MODELING OF THROUGH-THICKNESS INTRA-YARN FIBER VOLUME FRACTION GRADIENTS IN LAMINATED WOVEN FABRICS

M. Mühlstädt[1], W. Seifert[1], S. Maenz[1], K. D. Jandt[1] and J. Bossert[1],

[1]Otto Schott Institute of Materials Research, Chair of Materials Science, Friedrich Schiller University Jena, Löbdergraben 32, 07743 Jena, Germany
Email: joerg.bossert@uni-jena.de, Web Page: http://www.cms.uni-jena.de

Keywords: Polymer Matrix Composites (PMC); Fiber Volume Fraction Gradient, Woven Fabric

Abstract

Fiber-reinforced composites feature outstanding material properties, like high specific strength and specific modulus of elasticity linked to a high anisotropy. The optimization of load-bearing fiber-reinforced composite components to specific applications require the application of simulation techniques, e.g. micromechanics and finite element simulation. Therefore, a precise modeling of the fabric's meso-level structure and the arrangement of the filaments within the yarns are mandatory. Scanning electron microscopy images of polished micro-sections of laminated plain woven fabrics (PWF) showed intra-yarn fiber volume fraction gradients (IY-FVFG) in the yarn's through-thickness direction.

The aim of this study was to model the observed IY-FVFG within laminated PWFs according to the established meso-level model proposed by Naik and Shembekar and to investigate its formation. Here, we present details on the method to determine the IY-FVFG within laminated PWFs. All investigated laminates were manufactured by the vaccum-assisted resin transfer molding process. Polished sections of the laminates were investigated via scanning electron microscopy and analyzed by algorithms implemented in Matlab.

The proposed procedure is superior to more simple threshold based calculations, which are prone to preparation artifacts and the errors introduced by the boundary of the images. Moreover, the proposed procedure is not restricted to the PWF style.

1. Introduction

Fiber-reinforced polymeric composites (FRPC) feature outstanding material properties, like high specific strength and specific modulus of elasticity as well a high anisotropy. Mechanical and physical properties can be adjusted to technical requirements by an individual composite design, i.e. a variation of the fiber's or the fabric's orientation. Woven fabrics further increase the variety of possible properties due to an additional undulation of the rovings perpendicular to the fabric layer. In order to optimize load-bearing components to specific applications simulation techniques have to be applied, e.g. micromechanics [1-10] and finite element simulation (FES) [11,12]. A precondition for the micromechanical simulation is an accurate model of the fabric's geometric structure and the arrangement of the filaments within the yarns. Models describing the fabric's geometric structure were still investigated by several scientists [6,7]. One fourth of the representative volume element of a plain woven fabric (PWF) is illustrated in Figure 1.

Regarding the arrangement of the filaments the literature reports intra-yarn fiber volume fraction gradients (IY-FVFG) in woven FRPCs [13-16]. For example, Olave et al. found in the yarn's through-

Excerpt from ISBN 978-3-00-053387-7

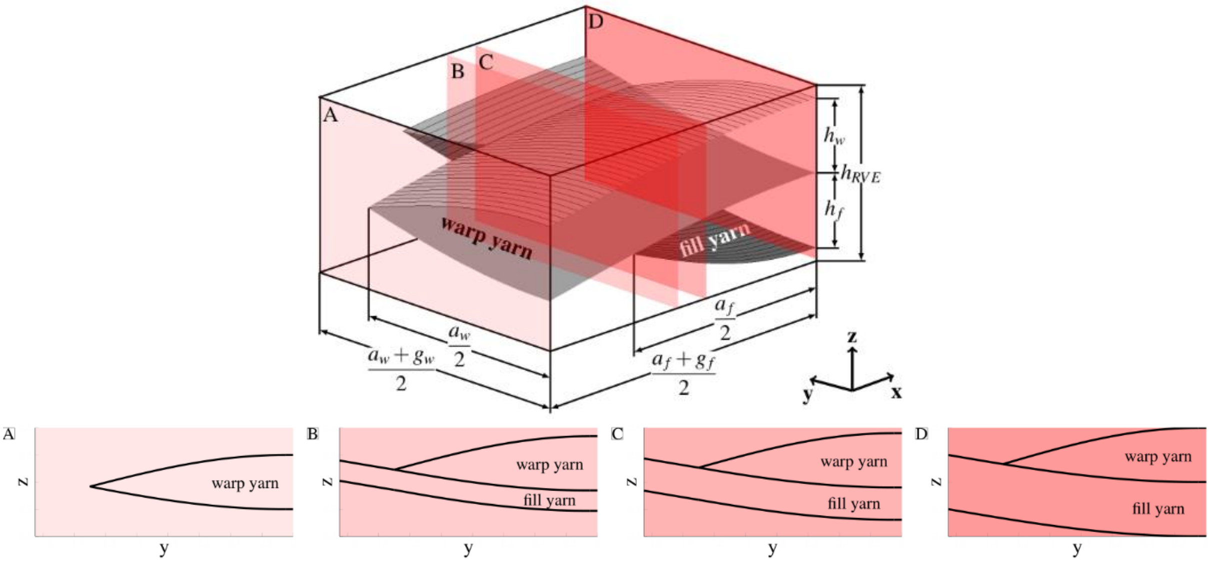

Figure 1: Schematical illustration of one fourth of the representative volume element (RVE) of a plain woven fabric (PWF) showing the geometrical parameters (a_f/a_w _ width of the fill/warp yarn, h_f/h_w _ height of the fill/warp yarn, h_{RVE} _ height of the RVE, g_f/g_w _ gap between two adjacent fill/warp yarns) and four section planes of the zy-plane (A-D).

thickness direction of a twill 2/2 woven fabric an IY-FVFG of approximately 50 vol% between the top and the bottom of the yarn [14]. However, to the best of our knowledge, methods to quantify the IY-FVFG within laminated plain weave woven composites are not available in the literature. In our preliminary study, we found an IY-FVFG in the through-thickness direction of the yarns of laminates made of a PWF. Here, to the best of our knowledge, a description of the IY-FVFG within laminated plain weave woven composites are not available in the literature.

1.1. Aim of the Study

The aims of this study were to develop a method to characterize and quantify as well as a model to describe the IY-FVFG in the yarn's through-thickness direction within laminated PWFs. In addition, the formation of these IY-FVFG will be investigated. Here, we present details on the method to characterize and quantify the IY-FVFG within laminated PWFs.

2. Materials

The resin system used in this study was prepared by mixing the epoxy resin Biresin CR81 (Sika Deutschland GmbH Stuttgart, Germany) with the hardener Biresin CH81-6 (Sika Deutschland GmbH Stuttgart, Germany). According to the manufacturers instructions, the resin and hardener were mixed in a ratio of 10:3 parts per weight.

As reinforcement an unidirectional glass-fiber PWF (Lange+Ritter GmbH, Gerlingen, Germany) was used, termed as E425UDL in the following. Unidirectional PWFs feature unequal masses of the fill and the warp yarn. The warp yarn of our unidirectional PWF takes 90 wt% and the fill yarn takes 10 wt% of the fabric's grammage. In addition, the manufacturer of the fabric E425UDL provided the grammage of the fabric (425 g/m²), the glass type (e-glass), and the number of fill (6.3 yarns/cm) and warp yarns (5.5 yarns/cm).

Excerpt from ISBN 978-3-00-053387-7

3. Sample Preparation

To investigate the IY-FVFG a laminate panel with a size of 250 mm × 330 mm was produced. The laminate panel was manufactured using the vacuum assisted resin transfer molding process. The infiltration of the PWF with the homogenized and degassed resin system was performed with a pressure excess of 2 bar. After the infiltration was completed, the riser pipes were closed and the excess pressure was increased to 5 bar inside the cavity of the mold. Afterwards, the gate pipe was closed and the filled mold was placed into an oven at 80 °C for 12 h. In this study we used a laminate with a fiber volume fraction of 42 vol% made of 10 layer E425UDL.

4. Characterization of IY-FVFG

The preliminary analysis of a laminated PWF (E425UDL) qualitatively suggested that the IY-FVFG changes in the direction of the yarns and in the yarns' through-thickness direction. Due to this presumption a 3D characterization of the yarns regarding their IY-FVFG is mandatory. The fact that established methods for a 3D morphology characterization, i.e. micro-computed tomography (µCT) and focused ion-beam scanning electron microscopy (FIB-SEM) are not applicable a suitable method need to be developed.
The µCT is restricted by its resolution, which is a minimum voxel size of approximately 0.3 µm × 0.3 µm × 0.3 µm. However, the filaments within a yarn possess spacings less than 0.3 µm, see Figure 2. The FIB-SEM method is basically suitable from the point of resolution, however, the size of the investigated volume is not large enough to cover a representative volume, in particular the whole yarn, see Figure 2 – it has a size of approximately 10 µm × 10 µm × 10 µm.

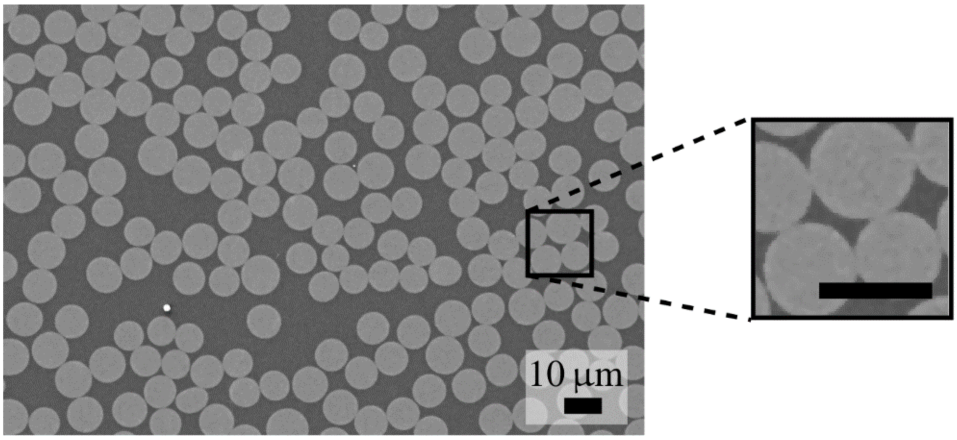

Figure 2: SEM image of a polished section of a laminated E425UDL fabric.

An alternative method to characterize the IY-FVFG in 3D will be presented in the following. This method is based on the analysis of section planes of the laminate in a tomographic manner. Thus, several polished section planes of the xz-plane and the yz-plane of each laminate need to be prepared. For each laminate several different yarns should be selected and characterized. The section planes were prepared by a step-wise grinding and polishing of the sample. After each preparation step (grinding and polishing) the middle of the yarns' cross-section were scanned in the through-thickness direction with a magnification of 350:1 by SEM. The width of the analyzed cross-sections of the yarns was approximately 130 µm containing approximately 600 filaments, see Figure 3 A.

In the following the quantification of the IY-FVFG will be detailed as well as the assignment of the section planes to the x-axis/y-axis intercept within the RVE of the laminated PWF E425UDL.

Excerpt from ISBN 978-3-00-053387-7

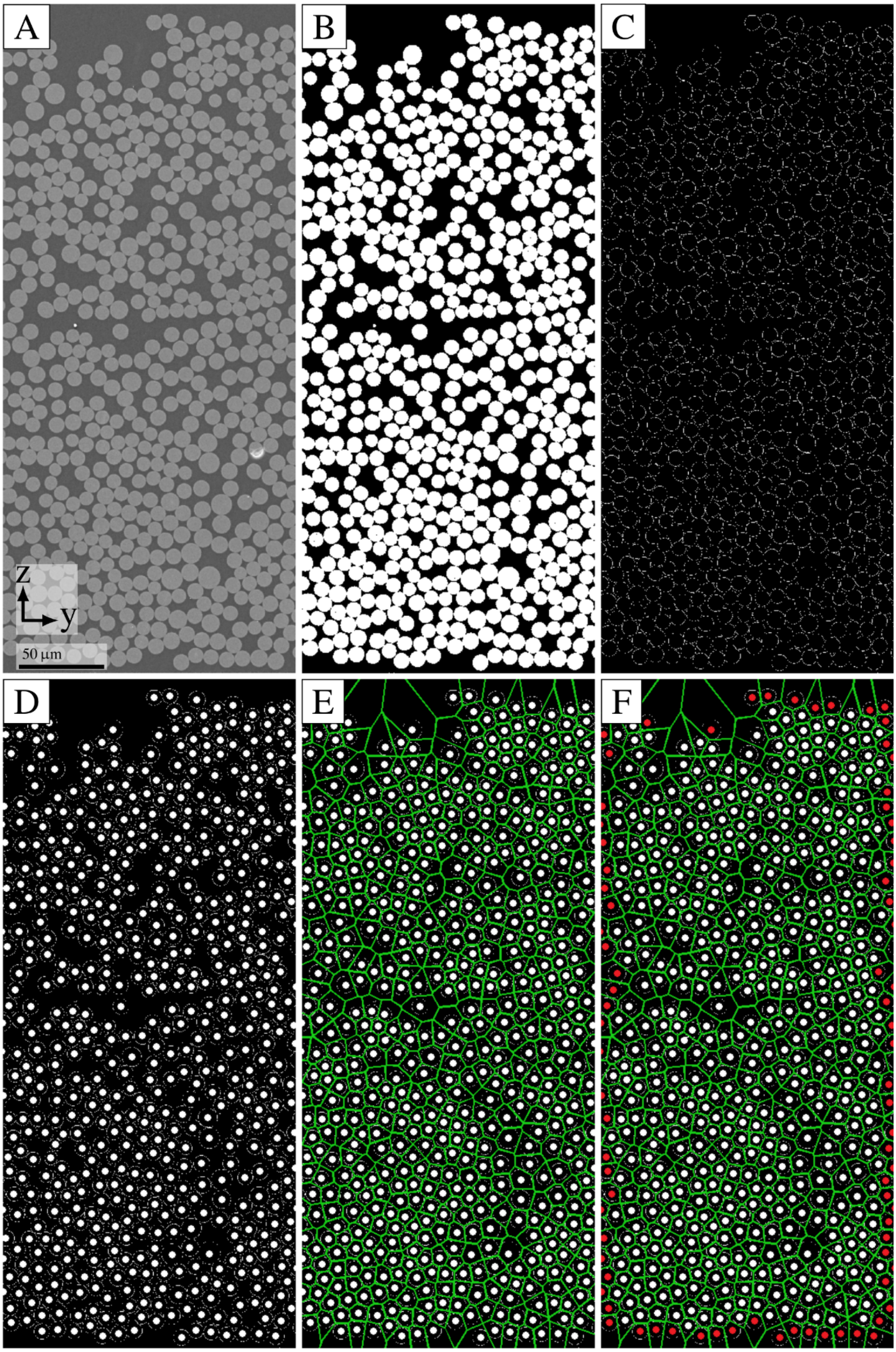

Figure 3: Procedure of the applied Voronoi cell method to dermine the IY-FVFG. A) original mapped SEM image; B-E) processed images: B) after binarization; C) after applying a find edge function; D) detected filaments (white circles); E) obtained Voronoi cells after applying Delaunay triangulation (green lines); F) removal of open Voronoi cells at the image boundaries (red circles).

Excerpt from ISBN 978-3-00-053387-7

4.1. Quantification of the IY-FVFG

The procedure to quantify the IY-FVFG in the yarn’s through-thickness direction can be distinguished in six steps, see Figure 3. At first, a median filter was applied to reduce any noise in the image, see Figure 3 A. After the binarization of the image (Figure 3 B) a modification of the Matlab script "Hough transform for circles" written by David Young[1] was applied to compute the center position and the size of the filaments. Therefore, the binarized image was filter to detect the edges of the filaments, firstly.

The result is given in Figure 3 C. After detecting the filament’s center positions (Figure 3 D) a Delaunay triangulation was applied to determine the Voronoi cells (Figure 3 E). At least, all open Voronoi cells at the boundary of the image were removed (Figure 3 F). The whole procedure is referred to as Voronoi cell method (VCM) in the following.

To determine the IY-FVFG the fiber volume fraction of each Voronoi cell was calculated by dividing the area of the enclosed filament with that of the whole Voronoi cell. The calculated fiber volume fraction was set to each pixel within the considered Voronoi cell. Subsequently, the mean fiber volume fraction in the yarn's width direction (x-direction for fill yarn, y-direction for a warp yarn) was determined via a pixel-wise integration of the fiber volume fractions and dividing the result with the number of the considered pixels. The determined mean fiber volume fractions in the yarn’s width direction were plotted as function of the converted pixel steps in micrometer – fiber volume fraction as function of the z-intercept, see Figure 4.

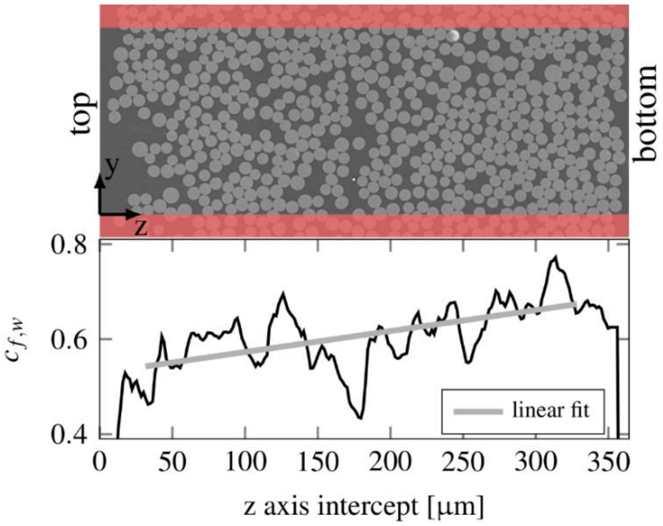

Figure 4: SEM image and the corresponding fiber volume fraction as function of the warp yarn’s through-thickness direction. $\boldsymbol{z} = \mathbf{0}$ is assigned to the warp yarn’s top. Slope of the linear fit: 422 m^{-1}. Fiber volume fraction at the warp yarn’s bottom and top were found to be 0.69 and 0.53, respectively.

The quantification of the IY-FVFG was done by determining the fiber volume fraction at the yarn’s top and bottom as well as the slope of the gradient. Therefore, the attained plot of the fiber volume fraction as function of the z-axis intercept was approximated by a linear function. This approximation provides the fiber volume fraction at the top and the bottom of the investigated yarn, see Figure 4. In this manner the fiber volume fraction in the top and the bottom of the yarn were determined for each yarn and all sections.

---

[1] David Young, http://www.mathworks.com/matlabcentral/fileexchange/26978-hough-transform-for-circles/content/html/circle$_$houghdemo.html, 17.03.2016

Excerpt from ISBN 978-3-00-053387-7

4.2. Assignment of Section Planes

The assignment of the prepared section planes to the PWF's RVE is mandatory to achieve the 3D characterization and to develop the IY-FVFG model in the direction of the yarn. The positions of the section planes within the PWF's RVE were determined by comparing the height of the crossing warp yarn respectively fill yarn with that in the geometrical model, see Figure 1. Therefore, the PWF's RVE was modeled and implemented into Matlab (Version 7.11.0.584 R2010b, © 1984-2010, Math-Works®). To model the RVE of the PWF E425UDL the established model proposed by Naik and Shembekar [7] was used. The required geometrical parameters (height of the fill/warp yarn, width of the fill/warp yarn, gap between adjacent fill/warp yarns, height of the RVE) were determined from overview SEM images of the polished samples. Figure 5 shows the slope of the IY-FVFG in the yarn's through-thickness direction for four different section planes as function of their x-axis intercept within the PWF's RVE.

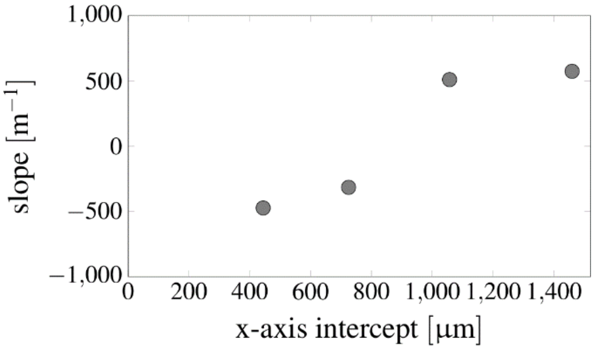

Figure 5: Experimentally determined slopes of the IY-FVFG in the warp yarn's through-thickness direction of four different section planes (same yarn). Each slope/section plane (zx-plane) was assigned to the x-axis intercept within the PWF's RVE.

5. Discussion

Established methods (µCT and FIB-SEM) for the characterization of the composite's IY-FVFG in 3D were not applicable, due to limitiations in the analyzed volume (FIB-SEM) and the resolution (µCT). The proposed Voronoi cell method was found to be suitable to characterize the IY-FVFG in 3D. In the following the quantification of IY-FVFG and the assignment of section planes to the fabric's RVE will be discussed in detail.

5.1. Quantification of the IY-FVFG

The determination of the IY-FVFG in the yarn's through-thickness direction was outlined in section 4.1. The procedure of determining the filaments' center positions, the generation of Voronoi cells and the removal of all open Voronoi cells at the boundary of the image can be considered as the fundamental steps of our method. Thus, the VCM is superior to a simple threshold based calculation, which is in particular prone to preparation artifacts, such as breaks at the filament edges. In addition, any errors introduced by the boundary of the image are eliminated.

The fact that Voronoi cells at the boundary of the image were removed prevents the direct measurement of the fiber volume fraction at the top and bottom of the yarn. Thus, a linear fit of the pixel-wise determined fiber volume fractions in the yarn's through-thickness direction was made. To quantify the fiber volume fraction at the yarn's top and bottom the fit was interpolated up to the top and the bottom of the investigated yarn. While observing SEM images of the yarn's cross-section it is obvious that there

Excerpt from ISBN 978-3-00-053387-7

is a loose arrangement of the filaments at the top of the yarn, compared to the bottom (fill yarn below), see Figure 4. Thus, the procedure to determine the fiber volume fraction at the yarn's top and bottom can be considered as an approximation.

In summary, the VCM enables the quantification of IY-FVFG in the yarn's through-thickness direction. As parameters to quantify the IY-FVFG in the yarn's through-thickness direction the slope of the gradient and the fiber volume fraction at the top and the bottom of the yarn were selected. The VCM is superior to a simple threshold based calculation, which is prone to preparation artifacts and any errors introduced by the boundary of the image.

5.2. Assignment of Section Planes

The assignment of the investigated section planes to the fabric's RVE is the fundamental step for the 3D characterization of IY-FVFG and the development of a suitable model. The procedure to determine the position of the section plane within the fabric's RVE is based on the determination of the height of the crossing yarn, see Figure 1. However, a precise assignment of the section plane is not possible for section planes showing no crossing yarn. This is the case in the region of the gap between two adjacent yarns. Thus, we used a series of section planes to ensure an accurate assignment. In addition, the measured height of the crossing yarn underlies fluctuations of the yarn's geometry due to the fabrication and handling of the fabric. This can lead to an incorrect assignment of the section plane to the x-axis/y-axis intercept within the fabric's RVE. Also, the fabric's RVE can trigger uncertainties in the position of the section planes within the fabric's RVE. However, the applied model can be considered as established and, thus, accurate as well as the use of several section planes prepared in distinct steps of the same yarn reduce these uncertainties to a minimum and permit their precise assignment.

6. Conclusions

This study presents a suitable method for the quantification of IY-FVFG of laminated PWFs in 3D. The proposed VCM was found to permit the quantification of IY-FVFG in the yarn's through-thickness direction. The VCM was found to be superior to a simple threshold based calculation, which is prone to preparation artifacts and any errors introduced by the boundary of the image. The quantification of the IY-FVFG in the yarn's through-thickness direction was accomplished via the determination of the slope and the fiber volume fraction at the yarn's top and bottom, which were provided by a linear fit of the pixel-wise determined fiber volume fractions within the yarn's cross-section.

The assignment of the analyzed yarn cross-sections to the fabric's RVE was achieved using a series of section planes of the same yarn. The RVE of the PWF build for the assignment was made using the established model proposed by Naik and Shembekar.

This study is the fundamental step for the development of a 3D model describing IY-FVFG within laminated woven fabrics.

Excerpt from ISBN 978-3-00-053387-7

Acknowledgments

The study received financial support by the Thuringian Aufbaubank (TAB) by the project FiLiMA (grant ID 2010 FE 9044).

References

[1] W. Voigt. Theoretische Studien über die Elasticitätsverhältnisse der Krystalle. *Abhandlungen der Königlichen Gesellschaft der Wissenschaften in Göttingen*, 53–100, 1892.

[2] A. Reuss. Berechnung der Fließgrenze von Mischkristallen auf Grund der Plastizitätsbedingung fiir Einkristalle. *Zeitschrift für angewandte Mathematik und Mechanik*, 9(1):49–58, 1929.

[3] C.C. Chamis. Simplified composite micromechanics equations for hygral, thermal, and mechanical properties. Sampe Quarterly, 1984.

[4] Z. Hashin, W. Rosen. The elastic moduli of fibre-reinforced materials. *Journal of Applied Mechanics*, 31(2):223-232, 1964.

[5] J.C. Halpin, J.L. Kardos. The Halpin-Tsai equations: a review. *Polymer Engineering & Science*, 16(5):344-52, 1976.

[6] T. Ishikawa, T.W. Chou. Stiffness and strength behaviour of woven fabric composites. *Journal of Materials Science*, 17(11):3211–20, 1982.

[7] N.K. Naik, P.S. Shembekar. Elastic Behavior of Woven Fabric Composites: I–Lamina Analysis. *Journal of Composite Materials*, 26(15):2196–225, 1992.

[8] D. Scida, Z. Aboura, M.L. Benzeggagh, E. Bocherens. A micromechanics model for 3D elasticity and failure of woven-fibre composite materials. *Composites Science and Technology*, 59:505–17, 1999.

[9] D. Scida, Z. Aboura. Prediction of the elastic behaviour of hybrid and non-hybrid woven composites. *Composites Science and Technology*, 57:1727–40, 1997.

[10] M.V. Donadon, B.G. Falzon, L. Iannucci, J.M. Hodgkinson. A 3-D micromechanical model for predicting the elastic behaviour of woven laminates. *Composites Science and Technology*, 11-12:2467–77, 2007.

[11] B.R. Kim, H.K. Lee. An RVE-based micromechanical analysis of fiber-reinforced composites considering fiber size dependency. *Composite Structures*, 90(4):418–27, 2009.

[12] H. Lin, M.J. Clifford, A.C. Long, M. Sherburn. Finite element modelling of fabric shear. *Modelling and Simulation in Materials Science and Engineering*, 17:1–16, 2009.

[13] E.J. Barbero, J. Trovillion, J.A. Mayugo, K.K. Sikkil. Finite element modeling of plain weave fabrics from photomicrograph measurements. *Composite Structures*, 73(1):41–52, 2006.

[14] M. Olave, A. Vanaerschot, S.V. Lomov, D. Vandepitte. Internal Geometry Variability of Two Woven Composites and Related Variability of the Stiffness. *Polymer Composites*, 33(8):1335–50, 2012.

[15] M.T. Cann, D.O. Adams, C.L. Schneider. Characterization of Fiber Volume Fraction Gradients in Composite Laminates. *Journal of Composite Materials*, 42:447–466, 2008.

[16] M. Karahan. The effect of fibre volume fraction on damage initiation and propagation of woven carbon-epoxy multi-layer composites. *Textile Research Journal*, 82(1):45–438, 2011.

Excerpt from ISBN 978-3-00-053387-7