

# CREEP-FATIGUE INTERACTION IN COMPOSITE MATERIALS

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## Abstract

A novel phenomenological model taking into account the creep-fatigue interaction in composite materials' fatigue performance is introduced in this paper. The model is based on the derivation of a master curve by using existing experimental fatigue data from a wide range of composite laminates and adhesively bonded composite joints. The developed model can subsequently be used in order to predict the fatigue behavior of additional composite materials systems under different loading conditions (stress ratios) when a limited amount of fatigue data is available. The model predictions were validated by comparisons to experimental data and relevant predictions provided by other existing models from the literature.

## 1. Introduction

Engineering structures are subjected to different mechanisms and loadings, such as corrosion, oxidation, impact, creep, and fatigue that deteriorate their structural integrity. Nowadays, it is widely accepted that fatigue is one of the most common failure types, especially in load-transferring parts of structures. In these components, the structural integrity is significantly vital and therefore, great efforts have been devoted by researchers to understand their fatigue behavior [1-3]. Investigation on the fatigue behavior of materials is possible by monitoring the evaluation of fatigue parameters and their effects on the fatigue life. This can be obtained by performing many experiments; however, this process is costly and in some cases cannot be followed in practice where numerous different loading patterns are applied on the structural elements. Therefore, various modeling techniques have been developed for more accurate prediction of the fatigue life of composite materials under different loading conditions [4-6]. These techniques are based on experimental data for basic loading conditions and are subsequently used for predicting the fatigue life under other complicated loading conditions [7-9]. In these methods, the damaging effects of fatigue has been solely taken into account for predicting the life of materials, however, the fatigue life of FRP composite materials can be affected by other types of material degrading mechanism such as creep. The susceptibility to creep in FRP materials, even at room temperature, is originated from their loading rate- and time-dependent mechanical responses. According to the above explanations, the term of the creep-fatigue interaction is introduced as a concerted action of creep and fatigue mechanisms under repeated loading [10]. Considering the creep-fatigue interaction leads to more accurate life predictions, and therefore, research efforts have been allocated to model this interaction in polymer matrix composite materials. There are some models in the literature for describing the creep-fatigue interaction in polymer matrix composites [11-16]. These methods use different particular hypotheses and concepts for predicting the fatigue behavior at different applied stresses and stress ratio, the ratio of the minimum over the maximum cyclic stresses ( $R=\sigma_{min}/\sigma_{max}$ ). However, each of them has some drawbacks, which restrict their extensive applications. In a model suggested by Miyano et al. [11-13] in order to predict the fatigue behavior at any arbitrary stress ratio, the S-N curve at the stress ratio of zero, for tension-tension fatigue, or infinity for compression-compression fatigue (pure fatigue) and the stress ratio of one (pure creep) are needed. The following equation for calculating fatigue strength as a function of stress ratio has been suggested.

$$\sigma_f(t_f, f, R, T) = R\sigma_{f,1}(t_f, f, T) + (1-R)\sigma_{f,0}(t_f, f, T) \quad (1)$$

where,  $R$  is stress ratio,  $\sigma_{f,1}$  stands for fatigue strengths for  $R=1$ , and  $\sigma_{f,0}$  denotes the fatigue strengths for  $R=0$ . In this model, creep is considered fatigue with arbitrary frequency. This assumption states that the difference between the damage mechanisms of creep and fatigue are virtually ignored. In Miyano's model, the linear cumulative damage law is taken into account for showing the progression of material degradation, however, some materials do not obey the linear cumulative damage law [14]. In addition, this assumption makes this model invalid for complex loading conditions, which induce nonlinear cumulative damage. In order to address these disadvantages, Guedes [14] suggested a new model based on the concept of evolution of strength integral (SEI) [17] as shown in the following equation:

$$F_r = 1 - R \int_0^{\tau_1} (1 - F_a) j_1 \tau^{j_1-1} d\tau - (1-R) \int_0^{\tau_2} (1 - F_a) j_2 \tau^{j_2-1} d\tau, \quad \tau_1 = \frac{t}{\tau_1}, \text{ and } \tau_2 = \frac{t}{\tau_2} \quad (2)$$

in which  $t$  is time,  $\bar{\tau}_1$  and  $\bar{\tau}_2$  are characteristic times associated with creep and fatigue,  $j_1$  and  $j_2$  are material parameters related to creep and fatigue,  $F_a$  is the normalized failure function that applies to a specific controlling failure mode, and  $F_r$  stands for the remaining strength. In this model, there are two material parameters ( $j_1$  and  $j_2$ ) for showing the damage progression; however, the relation between these parameters and different materials is not clear. Additionally, as the author showed, the predictions are not accurate as the stress ratio gets close to 1, e.g. 0.9, in which the participation of creep in the overall deformation is considerable.

The objective of this study is to present a new model for predicting the fatigue behavior of GFRP and CFRP composite materials and adhesively bonded composite joints. The model is based on the derivation of a master curve by using experimental data of a wide range of composite materials from the literature. The validity of this master curve is subsequently confirmed by comparisons of the model prediction to experimental results from additional experimental data and also comparisons to the predictions of the other existing models proposed by Miyano et al. [11-13] and Guedes [14].

## 2. Modeling

In order to describe the fatigue life, the classic power law relationship is used:

$$\sigma = \sigma_0 N_f^\alpha \quad (3)$$

in which  $N_f$  denotes the number of cycles,  $\sigma$  corresponds to the cyclic stress, while,  $\sigma_0$ ,  $\alpha$  are the fatigue model parameters, derived by linear regression analysis, after fitting the equations to the experimental fatigue data. According to the proposed model, parameter  $\alpha$ , can be expressed by the following equations:

$$\alpha = (1 - R^n) \alpha_f + R^n \alpha_c, \text{ where, } 0 < R < 1 \quad (4)$$

When  $0 < R < 1$ , Tensile-Tensile, (T-T)

$$\alpha = \left(1 - \left(\frac{1}{R}\right)^n\right) \alpha_f + \left(\frac{1}{R}\right)^n \alpha_c, \text{ where, } 1 < R < \infty \quad (5)$$

When  $1 < R < \infty$ , Compressive-Compressive (C-C)

where  $\alpha_f$  and  $\alpha_c$  are related to the contribution of fatigue and creep parts, respectively, and  $n$  is a material dependent parameter, which shows the dependency of  $\alpha$  with respect to the stress ratio  $R$ . In these equations, at  $R=1$ ,  $\alpha$  is governed only by the creep part and at  $R=0$  or  $R=\infty$  only fatigue is considered.

For the estimation of the parameter  $n$  a set of three S-N curves is required; in the following the application for the tension-tension fatigue will be presented, however, the process is exactly the same for the compression fatigue region as well.

$$\sigma_1 = f(\sigma_{0,1}, \alpha_1) \text{ at } R_1 \quad (6)$$

$$\sigma_2 = f(\sigma_{0,2}, \alpha_2) \text{ at } R_2 \quad (7)$$

$$\sigma_3 = f(\sigma_{0,3}, \alpha_3) \text{ at } R_3 \quad (8)$$

By substituting  $R_1$ ,  $\alpha_1$ , and  $R_2$ ,  $\alpha_2$  into Eq. 4, the following equations are derived:

$$\alpha_f = \frac{R_2^n \alpha_1 - R_1^n \alpha_2}{R_2^n (1 - R_1^n) + R_1^n (1 - R_2^n)} \quad (9)$$

$$\alpha_c = \frac{(1 - R_2^n) \alpha_1 - (1 - R_1^n) \alpha_2}{R_1^n (1 - R_2^n) - R_2^n (1 - R_1^n)} \quad (10)$$

The additional unknown parameter in these equations,  $n$ , can be numerically estimated by using the third available S-N curve:

$$\alpha_3 = (1 - R_3^n) \left( \frac{R_2^n \alpha_1 - R_1^n \alpha_2}{R_2^n (1 - R_1^n) + R_1^n (1 - R_2^n)} \right) + R_3^n \left( \frac{(1 - R_2^n) \alpha_1 - (1 - R_1^n) \alpha_2}{R_1^n (1 - R_2^n) - R_2^n (1 - R_1^n)} \right) \quad (11)$$

Another parameter showing the relation between the stress ratios and the corresponding slopes of the S-N curves can also be introduced as:

$$p = \sqrt{\frac{1 - R_1}{1 - R_2}} \frac{\alpha_2}{\alpha_1} \quad (12)$$

A study of the relationship between  $p$ ,  $n$  for several different material systems lead to the conclusion that if  $n$  is plotted against  $p$  the behavior can be sufficiently accurately be fitted by an exponential curve:

$$n = k_1 \exp(k_2 p) \quad (13)$$

where  $k_1$  and  $k_2$  are constants. This master curve equation is a unique representation, applicable, as shown herein for a wide range of composite materials and adhesively bonded composite joints, and can be used in the future for the calculation of the  $n$  parameter without the need of the existence of the third S-N curve experimental data.

For additional material investigations fatigue life at any stress ratio can be performed by using the introduced master curve. By having two sets of S-N curves, the  $p$  value for each material can be calculated and corresponding  $n$  values can be derived, allowing the estimation of S-N curves for each material at any stress ratio according to the following equations:

$$\sigma = \frac{\sigma_{0,1} + \sigma_{0,2}}{2} N_f^{(1-R^n)\alpha_f + R^n\alpha_c}, \text{ where } 0 < R < 1 \quad (14)$$

Following a similar procedure, the S-N curves under compression can be estimated by:

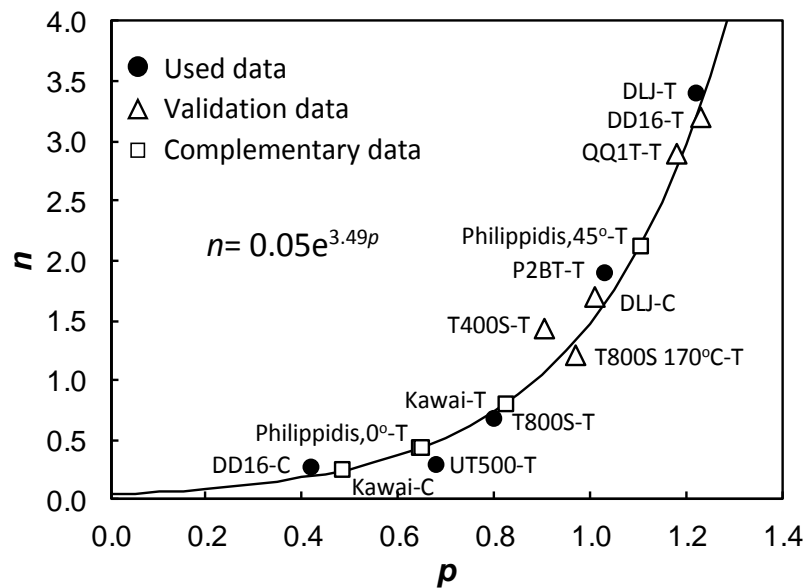
$$\sigma = \frac{\sigma_{0,1} + \sigma_{0,2}}{2} N_f^{(1-(\frac{1}{R})^n)\alpha_f + (\frac{1}{R})^n\alpha_c}, \text{ where } 1 < R < \infty, \quad (15)$$

### 3. Model application

#### 3.1. Master curve presentation and validation

Parameters of S-N curves along with calculated values of the  $p$  and  $n$  parameters for different materials from the literature are given in Table 1. The used materials are CFRP, GFRP, and double lap joints (DLJ). The first 5 material sets of data, namely P2BT [18], DD16 in C-C region [19], UT500/135 [13], DLJ in T-T region [20], and T800 [13] have been used for the derivation of the master curve, shown in Fig. 1. The coordinates of the used materials are represented by solid black circles. The two constants of  $k_1$  and  $k_2$  were estimated as 0.05 and 3.49, respectively by fitting Eq. (13) to these data set.

The rest of data shown Table 1 are employed for the validation of the master curve assumption and it is observed that the validation data (shown by open triangles) are corroborated very well by the derived master curve.



**Fig. 1.** Variation of parameter  $n$  against parameter  $p$ .

**Table 1.** List of composite materials, corresponding fatigue data ( $\sigma_0$  and  $\alpha$ ),  $n$  values (Eq. 13), and  $p$  values (Eq. 12).

Material	Material designation	R	$\sigma_0$	$\alpha$	$n$ (Eq. 13)	$p$ (Eq. 12)	Ref.
P2BT-T	Glass & carbon/epoxy [±45/90 <sub>4</sub> ]	0.10	87.2	-0.060	1.90	1.030	[18]
		0.50	91.4	-0.046			
		0.70	83.7	-0.026			
DD16-C	E-glass/polyester [90/0/±45/0] <sub>s</sub>	10.0	480.7	-0.068	0.27	0.416	[19]
		1.43	437.2	-0.021			
		1.10	490.7	-0.020			
UT500/135-T	Twill-woven UT500 carbon fiber and 135 epoxy	0.50	782.2	-0.074	0.30	0.680	[13]
		0.05	676.6	-0.036			
		1.00	649.0	-0.027			
DLJ-T	Pultruded GFRP bonded double lap joint	0.10	38.5	-0.083	3.40	1.218	[20]
		0.50	43.1	-0.075			
		0.90	31.7	-0.031			
T800S-25 °C-T	Carbon/epoxy [(45,-45)/(0,90)] <sub>s</sub>	0.05	1285.1	-0.051	0.67	0.800	[13]
		0.50	1134.6	-0.030			
		1.00	994.1	-0.018			
DD16-T	E-glass/polyester [90/0/±45/0] <sub>s</sub>	0.10	732.8	-0.102	3.20	1.230	[19]
		0.50	814.6	-0.094			
		0.90	688.2	-0.045			
DLJ-C	Pultruded GFRP bonded double lap joint	10.0	32.5	-0.043	1.70	1.009	[20]
		2.00	30.3	-0.032			
		1.10	30.3	-0.016			
T800S-170 °C-T	Carbon/epoxy [(45,-45)/(0,90)] <sub>s</sub>	0.05	757.5	-0.093	1.20	0.969	[13]
		0.50	690.6	-0.065			
		1.00	709.7	-0.058			
QQ1T-T	E-glass/epoxy [±45/0 <sub>2</sub> ] <sub>T</sub>	0.10	147.6	-0.082	2.90	1.180	[18]
		0.50	156.9	-0.073			
		0.70	144.5	-0.050			
T400/3601-T	Satin-woven CFRP laminates	0.10	1033.8	-0.040	1.43	0.905	[11]
		0.50	1026.8	-0.025			
		0.80	1048	-0.019			

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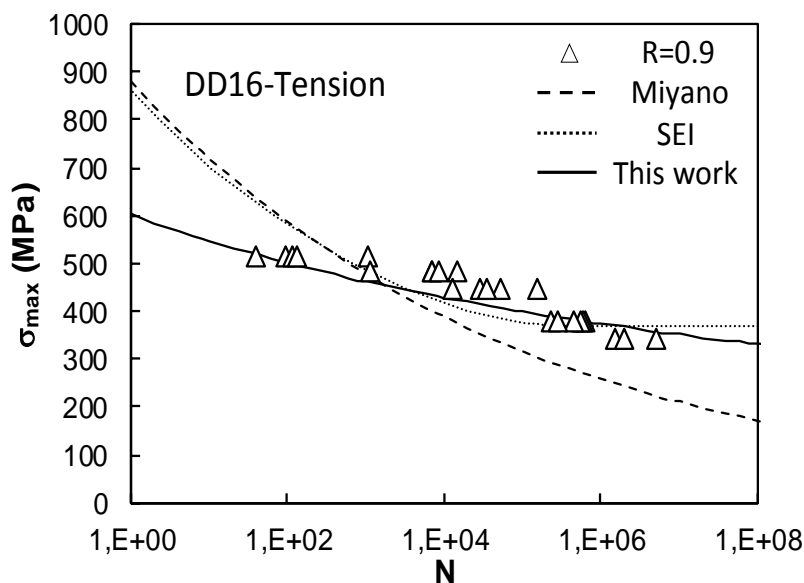
### 3.2. Model evaluation

Additional experimental data found in the literature (see Table 2) have been used (see open square symbols in Fig. 1) to validate the master curve hypothesis.

**Table 2.** List of composite materials, corresponding fatigue data ( $\sigma_0$  and  $\alpha$ ),  $n$  values, and  $p$  values.

Material	Material designation	R	$\sigma_0$	$\alpha$	$p$ (Eq. 12)	$n$ (Eq. 13)	Ref.
Kawai-T	Carbon/epoxy [45/90/-45/0] <sub>2s</sub>	2.00	903.3	-0.046	0.823	0.797	[21]
		10.0	860.3	-0.028			
Kawai-C	Carbon/epoxy [45/90/-45/0] <sub>2s</sub>	0.10	629.8	-0.026	0.483	0.243	[21]
		0.50	522.2	-0.009			
P2B-T	Glass & carbon/epoxy [0 <sub>4</sub> /±45]	0.10	1559.2	-0.023	0.645	0.428	[18]
		0.50	1450.4	-0.011			
Philippidis-0°-T	E-glass/polyester [0/±45 <sub>2</sub> /0]	0.10	613.8	-0.105	0.647	0.432	[22]
		0.50	393.3	-0.051			
Philippidis-45°-T	E-glass/polyester [0/±45 <sub>2</sub> /0]	0.10	178.2	-0.092	1.104	2.126	[22]
		0.50	187.4	-0.076			

Model predictions, are compared to available experimental data. Since the effect of creep is more pronounced for stress ratios approaching  $R=1$  in both T-T and C-C fatigue regions, the evaluation is carried out at the stress ratios close to 1. For example, the stress ratios of 0.7, 0.8, and 0.9 for T-T fatigue region, and 1.1 for C-C fatigue region are selected. Figs. 2-5 demonstrate the result of prediction and experimental validated results for studied CFRP, GFRP, and DLJ materials in both T-T and C-C fatigue regions. Accordingly, those five materials, which were used for validating the master curve, are also employed here. In addition, the predicted results are compared with two other suggested models of Miyano and SEI.



**Fig. 2.** Predicted S-N curves for  $R=0.9$ , DD16.

The fatigue date of DD16 at two different stress ratios of 0.1 and 0.5 in the T-T fatigue zone are used as the input data to predict the fatigue behaviour of same composite material with the stress ratio of 0.9. The predicted S-N curve is shown by solid line in Fig. 2, together with the predictions from the Miyano (dashed line) and the SEI (dotted line) models. In this case, the predicted S-N curve by the proposed approach is corroborated very well by the experimental data, in contrast to the two other model predictions that overestimate the fatigue life at low cycle fatigue (LCF). The Miyano model

underestimates the fatigue life at the high cycle fatigue (HCF) region, while the SEI model provides quite accurate predictions in the HCF range.

In order to evaluate the capability of this model on the C-C fatigue region, the model is applied to predict the fatigue behaviour of the pultruded GFRP bonded double lap joint (DLJ) at stress ratio of 1.1. In this case, the fatigue data of DLJ with the stress ratios of 2 and 10 are used as input data. The predicted S-N curves are shown in Fig. 3. For this material, the best prediction is obtained by using the suggested model that precisely follows the experimental data in both LCF and HCF, while both the Miyano and SEI models provide conservative predictions in nearly all stress ranges.

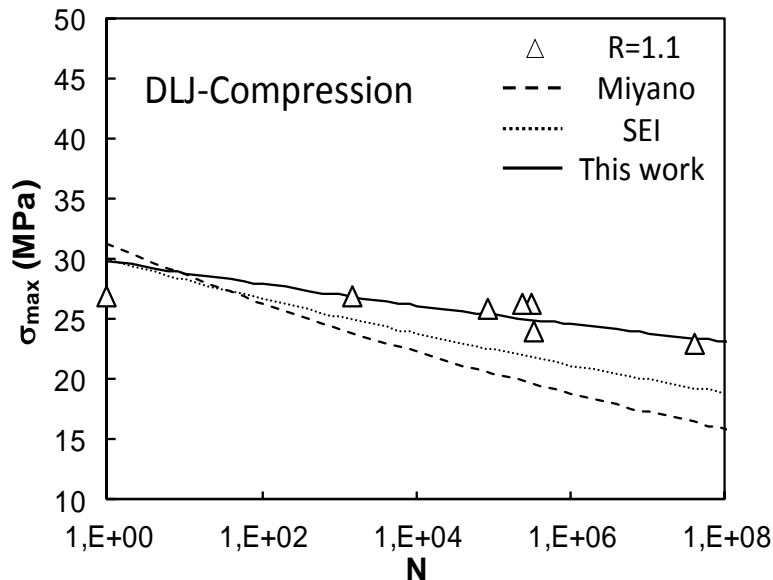


Fig. 3. Predicted S-N curves for R=1.1, DLJ.

The fatigue life prediction of QQ1T with stress ratio of 0.7 is done by applying two stress ratios of 0.1 and 0.5 into the models as the input data. In this material, both the suggested model and Miyano's model show good agreements with the experimental results, but SEI model predicts conservatively as demonstrated in Fig. 4.

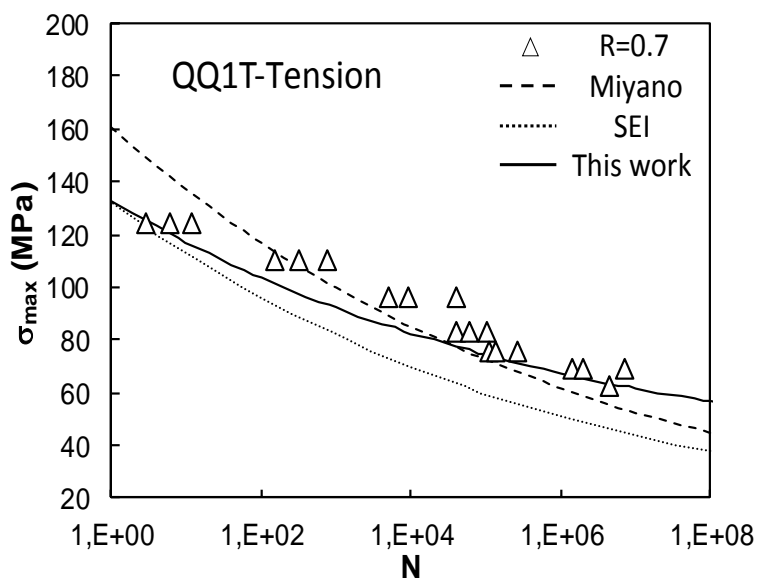


Fig. 4. Predicted S-N curves for R=0.7, QQ1T.

The applicability of the models is also examined in stain-woven T400/3601 material, see Fig. 5. Similarly, two sets of S-N curves with the stress ratios of 0.1 and 0.5 are used as input data to predict the fatigue behaviour of same materials with the stress ratio of 0.8. The results of prediction by the proposed approach in this work and Miyano show the highest accuracy, while, the SEI model is conservative.

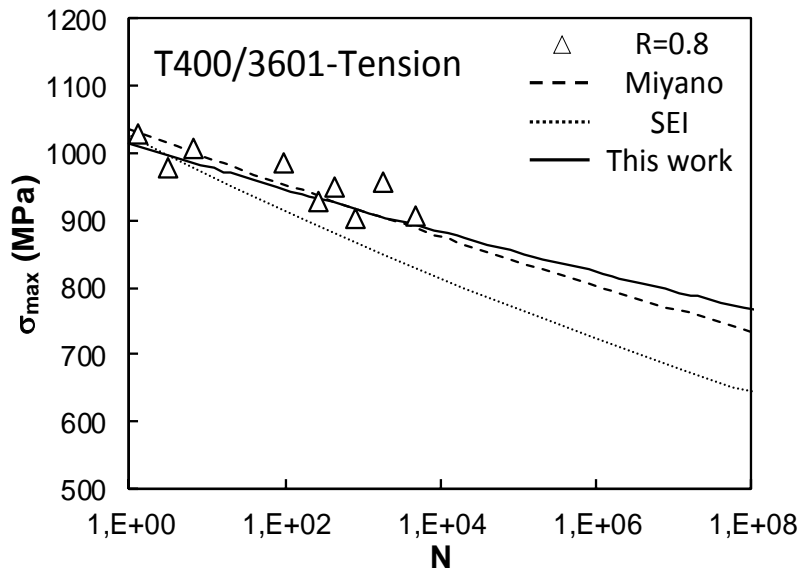


Fig. 5. Predicted S-N curves for R=0.8, T400/3601.

#### 4. Conclusions

In this study, a phenomenological model was developed to predict the fatigue behavior in polymer matrix composites including glass fiber-reinforced polymer (GFRP), carbon fiber-reinforced polymer (CFRP) and adhesively bonded double lap joints (DLJ). The introduced model is based on the derivation of a master curve describing the relationship of two introduced parameters,  $n$  and  $p$  for different material systems. The fatigue life at different stress ratios can be estimated by using this master curve considering the effect of creep-fatigue interaction. The model predictions were validated by comparison to available experimental data and also to predictions from two other existing models.

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#### References

- [1] R. Sarfaraz, A.P. Vassilopoulos, and T. Keller. Experimental investigation and modeling of mean load effect on fatigue behavior of adhesively-bonded pultruded GFRP joints. *International Journal of Fatigue*, 44:245–52, 2012.
- [2] M. Shahverdi, A.P. Vassilopoulos, and T. Keller. A total fatigue life model for the prediction of the R-ratio effects on fatigue crack growth of adhesively-bonded pultruded GFRP DCB joints. *Composites: Part A*, 43:1783–90, 2012.
- [3] T.P. Philippidis and A.P. Vassilopoulos. Life prediction methodology for GFRP laminates under spectrum loading. *Composite part A: Applied Science and Manufacturing*, 35.6:657-66, 2004.
- [4] I.P. Bond. Fatigue life prediction for GRP subjected to variable amplitude loading. *Composites Part A: Applied Science and Manufacturing*, 30.8 961-70, 1999.
- [5] M. Kawai, M. Koizumi. Nonlinear constant fatigue life diagrams for carbon/epoxy laminates at room temperature. *Composites Part A: Applied Science and Manufacturing*, 38.11 2342-53, 2007.

- [6] A.P. Vassilopoulos, B.D. Manshadi, and T. Keller. Piecewise non-linear constant life diagram formulation for FRP composite materials. *International Journal of Fatigue*, 32:10:1731-8, 2010.
- [7] A.P. Vassilopoulos, B.D. Manshadi, and T. Keller. Influence of the constant life diagram formulation on the fatigue life prediction of composite materials. *International Journal of Fatigue*, 32:659–69, 2010.
- [8] R. Sarfaraz, A.P. Vassilopoulos, and T. Keller. A hybrid S–N formulation for fatigue life modeling of composite materials and structures. *Composites: Part A*, 43:445–53, 2012.
- [9] R. Sarfaraz, A.P. Vassilopoulos, and T. Keller. Variable amplitude fatigue of adhesively-bonded pultruded GFRP joints. *International Journal of Fatigue*, 55:22-32, 2013.
- [10] B. Vieille, W. Albouy, L. Taleb. About the creep-fatigue interaction on the fatigue behaviour of off-axis woven-ply thermoplastic laminates at temperatures higher than T<sub>g</sub>. *Composites: Part B*, 58:478-86, 2014.
- [11] Y. Miyano, M. Nakada, M.K. McMurray, R. Muki. Prediction of flexural fatigue strength of CFRP composites under arbitrary frequency, stress ratio and temperature. *Journal of Composite Materials*, 31 619–38, 1997.
- [12] Y. Miyano, M. Nakada, and H. Kudoh. Prediction of tensile fatigue life for unidirectional CFRP. *Journal of Composite Materials*, 34:538–50, 2000.
- [13] Y. Miyano, M. Nakada, and K. Nishigaki. Prediction of long-term fatigue life of quasi-isotropic CFRP laminates for aircraft use. *International Journal of Fatigue*. 28:1217–25, 2006.
- [14] Guedes RM. Durability of polymer matrix composites: Viscoelastic effect on static and fatigue loading. *Composites Science and Technology*, 67:2574–83, 2007.
- [15] P.K. Mallick, and Y. Zhou. Effect of mean stress on the stress-controlled fatigue of a short E-glass fiber reinforced polyamide-6,6. *International Journal of Fatigue*, 26:941–946, 2004.
- [16] J. Petermann, K. Schulte. The effects of creep and fatigue stress ratio on the long-term behaviour of angle-ply CFRP. *Composite Structures*, 57 205–10, 2002.
- [17] K.K. Reifsnider, S.C. Case, J. Duthoi. The mechanics of composite strength evolution. *Composite Science Technology*, 60:2539–46, 2000.
- [18] T.J. Wilson. *Modeling of In-Plane and Interlaminar Fatigue Behavior of Glass and Carbon Fiber Composite Materials*. MS Thesis, Department of Mechanical Engineering, Montana State University, 2006.
- [19] J.F. Mandell, D.D. Samborsky. *DOE/MSU Composite Material Fatigue Database: Test Methods, Materials, and Analysis*, Contractor Report SAND97-3002, Sandia National Laboratories, Albuquerque, NM, 1997.  
[http://mfoserve.library.sandia.gov/sand\\_doc/1997/973002.pdf](http://mfoserve.library.sandia.gov/sand_doc/1997/973002.pdf).
- [20] R.S. Khabba. *Fatigue Life Prediction of Adhesively-Bonded Fiber-Reinforced Polymer Structural Joints under Spectrum Loading Patterns* (Doctoral dissertation, École Polytechnique Fédérale de Lausanne), 2012.
- [21] M. Kawai, T. Teranuma. A multiaxial fatigue failure criterion based on the principal constant life diagrams for unidirectional carbon/epoxy laminates. *Composites Part A: Applied Science and Manufacturing*, 43.8:1252-66, 2012.
- [22] A.P. Vassilopoulos, T. Keller. *Fatigue of fiber-reinforced composites*. Springer. 2011.