STIFFNESS PREDICTION OF DAMAGED LAMINATES BASED ON THE CONCEPT OF LAMINATE EFFECTIVE STIFFNESS

J. Varna[1], MS. Loukil[2]

[1]Department of Engineering Sciences and Mathematics, Luleå University of Technology, SE-97187 Luleå, Sweden
Email: janis.varna@ltu.se, Web Page: http://www.ltu.se
[2]Swerea SICOMP, Box 271, SE-94129, Piteå, Sweden
Email: mohamed.loukil@swerea.se, Web Page: http://www.swerea.se

Keywords: Effective transverse modulus, transverse cracking, FEM, Crack opening displacement

Abstract
The concept of the effective stiffness of a unidirectional layer with intralaminar cracks is revisited performing 3-D FEM parametric analysis of symmetric and balanced laminates with damaged 90-layer. The effective stiffness of the damaged layer is obtained from the difference between damaged and undamaged laminate stiffness. The effective longitudinal modulus and Poisson's ratio of the layer are equal to their initial values. A very simple expression for the effective transverse modulus change with normalized crack density has sufficient accuracy and generality to be used in laminate theory to predict macroscopic elastic property change with crack density in laminates with very different lay-ups and made of different UD composites.

1. Introduction

Usually the first damage mode in laminates is intralaminar cracking of layers with off-axis orientation with respect to the main load. Intralaminar cracks run parallel to fibres in the layer and the crack plane is perpendicular to the laminate middle-plane usually covering the entire thickness of the layer and the whole width of the specimen. The term "crack density" ρ_k (a number of cracks in the k-th layer over certain distance (cracks/mm)) is used to quantify the damage state. More appropriate for use is dimensionless crack density

$$\rho_{kn} = \rho_k t_k \tag{1}$$

The macroscopic stress applied to the laminate boundaries has a rule of mixtures relationship with average stresses in damaged and undamaged layers [1]. Therefore the stiffness of the damaged laminate, being calculated from macroscopic stresses and strains, can be expressed in terms of average stresses in damaged layer. Determination of the average stress change requires knowledge of the stress distribution between cracks.
The so called micromechanics modelling deals with stress perturbations caused by cracks. The simplest calculation scheme is based on shear lag assumptions, for example [2,3]. Variational models of varying degree of accuracy have been presented in [4-7]. A high accuracy model was presented in [7]. Unfortunately most of the analytical solutions are applicable to cross-ply type of laminates with cracks in 90-layers only. In [8] analysing the stress between cracks in one layer the "Equivalent constraint model" was introduced suggesting to replace the rest of damaged and undamaged layers by one homogenized layer which would have the same constraint effect on the damaged layer.
An alternative approach to account for the average stress change was suggested in [1,9]: in [10] exact relationships between the average stress change in layers and the average crack opening (COD) and crack sliding (CSD) displacements were established. That was a proof that the damaged laminate

Excerpt from ISBN 978-3-00-053387-7

stiffness can be expressed in terms of change of average stress in damaged layers or, if it is more convenient, in terms of normalized average CODs and CSDs. The latter is preferable when crack face displacements COD and CSD have been calculated, for example, using FEM. The COD and CSD formulation is used in the GLOB-LOC approach [1,9], obtaining exact expressions for thermo-elastic constants of a general symmetric damaged laminate with average COD and CSD as input parameters (fitting functions from FEM parametric analysis).
Both described approaches render stiffness of the damaged laminate. However, often (for example using shell elements in numerical damage analysis of composite structures) not the damaged laminate properties but properties of the layer with multiple cracks (geometrical discontinuities) are the input data. The most efficient approach to that is replacing the layer with cracks by undamaged homogenized material with "effective" elastic properties. An extreme case of this approach is the well know ply-discount model.
The effective stiffness matrix of the damaged layer may be back-calculated from known undamaged laminate stiffness and the stiffness of laminate with damage. Strictly speaking the effective stiffness dependence on crack density is not just a property of the layer: it depends on laminate lay-up and properties of the surrounding layers. The numerical results presented in this paper prove that at least the crack density dependence of the transverse effective modulus, E_T^{eff} is a very robust function with respect to material change or location of the damaged layer. Here the upper index "eff" stands for "effective". The effective longitudinal modulus E_L^{eff} and the Poisson's ratio ν_{LT}^{eff} of the damaged layer remain the same as for undamaged UD composite.
The study is limited to symmetrical laminates considered in global coordinate system where the cracked layer becomes 90-layer and the laminate in this system is balanced (for example cross-ply and quasi-isotropic laminates). Analysis of the effective shear modulus, which would require shear loading of laminates in FEM, is not considered in this paper.

2. Theoretical background

Symmetric N- layer laminate is considered. The k-th layer of the laminate is characterized by thickness t_k, fiber orientation angle with respect to the global x-axis θ_k and by stiffness in the local axes $[Q]$ (defined by elastic constants $E_L, E_T, G_{LT}, \nu_{LT}$). Notation $[\bar{Q}]_k$ is used for the stiffness matrix of this layer in global coordinates. The thickness of the laminate is $h=\sum_{k=1}^{N} t_k$. The crack density in a layer is ρ_k and the dimensionless crack density ρ_{kn} is introduced in (1).
It is assumed that the laminate remains symmetric in the damaged state: the crack density in corresponding symmetrically located layers is the same. The stiffness matrix of the damaged laminate is $[Q]^{LAM}$ and the stiffness of the undamaged laminate is $[Q]_0^{LAM}$. Notation for the compliance matrix of the undamaged laminate is $[S]_0^{LAM}=\left([Q]_0^{LAM}\right)^{-1}$. Elastic constants of the undamaged laminate are calculated using CLT.

2.1. Effective stiffness of the damaged layer

The study is limited to laminates with two symmetrically located damaged layers of the same orientation and with the same state of damage. In CLT the stiffness matrix $[Q]_0^{LAM}$ of the laminate is independent on the layer sequence in the laminate and, therefore, without losing generality we can assume that the two symmetric layers to be damaged have indexes 1 and N.
The stiffness matrix of the undamaged laminate is calculated as

$$[Q]_0^{LAM}=\sum_{k=2}^{N-1}[\bar{Q}]_k\frac{t_k}{h}+2[\bar{Q}]_N\frac{t_N}{h} \tag{2}$$

When damage with given dimensionless crack density ρ_{kn}, see (1), is introduced in the N-th layer the stiffness matrix of the laminate is $[Q]^{LAM}$

Excerpt from ISBN 978-3-00-053387-7

$$[Q]^{LAM} = \sum_{k=2}^{N-1}[\overline{Q}]_k \frac{t_k}{h} + 2[\overline{Q}]_N^{eff} \frac{t_N}{h} \quad (3)$$

Subtracting (3) from (2) we obtain

$$[Q]_0^{LAM} - [Q]^{LAM} = 2\left\{[\overline{Q}]_N - [\overline{Q}]_N^{eff}\right\}\frac{t_N}{h} \quad (4)$$

From here the effective stiffness of the damaged N-th layer is

$$[\overline{Q}]_N^{eff} = [\overline{Q}]_N - \frac{h}{2t_N}\left\{[Q]_0^{LAM} - [Q]^{LAM}\right\} \quad (5)$$

If the damaged layer is the central layer in the laminate its index is N1=(N+1)/2 and

$$[\overline{Q}]_{N1}^{eff} = [\overline{Q}]_{N1} - \frac{h}{t_{N1}}\left\{[Q]_0^{LAM} - [Q]^{LAM}\right\} \quad (6)$$

The global stiffness of the layer can be transformed to local axes using expression (index N or N1 is now omitted)

$$[Q]^{eff} = [T][\overline{Q}]^{eff}[T]^T \quad (7)$$

The effective compliance matrix and the effective engineering constants are

$$[S]^{eff} = \left([Q]^{eff}\right)^{-1}, \ E_L^{eff} = \frac{1}{S_{11}^{eff}}, \ E_T^{eff} = \frac{1}{S_{22}^{eff}}, \ G_{LT}^{eff} = \frac{1}{S_{66}^{eff}}, \ -\nu_{LT}^{eff} = E_L^{eff} S_{12}^{eff} \quad (8)$$

With coordinate transformation we can always achieve that the damaged layer in a laminate has 90-orientation. In these coordinates (assuming the laminate is balanced) the change in $E_x^{LAM}, E_y^{LAM}, \nu_{xy}^{LAM}$ will depend on $E_L^{eff}, E_T^{eff}, \nu_{LT}^{eff}$ of the damaged 90-layer and not on the effective shear modulus G_{LT}^{eff}. This is because in symmetric and balanced damaged laminates a) the shear stress-strain response is separated from the normal response; b) the shear modulus of the 90-layer (the initial or the effective) does not enter the laminate equations for the normal response. For similar reason G_{xy}^{LAM} does not depend on $E_L^{eff}, E_T^{eff}, \nu_{LT}^{eff}$ of the damaged 90-layer.
Interaction of cracks belonging to different layers is not accounted for in the present analysis. A simple iterative procedure can be suggested: cracks are explicitly introduced only in one couple of symmetric layers and the rest of damaged layers are homogenized .

2.2. Damaged laminate stiffness

The following exact expressions were obtained for symmetric and balanced laminate with damaged 90-layers in [1]:

$$\frac{E_x^{LAM}}{E_{x0}^{LAM}} = \frac{1}{1+2M\rho_{90n}\frac{t_{90}}{h}u_{2an}^{90}c_2} \qquad \frac{E_y^{LAM}}{E_{y0}^{LAM}} = \frac{1}{1+2M\rho_{90n}\frac{t_{90}}{h}u_{2an}^{90}c_4} \quad (9)$$

$$\frac{\nu_{xy}^{LAM}}{\nu_{xy0}^{LAM}} = \frac{1+2M\rho_{90n}\frac{t_{90}}{h}u_{2an}^{90}c_1\left(1-\frac{\nu_{LT}}{\nu_{yx0}^{LAM}}\right)}{1+2M\rho_{90n}\frac{t_{90}}{h}u_{2an}^{90}c_2} \qquad \frac{G_{xy}^{LAM}}{G_{xy0}^{LAM}} = \frac{1}{1+2M\rho_{90n}\frac{t_{90}}{h}u_{1an}^{90}\frac{G_{LT}}{G_{xy0}^{LAM}}} \quad (10)$$

$$c_1 = \frac{E_T}{E_{x0}^{LAM}}\frac{1-\nu_{LT}\nu_{xy0}^{LAM}}{(1-\nu_{LT}\nu_{TL})^2} \quad c_2 = c_1\left(1-\nu_{LT}\nu_{xy0}^{LAM}\right) \qquad c_4 = \frac{E_T}{E_{yo}^{LAM}}\frac{\left(\nu_{LT}-\nu_{yx0}^{LAM}\right)^2}{(1-\nu_{LT}\nu_{TL})^2} \quad (11)$$

In (9)-(11) u_{2an}^{90} is the average normalized opening and u_{1an}^{90} the average normalized sliding displacement of the 90-layer crack faces (letters "*a*" and "*n*" denote the average and "normalized" respectively). Index 90 is used also for thickness, crack density and COD in the 90-layer. The quantities with upper index LAM are laminate constants, quantities with additional lower index 0 are undamaged laminate constants. Parameter M is the number of equal damaged 90-layers in the laminate: for central 90-layer $M = 1$, for two symmetric damaged 90-layers $M = 2$.

Excerpt from ISBN 978-3-00-053387-7

According to (10) the laminate shear modulus is not related to u_{2an}^{90} and depends on sliding displacement only. On the other hand the sliding displacement u_{1an}^{90} does not enter expressions for $E_x^{LAM}, E_y^{LAM}, \nu_{xy}^{LAM}$: the change of the laminate shear modulus is not coupled with the change of other elastic constants.
The class of laminates covered by these expressions is broader than just cross ply laminates or balanced laminates with cracked 90-layers. Any quasi-isotropic laminate with an arbitrary damaged layer can be rotated to have the damaged layer as a 90-layer still keeping the laminate balanced. The limitation of (9)-(11) is that the laminate has zero coupling terms in $[S]_0^{LAM}$ and in $[S]^{LAM}$.
Expressions (9), (10) show that analyzing elastic constants for normal loading, only one constant for the damaged laminate has to be calculated: for example using 3-D FEM we find E_x^{LAM} only. From there we can use (9) to find the term with u_{2an}^{90} and use it to calculate ν_{xy}^{LAM} and E_y^{LAM}. A separate FEM calculation is required to find G_{xy}^{LAM}.

3. Effective transverse modulus: parametric FEM analysis

In this Section the effective elastic properties of damaged layers are analyzed. Calculations are performed for transversely isotropic CF/EP and GF/EP composites with elastic properties and prepreg layer thickness t_0 given in Table 1. Cross-ply laminates of configuration [0m/90$_8$]s and [90$_8$/0m]s and quasi-isotropic laminates with 0, 45, -45 and 90 orientation of layers, varying position of the 90-layer are considered. In all cases cracks are in 90-layers only.

Table 1 Elastic constants of used UD composites

Material	E_L (GPa)	E_T (GPa)	ν_{LT}	ν_{3T}	G_{LT} (GPa)	t_0 (mm)
GF/EP	45.0	15.0	0.3	0.4	5.0	0.5
CF/EP	150	10	0.3	0.4	5.0	0.5

3-D models, see Fig. 1, were created in ABAQUS with 3D (C3D8) 8-node linear brick elements. Mesh with 86400 elements was used in each FE model with refined mesh near the crack surfaces. Constant displacement corresponding to 1% average strain was applied to the repeating unit in x-direction. On the front edge (y=0) and the far-away edge (y=w) coupling conditions were applied for normal displacements (u_y = unknown constant). These conditions correspond to generalized plane strain case. The total force in x-direction was used to calculate E_x^{LAM}. Then the u_{2an}^{90} was obtained from first equation in (9) and used in (9),(10) to calculate $E_y^{LAM}, \nu_{xy}^{LAM}$.

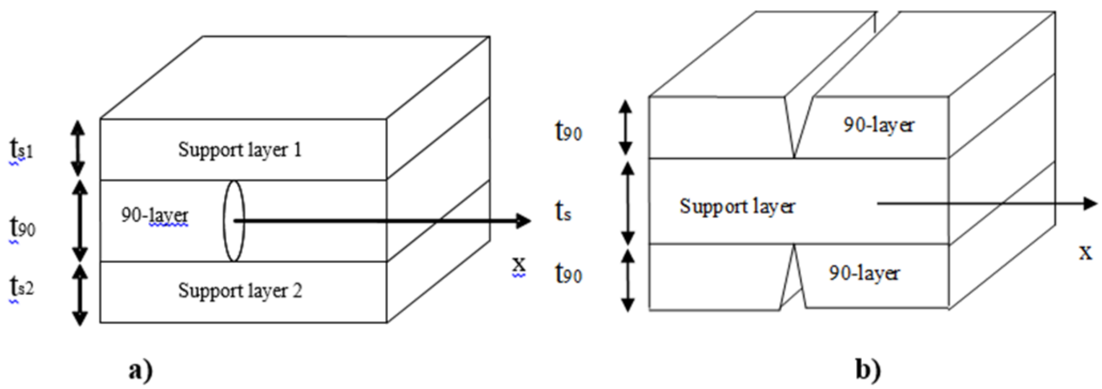


Figure 1. Models for COD studies with cracked 90-layers supported by domains with balanced lay-ups: a) crack in inside 90-layer; b) crack in surface 90-layer.

Excerpt from ISBN 978-3-00-053387-7

The stiffness matrix of the damaged laminate was used to calculate the effective elastic constants $E_L^{eff}, E_T^{eff}, \nu_{LT}^{eff}$ of the 90-layer following expressions in Section 2.1.
For all considered laminates the following numerical result was obtained: the effective longitudinal modulus and the major Poisson's ratio of the damaged layer do not change due to cracking ($E_L^{eff} = E_L$, $\nu_{LT}^{eff} = \nu_{LT}$). This result supports the modified ply-discount model in which only the transverse modulus and the shear modulus are reduced.
The obtained E_T^{eff} as a function of normalized crack density for $[0_m/90_8]$s cross-ply laminates is shown in Fig. 2. The effect of the constraint layer (0-layer) thickness on E_T^{eff} is very weak for both materials. The data in Fig.2 were fitted and the fitting functions were used for qualitative comparison between materials and lay-ups.
For GF/EP $[0_m/90_8]$s the fitting function is

$$E_T^{eff} = 9 \cdot e^{-2.5\rho_{90n}} + 6 \cdot e^{-0.9\rho_{90n}} \quad (12)$$

For CF/EP $[0_m/90_8]$s the fitting function is

$$E_T^{eff} = 9.5 \cdot e^{-1.4\rho_{90n}} + 0.5 \cdot e^{-1.3\rho_{90n}} \quad (13)$$

Similar relationships were obtained for $[90_8/0_m]$s laminates. The effect of the 0-layer thickness on E_T^{eff} even in this case is similar for both materials.

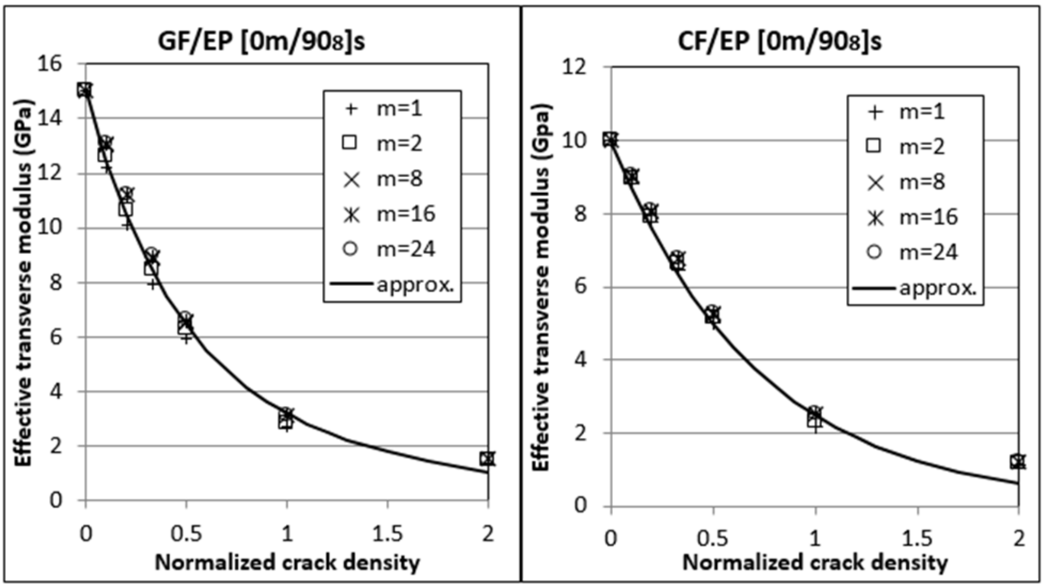


Figure 2. Effective transverse modulus E_T^{eff} versus the normalized crack density in the 90-layer of $[0m/90_8]$s laminates. Solid lines are approximations by functions (12) and (13) respectively for GF/EP and for CF/EP

Analyzing the E_T^{eff} of 90-layer in quasi-isotropic laminates with cracks in internal 90-layers, see Fig.3, we conclude that the difference between E_T^{eff} of the 90-layer in [45/-45/0/90]s and in [0/45/-45/90]s laminate is very small. The solid curves are the fitting for E_T^{eff} of the 90-layer in $[0m/90_8]$s laminate: the effective modulus of the 90-layer in quasi-isotropic laminate can be represented by the E_T^{eff} from cross-ply laminate. The E_T^{eff} data for the surface 90-layer in [90/0/45/-45]s and [90/-45/45/0]s laminates confirm this observation. These results show that the effective 90-layer transverse modulus in damaged cross-ply laminate may be of relevance for laminates with more general lay-up.

Excerpt from ISBN 978-3-00-053387-7

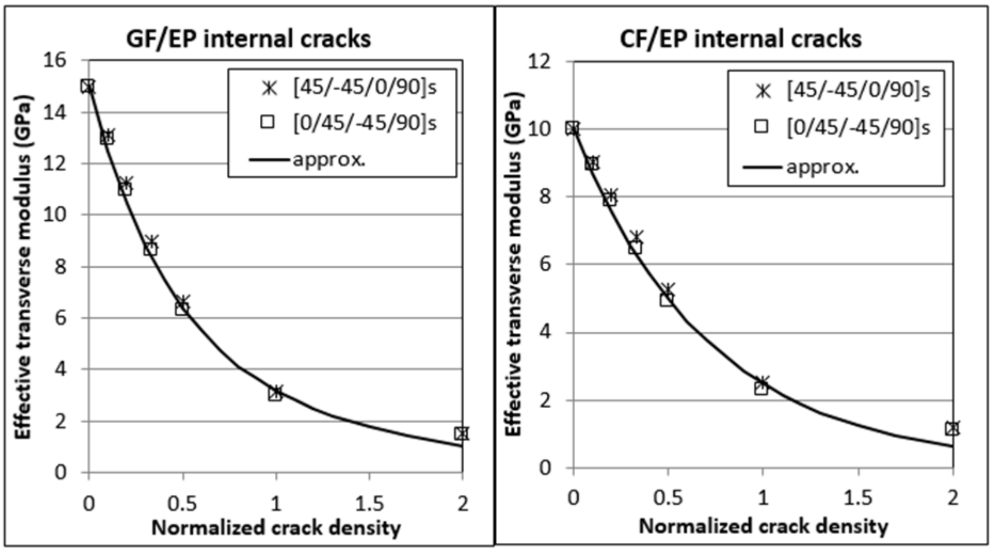
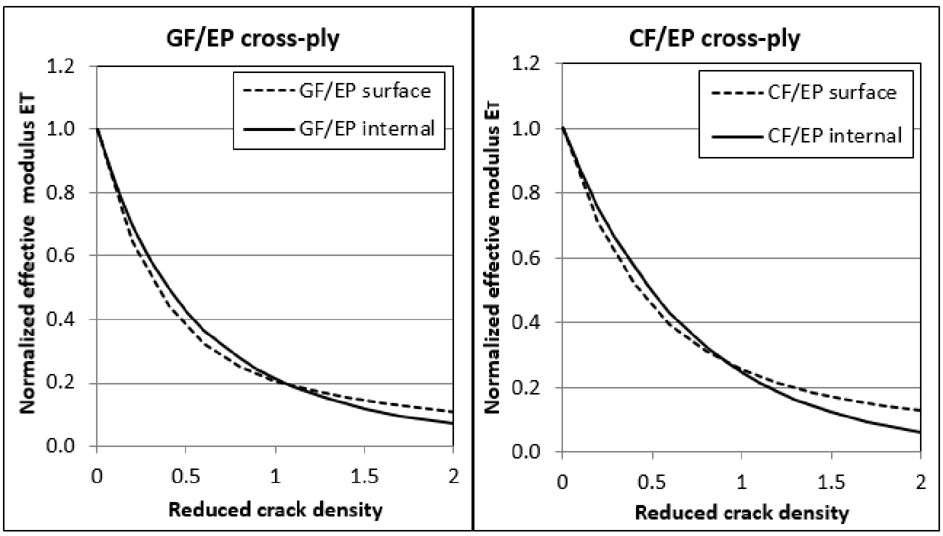


Figure 3. Effective transverse modulus E_T^{eff} versus normalized crack density in the 90-layer of [45/-45/0/90]s and [0/45/-45/90]s laminates. Solid lines are approximations (12) and (13) for [0m/90_8]s laminates

When the effective transverse modulus of the layer E_T^{eff} is normalized with respect to the initial value E_T, the E_T^{eff}/E_T dependence on normalized crack density is similar in GF/EP and CF/EP cross-ply and, for example, for internal 90-layer normalized (12) can be used for GF/EP and CF/EP

$$E_T^{eff} = E_T\left(\frac{3}{5}\cdot e^{-2.5\rho_{90n}} + \frac{2}{5}\cdot e^{-0.9\rho_{90n}}\right) \quad (14)$$

However, the difference in reduction rate is significant between surface and internal damaged 90-layers. It is because the crack opening u_{2an}^{90} in surface layer is about two times larger [1] leading to faster effective stiffness reduction. In the laminate stiffness expressions (9), (10) u_{2an}^{90} is always multiplied by normalized crack density ρ_{90n}. Hence it is possible to introduce "reduced crack density" ρ_{90n}^* to reduce the effect of the COD differences ($\rho_{90n}^* = \rho_{90n}$ for internal layers and $\rho_{90n}^* = 2\rho_{90n}$ for surface layers. The E_T^{eff} versus the "reduced crack density" is shown In Fig. 4: curves are very similar for surface and internal damaged layers.



Excerpt from ISBN 978-3-00-053387-7

Figure 4. Effective transverse modulus E_T^{eff} versus reduced crack density ρ^*_{90n} in the 90-layer of cross-ply laminates. Approximation functions (14) is used

Based on the above we suggest an extremely robust E_T^{eff} calculation scheme, where the normalized curve (14) for GF/EP $[0_m/90_8]s$ is used for CF/EP cross-ply, quasi-isotropic laminates, for inside and surface damaged layers . In other words

$$E_T^{eff} = E_T\left(\frac{3}{5}\cdot e^{-2.5k\rho_{90n}} + \frac{2}{5}\cdot e^{-0.9k\rho_{90n}}\right) \quad (15)$$

can be used for any damaged layer ($k = 1$ for internal layers and $k = 2$ for surface layers). In next section we will inspect the accuracy of this assumption predicting stiffness of laminates.

4. Damaged laminate stiffness prediction

The axial modulus E_x^{LAM} and Poisson's ratio ν_{xy}^{LAM} of damaged laminates may be calculated in any of the following ways: a) using 3-D FEM models as described in Section 3; b) using CLT with effective transverse modulus of the layer E_T^{eff} calculated according to (15). The CLT expression is

$$[Q]^{LAM} = \sum_{k=1}^{N} [\bar{Q}]_k^{eff}\frac{t_k}{h} \quad (16)$$

Undamaged layers have

$$[\bar{Q}]_k^{eff} = [\bar{Q}]_k \quad (17)$$

For damaged layers $[\bar{Q}]_k^{eff}$ is calculated using the effective modulus E_T^{eff} and initial elastic constants for E_L, ν_{LT} . Predicted laminate stiffness is in a very good agreement with FEM results for both composites and used lay-ups. An example is shown in Fig.5 where the GF/EP cross-ply laminate based (15) is used to predict the elastic properties reduction of quasi-isotropic laminates.

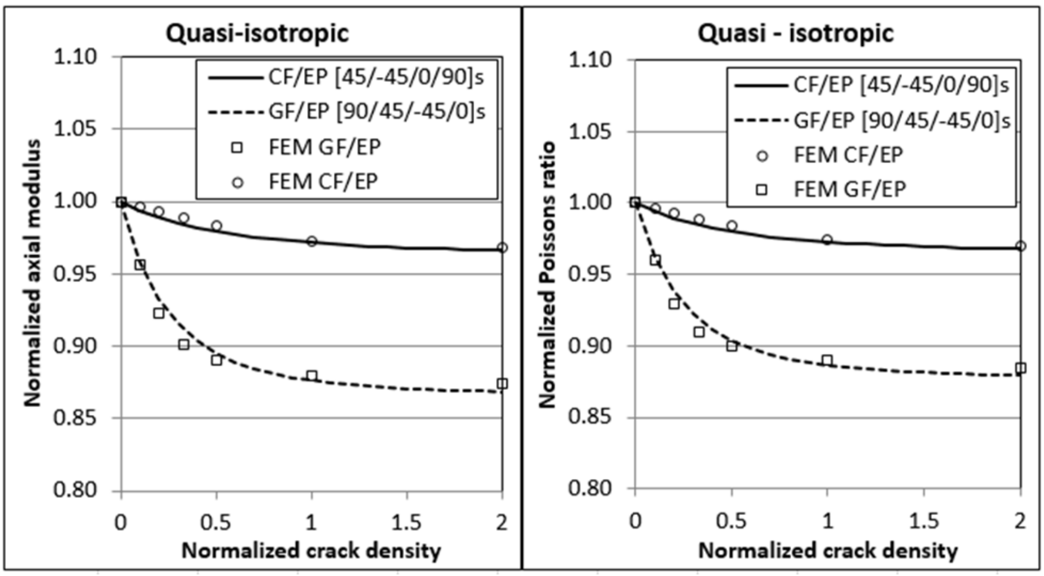


Figure 5. Normalized axial modulus E_x^{LAM} and Poisson's ratio ν_{xy}^{LAM} versus normalized crack density ρ_{90n} in the 90-layer of CF/EP and GF/EP quasi-isotropic laminates. In CLT predictions functions (15) is used.

This paper does not cover cases when the effective shear modulus reduction affects laminate stiffness because until now the effective shear modulus change has not been studied sufficiently.

4. Conclusions

Stiffness of symmetric and balanced laminates with intralaminar cracks in 90-layers was analyzed directly using 3-D FEM model and also using laminate theory with effective transverse modulus of the damaged layer. The effective transverse modulus of the 90-layer was calculated from the difference between the undamaged and damaged laminate stiffness. Calculations showed that the effective longitudinal modulus and the major Poisson's ratio of the damaged layer do not change at all due to intralaminar cracking.
The effective transverse modulus reduction in a normalized form is almost the same for glass fiber and carbon fiber/epoxy layers and the layer thickness ratio and lay-up (quasi-isotropic laminates) has very small effect on the modulus reduction rate. However, the effective transverse modulus reduction with normalized crack density is much faster in surface layers. Introducing "reduced" crack density the effective stiffness reduction curves for surface and internal layers almost collapse. A method was proposed in which the very robust effective transverse modulus dependence on the normalized crack density is described by one simple function. This expression was used in CLT to calculate stiffness of different damaged laminates reaching very good agreement with FEM results.

Acknowledgments

This work is performed within the Swedish Aeronautical Research Program (NFFP), Project 2014-00882, in co-operation with GKN Aerospace Sweden AB and jointly funded by the Swedish Armed Forces, Swedish Defence Materiel Administration and the Swedish Governmental Agency for Innovation Systems.

References

[1] J. Varna. Modeling Mechanical Performance of Damaged Laminates. *Journal of Composite Materials*, 47(20-21):2443-2475, 2013.

[2] P.A. Smith and J.R.Wood. Poisson's ratio as a damage parameter in the static tensile loading of simple cross-ply laminates. *Composite Science and Technology*, 38:85-93, 1990.

[3] S.C. Tan and R.J. Nuismer. A theory for progressive matrix cracking in composite laminates, Journal of Composite Materials, 23:1029-1047, 1989.

[4] Z. Hashin. Analysis of cracked laminates: a variational approach. *Mechanics of Materials*, North-Holland 4, 121-136, 1985.

[5] V. Vinogradov and Z. Hashin. Variational analysis of angle-ply laminates, *Composites Science and Technology*, (70):638-646, 2010.

[6] D.T.G. Katerelos, A. Krasnikovs and J. Varna. Variational models for shear modulus of symmetric and balanced laminates with cracks in 90-layer, *International journal of solids and structures*, in print, 2015

[7] L.N. McCartney, G.A. Schoeppner and W. Becker. Comparison of models for transverse ply cracks in composite laminates, *Composite Science and Technology*, 60:2347-2359, 2000.

[8] J. Zhang, J. Fan and C. Soutis. Analysis of multiple matrix cracking in $[\pm\theta_m/90_n]s$ composite laminates. Part 1. In-plane stiffness properties, Composites, 23(5):291-304, 1992.

[9] P. Lundmark and J. Varna, Crack face sliding effect on stiffness of laminates with ply cracks, *Composites Science and Technology*, 66:1444-1454, 2006.

[10] J. Varna. Strategies for stiffness analysis of laminates with microdamage: combining average stress and crack face displacement based methods, Journal of Applied Mathematics and Mechanics, 95(10):1081-1097, 2015

Excerpt from ISBN 978-3-00-053387-7