinal directions.

E_m and ν_m are the elastic constant and the Poisson's ratio for the matrix. E_{fa} and E_{ft} are the elastic modulus for the fibre in the axial and transverse direction, respectively. Similarly, ν_{fta} and ν_{ft} are the Poisson's ratio for the fibre in the transverse axial and transverse direction. G_f is the shear modulus for the fibre, and V_f is the fibre volume fraction.

Table 1. Material and Calibration Data for APC-2/AS4 Composite

E_m (GPa)	ν_m	E_fa (GPa)	E_ft (GPa)	ν_fta	ν_ft	G_f (GPa)	V_f	α
6.14	0.356	214.0	12.0	0.263	0.2	57.9	0.6	0.3

X_t (GPa)	X_c (GPa)	Y_t (GPa)	Y_c (GPa)	S_c (GPa)	k	l	m
1.37	1.2	0.079	0.214	0.136	0.9	1.15	0.4

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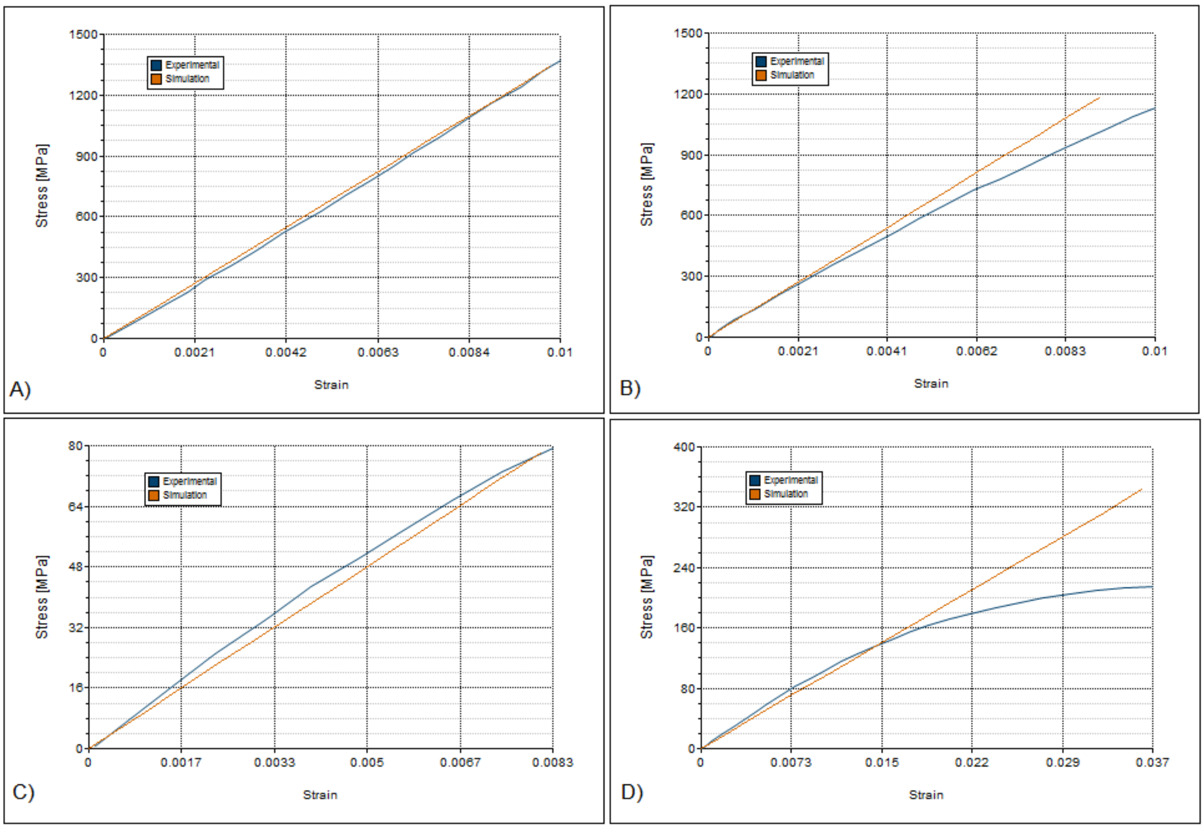

Figure 1. Experimental vs. Simulated Results for a) longitudinal tension b) longitudinal compression c) transverse tension d) transverse compression

It is shown that the experimental data for tension in the longitudinal direction is highly linear, whereas the other experimental data behaves plastically. When examining the simulated data, it is shown that there is good agreement between the trends for the simulated data and the experimental data. In the longitudinal directions, the simulation is slightly over-predicting the stress, where in the transverse direction, the simulation under-predicts. However, the two curves do begin to deviate when the matrix material begins to behave plastically. This strongly suggests that plasticity should be incorporated into the constitutive model in future works.

4. Conclusion

This paper provides a novel multi-scale approach to modeling failure in composites materials based on the micromechanics work developed by Sabiston et al. [13]. This model is able to use the individual constituent properties of the fibres and the matrix to model the overall composite behaviour. Failure is incorporated into this model through the use of the Chang and Chang Failure Criteria [11, 12].

Since it is known that a thermoplastic matrix behaves elasto-plastically, this model needs to be further developed in order to capture the plasticity of the overall composite in the transverse directions. The next steps for this model would be to incorporate plasticity into the constitutive law for the matrix.

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Excerpt from ISBN 978-3-00-053387-7